THE ASTROPHYSICAL JOURNAL, 203:52–62, 1976 January 1

EVOLUTIONARY SYNTHESIS OF THE STELLAR POPULATION IN ELLIPTICAL GALAXIES. I. INGREDIENTS, BROAD-BAND COLORS, AND INFRARED FEATURES*

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Received 1975 May 12

ABSTRACT

Broad-band photometric data and infrared line indices have been combined with new results on giant-branch luminosity functions to yield population syntheses for giant elliptical galaxies. If the main-sequence mass function is a power law of slope x, a value $x < 1$ is indicated. This yields rather rapid luminosity evolution and a large correction to the deceleration parameter q_0 as derived from the Hubble diagram for first-ranked cluster ellipticals. The uncertainties are discussed.

Subject headings: cosmology — galaxies: photometry — galaxies: stellar content

I. INTRODUCTION

Giant elliptical galaxies are used as probes of the past expansion rate of the Universe. Since cosmologically interesting look-back times are necessarily a considerable fraction of the galactic ages, systematic effects of their evolution must be fully understood. For example, evolution toward fainter luminosities may reduce the value of q_0 derived from the Hubble diagram by 1 or more. The rate of evolution has been shown to depend on essentially one parameter: the slope of the initial mass function near the main-sequence turnoff; this determines both the fraction of light due to "unevolving" dwarfs and the rate of decrease of the number of giants. Because the initial mass function also determines the integrated color of the population and the strengths of luminosity-dependent spectral features, one hopes by the technique of evolutionary population synthesis to derive the turnoff slope. But the snag is that the colors and spectrum are also affected by many *other* parameters, such as galactic age, metallicity, the time scale for star formation, rate of evolution on the giant branch, etc. Moreover, if the initial mass function deviates strongly from the adopted power-law form, conclusions drawn from the present ratio of giant to dwarf light can give incorrect evolutionary results. Even using a power law, earlier syntheses have been too beset by uncertainties in the stellar ingredients and have been compared with too coarse photometric data to yield a usefully accurate estimate of the slope x: an uncertainty of 2 in the value of x (Tinsley 1972*c*) leads to an uncertainty $\sim$0.8 in the value of q_0 (Gunn and Oke 1975). The above results and problems with earlier models are discussed by Tinsley (1975). Motivated by the need for more accurate evolutionary corrections, we have undertaken a further detailed synthesis of elliptical galaxy populations. This paper describes the stellar ingredients and the evolution of broad-band colors in the models. Strengths of the very luminosity-sensitive Wing-Ford (WF) band (Whitford 1973, 1974) and CO band at 2.3 μ (Frogel *et al.* 1975) are also computed, and used to derive useful upper limits to the value of x. We do not attempt in this paper to make a detailed "best fit" to the data considered, by adjusting parameters of the models and stellar ingredients. This will be left to a subsequent paper, in which we will describe synthesis on a narrow-band spectrophotometric system that gives complete coverage of the spectrum at 20–40 Å resolution from 0.31 to 1.07 μ.

The models discussed here are based on the simplest possible assumptions for the stellar population of giant elliptical galaxies. These are *consistent* with the broad-band colors and the infrared band strengths, but the data are not powerful enough to disentangle a usefully accurate value of x from even the minimal number of other parameters. We hope to be able to derive better constraints on x and to discuss the possibility and/or necessity of a more complicated population in the next paper. Some of the ambiguities will be mentioned in § IV below. The population assumed here is given by a single generation of stars (i.e., all made in less than about 10^9 years) with the chemical composition of old-disk stars in the solar neighborhood and with a power-law initial mass function,

$$\frac{dN}{dm} = Am^{-(1+x)}, \quad m_L \leq m \leq m_U \,. \qquad (1)$$

The upper mass limit, m_U, is of course unimportant

* Supported in part by the National Science Foundation [MPS 73-04673 A01, MPS 75-01398, GP-40482, GP-3143].

here as long as it exceeds the turnoff mass in the youngest galaxy seen (e.g., mass $\sim 1.15\ M_\odot$ with lifetime $\sim 5 \times 10^9$ yr); we have tacitly assumed that nucleosynthesis by massive stars has enriched the original gas fast enough to give most stars typical old-disk abundances, as for example in Larson's (1974*a*, *b*) models. The lower mass limit, m_L, affects only the mass-to-luminosity ratio of the model, as long as it is less than the smallest mass that contributes significantly to the light ($\sim 0.4\ M_\odot$); we can always adjust m_L, for a given x, to give an appropriate M/L ratio (§ III*b*). So, as far as the colors and spectrum are concerned, we have *formally* a one-parameter series of models, the parameter being x. In practice, uncertainties in stellar evolution will add more parameters, even for this simple population.

We want to synthesize the population that dominates the light in a 32 kpc projected diameter of giant elliptical galaxies for evolutionary corrections to Gunn and Oke's (1975) magnitudes. The data given in Table 3, from sources cited in the notes to the table, refer as far as possible to this population, but unfortunately are mostly relevant to rather smaller apertures. We feel justified, although not secure, in nevertheless comparing our models with the available data, since there is no convincing evidence for substantial color gradients outside the immediate nuclei. Aperture effects will be considered more carefully in the coming detailed synthesis.

II. STELLAR INGREDIENTS

a) *Unevolving Dwarfs*

Stars not massive enough to evolve significantly during galactic ages up to 20×10^9 yr are put in the models as an unevolving lower main sequence, defined empirically by nearby red dwarfs and joined smoothly to the theoretical main sequence for more massive stars. The evolutionary tracks used in § II*b* below show that stars with mass $m < 0.7\ M_\odot$ can be treated as unevolving: such stars become brighter by less than 0.3 mag and bluer by $\Delta(R - I) < 0.05$ (spectral type shift less than from say K3 to K2) in times of interest.

By using the nearby red dwarfs we are sampling low-mass stars at an average age $\sim 10^{10}$ years, so their properties can be used with sufficient accuracy for the elliptical galaxies at all ages. The *systematic* error caused by neglecting the slow brightening of late dwarfs might have been a worry, were it not for the strong spectroscopic evidence that their contribution to the light is extremely small. (Note that the giant branch *alone*, because of TiO blanketing, contributes a WF index that is 1 σ greater than the observed average; giants and evolving dwarfs and subgiants give a WF index too great by 1.5 σ even if there are *no* dwarfs later than K2! It can be estimated that, in consistent models, inclusion of the upward evolution of dwarfs below $0.7\ M_\odot$ would change the computed rate of evolution by less than 5 percent.)

Table 1 lists the masses, luminosities, colors, and feature strengths adopted for the lower main sequence from the following sources: (1) The relation between mass and M_V is from a least-squares fit (weighted to be an unbiased fit to the data if the dominant errors were in parallaxes) to the data given by Veeder (1974). Above $0.4\ M_\odot$, the empirical line was joined smoothly to the theoretical 10^{10}-year isochrone of the evolutionary tracks used in § II*b*. (2) Relations between M_V, $M_{\rm bol}$, $B - V$, $R - I$, and $\log T_e$ were also obtained from Veeder's data for M dwarfs and joined smoothly to the theoretical isochrone in the ($M_{\rm bol}$, $\log T_e$)-plane. The resulting main sequence for K dwarfs agrees well with empirical results for old-disk stars. (3) Other colors were obtained from the color-color relations of Veeder (1974) and Johnson (1966). The R, I system used in this paper is Johnson's. (4) The WF indices were derived from detailed stellar data kindly provided by Whitford. (5) The CO indices were derived from the stellar data used by Frogel *et al.* (1975) and kindly supplied by those authors.

The individual points from Table 1 are shown in Figure 1.

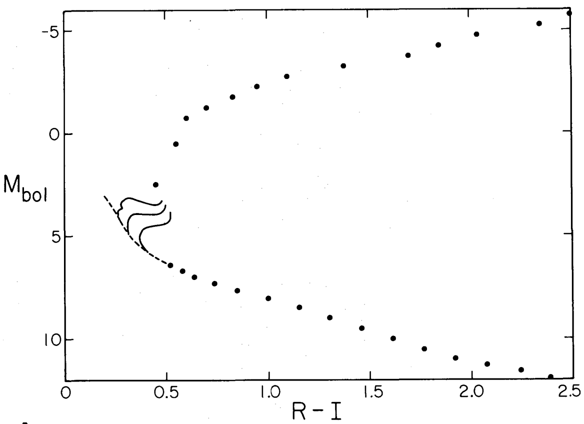
b) *Evolving Dwarfs and Subgiants*

Stars between 0.7 and $1.4\ M_\odot$ evolve significantly in look-back times of interest; more massive stars are

TABLE 1

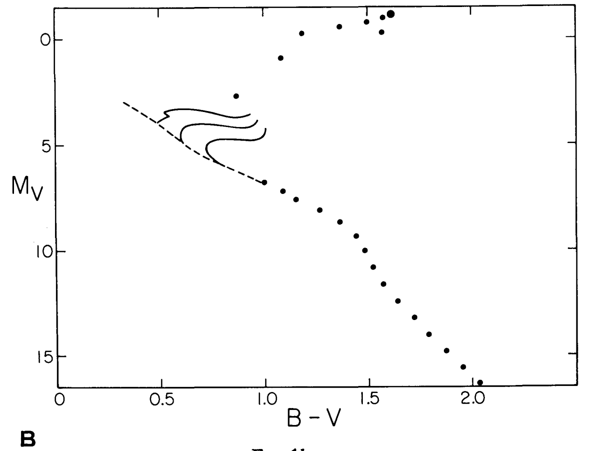
DATA FOR UNEVOLVING DWARFS

Mass ($M_\odot$)	$M_{\rm bol}$	M_V	$B - V$	$V - R$	$R - I$	$V - J$	$V - K$	$V - L$	WF	CO
0.091	11.90	16.39	2.03	2.58	2.39	6.28	7.27	7.53	0.26	0.05
0.110	11.57	15.59	1.95	2.38	2.23	5.87	6.83	7.10	0.21	0.04
0.132	11.24	14.80	1.87	2.20	2.08	5.44	6.42	6.70	0.18	0.03
0.158	10.91	14.01	1.79	2.04	1.92	4.94	5.98	6.25	0.14	0.02
0.190	10.53	13.21	1.72	1.91	1.77	4.49	5.57	5.82	0.11	0.01
0.228	10.01	12.42	1.64	1.80	1.62	4.19	5.15	5.45	0.08	0.00
0.274	9.50	11.63	1.57	1.68	1.46	3.90	4.75	4.98	0.06	0.01
0.328	8.98	10.84	1.52	1.57	1.31	3.62	4.40	4.60	0.05	0.01
0.394	8.47	10.05	1.48	1.46	1.15	3.29	4.05	4.24	0.04	0.02
0.460	8.02	9.33	1.44	1.34	1.00	2.95	3.75	3.91	0.03	0.02
0.515	7.64	8.66	1.36	1.22	0.85	2.58	3.45	3.60	0.03	0.04
0.565	7.28	8.09	1.27	1.11	0.74	2.32	3.13	3.26	0.02	0.05
0.605	6.98	7.60	1.15	1.01	0.64	2.03	2.72	2.86	0.02	0.05
0.650	6.68	7.18	1.09	0.94	0.58	1.84	2.47	2.61	0.02	0.05
0.690	6.40	6.78	1.00	0.81	0.52	1.66	2.22	2.36	0.02	0.04



A

FIG. 1*a*



B

FIG. 1*b*

FIG. 1.—(*a*) The adopted H-R diagram in M_{bol}, $R - I$ for the unevolving dwarfs, giants, and turnoff region for stars of mass 0.798, 0.956, and 1.146 $M_\odot$. See text for details of fitting these pieces together. (*b*) As for Fig. 1 (*a*) except in M_V, $B - V$.

dead before 2.5×10^9 yr. To ensure smooth evolution of galaxy properties in steps of 10^9 years, we have used 56 evolutionary tracks for stars with lifetimes (to the base of the giant branch) in 5×10^8 year steps from 2.5 to 30.0×10^9 yr. Anything coarser gives slightly zigzag paths for the evolution of integrated colors. These could be smoothed in plots of the output data in the present broad-band system, but the process would be too cumbersome when we later use the computed populations to synthesize multichannel scans.

The tracks were interpolated among those for composition $X = 0.70$, $Z = 0.02$ by Hejlesen *et al.* (1972), for which detailed listings were kindly supplied by Jørgensen. Twenty-four points on these tracks (of which seven were in the relevant mass range) corresponding to "equivalent" evolutionary stages were identified, and linear interpolations in log mass were made for the quantities log M_{bol}, log T_e, and log Δt at each point. This procedure failed for masses between 1.00 and 1.12 $M_\odot$, since the appearance of a convective core somewhere in this interval means that the tracks are not homologous. The interpolation scheme finally adopted here gives a smooth transition from one type of track to another, so it causes no discontinuities of galactic evolution between ages 5 and 8×10^9 yr. We do not know if the transition *is* in reality smooth, but luckily the stars burn little fuel in the troublesome stages of evolution and so contribute little to the galactic light.

Conversion from (M_{bol}, log T_e) to (M_V, $B - V$) and other colors was done from the calibrations of Morton and Adams (1968) and Johnson (1966). Sources for WF and CO indices are as given in § II*a*. The main sequence at age 10×10^9 yr joins smoothly to the points on the lower main sequence.

The zero-age main sequence for these stars and three of the evolutionary tracks (for masses 0.798, 0.956, and 1.146 $M_\odot$ which reach the base of the giant branch at 20.0, 10.0, and 5.0×10^9 yr) are shown in Figure 1.

c) *Giants*

Giant stars evidently dominate the light from elliptical galaxies at all wavelengths, so it is important to represent their colors, luminosities, and numbers as realistically as possible. We base our model giant populations on the nearby old-disk giants, as was done by Tinsley (1972*a*, *b*) and Rose and Tinsley (1974), using revised statistics to be detailed in a forthcoming paper (Tinsley and Gunn 1975; hereafter TG).

A common giant branch is used for all stars in these models, i.e., for the mass interval 0.8 to 1.4 $M_\odot$. Obviously this is a crucial assumption, strongly affecting the predicted changes of galaxy properties with redshift (look-back time), so it is unfortunate that detailed justification is lacking. There is strong evidence against systematic changes of the *position* of the giant branch in the H-R diagram: Eggen (1973, cf. Fig. 6) finds nearly identical positions for the old-disk and Hyades group stars, whose turnoffs suggest masses near 1 and 2 $M_\odot$, respectively; and Cannon (1970) finds no systematic change with age in the position of the red giant clump in open clusters with a similar range of turnoff masses. This agrees with the theoretical prediction that the locus of the Hayashi track depends on the envelope composition, but hardly at all on the total stellar mass. Likewise, theory predicts weak if any mass dependence of the *rate* of evolution on the giant branch, which is equivalent to the luminosity function; various stellar model-builders even differ as to the sign of the small effects they find; moreover, their relevance is dubious since the mass of an individual star undoubtedly decreases significantly during giant evolution. The best assumption, at least for now, seems to be to use a giant branch that is independent of the original stellar mass. Further support is derived from the fact that the whole range of interpolated tracks (§ II*b*) reaches the base of the giant branch with an almost identical core mass (0.19–0.22 $M_\odot$).[1]

For the luminosity function, we start from a smoothed composite of the theoretical, old-disk group and old-disk field (5th mag sample) giant functions derived in TG. These agree extremely well in shape where they overlap and provide a smooth curve, within the statistical uncertainties, from $M_{\text{bol}} = +2.5$ to -5.5. The theoretical function can be used to normalize star counts to time intervals with very little ambiguity, except that neither theory nor data determines the core helium-burning (clump) lifetime satisfactorily.

Unfortunately, what matters is not the time a star spends at a given *magnitude*, but the energy output at a given *color*, which we denote by a quantity $E \equiv \Delta t$ dex $(-0.4\,M_{\text{bol}})$. We have to derive the relation between E and $R - I$ from the luminosity function, but the scatter in the empirical (M_{bol}, $R - I$)-diagram (see TG) makes it impossible to draw a mean relation with any confidence. We have derived E as a function of $R - I$ by several approaches, none fully satisfactory, and used each function in model galaxies to test the importance of the ambiguity.

1. The first method was to forget about the distribution of luminosities and to compute the contribution to E of each old-disk giant (ODG) in a suitable sample, and then to sum these in 0.1 intervals of observed $R - I$. The group sample of TG was used, and the contribution of each ODG was found as follows: a star was assigned a weight equal to its weight in the luminosity function derived by the statistical method of TG (which effectively "corrects" a magnitude-limited sample to a distance-limited sample), and this was converted to Δt by the scale factor that gave the best fit between theoretical and empirical luminosity functions; then the contribution of the star to E was simply Δt dex $(-0.4\,M_{\text{bol}})$. This method suffers from uncertainties in group membership of the ODGs (which really only enters our statistics through the group parallax), small numbers in some color intervals, and variability of the late M giants. For the latter, we used time average luminosities in bolometric, R, and I light as in TG. The values of E are shown in Figure 2 (*dash-dot line*), after slight smoothing—always within $1.5\sqrt{N}$—to avoid gross small number effects; the worst uncertainty is at $R - I > 2.2$, where there are only two stars in the sample!

2. As an alternative, we have converted the composite (M_{bol}, Δt)-relation described above directly into (E, $R - I$)-relations using three alternative "mean" relations between M_{bol} and $R - I$ for the ODGs. These are shown as the "semitheoretical" values of E in Figure 2. Number 1 is derived from the best

[1] Ostriker and Thuan (1975) have used giant-branch parameters which depend on mass, resulting in rather different evolutionary rates. For reasons outlined above we prefer the present scheme. Further details will be discussed in TG.

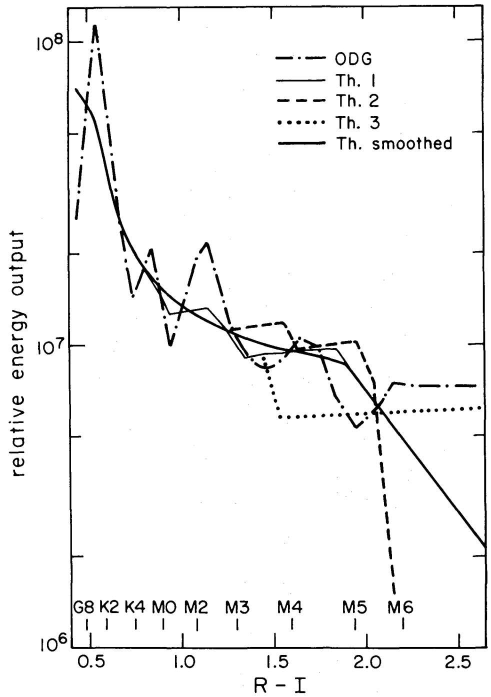


FIG. 2.—The relative energy output E ($\propto L_{\text{bol}}\Delta t$) in equal intervals of $R - I$, as a function of $R - I$ for the adopted alternative giant branch tracks. See text for discussion. The "best theoretical" curve (Th 1) is largely coincident with its smoothed version (*heavy line*) which is used in most models in this paper.

estimate of a mean line in the H-R diagram, and numbers 2 and 3 are from "limiting plausible" lines in the poorly populated M giant region. The heavy line in Figure 2, which is largely coincident with curve 1, is the smoothed semitheoretical relation to be used in most models. These curves do not include the core helium-burning (clump) stars. Following the discussion in TG we assign two alternative (extreme) estimates for the clump lifetime, 6.5 or 15.6×10^7 yr, then distribute the clump stars in *color* to agree with the spread of the ODG's clump. The crosses in Figure 2 show the total energy E in the bluest two color intervals for the semitheoretical case with core helium-burning included. The color distribution is treated with care since clump stars contribute significantly to the light at all wavelengths of interest, and are observed (in the ODGs) to have $B - V$ colors that range from bluer to redder than the integrated $B - V$ of a gE galaxy.

The main differences among the alternative functions in Figure 2 are at the late G giants, where the ODGs appear to be deficient in stars at the base of the giant branch and at the later M giants. The former may be due to incompleteness in the group membership lists (TG). If real, it leads to significantly redder model galaxies, but we have more confidence in the semitheoretical function at this point. Late M giants luckily contribute little light, even if they are put in

TABLE 2

DATA FOR GIANT BRANCH

Δt* (10^7 yr)	M_{bol}	M_V	$B-V$	$V-R$	$R-I$	$V-J$	$V-K$	$V-L$	WF	CO
73.3(77.9).....	+2.50	+2.70	0.87	0.69	0.45	1.50	2.00	2.10	0.00	0.02
9.9(14.4).....	+0.50	+0.92	1.09	0.81	0.55	1.75	2.47	2.60	0.00	0.09
1.62........	−0.75	−0.23	1.19	0.86	0.60	1.90	2.70	2.85	0.00	0.12
0.82........	−1.25	−0.52	1.37	0.97	0.70	2.24	3.08	3.22	0.01	0.15
0.43........	−1.75	−0.75	1.50	1.11	0.83	2.56	3.50	3.65	0.01	0.19
0.24........	−2.25	−0.99	1.58	1.25	0.95	2.87	3.87	4.05	0.02	0.21
0.21........	−2.75	−1.20	1.61	1.39	1.10	3.19	4.20	4.40	0.02	0.23
0.15........	−3.25	−1.21	1.62	1.59	1.38	3.72	4.85	5.01	0.03	0.23
0.068.......	−3.75	−1.08	1.61	1.85	1.70	4.47	5.62	5.85	0.04	0.26
0.030.......	−4.25	−1.22	1.60	2.03	1.85	4.83	6.04	6.26	0.05	0.26
0.021.......	−4.75	−1.18	1.58	2.33	2.04	5.37	6.60	6.88	0.06	0.27
0.0066......	−5.25	−0.40	1.57	3.21	2.35	6.44	7.80	8.03	0.06	0.28
0.0035......	−5.75	−0.25	1.57	3.70	2.50	6.93	8.49	8.55	0.07	0.29

* Times are for the smoothed semitheoretical case, including the clump with short (or long, in parentheses) lifetime estimates (§ II*c*).

the models at the level shown for ODGs, so the latter uncertainty is not serious.

The energy functions in Figure 2 are finally converted into evolutionary tracks by the (essentially arbitrary) assignment of an M_{bol} to each $R-I$. The time is clearly $\Delta t = E\,\text{dex}\,(0.4\,M_{bol})$. Table 2 gives the adopted points, with times for the smoothed theoretical case. Colors other than $R-I$ are derived from the color-color relations of Johnson (1966) and Lee (1970), and the WF and CO strengths are from the sources cited in § II*a*. The points are also shown in Figure 1. Here it appears as if the lowest giant point is too blue relative to the ends of the interpolated subgiant tracks, but for two reasons a point near this color may be called for: (1) Real ODGs have a broad, nearly vertical, giant branch in the color interval represented by the lowest two points, including the clump; as long as we have the correct (E, $R-I$)-relation, the positions of discrete points in the H-R diagram do not matter. (2) Theoretical tracks have quite uncertain temperatures at this stage, so the observed color at the base of the giant branch is to be preferred. Stars evolve rapidly across the nearly horizontal subgiant part of the theoretical tracks, contributing rather little light to the models, so the discrepant points are not very important. Whether or not the bluest giant point enters significantly at all depends on the choice of energy functions from Figure 2. The "ODG" function has a very small contribution from the bluest point, but the best estimate for a mean line in the H-R diagram leads to the much greater contribution in the other cases. The effects on model colors are discussed below.

III. MODEL GALAXIES

a) *Method of Calculation*

The evolutionary synthesis method (Tinsley 1968, 1972*a*) is simplified greatly here, with only a single generation of stars and without any calculations of gas content or composition.

The significant model parameters are just x, m_L (cf. § I), and the choice of giant branch lifetimes.

Given the set of discrete stellar masses, m_i (§ II*a*, *b*), we compute the number of each born, using the relation

$$b_i = \int_{m_{i-}}^{m_{i+}} A m^{-(1+x)} dm\,, \qquad (2)$$

where $m_{i+} = (m_i m_{i+1})^{1/2}$ and $m_{i-} = (m_i m_{i-1})^{1/2}$; the limits are replaced by m_L or m_U where appropriate, and the numbers are normalized to give a chosen total mass by means of the parameter A. Then the stellar population at times spaced by 10^9 years can be found from the adopted evolutionary tracks. The number of stars of mass m_i at each point along its track, at any time, depends on the rate of star formation during the assumed initial burst. We compute the numbers as though the rate were uniform (for 10^9 yr) because it can be shown that if the lifetimes τ_i of the set of masses are equally spaced, this procedure, together with formula (2), gives a population very close to that obtained in the limit of a continuous distribution of masses and a delta-function birthrate.

Numerical calculation of the population at each point on the interpolated tracks (56 tracks × 24 points, but most are unoccupied at a given time) gives adequately smooth isochrones from the main sequence to the base of the giant branch. The unevolving dwarf points are, of course, occupied by the full number b_i at all times.

The giant branch presents a problem, because very slight departures from equal spacing in the stellar lifetimes lead to unacceptable irregularities in the evolution of color, as certain short-lived but energetically important points on the giant branch are underpopulated and overpopulated at alternate time steps. Therefore we have chosen to calculate analytically the numbers of stars at each point on the common giant branch, rather than following individual stars numerically beyond the 24th point on their interpolated tracks. Consider a point on the common giant branch occupied from time τ' to $\tau' + \Delta t$ after a star leaves its 24th point. Stars here at the present time t_0 had lifetimes (τ_m) to point 24 of $t_0 - \tau'$ to $t_0 - \tau' - \Delta t$, and have masses given by the following relation, which holds for the tracks of Hejlesen *et al.* (1972)

$$m = c\tau_m{}^{-\Theta} ,$$

where $c = M_\odot (8.38 \times 10^{-9})^\Theta$, $\Theta = 0.261$. Thus the number of stars at the giant point of interest, which is the number of stars leaving the main sequence between $t_0 - \tau' - \Delta t$ and $t_0 - \tau'$, is approximately

$$\Delta N(t_0) = \Delta t \frac{dN}{dm} \frac{|dm|}{|d\tau_m|_{\tau_m = t}} = A\Theta c^{-x} \Delta t t^{-(1-\Theta x)} ,$$

where $t = t_0 - \tau' - \Delta t/2$. Numerical calculations would have given just these numbers for the giant population had we spaced our m_i more closely and evenly. Note that the total number of giants varies approximately as $t^{-(1-\Theta x)}$ and so is a *decreasing* function of time if $x < \Theta^{-1} = 3.8$.

The above methods were used to compute populations at times between 5 and 20 × 10^9 yr, and then integrated colors, etc., were found in an obvious way. The population numbers themselves can be listed for future syntheses on other photometric systems.

b) *Results*

Properties of a variety of models are given in Table 3, at a fiducial (not necessarily best) age of 15 × 10^9 yr, while Figure 3 shows the evolution of three of these models that have the same giant branch but different values of x. For estimates of properties of the other models at other ages, note that the rate of evolution is a function of almost only x. At the top of Table 3 are shown the range of values that we consider to be acceptable for models of present giant elliptical galaxies; further discussion is given in § I and in the notes to Table 3.

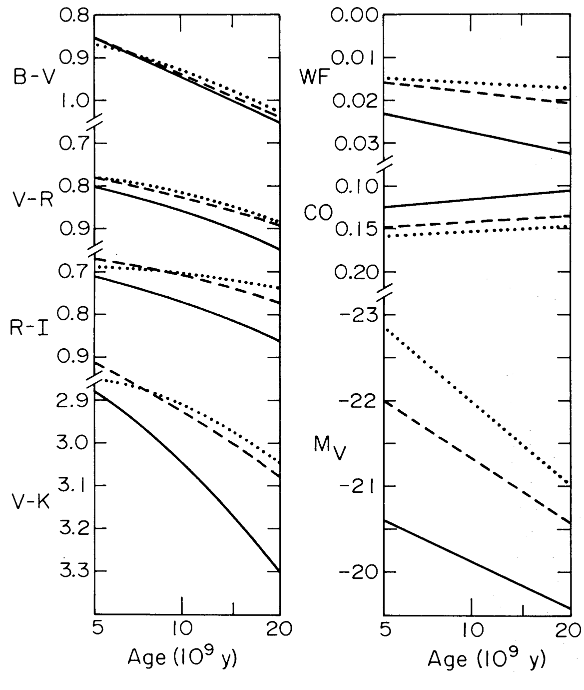


FIG. 3.—Evolution in colors, magnitude, and line indices for models *a*, *b*, and *c*, which are similar except for different values of x: 0 (*dotted*), 1 (*dashed*), and 2 (*solid*), respectively. The values of M_V are for a model with a total mass of 10^{11} $M_\odot$.

Table 4 lists the stellar population of model *a* at age 15 × 10^9 yr, scaled to a total mass of 10^{11} $M_\odot$.[2] As Table 3 shows, this model is consistent with the colors and line indices considered, although different populations might be preferred after more detailed spectrophotometry has been included.

i) *Effects of Changing the Values of x*

The evolutionary correction for cosmology depends on $dM_V/d \ln t$. Table 3 and Figure 3 show that this rate is very sensitive to x, but not to other parameters, and is given closely by

$$dM_V/d \ln t = 1.3 - 0.3x ,$$

in agreement with analytical models (Tinsley 1972*b*, 1973*b*) and earlier numerical models based entirely on different stellar evolutionary tracks (Tinsley 1972*c*; Rose and Tinsley 1974). The downward correction to q_0 is, in first order (Gunn and Oke 1975),

$$\Delta q_0 = \frac{1.4}{H_0 t} \frac{dM_V}{d \ln t} \approx 1.4 \frac{dM_V}{d \ln t} \approx 1.8 - 0.42x .$$

Thus if we want to know q_0 to an accuracy of a few tenths, we must know x within 0.5 or less.

Which of the observable quantities might give x without ambiguity? First of all, consider models *a*, *b*, and *c* (Table 3 and Fig. 3) that differ only in the value of x.

The M/L ratio is sensitive to x, but (even if it were accurately known) it would not be a useful constraint, since it depends strongly on m_L. For example, since $M/L \propto m_L{}^{(x-1)}$ if $x > 1$ (Tinsley 1973*b*), a change of m_L to 0.28 in model *c* with $x = 2$ would reduce its M/L to that of model *b* with $x = 1$. The light is much too giant-dominated for this change in the number of M dwarfs to be detectable.

The color $B - V$ is sensitive to x, but again not unambiguously. Since turnoff stars are important at short wavelengths, $B - V$ evolves rapidly—at a given $B - V$, a change ~ 1 in x corresponds to a change $\sim 10^9$ yr in the estimated age. Moreover, this color is very sensitive to some of our simplifying assumptions (normal metallicity, no stars above turnoff, etc.). The color $U - B$ could be interpreted with even less confidence in these simple models, so we have not considered it. The forthcoming detailed spectrophotometry will give much more information at short wavelengths.

The broad-band colors at longer wavelengths show improved sensitivity to x (as the red dwarf population appears with increasing x), together with slower evolution and presumably less sensitivity to some of

[2] This is the mass of stars formed, from 0.1 to 1.43 $M_\odot$. As explained in note (6) to Table 3, the mass of stars included in Table 4 is only 5.8 × 10^{10} $M_\odot$.

TABLE 3

PROPERTIES OF SOME MODELS

MODEL PARAMETERS			PROPERTIES AT AGE 15 × 10^9 YEARS					
Model No.	x / m_L*	Giant Branch†	$B-V$ / $V-R$	$R-I$ / $V-J$	$V-K$ / $V-L$	WF / CO	M/L_B* / $M/L^B{}_V$*	tdM_V/dt
Observed values‡			0.95–0.99 / 0.84–0.88	(~0.75) / 2.2 ± 0.2	3.2 ± 0.2 / ...	0.011 ± 0.004 / 0.197 ± 0.026	≳8 ± 5 / ≳6 ± 3	
Notes			(1)	(2), (3)	(3)	(4), (5)	(6)	
a	0	ST1 +	0.98	0.72	2.97	0.017	4.1	1.27
	0.1	short	0.85	2.11	3.13	0.151	3.0	
b	1	ST1 +	0.99	0.74	3.00	0.019	6.8	0.98
	0.1	short	0.86	2.15	3.17	0.138	4.9	
c	2	ST1 +	1.00	0.82	3.17	0.030	19.0	0.71
	0.1	short	0.90	2.31	3.35	0.111	13.5	
d	1	ST1 +	0.99	0.73	2.97	0.018	6.3	0.98
	0.1	long	0.86	2.12	3.14	0.136	4.5	
e	1	ODG	1.02	0.77	3.09	0.020	7.1	0.96
	0.1		0.88	2.20	3.26	0.147	4.9	
f	1	ST2 +	0.99	0.74	3.00	0.019	6.8	0.98
	0.1	short	0.86	2.15	3.17	0.138	4.9	
g	1	ST3 +	0.99	0.74	2.99	0.020	6.8	0.98
	0.1	short	0.86	2.13	3.16	0.138	4.9	

* Solar units.

† Notation used for luminosity functions defined in § II*c*: ST1 = smoothed semitheoretical case 1, ST2 = same except case 2 used for M giants, ST3 = same except case 3 used for M giants, short (long) = short (long) estimate of clump lifetime included, ODG = luminosity function from old-disk group giants.

‡ Sources cited in notes below.

NOTES TO TABLE 3

1. Sandage's (1973, cf. Table 6) mean colors corrected for redshift and reddening are $B - V = 0.975 \pm 0.039$ and $V - R = 0.861 \pm 0.031$; he finds $E(V - R)/E(B - V) = 0.80$ and zero reddening at the poles. The "allowed" colors for an average galaxy shown in Table 3 include the possibility of $E(B - V) = 0.03$ mag at the poles, and the uncertainty in how reddening and its fluctuations affect the stars and galaxies.
2. The $R - I$ color is a preliminary estimate, derived from absolute spectral energy distributions of galaxies; the calibration uses stars with the latter type of photometry and broad-band $(R - I)$ colors. A better estimate, from more data, will be available eventually. The uncertainty is about ±0.05 mag.
3. The J and K colors are from Frogel *et al.* (1975) and Grasdalen (1975). The uncertainties shown are representative of the observational errors.
4. The quoted WF is the average value for eight galaxies measured by Whitford (1974). These are nuclear values and there is no information on gradients, but the behavior of other spectral features makes any increase outwards unlikely.
5. The quoted CO is an unweighted mean of all galaxy values given by Frogel *et al.* (1975), and one other unpublished value, with K-corrections (supplied by the authors, private communication) where necessary. There is possibly evidence for a decrease with increasing aperture.
6. Faber and Jackson (1975) derive as a mean value for elliptical galaxies, $M/L_B \gtrsim (10 \pm 3)(H_0/75 \text{ km s}^{-1} \text{ Mpc}^{-1})$, where an upper limit indicates that effects of rotation have not been included. The value quoted in Table 3 is based on a Hubble constant $H_0 = 60 \pm 20 \text{ km s}^{-1} \text{ Mpc}^{-1}$. It should be noted that these values refer to the core only and may not be representative of the synthesized region.

The computed M/L values are for an initial model mass $10^{11} M_\odot$, i.e., including all stars formed between the limits 0.1 and 1.43 $M_\odot$. Since stars above 0.87 $M_\odot$ have evolved away by 15 × 10^9 yr, the luminosity is contributed by 0.58, 0.81, or 0.95 of the original mass, according as $x = 0$, 1, or 2, respectively. If the evolved stars become white dwarfs of 0.7 $M_\odot$, but the rest of their mass has been lost from the galaxy, its total mass at this age is 0.84, 0.93, or 0.98 of the origin l mass, respectively. Clearly, if the IMF had a shallow slope up to large masses in the initial burst, a considerable fraction of the initial mass could be lost, and a considerable fraction of the present mass could be in invisible stellar remnants.

the simplifying assumptions. Table 3 shows that the values are satisfactory. $V - R$ is more sensitive to x and less sensitive to age than $B - V$ (cf. Fig. 3); comparison with observed $V - R$ colors indicates $x < 2$. Colors at longer wavelengths would be even more useful, if more accurate data were available.

The general problem with continuum colors is, of course, that dwarfs can be substituted for giants (within the evolutionary constraints) with no color difference (Faber 1972). Elliptical galaxies have roughly early K colors in the blue and late K colors in the infrared. For this reason, it is important to find spectral features that differ widely in strength between K dwarfs and K giants for the same color; infrared features are especially useful since they are little affected by the evolution of turnoff stars. Many features have been studied in population syntheses (e.g., Spinrad and Taylor 1971; Faber 1972; Tinsley 1972*c*; O'Connell 1974), and most of these will be included later in our program. At present, we consider only the very sensitive infrared WF and 2.3 μ CO bands. These have the advantage that one (WF) is much stronger in dwarfs than giants and very weak in galaxies, while the other (CO) shows the opposite

TABLE 4

POPULATION OF MODEL *a* AT AGE 15 × 10^9 YEARS

A. UNEVOLVING DWARFS
(See data in Table 1)

Mass ($M_{\odot}$)	No. of Stars	Fraction of V Light
0.110	1.4E+10	2.1E−5
0.132	1.4E+10	4.2E−5
0.158	1.4E+10	8.8E−5
0.190	1.4E+10	1.8E−4
0.228	1.4E+10	3.8E−4
0.274	1.4E+10	7.9E−4
0.328	1.4E+10	1.6E−3
0.394	1.3E+10	3.1E−3
0.460	1.0E+10	4.8E−3
0.515	7.7E+9	6.8E−3
0.565	6.1E+9	9.1E−3
0.605	5.3E+9	1.2E−2
0.650	4.9E+9	1.7E−2
0.690	3.7E+9	1.8E−2

B. GIANTS
(See data in Table 2)

$R-I$	No. of Stars	Fraction of V Light
0.45	9.8E+8	2.1E−1
0.55	1.4E+8	1.5E−1
0.60	2.2E+7	7.2E−2
0.70	1.1E+7	4.7E−2
0.83	6.0E+6	3.1E−2
0.95	3.3E+6	2.1E−2
1.10	2.9E+6	2.3E−2
1.38	2.1E+6	1.6E−2
1.70	9.4E+5	6.6E−3
1.85	4.2E+5	3.3E−3
2.04	2.9E+5	2.2E−3
2.35	9.2E+4	3.4E−4
2.50	4.9E+4	1.6E−4

C. EVOLVING DWARFS AND SUBGIANTS

M_{bol}	M_V	$B-V$	$R-I$	Mass Range ($M_{\odot}$)	No. of Stars	Fraction of V Light
6.0	6.2	0.87	0.47	0.71–0.72	1.7E+9	1.4E−2
5.7	5.9	0.83	0.43	0.72–0.75	3.2E+9	3.5E−2
5.5	5.7	0.78	0.40	0.75–0.77	2.1E+9	3.0E−2
5.2	5.3	0.73	0.37	0.77–0.81	3.8E+9	7.6E−2
4.7	4.8	0.70	0.36	0.81–0.84	2.7E+9	8.2E−2
4.4	4.5	0.74	0.37	0.84–0.85	1.0E+9	4.0E−2
4.3	4.5	0.82	0.42	0.85–0.86	5.9E+8	2.5E−2
4.1	4.4	0.96	0.49	0.86–0.87	8.2E+8	3.8E−2

behavior; therefore it is unlikely that the apparent giant dominance is due to an overabundance or underabundance of heavy elements.

As expected, the WF and CO indices vary strikingly with x, especially between $x = 2$ and 1.

The WF index does not allow $x > 1$ (at the 2 σ level); as mentioned in the notes to Table 3, this constraint is expected to get stronger as measurements are made with greater apertures. The CO index is significantly better at $x = 0$ or 1 than at $x = 2$, but in this case comparison with nuclear values may exaggerate the constraint.

ii) *Effects of Changes in the Giant Branch*

How much do these constraints depend on the choice of giant luminosity function used in models *a–c*? Next we look at the effects of uncertainties in and alternatives to that function, which used the smoothed semi-theoretical contributions and short clump lifetime.

Model *d* differs from *b* only in having a long *clump lifetime*; the changes are very small, because most of the energy output at the colors of the clump comes from slow evolution up the almost vertical giant branch below it (cf. Fig. 2).

In model *e*, *the contribution function derived from the ODGs* is used. This gives redder colors at short wavelengths, because of the paucity of stars near the base of the giant branch compared with the theoretical function, representing a significant uncertainty for the choice of age and/or x to agree with observed colors. One possible source of error in the theoretical case is that the base of the giant branch may have been counted twice—at the end of the interpolated tracks and in the lowest giant-branch interval—but the possibly too red points at the end of the interpolated tracks contribute very little light (4 percent of the V light in model *a*, for example), so removing them altogether from the theoretical tracks would make models *a–c* redder by negligible amounts (by 0.002 mag in $B - V$ in model *a*, for example). A much more serious uncertainty is in *the color of the bluest giant point*, discussed at the end of § II; if it were redder, the discrepancy between models with the theoretical and ODG contribution functions would be much less. This point contributes 21 percent of the V light in model *a*, for example; so an error in its $B - V$ or $V - R$ color gives rise to an error about 20 percent as great in the integrated color of the model.

The extra population of very late M giants in model *e* makes a negligible contribution, but there are significant effects at long wavelengths due to the enhancement in *early M giants* (perhaps not a believable enhancement, since only five stars contribute to the peak at M2 seen in Fig. 2). The increases in $V-K$ and CO are formally improvements over model *b*, and even more early M giants would give values closer to the data quoted in Table 3—at the expense, however, of increasing WF somewhat. Since $V - K$ and CO could be affected by the presence of carbon stars (§ IV*a*), they do not yet provide detailed constraints on the early M giants.

Finally, models *f* and *g* show the effect of using the alternative semitheoretical contributions from *late M giants* (Fig. 2). The differences from model *b* are seen to be altogether negligible. This is fortunate, since the small numbers and erratic variability of these stars would make it almost impossible to estimate their contributions with more confidence.

The importance of the uncertainty in numbers and colors at the base of the giant branch serves as a warning that detailed predictions from a given giant contribution function should not be taken too literally. These differences reflect rather complicated astrophysical problems, even though the "smoothed semitheoretical" function (including some contribution from the bluest point) looks plausible and is consistent with the statistical uncertainties in the ODGs. It should be noted that if the models other than *e* have too much blue light from the base of the giant branch (the more likely sign of an error in the present populations), then we have *overestimated* the allowed values of age and/or x. If we regard the age as known (e.g., from the ages of globular clusters), then a correction to redder giant colors would reduce the consistent values of x, and so lead to greater evolutionary corrections. Similarly, if we determine x (e.g., from spectral features in the infrared), then redder giant colors would require smaller ages to give *BVR* colors consistent with those observed, and these would lead to greater evolutionary corrections in a given cosmological model. In this sense, the giant branch used in most models here leads to conservative conclusions.

c) *Formal Limits on x*

None of the ambiguities in giant evolution is enough to bring down the WF index to an acceptable value if $x > 1$. Even if $x < 1$, the index is always about $2\,\sigma$ greater than the mean observed value. This is actually due to TiO blanketing at 9910 Å in giants, not to the Wing-Ford band itself which only appears in dwarfs; so the extremely low value for galaxies is surprising (Whitford 1975).

Equally strong constraints on x can be inferred from the CO index, provided that future measurements at greater apertures do not give consistently smaller values.

A problem with earlier giant-dominated models has been the strength of the Na I lines at 8190 Å, which have been interpreted as indicating a substantial dwarf contribution to the red light in elliptical galaxies (Spinrad and Taylor 1971). We have not used this feature as a constraint in the present preliminary synthesis, since there are observational problems arising from an atmospheric water vapor band at that wavelength. However, we have calculated an approximate value of its strength in our model *a*, using Spinrad and Taylor's (1971) stellar data, the result being $w \approx 0.03$ on their system; this is probably not significantly smaller than their observed values of -0.03 and $+0.01$ in the nuclei of M32 and M31, respectively. (The giant-dominated models of Rose and Tinsley 1974 did have discordantly negative Na I 8190 indices, arising from the different giant luminosity functions adopted in that paper.)

If $x \leq 1$, we can use the computed values of $B - V$ to derive *provisional* age limits, keeping in mind that $B - V$ is sensitive to neglected complications: the youngest model with $B - V$ in the accepted range has age 8×10^9 yr ($B - V = 0.95$ with $x = 1$ and the ODG contribution function), while the oldest has age 16×10^9 yr ($B - V = 0.99$ with $x = 0$ and the semitheoretical giant contribution function). These are in good agreement with age estimates for globular clusters in the Galaxy.

And finally, if $x \leq 1$, the first-order estimate of the correction to q_0 has the startling limit $\Delta q_0 \gtrsim 1.4$.

IV. DISCUSSION

a) *Neglected Complications*

Although we chose the simplest likely assumptions to define the population in these models, because the data used here cannot disentangle even the minimum number of parameters, we must ask what systematic errors may have been caused by neglected complications.

i) The *metallicity* used here was chosen because it is consistent with available evidence from line strengths in giant elliptical galaxies (e.g., Faber 1973) and because we wanted to use old-disk giants to define an empirical giant branch. All colors and line indices would be affected by the shift of the whole giant branch to the blue or red if the average metal abundance were less or greater than that of the ODGs (for which the mean [Fe/H] ≈ -0.23 relative to the Hyades [Hansen and Kjaergaard 1972]). The only quantitative estimate we have is Larson and Tinsley's (1974), that a factor 2 decrease (for example) in metal abundance would make the integrated light of a single generation of stars bluer at a given age by about 0.04 in $B - V$. This would correspond to an increase in the *age* estimate, at a given $B - V$ and x, by 3 to 4×10^9 yr. More drastic reductions in metallicity could give a much bluer clump; although spectra of giant elliptical galaxies show no evidence for a significant contribution of A stars, we might expect some halo population with a blue horizontal branch to be seen in projection against the inner 32 kpc. We hope that the forthcoming detailed spectrophotometry will reveal any significant contributions from stars that are not of old-disk composition.

ii) *Stars above the main-sequence turnoff* have been neglected in spite of two possible sources: (1) *Younger stars* may be present. Although significant current star formation is unlikely since it would show as a blue nucleus if at all (Larson and Tinsley 1974), there could well be a considerable population of stars born after the rapid initial burst, yet before star formation was stopped by some mechanism such as establishment of a wind (Larson 1974*b*). (2) *Blue stragglers* are present in old populations of the kind that we imagine elliptical galaxies are made of. Whatever their origin, we have no idea how to predict their numbers or distribution in the H-R diagram. Presumably, if stars above turnoff contribute enough light to affect the continuum colors significantly, their presence will be betrayed in more detailed spectra. Ultraviolet photometry would be very helpful.

iii) The assumption of a *power-law initial mass function* (eq. [1]) is undoubtedly an oversimplification. For the synthesis of present-day ellipticals, the value of x can be regarded as an average slope over the small mass range between turnoff and the latest visible dwarfs, ~ 0.4–$1.0\ M_\odot$. The slope cannot become much steeper above about $0.2\ M_\odot$, or there will be more dwarf light in the infrared than allowed by the WF and CO indices; but at even smaller (including substellar) masses there could be any number of stars contributing only invisible weight to M/L. Another possibility is that the IMF turns over at early K, so that the infrared indices give no information about the slope at turnoff. The evolutionary correction depends mainly on the number of giants that were present in the past, and so on *extrapolation* of the IMF beyond the present turnoff. The slowness of changes of spectral energy distribution with look-back time (Oke 1971; Crane 1975) made it unlikely that any drastic changes—such as extreme depletion of giants due to a turnover in the IMF above the present turnoff—have occurred. Until we have enough data to make a detailed synthesis of a galaxy at a large redshift, the most reasonable extrapolation of the IMF is a linear one, with constant x, over the additional small mass interval to earlier turnoff masses ($m \lesssim 1.2\ M_\odot$).

iv) Our giant branch includes no *carbon stars*, although these obviously exist in the old-disk population. They have very strong CO bands and so could alter the interpretation of the observed CO strengths considerably. Using colors and the $2.3\ \mu$ band strength of a typical late carbon star (J. Frogel, private communication), we find that if more than about half of the K light in any model is from such stars, $V - K$ is redder than observed. (For example, in model *b*, if 50 percent of the K light is from a "typical" C star, $V - K$ increases from 3.00 to 3.45; the CO index increases from 0.14 to 0.24, which is less than $2\ \sigma$ away from the observed mean.) Can we test whether a significant population of carbon stars is present? The limit corresponds to an average star spending less than a few percent of its giant lifetime as a carbon star, or to a few percent of stars becoming such giants. There seems to be little prospect of determining the C star population of galaxies from stellar evolution theory. On the other hand, *if* C stars contribute importantly to the $2.3\ \mu$ CO band, they would be detectable in two other ways: (1) $K - L$ would be substantially redder than expected from M stars alone (~ 0.38 rather than 0.17 if half the K light is from a typical late C star); (2) the C_2 band at $1.77\ \mu$ would appear in the galaxies if and only if C stars contribute at $2.3\ \mu$ (cf. the spectra of C and M stars in Thompson *et al.* 1969*a*, *b*). Relevant infrared observations of elliptical galaxies would be very valuable.

b) *Empirical Constraints on the Evolution of Color*

Oke and Sandage (1968) and Sandage (1973) were unable to detect a change of color with redshift at a level that corresponds roughly to $d(B - V)/dt \lesssim 0.02$ ($H_0/100$ km s^{-1} Mpc^{-1}) per 10^9 yr. All of the models with $x \leq 2$ have $d(B - V)/dt \approx 0.01$ over the past several billion years and so are consistent with this limit.

The evolution of intrinsic colors derived by Crane (1975) from the data of Gunn and Oke (1975) can be transformed into approximate rates for broad-band colors: plotting Crane's rates of evolution versus wavelength, and interpolating at the effective wavelengths of the B, V and R bands, one estimates that between the age corresponding to redshift 0.4 and now, $B - V$ has increased by 0.09 ± 0.03 and $V - R$ has decreased by 0.06 ± 0.05 mag ($1\ \sigma$ errors). The models agree in $B - V$ but have the opposite sign of evolution in $V - R$. For example, from ages 9 to 15×10^9 yr, model *b* becomes redder in $B - V$ by 0.06 and redder in $V - R$ by 0.03 mag. The discrepancy in $V - R$ is only at the 2σ level; so in view of the sensitivity of Crane's results to small calibration errors in Gunn and Oke's data, it is not disturbing. In fact, firm evidence that galaxies were *redder* in the past would be extremely difficult to explain (shades of the Stebbins-Whitford effect!) unless perhaps they were redder at *all* wavelengths in which case intergalactic dust could be invoked.

V. CONCLUSION

The stellar ingredients described here have been used in a preliminary synthesis of the broad-band colors and Wing-Ford and $2.3\ \mu$ CO band strengths in elliptical galaxies. The giant dominance indicated by the weakness of the WF band and the strength of the CO band implies important evolutionary corrections to the luminosities of elliptical galaxies seen at large redshifts. The exact values of these corrections are sensitive to uncertainties in the shape of the IMF, the evolution of giant stars, and the assumptions used to define the age and metallicity distributions of the population. Models based on the present ingredients are to be used in subsequent syntheses with more detailed spectrophotometric data, which hopefully will enable us to set better constraints on all the uncertain parameters, and in particular to determine more accurate evolutionary corrections.

We are grateful to Drs. Jørgensen, Frogel, Persson, and Whitford for unpublished details of their calculations and data, and to Drs. Sandra Faber and Hyron Spinrad for helpful discussions and critical comments. This research was supported in part by National Science Foundation grants MPS 73-04673 A01, GP-40482, GP-3143, and MPS 75-01398.

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