PARAMETRIC BASED CONTROLLER FOR RAPID PROTOTYPING APPLICATIONS

Georges Fadel
Associate Professor

Ravi Ganti
Graduate Student

Center for Advanced Manufacturing
Mechanical Engineering Department
Clemson University

ABSTRACT

A methodology aiming at reproducing in Rapid Prototyping applications, exact parametric curves from CAD data is presented. The approach consists of converting the space-based parametric curves from the CAD system into time-base, such that the equations of the curve in terms of time are then fed to a controller directly. Optimization is used to solve the problem, which has both Rapid Prototyping process and scanning constraints. With information such as the equation of the curve, its first and second derivatives with respect to time, a real-time trajectory controller can be designed. The trajectory displays an increase in accuracy over traditional approaches using STL files, which is of the order of the chordal tolerance used to generate tessellations. The system model involves electrical and mechanical dynamics of the galvanometers and sensors. The controller, which acts on two mirrors, deflecting the laser beam of a stereolithography machine in the x and y directions respectively, should be easily substituted for current systems. Application of the methodology to freeform curves shows acceptable tracking and can be improved by judicious selection of the equation representing the spatial parameter as a function of time.

KEYWORDS

Controller, rapid prototyping, optimization, time parameterization, optimal control, galvanometers.

1. INTRODUCTION

All commercially available Rapid Prototyping (RP) machines are based on a process that converts CAD drawings to machine instructions. The latter drive some mechanism that works mostly by material addition and creates the prototype. All are layered-based processes, and the only control mechanism used to either drive a laser beam or a nozzle, is vector based. The advantages of such a controller are obvious: (1) Two points define the start and the end of the vector; (2) CAD software inherently generates vectors to drive plotters, and it is relatively easy to convert arbitrary surfaces to triangles which are sliced to form a series of vectors.

However, since accuracy of the prototype is becoming a major concern for the users of rapid prototyping hardware who aim at producing finished parts, or molds to produce finished parts, the vector based approach is a limiting factor when curved features are replaced by vectors. Furthermore, most CAD vendors have migrated to a parametric based representation of surfaces, and the ability to draw exact curves would significantly improve surface quality. Clemson University and others have developed algorithms to slice parametric surfaces and generate parametric curves. Presently, these parametric-curves are approximated by vectors, and the improvement in accuracy is already visible over triangle based tessellations. The next step is to directly draw the parametric curves to close the page on the accuracy question in a plane.

This paper illustrates the approach selected to perform the conversion from a space-based parameterization to time-base. A representative curve is illustrated, and the strengths and limitations of the approach are exposed.

2. LITERATURE SURVEY

Previous work related to geometric contour following for scanning control in solid freeform fabrication is by researchers at the University of Texas, Austin (Wu and Beaman, 1990). Their approach utilizes the arc length as a parameter to represent a general geometric path. The reason for using arc length is that the first and second derivatives of the arc length represent the tangential velocity and acceleration respectively. Their approach is applied to standard curve shapes such as a circle or an ellipse. Moreover, the control strategy adopted is based on a simple dynamic model of the system. This does not include all the external and internal factors affecting the dynamics of the system but it is straightforward and can be tailored to consider process based constraints as well as galvanometer constraints

An adaptive slicing algorithm (Vouzelaud, 1993) that makes use of AutoCAD slicing routines to obtain 2-D contours from a 3-D CAD model was developed at Clemson University. Vuyyuru (1994), and Ganesan (1994) also at Clemson, developed a process to slice solid models from I-DEAS directly. This results in the two-dimensional cross sections of the solid model.

The motion of the laser beam in two-dimensional space is analogous to the robot path-planning problem, in which the robot is constrained to follow an arbitrary path in space. This problem, subjected to different motion constraints, has been solved by several researchers (Shiller and Dubowsky, 1989; Bobrow *et al.*, 1985; Shin and Mckay, 1985). However, these researchers consider only one type of constraints (limits on the actuation torque) while neglecting the constraints on velocity, which are process-dependent. The method of Van Willigenburg, (1991) uses optimal control strategies, which take into consideration the constraints on actuation torque and process dependent velocities in both directions. In this method, the path of the robot is given by a set of coordinate pairs (x_P, y_P), not by any continuous function, and is ultimately a collection of discrete line segments between two successive points. The time taken to travel from one point to the other is obtained by solving the time optimal control problem.

This paper presents an alternative methodology to generate parametric curve equations as function of time, which can then be used to drive a controller.

3. PROPOSED METHODOLOGY

The proposed methodology can be applied to most RP technologies, but is applied in this paper to stereolithography. We first derive the necessary equations that are process dependent and that quantify the process constraints, then explain the optimization method used to generate the conversion from space-based to time-based parameterization.

The scanning systems of stereolithography machines consist of a single laser beam, which is deflected by mirrors to draw 2D curves on the resin vat and solidify the liquid. In current scanning systems, two mirrors, actuated by DC motors, deflect the laser beam in one direction each (X or Y), and the combination of the two provides the full required motion range of the laser beam. The dynamics of the DC motors can be represented by

$$U = LdI/dt + RI + K\omega \tag{1a}$$

$$T = KI \tag{1b}$$

where

L = motor inductance (Henrys)
R = motor electrical resistance (Ohms)
I = motor current (Amperes)
U = voltage applied to motor (Volts)
ω = motor angular speed (rad /s)
T = motor generated torque (N. m/rad)
K = motor voltage constant (Vs /rad)

The heat generation and the dissipation characteristics mainly contribute to limiting the capabilities of a DC motor. Heat generation is represented by the second term in equation (1a) and thus is proportional to the motor current. Furthermore, conventional DC motor controllers are built around a motor current controller, which is used to limit the motor current to prevent overheating. Thus, from both a practical and modeling viewpoint, it is convenient to choose the motor currents as the control variables for the dynamic system especially when assuming the motor current controller to be ideal. Also, assuming an ideal transmission from the DC motor to the galvanometer scanners, the motor current can be considered proportional to the torque (1b). Both these conditions can be translated into acceleration constraints on the DC motors, and have to be taken into account in our analysis.

The other critical process constraint is the scanning speed of the laser. Stereolithography machines typical scanning speeds are in the range of 80 to 90 cm/sec or30 to 35 in/sec. The scanning velocity depends directly on the power of the laser beam and resin material properties. The laser power varies from laser to laser and varies with time and age of the laser; hence, the velocity is a machine dependent factor. Note that the velocity referred to is the absolute scanning velocity.

The above-presented constraints are the main process constraints that have to be considered in the model for Stereolithography. However, to ensure convergence, and to add general process constraints such as the ones related to nozzle movement, minimum and maximum velocity constraints in the X and Y directions are also added.

3.1 Solving the Time Optimal Control Problem using the Method of Optimization

The problem of converting the equations from space-based parameterization to time-base is formulated as a standard optimization problem wherein the function to be minimized is the total time taken to track the desired trajectory subjected to desired velocity and acceleration constraints.

The path, as resolved from the CAD system slicing program, is a parametric curve representation (more and more NURBS based). Such a representation expresses Cartesian coordinates in two-dimensional space as a function of a spatial parameter P. A simpler parametric representation upon which much of the solid geometry has been built is the parametric-cubic representation. This representation is used in this paper without loss of generality. Hence, the equations of motion in terms of the spatial parameter P are:

$$\begin{aligned} x_p &= f(\mathrm{P}) = k_1\mathrm{P}^3 + k_2\mathrm{P}^2 + k_3\mathrm{P} + k_4, \\ y_p &= g(\mathrm{P}) = k_5\mathrm{P}^3 + k_6\mathrm{P}^2 + k_7\mathrm{P} + k_8 \end{aligned} \qquad 0 \le \mathrm{P} \le \mathrm{P}_{\max} \tag{2}$$

To build complex paths, the functions f and g (2) are assembled piecewise with other 3rd degree polynomials (parametric cubic) in terms of P and are continuous along with their derivatives. The derivation presented below does not consider the multiple segments, rather a single segment in which the parameter P varies between 0 and 1 (normalized).

With the equations of motion in the spatial parameter space (2) and the constraints above, the time optimal control problem boils down to determining a function of the parameter P in terms of time given by

$$\mathrm{P}(t) \qquad 0 \le t \le t_{\max} \tag{3a}$$

This relation should be a non-decreasing continuous function of time as the normalized parameter P goes from 0.0 to 1.0 i.e.,

$$\dot{\mathrm{P}}(t) \ge 0 \quad 0 \le t \le t_{\max} \tag{3b}$$

The initial and final conditions are given by

$$\mathrm{P}(0) = 0 \qquad 0 \le t \le t_{\max} \tag{3c}$$

$$\mathrm{P}(t_{\max}) = 1 \tag{3d}$$

such that $t_{\max}$ is optimal. Once (3) is known, the time optimal trajectory is given by:

$$x_p = f(\mathrm{P}(t)) \qquad 0 \le t \le t_{\max}, \tag{4a}$$

$$y_p = g(\mathrm{P}(t)) \qquad 0 \le t \le t_{\max}. \tag{4b}$$

Note that when $\dot{\mathrm{P}}(t) = 0$, the laser beam comes to a stand still. At these points we may split the path in two parts by demanding the laser beam to stand still at the end of the first path, as well as at the start of the second path. Therefore, except for possibly the initial and final time, $\dot{\mathrm{P}} > 0$.

Since $\mathrm{P}(t)$ in (3a) is a non-decreasing function of time, minimizing $t_{\max}$ is equivalent to maximizing $d\mathrm{P}/dt$ at all times t, $0 \le t \le t_{\max}$. From the first rules of differentiation, we adopt the following notation:

$$\begin{aligned} f_1 &= df/d\mathrm{P}, & f_2 &= d^2 f/d\mathrm{P}^2, \\ g_1 &= dg/d\mathrm{P}, & g_2 &= d^2 g/d\mathrm{P}^2 \\ \dot{\mathrm{P}} &= d\mathrm{P}/dt, & \ddot{\mathrm{P}} &= d^2\mathrm{P}/dt^2. \end{aligned} \tag{5}$$

From (2) and (5), the following relations are derived from the fundamental rules of differentiation:

$$\begin{aligned} \dot{x}_p &= f_1\dot{\mathrm{P}}, & \dot{y}_p &= g_1\dot{\mathrm{P}}, \\ \ddot{x}_p &= f_1\ddot{\mathrm{P}} + f_2\dot{\mathrm{P}}^2, & \ddot{y}_p &= g_1\ddot{\mathrm{P}} + g_2\dot{\mathrm{P}}^2. \end{aligned} \tag{6}$$

The quantity $\dot{\mathrm{P}}$ is referred to as the path velocity, because from the expressions, it is clear that the velocity with which the path is traveled in a given time is directly proportional to the slope of the curve with respect to the parameter at that point and $\dot{\mathrm{P}}$. Since the slope of the curve is specified by the equation of the curve in space, $\dot{\mathrm{P}}$ is the only variable. Similarly, the quantity $\ddot{\mathrm{P}}$ is referred to as the path acceleration.

A state trajectory determined by $x_p(t), y_p(t)$, $t_0 \le t \le t_{\max}$, can only be realized if both time domain functions are continuous and have continuous first derivatives. Now from (6), given the properties of f, g, and $\mathrm{P}(t)$ we observe that all of them are continuous functions.

The constraints imposed on the actuators, which supply the necessary power to the system, affect the acceleration of the laser beam in both directions. These constraint relations are obtained from the model of the laser beam in which the acceleration imparted to the beam is directly proportional to the torque supplied.

$$\ddot{x}_p = b_x\tau_x \qquad \ddot{y}_p = b_y\tau_y \tag{7}$$

The above expressions denote acceleration in the X and Y directions, where b_x and b_y are the coefficients associated with the mass and radius about which the torque is applied, τ_x and τ_y are the torque in the X and Y directions respectively.

To ensure the satisfaction of the constraints everywhere along the trajectory, the entire curve is divided into a finite number of segments, and the constraints are imposed on all the segments. The optimization problem is solved by assuming an arbitrary relation between parameter P and time t. In this study, this relation is assumed to be a sixth degree equation in time t without the constant term, which has been eliminated because of the initial condition. The following is the relation assumed to exist between parameter and time in this work.

$$\mathrm{P}(t) = a_0t^6 + a_1t^5 + a_2t^4 + a_3t^3 + a_4t^2 + a_5t \tag{8}$$

The sixth degree is selected in order to prevent the derivatives from vanishing by differentiation.

The objective of this problem is two folds. The time taken to complete the path is to be minimized and, at the same time, we need to resolve a best fit to find the coefficients of the expression (8) using a least squares method. The curve representing the path in parameter space should be identical to the curve in time space for a successful mapping. Hence, the relation between parameter and time should be such that the value of the parameter corresponding to any time should be equal to its true parametric value. The error can be defined as the difference between the parametric value corresponding to the time and the true parametric value. Hence the objective function is chosen such that the error obtained from calculating the value of the relation (8) and the value of P, over all the parametric locations from $\mathrm{P} = 0.0$ to $\mathrm{P} = 1.0$ should be minimum.

The optimization problem can be stated as follows:
Minimize :

$$f(t_i) = t_{\max} + \sum_{i=0}^{k} (a_0 t_i^6 + a_1 t_i^5 + a_2 t_i^4 + a_3 t_i^3 + a_4 t_i^2 + a_5 t_i - \mathrm{P}_i)^2$$

where f_i is the function to be minimized,

k is the number of segments into which the spatial curve is divided,

t_i is the time corresponding to the parametric value of P_i,

$a_{i=0..5}$ are the coefficients to determined by the optimizer.

Subject To:

1. Minimum and maximum absolute velocities given by $-(\sqrt{f_1^2 + g_1^2} * \dot{\mathrm{P}}) / \mathrm{V}_{\min} + 1.0 \le 0.0$ and $(\sqrt{f_1^2 + g_1^2} * \dot{\mathrm{P}}) / \mathrm{V}_{\max} - 1.0 \le 0.0$ which are $(\sqrt{f_1^2 + g_1^2} * \dot{\mathrm{P}}) \ge \mathrm{V}_{\min}$ and $(\sqrt{f_1^2 + g_1^2} * \dot{\mathrm{P}}) \le \mathrm{V}_{\max}$ normalized.
2. Minimum and maximum velocities in the X and Y are given by $-(f_1 * \dot{\mathrm{P}}) / \mathrm{V}_{\mathrm{xmin}} + 1.0 \le 0.0$, $(f_1 * \dot{\mathrm{P}}) / \mathrm{V}_{\mathrm{xmax}} - 1.0 \le 0.0$ and $-(g_1 * \dot{\mathrm{P}}) / \mathrm{V}_{\mathrm{ymin}} + 1.0 \le 0.0$, $(g_1 * \dot{\mathrm{P}}) / \mathrm{V}_{\mathrm{ymax}} - 1.0 \le 0.0$ (also normalized)
3. Minimum and maximum accelerations in the X and Y directions given by $-(f_1 \ddot{\mathrm{P}} + f_2 \dot{\mathrm{P}}^2) / \mathrm{A}_{\mathrm{xmin}} + 1.0 \le 0.0$, $(f_1 \ddot{\mathrm{P}} + f_2 \dot{\mathrm{P}}^2) / \mathrm{A}_{\mathrm{xmax}} - 1.0 \le 0.0$, $-(g_1 \ddot{\mathrm{P}} + g_2 \dot{\mathrm{P}}^2) / \mathrm{A}_{\mathrm{ymin}} + 1.0 \le 0.0$ and $(g_1 \ddot{\mathrm{P}} + g_2 \dot{\mathrm{P}}^2) / \mathrm{A}_{\mathrm{ymax}} - 1.0 \le 0.0$.

The optimizer used is CONMIN (Vanderplaatz, 1973), a commercial code based on the feasible directions method. It requires the calculation of gradients, which are automatically generated by the program using finite differences.

4. APPLICATION

The approach presented above is applied to the following parametric cubic curve. Additional examples were generated but cannot be presented because of space limitations.

$$x(\mathrm{P}) = 32.0 * (\mathrm{P}^3) - 48.0 * (\mathrm{P}^2) + 18.0 * (\mathrm{P}) \tag{9}$$

$$y(\mathrm{P}) = (32.0/3.0) * (\mathrm{P}^3) - (56.0/3.0) * (\mathrm{P}) + 12.0 * (\mathrm{P}) \tag{10}$$

The above equations are normalized such that the curve in the XY plane is completely described when the parameter P varies from zero to one. The number of segments into which the curve is divided is arbitrarily set to one hundred. The number of design variables is thus 107 with 6 variables representing the a_i parameters of equation (8) and the remaining accounting for the time increments, i.e. the time t values at each spatial P value.

The constraints applied in this example are: absolute velocity between 2 and 30 cm/s, acceleration between –2600 and 2600 cm/s^2 and individual velocities between + and – 80 cm/s.

Figure 1 shows the displacements in X and Y direction in space domain for the curve selected. Figure 2 shows the results of the optimization, which is the illustration of the spatial parameter P as a function of time t. Figure 3 illustrates the parametric cubic with tic marks representing equal time steps. This figure clearly shows how the algorithm slows the laser down towards its minimum absolute velocity at regions of high curvatures, and speeds it up at regions of lower curvature. Figure 4 further illustrates the individual velocities in the X and Y directions and the absolute velocity as a function of time elapsed.

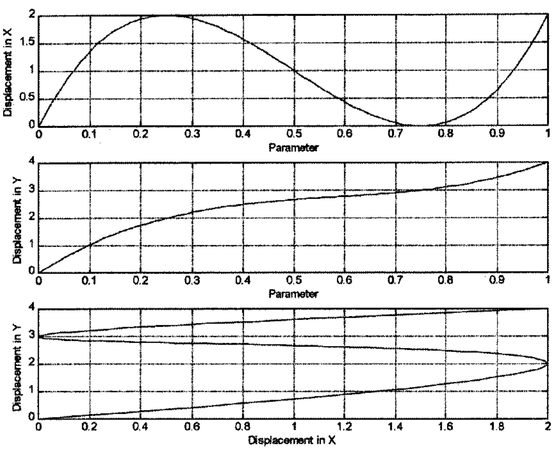

Figure 1. Displacements in X and Y Directions in Parametric Domain

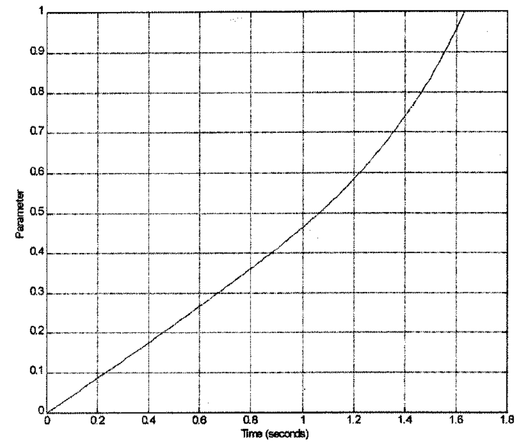

Figure 2. Parameter P versus Time

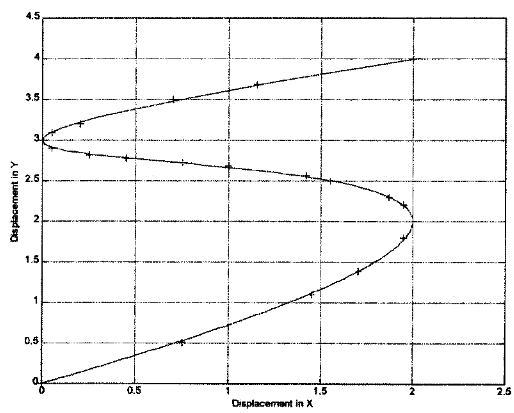

Figure 3. Cubic Spline Curve in Time Domain with (+) Signs Showing Trajectory Covered During Equal Intervals of Time.

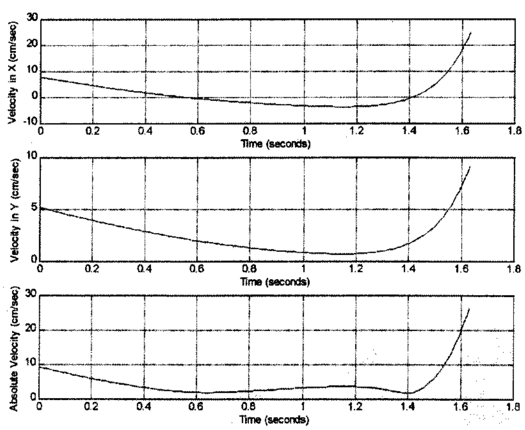

Figure 4. Absolute Velocity and Individual Velocities

This profile shows very well the expected behavior during the first lower part of the trajectory, but does not show as significant change in absolute velocity between the two changes in curvature when the trajectory becomes practically a straight line. The reasons may be the selection of the relation between parameter and time and the pressure of the optimizer to reduce the overall time.

The absolute velocity profile described above shows that the absolute velocity is within the bounds prescribed of 2 cm/sec and 30 cm/sec. The absolute and individual velocity profiles show an increase in the last portion of the trajectory. This is probably due to the pressures of the optimizer to reduce overall time. We would have liked figure 2 to show the monotonous increasing relationship between spatial parameter P and time t, which it did, but were expecting wiggles around the high curvature areas. The choice of a sixth order expression prevented this from clearly happening. Figure 5 shows again the increase in acceleration at the end of the trajectory, again in response to the pressures of the optimizer.

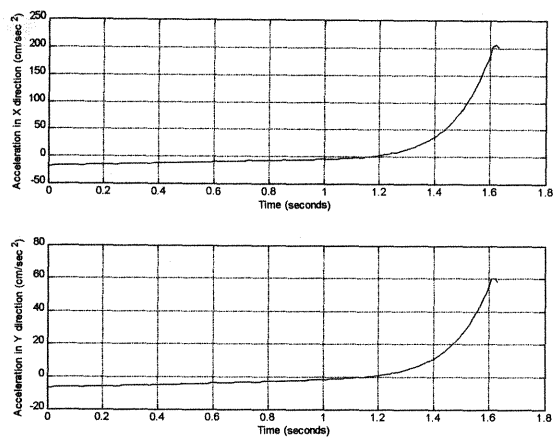

Figure 5. Individual Accelerations in X and Y Directions

Three different cases were tested on each of the curves selected. Table I shows that the time taken to track the trajectory reduces with the increase in velocity. The time taken to track the desired trajectory goes down from 1.65 seconds to 1.00 second with an increase in maximum absolute velocity from 30 cm/sec to 120 cm/sec.

Table I. Results of Maximum Velocity variation

Minimum Velocity (cm/sec)	Maximum Velocity (cm/sec)	Time (sec)
2	30	1.65
2	50	1.63089
2	120	1.00

4.1 Observations

The solution obtained in some of the cases is not in accordance with the desired philosophy of tracking the regions of low curvature at the maximum possible velocity and regions of high curvature with the lowest possible speed. The reasons for this behavior can be attributed to many factors. First and foremost, is the assumption of a relation between parameter and time. In this work, we assumed a polynomial expression of sixth order, this probably has some bearing on the solution. The initial conditions and the bounds on the design variables also effect the solution. The curvature is not explicitly tied to the solution.

Other practical issues such as sharp angle turns have not yet been considered. Also, the start and end velocities of the trajectories, as well as possible starts and stops, have not been incorporated as constraints but could be readily accomplished. These are issues for continued research especially when migrating to piecewise parametric representations and NURBS.

5. CONCLUSIONS

The traditional approach of vector scanning is replaced in this work by continuous parametric curves. The accuracy of tracking by time-based parametric representations is predicted to increase drastically due to the exact nature of the parametric curves fed to the controller. The approach presented in this paper is one of solving a dynamics and controls problem using a commercial optimization package. The previous approaches that attempted to solve the problem did not result in a continuous relation between parameter and time nor in a time-based parameterization of the original CAD drawing. The procedure developed and presented in this work converts parametric curves from a spatial parameter to a time parameter. The proposed approach has been tested on simple curves and curves with large radius of curvature. The results show that the relation can be found for different feasible and reasonable velocity limits. The solution obtained depends on factors like the selection of a relation between parameter and time and initial conditions. The methodology can be applied to a different combination of relations such as a linear superimposed by a sinusoidal or any higher order polynomial.

The time taken to track the desired trajectory is in the order of few seconds considering realistic process parameters. Reducing the time scale to milliseconds could better the results. Furthermore, scaling the objective function and considering multi-objective issues may produce even better results. Other considerations for the method include the significant reduction in data required to draw complex curves and better control ability. The parameters themselves are the only needed information, and, by providing the controller with an equation of a curve as a function of time, its velocity and acceleration at any point, a controller can be designed to accurately track the curve.

Present work is dealing with the following issues: piecewise parameterization, NURBS application, controller development, interface with CAD directly, and implementation in a stereolithography machine.

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