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Bayesian Hierarchical Linear Modeling of NFL Quarterback Rating

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Bayesian Hierarchical Linear Modeling of NFL Quarterback Rating

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Report

Presented to the Faculty of the Graduate School

of the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Statistics

The University of Texas at Austin

May 2015

Abstract

Bayesian Hierarchical Linear Modeling of NFL Quarterback Rating

by

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The University of Texas at Austin, 2014

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With endless amounts of statistics in American football, there are numerous ways to evaluate quarterback performance in the National Football League. Owners, general managers, and coaches are always looking for ways to improve quarterback play to increase overall team performance. In doing so, one may ask: Does the performance in the first quarter have any effect on the fourth quarter performance? This paper will investigate the linear dependence of the first quarter NFL QB rating on the fourth quarter NFL QB rating for 17 NFL starting quarterbacks from the 2014-2015 season. The aim is to use Bayesian hierarchical linear modeling to attain slope and intercept estimates for each quarterback in the study and attempt to determine what is causing the dependence, if any. Then, if a linear dependence is detected, investigating whether or not the statistic used is a viable measure of performance.

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Chapter 1: Introduction

In recent years, the use of advanced statistical practice has become more of a fixture in the world of sports. One could argue that the success of “MoneyBall” has had a domino effect across all major sports in the United States, resulting in increased use of more sophisticated statistical analysis for decision making within the sport. American Football, the nation’s most popular and lucrative sport, is no stranger to this current phenomenon. Organizations with in the National Football League (NFL) now have the ability to use the endless amounts of data of player statistics and game conditions to make better in game decisions in an effort to win more. Analyzing down and distance, the current down and how far until the team can reach the next set of downs, impacts play calling by coaches, measuring the effect of rush yards per games has on scoring, using overall player grades from scouting officials and expert projections to predict when a collegiate player will be drafted by an NFL team are just some examples of how NFL organizations or enthusiasts of the game are using statistics for more informed decision making (Almar and Mehrotra, 2011). However, no position in American football is more scrutinized that the quarterback position. This player is typically the leader of the team, the player who gets to much credit for a win, or all the blame for a loss (Farmer, 2012). That being said, this position is the one most often analyzed by NFL experts, scouts, and countless others in an effort to increase productivity from that position.

One popular game statistic is the NFL’s own quarterback rating. The NFL quarterback rating is a metric used to quantify a quarterback’s ability on an arbitrary scale of 0 to 158.3. It is comprised of four variables: percentage of completion per attempt, average yards gained per attempt, percentage of touchdown passes per attempt, and percentage of interceptions per attempt. Each variable is scaled between the values of 0 and 2.375 and each component a, b, c, and d represent individual summary statistics:

$$a = \left(\frac{\#\ of\ completions}{\#\ of\ passing\ attempts} - .3\right) \times 5 \tag{1}$$

$$b = \left(\frac{total\ passing\ yards}{\#\ of\ passing\ attempts} - 3\right) \times .25 \tag{2}$$

$$c = \left(\frac{\#\ of\ touchdown\ passes}{\#\ of\ passing\ attempts}\right) \times 20 \tag{3}$$

$$d = 2.375 - \left(\frac{\#\ of\ interceptions}{\#\ of\ passing\ attempts}\right) \times 25 \tag{4}$$

If the completion percentage in (1) is greater than 77.5%, then assign a value of 2.375. If the completion percentage in (1) is less than 30%, then assign a value of 0. If yards per attempt in (2) is greater than 12.5 or less than 3.0, then assign a value of 2.375 or 0 respectively. If touchdowns per pass attempt in (3) is greater than 11.875%, then assign a value of 2.375. If interceptions per pass attempt in (4) is greater than 9.5%, then assign a value of 0. Using the calculations above, an individual's NFL quarterback rating for a game, half, or quarter can be calculated by the following:

$$Passer\ Rating = \left(\frac{a+b+c+d}{6}\right) \times 100. \tag{5}$$

The purpose of this report will be to investigate the linear dependence of a quarterback's first quarter rating to his fourth quarter rating. Does the first quarter performance of an NFL quarterback greatly effect his fourth quarter performance? A sample of 17 of the 32 NFL quarterbacks who started at least 8 games was taken and their respective first and fourth quarter ratings were calculated for all complete games played (see

Appendix for Table 1). If any sort of linear dependence is found, it will be necessary to investigate what might be causing it. However, enthusiasts of the game would claim that performances on a quarter by quarter basis should be independent of one another. If a dependence is found it might cast doubt on the statistic itself, raising the question of whether or not the statistic is an accurate measure of performance for the quarterback position.

In order to quantify the linear dependence of a quarterbacks first and fourth quarter performance, a Bayesian hierarchical linear model will be implemented using the R statistical software package. Chapter two will introduce the basic notions of hierarchical data structures and a discussion of Bayesian inference and simulation, as well as a detailed description of the model and the Gibbs sampler for the model. Chapter three will present the results of the model. Chapter four will wrap up the discussion of the model, including possible limitations and extensions.

Chapter 2: Methodology

2.1 Bayesian Models

Bayesian inference is the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model and on the unobserved quantities such as predictors for new observations. The probability distribution on the parameters conditioned on the observed data, known as the posterior distribution, can simply be derived using Bayes' rule:

$$p(\theta|y) = \frac{p(\theta, y)}{p(y)} = \frac{p(\theta)p(y|\theta)}{p(y)}. \tag{6}$$

Where θ represents the population parameters of interest or a vector of unobservable quantities and y is a vector of the observed data. Typically, the denominator $p(y)$ is dropped since it does not depend on θ and is treated as a normalizing constant. Thus giving:

$$p(\theta|y) \propto p(\theta)p(y|\theta). \tag{7}$$

The components of the posterior distribution, $p(\theta)$ and $p(y|\theta)$, are known as the prior distribution and likelihood function respectively (Gelman et al., 2004). The prior distribution, $p(\theta)$, is one's belief about the distribution of the unknown parameter(s) before taking the observable data into account. The likelihood function, $p(y|\theta)$, is the distribution of the observed data given the parameters. These, seemingly simply, expressions envelope the foundation of Bayesian inference, whose primary task is to update probabilities of unknown quantities as more evidence is obtained.

2.2 Hierarchical Structure

A Bayesian hierarchical model is a statistical model that uses multiple levels to estimate the parameters of the posterior distribution using the methods established in the previous section. In general, a hierarchical structure consists of three stages:

I $\quad y_i \sim p(y_i|\theta_i, \phi)$ independent

II $\quad \theta_i \sim p(\theta_i|\phi)$ independent and identically distributed (i.i.d)

III $\quad \phi \sim p(\phi)$.

Where $p(y_i|\theta_i, \phi)$ is the likelihood, $p(\theta_i|\phi)$ is the prior of the unknown parameter θ and is conditioned on a hyperparameter (the parameter of a prior distribution) ϕ, and $p(\phi)$ is the distribution of the hyperparameter known as a hyperprior.

The basis of Bayesian hierarchical modeling is to think conditionally. Conditional independence often allows for easier representation of a joint posterior distribution. Using the ideas of Bayesian inference, posterior inference can be calculated by the following:

$$p(\theta_i, \phi|y_i) \propto p(y_i|\theta_i, \phi)p(\theta_i, \phi). \tag{8}$$

However, the above can be simplified further due to the conditional independence of the data, y_i, and ϕ for known values of θ. Additionally, $p(\theta_i, \phi)$ can be made into the product of $p(\theta_i|\phi)p(\phi)$ using Bayes' rule. Hence, posterior inference can be achieved by:

$$p(\theta_i, \phi|y_i) \propto p(y_i|\theta_i)p(\theta_i|\phi)p(\phi). \tag{9}$$

While the hierarchical form of analysis and organization helps in the understanding of multiparameter problems, the resulting posterior distribution is often not in the form of a well-known distribution and sampling from that target density requires simulation to achieve numerical approximations for complicated integrals (Gelman et al., 2004).

2.3 Simulation

As mentioned in the previous section, many times the joint posterior distribution is an unknown distribution and is challenging to sample directly (Hoff, 2009). However, there are several simulation algorithms, generally referred to as Markov Chain Monte Carlo (MCMC), that allow one to approximate the target posterior distribution. MCMC simulation is a general method based on drawing values of the unknown parameters from approximate distributions and then correcting those draws to better approximate the target posterior distribution (Gelman et al., 2004). The overall idea of an MCMC is that it is a collection of random variables in which the next draw is conditionally independent of all previous draws, i.e. only depends on the last value drawn. The effectiveness of this process is not so much the conditional independence but rather that with each step in the simulation the target distribution is being converged upon.

One particular MCMC that is common and relatively straightforward to implement is known as a Gibbs sampler. In certain cases, the joint posterior distribution can be factored into a set of full conditional distributions. In a Gibbs sampler, the target distribution can be approximated by sampling all of the full conditionals (Gelfand & Smith, 1900). The Gibbs sampler algorithm is defined by:

Start with a model, $p(\theta_1, ..., \theta_d|y) = p(\theta_1, ..., \theta_d)$

1. Initialize values for each conditional $\Theta^{(0)} = (\theta_1^{(0)}, ..., \theta_d^{(0)})$

2. Repeat for $j = 1, 2, ..., M$

   - Generate $\theta_1^{(j+1)}$ from $p(\theta_1|\theta_{-1}^{(j)})$
   - Generate $\theta_2^{(j+1)}$ from $p(\theta_2|\theta_{-2}^{(j)})$

     $\vdots$
   - Generate $\theta_i^{(j+1)}$ from $p(\theta_i|\theta_{-i}^{(j)})$

3. Return $\Theta^1, ..., \Theta^M$

Here $p(\Theta_i|\Theta_{-i})$ is the full conditional density. For example, suppose there exists normally distributed data $\boldsymbol{y} = (y_1, y_2, ..., y_i)^T$ and $y_i \sim N(\mu, \sigma^2)$ i.i.d. Next, define the likelihood and, for ease of calculation, a non-informative prior on the parameters.

Likelihood:

$$f(\boldsymbol{y}|\mu, \sigma^2) \propto (\tfrac{1}{\sigma^2})^{\frac{n}{2}} exp[-\tfrac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \mu)^2]$$

Prior:

$$p(\mu, \sigma^2) \propto \tfrac{1}{\sigma^2}.$$

Now define the joint posterior distribution and the appropriate conditional distribution for μ and σ^2.

Joint Posterior:

$$p(\mu, \sigma^2|\boldsymbol{y}) \propto (\tfrac{1}{\sigma^2})^{\frac{n}{2}+1} exp[-\tfrac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \mu)^2]$$

Conditionals: Let $\tau = \frac{1}{\sigma^2}$

$$p(\mu|\sigma^2, \boldsymbol{y}) \sim N(\bar{y}, (n\tau)^{-1})$$

$$p(\tau|\mu, \boldsymbol{y}) \sim Gamma(\tfrac{n}{2}, \tfrac{1}{2}\sum_{i=1}^{n}(y_i - \mu)^2).$$

Now that the conditionals are defined, the Gibbs sampler will be defined by the following:

1. Initialize $\mu^{(0)}$ and $\tau^{(0)}$

2. Sample:

   $\mu^{(t+1)} \sim N(\bar{y}, (n\tau^{(t)})^{-1})$

   $\tau^{(t+1)} \sim Gamma(\frac{n}{2}, \frac{1}{2}\sum_{i=1}^{n}(y_i - \mu^{(t+1)})^2)$
   with $\sigma^{2(t+1)} = \frac{1}{\tau^{(t+1)}}$

3. Return all sampled μ's and σ^2's.

Once the process has repeated itself for the chosen amount of iterations, the resulting μ and σ^2 vectors can be summarized using descriptive statistics to give estimates for the parameters. The above algorithm forms a Markov chain whose stationary distribution is the sought after target distribution. The Gibbs sampler will be the preferred simulation algorithm to estimate the linear dependence of an NFL quarterbacks first and fourth quarter performance.

2.4 Model Specification

The Bayesian hierarchical linear model chosen to quantify the dependence of a quarterbacks first and fourth quarter performance will take the form of:

$$y_{ij} = \beta_{0i} + \beta_{1i}x_{ij} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2), \tag{10}$$

where y_{ij} represents the the fourth quarter NFL quarterback rating of the ith quarterback in the jth game, x_{ij} represents the first quarter NFL quarter back rating of the ith quarterback in the jth game. Also, β_{0i} and β_{1i} are the true value coefficients for the ith quarterback with β_{1i} being the particular parameter of interest. For ease of notation, coefficients will be represented as a vector $\boldsymbol{\beta_i} = (\beta_{0i}, \beta_{1i})^T$ and claim that $\boldsymbol{\beta_i} \sim MVN(\boldsymbol{\mu}, \Sigma)$ where $\boldsymbol{\mu} = (\mu_0, \mu_1)^T$ and Σ is the co-variance matrix. The ϵ_i is the error term for each individual quarterback and it will be assumed that every quarterback will have equal variance. In order to proceed with Bayesian inference, a likelihood and prior will need to be established. Letting $\lambda = \frac{1}{\sigma^2}$ and Θ represent all the parameters of interest. Then likelihood will be defined as:

$$\mathcal{L}(\boldsymbol{y}|\Theta) = \prod_{i=1}^{n} \lambda^{\frac{m_i}{2}} exp[-\frac{\lambda}{2}\sum_{j=1}^{m_i}(y_{ij}-\beta_{0i}-\beta_{1i}x_{ij})^2] \times \prod_{i=1}^{n} |\Sigma^{-1}|^{\frac{1}{2}} exp[-\frac{1}{2}(\boldsymbol{\beta_i}-\boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{\beta_i}-\boldsymbol{\mu})].$$

This leads to the hierarchical nature of this model as appropriate priors will need to be set on the parameters $\lambda, \boldsymbol{\mu}, \Sigma^{-1}$.

The following priors were chosen:

1. $p(\lambda) \sim Gamma(a, b)$

   $p(\lambda) \propto \lambda^{a-1} exp[-b\lambda]$

2. $p(\boldsymbol{\mu}) \sim MVN(\boldsymbol{\nu}, \Omega)$

   $p(\boldsymbol{\mu}) \propto |\Omega|^{-\frac{1}{2}} exp[-\frac{1}{2}(\boldsymbol{\mu} - \boldsymbol{\nu})'\Omega^{-1}(\boldsymbol{\mu} - \boldsymbol{\nu})]$

3. $p(\Sigma^{-1}) \sim Wisart_p(V, r)$ Note: $r = p$

   $p(\Sigma^{-1}) \propto |\Sigma^{-1}|^{\frac{r-p-1}{2}} exp[-\frac{1}{2}tr(V^{-1}\Sigma^{-1})]$.

The Wishart distribution is a probability distribution that allows for sampling all the elements of the variance-covaraince matrix at once. It depends on a scale matrix, V, that must be non-negative and symmetric, the dimensions of the matrix, r, and degrees of freedom, p (Sawyer, 2007). Now, using the properties of hierarchical structures, the joint posterior distribution is simply the product of the likelihood function and the prior distributions for $\lambda, \boldsymbol{\mu}, \Sigma^{-1}$.

Joint Posterior distribution:

$$p(\Theta|\boldsymbol{y}) \propto \lambda^{\frac{\sum_{i=1}^{n} m_i}{2}} exp[-\frac{\lambda}{2}\sum_{i=1}^{n}\sum_{j=1}^{m_i}(y_{ij} - \beta_{0i} - \beta_{1i}x_{ij})^2] \times |\Sigma^{-1}|^{\frac{n}{2}} exp[-\frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{\beta_i} - \boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{\beta_i} - \boldsymbol{\mu})]$$
$$\times |\Omega|^{-\frac{1}{2}} exp[-\frac{1}{2}(\boldsymbol{\mu} - \boldsymbol{\nu})'\Omega^{-1}(\boldsymbol{\mu} - \boldsymbol{\nu})] \times |\Sigma^{-1}|^{\frac{r-p-1}{2}} exp[-\frac{1}{2}tr(V^{-1}\Sigma^{-1})] \times \lambda^{a-1} exp[-b\lambda]$$

The above joint posterior distribution is not a well known or studied distribution and therefore sampling directly from it would be difficult and computationally expensive. However, the joint posterior can be easily factored into various conditional distributions for each of the parameters being estimated.

Conditional Distributions:

1. $p(\lambda|\Theta_{-\lambda}, \boldsymbol{y}) \sim Gamma\left(a + \frac{1}{2}\sum_{i=1}^{n} m_i, b + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m_i}(y_{ij} - \beta_{0i} - \beta_{1i}x_{ij})^2\right)$

2. $p(\Sigma^{-1}|\Theta_{-\Sigma^{-1}}, \boldsymbol{y}) \sim Wishart_{r=p}(V^{-1} + \sum_{i=1}^{n}(\boldsymbol{\beta_i} - \boldsymbol{\mu})(\boldsymbol{\beta_i} - \boldsymbol{\mu})^T, n-1)$

3. $p(\boldsymbol{\mu}|\Theta_{-\boldsymbol{\mu}}, \boldsymbol{y}) \sim MVN(m, R)$, where $R = (\Omega^{-1} + n\Sigma^{-1})^{-1}$
   and $m = R(\Omega^{-1}\boldsymbol{\nu} + \Sigma^{-1}\sum_{i=1}^{n}\boldsymbol{\beta_i})$

4. $p(\boldsymbol{\beta_i}|\Theta_{-\boldsymbol{\beta_i}}, \boldsymbol{y}) \sim MVN(C^{-1}d, C^{-1})$, where $C = (\lambda\boldsymbol{X_i'}\boldsymbol{X_i} + \Sigma^{-1})$
   and $d = (\lambda\boldsymbol{X_i'}\boldsymbol{Y_i} + \Sigma^{-1}\boldsymbol{\mu})$.

since the all of the conditionals follow a well-known distribution they can each be sampled directly via a Gibbs sampler in an attempt to estimate the target joint distribution. First, each parameter must be initialized. Second, loop over the number of iterations (size=10000) sampling each conditional using updated values from the previous iteration. Lastly, store the results as a sequence of dependent vectors. Now that all of the model specifications are set, the next chapter will illustrate the results from [R].

Chapter 3: Results

The following are a series of tables and figures summarizing the results from the posterior draws for each parameter in the model. Basic descriptive statistics (mean, standard deviation, and a 95% Highest Posterior Density Interval) will be used to make inferences on the parameters of interest. It is also important to note that data was re-scaled in an effort to minimize working with large numbers and reduce the variance. Each observation was decreased by a factor of 10.

Figure 1: Plot of Variance, σ^2, by Iteration

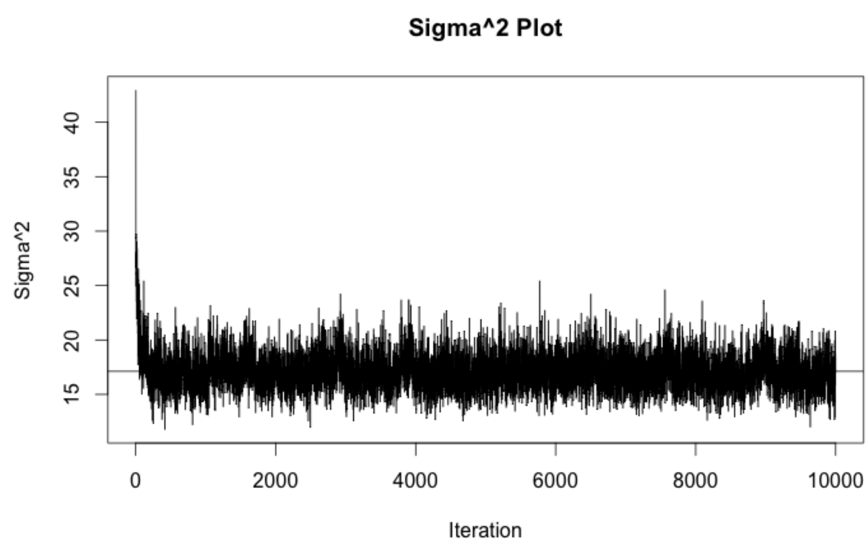


Table 2: Posterior summary for σ^2

Variable	Mean	St. Dev	95% HPD	Interval
			Lower	Upper
σ^2	17.12	1.96	13.86	20.67

Figure 1 shows that the Gibbs sampler was able to converge on a mean value of 17.12 for the variance of the error term of the model. Recall that constant variance for the error terms was assumed for each quarterback.

Figure 2: Mu Plot

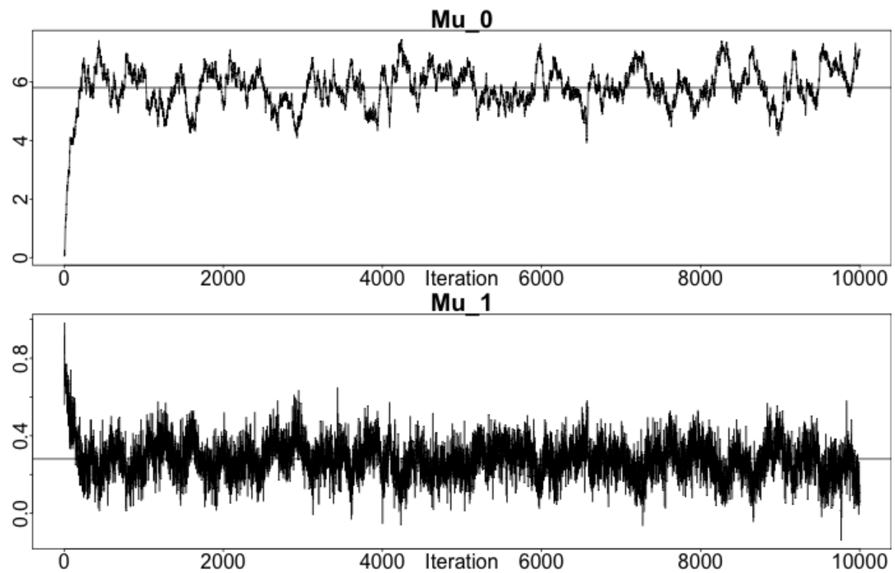


Table 3: Posterior Summary for Mu Vector

Variable	Mean	St. Dev	95% HPD	Interval
			Lower	Upper
μ_0	5.804	0.706	4.575	7.056
μ_1	0.281	0.098	0.093	0.465

The $\boldsymbol{\mu}$ is the overall mean vector for the $\boldsymbol{\beta_i}$ vector. Figure 2 shows that μ_0 converges to a value of 5.804 and μ_1 converges to a value of 0.281. Furthermore, the mean of the $\boldsymbol{\beta_i}$ vector from the simulation is $(5.828, 0.281)^T$. Figure 2 gives indication that Gibbs sampler is estimating the posterior density accurately.

Figure 3: Histogram of Intercept Estimate by Quarterback

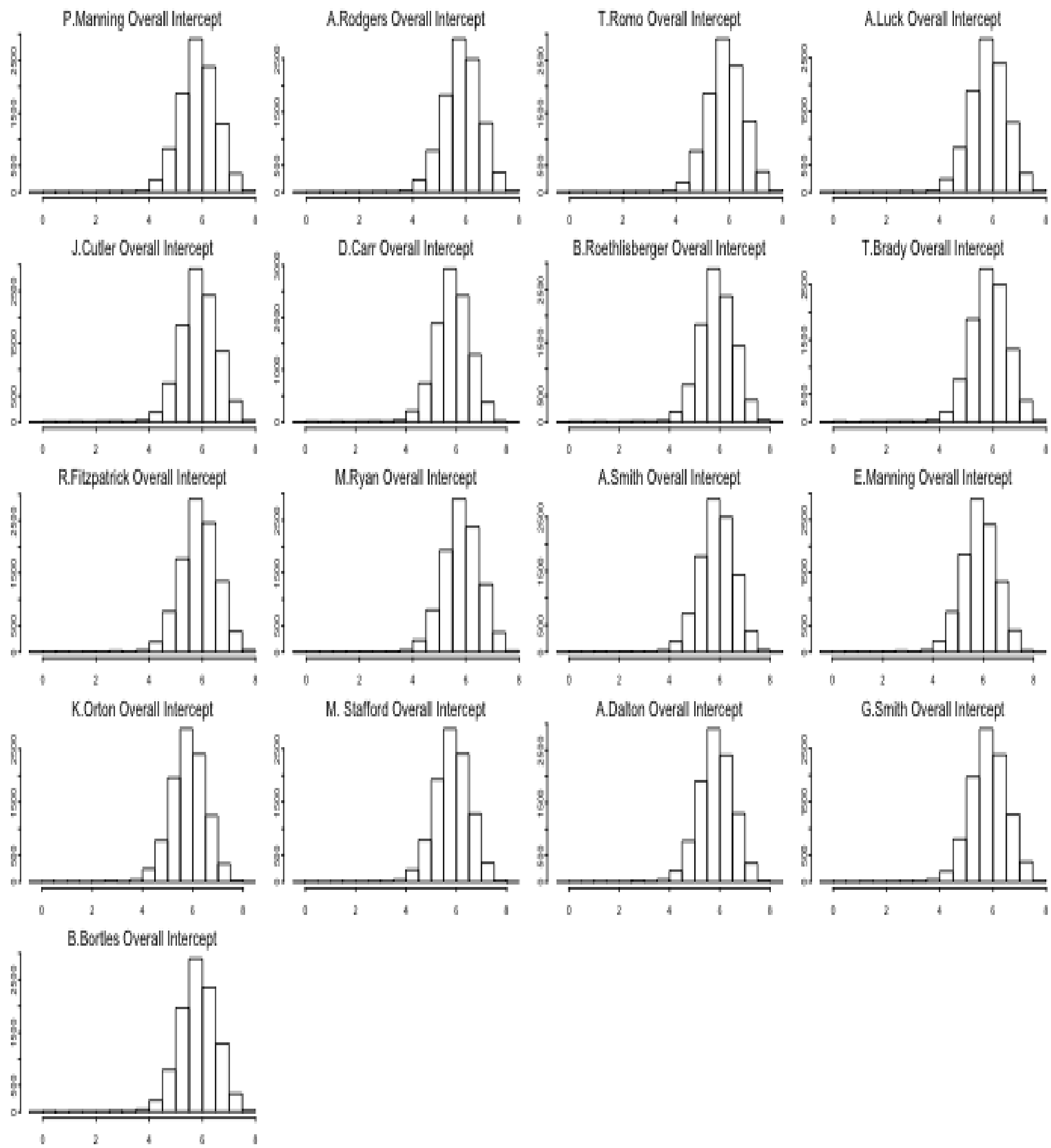
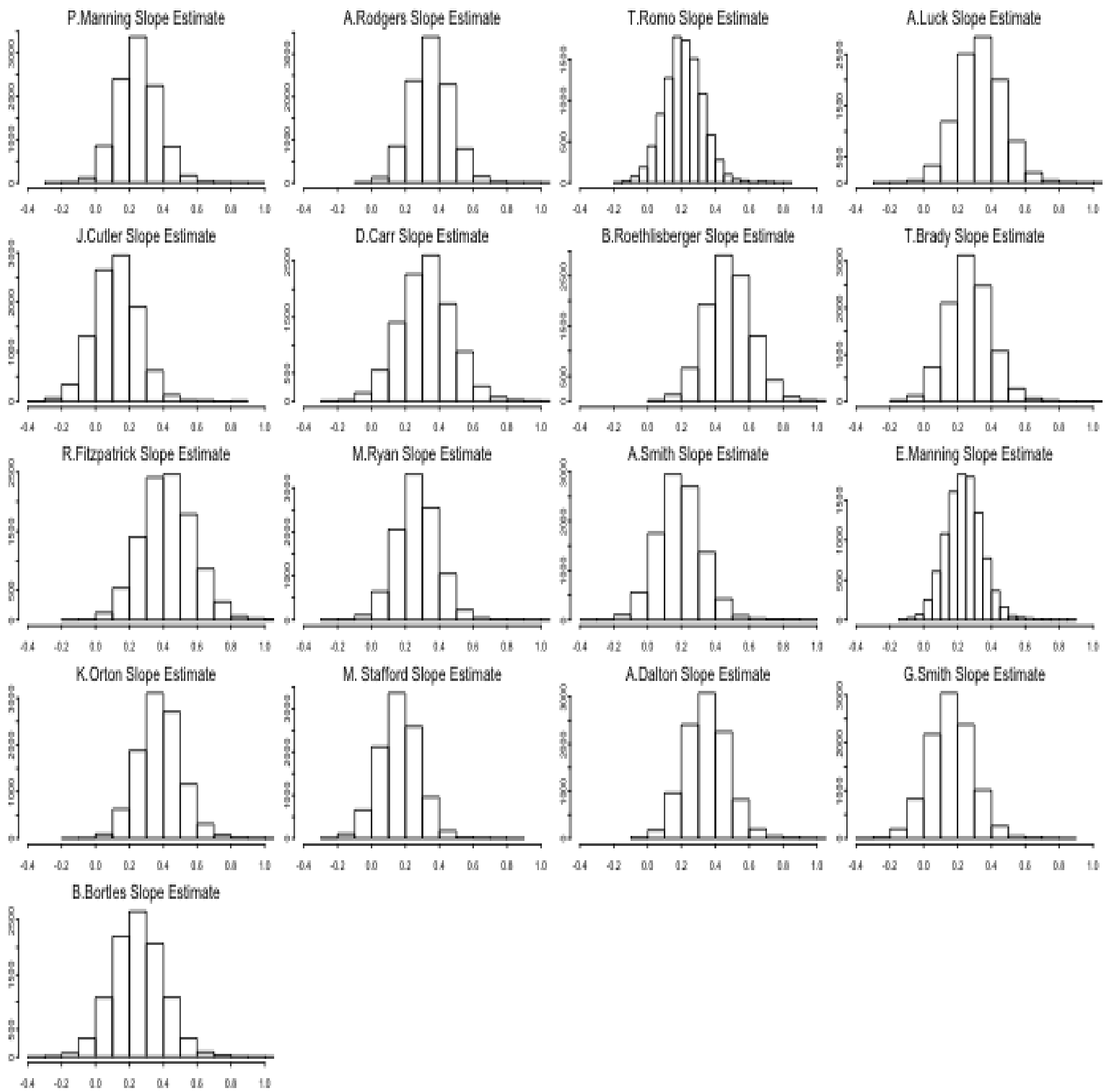


Figure 4: Histogram of Slope Estimate by Quarterback



The above histograms are for each quarterback and their respective $\boldsymbol{\beta_i}$. Figure 3 shows the distributions for β_{0i} appear uni-modal, symmetric, and centered around the same values. Figure 4 shows the distributions for β_{1i} also appear uni-modal and symmetric, however with some variation on where each is centered. Furthermore, Figure 4 suggests that the β_{1i} coefficients are all non-zero and different for each quarterback.

Table 4: Posterior Summary for Beta Coefficients

Quarterback	Beta Estimates	Mean	St. Dev
P. Manning	b0	5.809718	0.750604
	b1	0.251903	0.1191344
A. Rodgers	b0	5.833417	0.7464262
	b1	0.3502758	0.1169767
T. Romo	b0	5.837565	0.7452475
	b1	0.2128107	0.1144709
A. Luck	b0	5.815487	0.7522658
	b1	0.3340474	0.1359572
J. Culter	b0	5.844975	0.7456315
	b1	0.1219323	0.1305554
D. Carr	b0	5.829248	0.7425274
	b1	0.3236261	0.1560151
B. Roethlisberger	b0	5.862418	0.7515192
	b1	0.4819442	0.1359499
T. Brady	b0	5.839651	0.744044
	b1	0.2676116	0.1271415
R. Fitzpatrick	b0	5.845904	0.7485258
	b1	0.4248229	0.1536825
M. Ryan	b0	5.809979	0.7452962
	b1	0.2692171	0.1214298
A. Smith	b0	5.861901	0.7496621
	b1	0.1914642	0.1306859
E. Manning	b0	5.837468	0.752544
	b1	0.2388777	0.1073831
K. Orton	b0	5.804879	0.7495401
	b1	0.3804939	0.1256985

Table 4: Table continues below

Quarterback	Beta Estimates	Mean	St. Dev
M. Stafford	b0	5.814849	0.7464286
	b1	0.1647227	0.1176735
A. Dalton	b0	5.81679	0.7441632
	b1	0.3487378	0.1263923
G. Smith	b0	5.809777	0.7427529
	b1	0.1594236	0.1293962
B. Bortles	b0	5.806428	0.7475632
	b1	0.2506498	0.1501717

Table 4: Table continues below

95% HPD	Interval
Lower	Upper
4.489431	7.19387
0.018795444	0.4766624
4.54383	7.22966
0.12573386	0.5776482
4.512706	7.183581
-0.006634039	0.4363538
4.534302	7.242404
0.071489906	0.5972236
4.558264	7.218049
-0.117458089	0.382976
4.571297	7.224965
0.01503042	0.6185323
4.480187	7.178335

Table 4: Table continues below

95% HPD	Interval
0.213229403	0.7385129
4.546988	7.19978
0.018028702	0.5083352
4.505083	7.188078
0.120963795	0.7138439
4.496279	7.173408
0.031937846	0.5032389
4.493393	7.17052
-0.059172946	0.4483934
4.520174	7.210705
0.032005453	0.4472401
4.456427	7.149828
0.131520707	0.6201513
4.489762	7.168328
-0.061027333	0.3926259
4.466392	7.153525
0.099306186	0.5868898
4.510141	7.186309
-0.100862121	0.405886
4.48995	7.157375
-0.040856943	0.5386876

Table 4 outlines the $\boldsymbol{\beta_i}$ by quarterback, allowing for specific interpretation of linear dependence of first and fourth quarter performance by each individual. The β_0 parameter, in general, is the expected fourth quarter NFL QB rating for a specific individual when that person's first quarter rating is 0. The β_1 parameter, in general, is the expected increase in an individuals fourth quarter NFL QB rating for every one unit increase in that person's first quarter NFL QB rating. The HPD intervals for each quarterback are all generally positive, leading to the belief that and individual's first quarter performance does have positive influence their fourth quarter performance.

Figure 5: Scatter Plot NFL QB Rating by Beta Coefficient

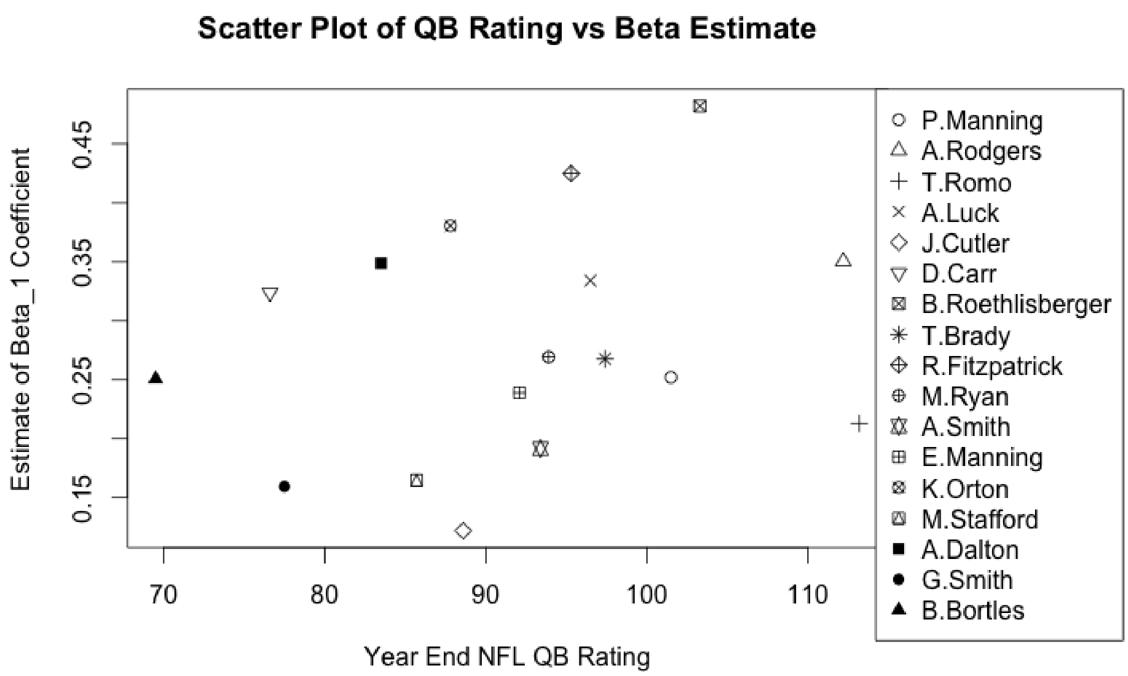


Figure 5 compares a quarterback's year end NFL QB rating to their respective β_1 estimate from the Gibbs sampler. The scatter plot, overall, has a weak positive association, r=.2420629. However, upon closer inspection, there appears to be two distinct groupings, a group of quarterbacks in the upper left portion and a group of quarterbacks in the lower right portion. The quarterbacks in the upper left portion all have relatively high β_1 coefficients, while the quarterbacks in the lower right portion have relatively low β_1 coefficients. The grouping of quarterbacks (D.Carr, B.Bortles, A.Dalton, R.Fitzpatrick, K.Orton, and B.Roethlisberger) with higher β_1 coefficients are all, for the exception of B.Roethlisberger, “journeymen” quarterbacks (quarterbacks who have played for multiple teams in their career) or quarterbacks with less than 5 years of NFL experience. This leads to a suggestion and belief that the end of game performance of more experienced and talented quarterbacks are less effected by how they start the game as opposed to the less experience and talented quarterbacks. This notation will be further elaborated on in the next chapter.

Chapter 4: Discussion and Extension

4.1 Discussion of Results

The overall goal of this paper was to investigate the linear dependence of an NFL quarterback's fourth quarter performance based on how that individual performs in the first quarter. As an enthusiast of the game, one would guess that the outcome of the first quarter would have absolutely no effect on the fourth quarter. Once players reach the NFL, they are considered to be some of the best athletes in their respective sport and are usually expected to perform at a high level. However, as is the case in any sport, there are some huge learning curves in adjusting to a higher level of competition. With this in mind, it is not unreasonable to think that some players are greatly effected by how they start a game.

The results in the previous chapter are intriguing because they suggest that, no matter the individual, there is some form of a positive matter of association between the performance in the first quarter and the performance in the fourth quarter. Summarizing Table 4, the $\boldsymbol{\beta_i}$ coefficients for each quarterback are all positive with averages ranging from 5.804879 to 5.862418 for β_0 and .1219323 to .4819442 for β_1. It is a safe assumption that the β_1 for each respective quarterback are all positive as 9 of the 17 95% HPD intervals fail to contain zero. The intervals for T. Romo, J. Cutler, A.Smith, M.Stafford, G.Smith, and B. Bortles do contain zero, however the majority of the interval contains positive values, confidently suggesting there is some positive effect of the first quarter performance on the fourth quarter performance for these individuals. All quarterbacks have approximately the same β_0 value, around 5.8, but there is more variation in the values of β_1. The value of the β_1 coefficient indicates the severity of dependence for an individual, the lower the value of β_1 the less dependent an individual is on their first quarter performance and the higher the values of β_i the more dependent an individual their first quarter performance. As stated in chapter 3 and illustrated in Figure 5, when the β_1 estimates of a quarterback are plotted against their respective end of year NFL quarterback rating it becomes clear which quarterbacks are more dependent of their beginning game performance.

While it should be reiterated that this is not a claim of causation, there is evidence that supports some dependency of a quarterbacks first quarter performance on their fourth quarter performance. Furthermore, it could be suggested that the value of the β_1 could be directly correlated to the level of quarterback. Based on this sample of 17 quarterbacks, the individuals whose NFL QB rating were near the top of the 2014 rankings tend to have a smaller β_1 coefficient. As one would hypothesize, these quarterbacks show minimal dependence of how they start a game to how they finish a game. On the contrary, players near the bottom of the 2014 NFL quarter ranking typically have minimal professional experience or are players who frequently change teams tend to have larger β_1 coefficients. These players, while still having tremendous athletic ability, seem to be more affected by recent in game outcomes. These results are promising and this information could potentially be used to help players improve their performance, influence in game coaching decisions for particular players, or even decisions about keeping the player on the roster.

In summary, steps have been taken in exploring the relationship of beginning and end game performance of an NFL starting quarterback. Those familiar with the game should not be surprised that more experienced and talented quarterbacks are less dependent on previous results. However, this study does show that, no matter of talent level, there is some positive matter of association and it would be of great interest to extend this further. Recall that the model only has one co-variate, the first quarter rating. It could prove beneficial to add more co-variates such as the second and third quarter ratings, years of experience, or the result of the game. Any additional information gathered could only benefit teams in their attempt in becoming a more successful NFL franchise.

4.2 Extension

It is not uncommon for those in the sports community to scrutinize and argue against the NFL quarterback rating as an accurate performance measure. The rating, established in 1973, has arbitrary bounds on all the factors outlined in equations (1), (2), (3), and (4) from chapter 1, resulting in an equally arbitrary number of 158.3 as a perfect passer rating. Critics claim that it is an overly complicated formula giving to much weight to completion percentage and touchdown percentage (Alberto, 2009). Passing yards per attempt is a component that can be easily inflated, leading to a higher non-representative passer rating. For example, a quarterback could throw a short pass, typically defined as an attempt less than 10 yards, that is then taken for many more yards by its recipient. The play, arguably, is influenced more so by the receiver as opposed to the quarterback but the result is still reflected in his passer rating. This example, and countless others, are reasons why many enthusiasts of the game feel the NFL quarterback rating is an archaic method used to measure the player's performance (Alberto, 2009). This realistically leads to the argument that the linear dependence found in this paper could be attributed to the inefficient summary given by the NFL QB rating.

Sports analysts would surmise that performances do not depend on a quarter by quarter basis, leading to the hypothesis that the $\beta_1 i$ coefficient should be zero for every quarterback. A possible solution to this problem could be investigating the more recent statistic created by ESPN, the prominent sports television channel in America, the Total Quarterback Rating (QBR). Total QBR measures much more than the four components outlined in the NFL QB rating. ESPN's Total QBR takes into account factors such as expected points, division of credit for a play, a "clutch" index, plus other in game situations not easily represented by basic football statistics (Oliver, 2011). The same methodology in this paper could be used to check the linear dependence of the first quarter performance on the fourth quarter performance using Total QBR as the measure of performance. If the results lead to the β_{1i} coefficients converging to zero, then the intercept estimate, the β_{0i}, would fully summarize the quarterback's performance and when plotted against the quarterbacks end of year Total QBR it should show a positive correlation, implying that ESPN's Total QBR is a statistic that properly quantifies a quarterback's performance. However, the reason this analysis has not been attempted in this paper is because the formula for computing each component has not been made public (Oliver, 2011). Making analysis on a quarter by quarter basis for each quarterback impossible.

Another possible solution to gain insight on the linear dependence of performance by quarter is to create a new statistic altogether. This statistic should be created in a way such that the β_{1i} is zero, i.e. that performances by quarter are independent of one another and the intercept estimate would be the predictor of quarterback performance. The overall goal being that when the intercept estimate is plotted against the individual's year end rating, it will show a positive correlation. While in principle this may seem like an easy task, the intangibles of the sport are incredibly difficult to quantify and therefore difficult to estimate and/or predict. Extensive time, along with trial and error, would be required to properly decide on the components needed to make up the statistic as well as weights for each component. With the amount of data continually produced and made available it is only a matter of time until more precise measures of player performance are created. It can be confidently stated that more sophisticated statistical techniques, similar to the methods mentioned in this paper, will continue to be used to gain more insight on players and their respective performance. Hopefully these potential insights will create better decision making on an organizational scale and thus leading to a even more exciting game for fans of the sport to watch.

Appendix

Table 1. First and Fourth Quarter NFL QB Ratings:

Quarterback	Game	First Qtr	Fourth Qtr
P. Manning	1	73.6	75
	2	148.4	112.5
	3	94.9	101.7
	4	112	150.4
	5	113.8	85.7
	6	109.6	93.2
	7	81.3	79.4
	8	92.8	40.3
	9	76	143.4
	10	141.7	42.6
	11	118.8	85.4
	12	116.7	98.8
	13	77.1	25
	14	119.8	56.6
A. Rodgers	1	82.3	125.2
	2	77.1	52.1
	3	108.9	95.1
	4	118.8	158.3
	5	158.3	93.8
	6	155.8	54.2
	7	126.5	113.9
	8	89.6	110.7
	9	130.9	78.1
	10	108.3	135.4
	11	29.5	46.5
	12	95.3	128.3

Quarterback	Game	First Qtr	Fourth Qtr
	13	2.1	94.3
T. Romo	1	58.8	107.7
	2	94.9	82.3
	3	99.2	139.6
	4	143.5	149
	5	87.2	84.4
	6	116.1	75.9
	7	131.3	113.3
	8	48.6	65.2
	9	119	62.5
	10	72.9	145.5
	11	103.6	47.9
	12	97.2	2.1
	13	104.2	145.4
	14	144.7	2.1
	15	158.3	41.3
A. Luck	1	31.3	86.1
	2	120.2	81.7
	3	119.9	149.6
	4	117.8	151.5
	5	30.6	126.4
	6	142.8	44.2
	7	79.9	132.9
	8	85.4	58.6
	9	64.6	52.1
	10	119.8	66.9
	11	70.1	68.8
	12	70.4	149.3

Quarterback	Game	First Qtr	Fourth Qtr
	13	30.5	70.4
	14	36.8	2.1
J. Cutler	1	153.3	47
	2	61.3	158.3
	3	109.7	77.8
	4	128.6	73.5
	5	94	36.6
	6	74.2	88.5
	7	86.8	54.6
	8	23.8	134.4
	9	24.8	3.8
	10	88.6	89.3
	11	63.9	49
	12	141.1	17.6
	13	74.7	97.9
	14	60	127.2
	15	81.7	47.2
D. Carr	1	126.3	96.1
	2	2.1	66.4
	3	75.3	14.9
	4	118.8	65.6
	5	52.8	96.7
	6	78.5	83.5
	7	24.2	106.5
	8	83.3	71
	9	64.2	42.4
	10	65.2	147.1
	11	50.7	83.6

Quarterback	Game	First Qtr	Fourth Qtr
	12	60.9	149.7
	13	83.3	141.3
	14	33.3	9.8
B. Roethlisberger	1	118.8	76.2
	2	88.1	37.1
	3	52.1	126.4
	4	117.9	84.9
	5	50.9	95.8
	6	107.6	84.7
	7	66.4	84.5
	8	104.6	138.9
	9	67.6	152.1
	10	69.8	143.9
	11	90.5	131.5
	12	58.8	104
	13	53.6	135.4
	14	118.8	108.8
	15	84.2	92.7
	16	57.2	158.3
T. Brady	1	88.3	33.5
	2	89.6	2.1
	3	52.1	68.1
	4	56.3	6.3
	5	158.3	90.3
	6	82.2	158.3
	7	149.3	116
	8	137	106.3
	9	83.7	120.8

Quarterback	Game	First Qtr	Fourth Qtr
	10	4.7	156.3
	11	108.5	79.6
	12	86.5	109.5
	13	70.8	158.3
	14	22.2	86
	15	78	13.1
R. Fitzpatrick	1	118.8	117.7
	2	0	72.2
	3	97	52.4
	4	7.8	105.4
	5	2.1	109.4
	6	97.2	123.5
	7	49.5	93.3
	8	85.4	61
	9	139	158.3
	10	68.8	105.2
M. Ryan	1	109.5	94.6
	2	70.1	65.6
	3	136.8	53.7
	4	95.3	83.6
	5	72.1	0
	6	83.6	81.7
	7	149.6	96.3
	8	72.2	126.2
	9	50	60.1
	10	103	85.4
	11	149	91.7
	12	114.2	129.2

Quarterback	Game	First Qtr	Fourth Qtr
	13	58.3	116
	14	100.8	94.2
	15	58.8	89.6
A. Smith	1	106	65.5
	2	62.5	89.6
	3	55.4	130.1
	4	86.8	150.7
	5	113.2	0
	6	71.9	104.7
	7	88	120.8
	8	152.1	59.2
	9	77.6	99.2
	10	92.5	42.4
	11	65.1	103.4
	12	28.1	123.6
	13	96.4	81.8
	14	113.5	20.4
	15	107.4	70.7
E. Manning	1	57.4	100
	2	64.6	27.9
	3	88.7	114.6
	4	149	75
	5	104.7	134.9
	6	129.2	84.7
	7	81.3	116.8
	8	43.2	116.9
	9	124	70.8
	10	139.2	23.3

Quarterback	Game	First Qtr	Fourth Qtr
	11	146.3	124.2
	12	97.9	74
	13	135.7	37.8
	14	108.1	130.1
	15	155.1	88.2
	16	94.4	89.8
K. Orton	1	80.8	121.4
	2	76.5	102.6
	3	87.5	120.4
	4	139	158.3
	5	116.3	80.9
	6	80.8	47.2
	7	154.2	118.8
	8	69.8	65.6
	9	76.4	76.6
	10	35.4	62.1
	11	88.1	113.5
	12	156.3	90.6
M. Stafford	1	149.3	112.5
	2	88.8	33.3
	3	19.3	93.8
	4	111.3	2.1
	5	129.2	97.9
	6	157.3	46.3
	7	41	101
	8	81.3	97.3
	9	116.3	98.4
	10	95.8	68.3

Quarterback	Game	First Qtr	Fourth Qtr
	11	71.7	31.7
	12	101.7	78
	13	124.6	119.2
	14	59.2	63.8
	15	87.5	53.8
	16	92.6	84.4
A. Dalton	1	92.2	189.6
	2	72.9	56.3
	3	117.4	116.7
	4	70.4	92.7
	5	92.6	141.1
	6	48.8	71.6
	7	210.4	61.3
	8	50	41.7
	9	23.3	4.6
	10	124.3	156.3
	11	94.9	73.4
	12	28.1	81.3
	13	87.5	39.6
	14	65.3	120.8
	15	47.3	50.7
	16	55.6	128.8
G. Smith	1	58.1	102.8
	2	141.1	46.4
	3	52.3	68.5
	4	72.9	55.3
	5	127.8	52.1
	6	104.5	99.3

Quarterback	Game	First Qtr	Fourth Qtr
	7	60.4	31.7
	8	142.1	51.2
	9	94.8	68.8
	10	34	86.6
	11	153.5	128.3
B. Bortles	1	98.1	89.6
	2	72.1	23.8
	3	122.2	133.3
	4	0	55.6
	5	63.3	158.3
	6	77.8	97.1
	7	92.7	59.8
	8	5	94.8
	9	104.2	89
	10	67.6	63.3
	11	77.1	25.5
	12	57.1	38.2
	13	89.2	55.9

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