INTERMEDIATION IN NETWORK ECONOMICS: THEORY AND APPLICATIONS

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DEDICATION PAGE

To my family, teachers and friends.

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ABSTRACT

The dissertation represents an attempt to study the role of intermediation in economic networks. Intermediation is widely observed in a variety of markets such as agriculture, transport, communication, international trade, and finance. Examples of intermediaries on networks include the traders in over the counter markets and dealers in artwork markets connecting buyers to sellers, not-for-profit associations connecting donors to recipients, and banks connecting lenders to borrowers. Such intermediaries often bring new trading opportunities between disconnected agents, potentially making markets more efficient than without intermediation. However, such intermediation may come at a cost that may include the intermediaries charging for their connecting abilities or the spreading of risks to other intermediaries and the whole system.

The main work includes three chapters that theoretically and empirically study the implications of intermediation in a variety of markets, such as the transmission of resources through networks and volatility spillover in the financial market. For markets of resource transmission, it studies equilibrium prices that intermediaries charge for the use of their connecting abilities, and conditions in the connecting abilities under which the cost of intermediation can be minimized. Such analysis is done in the complete and incomplete information setting with respect to the connecting abilities of intermediaries. The theoretical models illustrate how the network structure, the connecting abilities of intermediaries and information shape the outcomes of pricing mechanism in markets. This dissertation also provides an empirical study about how risk is spread in financial networks, especially for industry portfolios in the stock market. It shows that the network of industry portfolios' volatility evolves over time, and the financial industry is the main risk sender during the financial crisis of 2007 to 2009.

TABLE OF CONTENTS

1 THE INTERMEDIATION IN NETWORK ECONOMICS LITERATURE

While economic activities often involve the interactions between economic actors, it is only in the past two decades where network economics has been growing exponentially. Not only the social networks influence individual's economic behavior, but also the business and political interactions affect research and development, trading patterns and political alliances. There are growing research studies in the roles of networks in economic activity, and such studies fill gaps between local strategic behavior at the micro level and the outcomes at the macro level. Applying networks and tools in graph theory to characterize the interactions between economic participants, the literature on network economics in various contexts and applications has been exploded, so unifying the fragmented nature of this literature proves to be a time consuming but a worthy challenge. The following applications in fields of classic economics illustrate some nuances of how to model strategic interaction in network economics.

In classical economic research, we often assume the market is centralized, goods and services can be contracted between any pair of buyer and seller. All agents trade with each other at a common price. But in reality, individuals are not anonymous, typically able to trade only with a specific group of sellers. The bilateral relationships of trading are described by networks. For example, a manufacturer might only use raw materials from a few suppliers and compete with other manufacturers to sell the products to certain firms in downstream. The equilibrium prices depend on the interaction of global market and local trading relationships. Thus, there is a need to bring neoclassical economic paradigm to network analyses and develop a systemic understanding of exchange and trade on networks, including the effect of the network structure on pricing, surplus distribution, and efficiency. Jackson [83] and Jackson et al. [85] are good survey for researches in networks.

The questions then arise, how will local prices depend on network structure? When is does the law of one price hold? Is the outcome efficient? How will the price formation affect the payoffs and allocation? How will trading in networks evolve even if forming new links is costly?

Jackson [84] provides a good survey of research related to market on networks. Goyal [65] studies how the distributions of pricing are shaped by the network structure and price formation mechanism. Corominas-Bosch [47] studies the trade on networks with price formation via bargaining and provides a micro-foundation for the Walrasian benchmark. He finds that the law of one price happen only when local markets reflect the global balance of buyers and sellers. Corominas-Bosch [47] discusses the seller announce price to all connected buyers and Manea [99] studies the dynamic bargaining on random links. For a survey the research about network shaping bargaining, see Manea [101].

Besides the price formation through bargaining, results from auctions and posted prices are also influenced by a network. Kranton and Minehart [96] study markets of auctions with costly network formation, which captures characteristics of many industries, and surprisingly achieve efficiency.

Generally speaking, the research on network economics is very active. There is much work building new theoretical models to develop a systemic understanding of how network shape economic behaviors and contributing to applications in finance, international trade, labor market (more will be discussed below). On the other hand, there is a line of economic literature focusing on identifying the network effects to test the theoretical results.

Labor Economics

The labor market is another field where social network analysis plays an important role. Social networks are good channels to study information transmission, especially those through which firms avoid asymmetric information to save cost in screening. Networks also help the involuntarily unemployed worker to be employed again by obtaining the job information disseminated through social networks. The well-known observation in Granovetter [67] shows the strength of weak ties. Granovetter asked people how they found the job and how frequently they interacted with people who introduce the jobs to them. He defined the strong ties to be those who interacted at least twice per week, weak ties to be those who interacted less than once per year, and medium ties in between. More than half of jobs are referred by medium ties, and about 30% are introduced by weak ties. Weak ties play an important role in the transmission of job information.

Calvó-Armengol and Jackson [32, 33] study the labor market with social network and find there exists strategic complementarity in investment decisions on human capital for friends. Thus, the social network structure shapes the equilibrium wages distribution and employment rate on networks. It also gives individuals incentives to invest in human capital, like education, and build connections with friends and acquaintances of high human capital.

Game Theory

Starting from the seminal work by von Neumann and Morgenstern [134], game theory studies the strategic behavior of players. There are many decisions made by people influenced by the choice of friends or acquaintances, for instance, committing a crime, voting, job hunting, providing a public good, adopting a new technology. Jackson and Zenou [86] provide a summary of the literature analyzing games in which players are connected via a network structure, and the payoffs depend on the behaviors of neighbors. Though the games on network could be viewed as a special case for games in more general frameworks, there are interesting questions to ask, how will the equilibrium behavior of the game depend on the network structure of interactions and who is the most influential individual in a network where people choose their effort level depending on their neighbors.

For theoretical research, Bramoullé and Kranton [30] establish a common analytical framework for a wide class of games on networks. In particular, they provide game-theoretic foundations of network science notions including Bonacich centrality, maximal independent sets, and the lowest and largest eigenvalue, and study the effect of shocks to the system with heterogeneous players. There are still many open questions for theoretical research, including the existence of equilibria with large network effects, equilibrium multiplicity, and comparative statics on networks with strategic substitution.

There are also many applications for games on networks. One such question is related to the contagion on networks, Morris [106] studies the local structure for behaviors initially played by a finite number of players to spread to the whole population. Another question is related to finding the 'key player' in a network. Ballester et al. [11] study the noncooperative games with local payoff complementarities and characterize the key player with intercentrality measure based on his Bonacich centrality and his contribution to the centrality of others. More research on the key player could be applied to reduce crime by putting the head of criminals into prison.

Banerjee et al. [12] simulate the diffusion process of microfinance participation in a set of villages in rural India, and they find that patterns of microfinance are in line with peer effects. Based on that, they define the central player to be targeted to spread the information, which will maximize the participation. The results are closely related to the topics in seeding and targeting, to promote new technology adoption and local public good provision on a network[1]. Campbell [35] studies advertising with word-of-mouth communication between friends and finds the target for advertising are not the individuals with most friends.

As the advances in technology reduce the cost of communication, people become more connected to friends. It can be expected that the decisions will be increasingly influenced by neighbors. At the same time, people have freedom and face cost constraint in building a network, endogenous network formation is studied in Bala and Goyal [10]. R&D partnerships, such as joint R&D pacts and joint development agreements, is a good application for network formation, which has become

[1] Bloch [24] is a good survey for targeting and seeding problem on networks.

prevalent modes of inter-firm collaborations. Understanding how people are connected and the strategic behavior facing exogenous shock, like the advertisement, technology adoption and arrest of criminals can have important implications for policy. In the end, information diffusion and social learning on networks is another interesting topic related to games on networks.

Finance

The financial crisis is often attributed to the high connectedness of the financial system, the links between financial institutes are viewed as channels for propagation and amplification of shocks. The interlinkages between financial parties are complicated, many research study financial network from various perspectives, including the relationships of cross-shareholdings, counterparty risk, or liability.

As mentioned in Jackson [82], there is growing research in financial networks and macroeconomic performance. The interdependence of economic actors is necessary to construct a good understanding of the transmission of shocks, and the interdependence is naturally a network problem. As growing tools of network economic emerge, the gap between theoretical research about games on network and applications in finance has to be filled.

There are many interesting questions in the field of financial network. Is a financial institution too big to fail or too connected to fail? How to measure the interactions and counterparty risk between financial institutions to investigate the indirect effect of risk transmission? Which network structure is more stable against the risk propagation? What is a reasonable standard to regulate financial institutions to reduce systemic risk in a highly connected economy? On the contrary, what is the influence of globalization on economic fluctuation within certain industry or region? What are the patterns of financial network evolution?

This is an area in which network economics have many applications and provide policy suggestions for regulators. It is a promising area for future work to provide a measurement for financial networks, since the relationships between financial institutions contain multiple levels, including debt, input-output production, or even CEO's social network, cross-shareholding. The financial institutions adopt strategic behaviors for linkage formation in the longer horizon, thus, there is a need for researching the evolution of financial network structure and the influence on shocks transmission. Moreover, there is co-evolution of networks and behaviors. On one hand, the risk exposure relationships influence the decisions about investment and monitor. On the other hand, the investment strategies shape the risk exposure network. These co-dependencies could have broad policy implications.

Glasserman and Young [63] is a good survey about financial network and contagion. It covers both theoretical and empirical work about the interconnectedness of financial system and discusses how the network structure interacts with other factors, such as leverage, or size and its contribution to the susceptibility of the system to contagion. The influence of connectedness seems ambiguous, the links are a channel to diversify risk, but also increase the risk exposure to others, and face higher risk spillover from other financial partners. Elliott et al. [55] works on micro foundations for financial contagion with links as cross-shareholdings across institutions. It finds networks with an intermediate of diversification and integration can be the most problematic. The theoretical results about the influence of network structure on the propagation of shocks depend on assumptions about interactions between financial institutions. Acemoglu et al. [2] summarize seemingly contradictory results in the literature and develop a unified framework, based on games over the network, to study how the network interactions can be a channel for the propagation of shocks. They characterize the structure of equilibrium outcome and rank networks in terms of translating microeconomic shocks into macroeconomics.

There is also empirical work focusing on the structure of financial networks, including test the core-periphery networks and estimating networks about exposures between financial institutions.

Besides research about systemic risk based on networks between financial institutions, many finance research study the influence of CEO's social network on salary, the performance of the company, liability relationships, and investment decisions. The endogenous network of trading relationships in over the counter market is discussed in Babus and Hu [9].

International Trade

Trade occurs between firms within countries and across borders. International trade is another understudied field, in which the network of relationships and externalities play an important role. First, the complex structure of international trade can be intrinsically represented by a network, countries connected to each other by trading, firms are linked to other firms in foreign countries through import and export. The buyer-supplier relationships between firms have large impacts on production chain. The significant externalities and interdependencies across relationships in international trade is a perfect field for the application of network economics. At the same time, most records of international transactions are available for the empirical study of large-scale networks. The growing domestic and international transaction data inspire the development of new theories based on heterogeneous firms as both buyers and suppliers in production networks.

Chaney [40] is a recent survey of networks on international trade that summarizes three applications of network economics in trade. First, the information diffusion on social networks as a way for price discovering to be applied to study information frictions for trade between parties in different geographic regions. Chaney [39] study the trade frictions by building dynamic network formation what the firms actively searching for foreign partners, and apply French data to verify the predictions. Second, the ethnic networks of migrants are applied to explain the patterns of international trade through enforcement of contracts. The ethnic networks could be viewed as punishment mechanism on the default behavior. Finally, since most firms face a relatively small number of upstream suppliers and downstream customers, the trading relationships as the input-output network can partition the markets to firms only competing with peers. Bernard and Moxnes [19] reviews the literature on firm-to-firm connections in trade, which study the theoretical works based on a framework of many-to-many matching, and discuss both static and dynamic matching.

Unlike most traditional economic research that focuses on the study of markets with homogeneous participants or bilateral trading with two participants, international trade requires theoretical models beyond a two-at-a-time or market approach, given the complex relationships and large externalities. Country's decision about terms of trade depends heavily on the behaviors of other potential partners. For instance, a country can negotiate with multiple partners and consider all the available offers at the same time. Trade between two countries heavily depends on trade with other parties. Moreover, many negotiations relate to multiple goods. The problems for trade on firm-to-firm is similar with trade among countries. The application of network in international trade requires new models that capture the incentives for players and then use rich data to track networks of trade over time to test theoretical predictions.

The literature on trade and production networks is still preliminary, and there are many theoretical and empirical questions to be answered.

Macroeconomics

As mentioned above, network economics offers a way to study how microeconomic shocks can be transmitted via production network and propagate throughout the economic system, and establish a connection between local behavior and aggregate outcome. The intuition behind is that a big firm can be seen as the hub of the network and transmits the shock to other economic actors because of its high degree of connections and bridging role to bring others closer. Production networks provide a theoretical foundation for micro shocks to propagate across the real economic system through input linkages.

Carvalho [38] is a good survey and introduction about research related to production networks. Acemoglu et al. [1] provide a framework to illustrate how local sector-specific shocks may be amplified by the production network structure to generate large-scale aggregate fluctuations. Beyond the connection between sectors, the input linkages of firm-level are more complicated, since the decisions of firms depend on the competition with peers, the strategic substitution with supplier and buyers and the investment of capital as a sunk cost.

Empirical studies about how price pass-through using firm-to-firm data is a good application for network economics. The problem of link formation in production networks is also worth studying since the input-output flow is an endogenous outcome of strategic behavior of firms.

Comparing to social network research dealing with risk spillover in the channel of information diffusion and strategic behaviors, production network research builds the connection of information based micro problem with the macro issue of production. The comovement of information diffusion network and production network is another area to work on.

To summarize, economic activities take place at the intersection of global environment and local partners. The interaction relationships are key to understand many macro phenomena from micro behavior, which also comes up in classical work of Schelling [127].

1.1 Intermediation in Networks

As the networks describe relationships between agents, it is common to observe that some agents are not directly engaged in economic activity, but engaged indirectly with intermediaries in between. Intermediation is commonly observed in many areas of modern economics, including agriculture, international trade, finance, social learning, and also transport. For instance, financial brokers buy and resell in over the counter markets through networks of intermediaries. In supply chains, firms buy inputs, transform and resell intermediate goods to downstream companies. The underground trade of drugs is intermediated by many local dealers. Transporting good across borders via bribing for access, the local officers are intermediaries for transportation. This dissertation is an attempt to study the role of intermediation in network economics. It will study three issues related to the transmission of resource/risk in networks. For the first two issues, a novel problem is introduced that captures the transmission of the resource from a lanner to agents via intermediaries through links with various quality. A study is made that encompass a complete and an incomplete information setting. In particular, the first issue relates to the pricing strategies of intermediaries in competitive markets who try to maximize the price paid for the use of their connections (links). In such a case, the cost of intermediation may be high, and intermediaries may be able to fully extract the surplus. This section finds the sufficient and necessary condition for perfect competition between intermediaries, the existence of equilibrium, refinement of equilibrium and uniqueness of the equilibria found.

The second issue expands on the model developed on the first issue and focuses on the case of incomplete information about the connections of intermediaries. It discusses mechanisms on the environment where intermediaries report their connections and punishment for misreporting is possible. It characterizes the class of incentive compatible mechanisms (here interpreted as strategic-proofness) with a general class of punishment functions and discovers the minimal punishment function to make a given mechanism incentive compatible.

The third topic applies concepts of intermediation to financial networks. The network structure of industry portfolio risk-spillover changes over time. Both the total connectedness and centrality measurement are indicators of systemic stability against shocks.

These three issues altogether cover topics including intermediation, quality of links in resource transmission problems, and the network structure of volatility risk spillover in financial markets. This dissertation aims to explore these three topics by adopting game theoretical methods, mechanism design methods, and empirical methodology with time series techniques.

1.1.1 Game Theoretical Approach

Starting from von Neumann and Morgenstern [135], game theory becomes a commonly used tool to analyze the strategic behaviors of players, such as firms, individuals. Then definitions of equilibrium are introduced to predict the decisions made by players in non-cooperative game. Nash [117] introduces Nash equilibrium as a solution for non-cooperative game, in which players has no incentive to deviate to other strategies given others' strategies. Selten [128] studies the extensive form game and finds the refinement of equilibrium with subgame perfection. Harsanyi [73] discusses the game with incomplete information and refines Nash equilibrium with Bayesian Nash equilibrium. Gul [69] is a good summary for the contributions of John Nash, John Harsanyi and Reinhard Selten, who won the Nobel prize in 1994. Gul [69] also summarizes many applications of game theory, including design of auctions and topics in Industry Organization.

For the problem of intermediation on resource transmission network, Chapter 2 asks: what the strategy of pricing for intermediaries in the equilibrium looks like? When is there an equilibrium in which the planner can transmit the resource in a way maximizing its utility? Is there any way to refine the equilibrium? Chapter 2 builds a game theoretical model to study pricing strategies of the intermediaries, when the network structures are complete information. The intermediaries post prices to charge for using their ability to transmit the resource, and then the planner chooses intermediaries to pay and transmits the rest of resource. We prove the existence of subgame perfect Nash equilibrium, and study the conditions for equilibrium with perfect competition among intermediaries.

There are many applications for the theoretical model, including resource transmission with fixed proportional constraint, capacity constraint, unit capacities, and minimal cost spanning trees.

1.1.2 Mechanism Design Approach

Game theory study players' strategic behaviors given the rules of the games, while mechanism design[2] discusses various game structures according to designer's interest in the game's outcome based on the information of individuals. When the individuals' private information and actions are hard to monitor, the mechanism needs to give players the incentive to share their true information, and exert efforts according to their report.

The research about mechanism design approach starts from Hurwicz [81], who introduces the notion of incentive compatibility. In the 1970s, the revelation principle simplifies the analysis of mechanism design to focus on direct mechanisms, which provide individuals incentives to report truthfully about their private information. The revelation principle does not deal with the problem of multiple equilibria in implementation, Maskin [104] solves this problem and finds mechanisms with all equilibrium outcomes to be optimal. Myerson [115] applies the revelation principle to auction theory. The development of mechanism design theory has deepened our understanding of the optimal mechanism for allocation of resource, taking the individuals' private information into consideration.

For the problem of intermediation on resource transmission network, rather than asking how the intermediaries would compete in pricing for usage of their links as in the complete information environment of Chapter 2, Chapter 3 asks a related question: what mechanism provides incentives for intermediaries to report the truth if the quality of intermediation is private information? In the mechanism design approach, the intermediary reports its quality of intermediation, the planner chooses a rule for allocating resource among intermediaries, and the sharing rates from intermediaries to agents based on the report. The intermediaries will get punished if they are found to lie in their reports.

Chapter 3 uses this approach to investigate the strategy-proof, symmetric, budget balance mechanisms for the resource transmission problem on networks. It shows the allocation rule depends only on the aggregate quality of intermediation. Moreover, the class of strategy-proof mechanisms expands when the punishment increases. The minimal punishment for a mechanism to be strategy-proof given the punishment is discussed. Parallel to the Myerson–Satterthwaite theorem in Myerson and Satterthwaite [116], which finds there exists no mechanism which satisfying individual rationality, budget balance, incentive compatibility and ex-post efficiency, the result in this chapter shows there exists no strategy-proof, budget balance, symmetric and first-best efficient mechanism, and finds the punishment function to achieve first-best efficiency, strategy-proof with punishment and budget balance.

[2] For a longer introduction about mechanism design theory, see the document of Nobel prize committee [44].

1.1.3 Applications

Besides the intermediation for resource transmission on network above, there are many applications of intermediation for trade in over the counter market and liquidity provisions among banks. Both networks have core-periphery structure, in which the agents in the core are connected with each other, and agents on the periphery only link to few of the core agents without connection between each other. The core-periphery network is formed endogenously as the intermediation of core agents reduce the transaction cost. Similarly, at the industry level, the financial industry, which borrows money from agents who want to save and lends to firms that need resources for investment. Financial services are intermediaries that connect others and bring more efficient allocation of resources. On the other hand, the linkages between the industries may also increase the risk exposure, and result in vulnerability to shocks.

The network structure of risk exposure among industries develop over time, as such, it is worth studying the influences of the financial industry on the risk spillover in the stock market, comparing to the role of intermediation in services of a real economy.

Chapter 4 estimates the industry portfolios' volatility connectedness and applies network economics notions to estimate the risk of spillover across industries. It finds that the network structure of volatility spillover changes over time. Finance is the central industry, which transmits the highest risk to other industries, during the crisis in 2007-2009. But finance is not the largest risk sender in other periods. IT related industries grow in the past two decades, and become the center node of volatility spillover network.

2 INTERMEDIATION FREE EQUILIBRIUM IN RESOURCE TRANSMISSION GAMES

2.1 Introduction

There are many markets for which intermediaries play an essential role. The most common markets for which intermediaries are critical include the transmission of goods and resources to agents. For instance, the allocation of government resources to agents often require the use of private for-profit companies, called intermediaries, that are more closely connected to the agents than the government agency. Intermediaries therefore enable the government agency to more effectively target their agents. This top-down structure provides opportunity for competition between intermediaries with the potential for added benefits. Such benefits, however, are largely dependent on the way in which the intermediaries transmit the resources to the agents and the type of resource (i.e., divisible vs indivisible) that is to be distributed.

Although much attention has been paid to the case of intermediation for indivisible goods, few studies focus on intermediation for divisible goods and resources. Herein, we introduce a general model of intermediation where a planner is interested in transmitting a divisible resource to agents (such as money). Although the planner is not directly linked to the agents, it can do so via a group of intermediaries. Different groups of intermediaries have the ability to transmit different allocations of the goods to the agents. Thus, groups of intermediaries differ not only on the agents they can reach, but also the quality of their intermediation. For instance, two groups of intermediaries who can reach the same agents may be very different from the planner's perspective, since they may transmit different amounts to the agents.

We focus on the case where intermediaries are private and independent entities that can charge the planner for access to their agents. While intermediaries care about maximizing the amount paid by the planner, the planner has preferences over the different allocations of the resource to the agents as well as the total amount paid to the intermediaries employed.

We study the case of complete information where the planner and intermediaries are aware of the preferences of the planner as well as intermediaries' abilities to transmit the resource. The planner solicits bids from intermediaries to access their abilities and select a group of intermediaries to

[0]Joint work with Ruben Juarez

contract for the transmission of the resource. In order to model the behavior of the planner and intermediaries, we used a game theoretical approach.

In the first stage of our game, intermediaries independently and simultaneously report their fees for providing the planner with access to their agents. The fees might affect the transmission of the resource. In the second stage, the planner selects the intermediaries and feasible amounts of the resource allocated to each of them for transmission to agents. The intermediaries who are not selected do not get paid. The ultimate goal of the intermediary is to be contracted and maximize the price paid by the planner. The goal of the planner is to distribute as much resource to the agents in a way that maximizes his preferences. We use a subgame perfect Nash equilibrium (SPNE) to describe the result of the strategic behavior between the planner and the intermediaries. The equilibrium price of intermediaries depends on the utility function of the planner and the abilities of the intermediaries to transmit the resource to the agents.

An important challenge is to identify the necessary conditions for existence of a perfectly competitive equilibrium, where the intermediaries used by the planner earn zero profit. In particular, our first equilibrium concept, the *intermediation free equilibrium (IFE)* is a SPNE where the intermediaries used by the planner charge zero price. This equilibrium does not preclude the intermediaries who are not used by the planner to post a positive price. However, at an IFE, all intermediaries regardless of whether they are used by the planner earn zero profit. Thus, an IFE resembles a competitive equilibrium where the planner is directly transmitting the resource to the agents as if there are no intermediaries.

Even when an IFE exists, other SPNEs may also exist. This multiplicity of equilibrium is undesirable as it decreases the predictive power of equilibrium. We introduce a second refinement of the SPNE, the *robust SPNE*, where the group of intermediaries who are not selected by the planner price at zero. In particular, a robust SPNE is a refinement of a collusion-proof Nash equilibrium for the intermediaries who are not selected by the planner. One can imagine that if intermediaries are not selected, then they have the incentive to undercut their prices (individually or in groups) trying to get selected. Thus, a robust SPNE prevents group manipulation by the intermediaries who are not selected.[1] Note that there is a unique IFE that is robust; it requires all intermediaries to price at zero regardless of whether they are used by the planner.

The main contributions of the paper are two-fold. First, we provide the necessary and sufficient conditions on the utility function of the planner and the abilities of the intermediaries that guarantee the existence of an IFE. Second, we provide the necessary and sufficient conditions for the vector

[1]This does not prevent group manipulations by individuals who are selected by the planner. In fact, it is easy to see that a full coalition-proof Nash equilibrium does not exists for almost any intermediation problem.

of zero prices to be the unique robust SPNE. Our work is the first paper in the literature that works for a wide variety of planner's preferences and is able to encompass a large class of intermediation settings.

2.1.1 Overview of the Results

We describe the main results of our paper using a simple yet illustrative example. Consider a planner who is endowed with I units of a resource and seeks to transfer as much resource to two agents. His preferences over the allocation of the resource (x_1, x_2) are given by a perfect complement utility function $u(x_1, x_2) = \min\{x_1, x_2\}$. While the planner cannot directly connect to the agents, he can do so using a group of intermediaries. The intermediaries vary on their ability to transmit the resource to the agents. These differences come from the group of intermediaries selected as well as the price paid to them. This variation is captured by an *outcome possibility function* (OPF) F that assigns to every group of intermediaries and prices a set of potential outcomes available for the planner to select from. For this example, assume there are three intermediaries, and given the vector of prices $p = (p_1, p_2, p_3)$ of the intermediaries the OPF F is given by

$$F(\{1\}, p) = \{(x, 0) | 0 \leq x \leq \frac{6}{5}(I - p_1)\},$$

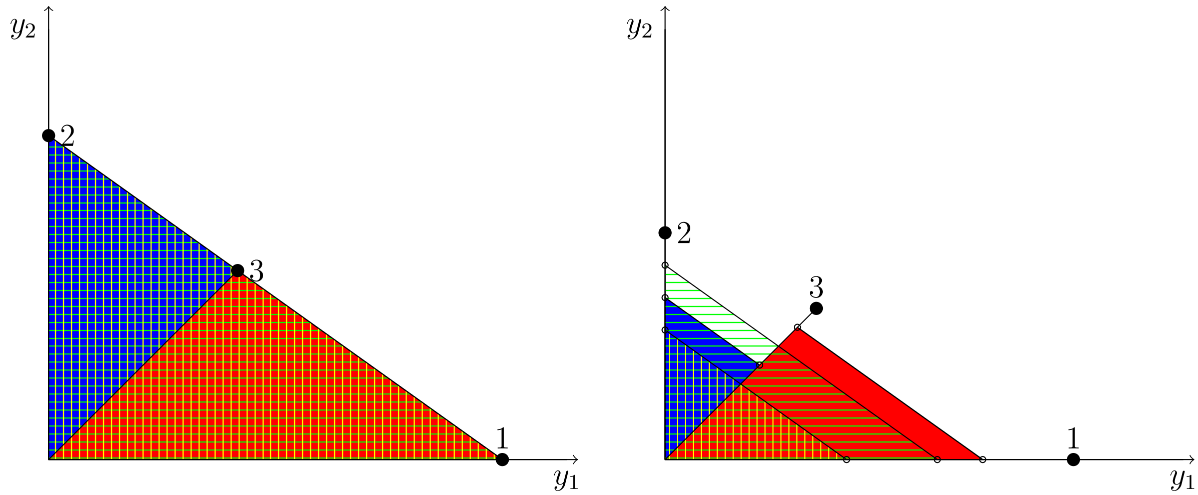
$$F(\{2\}, p) = \{(0, x) | 0 \leq x \leq \frac{6}{7}(I - p_2)\},$$

$$F(\{3\}, p) = \{(x, x) | 0 \leq x \leq \frac{I - p_3}{2}\},$$

$$F(S, p) = conv(\cup_{n \in S} F(\{n\}, (\sum_{i \in S} p_i) e^n)) \text{ for } S \subset \{1, 2, 3\},$$

where $conv$ is the convex hull of the sets and $e^n \in \mathbb{R}^3$ is the vector equal to 1 on the n-th coordinate and zero otherwise.

Thus, the outcome possibility function is such that intermediary 1 can only transmit the resource to agent 1 and intermediary 2 can only transmit the resource to agent 2. On the other hand, intermediary 3 can transmit the resource to agents 1 and 2, but it can only do so in equal proportions. Moreover, every unit of money sent to intermediary 1 is increased by 20%, whereas every unit sent to intermediary 2 is decreased by $\frac{1}{7}$. The ability of groups of intermediaries to transfer the resource is just the convex combination of the abilities of individual intermediaries at the prices of the entire group.



(a) Vector of prices $p = (0, 0, 0)$ (b) Vector of prices $p = (0.5, 1.5, 1)$

Figure 2.1: (a) and (b) illustrate the outcome possibility function F when $I = 5$ and prices are $p = (0, 0, 0)$ and $p = (0.5, 1.5, 1)$, respectively. The sets $F(\{1\}, p)$, $F(\{2\}, p)$ and $F(\{3\}, p)$ correspond to the line connecting the origin with the points 1, 2 and 3, respectively. The sets $F(\{2, 3\}, p)$, $F(\{1, 3\}, p)$, $F(\{1, 2\}, p)$ and $F(\{1, 2, 3\}, p)$ correspond to the shaded areas in blue, red, green/horizontal-lines and yellow/vertical-lines, respectively.

When the intermediaries price at zero, the planner can transmit the resource and achieve his maximal utility by using three potential groups. The planner can use intermediaries 1 and 2, and transmit $\frac{5I}{12}$ and $\frac{7I}{12}$ via intermediaries 1 and 2, respectively. The final allocation to the agents is $(\frac{I}{2}, \frac{I}{2})$. Alternatively, the planner can allocate all the resource to intermediary 3, and the agents will also receive the same allocation $(\frac{I}{2}, \frac{I}{2})$. Moreover, the planner can also use intermediaries 1, 2 and 3 to transmit the resource with maximal utility by selecting any convex combination of the above.

Now, assume that the intermediaries post prices for the use of their OPF, then the planner chooses a group of intermediaries and transmit the resource to the agents. Let (p_1, p_2, p_3) be the vector of prices, where p_n is the price that intermediary n reports. In this two stage dynamic game, there are two types of SPNEs. The first equilibrium prices $(0, 0, 0)$ is the *intermediation free equilibrium (IFE)*, where the planner is able to transmit the resource as if there are no intermediaries. That is, at the IFE, the planner fully transmits the resource to the agents without any amount paid to the intermediaries. This is an equilibrium because if an intermediary who is used by the planner increases its price above zero, then the planner will not select it, as it can use another group of intermediaries to transmit the resource more efficiently.

The second type of SPNE is a "planner-inefficient" equilibrium, (p_1, p_2, p_3), where $p_1 \geq I$, $p_2 \geq I$ and $p_3 = I$. In this equilibrium, the planner pays intermediary 3 an amount equal to I and transmits no resource to the agents. This is an equilibrium because neither intermediary 1 or 2 can decrease its price to undercut intermediary 3. Intermediary 3 has no incentive to decrease its price because it is being selected.

Two results of our study relate to the existence of an IFE, where the intermediaries used charge zero price at equilibrium. Theorem 1 shows that an IFE exists if and only if the intersection of the utility maximizing groups at prices $(0, 0, 0)$ (in our example above $\{1, 2\}$, $\{3\}$ and $\{1, 2, 3\}$) is empty.

For the second result, we introduce the *robust SPNE*, where the group of intermediaries who are not selected price at zero. In our example, $(0, 0, 0)$ is the unique robust SPNE, since in the second type of SPNE intermediaries 1 and 2 charge positive prices. Theorem 2 shows that if the problem (u, F) is monotonic and cross-monotonic,[2] then $(0, 0, 0)$ is the unique robust SPNE if and only if the intersection of the utility maximizing groups at prices $(0, 0, 0)$ is empty.

The paper also discusses specific classes of outcome possibility functions that guarantee the existence of IFE and uniqueness of a robust SPNE. In particular, Corollary 1 shows that either by replicating the existing intermediaries and their production functions, or by finding groups of intermediaries who perfectly complement in the OPF, will result in the existence of an IFE and unique robust SPNE.

Our work is the first paper in the literature that works for a wide variety of planner's preferences and is able to encompass a large class of intermediation settings where the abilities of groups of intermediaries to transmit the resource to agents can be represented by an OPF, including the four applications below.

2.1.2 Applications

The generality of our study provides a unified framework for the study of different literatures that seem disconnected, ranging from resource allocation problems in networks to minimal cost spanning tree models. Herein, we briefly discuss some of these applications, while Section 2.5 discusses the technical details.

Resource Transmission in a Network under Fixed Proportional Constraints: Consider the case where a planner is interested in transmitting a divisible resource to agents (such as money). The planner has preferences over the different allocations of the resource to the agents. The planner can reach the agents via a group of intermediaries that may differ in the *types* of agents they can reach as well as the *quality* in which they can reach the agents. The types of agents that intermediaries reach are represented by a network. The quality in which intermediaries reach the

[2] Monotonicity occurs, for instance, when the planner is strictly worse-off as all prices increase. Cross-monotonic preferences includes the case of homothetic preferences in prices, as well as a variety of other weaker conditions.

agents can be interpreted as the effective transmission of the resource from the intermediaries to the agents. This is represented by the total amount of the resource that an intermediary sends to the agents per unit of resource received, as well as by the proportions in which every agent receives a resource relative to another from a given intermediary.

This model can be applied to the transmission of advertising money in companies. A company looking to promote their product can use different media (the intermediaries) to reach the advertising target of their product; such intermediaries include TV channels, radio stations, Internet websites, newspapers, etc. The quality of the connections is important because, within the media, there are different channels that target to specific demographics of agents and may influence the planner's objective differently. Alternatively, this model can incorporate the allocation of government's money to people in need via charities. The government may decide to send the money via charities that will charge an indirect cost for the use of their services. The connections of the charities as well as their quality are exogenous information that the planner cannot control, and they are typically taken into account when making a decision on how to allocate the resources.

Resource Transmission in Networks under Unit-Capacities: Consider a planner interested in distributing a fix amount of a divisible resource to agents via a set of links owned by intermediaries. Multiple layers of intermediation are possible, and thus the planner might need to contract more than one intermediary to reach an agent. We assume that links have unit capacities, which decrease the amount transmitted to the agents by the product of the capacity of the links used.

A particular case of this problem occurs when intermediaries are directly connected to agents and have 'waste-constraints' where intermediaries are directly connected to a subset of agents but only transmit a portion of the amount sent through them. Such is the case of universities or charities, where an overhead cost is charged for every dollar sent to them, and the planner can choose where every charity spends the resources —unlike in the case of proportional constraints, where the charities have exogenous priorities. The problem can also be applied to more complex layers of intermediation arising in network flow problems. For instance, when there is ground water that must be distributed to agents via private canals (intermediaries) that have an evaporation loss or other conveyance losses[3] that are proportional to the amount of water transmitted and might be different across canals. The owners of the canals may charge the planner for the use of their canals, and therefore the planner should consider the trade-offs between allocating resources to cheap canals with high conveyance losses as opposed to more efficient but relatively more expensive canals.

[3]Conveyance losses are typical in these models, and typically depend as a proportion of the length of the canal and structure.

Resource Transmission under Capacity Constraints: Consider a planner interesting in distributing a fix amount of good to agents via a network of intermediaries. Intermediaries, who own the links, are constrained by the capacity of every link. Multiple layers of intermediation are possible, and thus the planner might need to contract more than one intermediary to reach an agent.

This model can be applied to the distribution of resources when natural disasters occur. For instance, an organization interested in transmitting the resources to regions in need may be faced with transportation capacities (such as cargo in ships and planes). Multiple layers of intermediation might be required as goods sent to remote regions might require more than one mode of transportation. This model also has applications to the transmission of data in the internet. Data transmitted in networks often goes through intermediaries which charge for the use of their links. These links are often capacity constrained and might require the user of the link to pay in order for the goods to flow in the network.[4]

Minimal Cost Trees and Related Models: The generality of our model also encompasses problems of network building that might not be explicitly used for the transmission of a divisible good. Such is the case for a planner seeking to build a minimum cost spanning tree that connect agents (nodes) using links in a network owned by intermediaries. When intermediaries post prices for the use of their links, the planner can choose any set that connects the agents at the minimal cost. Applications of this model include that construction of electricity and water networks.

2.1.3 Related Literature

The allocation of divisible resources has been prolific, especially in the network literature (see, Jackson [83] for the most comprehensive survey in networks). This includes Hougaard, et al. [76–80], Moulin [110, 111], Moulin et al. [112], Bochet et al. [28] and Juarez et al. [88, 89, 91–93]. However, we study the problem of transmitting a divisible good in networks with intermediaries, which not surprisingly creates substantial differences in the equilibria, strategies and difficulty of the model. A closely related paper is Moulin and Velez [114], which study the price of imperfect competition for the problem of spanning tree. Related results to the spanning tree model are specifically covered in Section 2.5.4, but our equilibrium results have broader applicability, mainly due to the generality on the abilities of the intermediaries (such as connections in the networks as well as quality of the connections) and utility function of the planner. Our companion paper, Han and Juarez [72], studies the transmission of a divisible resource when the abilities of the intermediaries are unknown to the planner. It characterizes a large class of strategy-proof mechanisms when the

[4] This can be seen in the recent dispute between Time Warner Cable Company (TWC) vs Netflix and other streaming devices, where TWC was interested in controlling the quality of streaming movies due to capacity constrains on its network. A recent agreement on the payment by Netflix to TWC has been reached.

planner elicits the intermediaries' abilities to transmit the resource to agents. Our work in this paper focuses in a more general game of complete information that includes the model from Han and Juarez [72] as a particular case.

There is also a large and growing literature in the transmission of indivisible goods and services with intermediaries. Condorelli and Galeotti [46] survey strategic models of intermediation in network. Manea [100] study dynamic game on bilateral bargaining in network with intermediation, Siedlarek [129] study a stochastic model of multilateral bargaining in a market with competition on different routes through the network. Kotowski and Leister [95] study intermediary traders in network with an auction mechanisms to set prices and analyze the welfare implications of stable and equilibrium networks. Blume et al. [27] study the effects of intermediation in markets with posted prices. Gale and Kariv [61] study a market with intermediaries and discover that the pricing behavior converges to the competitive equilibrium in an experiment. Choi, Galeotti and Goyal [42] study, theoretically and experimentally, pricing in complex structures of intermediation. In particular, their theoretical result can be obtained as a particular case of the results in Sections 2.5.2, 2.5.3 or 2.5.4.

Competition and pricing in networks has also been studied. For instance, Bloch [25] surveys targeting and pricing in social networks. Bloch and Querou [26] study the monopoly pricing in social networks with consumer externalities. Campbell [34] studies monopoly targeting and pricing with communication in the network of consumers. Chawla and Roughgarden [41] study the price of anarchy and price of stability in network pricing game, in which the sellers of links have price competition facing the demand of consumers. Goyal, Heidari and Kearns [66] study the competition between firms seeking the adoption of products by consumers in social network and find bounded price of anarchy under the property of decreasing returns to local adoption. Our paper is related to this literature as we study the competition behavior among the intermediaries in a targeting problem, including network settings. However, in our resource transmission problem, the intermediaries may have different quality of transmission of the resource, which generates substantially more difficulties in both the existence and computability of equilibrium.

Our model generalizes the classical Bertrand [20] price competition model and equilibrium in two dimensions. First, groups of intermediaries might have different abilities, which can be used to differentiate from other intermediaries. Second, the planner not only cares about the price paid to the intermediaries, but also about the quality of transmission of the resource. These differences generate substantial challenges regarding the existence of equilibrium, especially since our game is discontinuous on the strategy of the intermediaries. Simon and Zame [130] prove the existence of (mixed-strategy) Nash equilibrium in discontinuous games, including the Bertrand competition game, when the sharing rule is endogenous. Reny [125] proves the existence of a pure-strategy

Nash equilibrium in compact, quasi-concave and better-reply secure games. More recently, Bich and Laraki [21] extend Reny's work to obtain tighter conditions for the existence of approximate equilibria. They also show that many sharing rules, especially related to competition models like this paper, generate pure and mixed-strategy equilibria. Reny's result is used to prove the existence of equilibria in our setting.

2.1.4 Roadmap

Section 2.2 introduces the intermediation problem and the resource transmission game. Section 2.3 studies the sufficient and necessary conditions for the existence of an intermediation free e-quilibrium and uniqueness of a robust subgame perfect Nash equilibrium. Section 2.4 studies conditions on the OPFs that guarantee the existence of an intermediation free equilibrium. Section 2.5 discusses four applications and Section 2.6 concludes. All proofs are in the appendix.

2.2 The Model

Let $A = \mathbb{R}_+^M$ be the set of feasible outcomes, herein interpreted as the potential allocations of resource to the agents in M. The planner is interested in choosing one of these outcomes but cannot directly select it. Instead, a group of $\mathcal{N} = \{1, \ldots, N\}$ intermediaries are able to access subsets of the outcomes and set fixed prices $p = (p_1, \ldots, p_N)$ for the use of their ability. Given a group of intermediaries $S \subset \mathcal{N}$, the aggregate price of group S is denoted by $p_S = \sum_{n \in S} p_n$, and the projection of the vector of prices p over $\mathbb{R}_+^{|S|}$ is denoted by $p_{[S]} \in \mathbb{R}_+^{|S|}$. For simplicity, we denote $p_{-n} = p_{[\mathcal{N} \setminus \{n\}]}$. Given prices p and p', we say that $p' < p$ if $p'_i < p_i$ for all $i \in \mathcal{N}$. One important price vector is the vector of zero prices $\mathbf{0} = (0, \ldots, 0) \in \mathbb{R}_+^N$. In order to avoid confusion, we reserve the vector $(0, \ldots, 0) \in \mathbb{R}_+^M$ to represent the allocation of the agents, whereas $\mathbf{0}$ represents the vector of zero prices.

Definition 1 (Intermediation Problem)
An intermediation problem is a pair of functions (u, F) such that:

- $u : A \times \mathbb{R}_+ \to \mathbb{R}$ *represents the* ***planner's preferences*** *over the chosen outcome, as well as the aggregate price paid to the intermediaries chosen. We assume that u is continuous, monotonic in A and non-increasing in $\mathbb{R}_+$*[5]
- $F : 2^{\mathcal{N}} \times \mathbb{R}_+^N \to 2^A$ *is an* ***outcome possibility function*** *(OPF) that assigns to every group of intermediaries and vector of prices a set of potential outcomes. We assume that F satisfies the following conditions:*

[5]The utility function u is monotonic if for any $x > y$ and t we have that $u(x, t) > u(y, t)$. For the sake of brevity, we omit the other standard definitions for utility functions, but we follow standard definitions from Mas-Colell et al. [103], Chapter 3.

a. $F(S,p)$ *is a compact set for any group* $S \in 2^{\mathcal{N}}$ *and vector of prices* $p \in \mathbb{R}^N_+$*. Furthermore,* $F(\emptyset, \mathbf{0}) = \{(0,\ldots,0)\}$

b. $F(S,p)$ *is continuous in the vector of prices* p *for any group* $S \in 2^{\mathcal{N}}$[6]

c. F *is non-decreasing in the group of intermediaries at price* $\mathbf{0}$*. That is, if* $S \subseteq T$ *then* $F(S,\mathbf{0}) \subseteq F(T,\mathbf{0})$

d. F *only depends on the prices of the chosen group. That is,* $F(S,q) = F(S,p)$ *for* $q_{[S]} = p_{[S]}$ *and for any* $S \in 2^{\mathcal{N}}$

e. F *is non-increasing in prices. That is, if* $p \leq q$ *then* $F(S,q) \subseteq F(S,p)$ *for any* $S \in 2^{\mathcal{N}}$

An intermediation problem is composed of two functions u and F. First, the function u represents the planner's preferences over the chosen outcome as well as the aggregate price paid to the intermediaries who are contracted. We assume the planner's utility does not decrease as more resources are allocated to the agents and the planner's utility is non-increasing on the total amount paid to the intermediaries.

Second, intermediaries vary on their ability to transmit the resource to the agents. These differences come from the group of intermediaries selected as well as the prices paid to them. This variation is formally described by an outcome possibility function F that assigns a set of potential outcomes to every group of intermediaries and price vector. We interpret $F(S,p)$ as the outcomes available for the planner to use after he has contracted group S and paid prices $p_{[S]}$. We have five assumptions regarding F. The first two assumptions are technical assumptions needed to guarantee the existence of equilibrium. In particular, $F(\emptyset, \mathbf{0}) = \{(0,\ldots,0)\}$ gives the planner the possibility of inaction. Selecting more intermediaries when prices are $\mathbf{0}$ should lead to no fewer feasible outcomes, which is the spirit of the third assumption. The fourth assumption guarantees that the ability of a group of intermediaries should only depend on themselves, and not on the prices posted by intermediaries outside the group. The last assumption, which relates to monotonicity, represents the fact that higher prices paid to intermediaries lead to no more resources available to transmit by the planner.

We study a two-stage complete information game where at the first stage intermediaries choose simultaneously and independently a price p for having access to their outcome set. In the second

[6]For a given $x \in A$ and $\epsilon > 0$ the ball with center x and radius ϵ is denoted by $B_\epsilon(x) = \{x' \in A \mid \mid x - x' \mid < \epsilon\}$. In order to formally define continuity, for a given sequence of sets $\{\mathcal{M}^i\}_i$, the closure $cl(\{\mathcal{M}^i\}_i)$ is defined as $x \in cl(\{\mathcal{M}^i\}_i)$ if and only if for any $\epsilon > 0$ there exists $\delta > 0$ such that $B_\epsilon(x) \cap \mathcal{M}^i \neq \emptyset$ for any $i > \delta$. $F(S,p)$ is continuous in p whenever for any sequence in prices $\{p^i\}_i$ that converges to p, that is $\lim_{i\to\infty} p^i = p$, we have that $F(S,p) = cl(\{F(S,p^i)\}_i)$.

stage, after observing the price vector p charged by the intermediaries, the planner chooses a group of intermediaries $b(p) \subset \mathcal{N}$ and a feasible outcome $x(p) \in F(b(p), p)$.

Definition 2 (Resource Transmission Game)
*Given an intermediation problem (u, F), the **resource transmission game** is a sequential game of complete information such that:*

- *The strategy space of intermediary n is $[0, P_n]$, where $0 \leq P_n \leq +\infty$. The **strategy of intermediary** n is to set a fixed price $p_n \in [0, P_n]$ that the planner has to pay for the use of his ability. P_n is the maximum price that intermediary n is allowed to post, when $P_n = +\infty$ there is no upper bound on the price of intermediary n. Let $p = (p_1, ..., p_N)$ be the vector of strategies by the intermediaries.*
- *The **strategy of the planner** is a pair of functions $b : \mathbb{R}^N_+ \to 2^{\mathcal{N}}$ and $x : \mathbb{R}^N_+ \to A$ such that $x(p) \in F(b(p), p)$.*
- *The objective of each intermediary is to maximize the price paid by the planner. The **utility** $V^n(p, b, x)$ **of intermediary** n is $V^n(p, b, x) = p_n$ if $n \in b(p)$, and $V^n(p, b, x) = 0$ if $n \notin b(p)$. That is, only the intermediaries selected might get positive utility equal to their proposed price.*
- *We focus on the case where the planner only pays for the intermediaries used. Therefore, the **utility of the planner** equals $u(x(p), \sum_{n\in b(p)} p_n)$.*

In most of the paper, we impose no restriction on whether P_n is finite or infinite. We do impose a finite maximal price P_n, for every intermediary n, in Lemma 1.

Given prices p and a group of intermediaries S, the set of *utility maximizing allocations at* (S, p) is the set $x^*(S, p) \subset F(S, p)$ such that $x \in x^*(S, p)$ if and only if $u(x, p_S) \geq u(x', p_S)$ for any $x' \in F(S, p)$. Since $F(S, p)$ is compact and u is continuous, the set $x^*(S, p)$ is non-empty. The *maximal utility* $u^*(S, p)$ given prices p for group S equals $u(x, p_S)$ for $x \in x^*(S, p)$.[7] Given prices p, the group $S(p) \subset \mathcal{N}$ is a *utility maximizing group at* p if $u^*(S(p), p) \geq u^*(T, p)$ for any $T \in 2^{\mathcal{N}}$.

Given the complete information and sequentiality of the resource transmission game, we use a subgame perfect Nash equilibrium as a predictor of the behavior of the planner and intermediaries.

Definition 3 (Subgame Perfect Nash Equilibrium)
*The strategies from intermediaries $p \in \mathbb{R}^N_+$ and planner (b, x) are a **subgame perfect Nash equilibrium (SPNE)** if*

[7] The maximal utility $u^*(S, p) = max_{x\in F(S,p)} u(x, p_S)$ depends on prices p rather than p_S because $F(S, p)$ depends on p.

- $V^n(p, b, x) \geq V^n(\tilde{p}_n, p_{-n}, b, x)$ *for any* $n \in \mathcal{N}$ *and* $\tilde{p}_n \in \mathbb{R}_+$.
- *For any prices* p*, the selected group* $b(p)$ *is a utility maximizing group at* p *and* $x(p) \in x^*(b(p), p)$.

When there is no confusion, a SPNE (p, b, x) will simply be referred to as the vector of prices p.

The ideal for planner is to finding conditions under which there is no waste of resources used to pay the intermediaries. We capture this in the definition of an intermediation free equilibrium where the final allocation implemented as a SPNE is welfare-equivalent for the planner as if all the intermediaries price at zero.

Definition 4 (Intermediation Free Equilibrium)
An ***intermediation free equilibrium (****IFE****)*** (p, b, x) *is a vector of strategies such that* (p, b, x) *is a SPNE and* $u(x(p), p_{b(p)}) = \max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0)$.

Note that an IFE requires that the allocation to agents $x(p)$ and prices paid to intermediaries s-elected $p_{[b(p)]}$ are *planner-optimal*, that is, they achieve the maximal utility $\max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0)$. However, at an IFE not all intermediaries need to be pricing at zero.

Definition 5 (Indirect Utility Function)

- *The* ***indirect utility function*** $v(p) = max_{x \in F(S,p), S \in 2^{\mathcal{N}}} u(x, p_S)$ *is the maximal utility that the planner can achieve given the prices* p.
- *The* ***indirect utility function without intermediary*** n *is denoted by* $v_{-n}(p_{-n}) = v_{\mathcal{N} \setminus \{n\}}(p_{-n}) = max_{x \in F(S,p), S \in 2^{\mathcal{N} \setminus \{n\}}} u(x, p_S)$.

Note that since the OPF F and utility function u are non-increasing in prices, then the indirect utility function v is non-increasing in prices as well. Continuity of the indirect utility function is guaranteed, mainly due to the continuity of the utility function u and OPF F. This is proven in Lemma 2 and used in the main results.

Definition 6 (Intermediation Free Equilibrium)
An ***intermediation free equilibrium (****IFE****)*** (p, b, x) *is a vector of strategies such that* (p, b, x) *is a SPNE and* $u(x(p), p_{b(p)}) = \max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0)$.

Note that an IFE requires that the allocation to agents $x(p)$ and prices paid to intermediaries s-elected $p_{[b(p)]}$ are *planner-optimal*, that is, they achieve the maximal utility $\max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0)$. However, at an IFE not all intermediaries need to be pricing at zero.

2.3 Intermediation Free Equilibria

We formalize below a situation where a strict decrease in the prices of the intermediaries leads to a strict increase in the utility of the planner.

Definition 7 (Monotonicity)
The problem (u, F) is ***monotonic in prices*** *if for any $S \subseteq \mathcal{N}$, intermediary $n \in S$, and prices p and p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$ we have that for any $x \in F(S, p)$ such that $u(x, p_S) > u((0, \dots, 0), 0)$ there exists $y \in F(S, p')$ such that $u(y, p'_S) > u(x, p_S)$.*

When there is no confusion, we refer to the problem (u, F) as *monotonic* instead of monotonic in prices. Monotonicity in prices is a weak property that occurs in a large class of applications, including the four applications discussed in Section 2.5. Our definition of monotonicity of the intermediation problem (u, F) implies that the indirect utility function of the planner is monotonic in prices. This is proven in Lemma 3 and widely used in the proofs of the main results.

The monotonicity of the intermediation problem (u, F) may be coming from two forces. On one hand, it might be that the planner has a utility function u that is strictly monotonic in prices. On the other hand, the OPF F may be strongly monotonic in prices.

Remark 1
Either of the following conditions is sufficient for the monotonicity of the intermediation problem (u, F):

a. *u is strictly monotonic on the total price paid by the planner. That is, $u(x, t) > u(x, \tilde{t})$ for any $t < \tilde{t}$ and $x \in A$.*

b. *The outcome possibility function F is strongly monotonic in prices.*[8]

Lemma 3 also proves the claims in this remark. Conditions a and b provide simpler avenues to verify monotonicity of an intermediation problem. These conditions have important implications for the planner. They require that as price decreases, the planner should be strictly better off either due to paying less for intermediation (condition a) or having strictly better off options available for the planner to choose from (condition b). The example discussed in Section 2.5.4 satisfies condition a, whereas a large class of networks discussed in sections 2.5.1, 2.5.2 and 2.5.3 satisfy condition b.

[8] We say that F is strongly monotonic in prices if for any $S \subseteq \mathcal{N}$, intermediary $n \in S$, and prices p and p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$ we have that for any $x \in F(S, p)$ such that $u(x, p_S) > u((0, \dots, 0), 0)$ there exists $y \in F(S, p')$ such that $y > x$.

The next property relates to the utility of the planner at the limit of the strategy space of every intermediary in the resource transmission game. It requires that for every intermediary there exists a large enough price such that the planner is (weakly) better-off not selecting any group containing that intermediary.

Definition 8 (Price-Satiated)
The resource transmission game generated by (u, F) *is* ***price-satiated*** *if the strategy space of every intermediary is bounded and for any group of intermediaries* $S \subseteq \mathcal{N}$ *and intermediary* $n \in S$ *with maximum price* P_n*,* $u(x, P_n) \leq u((0, \ldots, 0), 0)$ *for any* $x \in F(S, (P_n, \mathbf{0}_{-n}))$.

Price-satiation guarantees that the planner has the choice of inaction, selecting no groups of intermediaries when their price is high enough. A particular case of price-satiation occurs when at the maximum price of every intermediary there is no remaining resource to transmit, that is $F(S, (P_n, \mathbf{0}_{-n})) = \{(0, \ldots, 0)\}$ for every $S \subset N$ and intermediary $n \in S$. This condition is satisfied in the first three applications discussed in Section 2.5.

In general, the existence of a SPNE in a resource transmission game is not guaranteed. This can be easily seen in a Bertrand competition game with producers who have different marginal cost and where the planner splits the resource equally in case of ties. However, under the appropriate tie-breaking rule chosen by the planner, an equilibrium exists (Reny [125]). The following results provide conditions for the existence of equilibrium in a large class of resource transmission games.

Lemma 1 (Existence of SPNE)
Every price-satiated resource transmission game generated by a monotonic problem (u, F) *has a SPNE.*

The proof of this result is based on Reny [125], who proves that a pure-strategy equilibrium exists in games that are better-reply secure. Contrary to the rest of the paper, the existence of equilibria requires a finite upper bound on the price that every intermediary can charge (implied by price-satiation), as Reny's result requires a compact strategy space for every player.

One important utility maximizing group of intermediaries occurs when the prices posted are zero. The structure of such groups, especially with regard to their intersection, is important to understand the existence of an intermediation free equilibrium.

Theorem 2 (Existence of IFE)
Assume the utility maximizing groups at zero prices are $S_1(\mathbf{0})$, $S_2(\mathbf{0})$, ..., $S_J(\mathbf{0})$. *There is an intermediation free equilibrium if and only if* $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$.

The existence of an IFE implies that there is no group of intermediaries who belong to all the utility maximizing groups at zero prices. The intuition is that if a group of intermediaries belong to this intersection, then these intermediaries will have sufficient market power to price above zero, thus creating an equilibrium that is not planner-optimal. The extreme case occurs in the traditional Bertrand competition model where symmetric producers with zero marginal cost of production compete for a price and the unique SPNE leads to an equilibrium price equal to zero. However, when producers have different marginal cost of production, a SPNE where producers price above zero is possible.

Theorem 2 provides testable conditions for the existence of an IFE. For most of the applications shown in Section 2.5, these conditions are simple to compute, and are typically not more difficult than computing 2^N utility maximization problems. Thus, for instance, if the only utility maximizing group is the grand coalition, then every intermediary has sufficient market power to price above zero, hence there is no IFE.

2.3.1 Robust SPNE

Multiplicity of equilibria often occurs, as will be seen in Example 1 and other examples in Section 2.5. However, we can argue that some of the equilibria might not be as likely to occur because there are groups of intermediaries who may gain by offering their abilities at a lower price.[9] In particular, intermediaries who are not chosen by the planner always have the incentive to undercut their prices in hopes of being chosen. In this section we look at a robustness of SPNE, where intermediaries who are not used by the planner cannot jointly decrease their prices and affect the equilibrium. Formally, a SPNE is a robust SPNE when the intermediaries who are not used by the planner charge prices equal to zero.

Definition 9 (Robust SPNE)
*The subgame perfect Nash equilibrium (p, b, x) is **robust** if intermediaries who are not used by the planner post zero prices. That is, the SPNE (p, b, x) is robust whenever $n \notin b(p)$ implies $p_n = 0$.*

A robust SPNE is an equilibrium refinement weaker than a collusion-proof Nash equilibrium, since intermediaries who are not selected by the planner cannot collude and gain by lowering their prices. Note that at a robust SPNE it is possible for intermediaries who are used by the planner to charge positive prices, thus a robust SPNE might not be an IFE. Furthermore, it is possible for a robust SPNE not to exist or for multiple robust SPNE to exist. Our analysis in this section will focus on

[9] In the example in Section 2.1.1, assuming intermediaries are playing the equilibrium prices $p = (I, I, I)$ where intermediary 3 is chosen by the planner, we can see that intermediaries 1 and 2 can gain by lowering their prices simultaneously to a vector of prices p such that $p_1 + p_2 < I$.

finding the conditions on the intermediation problem for the existence and uniqueness of the robust SPNE.

The intermediation problem is cross-monotonic whenever the ranking of groups at the maximal utility is maintained as prices change.

Definition 10 (Cross-Monotonic)
The problem (u, F) is ***cross-monotonic*** *if* $\max_{x \in F(S,\mathbf{0})} u(x, 0) \leq \max_{x \in F(T,\mathbf{0})} u(x, 0)$, *then* $\max_{x \in F(S,p)} u(x, p_S) \leq \max_{x \in F(T,p)} u(x, p_T)$ *for any* p *with* $p_S = p_T$.

Cross-monotonicity is satisfied by a variety of intermediation problems, including the cases of homothetic[10] preferences and product separable OPF.

Remark 3
Either of the following conditions on the intermediation problem guarantees that the problem is cross-monotonic:

a. *u is product separable: there exists functions $\alpha : \mathbb{R}_+^M \mapsto \mathbb{R}$ and $\beta : \mathbb{R}_+ \mapsto \mathbb{R}$ such that $u(x, t) = \alpha(x)\beta(t)$ for any x and t. Moreover, F is independent of prices: $F(S, p) = F(S)$ for any S and p.*

b. *F is product separable: there exists functions $\gamma : 2^{\mathcal{N}} \mapsto 2^A$ and $\delta : \mathbb{R}_+ \mapsto \mathbb{R}_+$ such that $F(S, p) = \gamma(S)\delta(p_S)$ for any S and p. Moreover, the utility function is independent of prices and homothetic: $u(x, t) = \tilde{u}(x)$ and $\tilde{u}(\lambda x) = \lambda \tilde{u}(x)$ for any x, $t \geq 0$ and $\lambda > 0$.*

The proof of this Remark is in the appendix.

Theorem 4 (Uniqueness of Robust SPNE)
Assume that the problem (u, F) is monotonic in prices and cross-monotonic. $\bigcap_{j=1}^J S_j(\mathbf{0}) = \emptyset$ if and only if the price vector $\mathbf{0}$ is the unique robust SPNE.

This Theorem complements the existence results of Theorem 2. Under the conditions of monotonicity in prices and cross-monotonicity of the intermediation problem, and when no group of intermediaries belongs to all the utility maximizing groups, there exists a unique robust SPNE.

The proof of the converse of this Theorem is readily seen. Indeed, if the prices $\mathbf{0}$ is a (robust) SPNE, then $\mathbf{0}$ is also an IFE. Therefore, by Theorem 2, $\bigcap_{j=1}^J S_j(\mathbf{0}) = \emptyset$. The other side of the Theorem is substantially more difficult than the proof of Theorem 2. Its proof requires a variety of intermediate results related to the continuity and monotonicity of the indirect utility function

[10]Preferences are *homothetic* if and only if there exists a utility function such that $u(\lambda x) = \lambda u(x)$ for any $\lambda > 0$ and $x \in \mathbb{R}_+^M$.

(Lemmas 2 and 3) as well as a result that provides conditions that a SPNE satisfies even when it is not an IFE (Lemma 4).

Remark 5
Cross-monotonicity and Monotonicity in prices are necessary for Theorem 4 to hold.

The proof of this Remark is in the appendix.

2.4 OPFs and IFE

We now turn our attention to outcome possibility functions that guarantee an IFE and unique robust SPNE. Consider the situation where every intermediary has an exact duplicate at prices $\mathbf{0}$. For instance, we can imagine a situation where an economy is replicated by doubling the intermediaries along with their abilities. The following definition formalizes this situation.

Definition 11 (Duplicated OPF)
*An outcome possibility function F is **duplicated** if it is defined for $N = 2k$ intermediaries and for any $S \subset \{1, \ldots, k\}$ and $T \subset \{k+1, \ldots, 2k\}$, we have that $F(S \cup T, \mathbf{0}) = F(S \cup T(-k), \mathbf{0})$, where $T(-k) = \{n - k | n \in T\}$.*

Under a minimally competitive OPF no intermediary is unique. That is, for any intermediary n, there is an intermediary n' that brings exactly the same outcome as n. In particular, this happens when the OPF is additive[11] and any intermediary has an exact replica.

Definition 12 (Minimally Competitive OPF)
*An outcome possibility function F is **minimally competitive** if for any intermediary n, there exists $n' \neq n$, such that $F(S \cup \{n\}, \mathbf{0}) = F(S \cup \{n'\}, \mathbf{0})$ for any group S.*

Corollary 1 (Sufficient Conditions that Guarantee IFE)
Suppose that the problem (u, F) is monotonic in prices and cross-monotonic. Any of the following conditions is sufficient to guarantee the existence of an intermediation free equilibrium and a unique robust SPNE:

- a. *The problem has a minimally competitive OPF.*
- b. *The problem has a duplicated OPF.*
- c. *There exists a group of intermediaries S such that $F(S, \mathbf{0}) = F(\mathcal{N} \setminus S, \mathbf{0}) = F(\mathcal{N}, \mathbf{0})$.*

[11] An OPF F is additive if $F(S, \mathbf{0}) = conv(\cup_{n \in S} F(\{n\}, \mathbf{0}))$ for any $S \subset \mathcal{N}$ and prices $\mathbf{0}$. Such is the case of the example discussed in Section 2.1.1.

This result implies that either by replicating the existing intermediaries and their OPFs or by finding a group of intermediaries that have the same abilities as their complement, will result in IFE and unique robust SPNE. Part (c) also illustrate comparative statics with respect to the addition of intermediaries: if a new group of intermediaries arrive and have exactly the same abilities as the original intermediaries, then an IFE and a unique robust SPNE will be created. The intuition behind this corollary is similar to Theorem 2: perfect competition among the intermediaries occurs when every intermediary can be substituted by another group of intermediaries that achieve an equal level of utility.

2.5 Applications

2.5.1 Resource Transmission in Networks under Proportional Constraints

Consider the case where there are fixed links between the intermediaries $\mathcal{N} = \{1, \ldots, N\}$ and the agents $\mathcal{M} = \{1, \ldots, M\}$. Every intermediary is connected to a group of agents and can transmit resources to the agents that it is connected with some fixed quality (ability), this is denoted by the *sharing-rate*. Let q_{nm} be the sharing-rate of intermediary n connected to agent m, where $q_{nm} \geq 0$ for each intermediary n. The matrix of sharing-rates is $Q = (q_{11}, \ldots, q_{NM})_{N \times M}$, and $Q_n = (q_{n1}, \ldots, q_{nM})$ is the ability of intermediary n to transmit the resource to the agents. We assume that if there is no link between intermediary n and agent m, then $q_{nm} = 0$. The sharing-rate distinguishes the way in which intermediaries transmit resources to agents per unit of money given.[12] Two intermediaries connected to the same group of agents might have different impacts on the agents, and thus one might be better aligned than the other to the planner's preferences.

The planner has a utility function $u(x, p) = u(x)$ that is independent of the price paid to the intermediaries. That is, the planner cares only about the final resource transmitted to the agents in $\mathcal{M}$. Assume that the total resource available for the planner to transmit is I. Given the matrix of sharing rates Q, the outcome possibility function is

$$F(S,p) = \{\sum_{n \in S} Q_n y_n | \sum_{n \in S} y_n \leq I - \sum_{n \in S} p_n \text{ and } y_n \geq 0\} \text{ if } \sum_{n \in S} p_n \leq I$$

$$F(S,p) = \{(0, \ldots, 0)\} \text{ if } \sum_{n \in S} p_n > I$$

[12] One application of this model includes in the allocation of resources to charities who have a pre-determined set of priorities among agents. When $\sum_{m=1}^{M} q_{nm} < 1$, we can interpret the intermediary (charity) as being inefficient. Such inefficiencies happen often in charities (and universities) where every dollar spent is often decreased due to indirect cost which serves to pay for the administration. The case of $\sum_{m=1}^{M} q_{nm} > 1$ implies that a dollar transmitted using that intermediary increases, for instance when charities or universities offer matching funds from donors. Previous results in the transmission of resource in networks do not distinguish in the quality of the links or assume that the sharing-rate is equal across intermediaries.

That is, the possibility set of a group S when posted prices are p is the transmission of not more than $I - \sum_{n \in S} p_n$ units of the resource using the abilities given by Q of the intermediaries in S.

Example 1 (Perfect Substitute Utility Function)
Consider a planner with utility function $u(x) = \sum_{m=1}^{M} \alpha_m x_m$, where α_m is the weight of the final resource allocated to agent m. Given the sharing-rates $\{q_{nm}\}_{\{n \in \mathcal{N}, m \in \mathcal{M}\}}$, the marginal utility of resource allocated to intermediary n is constant and given by $MU_n = \sum_{m=1}^{M} \alpha_m q_{nm}$. Without loss of generality we rename the intermediaries based on a non-increasing order of their marginal utility, that is $MU_1 \geq MU_2 \geq \cdots \geq MU_N$.

When $MU_1 = \cdots = MU_k > MU_{k+1}$ and $k \geq 2$, the planner is indifferent between allocating the resources to any of the intermediaries from $\{1, \ldots, k\}$ when their prices are zero. If only one intermediary from $\{1, \ldots, k\}$ has a price zero, then he can raise the price to slightly below the second lowest price posted by a different intermediary. Alternatively, if no intermediary from $\{1, \ldots, k\}$ has zero price, then at most one of them will be chosen, and the ones who are not chosen have the incentive to decrease their price. Therefore, a SPNE requires that at least two intermediaries from $\{1, \ldots, k\}$ have price zero. It is easy to verify that every price allocation such that $p_i = p_{i'} = 0$, for some $i, i' \in \{1, \ldots, k\}$ and $p_n \geq 0$, $\forall n \neq i, i'$ is a SPNE. Thus, in this example there are multiple IFEs.

When $MU_1 > MU_2$, the intermediary 1 has some market power to price above zero and continue being chosen. In a SPNE, $p_2 = 0$ and $p_1 = I(1 - \frac{MU_2}{MU_1})$, $p_n \geq 0$, $\forall n \geq 3$ and intermediary 1 is chosen to transmit $I - p_1$ units of resource. The planner's utility would be $I \cdot MU_2$, which is welfare equivalent to the utility given by allocating all resources to the intermediary with the second highest marginal utility when he prices at 0. In particular, there is no IFE.

An alternative way to prove the existence of a robust SPNE is by computing the utility maximizing groups at $\mathbf{0}$ and applying Theorem 4 (since monotonicity and homotheticity of the preferences are clearly satisfied). Indeed, if $1, \ldots, k$ are the intermediaries with marginal utility $MU_n = MU_1$, $\forall 1 \leq n \leq k$, then each of $\{1\}, \ldots, \{k\}$ is a utility maximizing group at $\mathbf{0}$. Therefore, if $k > 1$ then $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$, hence a unique robust SPNE exists. However, if $k = 1$, then $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \{1\}$, thus no IFE exists.

Example 2 (Symmetric Network)
Assume that the planner with utility function $u(x) = min\{x_1, x_2, x_3\}$ cares about the agent who is allocated the least resources. The network in Figure 2.2 represents the connections from intermediaries to agents given by the matrix of sharing-rates $Q = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

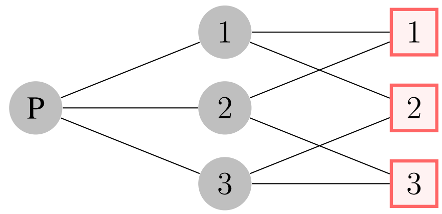


Figure 2.2: Network with three symmetric intermediaries

Every intermediary is connected to two agents and would always send the resource equally to the agents connected. Note that if the planner only uses two intermediaries, the optimal allocation is to transmit half the resources through each intermediary. Thus, the agent connected to both intermediaries would get half of the resource and each of the other two agents would get one quarter of the resource transmitted. In this case, the resource cannot be allocated equally to three agents and results in a waste of resources and an inefficiency for the planner. Thus, the planner-optimal allocation can only be achieved by using the three intermediaries in conjunction. Hence, every intermediary has some market power to post a positive price in equilibrium.

There is a symmetric equilibrium where every intermediary posts price $\frac{I}{6}$*, the planner would use all the intermediaries* $b(p) = \{1, 2, 3\}$*, and the allocation of resource to agents is* $x(p) = (\frac{I}{6}, \frac{I}{6}, \frac{I}{6})$*.*

There is another equilibrium price allocation which results when every intermediary posts price equal to total resource I*, that is* $p = (I, I, I)$*, and the planner pays one of the intermediaries (say, intermediary* 1*,* $b(p) = \{1\}$*) all the resource without transmitting anything, which means* $x = (0, 0, 0)$*. In this equilibrium, there is no incentive for intermediary* 1 *to deviate since it gets all the resource. For intermediary* 2 *or* 3*, even if one decreases his price, the planner cannot get positive utility because one intermediary is not connected to all the agents and at least one agent would receive* 0 *resource. Thus, paying all resource to intermediary* 1 *is still a best strategy for planner. The SPNE with planner's utility equal to* 0 *exists because intermediaries* 2 *and* 3 *cannot cooperate by lowering their prices simultaneously.*

There is an easier way to verify that no IFE exists in this case. Indeed, note that the only utility maximizing group at the vector of prices $\mathbf{0}$ *is* $\{1, 2, 3\}$*. Hence, the necessary conditions to guarantee an IFE in Theorem 2 do not hold.*

Let $conv(Q) = \{\sum_{n=1}^{N} \lambda_n Q_n | \sum_{n=1}^{N} \lambda_n = I, \lambda_n \geq 0, \forall n\}$ be the convex hull of the sharing rates $Q_1, \ldots, Q_N$ of intermediaries. The points in $conv(Q)$ are the feasible allocations of the resource to agents subject to the constraints Q given by the intermediaries. Let Q_{-n} be the matrix where the row Q_n is removed from Q. Let $conv(\mathbf{0}, Q)$ be the convex hull of Q and the vector of zeros. Let $x^\star(Q, u) = \{x \in conv(Q) | u(x) \geq u(x'), \forall x' \in conv(Q)\}$ be the set of allocations to the agents that maximize the planner's utility. Note that, when the planner's preferences are convex the set $x^\star(Q,u)$ is a convex set. Moreover, when the planner's preferences are strictly convex the set $x^\star(Q,u)$ contains a unique point.

The next result follows from the two main Theorems in the paper. We need to recognize that, due to the restrictions of the model, the assumptions in Theorem 4 regarding monotonicity and cross-monotonicity of a problem can be simply implied by the strong monotonicity and homotheticity of the planner's preferences, respectively.

Corollary 2

a. *Given the sharing rates of intermediaries $Q_1,\dots,Q_N$, there exists an IFE (or $\mathbf{0}$ is the unique robust SPNE) for any strongly monotonic*[13] *and homothetic preferences of the planner if and only if for every intermediary n, $Q_n \in conv(\mathbf{0}, Q_{-n})$.*

b. *Suppose that preferences of the planner are homothetic, strongly monotonic and strictly convex. An IFE exists (or $\mathbf{0}$ is the unique robust SPNE) if and only if the utility maximizing allocation $x^\star(Q,u)$ belongs to the intersection of $\bigcap_{n\in\mathcal{N}} conv(Q_{-n})$.*

Part (a) provides conditions for the existence of an IFE for any strongly monotonic and homothetic preferences of the planner. Such conditions imply that the ability Q_n to transmit the resource by intermediary n can be replicated by a subset of other intermediaries. On the other hand, part (b) focuses on a specific utility function u of the planner that is monotonic and strictly convex. It requires that the utility maximizing allocation belongs to $conv(Q_{-n})$ for any n. Thus, no intermediary is unique, as his ability can be replicated by the ability of others.

2.5.2 Resource Transmission in Networks under Unit-Capacities

We consider the problem of intermediation with unit-capacity constraints. A finite directed network $G=(V,E)$ without cycles that connects a single source P and sinks $\mathcal{M}=\{1,\dots,M\}\subset V$ is interpreted as connecting the planner with agents $\mathcal{M}$. The link $e\in E$ has a unit-capacity constraint c_e, which means that every unit of resource transmitted using link e would receive at most c_e units. Consider the case where the planner is endowed with I units of resource to distribute to the agents. Thus, for instance, if I units of good are transmitted in the sequence of links with unit capacities $c_1,\dots,c_l$, then $c_1\cdots c_l I$ is the maximal amount of resource that reaches its destination.

[13]The preferences represented by a utility function u are strongly monotonic if for any x and x' such that $x\geq x'$ and $x\neq x'$, $u(x)>u(x')$. While we use strong monotonicity in Corollaries 2 and 3, the same results apply for some non-monotonic preferences such as those represented by a perfect complements utility function $u(x)=\min_{i\in\mathcal{N}} x_i$.

Assume the intermediaries in the set $\mathcal{N} = \{1, \ldots, N\}$ own the links in the network. Let $E = \{E_1, \ldots, E_N\}$ be a partition of the links E, where E_n represents the links owned by intermediary n.[14]

The planner has preferences over allocations in $\mathbb{R}_+^M$ denoted by a utility function $u : \mathbb{R}_+^M \to \mathbb{R}$ that is independent of the prices p. Thus, for instance, if the planner only cares about the total allocation to the agents, then $u(x) = \sum_m x_m$, but in general the planner might care about the worst individual $u(x) = \min_{m \in \mathcal{M}} x_m$ or some other utility function. Assume the intermediaries post prices $p = (p_1, \ldots, p_N)$ for the use of their links.[15] Given the prices, the planner decides on the group of intermediaries to contract by paying the prices posted, and distributes the rest of the resources. Thus, for instance, if the planner is selecting group S, then he pays a total price of $\sum_{n \in S} p_n$ for the use of links in S, and $I - \sum_{n \in S} p_n$ units of resource are left for transmission to the agents.

For intermediaries $S \subset \mathcal{N}$ and agent $m \in \mathcal{M}$, let $PG(S, m)$ be the paths in G connecting the planner with agent m in the network where the capacities of the intermediaries in $\mathcal{N} \setminus S$ are zero. For a given path w with unit capacities $(c_1 \ldots c_l)$ on the links, let $c(w) = c_1 \cdots c_l$ be the unit capacity of the path. Given an agent m and intermediaries $S \subset \mathcal{N}$, let $\bar{c}^m(S) = \max_{w \in PG(S,m)} c(w)$ be the maximum unit capacity of the paths that connect agent m with the planner in the network. Note that since there is a finite number of paths, $\bar{c}^m(S)$ is easily computable. Given the group of intermediaries S, the maximal unit capacity is $\bar{c}^m(S)$ for agent m. Let $x^{m,S} = (0, \ldots, 0, \bar{c}^m(S), 0, \ldots, 0) \in \mathbb{R}_+^M$ be the vector representing the maximal transmission to the agent m using intermediaries in S. The OPF for group S and vector of prices p is

$$F(S,p) = \{x \in \mathbb{R}_+^M \mid x \leq \sum_{m=1}^{M} \lambda^m x^{m,S}(I - p_S), \sum_{m=1}^{M} \lambda^m = 1, \lambda^m \geq 0, \forall m\} \text{ if } \sum_{n \in S} p_n \leq I$$

$$F(S,p) = \{(0, \ldots, 0)\} \text{ if } \sum_{n \in S} p_n > I$$

[14]The canonical case of this model occurs when every intermediary owns one link. Another traditional case occurs when intermediaries own the span of links emanating from nodes. Moreover, the model where there is a single agent and every link has capacity 1 is discussed in Choi, Galeotti and Goyal [42]. Their results from Theorem 1 can be easily obtained from our Corollary 3 below.

[15]We focus on the case where each intermediary posts a single price for the use of all his links. We do not study the case of multiple pricing, but it is also an interesting case.

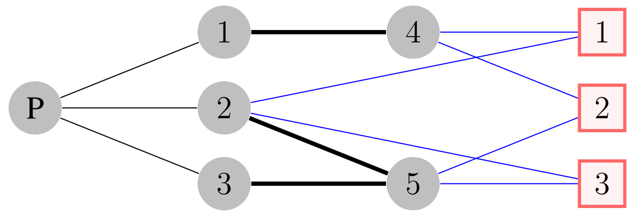


Figure 2.3: Network with Multiple Layers of Intermediation

Definition 13 (Non-zero Corners Utility Function)
The utility function has non-zero corners if for any $x \in \mathbb{R}_+^M$ *such that* $x_m = 0$ *for some* m*, then* $u(x) = 0$*; and if* $x > (0, \dots, 0)$*, then* $u(x) > 0$*. The preferences of the planner are non-zero corners if there exists a non-zero corners utility function that represents such preferences.*

The perfect complements utility function $u(x) = min\{x_1, \dots, x_M\}$ and the Cobb-Douglas utility function $u(x) = \prod_{m=1}^{M} x_m^{\alpha_m}$ satisfy non-zero corners. Given that the preferences of planner are homothetic, the problem (u, F) is monotonic if the preferences are strongly monotonic or the utility function has non-zero corners (we prove this in the proof of Corollary 3 below).

Example 3
Consider the network in Figure 2.3. The intermediaries $\mathcal{N} = \{1, \dots, 5\}$ *are represented by the middle nodes in the network. Each of them own the links that originate from their node. The agents* $\mathcal{M} = \{1, 2, 3\}$ *are in the final layer of network. The black (thick) links have a unit capacity of* 1*, while the blue links have a unit capacity* $c_j = 0.5$*. The planner has a perfect complement utility function* $u(x) = \min\{x_1, x_2, x_3\}$ *over the final allocation of the resource to the agents in* $\mathcal{M}$*.*

In this example no intermediary is fundamental. That is, the unit capacity of resource transmission to agent m *with all intermediaries except* n *is* $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N}) = 0.5$*,* $\forall m, n$*. There is an IFE and unique robust SPNE,* $p = \mathbf{0}$*,* $b(p) = \{1, 2, 4\}$*,* $x(p) = (\frac{I}{6}, \frac{I}{6}, \frac{I}{6})$*.*

Consequences of Theorems 2 and 4 in the problem of resource transmission under unit-capacities are described below.

Corollary 3

a. *Suppose that for any agent* $m \in \mathcal{M}$ *and intermediary* $n \in \mathcal{N}$ *we have that* $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N})$*. Then, for any homothetic preferences of the planner, the price vector* $p = \mathbf{0}$ *is an IFE and unique robust SPNE. Conversely, if for any strongly monotonic utility function of the planner there exists an IFE (or* $\mathbf{0}$ *is the unique robust SPNE) then* $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N})$ *for any agent* m *and intermediary* n*.*

b. *Suppose the planner's utility function is homothetic and has non-zero corners. An IFE exists (or* $\mathbf{0}$ *is the unique robust SPNE) if and only if* $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N})$ *for any* $m \in \mathcal{M}$ *and* $n \in \mathcal{N}$.

This corollary establishes the sufficient conditions for the existence of an IFE and for the prices $\mathbf{0}$ to be the unique robust SPNE. These conditions require that the maximal unit capacity that can be transmitted to an agent in the network should not change when any intermediary is removed. Part (a) shows that this property is necessary if we want the existence for any monotonic utility function of the planner. On the other hand, part (b) shows that the same condition is necessary when we restrict to a single set of preferences of the planner that satisfy non-zero corners.

2.5.3 Resource Transmission in Networks under Total-Capacities

We consider the case of intermediation with total capacity constraints on the links. A finite directed network $G = (V, E)$ without cycles that connects a single source P and sinks $\mathcal{M} = \{1, \ldots, M\}$ is interpreted as connecting the planner with agents $\mathcal{M}$. Every link $l \in E$ in the network has a capacity constraint c_l, which is the maximal capacity that can be transmitted in that link.[16] Assume the intermediaries in the set $\mathcal{N} = \{1, \ldots, N\}$ own the links in the network. Let $E = (E^1, \ldots, E^N)$ be a partition of G, where E^n represents the links owned by intermediary n.The planner is endowed with I units of the resource and has preferences over the final allocations of the agents, denoted by a utility function $u(x) : \mathbb{R}_+^M \to \mathbb{R}$. Unlike in the case of unit-capacities discussed above, the links have total capacities, therefore if I units of good are transmitted in the sequence of links with total capacities $(c_1, \ldots, c_l)$, then $\min\{c_1, \ldots, c_l, I\}$ reach their destination. The allocation of resource follows the same posting-price mechanism as in the case of unit-capacities.

Given an agent m and intermediaries $S \subset \mathcal{N}$, let $\bar{c}^m(S, I)$[17] be the maximal amount of resource that can be transmitted to agent m using the links owned by intermediaries in S[18] when I units are available for transmission. Notice $\bar{c}^m(S, I)$ is easily computable in the network, for instance the simple Ford-Fulkerson algorithm ([59]) computes the max-flow in a network. Let $x^{m,S} = (0, \ldots, 0, \bar{c}^m(S, I), 0, \ldots, 0) \in \mathbb{R}_+^M$ be the vector representing the maximal transmission to the agent m using intermediaries in S. The OPF for group S and vector of prices p is

[16] Similar models with capacity constraints in links have been studied in the literature, for instance Bochet, Ilkilic, Moulin and Sethuraman (2012) discuss the transmission of a divisible resources from suppliers to demanders in a network with similar capacity constraints over the links.

[17] Unlike in the previous section, the results under total capacity depend on the total resource I, see below.

[18] Alternatively, we can re-interpret this as saying that the capacities of all the links owned by the intermediaries in $\mathcal{N} \setminus S$ are changed to zero.

$$\tilde{F}(S,p) = \{x \in \mathbb{R}_+^M \mid x \leq \sum_{m=1}^{M} \lambda^m x^{m,S}(I - p_S), \sum_{m=1}^{M} \lambda^m = 1, \lambda^m \geq 0, \forall m\} \text{ if } \sum_{n \in S} p_n \leq I$$

$$\tilde{F}(S,p) = \{(0,\ldots,0)\} \text{ if } \sum_{n \in S} p_n > I$$

Unlike the previous two applications, the OPF in this example is not additive (see footnote 11). This can be readily seen in an example of two links l_1, l_2 owned by different intermediaries, where (l_1, l_2) is the only path connecting the planner to a single agent. Each link has capacity 1. If the planner selects l_1 or l_2, then he cannot transmit anything to the agent. However, if the planner selects l_1 and l_2, then he can transmit 1 unit.

Furthermore, unlike in the previous two applications, the OPF $\tilde{F}$ is not homothetic in the resource I. Thus, an increase in the amount of the resource I may change the multiplicity of equilibria and welfare of the planner at equilibrium, as illustrated in the following example.

Example 4

Consider a graph with three parallel links directly connecting the planner with a single agent. The links are owned by different intermediaries and have capacities 10, 10 *and* 11, *respectively. The planner cares about transmitting the maximal amount of the resource to the agent (i.e.,* $u(x) = x$*). If the planner has* $I = 18$ *units of resource, then every pair of links can transmit the full resource (thus every pair of intermediaries would maximize the utility). The prices* $p = (0,0,0)$, $b(p) = \{1,2\}$ *and* $x(p) = 18$ *is an IFE and a unique robust SPNE. At the same time, prices* $p = (8,8,8)$, $b(p) = \{3\}$ *and* $x(p) = 10$ *is also SPNE, since intermediaries* 1 *and* 2 *cannot coordinate to lower the prices and get higher utility.*

If the planner has $I = 40$ *units of resource, there is a SPNE with* $p = (0,0,20)$ *and intermediaries 1 and 3 (or 2 and 3) being used. Note that in this equilibrium, intermediary* 3 *has a link with a larger capacity constraint than intermediaries* 1 *and* 2, *but he posts a positive price and gets a larger benefit than intermediaries 1 and 2. There are multiple SPNE, for example* $p = (30,30,30)$ *and only intermediary 3 being used. However, there is a unique robust SPNE.*

This example also shows that when resource I *increases, the planner's utility at the equilibrium may not increase and the increase resource is paid to intermediaries.*

This example also illustrates that the problem $(u, \tilde{F})$ is not monotonic, hence the results in Theorem 2 might not apply. Indeed, once the full capacity of the network has been reached, a strict increase in one of the prices may not strictly decrease the OPF. Therefore, the results in Theorem 4 may not apply. Consequences of Theorems 2 in the problem of resource transmission under total-capacities are described below.

Corollary 4

a. *For any monotonic utility function u there exists an IFE if and only if the full transmission of the resource to any agent m without using the links of intermediary n is possible, that is $\bar{c}^m(\mathcal{N} \setminus \{n\}, I) = I$ for any agent m and intermediary n.*

b. *Suppose that the planner's utility function has non-zero corners. In the problem without capacities, i.e., capacities are infinity for every link, an IFE exists (or* $\mathbf{0}$ *is the unique robust SPNE) if and only if there is no intermediary who owns link(s) on every path from the planner to some agent.*

We use a simple argument of the max-flow min-cut Theorem to prove part a. A particular case of part b is discussed in Choi, Galeotti, Goyal [42], which proves the case that connects sellers and buyers, and they generate a surplus of 1 if they connect, and a surplus of 0 if they do not connect.

2.5.4 Separable Utility: Minimum Cost Spanning Trees and Related Models

In this section we restrict our attention to intermediation problems (u, F) with a separable utility function, $u(x, p_S) = u(x) - p_S$, and an outcome possibility function that is independent of the price p, $F(S, p) = F(S)$. Intermediation problems with such structure capture more stylistic settings previously discussed in the literature, as shown below.

Example 5 (MCST and Related Models, Moulin and Velez [114])

Let $\mathcal{B} = \{B_1, \ldots, B_c\} \subset 2^{\mathcal{N}}$ be a collection of acceptable subsets of intermediaries such that if $B_i \in \mathcal{B}$ and $B_i \subset D$ then $D \in \mathcal{B}$. Consider the outcome space $A = \mathbb{R}$, the utility of the planner $\bar{u}(x, p_S) = x - p_S$ and OPF equal to $F(S, p) = [0, 1]$ if $S \in \mathcal{B}$ and $F(S, p) = \{0\}$ if $S \notin \mathcal{B}$. Thus, the planner has a quasilinear utility function with numeraire good equal to the total price paid. The OPF has a positive element only when it is part of an acceptable set.

For instance, if $\mathcal{B}$ contains at least two individual intermediaries, say $\{i\}$ and $\{j\}$ are acceptable, then at a SPNE, the planner gets utility 1 and pays no money for the intermediaries. This is similar to a Bertrand competition model, where intermediaries lower their prices to zero in hopes to be chosen by the planner.

One particular case of this setting occurs in the minimal cost spanning tree (MCST) discussed in Moulin and Velez [114], where the links E in a network connecting a set of nodes $\mathcal{M}$ are owned by the group of intermediaries $\mathcal{N}$. Let $(E_1, \ldots, E_N)$ be a partition of the set of links E, where

E_n represents the links owned by intermediary n. The set of acceptable intermediaries $\mathcal{B} \subset 2^{\mathcal{N}}$ contain the groups of intermediaries whose links connect to all nodes in $\mathcal{M}$. Note this might not necessarily be a spanning tree. In the case where every intermediary owns exactly one link, the set $\mathcal{B}$ contains all spanning trees.

Other related models of interconnection in trees can be similarly encompassed by this analysis, including the Steiner tree problem where the shortest interconnect for a given set of objects is found.

Let $u_S = \max_{x \in F(S)} u(x)$ be the maximal utility achieved when using the intermediaries in S and $\bar{u} = u_{\mathcal{N}} = \max_{x \in F(\mathcal{N})} u(x)$ be the maximal utility achieved when using all the intermediaries. The straightforward consequence of Theorems 1 and 2 are discussed below.

Corollary 5

a. *Consider an intermediation problem (u, F) with a separable utility function, $u(x, p_S) = u(x) - p_S$, and an outcome possibility function that is independent of the prices p, $F(S, p) = F(S)$. An IFE exists (or $\mathbf{0}$ is the unique robust SPNE) exists if and only if the group of intermediaries who achieve the maximal utility, $\mathcal{S} = \{S_i \subseteq \mathcal{N} | u_{S_i} = \bar{u}\}$, satisfy $\bigcap_{S_i \in \mathcal{S}} S_i = \emptyset$.*

b. *For the model in Example 5, an IFE exists (or $\mathbf{0}$ is the unique robust SPNE) if and only if the intersection of the acceptable sets is empty, that is $\bigcap_{B_i \in \mathcal{B}} B_i = \emptyset$. Furthermore, in the MCST problem an IFE exists (or $\mathbf{0}$ is the unique robust SPNE) if and only if for every node $m \in \mathcal{M}$ there are at least two intermediaries with links to node m.*

2.6 Conclusion

This paper investigates how intermediation affects the resource transmission between a planner and agents. We build a game theory model to study the market power of intermediaries to charge the planner a price for the use of their abilities to transmit the resource. We discover and describe the necessary and sufficient conditions for the existence of IFEs and uniqueness of a robust SPNE. We demonstrate how properties in the OPFs can achieve an IFE, including one that replicates the economy.

The generality of our model allows for the application of the results to a wide variety of new and old intermediation problems, some of them described in Section 2.5. This paper is a start to the analysis of the transmission of a divisible resource from a planner to agents via intermediaries. Future work should include the case of incomplete information about the OPFs (Han and Juarez [72] has initial results where the quality of intermediation is elicited from the intermediaries in a more stylistic network setting), competition between multiple planners and more complex pricing structures that include variable-pricing instead of fixed-pricing (see also, Han and Juarez [72]).

3 INCENTIVE COMPATIBLE RESOURCE TRANSMISSION WITH PUNISHMENT

3.1 Introduction

Consider a planner interested in transmitting a divisible resource to agents (such as money). Although the planner is not directly linked to the agents, it can do so via a group of intermediaries. Intermediaries differ in their ability to transmit the resource. This ability is represented by the total amount of the resource that an intermediary sends to the agents per unit of resource received, as well as by the proportions in which every agent receives a resource relative to another from a given intermediary. Thus, for instance, two intermediaries might be able to reach different agents, and even when they reach the same group of agents, they may transmit different amounts to the agents.

We study the case where the intermediaries' abilities are private information.[1] Therefore, the planner uses a direct mechanism, where intermediaries report their abilities, which are then used to determine the actual transmission rate to the agents, as well as the distribution of the resource among the different intermediaries. In such cases, intermediaries might be able to game the planner by misrepresenting their ability to transfer the resource to agents. Therefore, incentive compatibility of the mechanism, in our case strategy-proofness, is a desirable requirement.

Strategy-proofness is a very robust property that prevents intermediary to misrepresent their ability regardless of the reports of other intermediaries. Restricting to strategy-proof mechanisms might come with a high cost to the planner.[2] In some settings, the planner might be able to alleviate such a cost by enforcing truthful reporting by other means. Indeed, consider the case of auditing, where the planner has the ability to audit the intermediaries in the game (perhaps with some probability) and assign a punishment (expressed in monetary terms) for the intermediaries who are found misreporting their ability. For a given set of abilities, there is always a large enough punishment such that the intermediaries should not feel compelled to misrepresent their preferences. Indeed, any punishment such that the expected punishment is larger than the expected rewards gained by

[0] Joint work with Ruben Juarez

[1] In our companion paper, Han and Juarez [71], we study the case where the abilities of the intermediaries are public information. The planner solicits bids from intermediaries to the use of their links and applies this information to select which intermediaries to contract for the transmission of the resource. The main result of Han and Juarez [71] is the necessary and sufficient conditions for the existence of a free intermediation equilibrium, where there is a perfect transmission of the resource to the agents as if there is no intermediation.

[2] This cost is typically measured in efficiency terms, but can also be measured in equity or other terms.

misrepresenting their preferences satisfies that. Thus a natural generalization of strategy-proofness extends the class of mechanisms that are strategy-proof when such a punishment are available for the planner.

The paper introduces a generalization of strategy-proofness when the planner has the ability to monitor and punish the intermediaries for misrepresenting their preferences. Indeed, in the domain of quasilinear preferences where money is available, we consider monetary punishments that will depend on an arbitrary function $ch(\alpha_i, \beta_i)$, where $c \in [0, 1]$ is a constant that can be interpreted as the planner ability of monitoring the intermediaries (e.g., probability) and $h(\alpha_i, \beta_i)$ is the punishment paid by the intermediary represented by the difference between his true ability α_i and reported ability β_i. Thus, $ch(\alpha_i, \beta_i)$ can be interpreted as the expected punishment that is paid if the intermediary is found lying. A mechanism is ch-strategy-proof if there is no incentive for any intermediary to misreport under the punishments ch. When the planner does not have the ability to monitor the intermediaries, $ch = 0$, our property boils down to the traditional strategy-proofness. On the other hand, when $ch > 0$ is large, the intermediaries will be punished a large amount $ch(\alpha_i, \beta_i)$ and the amount of misreporting will be substantially reduced. This allow us to capture all mechanisms, when $ch \to \infty$.[3]

The main contributions of the paper are three-fold. First, it introduces a notion of strategy-proofness with monitoring and punishment. Second, it introduces a new model of resource transmission and intermediation in networks when the abilities of intermediaries are incomplete information and characterizes the entire class of strategy-proof mechanisms when monitoring and punishment are available to the planner. Finally, the paper further refines the general characterization of strategy-proofness to particular punishment functions, which allow us to connect our mechanisms with old and new mechanisms.

3.1.1 Illustrative Example

To illustrate our mechanisms and main results, consider the example of a planner who is connected to three intermediaries, who themselves are connected to two agents (see Figure 3.1). The planner is interested in transmitting I units of a resource to the agents, but can only do so via the intermediaries. Intermediaries have different quality of intermediation, represented by the proportion in which they transmit their share to the agents for every unit of resource transmitted. In this case, the abilities of intermediaries are $\alpha_1 = (0.7, 0.4)$, $\alpha_2 = (0.6, 0.6)$ and $\alpha_3 = (0.5, 0.8)$, respectively.

In the absence of information, the planner will ask intermediaries to report their abilities to transmit the resource and determine (a) the amount of resource allocated to every intermediary for trans-

[3] This is true for punishment functions h such that $h(\alpha_i, \alpha_i) = 0$ for all α_i and $h(\alpha_i, \beta_i) = 0$ for $\beta_i \neq \alpha_i$.

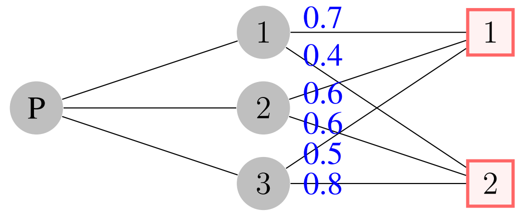


Figure 3.1: A network with three intermediaries and two agents.

mission to agents and (b) the sharing rate charged for every intermediary to transmit at every link based on the information of report.[4] The intermediary's profit is the difference between his true abilities to transmit the resource and his charged sharing rates multiplied by the amount of resource allocated to him.

For instance, consider the traditional first price auction. When intermediaries report abilities $(\beta_1, \beta_2, \beta_3)$, the planner selects the intermediary with highest reported aggregate ability to transmit all the resource with charged sharing rates $s_i(\beta) = \beta_i$. This mechanism is not strategy-proof. Indeed, when intermediaries report their true abilities $(0.7, 0.4)$, $(0.6, 0.6)$ and $(0.5, 0.8)$, intermediary 3 with aggregate ability 1.3 is selected to transmit all the resource and the charged sharing rates equals his ability $(0.5, 0.8)$. Thus, intermediary 3's profit equals 0. This is not strategy-proof, because he can decrease his report to $(0.5, 0.71)$, where he will get a positive profit equal to $[(0.5, 0.8) - (0.5, 0.71)] * (I, I)^T = 0.09I$.

Now, suppose the planner is able to audit the intermediaries. For instance, with the probability $c = 10\%$, the planner is able to observe the true abilities of the intermediaries. Furthermore, suppose that the planner punishes the intermediaries based on the deviation from their true reports with punishment function $h(\alpha_i, \beta_i) = 10 \sum_{m=1}^{M} |\beta_i^m - \alpha_i^m|$. In such a mechanism, intermediaries have no incentive to lie about their reports. Indeed, at the profile α above, when intermediary 3 reports $(0.5, 0.71)$ and planner finds that his true ability is $(0.5, 0.8)$, the expected punishment on intermediary 3 is $0.1 \times 10(|0.5-0.5|+|0.8-0.71|) = 0.09$. The expected payoff for intermediary 3 is $(0.09-0.09)I = 0$, the same as reporting $\beta_3 = \alpha_3$. Thus, there is no incentive for intermediary 3 to misreport, and first price auction is ch-strategy-proof. The set of ch-strategy-proof mechanisms expands the set of strategy-proof mechanisms.

3.1.2 Overview of the Results

We introduce the resource transmission problem of the planner in Section 2.2 and strategy-proof mechanisms in Section 3.3. We provide conditions for a mechanism to be ch-strategy-proof for the probability of punishment c and arbitrary punishment function h, when the punishment function is

[4]For instance, we can imagine the case where the planner might use a second price auction.

differentiable[5] at any truthfully report point (Proposition 1). The class of 0-strategy-proof mechanisms coincide with the class of strategy-proof mechanisms (Theorem 6), and any mechanism is ∞-strategy-proof (Corollary 7). Thus, the class of ch-strategy-proof mechanisms is largely depending on c and h, and the comparative static analysis studied in Proposition 2. Furthermore, we study the minimal punishment function for any mechanism in Section 3.4. Proposition 4 provides the necessary and sufficient condition for minimal punishment function, Position 3 and Corollary 8 shows the convexity and existence of minimal punishment function. Finally, we characterize the first-best efficient allocation in Section 3.5. Theorem 7 shows that there exists no symmetric, SP, budget balance and first-best efficient mechanism. The minimal punishment function to achieve first-best efficiency is provided.

3.1.3 Applications

An application of our game theoretical model is the transmission of advertising money in companies. A company looking to promote their product can use different media (the intermediaries) to reach the advertising target of their product; such intermediaries include TV channels, radio stations, Internet websites, and newspapers. The quality of the connections is relevant because, within the media, there are different channels that target to specific demographics of agents and may influence the planner's objective differently. For instance, two local TV stations based in the same city may be connected to all agents in the city, but the audience may be more biased based on demographics or political preferences —e.g. Fox News and CNN reach the same audience, but they target their programming to attract more conservative or liberal viewers, respectively. Nowadays, the printed version of newspapers are read heavily by older people instead of younger people, and the proportions of older to younger readers are typically available to potential purchasers of advertisements. Therefore, it is in the interest of the planner to choose the media channel that best aligns with his preferences.

Alternatively, consider the case of government contracting. For instance, the allocation of government's money to people in need via charities. The government may decide to send the money via charities that will charge an indirect cost for the use of their services. The connections of the charities, as well as their quality, are exogenous information that the planner cannot control, and they are typically taken into account when making a decision on how to allocate the resources. For instance, charities heavily funded by the government include UNICEF or the Red Cross. While both charities overlap in some of the agents that they serve (e.g. children in need), they also have

[5] More general punishment functions are discussed in the Appendix.

large difference in their recipients.[6] The quality of the connections of the charities is also important when picking a charity. For instance, inefficiencies happen often in charities and universities, where every dollar spent is often decreased due to indirect cost, which serves to pay for administration.[7] Thus, the planner should care about how their money is distributed to the agents and aligned with its preferences. Our model looks at the case of complete information, which is also the case in this example, as the priorities and activities of the charities are typically reported by them in advance.[8] As such, the planner can make an informed decision on how its money will transmit by the charities chosen.

Finally, the problem has applications to network flow problems. For instance, when there is groundwater that must be distributed to agents via private canals (intermediaries). The planner can decide how to route the water to the canals, but once the water reaches the canal it is distributed to the agents connected to these canals in some fixed proportions that may vary between canals. Conveyance losses are typical in models and may depend on how far the agents are from the source (Jandoc, Juarez, and Roumasset [87] study the optimal allocation of water networks in the presence of these losses). The owners of the canals may charge the planner for the use of their canals, and therefore the planner should consider the trade-offs between allocating goods to cheap canals as opposed to more efficient but expensive canals. The paper studies the case of exogenous quality of the intermediaries.

3.1.4 Related Literature

The literature on strategy-proofness when money is available has been widely explored. Indeed, the traditional VCG mechanisms in Vickrey [133], Clarke [43], Groves [68] are strategy-proof and efficient. However, one limitation of VCG mechanisms is that they are not budget balance, which does not apply to our model.

The large literature on social choice has been concerned with non-manipulable mechanisms, dating back from Arrow [6] and Gibbard [62], see Barberà [13] for an introduction to strategy-proof

[6]Thus, for instance, the Federal Emergency Management Agency may be more interested in allocating money to the Red Cross, which distribute a large percentage of their resources to helping domestic citizens affected by disasters, as opposed to UNICEF which helps children around the world.

[7]This factor in the quality of the charities is so important that all charities in the US are required by law to report the total percentage amount spent in their causes, as opposed to administrative costs. For instance, the current indirect costs for the Red Cross and UNICEF are 9.7% and 4.74%, respectively. Multiple online websites exist that rank charities based on the indirect costs, among other metrics.

[8]The Red Cross publishes at the end of each year 'its activities in the field and at the headquarters during the coming year,' which allow donors to make an informed decision on where the money will go. Earmarking is typically not allowed in such big charities, as 'experience shows that the more restrictive the earmarking policy (whereby donors require that their funds be allocated to a particular region, country, program, project or goods), the more limited the ICRC's operational flexibility, to the detriment of the people that the ICRC is trying to help.'

social choice functions. Such studies include the case of strategy-proof social choice functions in classical exchange economies (Barberà and Jackson [16]), matching with contracts (Hatfield and Kojima [74]), house allocation with prices (Miyagawa [105]), cost sharing (Moulin and Shenker [113], Moulin [109], Sprumont [131]), preference aggregation (Bossert and Sprumont [29]), social choice (Barberà, Dutta and Sen [15]).

Barberà, Berga and Moreno [14] study group strategy-proof mechanisms, in a general setting that includes the provision of private good and matchings such as house allocation. Moulin [108], Juarez [90] study group strategy-proof in cost sharing problems.

There is also strategy-proof mechanisms for restricted domain of preferences, such as the class of single-peaked preferences (Moulin [107]). Our focus in the paper is in the entire domain of quasi-linear preferences, where the class of strategy-proof mechanisms that satisfy desirable conditions tends to be small. Hence our work expands the class of strategy-proof mechanisms that the planner can use.

There is also a more recent literature dealing with various relaxations and strengthening of strategy-proof notions. There are approximately strategy-proof mechanisms in voting (Birrell and Pass [23]), matching (Pathak and Sönmez [121]), and more generally, Carrol [36] finds that local strategy-proof with single-crossing ordinal preferences implies full strategy-proof. Obviously strategy-proof mechanisms in Li [97] refine the strategy-proof mechanisms by requiring the s-trategy to be obviously dominant. Pathak and Sönmez [121] develops a rigorous methodology to compare mechanisms based on their vulnerability to manipulation. Unlike this literature on strategy-proofness, our notion of manipulation depends on two variables selected by the planner, the punishment function h as well as the probability of punishment c. This allows for a weaker notion of manipulation that expands the class of strategy-proof mechanisms that a planner can use, hence providing more flexibility when selecting mechanisms. Indeed, our more general version of strategy-proofness can be easily adapted to these settings.

In contrast with the literature on strategy-proofness, our mechanisms are specifically applied to a novel problem of resource transmission in a network. On this line of work, there is only one closely related, our companion paper, Han and Juarez [71], which study the strategic behavior of intermediaries in a more general resource transmission game. Unlike that paper, our model with incomplete information does not restrict the type of mechanisms to a first-price type of mechanism, instead, it characterizes a large class of mechanisms in a more specific resource transmission game in a network.

Townsend [132] first studies costly verification in a principal-agent model with a risk-averse agent. There is a growing interest in mechanism design problem with state verification, Ben-Porath et al. [18] study the principal allocating an indivisible good among agents with an ability to verify agents' type costly, and they don't allow transfer payments. They study the principal's trade-off between allocating the good more efficiently and incurring the cost of verification, and find the optimal mechanism to be a favored agent mechanism, where a pre-determined agent receives the good, unless another agent reports higher than threshold and agent with highest bid will get the good, if his report is verified to be true. Erlanson and Kleiner [57] study similar problem of costly verification in collective choice problem. Li [98] studies costly verification with limited punishment. On the other hand, Carroll and Egorov [37] studies the mechanism of minimal verification to elicit multidimensional information fully by using a randomized verification strategy and allowing severe punishment. However, we study the mechanism design problem with planner allocating divisible good with report of multidimensional information, allowing exogenous probabilistic verification, and study the minimal punishment as the transfer payments to induce the strategy-proof for a mechanism and also to achieve the first-best allocation.

3.2 The Model

A planner is endowed with I[9] unit of divisible good. He is interested in transmitting the resource to a group of agents $\mathcal{M} = \{1, \ldots, M\}$, but he can only do so via a set of intermediaries $\mathcal{N} = \{1, \ldots, N\}$. Every intermediary $i \in \mathcal{N}$ has a **quality of intermediation** (or simply refer to as quality) $\alpha_i = (\alpha_i^1, \ldots, \alpha_i^M) \in \mathbb{R}_+^M$ that represents the proportions in which intermediary i can transmit the good to agents. That is, if x_i units are assigned for transmission by intermediary i, then $x_i\alpha_i$ units are received by the agents. Let $\alpha = (\alpha_1, \ldots, \alpha_N)$ be the **quality of intermediaries**. Define $\alpha_i(\mathcal{M}) = \sum_{m \in \mathcal{M}} \alpha_i^m$ to be the aggregate intermediation quality of intermediary i and $\alpha_{-i}(\mathcal{M}) = (\alpha_j(\mathcal{M}))_{j \in \mathcal{N} \setminus i}$ the abilities of the intermediaries in $\mathcal{N} \setminus i$.

We assume that information is asymmetric, the quality of intermediation is private information. Intermediaries know their own quality but do not know others'. Furthermore, the quality of every intermediary is unknown to the planner. Therefore, a mechanism that split the resource to intermediaries for further transmission to the agents is needed. We focus on mechanisms where the planner makes an allocation of the resource to intermediaries based on their reports of quality of intermediation. Assume intermediary i reports his quality $\beta_i \in \mathbb{R}_+^M$, $\beta = (\beta_1, \ldots, \beta_N)$ and $\sum_{m=1}^M \beta_i^m = \beta_i(\mathcal{M})$. The planner also charges the sharing rates of the intermediaries to agents.[10]

[9] $I = 1$ in most of the discussion.

[10] Thus for instance, the planner might decide to use certain intermediaries to transmit resource to an agent while other intermediaries to transmit resource to a different agent. Since every agent is allowed a charge, this model extends the case where intermediaries are allowed to charge a single price for their links (Han and Juarez [71]) or the case where intermediaries are allowed a proportional constraint.

Definition 14 (Mechanism)
A mechanism $\phi = (x(\cdot), s(\cdot))$ is a pair of functions $(x(\cdot), s(\cdot))$ such that

i. *$x : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+^N$ allocates the share of a resource to every intermediary based on the reported quality of intermediation β. That is, for a quality of intermediation β and $x(\beta) = (x_1(\beta), \ldots, x_N(\beta))$, the amount $x_i(\beta) \in \mathbb{R}_+$ represents the resource allocated to intermediary i.*[11]

ii. *$s : \mathbb{R}_+^{NM} \mapsto \mathbb{R}^{NM}$ represents the rates at which the planner transmits resource through the intermediaries. Thus, for a quality of intermediation β and $s(\beta) = (s_1(\beta), \ldots, s_N(\beta))$, the vector $s_i(\beta) \in \mathbb{R}_+^M$ is the sharing rates charged by the planner to transmit via intermediary i.*[12]

For a mechanism $\phi(\cdot) = (x(\cdot), s(\cdot))$ and reported quality of intermediaries β, the final allocation to agents is $\sum_{i=1}^N s_i(\beta)x_i(\beta) \in \mathbb{R}_+^M$.

Intermediaries gain by the surplus of resource transmitted between the charged rates $s_i(\beta)$ and their quality of intermediation α_i. Thus, intermediary i receives a profit $(\alpha_i - s_i(\beta))^T \mathbf{1} x_i(\beta)$ when the reported quality of intermediaries is β. Note that $\mathbf{1} = (1, \ldots, 1)_{M\times 1}$.

The generality of a mechanism allows for a variety of properties not covered in previous literature. For instance, we do not assume that the charged rates of any intermediary i is always below the reported sharing rates, $s_i(\beta) \leq \beta_i$ for any i. Our analysis allows for some intermediaries to be charged to only transmit resource to some agents.

Example 6
The mechanism $\phi = (x(\cdot), s(\cdot))$ is the

i. ***Equally-Sharing (ES)**: $x_i(\beta) = \frac{1}{N}$, $s_i(\beta) = \beta_i$, $\forall i$.*

ii. ***Equally-Sharing rates (ESR)**: $x_i(\beta) = \frac{\beta_i(\mathcal{M})}{\sum_{i=1}^N \beta_i(\mathcal{M})}$, $s_i^m(\beta) = \min_{n\in\mathcal{N}} \beta_n^m$, $\forall i$.*

iii. ***Second price mechanism (SPM)**: $x(\beta)$ satisfies: there exists i, s.t. $\beta_i(\mathcal{M}) = \max_{n\in\mathcal{N}} \beta_n(\mathcal{M})$ and $x_i(\beta) = 1$, $\forall j \neq i$, $x_j(\beta) = 0$. $s(\beta)$ satisfies: $s_i(\beta) = \max_{n\neq i} \beta_n(\mathcal{M})$ and $s_j(\beta) = \beta_j$, $\forall j \neq i$.*

iv. ***First price mechanism (FPM)**: $x(\beta)$ satisfies: there exists i, s.t. $\beta_i(\mathcal{M}) = \max_{n\in\mathcal{N}} \beta_n(\mathcal{M})$ and $x_i(\beta) = 1$, $\forall j \neq i$, $x_j(\beta) = 0$. $s(\beta)$ satisfies: $s_i(\beta) = \beta_i$.*

[11] $x(\beta)$ is fully differentiable except some points with measurement 0.
[12] It is equivalent with intermediaries charging cost proportional to the amount allocated to intermediary i, rather than a fixed cost discussed in Han and Juarez [71].

ES always allocates the resource equally through each intermediary. The sharing rates $s_i(\beta)$ equal to the reported quality of intermediation β_i.

The ESR mechanism always allocates resource with the same sharing rates $s_i(\beta)$ through all intermediaries with the share equal to the ratio of intermediary i's aggregate intermediation quality $\beta_i(\mathcal{M})$ over the aggregate intermediation quality of all intermediaries $\sum_{i=1}^{N} \beta_i(\mathcal{M})$.

The second price mechanism always allocates the resource through intermediary with highest sum of intermediation quality and chooses the sharing rates $s_i(\beta)$ equal to second highest sum of quality.

A mechanism is budget balance when all the resource is allocated through intermediaries to the agents.

Definition 15 (Budget Balance)
The mechanism $\phi = (x(\beta), s(\beta))$ is budget balance if the resource allocated to the intermediaries sums up to the total resource, which means $\sum_{i=1}^{N} x_i(\beta) = 1$, for any β.

All the mechanisms discussed above are budget balance.

Definition 16 (Symmetric)
The mechanism $\phi = (x(\beta), s(\beta))$ is symmetric if the resource allocated to the intermediaries $x_i(\beta)$ and the sharing-rates $s_i(\beta)$ satisfy: $x_i(\beta) = x_j(\beta')$, $s_i(\beta) = s_j(\beta')$ for any $\beta_i = \beta'_j$, $\beta_j = \beta'_i$ and $\beta_m = \beta'_m$ for any $m \neq i, j$.

All the mechanisms discussed above are symmetric.

3.3 ch-Strategy-Proof Mechanisms

A punishment function $h : \mathbb{R}_+^{2M} \mapsto \mathbb{R}$, $h(a, b)$ can be interpreted as the punishment of an intermediary to report $b \in R_+^M$, if when the true quality of intermediation is $a \in R_+^M$. Assume there is no punishment for truthful report, $h(a, b) = 0$ if $b = a$.[13] $c \in [0, 1]$ is the probability of punishment.

Definition 17 (ch-Strategy-Proof)
The mechanism $\phi = (x(\cdot), s(\cdot))$ is ch-strategy-proof (ch-SP) if for any intermediary i and for any quality of intermediation α_i and β_i, there is

$$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$$

ch-strategy-proof mechanisms can be understood in a way that planner has probability c auditing the report β_i and finds out true value α_i. Planner imposes a punishment $h(a, b)$ when report and

[13] We do not assume that the punishment is negative, as will be illustrated below.

true value are different. The intermediaries choose to report the intermediation quality to maximize the expected profit.

Proposition 1

A mechanism $\phi = (x(\cdot), s(\cdot))$ is ch-SP, then there exists a function $\Phi : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+^N$ such that:

i. The aggregate rate that intermediary i charged equals $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$.

ii. Assume punishment function $h(\alpha, \beta)$ is differentiable at each point $h(\alpha, \alpha)$, where $\alpha = \beta$.[14] *For each i, m and α_i, $\frac{\partial \Phi_i(\alpha_i, \beta_{-i})}{\partial \alpha_i^m} = (1 + c \cdot h_{2m}(\alpha_i))x_i(\alpha_i, \beta_{-i})$ with $h_{2m}(\alpha_i) = \lim_{\beta_i^m \to \alpha_i^m} \frac{h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i)}{\alpha_i^m - \beta_i^m}$, for $\alpha_i^{-m} = \beta_i^{-m}$.*

iii. If $h_{2m}(\alpha_i) = d$, then $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})}(1+cd)\hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$, with $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$ and $\frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^m} = (1 + cd) \cdot x_i(\alpha_i, \beta_{-i})$.

From part i, the function

$$\Phi_i(\beta) = \sum_{m=1}^{M} (\beta_i^m - s_i^m(\beta))x_i(\beta)$$

is the profit of intermediary i when he truthfully reports $\beta_i = \alpha_i$ at the profile $\beta = (\beta_i, \beta_{-i})$. From part ii, the profit function Φ_i is monotonic as β_i increases.

These are local conditions of strategy-proof for deviation of α to β. The conditions are not sufficient for ch-SP, and global conditions of strategy-proof are needed.

Theorem 6

The following three conditions are equivalent:

i. A mechanism is 0-SP.

ii. For any probability of punishment $c \in [0, 1]$ and any punishment function $h(a, b)$, such that the derivative at the truthful report is zero,[15] *$h_{2m}(\alpha_i) = 0$ for any α_i.*

iii. There exists a function $\hat{x}_i : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$ non-decreasing in the first coordinate such that for any β: $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$ and $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

There are two important consequences of Theorem 6. On one hand, it provides precise conditions for a mechanisms to be 0-SP, the traditional strategy-proof condition discussed in the literature.

[14] The more general case, when h is not differentiable, will be discussed in Appendix.

[15] This happens, for instance, at the large class of polynomial punishment functions $h(\alpha_i, \beta_i) = \sum_{m=1}^M \gamma_m(\alpha_i^m - \beta_i^m)^{k_m}$ for some $k_m > 1$ for all m.

First, the allocation of the resource to an intermediary should depend on his aggregate intermediation quality instead of specific quality of transmission to agents. Second, the charged share to an intermediary depend on the average allocation over all the qualities $\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt$.

On the other hand, another consequence of Theorem 6 shows that any punishment function h, whose derivative at the truthful report is zero, is ineffective for any probability of punishment $c \in [0, 1]$. That is, such a punishment function h will generate exactly the same class as if there is no punishment, the set of mechanisms in 0-SP. There is a large number of functions that meet this condition, including the class of polynomial punishments.

Corollary 6
Given the payoff function of intermediaries $(\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i})$, the charge rate $s_i^m(\beta)$ is perfectly substituted among the links between intermediaries and agents when $\sum_{m=1}^M s_i^m(\beta)$ is fixed.

The following corollary complements the characterization above for the case where the punishment ch is infinite.

Corollary 7
A mechanism $\phi = (x(\cdot), s(\cdot))$ is ∞-SP, if it is ch-SP and $ch(a, b) = \infty, \forall b \neq a$. Any mechanism ϕ is ∞-SP.

Given the results in Theorem 6, the remaining discussion of the paper deals with specific punishment functions where the derivative at the truthful report is non-zero or where the derivative does nor exists (in Appendix).

Proposition 2
For any parameters c, c' and functions h, h', s.t. $ch(a, b) \leq c'h'(a, b), \forall a, b \in \mathbb{R}_+^M$. Then any ch-SP mechanism $\phi = (x(\cdot), s(\cdot))$ is $c'h'$-SP.

This proposition shows the result of comparative static analysis of ch-SP mechanisms. As probability of punishment c or punishment function h increases, the set of ch-SP mechanisms expands. The result is consistent with intuition that punishment would decrease the incentives of intermediaries to misreport.

3.4 Minimal Punishment Function

It is often the case that a mechanism to allocates goods and services is given, whereas the designer of the mechanism has the flexibility to design the punishment function (c, h). In this section, we ask the question: What is the class of punishment functions that makes the mechanism ch-strategy-proof. In this section, assume c does not depend of report β, without loss of generality, let $c = 1$, we study the properties of punishment function h.

The following proposition shows the linear combination of punishment functions, which make mechanisms ch-SP, also guarantees the linear combination of the mechanisms to be ch-SP.

Proposition 3 (Convexity of Punishment Function h)
Suppose the mechanism $\phi_1 = (x_1(\cdot), s_1(\cdot))$ and $\phi_2 = (x_2(\cdot), s_2(\cdot))$ satisfy $x_1(\beta) = x_2(\beta)$ for any β, and ϕ_1 is h-SP for punishment function h_1, ϕ_2 is ch-SP for punishment function h_2, then $\phi = (x(\cdot), s(\cdot)) = \lambda\phi_1 + (1-\lambda)\phi_2$, for which $x(\beta) = x_1(\beta) = x_2(\beta)$, and $s(\beta) = \lambda s_1(\beta) + (1-\lambda)s_2(\beta)$. Then ϕ is ch-SP for punishment function h, with $h(\alpha_i, \beta_i) = \lambda h_1(\alpha_i, \beta_i) + (1-\lambda)h_2(\alpha_i, \beta_i)$ for any α_i, β_i.

In the rest of this section, we are going to discuss the minimal punishment function h for any mechanism ϕ, such that ϕ is ch-SP.

For any mechanism $\phi = (x(\cdot), s(\cdot))$, assume the sum of sharing rates charged by planner is $s_i(\beta, \mathcal{M}) = \sum_{m=1}^{M} s_i^m(\beta)$, and the profit function of intermediary i is $v_i : \mathbb{R}_+^{M(N+1)} \mapsto \mathbb{R}_+$. If the intermediation quality of intermediary i is α_i and reports of all intermediaries are β, the profit of intermediary i is $v_i(\alpha_i, \beta) = (\alpha_i(\mathcal{M}) - s_i(\beta, \mathcal{M}))x_i(\beta)$. The profit of intermediary i for truthfully report is $v_i(\alpha_i, \alpha_i, \beta_{-i}) = (\alpha_i(\mathcal{M}) - s_i(\alpha_i, \beta_{-i}, \mathcal{M}))x_i(\alpha_i, \beta_{-i})$. So intermediary i has incentive to report truthfully if $v_i(\alpha_i, \alpha_i, \beta_{-i}) \geq v_i(\alpha_i, \beta) - h(\alpha_i, \beta_i)x_i(\beta)$, for any α, β.

Definition 18 (Minimal Punishment Function)
For any mechanism $\phi = (x(\cdot), s(\cdot))$, $h_i^{\min} : \mathbb{R}_+^{2M} \mapsto \mathbb{R}_+$ is minimal punishment function for intermediary i, if for any punishment function $h(\alpha_i, \beta_i)$, such that ϕ is ch-SP for intermediary i with punishment h, then $h(\alpha_i, \beta_i) \geq h_i^{\min}(\alpha_i, \beta_i)$, $\forall \alpha_i, \beta_i$.

From the definition of minimal punishment function, if mechanism ϕ is symmetric, the minimal punishment function is the same for all intermediaries. We focus on symmetric mechanism in the following. For simplicity, assume the minimal punishment function is denoted as $h^{\min} = h_i^{\min}$.

The following result shows that there exists a minimal punishment at every profile and misreport in order to achieve strategy-proofness.

Proposition 4 (Minimal Punishment Function)
Consider the mechanism $\phi = (x(\cdot), s(\cdot))$, the profit of intermediary i is $v_i(\alpha_i, \beta)$, $v(\alpha, \beta) = (v_1(\alpha_1, \beta), \ldots, v_N(\alpha_N, \beta))$ when the true profile is α and reported profile is β. The punishment function h, which guarantees mechanism ϕ to be ch-SP, satisfies: $h \geq h^{\min}(\alpha_i, \beta_i) = \max_{\beta_{-i}}[\frac{v_i(\alpha_i, \beta) - v_i(\alpha_i, \alpha_i, \beta_{-i})}{x_i(\beta)}]$, for any $\beta_i, \alpha_i, \beta_{-i}$, such that $x_i(\beta) > 0$.

We can interpret the function $h^{\min}(\alpha_i, \beta_i) = \max_{\beta_{-i}}[\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)}]$ as the minimal punishment that intermediary i needs to incur, when the true profile is α_i but he actually reports β_i.

In particular, we note that the minimal punishment for a strategy-proof mechanism satisfies $h(\alpha_i, \beta_i) = 0$, for any α_i, β_i. Thus any strategy-proof mechanism is ch-SP. On the other hand, if a mechanism is not strategy-proof, the minimal punishment function for the mechanism has to be non-zero.

Corollary 8 (Properties of Minimal Punishment Function $h^{\min}$)

i. For any symmetric and monotonic mechanism $\phi = (x(\cdot), s(\cdot))$, there exists minimal punishment function $h^{\min}$.

ii. If the mechanism is strategy-proof, then $h^{\min}(\alpha_i, \beta_i) = 0$, for any α_i, β_i.

iii. If the mechanism is not strategy-proof, then the minimal punishment function is nonzero. In other words, there exists α_i, β_i, such that $h^{\min}(\alpha_i, \beta_i) > 0$.

The following example discusses the minimal punishment function for first price mechanism and second price mechanism.

Example 7

Consider the first price mechanism $\phi_F = (x_F, s_F)$, and second price mechanism $\phi_S = (x_S, s_S)$, $x_S = x_F$ satisfies: for any i, $\beta_i(\mathcal{M}) < \max_{n\in\mathcal{N}} \beta_n(\mathcal{M})$, $x_{Si}(\beta) = 0$. For any i, $\beta_i(\mathcal{M}) = \max_{n\in\mathcal{N}} \beta_n(\mathcal{M})$, $x_{Si}(\beta) = \frac{1}{k(\beta)}$, $k(\beta)$ is the number of intermediaries with largest $\beta_i(\mathcal{M})$.

The sharing rates for first price mechanism is $s_F(\beta) = \beta$, which means the intermediaries are charged at the rates they report. For second price mechanism, the sharing rates $s_{Si}(\beta) = \beta_i$ for i with $\beta_i(\mathcal{M}) \leq \max_{j\neq i} \beta_j(\mathcal{M})$, and $s_{Si}(\beta) = \max_{j\neq i} \beta_j(\mathcal{M})$ for i with $\beta_i(\mathcal{M}) > \max_{j\neq i} \beta_j(\mathcal{M})$. The second price mechanism sharing rates s_S and allocation x_S satisfies $s_{Si}(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_{Si}(t,\beta_{-i})dt}{\hat{x}_{Si}(\beta_i(\mathcal{M}),\beta_{-i})}$, thus, second price mechanism is strategy-proof, so the minimal punishment function for the second price mechanism is $h(\alpha_i, \beta_i) = 0$.

The minimal punishment function for first price mechanism is $h(\alpha_i, \beta_i) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$ with $(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+ = \max\{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}), 0\}$.

Consider a mechanism ϕ, which is linear combination of ϕ_F and ϕ_S, satisfies $\phi = \epsilon\phi_F + (1-\epsilon)\phi_S$ with $\epsilon \in [0, 1]$. From Corollary 8, the minimal punishment function for the mechanism ϕ is $h^{\min}(\alpha_i, \beta_i) = \epsilon(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$.

Example 8

Consider the equally sharing rule of resource allocation, $x_i(\beta) = \frac{1}{N}$, which means the planner always allocates $\frac{1}{N}$ to each intermediary. The equally sharing mechanism ϕ_1 in Example 6, satisfying $s_{1i}(\beta) = \beta_i$, is not strategy-proof. The intermediary has higher profit reporting *lower than the true quality of intermediation. The ch-SP condition requires* $h(\alpha_i, \beta_i)x_i(\beta) \geq \max_{\beta_{-i}}[v_i(\alpha_i, \beta) - v_i(\alpha_i, \alpha_i, \beta_{-i})]$,*substitute allocation* x_i *and sharing rates* s_i *into the inequality,* $\frac{h(\alpha_i,\beta_i)}{N} \geq \frac{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})}{N} - 0$, *thus,* $h(\alpha_i, \beta_i) \geq \alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})$. *The minimal punishment function* $h^{\min}(\alpha_i, \beta_i) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$

The equally sharing strategy-proof mechanism ϕ_2 *satisfies* $x_{2i}(\beta) = \frac{1}{N}$ *and* $s_{2i}(\beta) = \mathbf{0}$. $\mathbf{0} = (0, \ldots, 0)_{M\times 1}$. *The minimal punishment function for this mechanism is* $h_2^{\min} = 0$, *but no resource will be transmitted to agents.*

Finally, notice that Example 8 shows the strategy-proof mechanism may transmit nothing to agents. The goal of planner is to send resource to agents in need via intermediaries, so the class of strategy-proof mechanisms may not be good in some circumstances, punishment based on verification is necessary to achieve larger resource allocated to agents. The following section will discuss the preferences of planner, how will the punishment help improving the resource sent to agents.

3.5 First-Best Efficiency

We need to emphasize that we are in a setting of transmitting scarce resource to people in need, where the planner does not care about intermediaries, but only the agents. The resource allocation to agents $y \in \mathbb{R}^M_+$ satisfies $y = \sum_{i=1}^N s_i(\beta)x_i(\beta)$. Assume planner's preferences $\succeq$ over the resource allocation to agents y is monotonic, and there exists utility function $u : \mathbb{R}^M_+ \mapsto \mathbb{R}$ to represent the preferences.

The following definition states the first-best outcome from the perspective of planner.

Definition 19 (First-best Efficient (FBE))
Given the preferences of planner $\succeq$ *and the utility function* $u : \mathbb{R}^M_+ \mapsto \mathbb{R}_+$. *A mechanism* ϕ *is first-best efficient (FBE), if for any profile of intermediaries* α*, the resource allocated to the agents* y *maximizes the planner's utility as if there is no intermediation, under the condition of individual rationality. The individual rationality means that intermediaries have nonnegative profit. If the preferences is strictly convex, then there exists a unique resource allocation that maximizes planner's utility given any profile* $\alpha = (\alpha_1, \ldots, \alpha_N)$.

Given the intermediation quality α, the maximal utility $\bar{u}(\alpha)$ equals $\max_x u(\sum_{i=1}^N x_i\alpha_i)$, such that $\sum_{i=1}^N x_i = 1$. Assume $\bar{x} : R^{MN}_+ \mapsto \mathbb{R}^N_+$ is the allocation of resource among intermediaries, which maximizes planner's utility when profile of intermediaries is α, $\bar{x}(\alpha) = (\bar{x}_1(\alpha), \ldots, \bar{x}_N(\alpha)) = \arg\max_x u(\sum_{i=1}^N x_i\alpha_i)$.

Notice that FBE implies the planner allocates resource through intermediaries optimally to achieve maximal utility with the true quality of intermediation.

Given mechanism $\phi = (x(\cdot), s(\cdot))$, and the quality of intermediation is α, the utility of planner is $u^*(\alpha, \phi) = u(\sum_{i=1}^N s_i(\alpha)x_i(\alpha))$, when intermediaries truthfully report their quality of intermediation $\beta = \alpha$.

Theorem 7

Assume the preferences of the planner $\succeq$ is strongly monotonic, continuous, and there exists utility function $u : \mathbb{R}_+^M \mapsto \mathbb{R}$ representing the preferences.

i. There is no symmetric, SP, budget balance and first-best efficient mechanism.

ii. For any FBE mechanism ϕ, the minimal punishment function $h^{\min}$ for ϕ to be ch-SP satisfies $h^{\min}(\alpha_i, \beta_i) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$. Any punishment function h that implements a FBE mechanism if and only if $h(\alpha_i, \beta_i) \geq h^{\min}(\alpha_i, \beta_i) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$ for any $\alpha_i \geq \beta_i$.

Theorem 7 shows there exists no mechanism satisfying symmetric, budget balance, 0-SP and FBE. It also provides the condition of minimal punishment function for mechanism to achieve the first-best efficient. The minimal punishment function is the same with the one in Example 7 to guarantee the first price mechanism to be ch-SP, which is not surprising because the first price mechanism is FBE for some special planner's preferences. The following example shows that the first price mechanism is the first-best efficient when the resource transmitted to agents is perfect substitute for planner, while the second price mechanism is SP, but not FBE.

Example 9

Consider the preferences of planner is perfect substitute and represented by utility function $u(y) = \sum_{i=1}^M y_i$. Then the first price mechanism ϕ_F is FBE but not SP, and the second price mechanism ϕ_S is SP but not FBE.

When the planner only cares about the sum of resource transmitted to all agents, the first-best outcome is to allocate the resource through a intermediary with largest aggregate intermediation quality $\alpha_i(\mathcal{M})$ and charge the sharing rates equal to his true quality of intermediation. Assume the aggregate intermediation quality ranking from high to low is $\alpha^1(\mathcal{M}) \geq \cdots \geq \alpha^N(\mathcal{M})$, the maximal utility $\bar{u}(\alpha) = \max_{i \in \mathcal{N}} \alpha_i(\mathcal{M}) = \alpha^1(\mathcal{M})$.

Thus, the first price mechanism, which allocates all resource to intermediary i with largest aggregate intermediation quality $\alpha_i(\mathcal{M})$ and transmit with sharing rates equal to report. The final allocation $y = \alpha^1$ and utility $u^*(\alpha, \phi_F) = \alpha^1(\mathcal{M})$, which is FBE. However, the second price mechanism, which is strategy-proof, but it has to pay the intermediary i with largest aggregate intermediation $\alpha_i(\mathcal{M}) - \max_{j \neq i} \alpha_j(\mathcal{M})$ as information rent, The planner's utility $u^*(\alpha, \phi_S) = \alpha^2(\mathcal{M})$. When $\alpha^2(\mathcal{M}) < \alpha^1(\mathcal{M})$, $u^*(\alpha, \phi_S) < \bar{u}(\alpha)$, the second price mechanism can not achieve FBE.

This section shows that punishment is necessary to achieve the first-best efficient for the planner, and the minimal punishment function for first-best efficient mechanism to be ch-SP coincides with the minimal punishment function for first price mechanism to be ch-SP. Verification and punishment could be used to expand the class of strategy-proof mechanisms and achieve higher efficiency for planner.

3.6 Conclusion

This paper investigates the class of strategy-proof mechanisms for the problem of resource transmission with intermediation on networks. The mechanism requires the intermediaries to report their quality of intermediation, transmits the resource according to the sharing rates based on the report, and imposes punishment for misreporting.

This paper is a start to study the strategy-proof mechanisms with punishments. We discover and describe the sets of strategy-proof mechanisms with various punishment functions. The conditions for strategy-proofness are provided based on the share of resource transmission and sharing rates of intermediaries to satisfy. We also demonstrate the class of strategy-proof, symmetric, budget balance mechanisms with three cases of punishment function. With linear punishment function, the strategy-proof, symmetric, budget balance mechanisms require the sharing rule to depend only on the sum of quality of links.

The mechanism design approach in this chapter is a complement to the game theoretical approach from chapter 2. This chapter shows how to provide incentives for intermediaries to report the true quality of intermediation. Along with chapter 2, we have provided a complete description of how to find the optimal mechanisms for the planner, both under complete and incomplete information.

4 FINANCIAL NETWORK AND INDUSTRY CONNECTEDNESS

4.1 Introduction

The financial crisis in subprime mortgage market spread through the collapse of big investment banks, like Lehman Brothers, Merrill Lynch, and finally turned into an international banking crisis from 2007 to 2009. After the financial crisis, regulations, like Basel III capital and liquidity standards, are adopted by countries all around the world to promote the financial stability. The regulation introduces requirements on liquid asset holdings and minimal capital ratios, but it does not take the cross holding of asset into consideration. The high connectedness of financial market is found to be the reason of risk spillover throughout the system in Yellen [136], and it motivates new regulation rules on the interactions between financial institutions. In 2014, the Basel Committee on Banking Supervision [45] introduces the requirement of caps to restrain large exposures between banks. This requirement limits the exposure and risk sharing between banks, especially global systemically important banks (G-SIBs). There is growing researches about the influence of the architecture of financial system on systemic risk, risk spillover and the extent of contagion. Glasserman and Young [63] is a great survey about financial networks and contagion, summarizes the progress of theoretical models of financial network and empirical tools in estimating systemic risk based on the balance sheets and comovements.

At the same time, the finance industry plays an important role of intermediation in economic growth with its services for other industries through the process of savings, borrowing, and investment. Financial institutions borrow from people who want to save money and lend to companies that need resources for investment. The collapse of bankings results in fewer channels for funding and new investment, and the decline of interest rates, increase of unemployment bring large uncertainty to other industries. It is interesting to study the risk contagion not only within the industry of finance but also between the industry of finance and other industries. The connectivity among industries is also important for amplifying and dissipating shocks from one sector to the whole economic system. Moreover, the development of empirical research studies in financial networks bring tools to study the interconnectedness of system. This chapter uses the tools in financial networks to study the interconnectedness between industries based on the comovements of industry portfolios.

[0]Joint work with Qianqiu Liu

Diebold and Yilmaz [53] study the interconnectedness of financial institutions and find that there is increasing connectedness during a financial crisis. We apply the network analysis method to investigate the interconnectedness among industry portfolios and discuss the problems: How will the risk of finance industry spread to other industries, especially during the financial crisis in 2007 to 2009? Will the interconnectedness increase during recessions for the industry network? How will the risk spillover relationships between industries change over time? Which industry is risk receiver/sender? Which industry has high/low centrality in the dynamic networks? To answer these questions, we collect the data of daily return of value-weighted and equal-weighted portfolios of industries from 1927 to 2017, and estimate the comovement of portfolios' volatility, then construct the networks of industries to investigate the connectedness and risk spillover between industries. The estimation of comovement adopts the generalized variance decomposition method in Demirer et al. [49].

There is substantial research done regarding the connections between financial institutions with different types of relationships, such as counter-party relationships related to liability positions in Eisenberg and Noe [54], cross-share holding relationships in Elliott et al. [55], and social network of CEO for information flow in Engelberg et al. [56], risk sharing relationships with counter party in Babus [7]. However, there is little empirical research on connectedness beyond the industry of finance, especially the relationships of risk spillover between different industries. For a thin industry categorization, the large number of industries require estimation of high dimensional networks. The high dimensional parameters are difficult to estimate with the traditional econometric method. Thus, in this chapter, we adopt the machine learning method to solve the problem of high dimensionality. We find the network of industry portfolios change in different business cycles, the networks of volatility connectedness show the finance industry is the central node for risk contagion in the financial crisis of 2007-2009.

4.1.1 Related Literature

As mentioned above, Glasserman and Young [63] offers a survey for literature related to financial network and discuss how the network structure could influence the vulnerability of the system to risk contagion together with other variables, such as risk exposures, short-term funding, and leverage. Allen and Gale [4] build a theoretical model to analyze the contagion of risk in financial systems by studying the liquidity risk sharing and funding runs, focusing on the trade-off between risk diversification and exposure through connections. They find the contagion in a network is driven by the relationship between network structure and the correlation in funding shocks across banks, so not only the level of connectivity matters.

Eisenberg and Noe [54] build model of interconnected balance sheets to study how shortfall of one node can spread to a series of defaults. Babus [8] considers the endogenous network formation with links for risk sharing between banks and calculate the systemic risk in equilibrium interbank networks. Erol and Vohra [58] introduces a model of endogenous network formation and systemic risk with a link representing potential benefits in trading opportunity if no default. They find that good shock may generate a higher probability of system-wide default because increased interconnectedness in the network offsets the effect of better fundamentals. Acemoglu et al. [3] find the dense network among small financial institutions enhances financial stability, while beyond a threshold, the dense network leads to a more fragile financial system. Elliott et al. [55] study the network of asset cross holding, and find that intermediate levels of diversification and integration can be the most problematic.

Based on the theoretical research of financial network, many empirical works on financial networks study the structure of financial networks, including testing core-periphery network of bank system based on balance sheets. Gofman [64] estimates the network-based model of the over-the-counter interbank lending market and show limits on interconnectedness improve stability and reduce efficiency.

Comparing to the empirical research relying on data covering the balance sheet of financial institutions with low frequency, some research studies adopt correlation-based measures and focus on the comovement of higher frequency return or volatility. The closest paper to our research is Diebold and Yilmaz [52], which uses the method of variance decompositions to estimate networks about the influences of one variable on another, with the weight of link equal to the off-diagonal element in the variance decomposition matrix. This method is used in a series of paper, Demirer et al. [49], Diebold and Yilmaz [53], [50], Diebold et al. [51], which is a generalized approach of Koop et al. [94] and Pesaran and Shin [122]. Billio et al. [22] estimates relationships between banks, insurance companies, hedge funds and brokers through Granger causality test in stock returns, which results in a large sparse network among financial institutions.

Besides the empirical research on estimation of networks, there is a line of research adopting network science method to study the features of networks. This line of work targets on identifying simple network features describing the vulnerability of network, like the connectivity and centrality. Nier et al. [120] simulate random networks in which each pair of banks has a fixed probability of connection through a loan. They study that shocks to a bank's assets spillover to other banks through cascading defaults and find a non-monotonic effect of increasing connection probability on the total number of defaults. Increasing connectivity also increases shock transmission and shock absorption, with the first effect dominating at low connectivity and the second effect dominating at higher connectivity. Gai and Kapadia [60] and Haldane and May [70] also find that higher connectivity is associated with more severe and less frequent crises. Some research study the effect of network centrality. Craig et al. [48] find that higher centrality predicts a lower probability of default. In their study of the Mexican banking system, Martinez-Jaramillo et al. [102] study the structure of payment and exposure networks of Mexican banks, they find the centrality is not necessarily determined by asset size, but also the contagion it may cause. Bech and Atalay [17] study the Fed funds market and find that centrality measures predict the interest rate banks charge each other.

Alter, Craig, and Raupach [5] simulate the loans propagation in a network with random shocks, according to the Rogers and Veraart [126] extension of the Eisenberg–Noe [54] algorithm. They find that reallocating the capital requirements from banks with low centrality to high centrality is effective in minimizing bankruptcy losses. But it is hard to set different capital requirements according to the centrality of each bank in reality because centrality changes endogenously over time.

Puhr et al. [123] use the data of Austrian banks as input to simulate bank failures, distinguish between contagiousness (to-degree) and vulnerability (from-degree), and find contagiousness increases with centrality, but vulnerability depends on several network measures, including clustering. It shows Katz centrality could be a useful indicator in measuring systemic risk, but it is still not clear how important is centrality as a feature in the simulation.

Overall, there is a hole in constructing measurements of networks to estimate the level of financial stability in empirical research. The low-frequency data for interbank balance sheets during the financial crisis may limit the research in this field. This paper contributes to the literature in studying the changes of network structure between industries and calculating the centrality of industries as indicators for the importance of risk contagion. In future research, it is worth developing new measurements for interbank networks based on theoretical analysis, instead of borrowing the measurements directly from network science.

Finally, this paper conveys an empirical study about relationships between the volatility of industry portfolios. The research is related to works about the correlation of industry portfolio returns, including Hong et al. [75] and Rapach et al. [124], who study how the return of one industry could be used to predict returns in other industries or the movements of stock markets. Hong et al. [75] finds the leading industry used to predict the market is correlated with various indicators of economic activity. Their findings suggest that stock markets react with a delay to information contained in industry returns about their fundamentals and that information diffuses gradually across markets. Rapach et al. [124] adopts machine learning method of OLS post-LASSO to study the dynamic dependence of return across industries. Comparing to the research about returns, the volatilities tend to move together during a crisis rather than during a normal period, while returns often move closely together in both crises and upswings. The research on industry portfolios volatility also reflects the dynamics of risk diffusion. Thus, it would be a good complement with research on industry portfolio returns. Brunnermeier and Sannikov [31] finds the volatility paradox, which means the exogenous risk is very low during a crisis, but the endogenous risk, driven by asset illiquidity, persists in crisis. Securitization and derivatives contracts improve risk sharing but bring a higher level of endogenous leverage, which results in the financial crisis.

4.1.2 Overview of the Results

In this paper, we focus on networks of volatility of industry portfolio. Volatility connectedness is of direct interest in financial markets. Since volatility is often used to track investors' fear (VIX - CBOE Volatility Index is a widely used measure of market risk, often referred to as the 'investor fear gauge'), then volatility connectedness measures the spreading process of investors' fear in the whole system. Figure 4.1 shows the CBOE VIX index from 1990 to 2018, the index reaches the peak in 2008, which captures the fear of investor in the financial crisis. The three dark areas in the figure represent three contraction periods of US business cycle, which are Jul 1990 to Mar 1991, Mar 2001 to Nov 2001, and Dec 2007 to Jun 2009. It can be observed that there is sharp increasing of VIX index in each period, so there are higher uncertainty about market return during the recession, but the fear of investors returns to normal level at the end of these periods.

Since the data on balance sheets is low frequency, and not easy to collect because of confidentiality, the network of balance sheets in Eisenberg and Noe [54] is hard to estimate. We collect the daily return of industry portfolio and then construct the network of volatility connectedness. The comovement of industries' risk can be reflected clearly in portfolio network. Based on the estimation of comovement, we can use volatility connectedness network as an indicator for real-time crisis monitoring, to set regulation on certain 'key risk sender' to reduce the influence of potential recession by defending the central node against exogenous shocks. On the other hand, it is interesting to study how risk transmission structures change during different business cycles. It will be helpful to understand the volatility paradox mentioned in Brunnermeier and Sannikovc [31], lower exogenous risk can lead to more extreme volatility spikes in the crisis regime. Total directional connectedness of the dynamic networks increases during the recessions in the early 1990s and financial crisis in 2007-2009. We measure the node centrality for each industry and find that the financial industry becomes the sender of risk and has the highest centrality during the financial crisis, but it is not the central node for most of the time. However, the industry of telecommunications grows fast in the centrality of the network over the past 30 years and becomes a central node in the network of industry volatility.

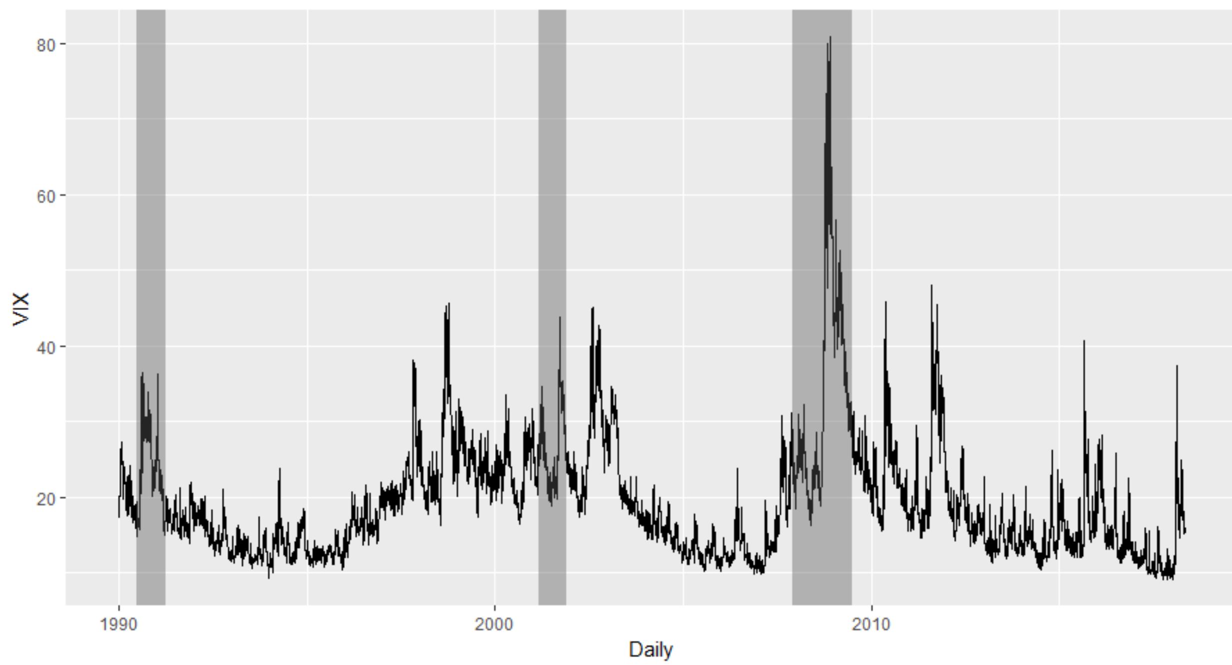


Figure 4.1: CBOE VIX Index: 1990/01/02 to 2018/04/30

Note: The shaded bars are recessions dated by NBER.

We proceed as follows. In Section 4.2, we briefly introduce the Generalized Variance Decomposition method to estimate the connectedness of network with portfolio volatility data and also introduce the centrality measurement of nodes. In Section 4.3, we discuss data source and provide static and dynamic analysis of industry portfolio volatility network. In Section 4.4, we provide robust assessments for different industry categories(48 industry instead), weight in constructing portfolio(equal-weighted instead of value-weighted). The results are consistent. Finally, we conclude in Section 4.5.

4.2 Model

In this section, we first introduce the Generalized Variance Decompositions (GVD) Method to estimate the industry portfolio volatility network based on variance decompositions with time series data. This method is used in a series of papers, Demirer et al. [49], Diebold and Yilmaz [53], [52], [50], Diebold et al. [51]. After constructing the industry portfolio volatility network from data, we apply methods in network science to analyze the structure of static and dynamic networks.

4.2.1 Generalized Variance Decompositions (GVD) Method

To avoid the identification problem for estimation dependent on ordering of variables in Cholesky factorization, we adopt the GVD method following the generalized identification framework of Koop, Pesaran and Potter [94] and Pesaran and Shin [122]. They study the variance decompositions independent of ordering of variables. This method considers correlation, and allows correlated shocks without orthogonalization.

The GVD method studies the comovement of time series to uncover potential links between different variables. The comovement is estimated through both the cross-variable dependence in Vector Auto Regression (VAR) and the shock dependence in the VAR disturbance covariance matrix.

The GVD method is based on N-variable VAR(p), $x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + \epsilon_t$, written in the MA representation as $x_t = \sum_{t=0}^{\infty} A_i \epsilon_{t-i}$. The $N \times N$ matrix A_i satisfies $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \cdots + \Phi_p A_{i-p}$, and $A_i = 0$ for $i < 0$.

j's contribution to i's H steps ahead generalized forecast error variance is

$$\theta_{ij}^{g}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \Sigma A_h' e_i)}, for\ H = 1, 2, \ldots, .$$

where Σ is the covariance matrix of vector ϵ, σ_{jj} is the j^{th} diagonal element of Σ, which is variance of disturbance ϵ_j of equation j. $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)$, $e_{ii} = 1$, $e_{ij} = 0$ for $j \neq i$.

After calculating the generalized variance decomposition matrix $\theta_{ij}^{g}(H)$, we normalize the sum of variance shares of directional connectedness from j to i to 1. The normalized directional connectedness is $\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N} \theta_{ij}^{g}(H)}$. So $\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = 1$.

$\tilde{\Theta}$ is the variance decomposition matrix with element of i^{th} row, j^{th} column element satisfying $\tilde{\Theta}_{ij} = \tilde{\theta}_{ij}^{g}(H)$.

The GVD method is usually used in estimating the comovement network of return or return volatility. We focus on the network of portfolio return volatility in this chapter.

4.2.2 Network Science

Based on the normalized variance decomposition matrix $\tilde{\Theta}$ estimated from GVD method, we introduce the network measurement of connectedness and centrality.

$\tilde{\theta}_{ij}^{g}(H)$ measures the pairwise directional connectedness from j to i at horizon H. Construct the volatility network using $N \times N$ variance decomposition matrix $\tilde{\Theta} = [\tilde{\theta}_{ij}^{g}(H)]$ to calculate the adjacency matrix $C(H)$ of a weighted directed network. The element of adjacent matrix $C_{i \leftarrow j}^{H} =$

$\tilde{\theta}^g_{ij}(H)$, $j \neq i$, measures the connectedness of volatility from j to i. In short, the adjacency matrix $C(H) = \tilde{\Theta} - diag(\tilde{\Theta}_{11}, \ldots, \tilde{\Theta}_{NN})$.

Connectedness

Given the volatility connectedness network with adjacency matrix $C(H)$, here we define the total connectedness (or system-wide connectedness) measurement of the network C^H as following:

$$C^H = \frac{\sum_{\substack{i,j=1 \\ j \neq i}}^{N} \tilde{\theta}^g_{ij}(H)}{N}.$$

C^H is an index for total network connectedness, and it is calculated as the ratio of sum of the off-diagonal elements of variance decomposition matrix $\tilde{\Theta}$ to the sum of all elements.

Given the connectedness of volatility from j to i, as the i^{th} row, j^{th} column element of adjacent matrix $C^H_{i \leftarrow j}$, we define the index for **total directional connectedness from**, **total directional connectedness to**, net directional connectedness, and total net directional connectedness.

The **total directional connectedness from** of node i is $C^H_{i \leftarrow \bullet}$, which measures the total connectedness to i spillover from others. $C^H_{i \leftarrow \bullet}$ equals to the sum of i^{th} row:

$$C^H_{i \leftarrow \bullet} = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{N} \tilde{\theta}^g_{ij}(H)}{N}.$$

The **total directional connectedness to** of node j is $C^H_{\bullet \leftarrow j}$, which measures the total connectedness spillover from j to others. $C^H_{\bullet \leftarrow j}$ equals to the sum of j^{th} column:

$$C^H_{\bullet \leftarrow j} = \frac{\sum_{\substack{i=1 \\ i \neq j}}^{N} \tilde{\theta}^g_{ij}(H)}{N}.$$

In short, the row sums of the adjacency matrix, which is node i's in-degree, is the **total directional connectedness from** of node i. The column sums of the adjacency matrix, which is node i's out-degree, is the **total directional connectedness to** of node i.

We define the net pairwise directional connectedness as $C^H_{ij} = C^H_{j \leftarrow i} - C^H_{i \leftarrow j}$, which measures the net connectedness of i to j. The net total directional connectedness of C^H_i node i satisfies $C^H_i = C_{\bullet \leftarrow i} - C_{i \leftarrow \bullet}$. When $C^H_i > 0$, then node i is a source of diffusion rather than sink. From the net directional connectedness, we can tell whether a node diffuses more to others, or receives more flow from others.

Centrality

The connectedness measurements above provide a general description of the features of network, including the in-degree, out-degree of nodes, and average number of links. Here we introduce three measurements of node centrality, which provides index for the importance of nodes. It is especially useful in identifying the most influential node on a network.

Since the connectedness network is a directed graph, the **degree centrality** include in-degree centrality and out-degree centrality. The total directional connectedness $C^H_{i\leftarrow\bullet}$ to i is equivalent with the in-degree centrality of node i. The total directional connectedness $C^H_{\bullet\leftarrow i}$ from i is equivalent with the out-degree of node i. The degree centrality measures the direct influence of a node on its neighbors, which is the sum of links with ('to' or 'from') the neighbors, with equal-weighted on every neighbor.

Given the adjacency matrix of network is $C(H)$, assume κ_1 is largest eigenvalue of matrix $C(H)$, and the $v \in \mathbb{R}^N$ is the eigenvector of $C(H)$ for eigenvalue κ_1, satisfying $C(H)v = \kappa_1 v$. v is the **eigenvector centrality** of the network. The eigenvector centrality measures the influence of a node on neighbors with higher weighted on the link with more central node.

Besides degree centrality and eigenvector centrality, Newman [118] introduces other centrality measures, including Katz centrality, pagerank centrality.

Katz centrality is a general version of eigenvector centrality. v satisfies $v = \alpha C(H)v + \beta\mathbf{1}$. When $\beta = 0$, the Katz centrality $x = (\mathbf{I} - \alpha\mathbf{A})^{-1}\mathbf{1}$ is the case of eigenvector centrality.

However, the **pagerank centrality** of a node is derived from neighbors' centrality divided by their out degree. Thus, the node links to many other nodes passes only a small amount of centrality to each of the node connected. In mathematics, the pagerank centrality is defined as $v_i = \alpha \sum_j C^H_{ij} \frac{v_j}{k^{out}_j} + \beta$. Assume the matrix $\mathbf{D} = diag_i\{\max(k^{out}_i, 1)\}$, the centrality vector $x = \mathbf{D}(\mathbf{D} - \alpha\mathbf{A})^{-1}\beta$. k^{out}_j is the out-degree of node j.

The measurement of centrality will be an indicator for the importance of node in the network, which may also be helpful for further research about information diffusion on network.

4.2.3 Estimation

Thus far we have discussed calculation for the network of direction connectedness and centrality measurements of graph structure in network science. Now we discuss sample estimation of the networks from data. The connectedness assessment is based on estimation of VAR model, which is usually done by selection with traditional standard like AIC. But estimating VAR with high dimensions takes long time and results in low degree of freedom. Least absolute shrinkage and selection operator (LASSO) estimation is helpful for estimating parameters with blending shrinkage and selection.

To understand LASSO in this problem, the VAR estimation solves the following:

$$\hat{\beta} = \arg\min_{\beta}(\sum_{t=1}^{T}(y_t - \sum_{i}\beta_i x_{it})^2).$$

However, LASSO estimation of the parameters solves the following:

$$\hat{\beta} = \arg\min_{\beta}(\sum_{t=1}^{T}(y_t - \sum_{i}\beta_i x_{it})^2) \quad s.t. \sum_{i=1}^{K}|\beta_i|^q \leq c.$$

Equivalently, the estimation problem with Lagrange method becomes:

$$\hat{\beta} = \arg\min_{\beta}(\sum_{t=1}^{T}(y_t - \sum_{i}\beta_i x_{it})^2 + \lambda\sum_{i=1}^{K}|\beta_i|^q).$$

In this model, the LASSO estimation of parameter $\Phi = (\Phi_1, \ldots, \Phi_p)$ is:

$$\hat{\Phi} = \arg\min_{\Phi}(\sum_{t=1}^{T}(x_t - \sum_{i=1}^{p}\Phi_i x_{t-i})^T(x_t - \sum_{i=1}^{p}\Phi_i x_{t-i}) + \lambda\sum_{i=1}^{p}||\Phi_i||^q).$$

Nicholson et al. [119] introduce the package "BigVAR" in R to calculate the LASSO estimators of vector autoregression with exogenous variables (VARX) frameworks. In this chapter, we will estimate LASSO parameters with penalty function $||\Phi_i||_1 = \sum_{i=1}^{N}\sum_{j=1}^{N}|\Phi_{ij}|$.

The penalty parameter $\lambda \geq 0$ and is estimated according to sequential cross validation, the details of parameter selection process are introduced in Nicholson et al. [119].

Before closing this section, the GVD method estimates the sparse matrix in approximating VAR through LASSO. The variance decomposition matrix transformed from the sparse estimation of VAR coefficients are generally not sparse, which is a better measurement of directional connectedness for network than estimating the sparse correlation directly by Granger causality standard in Billio et al. [22]

4.3 Industry Connectedness

In this chapter, we investigate whether the comovements of industry portfolios volatility are able to reflect the uncertainty or exogenous shock into the market and information diffusion across industry. We analyze industry portfolios assigned with the NYSE, AMEX, and NASDAQ stocks based on their four-digit SIC code at that time. Over the period 1927-2017, we study 12 industries, including consumer non-durables, consumer durables, manufacturing, energy, chemicals, business equipment, telecommunications, utilities, shops, health, finance, and other.[1]

In this section, we describe our data, then show the results of static and dynamic network of industry portfolio volatility connectedness.

4.3.1 Data

The daily return of value-weighted and equal-weighted industry portfolios for the years 1927–2017 is collected from Ken French's website[2].

Figure 4.2 shows the monthly return for 12 value-weighted portfolios from 1927 to 2017. The returns of portfolios fluctuate around 0, in the interval of $[-20, 20]$. They have different upper bounds and lower bounds, and returns of all industry portfolios reach beyond the bounds in 1930s, which represent the Great Depression and recovery. There are fluctuations for health in 1970s, energy in 1980s, telecommunications around the dot-com bubble around 2000 and durable good in 2007 to 2009 beyond this interval. The return of industry portfolios move towards the mean after business cycle.

The monthly volatility of industry portfolios is calculated as the standard deviation of return in a month. While the industry portfolio volatility is distributed right skewed, the distribution is presented in appendix C.1. We take the natural logarithms of volatility before estimation of volatility connectedness network, and the distribution is approximately symmetric in appendix C.1. To be more precise, we study the network (connectedness) of industry portfolios' logarithmic volatility.

[1]There are other categories of industry, including 17 industries, 48 industries. In this paper, we analyze based on the data of 12 industries in main results, and 48 industries in robust analysis. As for the data of 48 industries, there are 40 industries portfolios data available for the whole time period 1927 to 2017. There are some industry portfolios return data missing from July 1927 to June 1969. To be more clear about the categories, the industry of consumer non-durables include food, tobacco, textiles, apparel, leather, toys. The industry of consumer durables covers the cars, TV, furniture, household appliances. The Manufacturing industry includes the machinery, trucks, planes, office furniture, paper, printing. The industry of energy covers the oil, gas, and coal products and extraction. The industry of chemistry includes chemicals and allied products. The industry of business equipment covers computers, software, and electronic equipment. The telecommunications includes both telephone and television transmission. The industry of shops covers the services in wholesale, retail. The industry of health includes health care, medical equipment, and drugs. The industry category of other covers the mines, construction, transportation, hotels, bus service, entertainment.

[2]See the website: $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$. The data can be downloaded under the category of "Industry Portfolios".

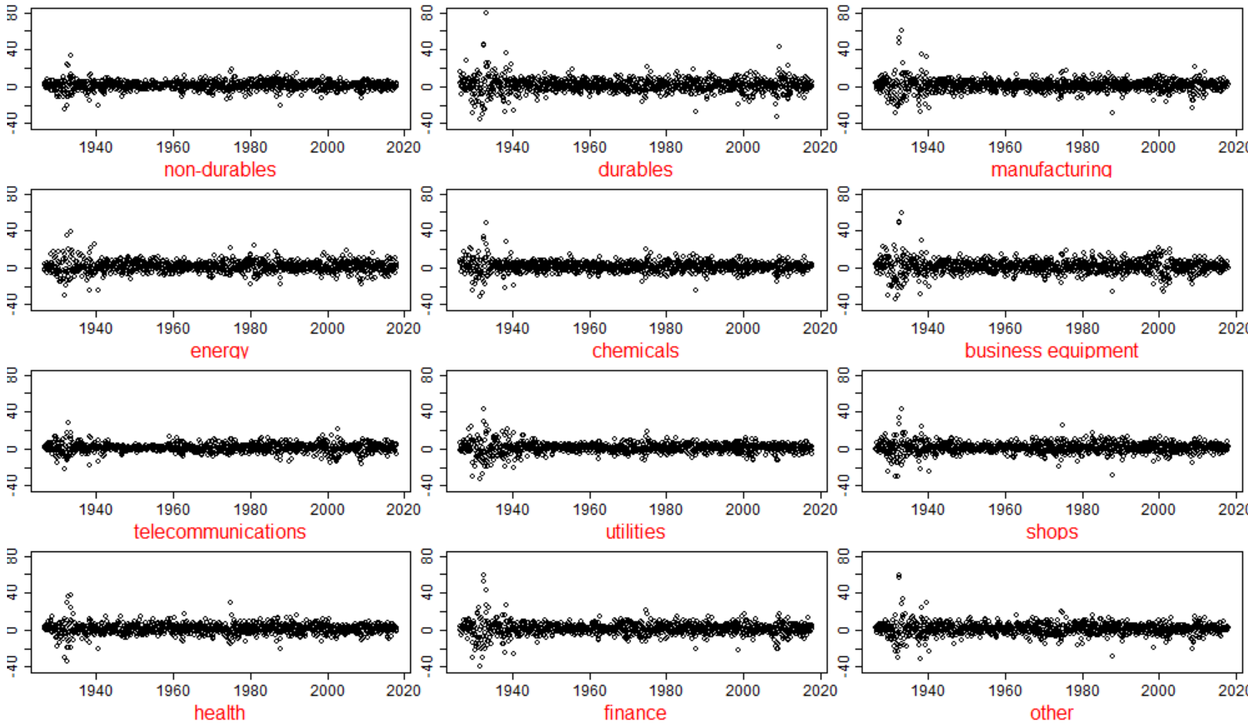


Figure 4.2: Return for 12 Industries (Value-Weighted Portfolios): 1926/07 to 2017/12

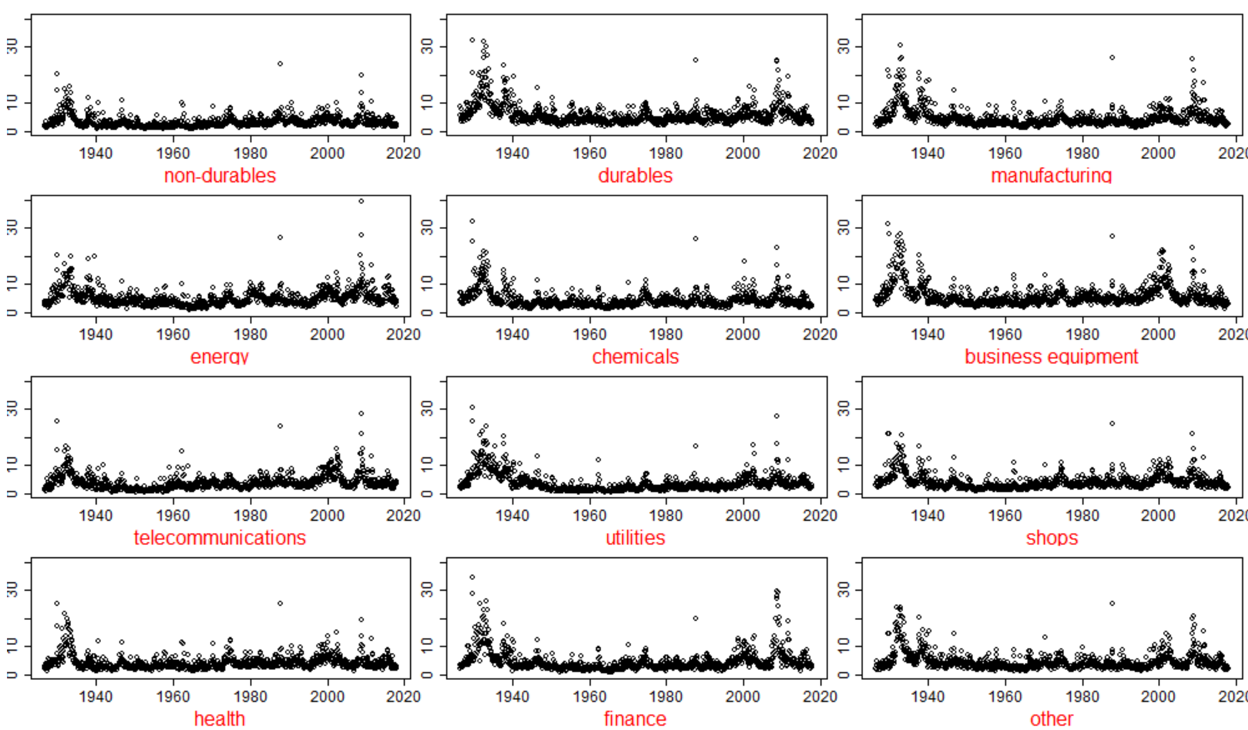


Figure 4.3: Volatility for 12 Industries (Value-Weighted Portfolios): 1926/07 to 2017/12

Figure 4.3 presents the monthly volatility for 12 value-weighted portfolios from 1927 to 2017. From this figure, it can be observed that the series of volatility are stable most of the time. There are several peaks during these 90 years, and the volatility of all industries reach peaks in the same period. As it is mentioned, the volatility is used as a measure of market risk. From the figure, business cycles could be observed clearly from the peak of dynamic industry portfolios volatility, and some are widely spread, some are centralized in few industries. There are peaks of volatility around 1930, 1990 and 2008 for all industries, corresponding to Great Depression, early 1990s recession, and financial crisis. For the dot-com bubble around 2000, the volatility of business equipment, telecommunications, utility, chemicals industries' portfolios experience larger fluctuations than the energy, nondurable good industries. Thus, it is interesting to study how the network structure of industry portfolio connectedness changes in the business cycles. The periods of expansions and contractions of US business cycles are based on results of NBER[3].

4.3.2 Static (Full-Sample) Analysis

We use the full sample of industry portfolio volatility from 1927 to 2017 to estimate the network of connectedness. The adjacency matrix is 12×12, so detail of the connectedness is presented in appendix C.2. For the static network of the whole period of 90 years, we calculate the degree to, degree from, and total direct connectedness of each industry, the results are presented in Figure 4.4. The total connectedness is very different to commodity network in Diebold et al. [51], but similar to global banks network in Demirer et al. [49]. The total connectedness of industry portfolio volatility network is 86%, much higher than 40% of system-wide connectedness. It means the risk of industry portfolio spillover to each other, and most of the risk comes from outside rather than idiosyncratic fluctuations. Generally speaking, the industry network is highly connected, thus, the shocks coming to one industry will spread to others and diversify through all other industries.

In figure 4.4, the green bars, which represent the in-degree centrality, have similar heights between 6 and 8. However, the blue bars, which represent the out-degree centrality, have larger variation. For example, the overall out-degree centrality of telecommunications industry is smaller than 5, but the out-degree centrality of manufacturing is larger than 8. From the static network analysis, we can also observe a relatively small variation of in-degree centrality, but there are larger variations for out-degree centrality across the industries. For net directional connectedness, which is the yellow bar in the figure, the durable goods, energy and telecommunications are negative, and nondurable, manufacturing, and other are positive. Thus, the durables, energy, and telecommunications are risk receivers, and the nondurable, manufacturing, and other send the uncertainty to others.

[3]See $http://www.nber.org/cycles.html$

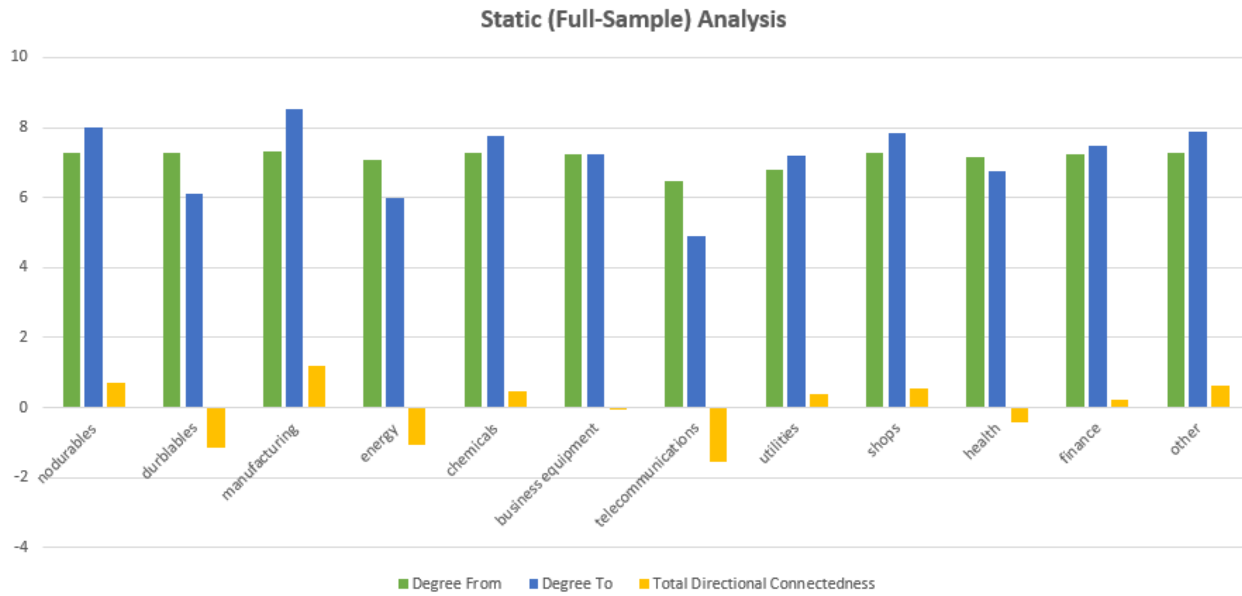


Figure 4.4: Directional Connectedness for 12 Industries: 1926/07 to 2017/12

Since there are technology developments in the past 90 years, including the information revolution, not only the production structure within one industry develops, but also the relationship between industries adjust over the time. Thus, it is hard to use one single network to capture the whole picture of comovement between the industries. We turn to study the features of dynamic network in the following.

4.3.3 Dynamic (Rolling-Sample) Analysis

In this section, we study time series of volatility connectedness, estimated using a rolling window with a width of 200 months. Both total system-wide, total directional (to and from) connectedness, and centrality are discussed.

System Wide Connectedness

Figure 4.5 is the dynamics of total connectedness C^H of networks for 12 industry based on the volatility of value-weighted portfolios. The total connectedness of the volatility network is around 0.88 for most of the time from 1927 to 2017, but it starts to go down in 1990s, and reach the bottom about 0.80 in 2007. The overall total connectedness level is higher than the commodity network in Diebold et al. [51]. There are two 'U' shape curve for total connectedness, one from 1950 to 1980, another 2004-2009. It is interesting to discover how the network structure changes among industries during these period. There are 12 shading areas in the figure showing recession periods of US business cycles from 1943 to 2017. There are fluctuations of total connectedness during all

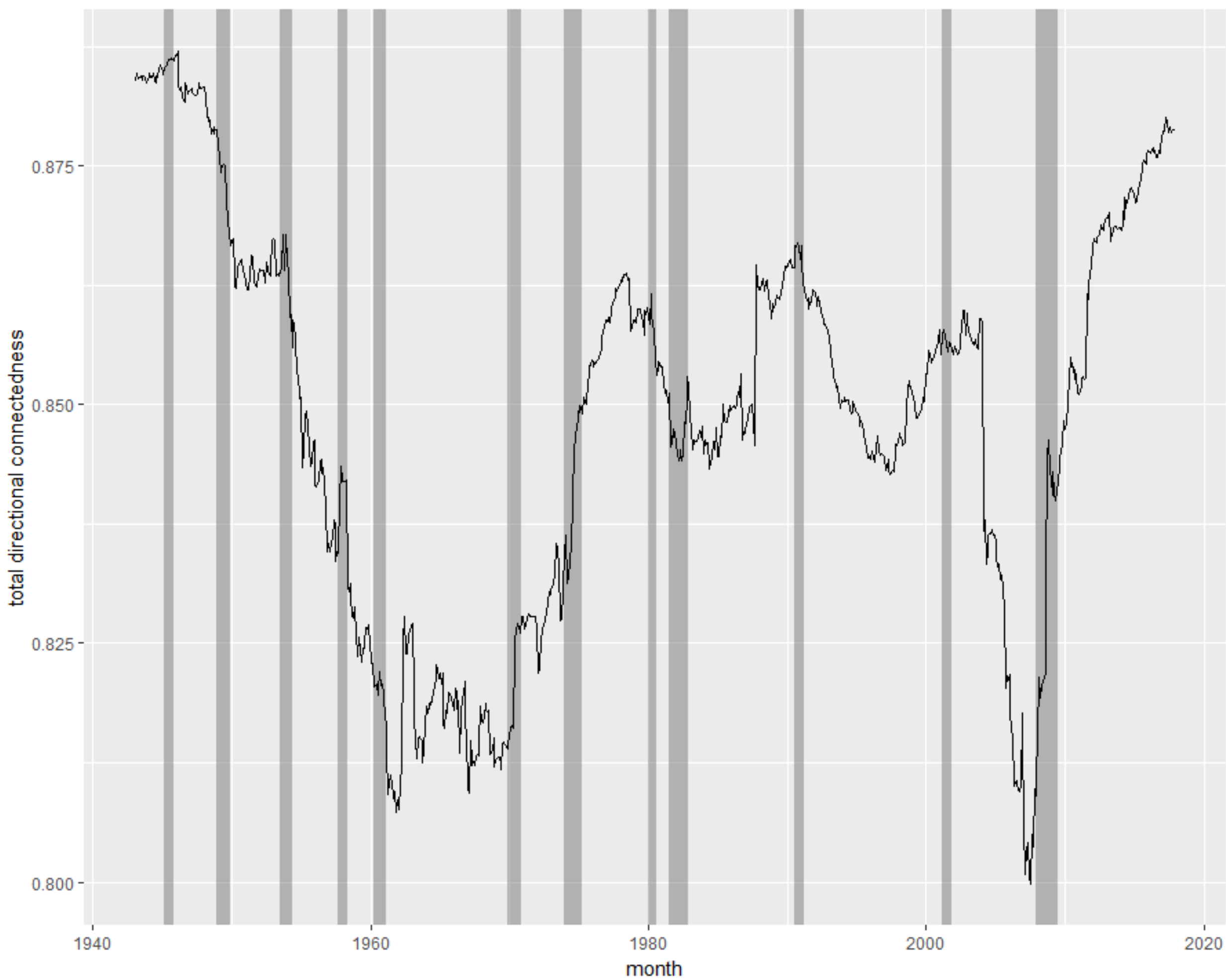


Figure 4.5: System Wide Total Connectedness (Value-Weighted): 1943/02 to 2017/12

Note: The shaded bars are recessions dated by NBER.

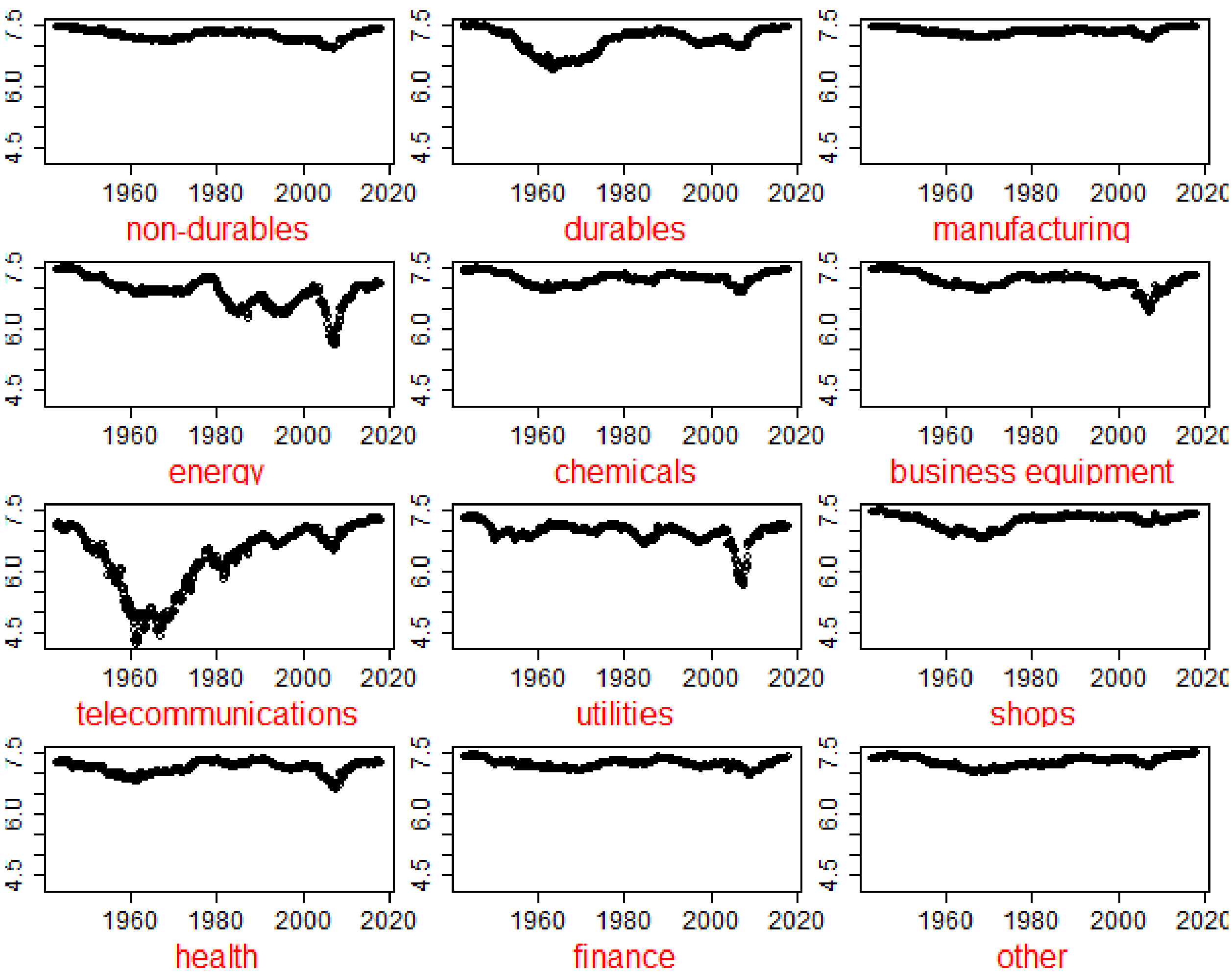


Figure 4.6: Degree From for 12 Industries (Value-Weighted): 1926/07 to 2017/12

recession periods. Most of them are increasing, and few are decreasing. It is worth further research to investigate reasons behind observed patterns.

Total Directional Connectedness

Figure 4.6 presents the in-degree centrality of 12 industries for dynamic network. We can observe that there is a similar pattern for all the industries to receive risk from others. They reach the peak of risk inflow in 1980 and go down smoothly until 1990, then some industries, including non-durable, durable, manufacturing, energy, chemicals business equipment, finance, experience steep decreases in receiving risk, which means the they are more independent and influenced by idiosyncratic fluctuations. After that, they move back to be more connected, and go down again until the financial crisis, which brings industry to be more connected again at a similar level of peaks in historical observations. Overall, the telecommunications, energy, and utility are less influenced by other industries.

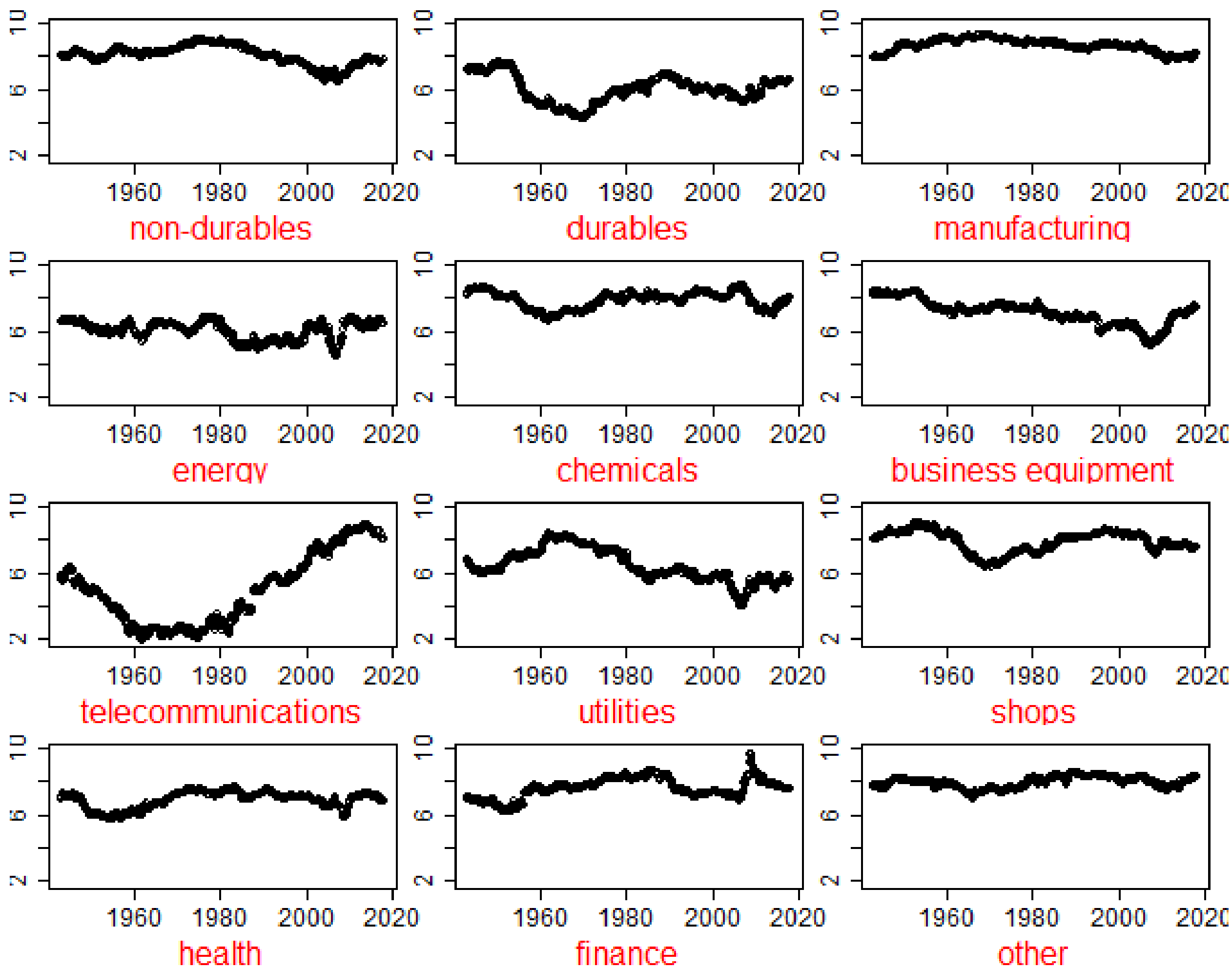


Figure 4.7: Degree To for 12 Industries (Value-Weighted): 1926/07 to 2017/12

Figure 4.7 shows the out-degree centrality of 12 industries for dynamic network. The dynamics of out-degree centrality are very different from the dynamics of in-degree centrality. Durables, chemicals, shops, and utility have similar patterns. Non-durables, manufacturing share similar patterns. The energy industry experiences more fluctuations than others.

The sharp increase of out-degree centrality of finance in 2007-2009 reflects that the industry finance becomes risk sender in financial crisis. However, industry of finance is not the biggest source of investors' fear with highest out-degree in other periods. The volatility network structures change during the financial crisis. The failures of financial system spread to the rest, and cause potentially large impact. The graph also shows that the risk sent from telecommunications industry keeps increasing since 1980, including the Dot-com bubble period. We can notice that the out-degree of telecommunications is increasing since 1980, from the lowest of all industries in 1980 to highest in 2017, which is not surprising, because of the growth of information technology services. The future

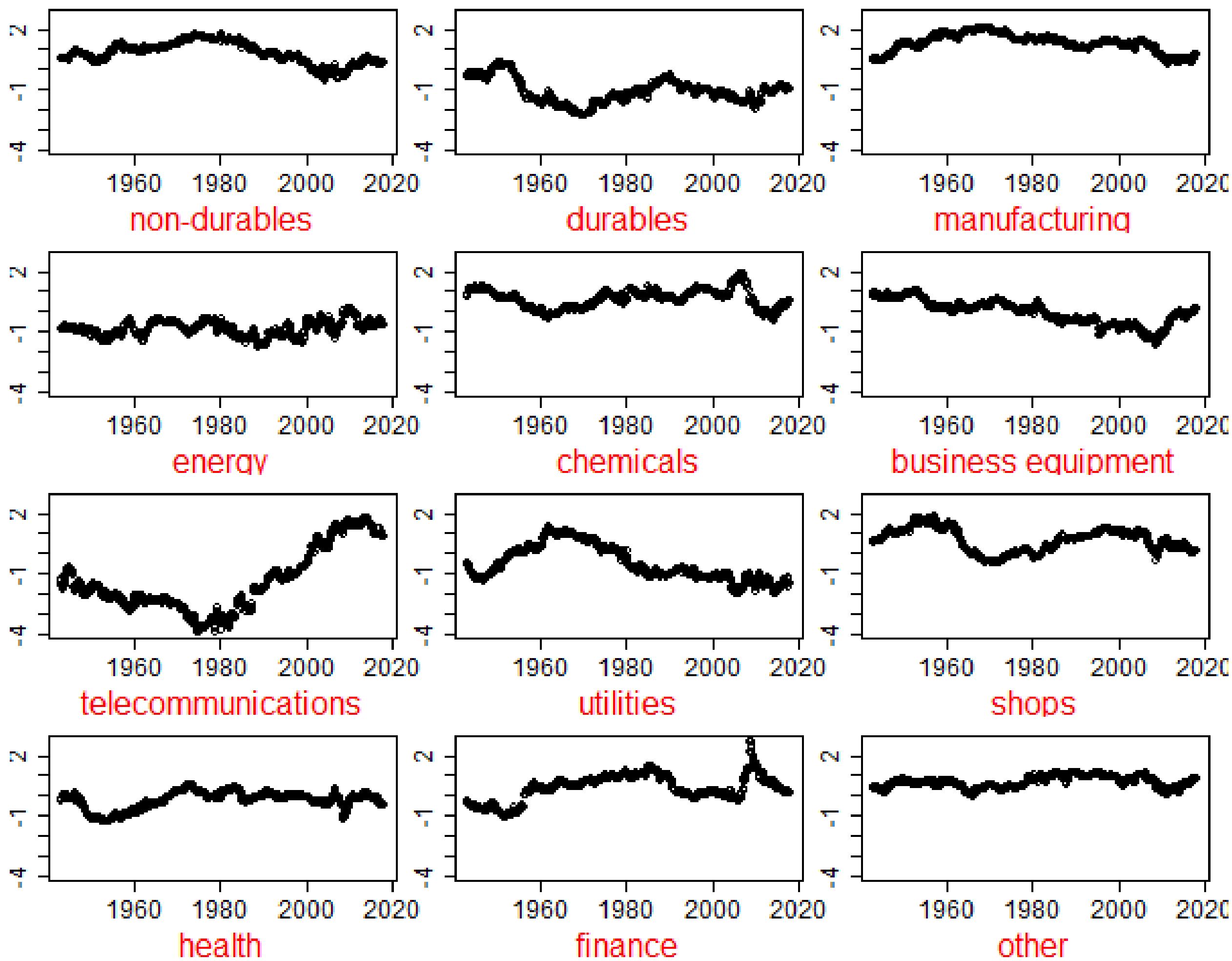


Figure 4.8: Total Direct Connectedness for 12 Industries (Value-Weighted): 1926/07 to 2017/12

development in this industry may bring new revolution to other industries, so the uncertainty in telecommunications is more likely to spread to others. The telecommunications industry becomes a source of volatility fluctuation.

Figure 4.8 presents the total directional connectedness C_i^H of dynamic network of 12 industries based on volatility of value-weighted portfolios, which is the difference between the in-degree and out-degree. The positive value means the industry is a risk receiver, and the negative value means the industry is a risk sender. From the figure, we can see durables, energy, and health are positive in total direct connectedness for almost all rolling sample windows, which indicates that their portfolios prices on average are influenced by other industries and receive shocks from others.

We can also focus on the period of financial crisis, except telecommunications and finance, the total direct connectedness of all industries increase, which means they receive more risk from others. Health, business equipment, and shops et al. experience a temporary shock and then recover. The total direct connectedness of finance also experiences a short period peak and then come back to around 0 in the end of 2017. The degree centrality of the dynamic networks of 12 industries reflects the risk sent from industry of finance to other industries during crisis in 2007-2009, with the phenomenon of bankruptcy of many financial institutions. It is consistent with the intuition that financial services works as intermediary for other industry through borrowing and lending resources, and the widely defaults of banks bring fear and uncertainty to investors in other industry. At the same time, the relationships of risk spillover change over time, and industry of finance is not always the main source of risk.

Centrality

Here we discuss two centrality measurements of the node in connectedness network, eigenvector centrality and pagerank centrality, which are defined in Section 4.2.2 to find which industry is the 'key player' in the volatility connectedness network, which brings risk to others.

The red curves in Figure 4.9 present the eigenvector centrality of 12 industries in the dynamic networks. For most of the industries, there are relatively small fluctuations for eigenvector centrality. For example, the eigenvector centrality of industries of non-durables, manufacturing, chemicals, shops, health, finance, other fall in $[0.8, 1]$, $[0.9, 1]$, $[0.8, 1]$, $[0.8, 1]$, $[0.8, 1]$, $[0.8, 0.95]$, $[0.8, 1]$, and $[0.8, 1]$ respectively. The durables industry experiences a decrease in 1950s and increase in 1970s. The industries of energy, business equipment and utilities experience a downward shock and recover in 2000s. Manufacturing has highest eigenvector centrality from 1940s to 2004. For the industry of telecommunications, the eigenvector centrality increases from the lowest about 0.4 in 1960s to highest about 1.0 after 2010. Finance industry has two peaks for eigenvector centrality, one around 1990, the other around financial crisis 2007-2009, both are recession periods. Similar with total directional connectedness, eigenvector centrality also captures the structure change in financial crisis 2007-2009. The finance industry becomes the central node, while the industries of manufacturing, energy, business equipment, utilities and health become less central in economy.

The blue curves in Figure 4.9 present the dynamics of 12 industries pagerank centrality of network of volatility connectedness. By definition of pagerank centrality, the value is calculated with neighbors centrality divided by the out-degree, which means the industry connected closely to the sender of risk will have lower pagerank centrality than eigenvector centrality. Intuitively, the pagerank centrality gives less weight to industries connected with key senders of risk. The discount of out-degree of node is the reason that the blue curves are always below the red curves, but it is clearly seen that the trends of blue curves and red curves are highly correlated. Generally speaking, the results of pagerank centrality of all industries are similar with eigenvector centrality. The industry of telecommunications still has largest variation, increasing from 0.04 in 1960s to around 0.1 in

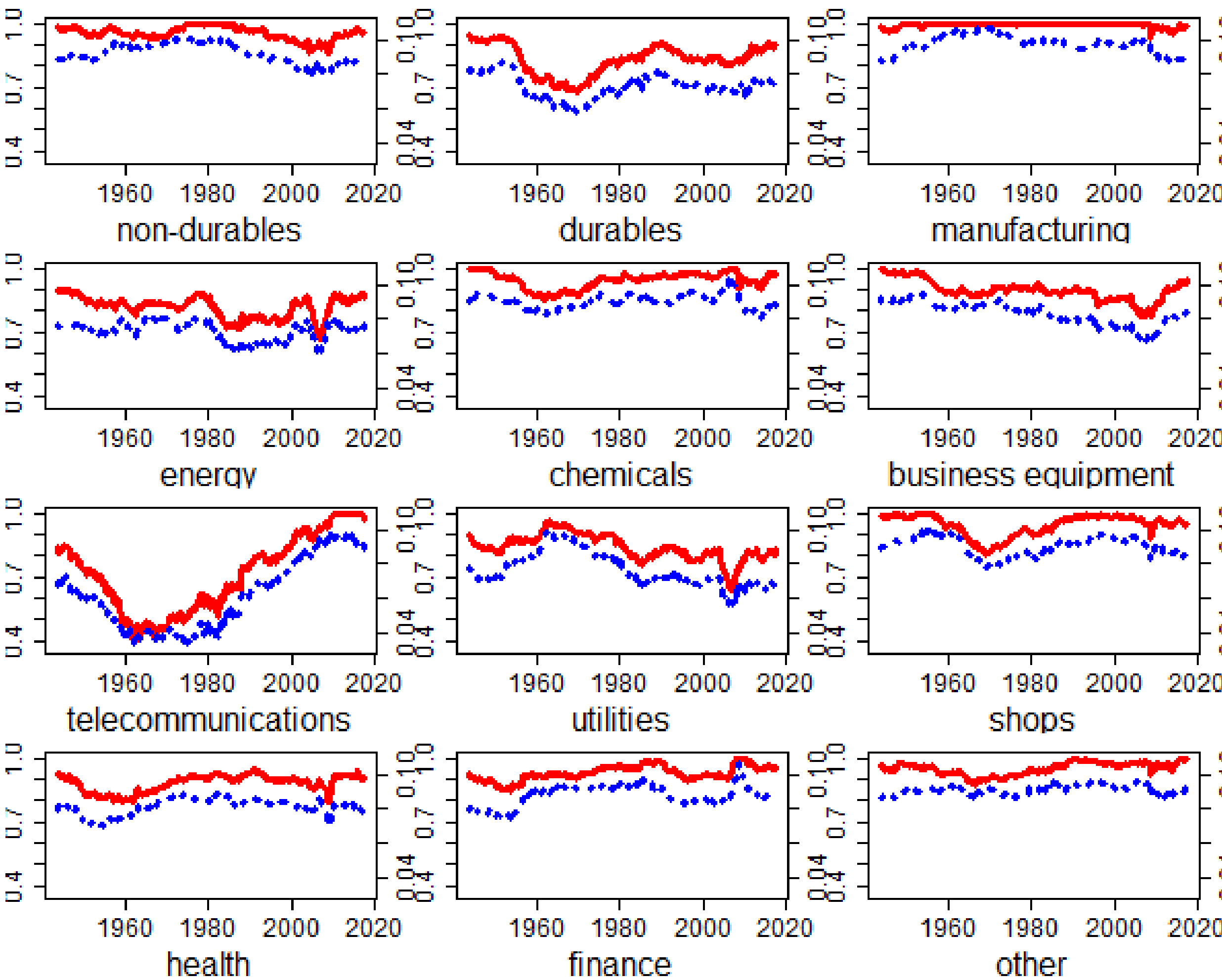


Figure 4.9: Centrality for 12 Industries (Value-Weighted): 1926/07 to 2017/12

Note: Red: Eigenvector centrality. Blue: Pagerank centrality.

2010s. Similar with eigenvector centrality, the fluctuations for pagerank centrality of industries of non-durables, manufacturing, chemicals, shops, health, finance are smaller than other industries. The peak of finance industry is clearer for pagerank centrality in financial crisis 2007-2009, while the industries of energy, chemicals, business equipment, utilities, and health experience a drop for pagerank centrality.

4.4 Robustness

Following the results of network of 12 industries based on value-weighted portfolios, we discuss the robustness of results to the choice of dataset. In particular, we estimate the networks for two alternative datasets. We first use the data of equal-weighted portfolios of 12 industries, which is commonly used in the literatures to compare with results of value-weighted portfolios. At the same time, we study the data of another category of industry, 48 industries, and LASSO is able to deal with the estimation of high dimensional parameters. All data are collected from Ken French's website.

4.4.1 Equal-Weighted Portfolios of 12 Industries

Figure 4.10 shows the total network connectedness C^H of dynamic networks of 12 industry based on equal-weighted portfolios. Comparing to the results of value-weighted portfolio in Figure 4.5, the fluctuations of total connectedness before 1990 is smaller in the case of equal-weighted portfolio than value-weighted portfolio, but the trend is similar after 1990. Moreover, the patterns of total connectedness are clear. Except the recession in 1960s, total connectedness is almost flat with small fluctuation. The total connectedness are either experiencing big increase in Dec 1969 to Nov 1970, Jan 1980 to Jul 1980, and Dec 2007 to Jun 2009, or climbing to a peak and then falling in the rest of recessions.

The total directional connectedness of 12 industries is presented in Figure 4.11. For non-durables, durables, manufacturing, shops and other, their total directional connectedness fall into the interval of $[0, 1]$ most of the time from 1926 to 2017.

Similar to the results of value-weighted portfolios in section 4.3.3, there is a decline of total directional connectedness of business equipment and health, and the total directional connectedness of finance climb up sharply during crisis then decrease. The finance industry becomes the sender of risk in financial crisis 2007-2009, and industry of chemicals, business equipment, health are risk receivers. Thus, these results are robust for the data of equal-weighted portfolios. At the same time, the increasing trend of telecommunications industry since 1960 is also observed in the equal-weighted case, the total directional connectedness increases from -3 to 1. The industry of telecommunications switches from risk receiver to risk sender with in past 50 years. The fluctu-

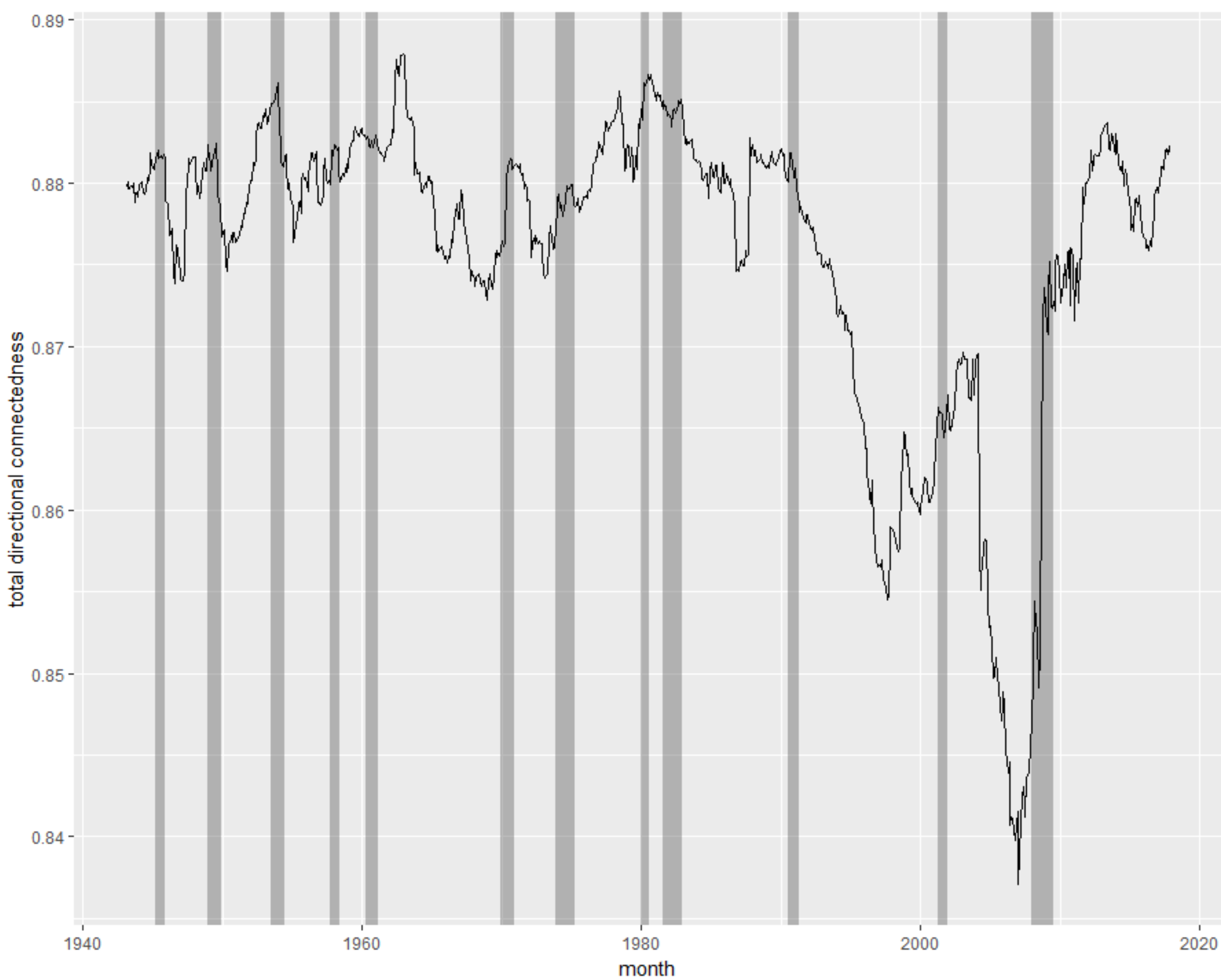


Figure 4.10: Total Connectedness of 12 Industries (Equal-Weighted): 1926/07 to 2017/12

Note: The shaded bars are recessions dated by NBER.

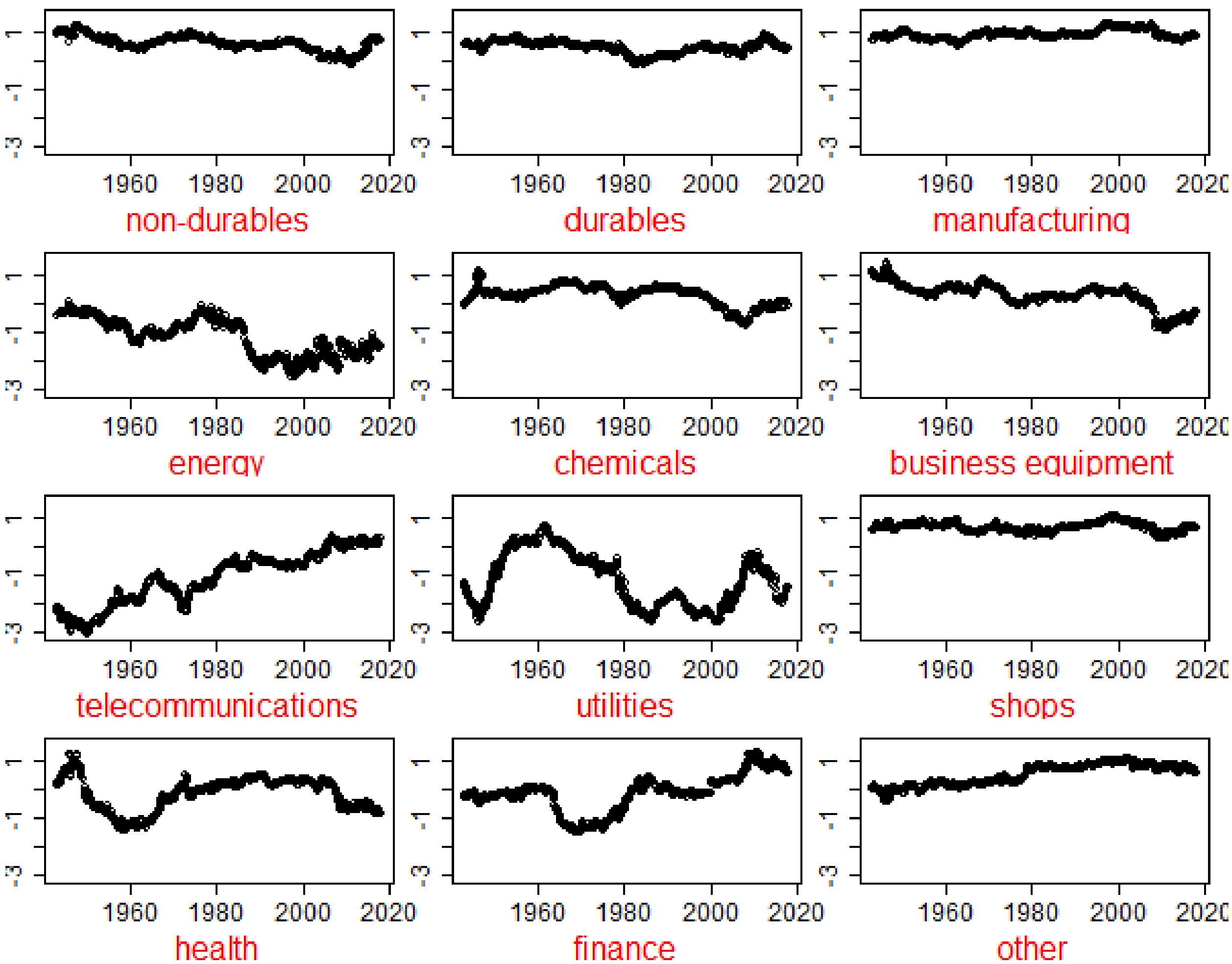


Figure 4.11: Directional Connectedness for 12 Industries (Equal-Weighted): 1926/07 to 2017/12

ations of utilities industry is larger in equal-weighted portfolio case, a possible explanation is the companies in this industry perform very differently according to the scale.

For equal-weighted portfolios, the eigenvector centrality and pagerank centrality of 12 industries are presented in 4.12. The red curves represent the eigenvector centrality of dynamic networks of 12 industries, and the blue curves represent the pagerank centrality. Comparing to the results of value-weighted portfolios, the eigenvector centrality and pagerank centrality are closer in the equal-weighted case. For industry of telecommunications, utilities, energy, and finance, the increasing and decreasing trends of eigenvector centrality is highly correlated with total directional connectedness. The centrality of a node is closely related with indicator for risk sent through it.

For eigenvector centrality, manufacturing has highest value since 1940s. The eigenvector centrality of industry of telecommunications increases from the least central industry to one of most central. For industry of finance, the eigenvector centrality climb up sharply during financial crisis 2007-

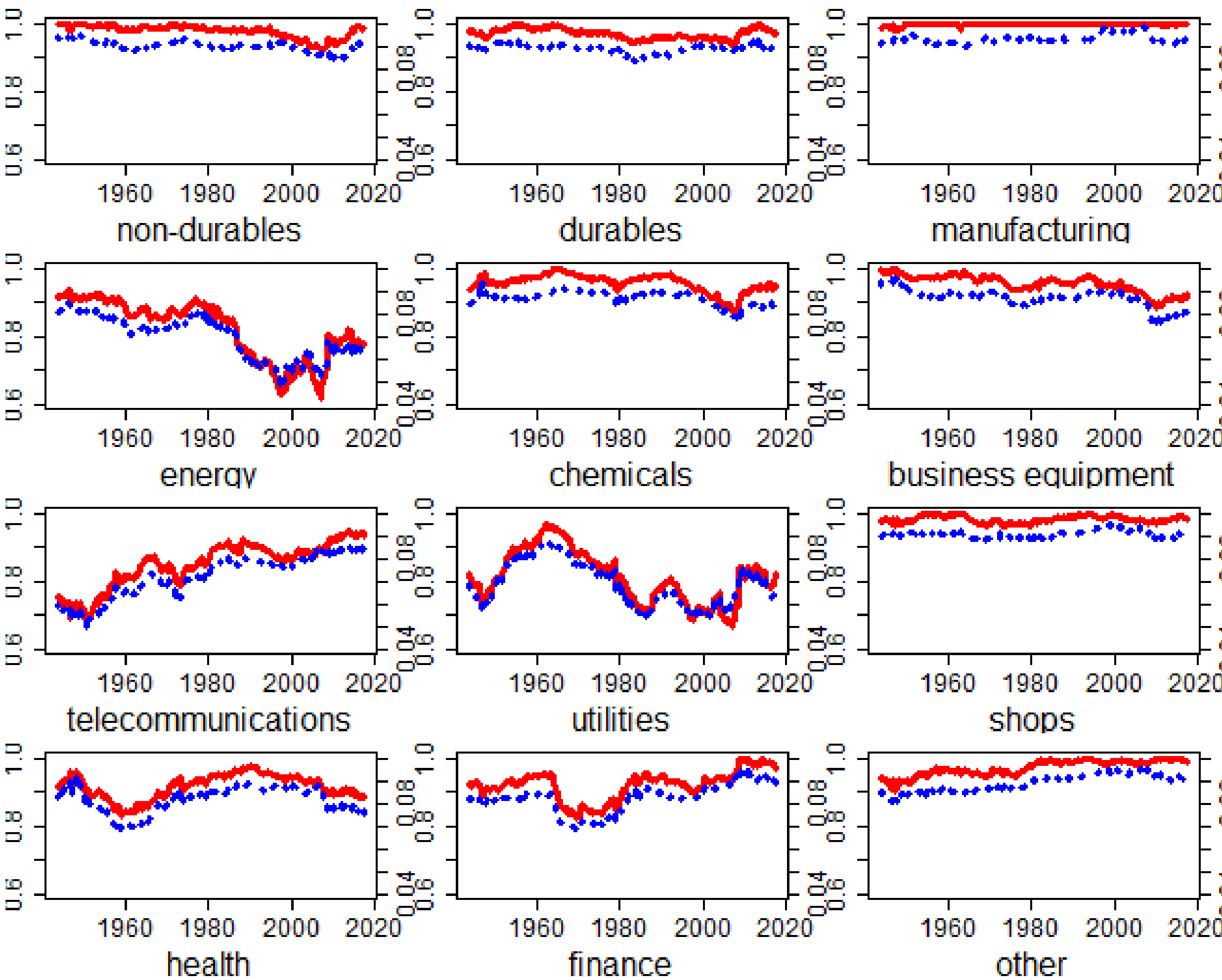


Figure 4.12: Centrality for 12 industries (Equal-Weighted): 1926/07 to 2017/12

Note: Red: Eigenvector centrality. Blue: Pagerank centrality.

2009. The shocks of financial crisis to other industries are less clear in this case. The industries of energy, utilities experience sharp decrease in 2007, and industry of business equipment decreases more smoothly than value-weighted case. Since the pagerank centrality is highly correlated with eigenvector centrality, the same patterns for eigenvector centrality could be observed in pagerank centrality. Although the results for equal-weighted portfolios differ from value-weighted portfolios, both of them captures the similar fluctuations for some industries. For example, telecommunications is becoming more and more central, financial crisis brings a shock ot the network, and industry of finance becomes central as risk sender during 2007-2009. The industry of non-durables, manufacturing, other have small variations in centrality.

4.4.2 Value-Weighted Portfolios of 48 Industries

There is missing data of value-weighted portfolios for 48 industries at the beginning of the sample in 1927, and the complete data starts after 1969 July. Since the relationships between industry change over time, the static analysis of the network of 48 industry may not give complete descriptions for relationships between industry for the past 50 years. We estimate the dynamic networks of rolling windows with width of 200 months. Cutting off the data of 48 industries before 1969 July will not affect the rolling window results after that.

Figure 4.13 shows the total connectedness C^H of dynamic network of 48 industry based on volatility of value-weighted portfolios. Total connectedness of the network is higher than 0.94 most of the time, there is a shock starting in 2004 and reaching the bottom point in 2007 before financial crisis. The total direct connectedness increases to 0.94 at the end of contraction period. After dividing the industry category to thinner, the rolling window analysis captures more details about comovements than 12 industries, the overall connectedness level is higher than the network of 12 industries. However, the fluctuations of total connectedness of 48 industries are similar with Figure 4.5 and Figure 4.10. There are two periods of total connectedness to decrease first and then increase to the level of 0.945 between recessions. Then we look at the behaviors of total connectedness in recessions. The dark areas represent three contraction periods of business cycles in the sample from 1986 to 2017: Jul 1990 to Mar 1991, Mar 2001 to Nov 2001, and Dec 2007 to Jun 2009. The trends of total connectedness are different in these periods. Total connectedness increases first and then decreases with a local peak in the recession from 1990 to 1991, stays at same level in dot-com bubble Mar 2001 to Nov 2001, and and increases sharply in financial crisis of 2007-2009.

Figure 4.14 presents the total direct connectedness of dynamic networks of 48 industries. For many industries, the trend of direct connectedness is flat with limited fluctuation from 1986 to 2017, including the industries of beer&liquor, entertainment, apparel, drugs, chemicals, textiles,

Figure 4.13: Total Connectedness of 48 Industries (Value-Weighted): 1986/02 to 2017/12

Note: The shaded bars are recessions dated by NBER.

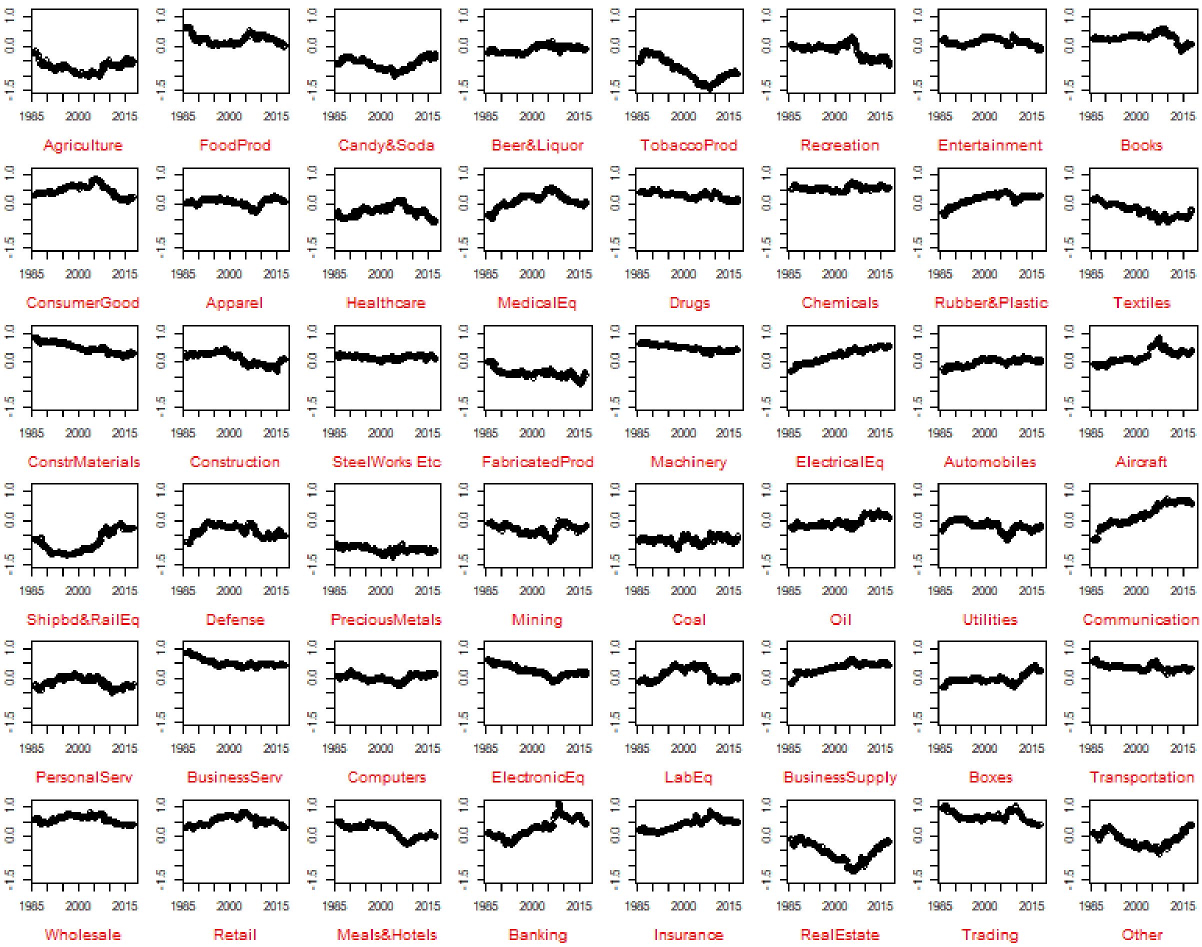


Figure 4.14: Total Direct Connectedness for 48 Industries (Value-Weighted): 1986/02 to 2017/12

steelworks, machinery, automobiles, precious metals, transportation, computers,and wholesales. The industries of plastic, aircraft, business supply, banking and insurance have upward sloping total direct connectedness. The industries of tobacco, real estate, and other experience decline, and then climb up. Based on thinner categorization of industry, there exists some substitution and complementarity relationships between different industries. For example, the curves of soda and tobacco products, health and medical equipment, banks and insurance are correlated in trend. Banks and insurance have similar patterns in the past 30 years. Although the window of sample for 48 industries is shorter than 12 industries, 1986-2017 comparing to 1926-2017, there are some results robust to the categorization during past 30 years. The total direct connectedness of telecommunications industry switches from negative to positive and stays at high level, meaning that it changes from risk receiver to one of main risk sender. The trend is similar for electrical equipment. Gold, as industry of precious metals, has highest total directional connectedness, so it is the biggest risk receiver. Among 48 industries, we observe the industries of mining, utilities, meals and hotels experiencing sharp decrease and recover during financial crisis 2007-2009. These industries receive short-term shock to the risk spillover relationships. Consistent with the evidence of bankruptcy of investment banks, such as Merrill Lynch, Lehman Brothers, and insurance company AIG, the total direct connectedness of industries of banking and insurance jumps to a high value and decreases to previous level, reflecting the risk within these two industries contagion to the rest of economy.

Figure 4.15 shows the eigenvector centrality and pagerank centrality of dynamic networks of 48 industries. The red curves are eigenvector centrality, and blue curves are pagerank centrality. Same as the analysis of 12 industries, the curves of pagerank centrality are below curves of pagerank centrality, and the trends are correlated, even though the value of these two centrality measurements are very different. The eigenvector centrality of industries of banks, insurance, telecommunications, electronic equipment, aircraft are increasing during the last 30 years. Tobacco products, precious metals and agriculture have low eigenvector centrality about 0.7. Oil's centrality increases from 0.7 to 0.9 in the last 30 years. Industries of trading, business services, transportation, retails, wholesale, machinery experience fluctuations, but their centrality stays around 0.9. Some industries have 'U' shape of eigenvector centrality curves, such as Candy&Soda, tobacco products, apparel, real estate and other. The results for centrality measurements robust to the 48 industry category are: The centrality of industries of banking, and insurance increase sharply, and industries of real estate, tobacco, utilities, meals&hotels are risk receivers during financial crisis 2007-2009. The industry of telecommunications is increasing in centrality of network for risk spillover. Some industries, such as agriculture, and precious metals, have low centrality in the sample.

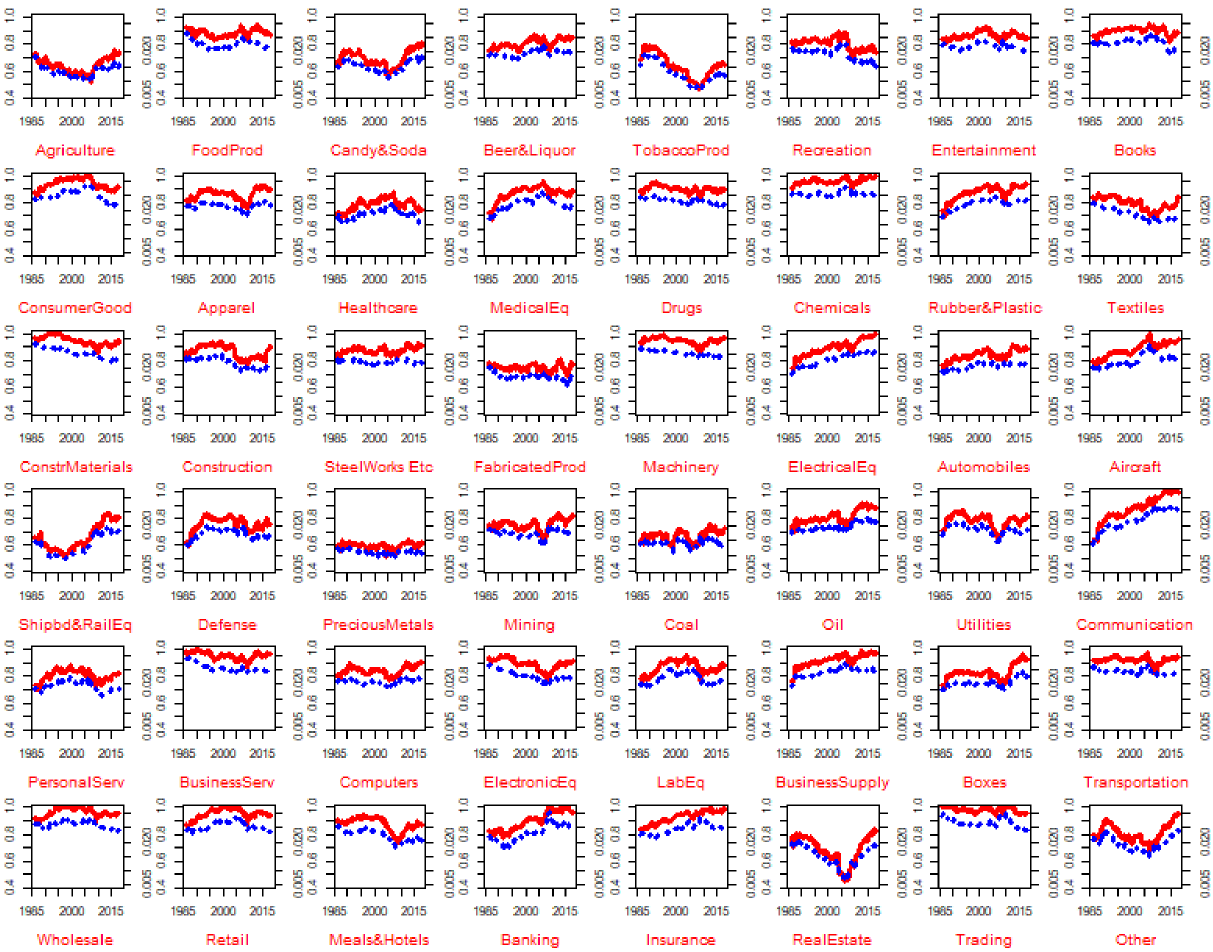


Figure 4.15: Centrality for 48 Industries (Value-Weighted): 1986/02 to 2017/12

Red: Eigenvector centrality. Blue: Pagerank centrality.

It will be interesting to do further research about the relationships between centrality of volatility network and centrality of production network with input output relationship. The relationships between risk spillover of industry portfolios and the gradual information diffusion between industries is worth studying.

4.5 Conclusion

We use LASSO to estimate networks of industry based on the volatility of value-weighted and equal-weighted portfolios. The static analysis describes the connectedness of risk between industries during the whole period of sample. It shows the industry of durable goods, energy and telecommunications are risk receivers, and industry of nondurable, manufacturing and other send the uncertainty to others. However, the dynamic analysis of rolling window provides a better picture for understanding the development of relationships of risk spillover between industries. The structure of dynamic networks are different from static networks, such as the industry of telecommunications is not risk receivers for all 90 years, and it actually becomes a risk sender after 2000. Moreover, the dynamic network is a tool to study the connections of industry in different business cycles. The fluctuations of total connectedness behave differently during recessions. The total connectedness increases sharply in the financial crisis of 2007-2009, decreases during the recession in the 1990s, while stays at the same level during the dot-com bubble in 2001. There is no clear trend of connections in business cycles, other factors are needed to explain the behavior of the network dynamics. There are also results in the relationships between specific industries. Although the industry of finance operates as an intermediary in the real economy by offering services of borrowing and lending resources to other industries, its centrality is not the highest among all industries, thus it is not in the 'middle' of relationships for risk spillover in the static analysis of the whole sample. The robust results of the dynamic analysis show the structure of industry networks changes over time. Finance industry becomes the biggest source of risk spreading to others during the financial crisis of 2007-2009, while the industries of real estates, utilities, shops such as restaurants and hotels, energy such as coal and oil have a short-term decline in total direct connectedness. The industry of manufacturing has a high centrality, and precious metals has low centrality in networks. Moreover, the industry of telecommunications has negative total direct connectedness in static analysis but has increased total direct connectedness from negative to positive in the analysis of dynamic networks with rolling window. To summarize, this chapter is the first try to adopts the method of generalized variance decomposition to construct dynamic networks measuring the comovements of volatility of industry portfolios, and uses the comovements to study the relationships of risk spillover between industries, and presents the patterns observed in dynamic networks. There are many directions for potential extensions of this chapter: 1, the relationships between real production activity of industry and the behavior of portfolios are not discussed. 2, the effect of changes in network structure on the return of industry portfolios is not studied. 3, the stability of network structure against exogenous shocks and policy implications for regulation, supervision and risk management are interesting to be investigated.

5 SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

5.1 Summary

Interactions between agents is a commonly observed phenomenon in daily life. Agents can take the form of firms, people, etc. Such interactions are studied by network economics and have a wide range of applications ranging from job references of the social network in labor markets, ethnic connection in international trade, information diffusion, and social learning, production relationships of firms, assets trading with dealers as intermediaries, share cross-holdings between financial institutions. All of these research rest on the assumption that local interactions influence the decisions and future outcomes. One of such relationships is intermediation, which describes the situation where two agents are not directly connected but reach each other indirectly via a common agent in a network. The agent connecting both agents is the intermediary. The intermediation in a network reduces the transaction cost and improves the efficiency of outcomes by connecting separate agents and bringing new trading opportunities, by transmitting information and resources from one to the other. However, the intermediaries are likely to have high bargaining power, and could potentially gain high benefits from the improvement of efficiency of the outcomes. The intermediaries linked to many periphery agents could also become a source of risk and shocks to spread and transmit to all agents.

The organizing theme of this dissertation regards intermediation on networks: When do intermediaries have the market power to charge for the use of their services to transmit resource to agents? The answer depends on how the network is formed, who intermediaries connect, the quality of such connection and the preferences of the sender of the resource. Is the intermediary of real economic services also the central node for risk transmission? This dissertation studies the role of intermediation on networks, both of the questions about intermediaries are examined. The first question is studied in the setting of resource transmission on networks. The second question discusses an application of financial network.

Chapter 2 investigates the problem of planner sending resource via intermediaries to agents. The intermediaries have exogenous weighted links to agents describing their quality of intermediation in transmitting resource, and they post a price for using their services to transmit resource. Planner chooses the way of resource allocation based on their prices and quality of intermediation to maximize his utility depending on the resource allocated to final agents. In this chapter, the information about the network structure, including links and weight of connection, is complete information. The main factors determining the equilibrium outcomes are the influences of intermediation on allocation possibility of resource and the preferences of planner over the resource allocations. The subgame perfect Nash equilibrium is defined to characterize the outcomes. The research focuses on free intermediation equilibrium, which means perfect competition in intermediation services, and the resource is transmitted as if there are no intermediaries charging any fees for their services. The main theorem shows that the necessary and sufficient condition for the existence of free intermediation equilibrium is that there exists no *essential* intermediary, which individually improves the utility of the outcome.

This chapter also discusses a refinement of equilibrium, called robust equilibrium, in which case, intermediaries can coordinate their prices to achieve higher transmission and get the higher payoff. The theorem discovers necessary and sufficient condition for the free intermediation equilibrium to be a unique robust equilibrium. The findings in this chapter can be applied to many network problems, including flows on the network, and minimal cost spanning tree problems.

Chapter 3 studies the resource transmission problem in the situation that planner has no information about connection of intermediaries, including their connections and how well they are connected with the agents. The research focuses on a specific network structure of intermediation, which is the bipartite network between intermediaries and agents. In this situation, a planner who owns the resource needs to ask the intermediaries to report their quality of links and then allocates resource based on the reports. The mechanism includes two parts, the distribution of resource from planner to intermediaries and the sharing rates that intermediaries are charged to send to agents per unit resource received from the planner. Moreover, exogenous verification and punishment are taken into consideration.

This chapter focuses on the class of strategy-proof mechanism, which requires the allocation rule to satisfy that intermediaries have no incentive to lie no matter what the others' reports are. For instance, the second price mechanism is strategy-proof. It also characterizes the punishment function which does not effectively expand the class of strategy-proof mechanisms. Moreover, it discusses the minimal punishment function which poses the least punishment to incentivize intermediaries to report truthfully their quality of intermediation given a certain mechanism. Finally, the efficiency is defined as the first-best outcome for a planner, and it is showed that there exists no mechanism that is strategy-proof, symmetric, budget balance and first-best efficient. The minimal punishment function to achieve first-best efficient is equivalent to the one for the first price mechanism.

Chapter 4 investigates the role of industry in risk spillover through estimating the comovement of industry portfolios' volatility. The efficient-market hypothesis in financial economics states that the asset prices fully reflect all available information, which means the industry portfolios' prices fluctuation reflect shocks to the industry and relationships with risk of other industries. This chapter uses variance decomposition method to estimate the connectedness of industry portfolios' volatility, constructs network based on the estimation, and measures the centrality of each industry in the dynamic networks.

Since finance is the intermediary between borrowing and lending money to other industries, it is highly connected to other industry. The results show that network structures change over time, especially in response to shocks and business cycles. The total connectedness of system level increases during a recession, and the industry related to the source of the crisis has higher centrality during that period. During the financial crisis of 2007 to 2009, the out-degree and centrality of finance increases sharply and becomes the largest sender of risk. Similarly, the centrality of IT related industry increases during the Dot-com bubble. It is worth noticing that as the demand for IT related services grows fast, the centrality of IT related portfolios have increasing centrality during the past two decades, and the industry change from risk receiver to risk sender. More research about the relationship of the performance of the industry in the stock market and real economy should be done to fill the gap, also some hints about efficient-market hypothesis can be obtained by the research.

5.2 Further Directions

There are several open questions for future research that thread through the chapters in this dissertation. Chapters 2 and 3 utilize the game theory and mechanism design approach to study the intermediation network described in Chapter 1. In particular, Chapter 2 studies the price competition of intermediaries on networks with complete information about network structure, and it covers a general network structure that expands the literature for minimal cost spanning tree problem, network flow problem with multiple layers. Chapter 3 studies mechanisms of a planner to ask the intermediaries to report their true connection to agents, thus it studies the bipartite network structure. One interesting extension for these Chapters is to extend the analysis by allowing the network to more general than the bipartite network with incomplete information about the intermediaries' network. Another interesting extension for both of these Chapters is to consider the endogenous network formation, under which condition, the intermediaries could choose which agent to link with and invest on the quality of the connection. Doing this can open the analysis to endogenous network formation, which can be seen as the long run outcome of strategic behavior adopted by intermediaries since the intermediaries have more flexibility in choosing links and adjusting cost in resource transmission in a longer period. Moreover, an extending strategy-proof mechanism with endogenous probabilistic punishment in Chapter 3 open the analysis on the frontier of mechanism design literature, which studies efficient mechanisms with costly verification or limited space for verification, and limited punishment. This mechanism will work, for example, in real life situation, there is a cost in information acquisition, and verification based on a targeted central agent, rather than exogenous verification and punishment rule.

Another interesting development of this model is to extend resource transmission process in these chapters to the case that multiple layers of intermediaries bargain in a different period based on the network they build. This could be achieved, for example, by developing a dynamic game theory model including the decisions about connection and price of transmitting resource to study the network structure in a stable state. For instance, the resource allocation through intermediaries is usually not finished once, the learning process in building a partnership for transmission is also worth taking into consideration. The dynamic game will be not only interesting to characterize the equilibrium of price competition as Chapter 2, but also provide another source of incentives for intermediaries to report their true connection with agents for the element of repeated cooperation, which will expand the class of strategy-proof mechanism.

Moreover, in our models, we assume there exists a planner to allocate the resource through bargaining with all intermediaries at the same time, so there is no coordination among intermediaries. In real life, it may take time for a planner to bargain with each intermediary, and the intermediaries may form a coalition to cooperate in pricing their services. To solve this problem, the mechanism is required to be group strategy-proof, which is not discussed.

Chapter 4 studies the application of network economics on empirical research, especially in the financial market of industry portfolios. The result in this chapter shows the potential for research in the field of network economics to be powerful in explaining the abnormal fluctuation of the financial market, especially for volatility comovement in the stock market. This chapter can be extended to study multigraph of the industry by including another layer of the network among industries with the input-output relationship, and study how these two layers of network correlate with each other. The result of research in this field will be helpful for government's fiscal policy interested in subsidy or taxation for the different industry to avoid a large crisis.

Finally, with regards to real-world phenomena, the problems about intermediation not only in transmitting resource, volatility comovement in the financial network but also in another area of information transmission, social learning, trading, etc. How will the role of intermediaries change based on the network structure for the different environment? From the planner's perspective, how could he design a mechanism to maximize the social welfare given that intermediaries adopt strategic behavior for their own interests? What kind of intervention can planner use to reduce the systemic risk through regulations on financial intermediaries? This is a fruitful direction that will enrich the models of intermediation described in this dissertation.

As the research on networks is diverse, so are the perspectives and approaches to understanding it. Mathematicians and computer scientists study 'what' the network looks like based on measurements, like degree, centrality, clustering, disclosure, community, including the observation that the degree distribution of many networks satisfies power law. They also work on 'how' to generate the graph similar to approximate the network in real life from a random network, like Erdős–Rényi model, preferential attachment model. Economists, on the other hand, look at the problem as a rational agent's strategic behavior based on the cost and benefit obtaining from the network structure and try to understand 'why' rational agents form network like this.

While methodological approaches in economics, especially from game theory and mechanism design, have been used in different chapters in this dissertation, they can be complemented by the approaches of other fields as well. Such approaches, like the ones on computability brought by computer scientists, graph theory brought by mathematicians, and other-regarding preferences brought by other social scientists, should help make the models and results in this dissertation even more aligned to real life problems. This should make network economics an even more relevant and active research area for years to come, that will bridge fields and use concepts from different fields beyond economics, computer sciences, and mathematics.

Appendix A PROOFS OF CHAPTER 2

In order to prove Lemma 1 and Theorem 4, we introduce three intermediate results related to the continuity and monotonicity of the indirect utility function as well as a result that provides necessary and sufficient conditions for a SPNE.

Lemma 2 (Continuity of the Indirect Utility Function)
For any problem (u, F), the indirect utility function $v(p)$ is continuous in p. Furthermore, the maximal utility $u^*(S, p)$ given group S is continuous in prices p.

Proof. We first show that the indirect utility function is continuous. We prove this in five steps, breaking the problem into converging sequences bounded from above and below the limit price.

Consider a decreasing sequence $\{p^i\}$, s.t. $\lim_{i\to\infty} p^i = p$, $p^i \geq p$. The indirect utility function $v(p)$ is non-increasing in p, since u is non-increasing in the aggregate prices and F is non-increasing in prices. Thus, $v(p^i) \leq v(p)$ for any i. Note that $\lim_{i\to\infty} v(p^i)$ exists by the monotonicity of $\{v(p^i)\}_i$. Therefore, $\lim_{i\to\infty} v(p^i) \leq v(p)$. From Definition 1, for any S, $F(S, p^i) \subseteq F(S, p)$. Let $S(p)$ and $x(p)$ are a utility maximizing group and optimal allocation at prices p, respectively. Let $S = S(p)$, then $v(p) = u(x(p), p_S)$. Since $F(S, p)$ is continuous in p, there exists a sequence of allocations $\{x^i\}_i$, s.t. $x^i \in F(S, p^i)$ for all i, $\lim_{i\to\infty} x^i = x(p)$. Since the utility function $u(x, t)$ is continuous, and $\lim_{i\to\infty} p^i_S = p_S$, then $\lim_{i\to\infty} u(x^i, p^i_S) = u(x(p), p_S)$. Since $v(p^i) \geq u(x^i, p^i_S)$, then $\lim_{i\to\infty} v(p^i) \geq \lim_{i\to\infty} u(x^i, p^i_S) = u(x(p), p_S) = v(p)$. This inequality together with the inequality above imply that $\lim_{i\to\infty} v(p^i) = v(p)$.

For any sequence $\{p^i\}$ with $\lim_{i\to\infty} p^i = p$, $p^i \geq p$. We can find a decreasing sequence p'^i, s.t. $p'^i \geq p^i$ and $\lim_{i\to\infty} p'^i = p$. Thus, $\lim_{i\to\infty} v(p'^i) \leq \lim_{i\to\infty} v(p^i) \leq v(p)$, and since $\lim_{i\to\infty} v(p'^i) = v(p)$, we have $\lim_{i\to\infty} v(p^i) = v(p)$.

Consider a increasing sequence $\{p^i\}$, with $\lim_{i\to\infty} p^i = p$, $p^i \leq p$. There exists limit for the monotonic decreasing sequence $v(p^i)$ and $v(p^i) \geq v(p)$, so $\lim_{i\to\infty} v(p^i) \geq v(p)$. Assume $v(p^i) = u^*(S^i, p^i)$, the group of intermediary $S^i \in 2^{\mathcal{N}}$, there exists group S, s.t. $\{p^k\}$ which is a subsequence of $\{p^i\}$, $S^k = S$ and $\lim_{k\to\infty} u^*(S, p^k) = \lim_{i\to\infty} v(p^i)$. Assume x^k is the utility maximizing allocation in $F(S, p^k)$, $u^*(S, p^k) = u(x^k, p^k_S)$, $x^k \in F(S, p^k) \subset F(\mathcal{N}, \mathbf{0}) \subset \mathbb{R}^M_+$. $F(\mathcal{N}, \mathbf{0})$ is compact, so $F(\mathcal{N}, \mathbf{0})$ is sequentially compact, there exists a convergent subsequence of x^k, name as x^h, s.t. $\lim_{h\to\infty} x^h = x$, $x^h \in F(S, p^h)$. OPF is continuous in prices p, so $x \in F(S, p)$. Since $\lim_{h\to\infty} x^h = x$ and $\lim_{h\to\infty} p^h = p$, $\lim_{h\to\infty} v(p^h) = \lim_{h\to\infty} u(x^h, p^h_S) = u(x, p_S) \leq u^*(S, p) \leq v(p)$. While $\lim_{h\to\infty} v(p^h) = \lim_{i\to\infty} v(p^i) \geq v(p)$, we have $\lim_{i\to\infty} v(p^i) = v(p)$.

For any sequence $\{p^i\}$ with $\lim_{i\to\infty} p^i = p$, $p^i \leq p$. We can find a increasing sequence p'^i, s.t. $p'^i \leq p^i$ and $\lim_{i\to\infty} p'^i = p$. Thus, $\lim_{i\to\infty} v(p'^i) \geq \lim_{i\to\infty} v(p^i) \geq v(p)$, and since $\lim_{i\to\infty} v(p'^i) = v(p)$, we have $\lim_{i\to\infty} v(p^i) = v(p)$.

Finally, we prove that for any sequence $\{p^i\}$ such that $\lim_{i\to\infty} p^i = p$, we have that $\lim_{i\to\infty} v(p^i) = v(p)$. Construct two sequences $\{p^{1i}\}$ and $\{p^{2i}\}$, let $p_n^{1i} = \min\{p_n^i, p_n\}$, and $p_n^{2i} = \max\{p_n^i, p_n\}$, then $p^{1i} \leq p$, $p^{2i} \geq p$ and $p^{1i} \leq p^i \leq p^{2i}$, $\lim_{i\to\infty} p^{1i} = \lim_{i\to\infty} p^{2i} = p$. Thus, $\lim_{i\to\infty} v(p^{1i}) \geq \lim_{i\to\infty} v(p^i) \geq \lim_{i\to\infty} v(p^{2i})$. From above, $\lim_{i\to\infty} v(p^{1i}) = \lim_{i\to\infty} v(p^{2i}) = v(p)$, then $\lim_{i\to\infty} v(p^i) = v(p)$. Thus, the indirect utility function is continuous in price p.

We can similarly show that the maximal utility $u^*(S,p)$ is continuous in prices p, given group S. Indeed, consider a decreasing sequence $\{p^i\}$, with $p^i \geq p$ and $\lim_{i\to\infty} p^i = p$. $u^*(S,p^i) = \max_{x\in F(S,p^i)} u(x, p_S^i)$. $F(S,p^i) \subseteq F(S,p^j)$, $u(x,p_S^i) \leq u(x,p_S^j)$ for $i \leq j$, $p^i \geq p^j$. Thus, $u^*(S,p^i) \leq u^*(S,p^j)$, $u^*(S,p^i) \leq u^*(S,p)$, and $\lim_{i\to\infty} u^*(S,p^i) \leq u^*(S,p)$. Assume $u^*(S,p) = u(x,p_S)$, $x \in F(S,p)$. Since $F(S,p)$ is continuous in p, there exists a sequence of allocations $\{x^i\}_i$, s.t. $x^i \in F(S,p^i)$ for all i, $\lim_{i\to\infty} x^i = x(p)$. Since the utility function $u(x,t)$ is continuous in x and t, and $\lim_{i\to\infty} p_S^i = p_S$, then $\lim_{i\to\infty} u(x^i, p_S^i) = u(x(p), p_S)$. Since $u^*(S,p^i) \geq u(x^i, p_S^i)$, then $\lim_{i\to\infty} u^*(S,p^i) \geq \lim_{i\to\infty} u(x^i,p_S^i) = u(x(p),p_S) = u^*(S,p)$. This inequality together with the inequality above imply that $\lim_{i\to\infty} u^*(S,p^i) = u^*(S,p)$.

For an increasing sequence $\{p^i\}$, with $p^i \leq p$ and $\lim_{i\to\infty} p^i = p$. By definition of $u^*(S,p)$, $u^*(S,p) \leq u^*(S,p^i)$, there exists limit of the sequence $\lim_{i\to\infty} u^*(S,p^i)$, and $u^*(S,p) \leq \lim_{i\to\infty} u^*(S,p^i)$. Assume $u^*(S,p^i) = u(x_i, p_S^i)$, $F(\mathcal{N}, \mathbf{0})$ is sequentially compact, there exists a convergent subsequence of x^i, name as x^k, s.t. $\lim_{k\to\infty} x^k = x$, $x^k \in F(S,p^k)$. OPF is continuous in prices p, so $x \in F(S,p)$. Since $\lim_{k\to\infty} x^k = x$ and $\lim_{k\to\infty} p^k = p$, $\lim_{k\to\infty} u^*(S,p^k) = \lim_{k\to\infty} u(x^k, p_S^k) = u(x,p) \leq u^*(S,p)$. Thus, we have $\lim_{i\to\infty} u^*(S,p^i) = u^*(S,p)$.

By repeating the same strategy, used in the continuity of the indirect utility function, that bounds an arbitrary converging sequence with monotonic convergent sequences, we have that for any sequence p^i such that $\lim_{i\to\infty} p^i = p$, $p^i \leq p$, there is $\lim_{i\to\infty} u^*(S,p^i) = u^*(S,p)$. And for any sequence p^i, with $\lim_{i\to\infty} p^i = p$, $p^i \geq p$, there is $\lim_{i\to\infty} u^*(S,p^i) = u^*(S,p)$. Hence, since any convergent sequence $\{p^i\}$ to p can be split into two convergent sub-sequences with values above or below p, we have that $\lim_{i\to\infty} u^*(S,p^i) = u^*(S,p)$. Hence, the function $u^*(S,p)$ is continuous in prices p. ■

Lemma 3 (Monotonicity of the Indirect Utility Function)

a. *If the problem (u, F) is monotonic, then the indirect utility function v is monotonic in prices. That is, if $p' < p$ and $v(p) > u((0,\ldots,0),0)$ then $v(p') > v(p)$.*

b. Consider a utility maximizing group S at prices p, intermediary $n \in S$ and prices p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$. If $v(p) > u((0, \ldots, 0), 0)$, then $v(p') > v(p)$.

c. Consider any group S, intermediary $n \in S$ and prices p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$. If $u^*(S, p) > u((0, \ldots, 0), 0)$, then $u^*(S, p') > u^*(S, p)$.

d. Either of the following conditions is sufficient for the monotonicity of the problem (u, F):

- *u is strictly monotonic on the total price paid by the planner. That is, $u(x, t) > u(x, \tilde{t})$ for any $t < \tilde{t}$ and $x \in A$.*
- *The outcome possibility function F is strongly monotonic in prices.*

Proof. First note that part a is clearly implied by part b.

To prove part b, assume S is a utility maximizing group at prices p and such that $v(p) > u((0, \ldots, 0), 0)$. Let x be a utility maximizing allocation in $F(S, p)$. Then, $v(p) = u(x(p), p_S)$. Given that the problem (u, F) is monotonic, for $n \in S$, and prices p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$, there exists $y \in F(S, p')$ such that $u(y, p'_S) > u(x, p_S)$, so $v(p') \geq u(y, p'_S) > u(x, p_S) = v(p)$.

To prove part c, consider the group S and prices p such that $u^*(S, p) > u((0, \ldots, 0), 0)$. Let x be a utility maximizing allocation in $F(S, p)$. Then, $u^*(S, p) = u(x, p_S)$. Given that the problem (u, F) is monotonic, for each $n \in S$ and prices p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$, there exists $y \in F(S, p')$ such that $u(y, p'_S) > u(x, p_S)$, so $u^*(S, p') \geq u(y, p'_S) > u(x(p), p_S) = u^*(S, p)$.

In order to prove part d, suppose that $u(x, t)$ is monotonic in t. Let p, p' and S defined as above. By the monotonicity of the OPF, $F(S, p) \subseteq F(S, p')$. Thus, any allocation of resource $x \in F(S, p)$, is also feasible for prices p', that is $x \in F(S, p')$. Let $y = x$, $u(x, p'_S) > u(x, p_S)$, the problem (u, F) is monotonic.

On the other hand, consider a price p and a utility maximizing group S at prices p. Consider intermediary $n \in S$ and prices p' such that $p'_n < p_n$ and $p_{-n} = p'_{-n}$. Let x be a utility maximizing allocation in $F(S, p)$ and assume that $u(x, p_S) > u((0, \ldots, 0), 0)$. Since F is strongly monotonic in p, for $v(p) = u(x(p), p_S)$ and $x(p) \in F(S, p)$, there is $\epsilon > 0$, s.t. $B_\epsilon(x(p)) \subset F(S, p')$. Since the preferences represented by $u(x, t)$ are monotonic in A, then there exists $y \in B_\epsilon(x(p))$, s.t. $u(y, p_S) > u(x(p), p_S) \geq u(x, p_S)$. Therefore, $u(y, p'_S) \geq u(y, p_S) > u(x, p_S)$. ■

Lemma 4 (Conditions for Existence of SPNE)

Consider prices and allocation (p, b, x) that is a SPNE. Assume that the utility maximizing groups at prices p are $S_1(p), ..., S_J(p)$. Then,

a. $\bigcap_{j=1}^{J} S_j(p) = \emptyset$.

b. *If the problem* (u, F) *is monotonic in prices, then there exists a utility maximizing group, without loss of generality, assume it is* $S_1(p)$*, s.t.* $\forall n \in \bigcup_{j=1}^{J} S_j(p) \setminus S_1(p)$, $p_n = 0$.

c. $\forall n \in \mathcal{N} \setminus \bigcup_{j=1}^{J} S_j(p)$*, it would not increase the planner's utility even if its price decreases to* 0*. That is, for* $p'_n = 0$ *and* $p' = (p'_n, p_{-n})$*, we have that* $v(p) = v(p')$.

Conversely, if there exists a vector of prices p *that satisfies conditions (a)-(c), then* p *is supported by a SPNE. (with* $b(p) = S_1(p)$*)*

Proof. a. Suppose $\bigcap_{j=1}^{J} S_j(p) \neq \emptyset$, then there exists intermediary $n \in \bigcap_{j=1}^{J} S_j(p)$. The indirect utility function without using intermediary n is $v_{-n}(p_{-n})$. Since intermediary n is in every utility maximizing group, the utility maximizing group in $\mathcal{N} \setminus \{n\}$ would achieve lower utility at prices p_{-n}, thus $v_{-n}(p_{-n}) < v(p)$. By Lemma 2, the indirect utility function is continuous, there exists ϵ small enough, such that intermediary n increases its price by ϵ, $p'_n = p_n + \epsilon$, $p' = (p'_n, p_{-n})$, s.t. $v(p) \geq v(p') > v_{-n}(p_{-n})$. Intermediary n would still be paid by the planner after increasing price by ϵ, hence there is incentive for intermediary n to deviate, thus p cannot be a SPNE. Hence, $\bigcap_{j=1}^{J} S_j(p) = \emptyset$.

b. Suppose that we cannot find such a utility maximizing group $S(p)$ at price p that includes all intermediaries with positive price in $\bigcup_{j=1}^{J} S_j(p)$. Assume the planner allocates resource with intermediaries in $S_1(p)$, and intermediary $n \in \bigcup_{j=1}^{J} S_j(p) \backslash S_1(p)$ posts a price $p_n > 0$. Without loss of generality, assume that $n \in S_2(p)$. Since intermediary n is not used by planner, it receives 0 utility. Consider the price vector $p' = (p_n - \epsilon, p_{-n})$ for some $\epsilon > 0$ such that $p_n > \epsilon$. Since the problem (u, F) is monotonic, by Lemma 3, if $v(p) > u((0, \ldots, 0), 0)$, $v(p') \geq u^*(S_2(p), p') > u^*(S_2(p), p) = v(p)$. Since $v(p) = v_{-n}(p_{-n}) = v_{-n}(p'_{-n})$, then intermediary n will be in the utility maximizing group at price p and selected by the planner at prices p'. Thus, the payoff of intermediary n increases from 0 to p'_n, which is a contradiction. If $v(p) = u((0, \ldots, 0), 0)$, then for any group $S \subseteq \mathcal{N}$, $u^*(S, p) \leq u((0, \ldots, 0), 0)$, $S_1(p) = \mathcal{N}$ satisfies the condition.

c. For intermediary n not to be in any utility maximizing group $S_j(p)$ at price p, it will not be used by the planner. If it lowers its price and improves the maximal utility achieved by the planner, it has incentive to lower its price and get paid. To make sure this case will not happen in a SPNE, it requires that even when the price decreases to 0, the maximal utility of the planner would not increase. Thus, if $p'_n = 0$, $p' = (p'_n, p_{-n})$, then $v(p) = v(p')$.

In order to prove the converse, consider p satisfying conditions (a), (b) and (c) and assume that the planner chooses the group $S_1(p)$ in condition (a), paying all intermediaries in $\bigcup_{j=1}^{J} S_j(p)$ with positive price. Since $S_1(p)$ is a utility maximizing group, the planner will achieve the maximal utility under p. For intermediary n, $n \in \mathcal{N} \setminus \bigcup_{j=1}^{J} S_j(p)$, it has no incentive to increase price based on the monotonicity of problem, the utility from groups of intermediaries including n will not be maximal for planner, n will not be paid. At the same time, condition (c) shows n has no incentive to decrease price. If $n \in \bigcup_{j=1}^{J} S_j(p)$, intermediary n has no incentive to increase its price by condition (a), because the planner will use a different utility maximizing group of intermediaries if its price is higher. This intermediary will not decrease its price by condition (b), since by lowering the price the intermediary receives less profit. Thus, any price p satisfying conditions (a),(b),(c) is a SPNE. ■

Proof of Lemma 1

Proof. Consider the intermediation problem (u, F) and let $(b(p), x(p))$ be a strategy of the planner that maximizes u given p. Let $S_1(p), \ldots, S_J(p)$ be the utility maximizing groups at prices p. Without loss of generality, we assume that the strategy of planner $(b(p), x(p))$ satisfies the following two tie-breaking rules: (1) If $\emptyset$ is a utility maximizing group at prices p, let $\underline{p}_n = \min\{q_n \in [0, P_n] | v(q_n, p_{-n}) = u((0, \ldots, 0), 0)\}$ be the minimal price of agent n at which the empty coalition is efficient (by continuity of the indirect utility function such price exists). We require that if $p_n = \underline{p}_n$ then $n \in b(p)$, whereas if $p_n > \underline{p}_n$ then $n \notin b(p)$.

(2) if $\emptyset$ is not a utility maximizing group at prices p and there exists a utility maximizing group $S_1(p)$ such that $p_n = 0, \forall n \in \bigcup_{j=1}^{J} S_j(p) \setminus S_1(p)$ then $b(p) = S_1(p)$ (if there are multiple groups satisfying this condition then the planner chooses one among them). These tie-breaking rules guarantees that inaction is taken when it is optimal for the planner to do so. Furthermore, when there exists one or more utility maximizing groups containing all intermediaries who post positive prices, then the planner chooses one of such groups.

Let $\mathcal{G}$ be the simultaneous move game of the intermediaries, where the strategy of intermediary n is p_n and the payoff to intermediary n is $V^n(p) = V^n(p, b(p), x(p)), \forall n \in \mathcal{N}$, given $(b(p), x(p))$. Clearly, a Nash equilibrium price vector p^* in game $\mathcal{G}$ is a SPNE of the resource transmission game generated by (u, F). By price-satiation, the strategy space $[0, P_n]$ of intermediary n satisfies $P_n < +\infty$.

In order to prove that the game $\mathcal{G}$ has a Nash equilibrium, we verify the conditions of Theorem 3.1 in Reny [125]. Indeed, the strategy is non-empty, compact, convex subset are assumed. We show that the utility function $V^n(p)$ is quasi-concave in p_n, and that the game $\mathcal{G}$ is better reply secure.

Step 1. The payoff function $V^n(p)$ of intermediary n is quasi-concave in p_n.

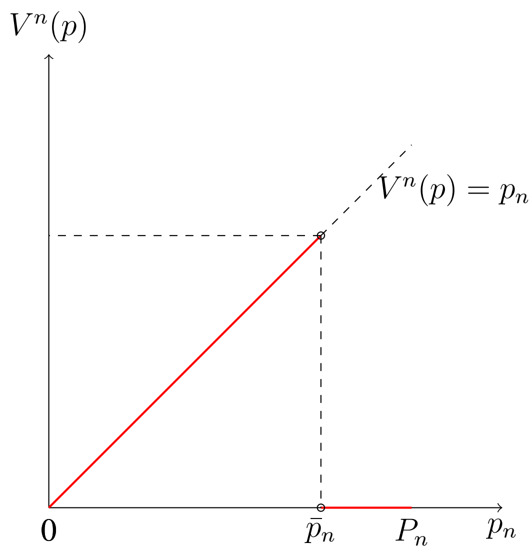


Figure A.1: The payoff function $V^n(p)$ is quasi-concave in prices.

First, we show that the maximal utility of the group of intermediaries S at prices p (denoted by $u^*(S,p)$): (i) is decreasing in p_n for $n \in S$, and (ii) is independent of p_n for $n \notin S$.

Consider prices p and p', with $p_{-n} = p'_{-n}$, $p_n < p'_n$ and $n \notin S$. In order to show (i), for $n \in S$, since the problem (u, F) is monotonic, from Lemma 3, if $u^*(S,p') > u((0,\ldots,0),0)$, $u^*(S,p') < u^*(S,p)$. In order to show (ii), for $n \notin S$, $F(S,p) = F(S,p')$ from assumption (d) of OPF in definition 1. Furthermore, $u(x,p_S) = u(x,p'_S)$ since $p_S = p'_S$, $\forall x \in F(S,p)$. Hence, $u^*(S,p) = u^*(S,p')$.

Assume the upper bound of the intermediary n's price is large enough that $u^*(S,p) < 0$ for $p = (P_n, \mathbf{0}_{-n})$ and $\forall S$ with $n \in S$, which means price p_n will not be paid if the price is too high.

Given the strategy of planner $(b(p), x(p))$, a utility maximizing group would be chosen. For any group of intermediaries S and T, $n \in S$ and $n \notin T$. As price p_n increases, $u^*(S,p)$ decreases and becomes negative if $p_n = P_n$, while $u^*(T,p)$ does not change as p_n changes. Given p_{-n}, define $\bar{p}_n$, s.t. $\max_S u^*(S,(\bar{p}_n, p_{-n})) = \max_T u^*(T,p)$, otherwise $\bar{p}_n = 0$, if $\max_S u^*(S,(0,p_{-n})) \leq \max_T u^*(T,p)$. When $p_n < \bar{p}_n$, $u^*(S,p) > u^*(T,p)$, for some S and $\forall T$. When $p_n > \bar{p}_n$, $u^*(S,p) \leq u^*(T,p)$ for some T and $\forall S$. Thus, the payoff function $V^n(p) = p_n$ when $p_n < \bar{p}_n$, and $V^n(p) = 0$ when $p_n > \bar{p}_n$. The function $V^n(p)$ is shown in as shown in Figure A.1. If $p_n = \bar{p}_n$, then the planner is indifferent between choosing or not intermediary n.

In order to show the quasi-concavity of $V^n(p)$, consider constant $c \geq 0$. If $n \in b(\bar{p}_n, p_{-n})$, $V^n(\bar{p}_n, p_{-n}) = \bar{p}_n$. The upper contour set $\{p_n | V^n(p) \geq c\}$ equals $[c, \bar{p}_n]$ when $c \leq \bar{p}_n$, and $\{p_n | V^n(p) \geq c\} = \emptyset$ when $c > \bar{p}_n$. If $n \notin b(\bar{p}_n, p_{-n})$, $V^n(\bar{p}_n, p_{-n}) = 0$. The upper contour set

$\{p_n | V^n(p) \geq c\}$ equals $[c, \bar{p}_n)$ when $c \leq \bar{p}_n$, and $\{p_n | V^n(p) \geq c\} = \emptyset$ when $c > \bar{p}_n$. Thus, the upper contour set $\{p_n | V^n(p) \geq c\}$ is convex for any constant c.

Step 2. The game $\mathcal{G}$ for intermediaries is better reply secure. That is, for any prices p, which is not a Nash equilibrium, there exists intermediary n who can secure a payoff strictly above $V^n(p)$ at p.

Assume the utility maximizing group of intermediaries at prices p are $S_1(p), \ldots, S_J(p)$. From Lemma 2, the maximal utility $u^*(S,p)$ is continuous in p. The problem (u, F) is monotonic, from Lemma 4, the price vector p satisfies condition (a) to (c) if it is a Nash equilibrium. So, if prices p is not Nash Equilibrium, at least one condition is not satisfied.

If condition (a) is not satisfied, then there exists intermediary $n \in \bigcap S_j(p)$. This implies that $u^*(S_j(p), p) > u^*(T, p)$, $\forall T \neq S_j(p)$, $\forall j$. Since $u^*(S,p)$ is continuous in p, for p'_{-n} in a small neighborhood of p_{-n}, $u^*(S_j(p), (p_n, p'_{-n})) > u^*(T, (p_n, p'_{-n}))$, $\forall T$. Thus, there exists $p'_n > p_n$, s.t. $u^*(S_j(p), (p'_n, p'_{-n})) > u^*(T, (p'_n, p'_{-n}))$ for some $S_j(p)$. Thus, some group of intermediaries $S_j(p)$ with $n \in S_j(p)$ is utility maximizing group for prices $p' = (p'_n, p'_{-n})$, intermediary n will secure payoff $V^n(p') = p'_n > p_n = V^n((p_n, p'_{-n}))$.

If condition (b) is not satisfied, then there exists intermediary $n \in \bigcup S_j(p)$ who is not used by the planner, and $p_n > 0$. Since problem (u, F) is monotonic in prices, there is $0 < p'_n < p_n$, s.t. for some group of intermediaries $S_j(p)$ where $n \in S_j(p)$, $u^*(S_j(p), (p'_n, p_{-n})) > u^*(T, (p'_n, p_{-n}))$, $\forall T$, $n \notin T$, because the maximal utility of group T without n does not change as price p_n changes to p'_n, while $u^*(S_j(p), p)$ increases and $S_j(p)$ is utility maximizing group at prices p. Assume the group of intermediaries S, where $n \in S$, achieves maximal utility at prices (p'_n, p_{-n}), $v((p'_n, p_{-n})) = u^*(S, (p'_n, p_{-n}))$, then for p'_{-n} close enough to p_{-n}, we have $u^*(S, (p'_n, p'_{-n})) > u^*(T, (p'_n, p'_{-n}))$, $\forall T$, $n \notin T$. Thus, for neighborhood of p_{-n}, intermediary n chooses price p'_n would secure payoff $V^n(p') = p'_n > 0 = V^n((p_n, p'_{-n}))$.

If condition (c) is not satisfied, then there exists intermediary n who could decrease the price from p_n to $p'_n > 0$ to be used by the planner, so there is some utility maximizing group S at prices (p'_n, p_{-n}) with $n \in S$, and $u^*(S, (p'_n, p_{-n})) > u^*(T, (p'_n, p_{-n}))$, $\forall T$, $n \notin T$.

Thus, for prices p'_{-n} in the neighborhood of p_{-n}, $u^*(S, p') > u^*(T, p')$. Thus, intermediary n chooses prices p'_n would secure payoff $V^n(p', b, x) = p'_n > 0 = V^n((p_n, p'_{-n}))$.

Thus, for all prices p not an Nash equilibrium, some intermediary n can secure a strictly higher payoff. The game $\mathcal{G}$ is better reply secure.

Hence, game $\mathcal{G}$ has at least one pure-strategy Nash equilibrium, and there exists a SPNE in the resource transmission game generated by problem (u, F). ■

Proof of Theorem 2

Proof. $\Leftarrow$) Suppose that $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$, $\forall n$. Consider intermediary n who posts price $p_n > 0$. Let $p = (p_n, \mathbf{0}_{-n})$. Therefore, for any group S^n such that $n \in S^n$, $u^*(S^n, p) \leq u^*(S^n, \mathbf{0}) \leq u^*(S_j(\mathbf{0}), \mathbf{0})$ for any j. Since $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$ then there exists group $S_j(\mathbf{0})$ such that $n \notin S_j(\mathbf{0})$. Hence intermediary n will not be chosen by the planner at p. Note that in the case where $u^*(S^n, p) = u^*(S_j(\mathbf{0}), \mathbf{0})$, the planner chooses a group of intermediaries who post zero prices even though it is indifferent with some group of intermediaries with positive price.

$\Rightarrow$) Let $S_1(p), \ldots, S_{J'}(p)$ be the set of utility maximizing groups at prices p. First, we show that if $\bigcap_{j=1}^{J'} S_j(p) \neq \emptyset$, then p is not a SPNE. Indeed, pick $n \in \bigcap_{j=1}^{J} S_j(p)$. Without using intermediary n, given the prices of other intermediaries p_{-n}, the indirect utility function without intermediary n (Definition 5) is $v_{-n}(p_{-n})$. Since n is used in all utility maximizing groups $S_j(p)$, then $v(p) > v_{-n}(p_{-n})$. Consider the decreasing sequence of prices $\{p^i\}$ such that $p^i_n = p_n + \frac{1}{i}$ and $p^i_{n'} = p_{n'}$ for any $n' \neq n$. Since $p^i \geq p$ and $\lim_{i\to\infty} p^i = p$, by Lemma 2, $\lim_{i\to\infty} v(p^i) = v(p)$. Moreover, since $v(p) > v_{-n}(p_{-n})$, then there exists i large enough such that $v(p) \geq v(p^i) > v_{-n}(p_{-n})$. Thus, at the price vector p, intermediary n can increase his price by $\frac{1}{i}$ and get higher profit.

Second, let p be an IFE. We show that $S = S_j(p)$ is also a utility maximizing group at prices $\mathbf{0}$. Since p is IFE, there exists a planner-optimal allocation x such that $x \in F(S, p)$. Furthermore, $u(x, 0) = u^*(S_k(\mathbf{0}), \mathbf{0})$ for any k. Since $x \in F(S, p)$ and $F(S, p) \subseteq F(S, \mathbf{0})$ by the monotonicity of F, then $x \in F(S, \mathbf{0})$. Thus, x is feasible and utility maximizing allocation in $F(S, \mathbf{0})$. Therefore, $u^*(S, \mathbf{0}) = u(x, 0) = u^*(S_k(\mathbf{0}), \mathbf{0})$. Hence, S is a utility maximizing group at prices $\mathbf{0}$.

Finally, if p is an intermediation free equilibrium, then it is a SPNE. Thus, by the first step above, $\cap_{i=1}^{J'} S_i(p) = \emptyset$. By the second step, $\cap_{i=1}^{J} S_i(\mathbf{0}) \subseteq \cap_{i=1}^{J'} S_i(p)$. Hence, $\cap_{i=1}^{J} S_i(\mathbf{0}) = \emptyset$ ■

Proof of Remark 3

Proof. a. $F(S, p) = F(S)$, $u^*(S, p) = \max_{x\in F(S)} u(x, p_S) = \max_{x\in F(S)} \alpha(x)\beta(p_S)$. $u^*(S, \mathbf{0}) \leq u^*(T, \mathbf{0})$, which is $\max_{x\in F(S)} \alpha(x)\beta(0) \leq \max_{x\in F(T)} \alpha(x)\beta(0)$, equivalent with $\max_{x\in F(S)} \alpha(x) \leq \max_{x\in F(T)} \alpha(x)$. Then, $\max_{x\in F(S)} \alpha(x)\beta(p_S) \leq \max_{x\in F(T)} \alpha(x)\beta(p_T)$ for $p_S = p_T$. Therefore, $u^*(S, p) \leq u^*(T, p)$ whenever $p_S = p_T$.

b. $u(x, p) = u(x)$, $u^*(S, p) = \max_{x\in F(S,p)} u(x, p_S) = \max_{x\in \gamma(S)\delta(p_S)} u(x)$. $u^*(S, \mathbf{0}) \leq u^*(T, \mathbf{0})$, means $\max_{x\in\gamma(S)\delta(0)} u(x) \leq \max_{x\in\gamma(T)\delta(0)} u(x)$. Assume x^1 and x^2 solves $\max_{x\in\gamma(S)\delta(0)} u(x)$ and $\max_{x\in\gamma(T)\delta(0)} u(x)$ respectively, there is $u(x^1) \leq u(x^2)$. Since the preferences are homothetic, $u(\delta(t)x^1) \leq u(\delta(t)x^2)$, and $\delta(t)x^1$, $\delta(t)x^2$ solves the problem $\max_{x\in\gamma(S)\delta(t)} u(x)$ and $\max_{x\in\gamma(T)\delta(t)} u(x)$, thus $u^*(S, p) \leq u^*(T, p)$, with $t = p_S = p_T$. ■

Proof of Theorem 4

Proof.

The converse has already been shown in the text. To prove the forward part, we will use Lemmas 3 and 4.

Similar with Lemma 1, assume that the strategy of planner $(b(p), x(p))$ satisfies the following two tie-breaking rules: (1) If $\emptyset$ is a utility maximizing group at prices p, let $\underline{p}_n(p_{-n}) = \min\{q_n \in [0, P_n] | v(q_n, p_{-n}) = u((0, \ldots, 0), 0)\}$ be the minimal price of agent n at which the empty coalition is efficient (by continuity of the indirect utility function such price exists). We require that if $p_n = \underline{p}_n(p_{-n})$ then $n \in b(p)$, whereas if $p_n > \underline{p}_n(p_{-n})$ then $n \notin b(p)$. (2) if $\emptyset$ is not a utility maximizing group at prices p and there exists a utility maximizing group $S_1(p)$ such that $p_n = 0$, $\forall n \in \bigcup_{j=1}^{J} S_j(p) \setminus S_1(p)$, then $b(p) = S_1(p)$ (if there are multiple groups satisfying this condition then the planner chooses one among them).

Recall that $u^*(S, p)$ is the maximal utility at prices p and group S. Let $\bar{u} = u^*(S_1(\mathbf{0}), \mathbf{0}) = \cdots = u^*(S_J(\mathbf{0}), \mathbf{0})$ be the maximal utility achieved by the planner when the prices are zero. Assume there is a robust SPNE with $p \neq \mathbf{0}$. Then, $\forall p_n > 0$, intermediary n is used in the utility maximizing group of intermediaries at price p. From part (b) of Lemma 4, there exists a utility maximizing group S_1 at price p, such that all the intermediaries with positive prices are in the group S_1, that is, $\forall p_n > 0$, intermediary $n \in S_1$. The maximal utility of the planner for the use of group S_1 is $u^*(S_1, p)$. Since $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$, for any intermediary k with $p_k > 0$, there exists $S_j(\mathbf{0})$ such that $k \notin S_j(\mathbf{0})$. Let $S_2 = S_j(\mathbf{0})$. $p_{S_1} = \sum_{p_n > 0} p_n > \sum_{n \in S_2} p_n = p_{S_2}$. S_2 is utility maximizing group at prices $\mathbf{0}$. $\bar{u} = u^*(S_2, \mathbf{0}) \geq u^*(S_1, \mathbf{0})$.

Since intermediary $k \in S_1$ and $k \notin S_2$, there are two cases: $S_2 \subset S_1$ or $S_2 \setminus S_1 \neq \emptyset$.

If $S_2 \subset S_1$, let the prices p^1 satisfy $p^1_{[S_2]} = p_{[S_2]}$ and $p^1_{[\mathcal{N} \setminus S_2]} = 0$, thus $u^*(S_2, p^1) = u^*(S_2, p)$. Then, $p_{S_2} = p^1_{S_2} = p^1_{S_1}$. From cross-monotonicity, $u^*(S_2, p^1) \geq u^*(S_1, p^1)$. At the same time, $p^1_{[S_1]} \leq p_{[S_1]}$ and $p^1_{[S_1]} \neq p_{[S_1]}$. The problem (u, F) is monotonic, by Lemma 3, if $u^*(S_1, p) > u((0, \ldots, 0), 0)$, $u^*(S_1, p^1) > u^*(S_1, p)$ holds. Thus, $u^*(S_2, p) > u^*(S_1, p)$, which contradicts that S_1 is a utility maximizing group at price p. If $v(p) = u((0, \ldots, 0), 0)$, then $\forall n$, $p_n > 0$, $p_n \geq \underline{p}_n(p_{-n})$, because $\underline{p}_n(p_{-n}) = \min\{q_n \in [0, P_n] | v(q_n, p_{-n}) = u((0, \ldots, 0), 0)\}$, and $n \in b(p)$. Given the tie-breaking rules above, if $p_n > \underline{p}_n(p_{-n})$, intermediary n will not be paid, so $p_n = \underline{p}_n(p_{-n})$. Problem (u, F) is monotonic, $\forall 0 < p'_n < p_n$, there is $v(p'_n, p_{-n}) > u((0, \ldots, 0), 0)$. Thus, utility strictly increases as p_n decreases, $u^*(S_1, p^1) > u^*(S_1, p)$ holds and $u^*(S_2, p) > u^*(S_1, p)$

If $S_2 \setminus S_1 \neq \emptyset$, assume intermediary $i \in S_2$ and $i \notin S_1$, let prices p^2 satisfy $p^2_{[S_1]} = p_{[S_1]}$ and $p^2_i = p_{S_1} - p_{S_2}$ and $p^2_{[\mathcal{N}\setminus(S_1\cup\{i\})]} = 0$. Then there is $p^2_{S_1} = p^2_{S_2}$. By cross-monotonicity, $u^*(S_2, p^2) \geq u^*(S_1, p^2) = u^*(S_1, p)$. At the same time, $p^2_{[S_2]} \geq p_{[S_2]}$ and $p^2_{[S_2]} \neq p_{[S_2]}$, thus $u^*(S_2, p) > u^*(S_2, p^2)$ by monotonicity of the problem (u, F). So $u^*(S_2, p) > u^*(S_1, p)$ which is a contradiction with S_1 being a utility maximizing group at price p. Hence, $p = \mathbf{0}$ is the unique robust SPNE. ■

Proof of Remark 5

Proof. Cross-monotonicity is necessary for Theorem 4 to hold.

Consider the planner's utility function $u(x, t) = \tilde{u}(x)$ that is independent of the prices paid to the intermediaries. Also, consider the OPF F that is strongly monotonic in prices such that for intermediaries 1 and 2, $F(\{1\}, \mathbf{0}) = F(\{2\}, \mathbf{0}) = F(\mathcal{N}, \mathbf{0})$. Moreover, for some price vector $p = (p_1, p_2, 0, \ldots, 0)$ where $p_1, p_2 > 0$, we have that $F(S, p) \subseteq F(\{1, 2\}, p) = F(\{1\}, p) = F(\{2\}, p)$ for any $S \subseteq \mathcal{N}$. First note that problem (u, F) is monotonic because F is strongly monotonic in prices. However, (u, F) is not cross-monotonic. To see this, assume that (u, F) is cross-monotonic. Then, $u^*(\{1\}, \mathbf{0}) = u^*(\{1, 2\}, \mathbf{0})$ implies that $u^*(\{1\}, p) = u^*(\{1, 2\}, (p_1, \mathbf{0}_{-1}))$. Furthermore, $u^*(\{1, 2\}, (p_1, \mathbf{0}_{-1})) > u^*(\{1, 2\}, p)$ by strong monotonicity in prices of F. Hence, $u^*(\{1\}, p) > u^*(\{1, 2\}, p)$, which contradicts $F(\{1, 2\}, p) = F(\{1\}, p)$. Finally, note that $\mathbf{0}$ and p are prices that are robust SPNE in the problem (u, F), since group $\{1, 2\}$ is a utility maximizing group at both prices, hence when such group is chosen by the planner no intermediary has the incentive to deviate by strong monotonicity in prices of F.

Monotonicity in prices is necessary for Theorem 4 to hold.

Consider the planner's utility function $u(x, t) = \tilde{u}(x)$ that is independent of the prices paid to the intermediaries. Also, consider an OPF F that satisfies the following conditions: (1) $F(\mathcal{N}, \mathbf{0}) = F(\mathcal{N} \setminus \{n\}, \mathbf{0})$, $\forall n$; (2) $F(\mathcal{N}, \mathbf{0}) = F(S, p)$ for some prices $p = (p_1, \mathbf{0}_{-1})$ such that $p_1 > 0$ and a group S such that $1 \in S$; (3) $\forall n \neq 1$, for prices p^n with $p^n = (p_1, 0, \ldots, p_n, 0, \ldots, 0)$, $p_n > 0$, $x^*(\mathcal{N}, \mathbf{0}) \cap F(T, p^n) = \emptyset \ \forall T$ with $n \in T$, and $F(\mathcal{N} \setminus \{n\}, p^n) = F(\mathcal{N} \setminus \{n\}, \mathbf{0})$; finally, (4) for any price vector $p' = (p'_1, \mathbf{0}_{-1})$, $p'_1 > p_1$, $x^*(\mathcal{N}, \mathbf{0}) \cap F(T, p') = \emptyset$, $\forall T$ with $1 \in T$. The problem (u, F) meeting these conditions is not monotonic in prices since u does not depend on t and F is not strongly monotonic in prices by condition (2).

We now see that the problem (u, F) has multiple robust SPNE. Indeed, first notice that since condition (1) is satisfied, $\mathbf{0}$ is an IFE by Theorem 2, and thus it is a robust SPNE. We now show that $p = (p_1, \mathbf{0}_{-1})$ is also a robust SPNE. Indeed, by condition (2), S is a utility maximizing group at prices $\mathbf{0}$. Assume that the planner chooses S and pays p_1 to intermediary 1, and the maximal utility that the planner could achieve at price $\mathbf{0}$ is $\bar{u}$. Then, intermediary 1 has no incentive to decrease its price. On the other hand, if intermediary 1 increases its price to p_1', from condition (4), the planner cannot get any utility maximizing allocation when choosing a group that contains intermediary 1. However, at price p' the planner can get utility $\bar{u}$ by choosing group $\mathcal{N} \setminus \{1\}$, since by condition (1), $F(\mathcal{N}, \mathbf{0}) = F(\mathcal{N} \setminus \{1\}, \mathbf{0}) = F(\mathcal{N} \setminus \{1\}, p')$. Thus, the planner will not choose intermediary 1 if his price increases to p_1'. Alternatively, consider the case where intermediary $n \neq 1$ deviates to a price p_n. By condition (3), the planner will have utility less than $\bar{u}$ using any group with intermediary n and get $\bar{u}$ with $\mathcal{N} \setminus \{n\}$. Thus, if intermediary n charges a positive price he will not be used by planner. So no intermediary has incentive to deviate from $p = (p_1, 0, \ldots, 0)$, and hence p is a robust SPNE. ■

Proof of Corollary 1

Proof. a. We show that $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$ is satisfied. For any intermediary n, let $S_n = \mathcal{N} \setminus \{n\}$. From the condition of minimally competitive outcome, $F(\mathcal{N}, \mathbf{0}) = F(S_n, \mathbf{0})$, for any n. $\max_{x \in F(S_n, \mathbf{0})} u(x, 0) = \max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0)$, thus $S_n = \mathcal{N} \setminus \{n\}$ is a utility maximizing group. Then $\bigcap_n S_n = \emptyset$, so the intersection of utility maximizing group at prices $\mathbf{0}$ is $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$. From Theorems 2 and 4, there exists IFE and unique robust SPNE.

b. From the definition of duplicate OPF, $F(\mathcal{N}, \mathbf{0}) = F(\{1, \ldots, k\}, \mathbf{0})$. Hence, for any intermediary i, $F(\mathcal{N} \setminus \{i\}, \mathbf{0}) = F(\{1, \ldots, k\}, \mathbf{0}) = F(\mathcal{N}, \mathbf{0})$. Similar with part (a), from Theorem 2 and 4, there exists IFE and unique robust SPNE.

c. There exists a group of intermediaries S s.t. $F(S, \mathbf{0}) = F(\mathcal{N} \setminus S, \mathbf{0}) = F(\mathcal{N}, \mathbf{0})$. Let $S_1 = S$ and $S_2 = \mathcal{N} \setminus S$. There is $\max_{x \in F(S_1, \mathbf{0})} u(x, 0) = \max_{x \in F(\mathcal{N}, \mathbf{0})} u(x, 0) = \max_{x \in F(S_2, \mathbf{0})} u(x, 0)$. So S_1 and S_2 are utility maximizing group at prices $\mathbf{0}$. $S_1 \cap S_2 = \emptyset$, from Theorem 2 and 4, there exists IFE and unique robust SPNE. ■

Proof of Corollary 2

Proof. We prove that the problem (u, F) is monotonic and cross-monotonic.

Recall that the preferences of the planner are independent of price $u(x, p) = u(x)$, strongly monotonic and homothetic in x. Consider a price p and group S such that $\sum_{n \in S} p_n \leq I$. Then, the OPF equals $F(S, p) = \{\sum_{n \in S} Q_n y_n | \sum_{n \in S} y_n \leq I - \sum_{n \in S} p_n \text{ and } y_n \geq 0\}$. Consider prices p', s.t. $p_n' < p_n$, $p_{-n}' = p_{-n}$ for $n \in S$. For any $x \in F(S, p)$, assume $x = \sum_{n \in S} Q_n y_n$, $\sum_{n \in S} y_n \leq I - \sum_{n \in S} p_n < I - \sum_{n \in S} p_n'$. Thus, for any y', s.t. $\sum_{n \in S} y_n' \leq I - \sum_{n \in S} p_n'$,

$x' = \sum_{n \in S} Q_n y'_n \in F(S, p')$, so there exists $x' \in F(S, p')$ such that $x' > x$. By monotonicity of u, $u(x') > u(x)$. Thus the problem (u, F) is monotonic in prices p.

In order to show that the problem (u, F) is cross-monotonic, note that

$$F(S,p) = \{\sum_{n \in S} Q_n y_n | \sum_{n \in S} y_n \leq I - \sum_{n \in S} p_n \text{ and } y_n \geq 0\}$$

$$= (I - \sum_{n \in S} p_n)\{\sum_{n \in S} Q_n y_n | \sum_{n \in S} y_n \leq 1 \text{ and } y_n \geq 0\}.$$

Hence, remark 3 (b) is satisfied.

a. First, if $\forall n$, $Q_n \in conv(\mathbf{0}, Q_{-n})$, then the OPF $F(\mathcal{N}, \mathbf{0}) = F(\mathcal{N} \setminus \{n\}, \mathbf{0})$. Thus, for each n there exists a utility maximizing group S_n at prices $\mathbf{0}$, s.t. $n \notin S_n$. Therefore, $\bigcap_{n \in \mathcal{N}} S_n = \emptyset$. So, from Theorem 2 and 4, there exists IFE and unique robust SPNE.

Second, if there exists IFE for any monotonic and homothetic preferences. Suppose there is intermediary n, s.t. $Q_n \notin conv(\mathbf{0}, Q_{-n})$. Consider the utility function $u(x) = \min\{\frac{x_1}{q_{n1}}, \ldots, \frac{x_M}{q_{nM}}\}$, then the indirect utility function satisfies $v(\mathbf{0}) > v_{-n}(\mathbf{0}_{-n})$. Hence, $p = \mathbf{0}$ is not equilibrium price allocation. Given prices $\mathbf{0}_{-n}$, intermediary n has incentive to deviate and post positive price $p'_n > 0$ with $p' = (p'_n, \mathbf{0}_{-n})$ and $v(p') > v_{-n}(\mathbf{0}_{-n})$. Thus, intermediary n would a higher payoff, which is a contradiction. Hence, $Q_n \in conv(\mathbf{0}, Q_{-n})$, $\forall n$.

b. When the planner has strictly convex preferences, there is a unique point $x \in F(\mathcal{N}, \mathbf{0})$ that maximizes the utility at prices $\mathbf{0}$. Let $x = x^\star(Q, u)$. Note that $x \in \bigcap_{n \in \mathcal{N}} conv(Q_{-n})$ is equivalent for the group $S_n = \mathcal{N} \setminus \{n\}$ to be a utility maximizing group at prices $\mathbf{0}$. Hence, $\bigcap_{n \in \mathcal{N}} S_n = \emptyset$. Thus, from Theorem 2 and 4, there exists IFE and unique robust SPNE.

For the converse, if an IFE exists, then by part a, $Q_n \in conv(0, Q_{-n})$ for every intermediary n. Thus, $F(N, \mathbf{0}) = conv(0, Q) = conv(0, Q_{-n})$. Therefore, $x^\star(Q, u) \in conv(0, Q_{-n})$ for all n. Hence, $x^\star(Q, u) \in \bigcap_{n \in \mathcal{N}} conv(0, Q_{-n})$. By monotonicity of u, $x^\star(Q, u)$ is in the boundary of $conv(0, Q_{-n})$, then $x^\star(Q, u) \in \bigcap_{n \in \mathcal{N}} conv(Q_{-n})$ ■

Proof of Corollary 3

Proof. a. For any agent m and intermediary n we have that $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N})$, which implies that $F(\mathcal{N} \setminus \{n\}, \mathbf{0}) = F(\mathcal{N}, \mathbf{0})$. Thus, group of intermediaries $S_n = \mathcal{N} \setminus \{n\}$ is utility maximizing group at prices $\mathbf{0}$, there exists $S_j(\mathbf{0}) = S_n$, so $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$.

We now show that the problem (u, F) is monotonic. Indeed, since preferences are strongly monotonic and homothetic, for price p and p', with $p'_n < p_n$, $p'_{-n} = p_{-n}$, $n \in S$, $p_S \leq I$, then $p'_S < I$.

Thus, there exists $y \in F(S, p')$, s.t. $y \geq x$ and $y \neq x$. Since the preferences are strongly monotonic, $u(y) > u(x)$.

In order to show that (u, F) is cross-monotonic, note that the preferences are homothetic and $F(S, p) = \gamma(S)\delta(p_S)$ for a set $\gamma(S) \subset \mathbb{R}^M$. Thus, it satisfies cross-monotonicity from Remark 3 (b). From Theorem 2 and 4, there exists IFE and unique robust SPNE.

Conversely, for any monotonic utility function, there exists an IFE (or unique robust SPNE). Suppose $\bar{c}^m(\mathcal{N} \setminus \{n\}) < \bar{c}^m(\mathcal{N})$, then if utility function $u(x) = x_m$, the planner only cares about the resource allocated to agent m, and deleting intermediary n would decrease the maximal unit capacity allocated to agent m. Thus intermediary n could post price $p_n > 0$ and get positive benefit. So there is no IFE.

b. Similar to part (a), the problem (u, F) is cross-monotonic due to the homotheticity of preferences. We now show that the problem (u, F) is monotonic when preferences are homothetic and the utility function is non-zero corners. Indeed, we simply prove that the outcome possibility function F is strongly monotonic in prices (which implies the monotonicity of (u, F) by Lemma 3). For price p and p', with $p'_n < p_n$, $p'_{-n} = p_{-n}$, $n \in S$, for $x \in F(S, p)$, $u(x) > u((0, \ldots, 0))$, thus $x_m > 0$, $\forall m$, there are links to all agents with group S. $I - p_S < I - p'_S$, there exists $y \in F(S, p')$ and $y > x$.

Since preferences are homothetic and the utility function has non-zero corners, $\bar{c}^m(\mathcal{N} \setminus \{n\}) = \bar{c}^m(\mathcal{N})$ for any $m \in \mathcal{M}$ and $n \in \mathcal{N}$, then we have that an IFE or a unique robust SPNE exists.

Conversely, suppose that x is an IFE (or a unique robust SPNE exists). By Theorem 4, $\bigcap_{j=1}^{J} S_j(\mathbf{0}) = \emptyset$, since the problem (u, F) is monotonic and cross-monotonic. Suppose that $x = (x_1, \ldots, x_M) \in F(S, \mathbf{0})$. $x_m > 0$, for any m, so $x = \lambda^m x^{m,S} I$.

Assume $\exists m, n$ such that $\bar{c}^m(\mathcal{N} \setminus \{n\}) < \bar{c}^m(\mathcal{N})$. To prove $v(\mathbf{0}) > v_{-n}(\mathbf{0}_{-n})$, there is $x \in F(\mathcal{N} \setminus \{n\}, \mathbf{0})$ s.t. $u(x) = v_{-n}(\mathbf{0}_{-n})$. Here to prove $\exists x' \in F(\mathcal{N}, \mathbf{0})$, s.t. $x' > x$ which means $x'_i > x_i$, $\forall i$. $x^i = (0, \ldots, 0, \bar{c}^i(\mathcal{N}), 0, \ldots, 0) \in \mathbb{R}^M$, assume $x^i_{-n} = (0, \ldots, 0, \bar{c}^i(\mathcal{N} \setminus \{n\}), 0, \ldots, 0) \in \mathbb{R}^M$. Since the preferences are non-zero corners, $x_i > 0$, $\forall i$. Thus there is $\lambda^i > 0$ with $\sum_i \lambda^i = 1$, s.t. $x = \sum_{i=1}^{M} \lambda^i x^i_{-n} I$. $\exists \epsilon > 0$ small enough, with $\lambda'^m = \lambda^m - \epsilon \geq 0$, s.t. $\lambda'^m \bar{c}^m(\mathcal{N}) > \lambda^m \bar{c}^m(\mathcal{N} \setminus \{n\})$. Let $\lambda'^i = \lambda^i + \frac{\epsilon}{M-1}$, $\forall i \neq m$, $\sum_{i=1}^{M} \lambda'^i = 1$, there is $\lambda'^i \bar{c}^i(\mathcal{N}) > \lambda'^i \bar{c}^i(\mathcal{N} \setminus \{n\})$. Then $x' = \sum_{i=1}^{M} \lambda'^i x^i_{-n} I > x$ and $x' \in F(\mathcal{N}, \mathbf{0})$. The preferences are monotonic and $x' > x$, then $v(\mathbf{0}) \geq u(x', 0) > u(x, 0) = v_{-n}(\mathbf{0}_{-n})$. At prices $p_{-n} = \mathbf{0}_{-n}$, from Lemma 2, intermediary n has incentive to deviate and charge positive price $p_n > 0$. So there is no IFE. ■

Proof of Corollary 4

Proof.

a. First, we prove $\bar{c}^m(\mathcal{N}\setminus\{n\},I) = \bar{c}^m(\mathcal{N},I)$ for every intermediary n and agent m if and only if $\bar{c}^m(\mathcal{N}\setminus\{n\},I) = I$ for every intermediary n and agent m. To see that, if $\bar{c}^m(\mathcal{N}\setminus\{n\},I) = I$, then $\bar{c}^m(\mathcal{N},I) \geq \bar{c}^m(\mathcal{N}\setminus\{n\},I) = I$ and $\bar{c}^m(\mathcal{N},I) \leq I$, so $\bar{c}^m(\mathcal{N}\setminus\{n\},I) = \bar{c}^m(\mathcal{N},I)$. To prove the converse, consider the problem of maximal flow from the planner to agent m, then $\bar{c}^m(\mathcal{N},I)$ is the maximal flow. Thus, there exists intermediary n who owns a link in the minimal cut such that after deleting his link, the maximal flow decreases. Hence, $\bar{c}^m(\mathcal{N},I) > \bar{c}^m(\mathcal{N}\setminus\{n\},I)$, which is a contradiction.

Note that $\bar{c}^m(\mathcal{N}\setminus\{n\},I) = \bar{c}^m(\mathcal{N},I)\ \forall m,n$ if and only if coalition $N\setminus n$ is a utility maximizing coalition at prices $\mathbf{0}$ for all n. Hence, by Theorem 2, an IFE exists.

Conversely, for any monotonic utility function, there exists an IFE (or unique robust SPNE). Suppose $\bar{c}^m(\mathcal{N}\setminus\{n\},I) < \bar{c}^m(\mathcal{N},I)$ and planner's utility function is $u(x) = x_m$, then if $p_{-n} = \mathbf{0}_{-n}$, since $u^*(\mathcal{N}\setminus\{n\},\mathbf{0}) = \bar{c}^m(\mathcal{N}\setminus\{n\},I) < \bar{c}^m(\mathcal{N},I) = u^*(\mathcal{N},\mathbf{0})$, intermediary n has incentive to post positive price $p_n > 0$, s.t. $p' = (p_n, \mathbf{0}_{-n})$ and $u^*(\mathcal{N}\setminus\{n\},\mathbf{0}) < u^*(\mathcal{N},p')$, intermediary n gets p_n. Thus, $\mathbf{0}$ is not an equilibrium.

b. Given the non-zero corner utility function, the planner needs to use path to each agent. If there exists intermediary n that owns link on every path to agent m, then $c^m(\mathcal{N}\setminus\{n\},I) = 0 < c^m(\mathcal{N},I) = I$, so posting a zero price for intermediary n is not optimal for him, hence there is no IFE. Conversely, if there is no intermediary n that owns link on every path connected to agent m, then $c^m(\mathcal{N}\setminus\{n\},I) = c^m(\mathcal{N},I) = I$ (due to the infinite capacities). From Theorem 2, there exists an IFE. ■

Proof of Corollary 5

Proof.

a. Since F is independent of the price, any group $S_j \subseteq \mathcal{N}$ such that $u_{S_j} = \bar{u}$ is a utility maximizing group $S_j(\mathbf{0})$ at prices $\mathbf{0}$. Then $\bigcap_j S_j = \emptyset$ is equivalent with $\bigcap_j S_j(\mathbf{0}) = \emptyset$. Furthermore, the utility function $u(x,t) = u(x) - t$ is strictly decreasing in total price paid t. Hence, from Lemma 3, the problem (u,F) is monotonic. $u^*(S,\mathbf{0}) = u_S \leq u^*(T,\mathbf{0}) = u_T$ if and only if $u^*(S,p) = u_S - p_S \leq u^*(T,p) = u_T - p_T$ with $p_S = p_T$; from Definition 10, the problem is cross-monotonic. Therefore, $\bigcap_{S_j\in\mathcal{S}} S_j = \emptyset$ if and only if there exists IFE and unique robust SPNE.

b. The monotonic and cross-monotonicity of the problem (u,F) is trivial. Note that for any group of intermediaries $B_k \in \mathcal{B}$, $u^*(B_k,\mathbf{0}) = 1 = u^*(\mathcal{N},\mathbf{0})$. So B_k is a utility maximizing group at prices $\mathbf{0}$. From Theorem 2 and 4, there exists IFE and unique robust SPNE if and only if $\bigcap_{B_k \in \mathcal{B}} B_k = \emptyset$.

We now show that in the MCST, $\bigcap_{B_k \in \mathcal{B}} B_k = \emptyset$ is equivalent to have every node linked via at least two intermediaries. If every node is linked to at least two intermediaries, then $\mathcal{N} \setminus \{n\}$ is an acceptable set, $\mathcal{N} \setminus \{n\} \in \mathcal{B}$. Hence $\bigcap_{B_k \in \mathcal{B}} B_k = \emptyset$. On the other hand, if m is uniquely linked to intermediary n, then $\mathcal{N} \setminus \{n\}$ is not acceptable, thus $u_{\mathcal{N}} = 1 > 0 = u_{\mathcal{N} \setminus \{n\}}$, intermediary n could charge positive price at equilibrium. Thus, there is no IFE. ■

Appendix B PROOFS OF CHAPTER 3

B.1 Proof of Main Results

In order to prove Theorems and Corollaries, first introduce the follow Lemmas.

Lemma 5

For function $f(x_1, \ldots, x_M)$ *satisfying* $\frac{\partial f(x)}{\partial x_i} = \frac{\partial f(x)}{\partial x_j}, \forall i, j$. *Then there exists function* $g : \mathbb{R} \mapsto \mathbb{R}$, *s.t.* $f(x) = g(\sum_{i=1}^{M} x_i)$.

Proof.

Assume $u_j = \sum_{i=1}^{j} x_i, \forall j$. $x_1 = u_1$, $x_j = u_j - u_{j-1}$ for $j \geq 2$. Define function $\hat{f}(u_1, \ldots, u_M)$ s.t. $\hat{f}(u_1, \ldots, u_M) = f(x_1, \ldots, x_M)$.

Then $\frac{\partial f(x)}{\partial x_j} = \sum_{i=1}^{M} \frac{\partial \hat{f}(u)}{\partial u_i} \frac{\partial u_i}{\partial x_j} = \sum_{i=j}^{M} \frac{\partial \hat{f}(u)}{\partial u_i}$.

$\frac{\partial f(x)}{\partial x_i} = \frac{\partial f(x)}{\partial x_j}, \forall i, j$.

$\sum_{i=j}^{M} \frac{\partial \hat{f}(u)}{\partial u_i} = \sum_{i=j+1}^{M} \frac{\partial \hat{f}(u)}{\partial u_i}$, then $\frac{\partial \hat{f}(u)}{\partial u_j} = 0, \forall j = 1, \ldots, M-1$.

So $\hat{f}(u)$ only depends on $u_M = \sum_{i=1}^{M} x_i$.

Thus, there exists function $g : \mathbb{R} \mapsto \mathbb{R}$, s.t. $f(x) = \hat{f}(u) = g(\sum_{i=1}^{M} x_i)$. ■

Lemma 6

For function $f(x_1, \ldots, x_N)$ *satisfying* $\int_0^{x_1} f(t, 0)dt + \int_0^{x_2} f(t, x_1, 0)dt + \cdots + \int_0^{x_N} f(t, x_{-N})dt = \int_0^{\sum_{i=1}^{N} x_i} f(t, 0)dt, \forall x$. *Then there exists function* $g : \mathbb{R} \mapsto \mathbb{R}$, *s.t.* $f(x) = g(\sum_{i=1}^{N} x_i)$.

Proof.

For any $x = (x_1, \ldots, x_N)$, $\int_0^{\sum_{i=1}^{N} x_i} f(t, 0)dt = \int_0^{x_1} f(t, 0)dt + \int_0^{x_2} f(t, x_1, 0)dt + \cdots + \int_0^{x_N} f(t, x_{-N})dt$.

Consider $\Delta > 0$, $\int_0^{\sum_{i=1}^{N} x_i + \Delta} f(t, 0)dt = \int_0^{x_1} f(t, 0)dt + \int_0^{x_2} f(t, x_1, 0)dt + \cdots + \int_0^{x_N + \Delta} f(t, x_{-N})dt$.

Then $\int_{\sum_{i=1}^{N} x_i}^{\sum_{i=1}^{N} x_i + \Delta} f(t, 0)dt = \int_{x_N}^{x_N + \Delta} f(t, x_{-N})dt$. Let $\Delta \to 0$, $f(\sum_{i=1}^{N} x_i, 0) = f(x)$.

Thus, $f(x)$ could be written as a function of $\sum_{i=1}^{N} x_i$. There exists function $g : \mathbb{R} \mapsto \mathbb{R}$ and $f(x) = g(\sum_{i=1}^{N} x_i)$. ■

Lemma 7

For the mechanism of planner $\phi = (x(\beta), s(\beta))$*, functions* $x_i(\beta), s_i(\beta)$ *satisfying the First Order Condition, and* $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$*,* $s_i(\beta) = \hat{s}_i(\beta_i(\mathcal{M}), \beta_{-i})$*. If the mechanism is strategy-proof for each dimension of quality of intermediation* β_i^m*, then it is strategy-proof for any* $\beta_i = (\beta_i^1, \ldots, \beta_i^M)$*.*

Proof.

Since $x_i(\beta)$, $s_i(\beta)$ depend only on $\beta_i(\mathcal{M})$ of the report β_i of intermediary i, deviation of intermediary i to β_i is equivalent as deviating to $\bar{\beta}_i$, with $\bar{\beta}_i(\mathcal{M}) = \sum_{m=1}^{M} \bar{\beta}_i^m$. $\bar{\beta}_i^m = \alpha_i^m$ for $m \neq j$. Since the mechanism ϕ satisfies F.O.C., it is strategy-proof for deviation in each dimension of sharing-rates β_i^m. Thus, it is strategy-proof for deviation to any β_i. ■

Lemma 8

When $x_i(\beta)$ *is symmetric, then there exists* $f : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+$*, s.t.* $x_i(\beta) = \frac{f(\beta_i, \beta_{-i})}{\sum_{i=1}^{N} f(\beta_i, \beta_{-i})}$*. If* $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$*, and the mechanism is symmetric, then there exists function* $f : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$*, s.t.* $x_i(\beta) = \frac{f(\beta_i(\mathcal{M}), \beta_{-i})}{\sum_{j=1}^{N} f(\beta_j(\mathcal{M}), \beta_{-j})}$*.*

Proof. $x_i(\beta) = x_i(\beta_i, \beta_{-i})$, there exists f_i, s.t. $x_i(\beta_i, \beta_{-i}) \times k = f_i(\beta_i, \beta_{-i})$, k is a constant.

For β and $\hat{\beta}$ with $\beta_i = \hat{\beta}_j$, $\beta_j = \hat{\beta}_i$ and $\beta_m = \hat{\beta}_m, \forall m \neq i, j$.

Since the mechanism is symmetric, $x_i(\beta) = x_j(\hat{\beta})$, by definition of f_i, $f_i(\beta_i, \beta_{-i}) = f_j(\hat{\beta}_j, \hat{\beta_{-j}})$ and $\sum_{n=1}^{N} f_n(\beta) = \sum_{n=1}^{N} f_n(\hat{\beta})$.

Thus, $f_i(\beta_i, \beta_{-i}) = f_j(\hat{\beta}_j, \hat{\beta_{-j}}) = f(\beta_i, \beta_{-i})$.

$x_i(\beta_i, \beta_{-i}) = \frac{f_i(\beta_i, \beta_{-i})}{k}$, budget balance requires $\sum_{i=1}^{N} x_i(\beta_i, \beta_{-i}) = 1$. $\sum_{i=1}^{N} \frac{f_i(\beta_i, \beta_{-i})}{k} = 1$, and $k = \sum_{i=1}^{N} f_i(\beta_i, \beta_{-i})$. So $x_i(\beta_i, \beta_{-i}) = \frac{f_i(\beta_i, \beta_{-i})}{\sum_{i=1}^{N} f_i(\beta_i, \beta_{-i})}$.

The proof for the case of $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$ is the same.

$x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$, there exists f_i, s.t. $\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) \times k = f_i(\beta_i(\mathcal{M}), \beta_{-i})$, k is a constant.

If there exists β, $\hat{\beta}$ with $\beta_i = \hat{\beta}_j$, $\beta_j = \hat{\beta}_i$ and $\beta_m = \hat{\beta}_m, \forall m \neq i, j$.

s.t. $f_i(\beta_i(\mathcal{M}), \beta_{-i}) \neq f_j(\hat{\beta}_i(\mathcal{M}), \hat{\beta}_{-j})$.

Since $\sum_{n=1}^{N} f_n(\beta_n(\mathcal{M}), \beta_{-n}) = \sum_{n=1}^{N} f_n(\hat{\beta}_n(\mathcal{M}), \hat{\beta}_{-n})$, then $x_i(\beta) = \frac{f_i(\beta_i(\mathcal{M}), \beta_{-i})}{\sum_{n=1}^{N} f_n(\hat{\beta}_n(\mathcal{M}), \beta_{-n})} \neq \frac{f_j(\hat{\beta}_j(\mathcal{M}), \hat{\beta}_{-j})}{\sum_{n=1}^{N} f_n(\hat{\beta}_n(\mathcal{M}), \hat{\beta}_{-n})} = x_j(\hat{\beta})$, this is contradiction with symmetric mechanism.

Thus, $f_i(\beta_i(\mathcal{M}), \beta_{-i}) = f_j(\hat{\beta}_j(\mathcal{M}), \hat{\beta}_{-j}) = f(\beta_i(\mathcal{M}), \beta_{-i})$. ■

Proof of Proposition 1

Proof.

Part i. Given the mechanism $\phi = (x(\cdot), s(\cdot))$, assume $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$, which represents the profit of intermediary i.

Part ii. ch-SP mechanism satisfies $(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$.

For β, with $\beta_i^j > \alpha_i^j$, and $\beta_i^{-j} = \alpha_i^{-j}$. Then ch-SP requires $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j} \leq x_i(\beta_i, \beta_{-i}) + \frac{c\cdot(h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i))\cdot x_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j}$.

For β, with $\beta_i^j < \alpha_i^j$, and $\beta_i^{-j} = \alpha_i^{-j}$. Then ch-SP requires $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j} \geq x_i(\beta_i, \beta_{-i}) + \frac{c\cdot(h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i))\cdot x_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j}$.

From these two inequalities: $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j} = x_i(\beta_i, \beta_{-i}) + \frac{c\cdot(h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i))\cdot x_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j}$.

Let $\beta_i \to \alpha_i$, with $\beta_i^{-j} = \alpha_i^{-j}$. The equation is equivalent with $\frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^m} = x_i(\beta_i, \beta_{-i}) + c \cdot h_{2m}(\alpha_i) \cdot x_i(\beta_i, \beta_{-i}) = (1 + ch_{2m}(\alpha_i))x_i(\alpha_i, \beta_{-i}), \forall m$.

Part iii. If $h_{2m}(\alpha_i) = h_{2k}(\alpha_i)$, $\frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^m} = \frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^k}, \forall m, k$.

From Lemma 5, there exists $\hat{\Phi}_i(\beta_i(\mathcal{M}), \beta_{-i}) = \Phi_i(\beta)$.

$\frac{\partial \hat{\Phi}_i(\beta_i(\mathcal{M}),\beta_{-i})}{\partial \beta_i^m} = x_i(\beta_i, \beta_{-i}) + c \cdot h_{2m}(\alpha_i) \cdot x_i(\beta_i, \beta_{-i}) = (1 + ch_{2m}(\alpha_i))x_i(\beta)$.

If $h_{2m}(\alpha_i) = d$, $x_i(\beta) = \frac{1}{1+cd} \cdot \frac{\partial \hat{\Phi}_i(\beta_i(\mathcal{M}),\beta_{-i})}{\partial \beta_i^m}$, there exists $\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) = x_i(\beta)$.

$\frac{\partial \Phi_i(\beta_i,\beta_{-i})}{\partial \beta_i^m} = (1 + cd)\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$, from Lemma 6, $\Phi_i(\beta) = \int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt$.

From definition of $\Phi_i(\beta)$, $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$.

So $\sum_{m=1}^{M} s_i^m(\beta) = \frac{\beta_i(\mathcal{M})x_i(\beta) - \Phi_i(\beta)}{x_i(\beta)} = \frac{\beta_i(\mathcal{M})\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i}) - \hat{\Phi}_i(\beta_i(\mathcal{M}),\beta_{-i})}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$ depend only on $\beta_i(\mathcal{M})$ instead of β_i.

There exist functions $\hat{s}_i^m : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$, $\sum_{m=1}^{M} s_i^m(\beta) = \sum_{m=1}^{M} \hat{s}_i^m(\beta_i(\mathcal{M}), \beta_{-i})$.

Thus, $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{(1+cd)\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

From Lemma 7, the mechanisms $\phi = (x(\cdot), s(\cdot))$ with functions $x_i(\beta)$ and $s_i(\beta)$ satisfying the conditions above are strategy-proof for intermediary i. ■

Proof of Theorem 6

Proof.

When $c = 0$, consider report β with $\beta_i^j > \alpha_i^j$ and $\beta_i^{-j} = \alpha_i^{-j}$. $\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j-\beta_i^j} \le x_i(\beta_i, \beta_{-i})$. Take the limit, $\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} \le x_i(\beta)$.

Consider report β with $\beta_i^j < \alpha_i^j$, $\beta_i^{-j} = \alpha_i^{-j}$, $\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j-\beta_i^j} \ge x_i(\beta_i, \beta_{-i})$. Take the limit, $\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} \ge x_i(\beta)$.

From the inequalities above, $\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} = x_i(\alpha_i, \beta_{-i})$, $\forall m$ with $\beta_i \to \alpha_i$.

From Lemma 5, there exists $\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) = x_i(\beta)$.

$\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$, from Lemma 6, $\Phi_i(\beta) = \int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt$.

Since $\Phi_i(\beta) = \sum_{m=1}^M (\beta_i^m - s_i^m(\beta))x_i(\beta)$, then $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

From Lemma 7, the mechanisms $\phi = (x(\cdot), s(\cdot))$ with functions $x_i(\beta)$ and $s_i(\beta)$ satisfying the conditions above are strategy-proof for intermediary i. A mechanism is 0-SP is equivalent with the condition $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

From Proposition 1, ch-SP mechanism satisfies $\frac{\partial\Phi_i(\beta_i,\beta_{-i})}{\partial\beta_i^m} = (1 + ch_{2m}(\alpha_i))x_i(\beta)$.

When $h(\alpha_i, \beta_i) = \sum_{m=1}^M \gamma_m(\alpha_i^m - \beta_i^m)^{k_m}$, there is $h_{2m}(\alpha_i) = 0$. For any $h_{2m}(\alpha_i) = 0$, the condition is equivalent with $\frac{\partial\Phi_i(\beta_i,\beta_{-i})}{\partial\beta_i^m} = x_i(\alpha_i, \beta_{-i})$. ■

Results for General Punishment Function h

- For $\beta_i(\mathcal{M}) > \alpha_i(\mathcal{M})$:

  $\sum_{m=1}^M \frac{(\alpha_i^m-\beta_i^m)}{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})} \frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} \le (1 + \lim_{t\to 0} \frac{c\cdot(h(\alpha_i,\alpha_i)-h(\alpha_i,\beta_i(t)))}{t(\alpha_i(\mathcal{M})-\beta_i(\mathcal{M}))})x_i(\alpha_i, \beta_{-i})$.

- For $\beta_i(\mathcal{M}) < \alpha_i(\mathcal{M})$:

  $\sum_{m=1}^M \frac{(\alpha_i^m-\beta_i^m)}{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})} \frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m} \ge (1 + \lim_{t\to 0} \frac{c\cdot(h(\alpha_i,\alpha_i)-h(\alpha_i,\beta_i(t)))}{t(\alpha_i(\mathcal{M})-\beta_i(\mathcal{M}))})x_i(\alpha_i, \beta_{-i})$. [1]

Proof.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \ge (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall\beta_{-i}$

$\Phi_i(\alpha_i, \beta_{-i}) = (\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i})$, $\Phi_i(\beta_i, \beta_{-i}) = (\beta_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i})$.

$(\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) = (\alpha_i - \beta_i)^T \mathbf{1} x_i(\beta_i, \beta_{-i}) + \Phi_i(\beta_i, \beta_{-i})$.

Thus, ch-SP is equivalent with $\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i}) \ge (\alpha_i - \beta_i)^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i})$.

[1] $\beta_i(t) = t\beta_i + (1 - t)\alpha_i$.

$\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i}) \geq (\alpha_i - \beta_i)^T \mathbf{1} x_i(\beta_i, \beta_{-i}) + c \cdot (h(\alpha_i, \alpha_i) - h(\alpha_i, \beta_i)) \cdot x_i(\beta_i, \beta_{-i})$.

If $\beta_i(\mathcal{M}) > \alpha_i(\mathcal{M})$, $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} \leq x_i(\beta_i, \beta_{-i}) + \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i)) \cdot x_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})}$.

If $\beta_i(\mathcal{M}) < \alpha_i(\mathcal{M})$, $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} \geq x_i(\beta_i, \beta_{-i}) + \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i)) \cdot x_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})}$.

Let $\beta_i(t) = t\beta_i + (1-t)\alpha_i$, with $0 \leq t \leq 1$. As $t \to 0$, $\beta_i(t) \to \alpha_i$.

If $\beta_i(\mathcal{M}) > \alpha_i(\mathcal{M})$, $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i(t),\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})(t)} \leq x_i(\beta_i(t), \beta_{-i}) + \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i(t))) \cdot x_i(\beta_i(t),\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})(t)}$.

Equivalent with $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i(t),\beta_{-i})}{t(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))} \leq x_i(\beta_i(t), \beta_{-i}) + \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i(t))) \cdot x_i(\beta_i(t),\beta_{-i})}{t(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))}$.

Take the limit $t \to 0$, $\sum_{m=1}^{M} \frac{(\alpha_i^m - \beta_i^m)}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} \frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^m} \leq (1 + \lim_{t \to 0} \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i(t)))}{t(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))}) x_i(\alpha_i, \beta_{-i})$.

If $\beta_i(\mathcal{M}) < \alpha_i(\mathcal{M})$, $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i(t),\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})(t)} \geq x_i(\beta_i(t), \beta_{-i}) + \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i(t))) \cdot x_i(\beta_i(t),\beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})(t)}$.

Take the limit $t \to 0$, $\sum_{m=1}^{M} \frac{(\alpha_i^m - \beta_i^m)}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} \frac{\partial \Phi_i(\alpha_i,\beta_{-i})}{\partial \beta_i^m} \geq (1 + \lim_{t \to 0} \frac{c \cdot (h(\alpha_i,\alpha_i) - h(\alpha_i,\beta_i(t)))}{t(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))}) x_i(\alpha_i, \beta_{-i})$.

■

Proof of Corollary 7

Proof.

$ch = \infty$, when $\beta_i \leq \alpha_i$, same with Proposition 1. $x_i(\beta_i, \beta_{-i})(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})) \leq \Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})$.

When $\beta_i^m > \alpha_i^m$, ch-SP requires $\Phi_i(\beta_i, \beta_{-i}) + x_i(\beta_i, \beta_{-i})(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})) + c(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))x_i(\beta_i, \beta_{-i}) \leq \Phi_i(\alpha_i, \beta_{-i})$. If $x_i(\beta_i, \beta_{-i}) = 0$, $\Phi_i(\beta_i, \beta_{-i}) = 0$, the inequality of ch-SP satisfies. If $x_i(\beta_i, \beta_{-i}) > 0$, $c(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))x_i(\beta_i, \beta_{-i}) \to -\infty$, the inequality of ch-SP satisfies.

Thus, ∞-SP is equivalent with $\Phi_i(\beta_i, \beta_{-i}) - \Phi_i(\hat{\beta}_i, \beta_{-i}) \geq x_i(\hat{\beta}_i, \beta_{-i})(\beta_i(\mathcal{M}) - \hat{\beta}_i(\mathcal{M})), \forall \hat{\beta}_i \leq \beta_i$.

From Lemma 7, the functions $x_i(\beta)$ and $s_i(\beta)$ satisfying this inequality are strategy-proof for intermediary i.

For any quality of intermediation α_i and β_i, $\alpha_i \neq \beta_i$.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1}, \forall \beta_{-i}$.

When $ch = \infty$, for $\alpha \neq \beta$, $h(\alpha, \beta) > 0$, then $c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}) \to \infty$ if $x_i(\beta_i, \beta_{-i}) > 0$, and the ch-SP holds. The inequality holds when $x_i(\beta_i, \beta_{-i}) = 0$.

$c = 0$, consider report β with $\beta_i^j > \alpha_i^j$ and $\beta_i^{-j} = \alpha_i^{-j}$. From Proposition 1, $\frac{\Phi_i(\alpha_i,\beta_{-i}) - \Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j - \beta_i^j} \leq x_i(\beta_i, \beta_{-i})$. Take the limit, $\frac{\partial \Phi_i(\hat{\beta}_i,\beta_{-i})}{\partial \beta_i^m} \leq x_i(\beta)$.

Consider report β with $\beta_i^j < \alpha_i^j$, $\beta_i^{-j} = \alpha_i^{-j}$, $\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i^j-\beta_i^j} \leq x_i(\beta_i, \beta_{-i})$. Take the limit, $\frac{\partial \Phi_i(\hat{\beta}_i,\beta_{-i})}{\partial \beta_i^m} \geq x_i(\beta)$.

From above, $x_i(\beta) = \frac{\partial \Phi_i(\hat{\beta}_i,\beta_{-i})}{\partial \beta_i^m}, \forall m$.

From Lemma 5, there exists $\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) = x_i(\beta)$.

$\frac{\partial \Phi_i(\hat{\beta}_i,\beta_{-i})}{\partial \beta_i^m} = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$, from Lemma 6, $\Phi_i(\beta) = \int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt$.

From definition of $\Phi_i(\beta)$, $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$.

Thus, $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

From Lemma 7, the functions $x_i(\beta)$ and $s_i(\beta)$ satisfying F.O.C. are strategy-proof for intermediary i. ■

Proof of Proposition 2

Proof.

For probability of punishment c and c', punishment function h and h'. $ch(a, b) \leq c'h'(a, b)$, for any a and b.

Any mechanism ϕ is ch-SP, satisfies: $(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$.

Since $c \cdot h(\alpha_i, \beta_i) \leq c' \cdot h'(\alpha_i, \beta_i)$, $(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c' \cdot h'(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$.

So the mechanism $\phi = (x(\cdot), s(\cdot))$ is $c'h'$-SP. ■

Proof of Proposition 3

Proof.

ϕ_1 is ch-SP for punishment function h_1, so $(\alpha_i(\mathcal{M}) - \sum_{m=1}^{M} s_{1i}^m(\alpha_i, \beta_{-i}))x_{1i}(\alpha_i, \beta_{-i}) \geq (\alpha_i(\mathcal{M}) - \sum_{m=1}^{M} s_{1i}^m(\beta))x_{1i}(\beta) - ch_1(\alpha_i, \beta_i)x_i(\beta)$.

ϕ_2 is ch-SP for punishment function h_2, so $(\alpha_i(\mathcal{M}) - \sum_{m=1}^{M} s_{2i}^m(\alpha_i, \beta_{-i}))x_{2i}(\alpha_i, \beta_{-i}) \geq (\alpha_i(\mathcal{M}) - \sum_{m=1}^{M} s_{2i}^m(\beta))x_{2i}(\beta) - ch_2(\alpha_i, \beta_i)x_i(\beta)$.

Then the linear combination of the two inequalities with weighted of λ and $(1-\lambda)$ results $\lambda[(\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_{1i}^m(\alpha_i,\beta_{-i}))x_{1i}(\alpha_i,\beta_{-i})]+(1-\lambda)[(\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_{2i}^m(\alpha_i,\beta_{-i}))x_{2i}(\alpha_i,\beta_{-i})] \geq \lambda[(\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_{1i}^m(\beta))x_{1i}(\beta)-ch_1(\alpha_i,\beta_i)x_i(\beta)]+(1-\lambda)[(\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_{2i}^m(\beta))x_{2i}(\beta)-ch_2(\alpha_i,\beta_i)x_i(\beta)]$.

$x_1(\beta)=x_2(\beta)$ and $x_1(\alpha_i,\beta_{-i})=x_2(\alpha_i,\beta_{-i})$, so rearrange the inequality above, there is $(\alpha_i(\mathcal{M})-\lambda\sum_{m=1}^{M} s_{1i}^m(\alpha_i,\beta_{-i})-(1-\lambda)\sum_{m=1}^{M} s_{2i}^m(\alpha_i,\beta_{-i}))x_i(\alpha_i,\beta_{-i}) \geq (\alpha_i(\mathcal{M})-\lambda\sum_{m=1}^{M} s_{1i}^m(\beta)-(1-\lambda)\sum_{m=1}^{M} s_{2i}^m(\beta))x_i(\beta)-c[\lambda h_1(\alpha_i,\beta_i)+(1-\lambda)h_2(\alpha_i,\beta_i)]x_i(\beta)$.

It is equivalent with $(\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_i^m(\alpha_i,\beta_{-i}))x_i(\alpha_i,\beta_{-i}) \geq (\alpha_i(\mathcal{M})-\sum_{m=1}^{M} s_i^m(\beta))x_i(\beta)-c[\lambda h_1(\alpha_i,\beta_i)+(1-\lambda)h_2(\alpha_i,\beta_i)]x_i(\beta)$. Thus, ϕ is ch-SP for punishment function h, with $h(\alpha_i,\beta_i)=\lambda h_1(\alpha_i,\beta_i)+(1-\lambda)h_2(\alpha_i,\beta_i)$. ■

Proof of Proposition 4

Proof.

A mechanism is ch-SP if and only if $v_i(\alpha_i,\alpha_i,\beta_{-i}) \geq v_i(\alpha_i,\beta)-ch(\alpha_i,\beta_i)x_i(\beta)$, for any β,α_i. It is equivalent with $ch(\alpha_i,\beta_i)x_i(\beta) \geq v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})$, for any β,α_i.

So if $ch(\alpha_i,\beta_i)x_i(\beta) \geq \max_{\beta_{-i}}[v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})]$ for any β,α_i, mechanism ϕ is ch-SP.

On the other side, if $ch(\alpha_i,\beta_i)x_i(\beta) < \max_{\beta_{-i}}[v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})]$ for some β,α_i. Then there exists β_{-i}, when intermediary i has ability of transmission α_i, he will deviate to report β_i and achieve higher profit $v_i(\alpha_i,\beta)-ch(\alpha_i,\beta_i)x_i(\beta)$, such that mechanism is not ch-SP. ■

Proof of Corollary 8

Proof.

i. By definition, the minimal punishment function $h^{\min}(\alpha_i,\beta_i)=\max_{\beta_{-i}}[\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)}]$, for any $\beta_i,\alpha_i,\beta_{-i}$, such that $x_i(\beta)>0$.

$\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)}=\alpha_i(\mathcal{M})-s_i(\beta,\mathcal{M})-\frac{(\alpha_i(\mathcal{M})-s_i(\alpha_i,\beta_{-i},\mathcal{M}))x_i(\alpha_i,\beta_{-i})}{x_i(\beta)}$.

Individual rationality for intermediary to participate, $(\alpha_i(\mathcal{M})-s_i(\alpha_i,\beta_{-i},\mathcal{M}) \geq 0$, so $\frac{(\alpha_i(\mathcal{M})-s_i(\alpha_i,\beta_{-i},\mathcal{M}))x_i(\alpha_i,\beta_{-i})}{x_i(\beta)}$ 0. Then $\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)} \leq \alpha_i(\mathcal{M})$.

Thus, $h^{\min}(\alpha_i,\beta_i)=\max_{\beta_{-i}}[\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)}] \leq \alpha_i(\mathcal{M})$. For any α_i, β_i, the upper bound exists. There exists minimal punishment function $h^{\min}(\alpha_i,\beta_i)$.

ii. Consider mechanism $\phi=(x(\cdot),s(\cdot))$ is strategy-proof. Then $v_i(\alpha_i,\beta) \leq v_i(\alpha_i,\alpha_i,\beta_{-i})$, for any β, α_i. So $0 \geq \max_{\beta_{-i}}[v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})]$ for any β, α_i. Then the minimal punishment function $h^{\min}(\alpha_i,\beta_i)=0$, for any β_i, α_i.

iii. Consider mechanism $\phi = (x(\cdot), s(\cdot))$ is not strategy-proof, there exists α_i, β, such that $v_i(\alpha_i, \alpha_i, \beta_{-i}) < v_i(\alpha_i, \beta)$. Suppose $h^{\min}(\alpha_i, \beta_i) = 0$, for any β_i, α_i. Then $ch^{\min}(\alpha_i, \beta_i)x_i(\beta) = 0 < v_i(\alpha_i, \beta) - v_i(\alpha_i, \alpha_i, \beta_{-i})$, contradicts with the ch-SP. So $h^{\min}$ is nonzero. ■

Proof of Theorem 7

Proof.

i. To prove there does not exist SP mechanism, such that it is FBE.

Assume mechanism $\phi = (x(\cdot), s(\cdot))$ is SP. Given the allocation $x(\beta)$, the sharing rates $s(\beta)$ of SP mechanism satisfies $s_i(\beta, \mathcal{M}) = \beta_i(\mathcal{M}) - \frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})}$ from Theorem 6.

Then aggregate resource transmitted to agents is $y(\mathcal{M}) = \sum_{j=1}^{M} y_j$. ϕ is strategy-proof, intermediaries will report truthfully, $\beta = \alpha$. The aggregate resource allocated to agents equals $y(\mathcal{M}) = \sum_{i=1}^{N} s_i(\alpha, \mathcal{M})\hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i}) = \sum_{i=1}^{N} \alpha_i(\mathcal{M})\hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i}) - \sum_{i=1}^{N} \int_0^{\alpha_i(\mathcal{M})} \hat{x}_i(t, \alpha_{-i})dt$.

Given mechanism ϕ, resource allocation is $y = \sum_{i=1}^{N} s_i(\alpha)x_i(\alpha)$. By definition of FBE, $\bar{u} = \max_x u(\sum_{i=1}^{N} x_i\alpha_i)$.

To prove, there exists α, such that $\sum_{i=1}^{N} \int_0^{\alpha_i(\mathcal{M})} \hat{x}_i(t, \alpha_{-i})dt > 0$. Mechanism ϕ is budget balance, $\sum_{i=1}^{N} x_i(\alpha) = 1$. Assume $x_i(\tilde{\alpha}) > 0$, then let $\alpha_{-i} = \tilde{\alpha}_{-i}$ and $\alpha_i > \tilde{\alpha}_i$. $\hat{x}_i(t, \alpha_{-i})$ is monotonic in t, so for any $t > \tilde{\alpha}_i(\mathcal{M})$, $\hat{x}_i(t, \alpha_{-i}) > 0$, thus, $\sum_{i=1}^{N} \int_0^{\alpha_i(\mathcal{M})} \hat{x}_i(t, \alpha_{-i})dt > 0$. The aggregate resource allocated to agents under mechanism ϕ is $y(\mathcal{M}) < \sum_{i=1}^{N} \alpha_i(\mathcal{M})\hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i})$, when quality of intermediation is α.

By definition of maximal utility $\bar{u}(\alpha)$, $\bar{u}(\alpha) \geq u(\sum_{i=1}^{N} \alpha_i\hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i})) > u(y)$. The first inequality is from definition of maximal utility $\bar{u}$, and the second inequality comes from strongly monotone of preferences.

So there is no symmetric, SP, budget balance, and FBE mechanism.

ii. Consider ϕ is FBE mechanism. If the quality of intermediation is α, $\beta = \alpha$, the intermediaries report truthfully about their quality of intermediation. The first-best allocation of resource is $\bar{x}(\alpha)$. To achieve the fist best efficient, the sharing rates s_i should equal to the true quality of intermediation α_i if intermediary i is used by the planner, which means $s_i(\beta) = \beta_i$, if $x_i(\beta) > 0$.

Assume the final allocation of resource to agents that maximizes planner's utility is $\bar{y}(\alpha) = \sum_{i=1}^{N} \bar{x}_i(\alpha)\alpha_i$ maximizes planner's utility. Suppose there exists i, sharing rates $s_i(\beta) \leq \beta_i = \alpha_i$, $s_i \neq \beta_i$ and $x_i(\beta) > 0$, then $y = \sum_{i=1}^{N} \bar{x}_i(\alpha)s_i(\beta) \leq \bar{y}(\alpha)$ and $y \neq \bar{y}(\alpha)$. If the preferences of planner is strongly monotone, then $u(y) < u(\bar{y}(\alpha))$, the fist best efficient will not be achieved. For FBE mechanism ϕ, the sharing rates $s_i(\beta) = \beta_i$, if $x_i(\alpha) > 0$, $\beta = \alpha$.

If $x_i(\alpha_i, \beta_{-i}) = 0$, $(\alpha_i(\mathcal{M}) - s_i(\alpha_i, \beta_{-i}, \mathcal{M}))x_i(\alpha_i, \beta_{-i}) = 0$. If $x_i(\alpha_i, \beta_{-i}) > 0$, there is $s_i(\alpha_i, \beta_{-i}, \mathcal{M}) = \alpha_i(\mathcal{M})$. So $v_i(\alpha_i, \alpha_i, \beta_{-i}) = (\alpha_i(\mathcal{M}) - s_i(\alpha_i, \beta_{-i}, \mathcal{M}))x_i(\alpha_i, \beta_{-i}) = 0$. And $v_i(\alpha_i, \beta) = (\alpha_i(\mathcal{M}) - s_i(\beta, \mathcal{M}))x_i(\beta) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))x_i(\beta)$.

Then $v_i(\alpha_i, \beta) - v_i(\alpha_i, \alpha_i, \beta_{-i}) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))x_i(\beta)$.

The minimal punishment function $h^{\min} = \max_{\beta_{-i}}[\frac{v_i(\alpha_i,\beta)-v_i(\alpha_i,\alpha_i,\beta_{-i})}{x_i(\beta)}] = \max_{\beta_{-i}}[\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})] = \alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})$.

From Proposition 4, any punishment function h that implements a FBE mechanism if and only if $h(\alpha_i, \beta_i) \geq h^{\min}(\alpha_i, \beta_i) = (\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))_+$ for any $\alpha_i \geq \beta_i$. ■

B.2 Special Cases of Punishment Function

We now study the class of ch-SP for different punishment functions. We focus the analysis in three of them:

Consider the case where intermediaries are punished for report a larger capacity than their true capacity, but equally rewarded when they report a capacity that is less than their true capacity. This is achieved with the linear punishment function that is additive:

$$\tilde{h}(a, b) = \sum_{m=1}^{M} b_i^m - a_i^m$$

Such punishment function can be found in applications such as tax return. When someone reports a lower tax rate and pay less, the government would charge him the difference to get the correct amount of tax. When someone reports a higher tax rate and pay more, the government would return him the difference between the amount reported and correct amount.

Alternatively, consider the case where intermediaries are punished based on the difference from their reported quality of intermediation and their true quality of every link. The absolute punishment function can be achieved by taking:

$$\bar{h}(a, b) = \sum_{m=1}^{M} |b_i^m - a_i^m|$$

Such punishment function can be found in applications such as auditing, there is cost in auditing the ability reported, the government can punish the company by misreporting on every link based on the deviation in quality of links.

The upper deviation punishment measure that we consider includes a punishment for reporting above their ability at every link. There is no punishment if intermediaries report below their ability, and contrary to the previous punishment, there is no compensation if reporting above for one link, but below for a different link:

$$h^*(a,b) = \sum_m (b_i^m - a_i^m)_+$$

Such punishment function can be found in applications such as competition, the award will be given to the one with highest ability. The intermediary report a higher ability to win will be punished, but if intermediary reports a lower ability and still win, there is no punishment.

Note that the punishment is increasing:

$$\tilde{h}(a,b) \leq h^*(a,b) \leq \bar{h}(a,b)$$

It is important to note that any mechanism that is ch-SP for $\tilde{h}$ is also ch-SP for $\bar{h}$ and h^*. Any mechanism that is ch-SP for h^* is also ch-SP for $\bar{h}$.

When $c = 0$, all these three cases are equivalent, including c-proportional mechanism as a special example.

B.2.1 $\tilde{h}(a,b) = \sum_{m=1}^M b_i^m - \sum_{m=1}^M a_i^m$

Example 10 (c-Proportional Mechanism)

Consider a function $f : \mathbb{R} \times \mathbb{R}^{n-1} \mapsto \mathbb{R}$ that is non-decreasing in the first coordinate and symmetric[2] in R^{n-1}. The c-proportional mechanism (x, s) that satisfies:

- $x_i(\beta) = \frac{f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M}))}{\sum_{j=1}^m f(\beta_j(\mathcal{M}), \beta_{-j}(\mathcal{M}))} = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M}))$
- $\sum_{m=1}^M s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\sum_{m=1}^M \beta_i^m} \hat{x}_i(t, \beta_{-i}(\mathcal{M}))dt}{\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M}))}$

Under a proportional sharing mechanism, the intermediaries are assigned a share of the resource in proportion to their abilities using the function f to determine their proportion. This mechanism is budget balance, as exactly one unit of the resource is distributed and symmetric.

[2] That is, invariant to permutations of the elements in R^{n-1}

For instance, when $f(x_i, x_{-i}) = 1$ *if* $x_i > \max_{j \in N \setminus \{i\}} x_j$; *and* $f(x_i, x_{-i}) = 0$ *if* $x_i \leq \max_{j \in N\{i\}} x_j$, *we obtain the generalized second price auction.*

Alternatively, when $f(x) = \frac{1}{n}$ *is the constant function, we obtain the equal sharing mechanism.*

In general, the share of intermediary i *is generated by the function* f. *The aggregate charged rate that intermediary* i *is requested to transmit* $\sum_{m=1}^{M} s_i^m(\beta)$ *equals* $\beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i}(\mathcal{M}))dt}{\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M}))}$. *At this value, the profit of such intermediary equals* $(1+c)$ *times* $\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i}(\mathcal{M}))dt$.

A special case of c*-proportional mechanism is* c*-second price auction.* $f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M})) = 0$, *if* $\beta_i(\mathcal{M}) < \max_{j \neq i} \beta_j(\mathcal{M})$ *and* $f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M})) = 1$, *if* $\beta_i(\mathcal{M}) > \max_{j \neq i} \beta_j(\mathcal{M})$. *One tie breaking rule could be* $f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M})) = 1$, *if* $\beta_i(\mathcal{M}) = \max_{j \neq i} \beta_j(\mathcal{M})$, *then* $x_i(\beta) = \frac{1}{k}$, *in which,* k *is the number of intermediaries with largest* $\beta_j(\mathcal{M})$. $c = 0$ *is the case of second price auction.*

Proposition 5

Consider the punishment function $\tilde{h}$. *A mechanism satisfies* ch*-SP if and only if there exists a function* $\hat{x}_i : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$ *non-decreasing in the first coordinate such that for any* β:

i. $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})$

ii. $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})}$

Same as before, we need to discuss how these mechanisms change as c increases, clearly this is a subset of the mechanisms discussed before. The planner can transmit larger amount of resource to agents as c increases. When $ch = \infty$, the planner punishes any deviation harshly, so intermediaries have no incentive to deviate, and they will always report exactly their quality of intermediation with $\beta_i = \alpha_i$.

Corollary 9

Consider the punishment function $\tilde{h}$. *A mechanism is* ch*-SP, BB, SYM, if and only if there exists a function* $f : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$ *non-decreasing in the first coordinate such that for any* β:

i. $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) = \frac{f(\beta_i(\mathcal{M}), \beta_{-i})}{\sum_{m=1}^{N} f(\beta_i(\mathcal{M}), \beta_{-i})}$

ii. $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t, \beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i})}$

Note that, the function f depends on the aggregate ability of the intermediaries, but also depends on the abilities of other intermediaries' resource transmission to all agents.

Definition 20 (Non-bossiness)

A mechanism $\phi = (x(\cdot), s(\cdot))$ *satisfies non-bossiness if for any* $i \in \mathcal{N}$, $\phi_i(\beta) = \phi_i(\hat{\beta}_i, \beta_{-i})$ *implies* $\phi(\beta) = \phi(\hat{\beta}_i, \beta_{-i})$.

Corollary 10

Consider the punishment function $\tilde{h}$, any mechanism satisfies ch-SP, NB and SYM is a c-proportional mechanism.

B.2.2 $\bar{h}(a, b) = \sum_{m=1}^{M} |b_i^m - a_i^m|$

Proposition 6

Consider the absolute punishment function $\bar{h}$. A mechanism satisfies ch-SP, then there exists a function $\Phi : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+^N$ such that:

i. The aggregate rate that intermediary i is charged equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$

ii. $(1-c)x_i(\beta) \leq \frac{\partial \Phi_i(\hat{\beta}_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$

The ch-SP results in Φ to satisfy, $(1-c)x_i(\beta) \leq \frac{\partial \Phi_i(\hat{\beta}_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$. So when c increases, the set of ch-SP mechanisms expands. Φ_i is the profit of intermediary i for truthfully reporting $\beta_i = \alpha_i$.

Corollary 11

Consider the punishment function $\bar{h}$. A mechanism is ch-SP, BB, SYM, if and only if there exists a function $f : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+$ non-decreasing in the first M coordinates, such that for any β:

i. The aggregate rate that intermediary i charges equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$

ii. $(1-c)x_i(\beta) \leq \frac{\partial \Phi_i(\hat{\beta}_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$

iii. The sharing rule $x_i(\beta) = \frac{f(\beta_i, \beta_{-i})}{\sum_{j=1}^{N} f(\beta_j, \beta_{-j})}$

B.2.3 $h^*(a, b) = \sum_m (b_i^m - a_i^m)_+$

Consider the function $h^*(a, b) = \sum_m (b_i^m - a_i^m)_+$. In this case, we punish intermediaries who report above, but we do not punish intermediaries who are below.

Proposition 7

Consider the upper deviation punishment function h^*. The mechanism $\phi = (x(\cdot), s(\cdot))$ is ch-SP, then there exist a functions $\Phi : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+^N$ such that:

i. The aggregate rate that intermediary i charges equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$

ii. $x_i(\beta) \leq \frac{\partial \Phi_i(\beta_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$ for each i and m

Corollary 12

Consider the the punishment function h^*. A mechanism is ch-SP, BB, SYM if and only if there exist a function $\Phi : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+^N$ and function $f : \mathbb{R}_+^{NM} \mapsto \mathbb{R}_+$ such that:

i. The aggregate rate that intermediary i charges equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$

ii. $x_i(\beta) \leq \frac{\partial \Phi_i(\beta_i,\beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$ for each i and m

iii. The sharing rule $x_i(\beta) = \frac{f(\beta_i,\beta_{-i})}{\sum_{j=1}^{N} f(\beta_j,\beta_{-j})}$

Example 11

Consider second price auction similar with ϵ-Proportional mechanism, $0 < \epsilon < c$.

- $x_i(\beta) = \frac{f(\beta_i(\mathcal{M}),\beta_{-i}(\mathcal{M}))}{\sum_{j=1}^{m} f(\beta_j(\mathcal{M}),\beta_{-j}(\mathcal{M}))}$
- $\hat{x}_i(\beta_i(\mathcal{M}), \beta_{-i}) = x_i(\beta)$, *and* $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+\epsilon)\frac{\int_0^{\beta_i(\mathcal{M})} \hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$
- $f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M})) = 1$, *if* $\beta_i(\mathcal{M}) \geq \max_{j\neq i} \beta_j(\mathcal{M})$. *And* $f(\beta_i(\mathcal{M}), \beta_{-i}(\mathcal{M})) = 0$, *if* $\beta_i(\mathcal{M}) < \max_{j\neq i} \beta_j(\mathcal{M})$.

The mechanism in the example above is ch-SP for punishment function $\bar{h}$, but not for punishment function h^*.

Corollary 13

i. For any $c \geq 0$, the group of mechanisms ch-SP for absolute punishment $\bar{h}$ and upper deviation punishment h^* include the mechanism ch-SP for overall deviation punishment $\tilde{h}$.

ii. When $c = 0$, all these three cases are equivalent, including c-Proportional mechanism as a special example.

iii. When $c > 0$, the group of mechanisms ch-SP for absolute punishment $\bar{h}$ includes the mechanisms ch-SP for upper deviation punishment h^*.

The intuition behind this corollary is: when $c > 0$, there are larger punishments to intermediaries in the case of absolute punishment than upper deviation punishment, so there are more ch-SP mechanisms for absolute punishment $\bar{h}$.

Proof of Proposition 5

Proof.

$\tilde{h}(a, b) = \sum_{m=1}^{M} b_i^m - \sum_{m=1}^{M} a_i^m$.

The mechanism $\phi = (x(\cdot), s(\cdot))$ is ch-SP if for any intermediary i and for any quality of intermediation α_i and β_i.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c\sum_m(\beta_i^m - \alpha_i^m)x_i(\beta_i, \beta_{-i})$, $\forall \beta_{-i}$

The profit of intermediary i is $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$.

Assume $\alpha_i^{-m} = (\alpha_i^1, \dots, \alpha_i^{m-1}, \alpha_i^{m+1}, \dots, \alpha_i^M)$.

For strategy-proof mechanism, intermediary i truthfully reports $\beta_i = \alpha_i$ maximizing the profit.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T\mathbf{1}x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T\mathbf{1}x_i(\beta_i, \beta_{-i}) - c\sum_m |\beta_i^m - \alpha_i^m| x_i(\beta_i, \beta_{-i})$

For $\beta_i^j > \alpha_i^j$, $\beta_i^{-j} = \alpha_i^{-j}$, ch-SP requires $(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T\mathbf{1}x_i(\alpha_i, \beta_{-i}) - (\alpha_i - s_i(\beta_i, \beta_{-i}))^T\mathbf{1}x_i(\beta_i, \beta_{-i}) \geq -c(\beta_i^j - \alpha_i^j)x_i(\beta_i, \beta_{-i})$.

Equivalent with $\frac{(\alpha_i - s_i(\alpha_i,\beta_{-i}))^T\mathbf{1}x_i(\alpha_i,\beta_{-i}) - (\alpha_i - s_i(\beta_i,\beta_{-i}))^T\mathbf{1}x_i(\beta_i,\beta_{-i})}{\beta_i^j - \alpha_i^j} \geq -cx_i(\beta_i, \beta_{-i})$.

Take the limit $\beta_i^j \to \alpha_i^j$, $[\sum_{m=1}^{M} s_i^m(\beta) - \alpha_i(\mathcal{M})]\frac{\partial x_i(\beta)}{\partial \beta_i^j} + \frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]}{\partial \beta_i^j}x_i(\beta) \geq -cx_i(\beta_i, \beta_{-i})$.

$[\sum_{m=1}^{M} s_i^m(\beta)]\frac{\partial x_i(\beta)}{\partial \beta_i^j} + \frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]}{\partial \beta_i^j}x_i(\beta) = \frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]x_i(\beta)}{\partial \beta_i^j}$.

$\alpha_i(\mathcal{M})\frac{\partial x_i(\beta)}{\partial \beta_i^j} = \frac{\partial x_i(\beta)(\sum_{m=1}^{M}\beta_i^m)}{\partial \beta_i^j} - x_i(\beta)$ at $\beta = \alpha$.

$-\frac{\partial x_i(\beta)(\sum_{m=1}^{M}\beta_i^m)}{\partial \beta_i^j} + x_i(\beta) + \frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]x_i(\beta)}{\partial \beta_i^j} \geq -cx_i(\beta_i, \beta_{-i})$.

Thus, $\frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]x_i(\beta)}{\partial \beta_i^j} \geq \frac{\partial x_i(\beta)(\sum_{m=1}^{M}\beta_i^m)}{\partial \beta_i^j} - (1+c)x_i(\beta)$.

The same way, for $\beta_i^m < \alpha_i^m$, $\beta_i^{-m} = \alpha_i^{-m}$.

Take $\beta_i^m \to \alpha_i^m$, $\frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]x_i(\beta)}{\partial \beta_i^j} \leq \frac{\partial x_i(\beta)(\sum_{m=1}^{M}\beta_i^m)}{\partial \beta_i^j} - (1+c)x_i(\beta)$.

From the inequality above, there is $\beta_i^m \to \alpha_i^m$, $\frac{\partial[\sum_{m=1}^{M} s_i^m(\beta)]x_i(\beta)}{\partial \beta_i^j} = \frac{\partial x_i(\beta)(\sum_{m=1}^{M}\beta_i^m)}{\partial \beta_i^j} - (1+c)x_i(\beta)$.

For the mechanism to be ch-SP, it is optimal to report $\beta_i^j = \alpha_i^j$. Thus, substitute $\beta = \alpha = (\alpha_i^j, \alpha_i^{-j}, \alpha_{-i})$, integral β_i^j from 0 to α_i^j on both sides.

$s_i^m(\alpha)x_i(\alpha) - s_i^m(0, \alpha_i^{-j}, \alpha_{-i})x_i(0, \alpha_i^{-j}, \alpha_{-i}) = \alpha_i(\mathcal{M})x_i(\alpha) - \sum_{m\neq j}\alpha_i^m x_i(0, \alpha_i^{-j}, \alpha_{-i}) - (1+c)\int_0^{\alpha_i^j} x_i(t, \alpha_i^{-j}, \alpha_{-i})dt$

Rearrange the equation:

$[\sum_{m=1}^{M}\alpha_i^m - \sum_{m=1}^{M} s_i^m(\alpha)]x_i(\alpha) = [\sum_{m\neq j}\alpha_i^m - \sum_{m=1}^{M} s_i^m(0, \alpha_i^{-j}, \alpha_{-i})]x_i(0, \alpha_i^{-j}, \alpha_{-i}) + (1+c)\int_0^{\alpha_i^j} x_i(t, \alpha_i^{-j}, \alpha_{-i})dt$.

Assume $[\sum_{m=1}^{M}\alpha_i^m - \sum_{m=1}^{M} s_i^m(\alpha)]x_i(\alpha) = h_i(\alpha)$, $h_i(\alpha) = [\sum_{m\neq j}\alpha_i^m - \sum_{m=1}^{M} s_i^m(0, \alpha_i^{-j}, \alpha_{-i})]x_i(0, \alpha_i^{-j}, \alpha_{-i}) + (1+c)\int_0^{\alpha_i^j} x_i(t, \alpha_i^{-j}, \alpha_{-i})dt$.

Similar with above, F.O.C. for partial derivative to β_i^k.

$h_i(\alpha) = [\sum_{m\neq k}\alpha_i^m - \sum_{m=1}^{M} s_i^m(0, \alpha_i^{-k}, \alpha_{-i})]x_i(0, \alpha_i^{-k}, \alpha_{-i}) + (1+c)\int_0^{\alpha_i^k} x_i(t, \alpha_i^{-k}, \alpha_{-i})dt$.

$\frac{\partial h_i(\alpha)}{\partial \alpha_i^j} = \frac{\partial[\sum_{m\neq j}\alpha_i^m - \sum_{m=1}^M s_i^m(0,\alpha_i^{-j},\alpha_{-i})]x_i(0,\alpha_i^{-j},\alpha_{-i}) + (1+c)\int_0^{\alpha_i^j} x_i(t,\alpha_i^{-j},\alpha_{-i})dt}{\partial \alpha_i^j} = x_i(\alpha).$

$\frac{\partial h_i(\alpha)}{\partial \alpha_i^k} = \frac{\partial[\sum_{m\neq k}\alpha_i^m - \sum_{m=1}^M s_i^m(0,\alpha_i^{-k},\alpha_{-i})]x_i(0,\alpha_i^{-k},\alpha_{-i}) + (1+c)\int_0^{\alpha_i^k} x_i(t,\alpha_i^{-k},\alpha_{-i})dt}{\partial \alpha_i^k} = x_i(\alpha).$

$\frac{\partial h_i(\alpha)}{\partial \alpha_i^k} = \frac{\partial h_i(\alpha)}{\partial \alpha_i^j} = x_i(\alpha), \forall j, k.$

From Lemma 5, $h_i(\alpha) = \hat{h}_i(\alpha_i(\mathcal{M}), \alpha_{-i})$.

$h_i(\alpha) = (1+c)[\int_0^{\alpha_i^1} x_i(t, \alpha_i^{-1}, \alpha_{-i})dt + \int_0^{\alpha_i^2} x_i(0, t, \alpha_i^{-1,2}, \alpha_{-i})dt + \cdots + \int_0^{\alpha_i^M} x_i(0, \ldots, 0, t, \alpha_{-i})dt].$

For multiple dimension case, F.O.C. partial derivative to β_i^m, and at the point $\beta = \alpha$,

$[\alpha_i(\mathcal{M}) - \sum_{m=1}^M s_i^m(\alpha)]x_i(\alpha) = [\sum_{j\neq m}\alpha_i^j - \sum_{m=1}^M s_i^m(0, \alpha_i^{-m}, \alpha_{-i})]x_i(0, \alpha_i^{-m}, \alpha_{-i}) + (1 + c)\int_0^{\alpha_i^m} x_i(t, \alpha_i^{-m}, \alpha_{-i})dt$

$[\sum_{j\neq m}\alpha_i^j - \sum_{m=1}^M s_i^m(0, \alpha_i^{-m}, \alpha_{-i})]x_i(0, \alpha_i^{-m}, \alpha_{-i})$

$= [\sum_{j\neq m,k}\alpha_i^j - \sum_{m=1}^M s_i^m(0, 0, \alpha_i^{-m,k}, \alpha_{-i})]x_i(0, 0, \alpha_i^{-m,k}, \alpha_{-i}) + (1+c)\int_0^{\alpha_i^k} x_i(0, t, \alpha_i^{-m,k}, \alpha_{-i})dt$

Continue to substitute the equality, there is

$[\alpha_i(\mathcal{M}) - \sum_{m=1}^M s_i^m(\alpha)]x_i(\alpha) = (1+c)[\int_0^{\alpha_i^1} x_i(t, \alpha_i^{-1}, \alpha_{-i})dt + \int_0^{\alpha_i^2} x_i(0, t, \alpha_i^{-1,2}, \alpha_{-i})dt + \cdots + \int_0^{\alpha_i^M} x_i(\mathbf{0}_{-M}, t, \alpha_{-i})dt].$

For any permutation of $\{1, 2, \ldots, M\}$, denoted by $\{j1, j2, \ldots, jM\}$, we have

$[\alpha_i(\mathcal{M}) - \sum_{m=1}^M s_i^m(\alpha)]x_i(\alpha) = (1+c)[\int_0^{\alpha_i^{j1}} x_i(t, \alpha_i^{-j1}, \alpha_{-i})dt + \int_0^{\alpha_i^{j2}} x_i(0, t, \alpha_i^{-j1,j2}, \alpha_{-i})dt + \cdots + \int_0^{\alpha_i^{jM}} x_i(\mathbf{0}_{-jM}, t, \alpha_{-i})dt].$

From Lemma 6, $x_i(\alpha) = \hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i})$, which means $x_i(\alpha)$ does not change when $\alpha_i(\mathcal{M})$ is the constant.

Since $[\alpha_i(\mathcal{M}) - \sum_{m=1}^M s_i^m(\alpha)]x_i(\alpha) = h_i(\alpha)$, there is $\sum_{m=1}^M s_i^m(\alpha) = \alpha_i(\mathcal{M}) - \frac{h_i(\alpha)}{x_i(\alpha)} = \sum_{m=1}^M \alpha_i^m - \frac{\hat{h}_i(\alpha_i(\mathcal{M}), \alpha_{-i})}{\hat{x}_i(\alpha_i(\mathcal{M}), \alpha_{-i})}$.

Thus, $\sum_{m=1}^M s_i^m(\alpha) = g_i(\alpha_i(\mathcal{M}), \alpha_{-i})$.

To prove these conditions are sufficient for the mechanism to be ch-SP.

From Lemma 7, the functions $x_i(\beta)$ and $s_i(\beta)$ satisfying F.O.C. are strategy-proof for intermediary i.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T \mathbf{1} x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T \mathbf{1} x_i(\beta_i, \beta_{-i}) - c \cdot h(\alpha_i, \beta_i) \cdot x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$

Substitute the condition ii into the inequality, there is

$(1+c)\int_0^{\alpha_i(\mathcal{M})}\hat{x}_i(t,\beta_{-i})dt \geq (1+c)\int_0^{\beta_i(\mathcal{M})}\hat{x}_i(t,\beta_{-i})dt + \sum_{m=1}^{M}(\alpha_i^m - \beta_i^m)x_i(\beta) - c\sum_{m=1}^{M}(\beta_i^m - \alpha_i^m)x_i(\beta)$.

Equivalent with $(1+c)\int_{\beta_i(\mathcal{M})}^{\alpha_i(\mathcal{M})}\hat{x}_i(t,\beta_{-i})dt \geq (1+c)(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))x_i(\beta)$.

$x_i(\beta)$ is monotonic in β_i, so for $\beta_i(\mathcal{M}) \leq t \leq \alpha_i(\mathcal{M})$, $\hat{x}_i(t,\beta_{-i}) \geq x_i(\beta)$.

It satisfies the inequality, thus the condition is sufficient. ■

Proof of Corollary 9

Proof.

From Proposition 5, $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})$, $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\beta_i(\mathcal{M})}\hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$, $\hat{s}_i(\beta_i(\mathcal{M}),\beta_{-i}) = \sum_{m=1}^{M} s_i^m(\beta)$.

From Lemma 8, there exists function $f : \mathbb{R}_+ \times \mathbb{R}_+^{M(N-1)} \mapsto \mathbb{R}_+$, s.t. $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i}) = \frac{f(\beta_i(\mathcal{M}),\beta_{-i})}{\sum_{i=1}^{N} f(\beta_i(\mathcal{M}),\beta_{-i})}$. ■

Proof of Corollary 10

The mechanism satisfying ch-SP and NB, then $\phi_i(\beta) = \bar{\phi}_i(\beta_1(\mathcal{M}),\ldots,\beta_N(\mathcal{M}))$.

Proof.

From Proposition 5, the allocation $x_i(\beta) = \hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})$.

The sharing rates $\hat{s}_i(\beta_i(\mathcal{M}),\beta_{-i}) = \sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - (1+c)\frac{\int_0^{\beta_i(\mathcal{M})}\hat{x}_i(t,\beta_{-i})dt}{\hat{x}_i(\beta_i(\mathcal{M}),\beta_{-i})}$.

Thus, $\phi_i(\beta) = \hat{\phi}_i(\beta_i(\mathcal{M}),\beta_{-i})$.

For any i, β and $\hat{\beta}_i$ with $\beta_i(\mathcal{M}) = \hat{\beta}_i(\mathcal{M}) = \sum_{m=1}^{M}\hat{\beta}_i^m$, there is $\phi_i(\beta) = \hat{\phi}_i(\beta_i(\mathcal{M}),\beta_{-i}) = \hat{\phi}_i(\hat{\beta}_i(\mathcal{M}),\beta_{-i}) = \phi_i(\hat{\beta}_i,\beta_{-i})$.

The non-bossiness requires $\phi_i(\beta) = \phi_i(\hat{\beta}_i,\beta_{-i})$, then $\phi(\beta) = \phi(\hat{\beta}_i,\beta_{-i})$. So $\hat{\phi}(\beta_i(\mathcal{M}),\beta_{-i}) = \hat{\phi}(\hat{\beta}_i(\mathcal{M}),\beta_{-i})$. Thus, $\phi_n(\beta)$ depends only on $\beta_i(\mathcal{M})$ rather than β_i, $\forall n$.

So $\phi_i(\beta) = \bar{\phi}_i(\beta_1(\mathcal{M}),\ldots,\beta_N(\mathcal{M}))$. ■

Proof of Proposition 6

Proof.

Profit of intermediary i is $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$ when he truthfully reports $\beta_i = \alpha_i$. Thus, the aggregate rate that intermediary i charges equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$.

Punishment function $\bar{h}(a,b)=\sum_{m=1}^{M}|b_i^m-a_i^m|$, mechanism $\phi=(x(\cdot),s(\cdot))$ is ch-SP if for any intermediary i and for any quality of intermediation α_i and β_i.

$$(\alpha_i-s_i(\alpha_i,\beta_{-i}))^T\mathbf{1}x_i(\alpha_i,\beta_{-i})\geq(\alpha_i-s_i(\beta_i,\beta_{-i}))^T\mathbf{1}x_i(\beta_i,\beta_{-i})-c\sum_m|\beta_i^m-\alpha_i^m|x_i(\beta_i,\beta_{-i}),\forall\beta_{-i}$$

To prove $(1-c)x_i(\beta)\leq\frac{\partial\Phi_i(\hat{\beta}_i,\beta_{-i})}{\partial\beta_i^m}\leq(1+c)x_i(\beta)$ for each i and m.

ch-SP requires $x_i(\beta_i,\beta_{-i})(\alpha_i(\mathcal{M})-\beta_i(\mathcal{M}))-c\sum_m|\beta_i^m-\alpha_i^m|x_i(\beta_i,\beta_{-i})\leq\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})$.

When intermediary's behavior to report $\beta\leq\alpha$. Intermediary i can only deviate to a lower β_i, $\beta_i\leq\alpha_i$, $\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})\geq 0$. Then $(1-c)x_i(\beta_i,\beta_{-i})\leq\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})}$.

Let $\beta_i^{-m}=\alpha_i^{-m}$, and $\beta_i^m\leq\alpha_i^m$. Then $(1-c)x_i(\beta_i,\beta_{-i})\leq\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})}=\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i^m-\beta_i^m}$. Take the limit $\beta_i^m\to\alpha_i^m$, $(1-c)x_i(\alpha_i,\beta_{-i})\leq\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m}$.

Let $\beta_i^{-m}=\alpha_i^{-m}$, and $\beta_i^m\geq\alpha_i^m$. The ch-SP requires $\Phi_i(\beta_i,\beta_{-i})+x_i(\beta_i,\beta_{-i})(\alpha_i(\mathcal{M})-\beta_i(\mathcal{M}))-c(\beta_i(\mathcal{M})-\alpha_i(\mathcal{M}))x_i(\beta_i,\beta_{-i})\leq\Phi_i(\alpha_i,\beta_{-i})$, when intermediary i's payoff of misreport β_i lower than truthfully report α_i.

Rearrange the inequality, $\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})\geq(1+c)x_i(\beta_i,\beta_{-i})(\alpha_i(\mathcal{M})-\beta_i(\mathcal{M}))$, which means $\frac{\Phi_i(\alpha_i,\beta_{-i})-\Phi_i(\beta_i,\beta_{-i})}{\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})}\leq(1+c)x_i(\beta_i,\beta_{-i})$, with $\alpha_i^m\leq\beta_i^m$, and $\alpha_i(\mathcal{M})-\beta_i(\mathcal{M})=\alpha_i^m-\beta_i^m$.

Take the limit $\beta_i^m\to\alpha_i^m$, $\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m}\leq(1+c)x_i(\alpha_i,\beta_{-i})$.

So $(1-c)x_i(\alpha_i,\beta_{-i})\leq\frac{\partial\Phi_i(\alpha_i,\beta_{-i})}{\partial\beta_i^m}\leq(1+c)x_i(\alpha_i,\beta_{-i})$. ∎

Proof of Corollary 11

Proof.

From Proposition 6, part i and ii are proved.

BB requires $\sum_{i=1}^{M}x_i(\beta)=1$, SYM requires $x_i(\beta_i,\beta_{-i})=x_j(\beta'_j,\beta'_{-j})$, with $\beta_i=\beta'_j$ and $\beta_{-i}=\beta'_{-j}$.

From Lemma 8, there exists function f, s.t. the sharing rule $x_i(\beta)=\frac{f(\beta_i,\beta_{-i})}{\sum_{j=1}^{N}f(\beta_j,\beta_{-j})}$. Part iii satisfies.
∎

Proof of Proposition 7

Proof.

Profit of intermediary i is $\Phi_i(\beta) = \sum_{m=1}^{M}(\beta_i^m - s_i^m(\beta))x_i(\beta)$ when he truthfully reports $\beta_i = \alpha_i$. Thus, the aggregate rate that intermediary i charges equals $\sum_{m=1}^{M} s_i^m(\beta) = \beta_i(\mathcal{M}) - \frac{\Phi_i(\beta)}{x_i(\beta)}$.

Punishment function $h^*(a,b) = \sum_{m=1}^{M}(b_i^m - a_i^m)_+$, mechanism $\phi = (x(\cdot), s(\cdot))$ is ch-SP if for any intermediary i and for any quality of intermediation α_i and β_i.

$(\alpha_i - s_i(\alpha_i, \beta_{-i}))^T\mathbf{1}x_i(\alpha_i, \beta_{-i}) \geq (\alpha_i - s_i(\beta_i, \beta_{-i}))^T\mathbf{1}x_i(\beta_i, \beta_{-i}) - c\sum_m(\beta_i^m - \alpha_i^m)_+ x_i(\beta_i, \beta_{-i}), \forall \beta_{-i}$

To prove $x_i(\beta) \leq \frac{\partial \Phi_i(\hat{\beta}_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\beta)$ for each i and m.

ch-SP requires $x_i(\beta_i, \beta_{-i})(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})) - c\sum_m(\beta_i^m - \alpha_i^m)_+ x_i(\beta_i, \beta_{-i}) \leq \Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})$.

When intermediary's behavior to report $\beta \leq \alpha$. Intermediary i can only deviate to a lower $\hat{\beta}_i$, $\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}) \geq 0$. Then $x_i(\hat{\beta}_i, \beta_{-i}) \leq \frac{\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})}, \forall \beta_i \leq \alpha_i$.

Let $\beta_i^{-m} = \alpha_i^{-m}$, and $\beta_i^m \leq \alpha_i^m$. Then $x_i(\beta_i, \beta_{-i}) \leq \frac{\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} = \frac{\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})}{\alpha_i^m - \beta_i^m}$. Take the limit $\beta_i^m \to \alpha_i^m$, $x_i(\alpha_i, \beta_{-i}) \leq \frac{\partial \Phi_i(\alpha_i, \beta_{-i})}{\partial \beta_i^m}$.

Let $\beta_i^{-m} = \alpha_i^{-m}$, and $\beta_i^m \geq \alpha_i^m$. The ch-SP requires $\Phi_i(\beta_i, \beta_{-i}) + x_i(\beta_i, \beta_{-i})(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})) - c(\beta_i(\mathcal{M}) - \alpha_i(\mathcal{M}))x_i(\beta_i, \beta_{-i}) \leq \Phi_i(\alpha_i, \beta_{-i})$, when intermediary i's payoff of misreport β_i lower than truthfully report α_i.

Rearrange the inequality, $\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i}) \geq (1+c)x_i(\beta_i, \beta_{-i})(\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M}))$, which means $\frac{\Phi_i(\alpha_i, \beta_{-i}) - \Phi_i(\beta_i, \beta_{-i})}{\alpha_i(\mathcal{M}) - \beta_i(\mathcal{M})} \leq (1+c)x_i(\beta_i, \beta_{-i})$, with $\alpha_i^m \leq \beta_i^m$.

Take the limit $\beta_i^m \to \alpha_i^m$, $\frac{\partial \Phi_i(\alpha_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\alpha_i, \beta_{-i})$.

So $x_i(\alpha_i, \beta_{-i}) \leq \frac{\partial \Phi_i(\alpha_i, \beta_{-i})}{\partial \beta_i^m} \leq (1+c)x_i(\alpha_i, \beta_{-i})$. ■

Proof of Corollary 12

Proof.

From Proposition 7, part i and ii are proved.

BB requires $\sum_{i=1}^{M} x_i(\beta) = 1$, SYM requires $x_i(\beta_i, \beta_{-i}) = x_j(\beta'_j, \beta'_{-j})$, with $\beta_i = \beta'_j$ and $\beta_{-i} = \beta'_{-j}$.

From Lemma 8, there exists function f, s.t. the sharing rule $x_i(\beta) = \frac{f(\beta_i, \beta_{-i})}{\sum_{j=1}^{N} f(\beta_j, \beta_{-j})}$. Part iii satisfies. ■

Proof of Corollary 13

Proof.

Part i. Since $\tilde{h}(a,b) = \sum_{m=1}^{M} b_i^m - \sum_{m=1}^{M} a_i^m$, $\bar{h}(a,b) = \sum_{m=1}^{M} |b_i^m - a_i^m|$, $h^*(a,b) = \sum_m (b_i^m - a_i^m)_+$.

For any α_i and β_i, there is $\bar{h}(\alpha_i,\beta_i) \geq h^*(\alpha_i,\beta_i) \geq \tilde{h}(\alpha_i,\beta_i)$.

For any mechanism $\phi = (x(\cdot), s(\cdot))$, which is ch-SP for overall deviation punishment $\tilde{h}$. Then for any quality of intermediation α_i and β_i, there is $(\alpha_i - s_i(\alpha_i,\beta_{-i}))^T \mathbf{1} x_i(\alpha_i,\beta_{-i}) \geq (\alpha_i - s_i(\beta_i,\beta_{-i}))^T \mathbf{1} x_i(\beta_i,\beta_{-i}) - c \cdot \tilde{h}(\alpha_i,\beta_i) \cdot x_i(\beta_i,\beta_{-i}) \geq (\alpha_i - s_i(\beta_i,\beta_{-i}))^T \mathbf{1} x_i(\beta_i,\beta_{-i}) - c \cdot h^*(\alpha_i,\beta_i) \cdot x_i(\beta_i,\beta_{-i}) \geq (\alpha_i - s_i(\beta_i,\beta_{-i}))^T \mathbf{1} x_i(\beta_i,\beta_{-i}) - c \cdot \bar{h}(\alpha_i,\beta_i) \cdot x_i(\beta_i,\beta_{-i})$, $\forall \beta_{-i}$.

Thus, for any mechanism $\phi = (x(\cdot), s(\cdot))$ satisfying ch-SP for punishment function $\tilde{h}$ will also satisfy ch-SP for $\bar{h}$ and h^*.

Part ii. $c = 0$, ch-SP in 3 cases are equivalent with traditional SP: $(\alpha_i - s_i(\alpha_i,\beta_{-i}))^T \mathbf{1} x_i(\alpha_i,\beta_{-i}) \geq (\alpha_i - s_i(\beta_i,\beta_{-i}))^T \mathbf{1} x_i(\beta_i,\beta_{-i})$.

From Corollary 6, c-Proportional mechanism satisfies the 0-SP.

Part iii. Same with Part i. For any α_i and β_i, there is $\bar{h}(\alpha_i,\beta_i) \geq h^*(\alpha_i,\beta_i)$. So any mechanism $\phi = (x(\cdot), s(\cdot))$ satisfying ch-SP for punishment function $\bar{h}$ will also satisfy ch-SP for h^*. ■

Appendix C CHAPTER 4

C.1 Distribution of Volatility

Figure C.1 represents the histogram for volatility of 12 industry value-weighted portfolios. The distribution is right screwed.

Figure C.2 represents the histogram for logarithmic volatility of 12 industry value-weighted portfolios. This distribution is closer to normal distribution. The estimation of connectedness of industry portfolios are based on the logarithmic volatility, rather than volatility.

C.2 Static Analysis (Full Sample)

The adjacent matrix of industry is presented in Table C.1. The sample is Jul 1927 through Dec 2017, with 1086 monthly observations. The ij entry of the 12×12 matrix in the top left corner shows the ij pairwise directional connectedness of industry, which measures the percent of 10 month ahead ($H = 10$) forecast error variance of industry i due to shocks from industry j. The 'From' column equals to the row sum, and measures the total directional connectedness from all other industries to industry i. The 'To' row equals to the column sum, and measures the total directional connectedness from industry j to other industries. The 'NET' row equals the difference between the 'To' and 'From' total directional connectedness. Finally, number in the bottom right corner measures the systemic connectedness among 12 industries.

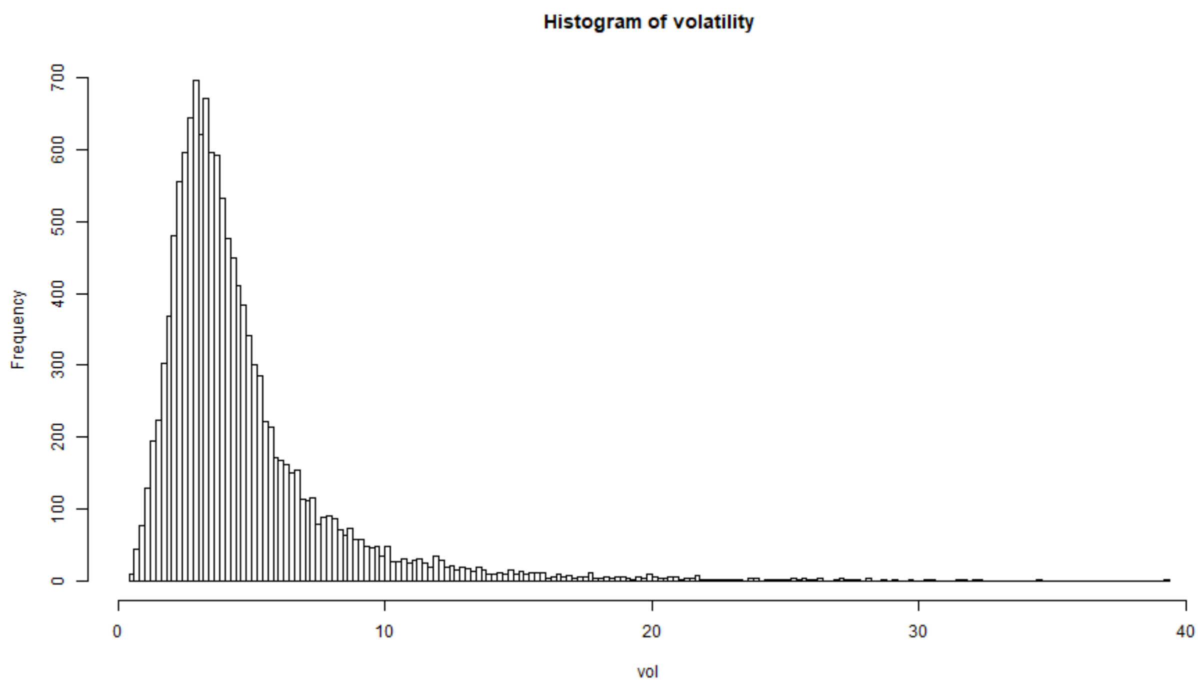


Figure C.1: Histogram of Volatility: 1926/07 to 2017/12

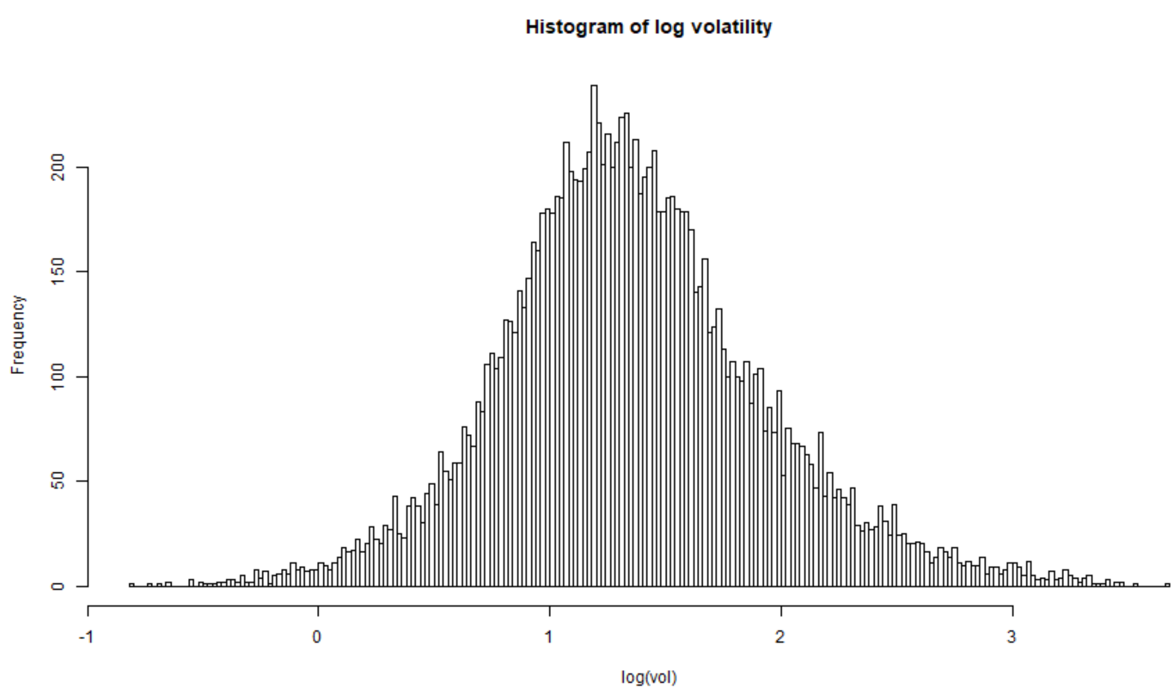


Figure C.2: Histogram of Logarithmic Volatility: 1926/07 to 2017/12

Table C.1: Static Analysis (Jul 1927 to Dec 2017)

	non-dur	durables	manu	energy	chem	busi	tele	util	shops	health	finance	other	From
non-dur		0.520	0.776	0.550	0.725	0.644	0.523	0.641	0.793	0.699	0.689	0.711	7.272
durables	0.666		0.830	0.521	0.751	0.672	0.403	0.658	0.709	0.592	0.700	0.761	7.263
manu	0.738	0.627		0.575	0.751	0.701	0.374	0.684	0.711	0.605	0.739	0.831	7.336
energy	0.711	0.518	0.776		0.683	0.610	0.399	0.702	0.673	0.609	0.680	0.697	7.058
chem	0.746	0.623	0.827	0.570		0.696	0.403	0.597	0.733	0.647	0.686	0.738	7.268
busi	0.698	0.585	0.808	0.517	0.732		0.501	0.630	0.722	0.642	0.659	0.739	7.232
tele	0.692	0.453	0.610	0.457	0.589	0.607		0.639	0.680	0.553	0.580	0.593	6.453
util	0.704	0.486	0.722	0.571	0.639	0.606	0.477		0.663	0.582	0.685	0.678	6.812
shops	0.786	0.572	0.764	0.534	0.718	0.672	0.522	0.662		0.643	0.697	0.716	7.287
health	0.797	0.552	0.744	0.562	0.752	0.690	0.469	0.557	0.740		0.634	0.660	7.156
finance	0.740	0.553	0.792	0.565	0.685	0.643	0.447	0.759	0.714	0.591		0.761	7.250
other	0.720	0.619	0.897	0.551	0.720	0.688	0.387	0.661	0.710	0.575	0.737		7.267
To	7.997	6.109	8.546	5.974	7.745	7.230	4.905	7.190	7.849	6.738	7.487	7.883	
From	7.272	7.263	7.336	7.058	7.268	7.232	6.453	6.812	7.287	7.156	7.250	7.267	
Net	0.725	-1.154	1.210	-1.083	0.477	-0.003	-1.548	0.379	0.562	-0.418	0.237	0.616	85.652

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