Disclosure and Rollover Risk

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Abstract

This paper studies whether and to what extent transparent disclosure prevents inefficient liquidation arising from rollover risk. We model an illiquid but solvent borrower who can design a public signal about what creditors can recover from forcing liquidation, and what their claims would be worth if the firm survives. We find that the signal structure that minimizes rollover risk never identifies liquidation or continuation values, and that borrowers can commit to this structure. Moreover, if creditors can impose disclosure requirements, they may increase inefficient liquidation, in order to pool states to increase the amount they expect to recover from defaults.

1 Introduction

This paper addresses the concern that a profitable firm can go bankrupt as a result of creditors seizing collateral rather than providing continued funding, out of fear that too few other claimants will agree to roll their debts over. Our primary interest is in what information an illiquid yet solvent firm can provide its creditors in order to minimize this rollover risk. Does disclosing information about what creditors stand to recover from seizing their collateral provide reassurance, or does it destabilize? What about disclosures concerning the expected value of a creditor's claim if the firm survives? If a borrower or policymaker interested in avoiding inefficient liquidation could design the type of information that creditors would receive in the event of a liquidity shock, what information would the ideal signal convey to creditors?

Secondarily, we are interested in whether creditors would necessarily prefer having borrowers' disclosures minimize rollover risk. In other words, in the absence of other motives such as contracting benefits, would creditors necessarily adopt the viewpoint that no one gains from inefficient liquidation?

We show that the optimal signal structure for preventing inefficient liquidation is imprecise, identifying neither the amount creditors can recover from forcing liquidation nor the expected value of their claims should the firm continue to operate. The intuition is that an imprecise disclosure pools states with low rollover risk with those in which rollover risk is high. In fact, this relationship turns out to be quite strong: as fundamental risk increases in a sense we make precise, the optimal signal structure (from the viewpoint of minimizing inefficient liquidation) becomes less informative, and the probability of inefficient liquidation falls.

Although this line of argument is familiar from the global games literature (Carlsson and van Damme 1993, Morris and Shin 2002, 2004, among others), it is natural to expect the borrower's disclosure problem to unravel, that is, for the borrower to disclose everything due to creditor skepticism over omissions. We find that this is not the case, and instead find that

borrowers can credibly choose disclosure policies that minimize rollover risk.

If a borrower can completely eliminate rollover risk through a (non)disclosure policy, then this policy is also optimal for the firm's creditors. However, if some chance of inefficient liquidation remains after any disclosure policy, then creditors' interests diverge from borrowers' interests, with creditors preferring disclosures that bring about some avoidable inefficient liquidation. Creditors care about the circumstances under which they face rollover risk, and trade off the probability of inefficient liquidation against the amount they stand to lose. The message is that we cannot take for granted that creditor-backed disclosure requirements generate stability, or that disclosures that reduce inefficient liquidation increase social welfare.

We develop these results in a disclosure and rollover risk game between a representative borrower ('she') and her creditors (a generic one labeled 'he'). The borrower, having suffered a liquidity shock prior to the start of play, depends on the creditors' willingness to roll their obligations over. Each creditor independently decides whether to roll his debts over or to refuse and seize any collateral he can, forcing the borrower into bankruptcy. As in the rollover risk model of Morris and Shin (2004), a creditor who rolls the debt over when another creditor forces the borrower into liquidation receives part of the bankruptcy settlement, which is less than what the creditor would have received from seizing the collateral before the bankruptcy process begins. If no one forces inefficient liquidation, the borrower and the creditors all benefit. In sum, the skeleton of our game, prior to incorporating asymmetric information and disclosure, is the stag hunt game of Rousseau (discussed in van Huyck et al. 1990, Crawford 1991, van Huyck et al. 1993, Battalio et al. 2001, Skyrms 2001).

Our game builds on this stag hunt structure by introducing an earlier stage, in which the borrower can costlessly obtain private information about two distinct but related values. The borrower observes her creditors' expected payoff from rolling their debts over in the event of no forced liquidation. We call this the creditors' continuation value. In addition, the borrower learns the market value of the collateral that creditors can seize, which we call the creditors' liquidation value. Intuitively, a borrower who faces a liquidity shock would

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try to discover how strong her creditors' temptation to liquidate is, and how easily she can entice her creditors to allow her to pay them later. The borrower issues a public disclosure about these values, which may be vague or incomplete but must be non-fraudulent. The creditors then make their rollover decisions, using risk dominance as an equilibrium selection criterion (Harsanyi and Selten 1988). We focus on risk dominance because it is grounded in the literature on evolutionary game theory (Crawford 1991, Ellison 1993, Kandori et al. 1993, Young 1993, Temzelides 1997) and on global games (Carlsson and van Damme 1993, Morris and Shin 2004 in a rollover risk context), and has support from laboratory experiments (Schmidt et al. 2003, Anctil et al. 2004, 2010). Risk dominance captures the idea of creditors minimizing the negative effects of being wrong about equilibrium selection.

Our borrower's problem is one of information design, a recent literature which Taneva (2016) develops fully as a multiple receiver extension of the Bayesian persuasion literature that Kamenica and Gentzkow (2011) pioneer. Bergemann and Morris (2016b) provide a good overview, and give details on the important building blocks in Bergemann and Morris (2016a). Information design, like mechanism design, has two components. A basic game consists of a set of players, their payoff functions, and their available actions, along with a common prior over a set of payoff states. An information structure can be thought of as a distribution of a signal about the state.[1] In mechanism design, the designer chooses the basic game, taking the information structure as given. Information design is dual to mechanism design, in the sense that the designer chooses the information structure and takes the basic

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[1]We limit attention to the case in which the borrower designs a public signal, and in which the creditors do not have additional private information. As Wang (2015), Bergemann and Morris (2016a,b) point out, an information designer prefers public communication when trying to increase coordination and prefers private communication when the receivers' actions are strategic substitutes. For examples of the latter case, see Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985), who show the limitations on decentralized information sharing in oligopoly, and the more recent information design approach in Bergemann and Morris (2013) and Michaeli (2017), in which an informed designer can increase aggregate information with private signals. A thorough discussion of privately informed receivers is beyond our scope, and we refer the interested reader to Kolotilin et al. (2015).

game as given.

Our finding that the solution to the borrower's information design problem is to provide an imprecise report goes against standard unraveling arguments (Grossman and Hart 1980, Milgrom 1981, Grossman 1981). The borrower is commonly known to be informed, and therefore cannot avoid disclosure by pleading ignorance, as in the models of Dye (1985), Jung and Kwon (1988), Ben-Porath et al. (2014). Disclosure for her is costless, so she cannot justify omissions by appealing to direct or indirect disclosure costs, as in the setting Verrecchia (1983) studies. We usually expect a fully informative report from a sender, given an objective of maximizing firm value, as Beyer et al. (2010) discuss in their survey article. Hedlund (2017) shows that a form of this argument applies in Bayesian persuasion models.

A borrower's payoff, however, is not strictly monotone in how encouraging her news is. Providing slightly better news does not benefit the borrower unless doing so can alter her creditors' liquidation decisions. The flat regions in a borrower's payoff make imprecise disclosure credible, similar to an observation that Milgrom (2008) makes. The creditors thus do not need to interpret omissions skeptically (as in Milgrom and Roberts 1986, Okuno-Fujiwara et al. 1990). Analogous to the argument Chen et al. (2008) make in a cheap-talk setting based on Crawford and Sobel (1982), we find that a lack of incentive to separate limits the amount of voluntary disclosure.

Our finding that creditors may prefer disclosures that do not minimize rollover risk is to the best of our knowledge new. As we note above, this result arises if the borrower's disclosure cannot completely eliminate rollover risk; otherwise, the game is one of pure common interest. Related results are in Bouvard et al. (2015), who study systemic risk. In their setting, if some rollover risk is inevitable, the sender may release information that deliberately forces some borrowers into inefficient liquidation, in order to avoid system-wide collapse. Like us, Bouvard et al. therefore show that common interest between the designer and (some of) the other players may vanish if only some coordination failure is avoidable.

Even if our borrower cannot eliminate rollover risk, she always opts for a imprecise disclosure. This result may seem to conflict with Lehrer et al. (2010), who argue that increased information precision is welfare improving in common interest games. The reason for this apparent difference is that their notion of welfare is based on the best equilibrium attainable under a given information structure. Our results, on the other hand, are based on an equilibrium selection criterion: rather than focusing on the best possible equilibrium, we concentrate on the highest Pareto-ranked risk dominant equilibrium.

The remainder of this paper is as follows. Section 2 presents the model. Section 3 gives the results on the optimal choice of information structure. Section 4 gives results on the effects of fundamental risk on the optimal information structure. Section 5 concludes. Proofs are in an appendix.

2   The Model

We show our results in a model, which consists of a basic game G and an information structure S, following the notation of Bergemann and Morris (2016a). The basic game G includes the economic environment (players, payoff states, and a common prior over the payoff states) as well as a characterization of the strategic interaction (utilities for the players and their available actions). The information structure S consists of a public signal $\widetilde{t}$ that depends on the payoff state and takes values in some set T.

There are three agents in the model, a borrower (she) and two creditors (both he). The creditors are players in the basic game G, and are both risk neutral and have rational expectations. The borrower is a designer, who chooses the information structure S. Her payoff depends on the equilibrium that the players in the basic game select. We might think of the borrower as player 0, but in terms of G the set of players includes only the two creditors. We denote the players in basic game G as $I = \{1, 2\}$.

Our interest is in rollover risk, rather than the cost of capital, credit rationing, or capital structure choices. We therefore assume that the creditors have loans in place to the borrower at the beginning of play. As in the rollover risk model of Morris and Shin (2004), we assume that the loans are backed by collateral. Unlike them, but like the study of asset impairment reporting by Göx and Wagenhofer (2009), we treat the value of the collateral $\widetilde{\ell}$ as stochastic. For simplicity, we assume the collateral for both creditors is of an identical nature and perfectly correlated. Thus, a single value $\widetilde{\ell}$ represents the value of each creditor's collateral. We refer to this value as the liquidation value.

If the creditors do not seize their collateral and instead roll their debts over, the borrower can remain in business. In this case, each creditor owns a claim with expected net present value $\widetilde{c}$, which we refer to as the continuation value. This continuation value is also stochastic, as is common in bank run models based on Bryant (1980) and Diamond and Dybvig (1983) (examples include Chari and Jagannathan 1988, Villamil 1991, Alonso 1996, Hazlett 1997, Kaplan 2006, Böckem and Schiller 2017).[2]

The realized pair (ℓ, c) is the payoff state. We assume that $(\widetilde{\ell}, \widetilde{c})$ has a commonly known prior ψ. For some $a, b \in \overline{\mathbb{R}}$, we let the support of ψ be $[a, b] \times [a, b]$. The payoff state is known to the designer, our borrower, and affects the utilities of the creditors when they play the basic game the G.

Prior to the realization of (ℓ, c), the borrower chooses the set of signals that the creditors can observe, which we call T, and a function t that maps each realized payoff state (ℓ, c) to an outcome in T. This is thought of as the borrower's disclosure policy. If t is a constant function, then the borrower discloses nothing about the payoff state. If $\#(T)$ is at least as large as the set of feasible values of $(\widetilde{\ell}, \widetilde{c})$ and t is injective, then the borrower's policy is full disclosure about the payoff state. If for some positive integer $n \geq 2$, $T = \{T_1, \ldots, T_n\}$

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[2]But not always. Adão and Temzelides (1998) study a setting with a riskless continuation value and no aggregate risk at an interim stage. They obtain non-sunspot coordination failures because a mixed-strategy equilibrium survives a forward induction refinement, whereas a no-run equilibrium does not.

and for all $i \in \{1, \ldots, n\}$, the prior probability of $\{(\ell, c) | t(\ell, c) = T_i\}$ is less than 1, then the borrower's policy is a nondegenerate finite partition of the set of payoff states. We let $S = (t, T)$ denote the borrower's chosen information structure. The creditors learn the realization of t and update their prior on the state before playing the basic game G. We let $\widetilde{t}$ denote the random variable t prior to the realization of $(\widetilde{\ell}, \widetilde{c})$.

Two remarks are in order. First, in a study of illiquidity, it is natural to impose that $\widetilde{\ell} \leq \widetilde{c}$, that is, that the borrower is never insolvent. For most of the analysis below, we make this assumption, restricting the support of ψ to $\{(\ell, c) \in [a, b] \times [a, b] | \ell \leq c\}$.

An alternative approach would allow for the possibility of insolvency ex ante, say by mandating full disclosure whenever $c < \ell$, i.e., requiring an insolvent borrower to declare bankruptcy. Absent a bankruptcy declaration, the creditors know that the borrower is solvent, though rollover risk could remain. This approach introduces extra complexity, because even if $\widetilde{\ell}$ and $\widetilde{c}$ are ex ante independent and identically distributed (iid), at an interim stage they would no longer be so. However, this extra complexity makes it simple for us to describe an increase in fundamental risk in the sense of second-order stochastic dominance, enabling us to address whether an increase in fundamental risk makes transparency more or less desirable. When we turn to this issue, we take this alternative approach.

Second, in most of what follows, we do not require the set of feasible values of $(\widetilde{\ell}, \widetilde{c})$ to be unbounded or to allow for the possibility of unlimited liability. In order to understand the robustness of our results, however, we find it useful to provide conditions on ψ under which fully disclosing either $\widetilde{\ell}$ or $\widetilde{c}$ (but as we shall see, not both) causes no harm. Additionally, in discussing the effects of increases in risk, allowing the set of outcomes to be unbounded enables us to discuss the case where $(\widetilde{\ell}, \widetilde{c})$ are jointly normally distributed. This special case enables us to make some observations related to first-order stochastic dominance. Other than when addressing these issues, we fix attention on the case in which $0 < a < b < \infty$, as would be expected for the problem we study.

To complete the description of the basic game G, we need to specify the actions available to the players and the payoffs each player receives from a given action profile and realized payoff state. Each player's set of possible actions consists of two pure strategies: $A = \{R, W\}$, with strategy R interpreted as rolling the debt over and W as withdrawing and seizing the collateral $\widetilde{\ell}$. In state (ℓ, c), the payoff to player $i \in I$ of strategy profile $(a_i, a_{-i}) \in A^2$ is

$$u_i(a_i, a_{-i}, \ell, c) = \begin{cases} \ell, & \text{if } a_i = W \\ 0, & \text{if } (a_i, a_{-i}) = (R, W) \\ c, & \text{if } (a_i, a_{-i}) = (R, R) \end{cases} \tag{1}$$

We view the payoff to a creditor who rolls his debt over when the other creditor forces liquidation as the amount received in the bankruptcy settlement. In (1), we normalize this amount to 0. One could justify instead setting this value to some amount $\varepsilon \in [0, \ell)$, so that the temptation to seize collateral rather than risk receiving a bankruptcy settlement is not as strong, though still present. For our purposes, this change would not matter. However, a researcher concerned with the rate of convergence to the risk dominant equilibrium might be interested in how strong a basin of attraction it is. Binmore et al. (1995) and Ellison (2000) address this issue, and we refer the interested reader there.

It is also noteworthy that a creditor who forces liquidation receives the same amount, given the state, regardless of whether the other creditor seizes his collateral or rolls his obligation over. This implies that a creditor who seizes his collateral does not receive any additional payment from the bankruptcy settlement; the pledged assets essentially resolve his claims. This is purely a convenience; we need not go quite this far. It is enough to assume that, for some $\delta \in [0, \ell]$, a creditor who seizes his collateral also receives δ from the bankruptcy settlement. That is, we can modify (1) to make $u_i(W, R, \ell, c) = \ell + \delta$. Any value of δ within this region would not fundamentally alter our analysis.[3]

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[3]To see this, fix a realization ℓ of the liquidation value, let $\delta \in [0, \ell]$, and for each $i \in I$, let the payoff $u_i(W, R, \ell, c) = \ell + \delta$. Suppose c is the realized continuation value. As we discuss below, strategy profile (R, R) is risk dominant under full disclosure if and only if c/ℓ exceeds a threshold. It is straightforward to

The borrower's objective is to keep her job. Her payoff depends on the action profile $(a_1, a_2) \in A^2$ that the creditors choose in basic game G as follows:

$$u_0(a_1, a_2) = \begin{cases} 1, & \text{if } a_1 = a_2 = R \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Her payoff is thus a function of the equilibrium the creditors play, and depends only indirectly on the state through its effect on the equilibrium in the basic game G. To try to affect equilibrium behavior, the borrower chooses the information structure $S = \{T, t\}$, as described above.

If the set of feasible payoff states includes realizations with $\ell > c$, then we include in T a trivial disclosure corresponding to a bankruptcy declaration in the event $\widetilde{\ell} > \widetilde{c}$, in which case the creditors would not play basic game G. Otherwise, given (G, S), the creditors observe the realized signal t, update their beliefs about the payoff state $(\widetilde{\ell}, \widetilde{c})$ using t and their common prior ψ, and choose their strategies.

It is commonly known that the creditors choose their strategies based on risk dominance, given their posterior beliefs about the state (using payoff dominance to break ties). That is, there is a decision rule σ that recommends strategies to the creditors, which depends only on the signal:

$$\sigma((R, R)|t) = \begin{cases} 1, & \text{if } (E[(u_i(W, R, \widetilde{\ell}, \widetilde{c})|t] - E[u_i(R, R, \widetilde{\ell}, \widetilde{c})|t])^2 \\ & \geq (E[(u_i(R, W, \widetilde{\ell}, \widetilde{c})|t] - E[u_i(W, W, \widetilde{\ell}, \widetilde{c})|t])^2 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and

$$\sigma((W, W)|t) = 1 - \sigma((R, R)|t)$$

In words, the decision rule is to play equilibrium (R, R) if and only if the product of the creditors' losses of a unilateral deviation from (R, R) weakly exceeds the product of the

show that, as δ increases to ℓ, this threshold increases to 3, and as δ decreases to 0, this threshold falls to 2. By setting $\delta = 0$, we stack the deck against our finding that full disclosure is efficient, as this assumption makes the probability that full disclosure leads to selection of the payoff dominant equilibrium as large as possible.

deviation losses from (W, W). After observing signal t, the creditors update their beliefs about the state and base their decisions on their expected utilities. Substituting (1) and rearranging, we obtain that $\sigma((R, R)|t) = 1$ if and only if

$$E[\widetilde{c}|t] \geq 2E[\widetilde{\ell}|t] \qquad (4)$$

Given that the borrower cannot control the decision rule σ, her information design problem is therefore to choose the information structure S in order to maximize the prior probability ψ that (4) holds.

Note that the signal $\widetilde{t}$ and the decision rule σ correlate the strategies of the creditors, and that each creditor finds it optimal to obey decision rule σ given the realization of t. Bergemann and Morris (2013, 2016a,b) dub this equilibrium concept *Bayes correlated equilibrium*, and the willingness to follow σ an *obedience condition,* analogous to an incentive compatibility condition in mechanism design. In our game, σ perfectly correlates the creditors' actions: they always choose the risk-dominant equilibrium, and they always have the same (first- and higher-order) beliefs about which equilibrium is risk dominant. Thus, while σ does not always choose the Pareto-dominant equilibrium, it never leads to mis-coordination (i.e., mismatched strategies).

3   Results

3.1   Benchmark: Full disclosure

We begin by considering the benchmark of full disclosure, that is, of setting the signal $\widetilde{t} = (\widetilde{\ell}, \widetilde{c})$. For now, we will assume that the problem is one of pure liquidity risk, i.e., that the set of feasible outcomes is $\{(\ell, c) \in [a, b] \times [a, b] | \ell \leq c\}$. We will also restrict attention to the case with $0 \leq a < b < \infty$,

It is immediate that in any nondegenerate setting, full disclosure cannot attain first-best: there is always some inefficient liquidation if the the borrower reveals all her information. Figure 1 illustrates.
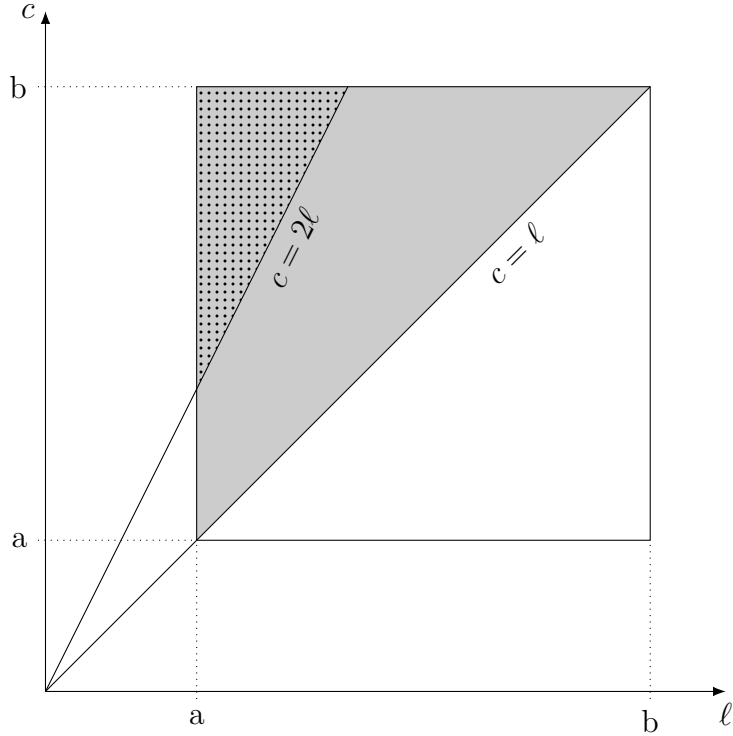

Figure 1: Continuation value c versus liquidation value ℓ. Above the 45° line, the borrower is solvent and continuation is efficient (shown in gray). It is risk dominant to roll the debts over if and only if (ℓ, c) is above the $c = 2\ell$ line (dotted gray area). With full disclosure, inefficient liquidation occurs whenever (ℓ, c) lies in the region between the $c = \ell$ and the $c = 2\ell$ lines (gray quadrilateral).

In the figure, the right triangle with vertices $\{(a, a), (a, b), (b, b)\}$, shown in gray, represents the entire region in which the borrower is solvent and rolling the debt over is Pareto-dominant. From (4), we see $\sigma((R, R)|\ell, c) = 1$ if and only if $c \geq 2\ell$. This is shown in the dotted triangle, bounded by the $c = 2\ell$ line, with vertices $\{(a, 2a), (a, b), (b/2, b)\}$. The gray (undotted) irregular quadrilateral with vertices $\{(a, a), (a, 2a), (b/2, b), (b, b)\}$ is the region in which σ selects the inefficient liquidation equilibrium (W, W). Unless ψ assigns zero

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probability to this region, a full disclosure information must have efficiency loss.

It also immediate that there are information structures that achieve coordination on (R, R) with higher probability than full disclosure. To the extent that the borrower can avoid an unraveling problem, she can generically do better than choosing full disclosure.

Refer again to Figure 1. The centroid of the full disclosure region has coordinates

$$(E[\widetilde{\ell}|\widetilde{c} \geq 2\widetilde{\ell}], E[\widetilde{c}|\widetilde{c} \geq 2\widetilde{\ell}])$$

Except in the degenerate case in which ψ assigns zero probability to every point strictly above the $c = 2\ell$ line, this centroid is in the interior of the full disclosure region. The borrower can then consider an information structure that pools the entire full disclosure region with a sufficiently small region adjacent to and just below the $c = 2\ell$ line. Call the full disclosure region Φ and the additional small region adjacent to it Δ, choosing Δ so that the centroid of $\Phi \cup \Delta$ is still on or above the $c = 2\ell$ line and so that ψ assigns nonzero probability to Δ.

Define a new random variable $\widetilde{\tau}$ by

$$\tau(\ell, c) = \begin{cases} 1, & \text{if } (\ell, c) \in \Phi \cup \Delta \\ 0, & \text{otherwise} \end{cases}$$

Suppose the borrower chooses signal $\widetilde{\tau}$ instead of $\widetilde{t}$, and announces at the start of play that she will use this information structure. After observing the realized signal τ, the creditors choose their strategies according to risk dominance. Then (3) becomes

$\sigma((R, R)|\tau = 1) = 1$ if $E[\widetilde{c}|\tau = 1] > 2E[\widetilde{\ell}|\tau = 1]$, which holds by construction.

Similarly, because outside of $\Phi \cup \Delta$ it is always the case that $c < 2\ell$, we have $\sigma(W, W|\tau = 0) = 1$. Therefore, decision rule σ selects equilibrium (R, R) with the probability ψ assigns to $\Phi \cup \Delta$, which is strictly greater than the probability assigned to Φ, i.e., of selecting the payoff-dominant equilibrium under full disclosure.

3.2 Optimal information structure for minimizing rollover risk

The analysis above shows that full disclosure cannot be the borrower's optimal decision choice unless the borrower's problem unravels. We now turn to the problem of finding a disclosure that maximizes the probability that both creditors roll their debts over, and then verifying that this information structure is one that does not unravel.

To limit the scope of the borrower's problem, we first observe that σ recommends decisions in pure strategies for each player, that these strategies are perfectly correlated, and that the number of pure strategies available to each player is 2. Therefore, it suffices to restrict attention to information structures with $T = 0, 1$, so that $\tilde{t}$ partitions the set of payoff states into two subsets. Further, we will treat partitions as equivalent if they are equal on all but a set of measure zero, as the creditors maximize their expected payoffs given their information.

The following example illustrates the idea. As we discuss above, an information structure that provides full disclosure is one in which the realized signal t is a sufficient statistic for (ℓ, c). It is possible to replicate what the borrower achieves under full disclosure, in every payoff state, with the following information structure:

$$t(\ell, c) = \begin{cases} 1, & \text{if } c \geq 2\ell \\ 0, & \text{if } c < 2\ell \end{cases} \tag{5}$$

Substituting the realized signal t defined in (5) into (4), we immediately obtain that $\sigma((R, R)|t) = 1$ if and only if $c \geq 2\ell$, exactly as in the full disclosure case.

We now show that the optimal information structure for the borrower in general does not identify the liquidation value $\tilde{\ell}$ or the continuation value $\tilde{c}$.

Theorem 1 *The optimal information structure from the borrower's viewpoint, that is, the information structure that minimizes rollover risk, is either nondisclosure or a threshold disclosure. In the latter case, the threshold depends on the relationship between c and ℓ but not the individual realizations of c or ℓ.*

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Figure 2 illustrates the idea of Theorem 1 in the case in which ψ is a uniform distribution over the triangle M with vertices $\{(a,a),(a,b),(b,b)\}$ (the gray region in the figure). By pooling states below the $c = 2\ell$ line with those above it, the borrower increases the probability of continuation. She can increase this pooled region R until its centroid meets the $c = 2\ell$ line. Beyond that point, pooling R with any additional subset of M with positive probability measure moves the centroid of R below the $c = 2\ell$ line, causing inefficient liquidation.
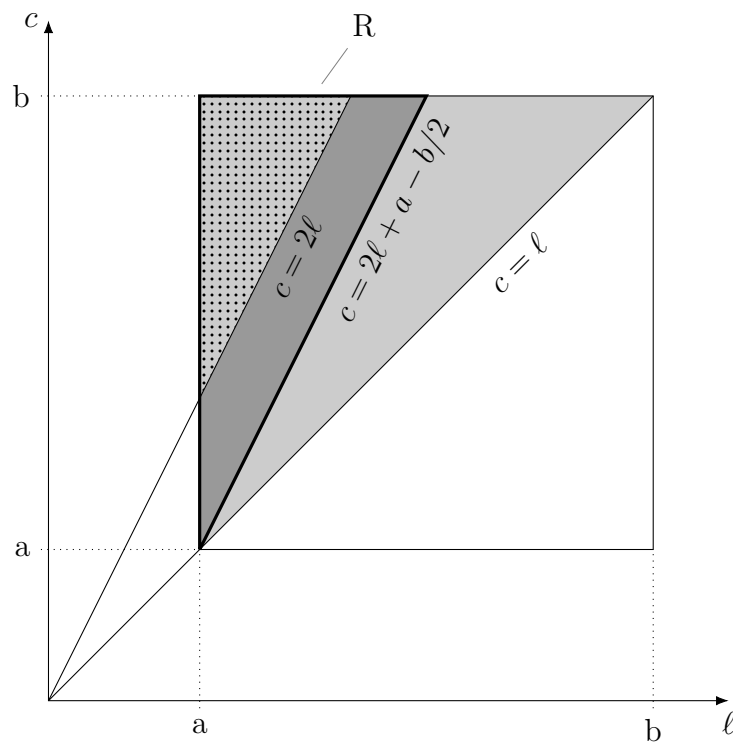

Figure 2: Special case in which $(\tilde{\ell}, \tilde{c})$ is uniformly distributed over the upper gray triangle. In the trapezoid between the $c = 2\ell$ and the $c = 2\ell + a - b/2$ lines (dark gray), full disclosure leads to inefficient liquidation. However, restricting disclosure to whether $c \geq 2\ell + a - b/2$ prevents inefficient liquidation in the dark gray and dotted regions.

The boundary of region R in Figure 2, as shown in the proof of Theorem 1, is the line $c = 2\ell + a - b/2$. Observe that, if ψ is uniform over the triangle M, then this shifted boundary is parallel to the $c = 2\ell$ line that bounds the full disclosure region. A strategy of fully disclosing ℓ and not c, by contrast, would appear in this picture as a vertical line. Analogously, a

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disclosure of c and not ℓ would appear as a horizontal line. For any distribution that is absolutely continuous with respect to Lebesgue measure, disclosure of ℓ or of c and not the other value would therefore be a report of a set that is thin, i.e., of measure zero. Thus, disclosure of a single component of (ℓ, c) provides little value to the borrower. Either nondisclosure is also optimal, so that distinguishing a set of measure zero is inconsequential, or nondisclosure is suboptimal, in which case distinguishing a set of measure zero does not help improve the probability of inefficient continuation.

3.3 Optimal information structure, creditors' viewpoint

We now consider the information structure that the creditors would prefer, if they could impose having someone acting on their behalf as the information designer.

Although the creditors would also like to prevent inefficient liquidation, their objective is different. They care about how much they recover, and this gives them a preference for inefficient liquidation in some states over others. Letting $Pr(\cdot)$ be the probability measure associated with ψ, we can write the creditors' ideal information design as the solution of the following problem

$$\max_{\hat{R} \subseteq M} E[\widetilde{c}|\hat{R}]Pr(\hat{R}) + E[\widetilde{\ell}|M\backslash\hat{R}](1 - Pr(\hat{R})) \tag{6}$$

subject to the obedience condition

$$E[\widetilde{c}|\hat{R}] \geq 2E[\widetilde{\ell}|\hat{R}]$$

i.e., the centroid of $\hat{R}$ must lie on or above the $c = 2\ell$ line. If $M\backslash\hat{R}$ has positive probability, then this constraint binds (the creditors do not benefit from waste).

We now show that this difference in objective means that the creditors would prefer a different disclosure region, unless rollover risk is completely avoidable (in which case there is always nondisclosure) or completely unavoidable (in which case disclosure is irrelevant). We illustrate this in the case in which ψ is uniform over the efficient continuation region M. To

rule out the cases in which all parties prefer nondisclosure and those in which disclosure is irrelevant, we assume $0 < 2a < b$. Purely as a technical convenience, we also assume $b \leq 4a$ (although the argument goes through provided $b < \infty$. We then have the following:

Theorem 2 *Under the assumption that ψ is uniform over M and $0 < 2a < b < 4a$, the optimal information structure from the creditors' viewpoint differs from the optimal information structure from the borrower's viewpoint.*

Figure 3 illustrates the conflict between the borrowers and creditors. The optimal region $\hat{R}$ from the creditors' viewpoint is the right triangle with vertices $\{(a, \hat{y}), (a, b), (\hat{x}, b)\}$. We see from the figure that, compared with the borrower, the creditors are willing to tolerate more inefficient liquidation when both ℓ and c are low, if in exchange they can avoid some additional inefficient liquidation when c is high and ℓ is not too high.

Lastly, we note the following:

Proposition 1 *Suppose that either $t = 1$ if and only if the payoff state (ℓ, c) is in the borrower's optimal disclosure region, or $t = 1$ if and only if the payoff state (ℓ, c) is in the creditors' optimal disclosure region. Then there is no unraveling: after the creditors observe any realization of t, the borrower has no incentive to reveal any additional information.*

To summarize, we find that full disclosure is never optimal. Nondisclosure, on the other hand, may be optimal, and if it is, it is optimal from both the borrower's and the creditors' viewpoint. If nondisclosure is not optimal, then the best information structure from the borrower's viewpoint, that which minimizes rollover risk, generally is suboptimal from the creditors' viewpoint. This is because creditors are concerned with the expected costs of rollover risk, whereas borrowers are concerned with the overall amount of rollover risk, i.e., the probability of coordination failure.
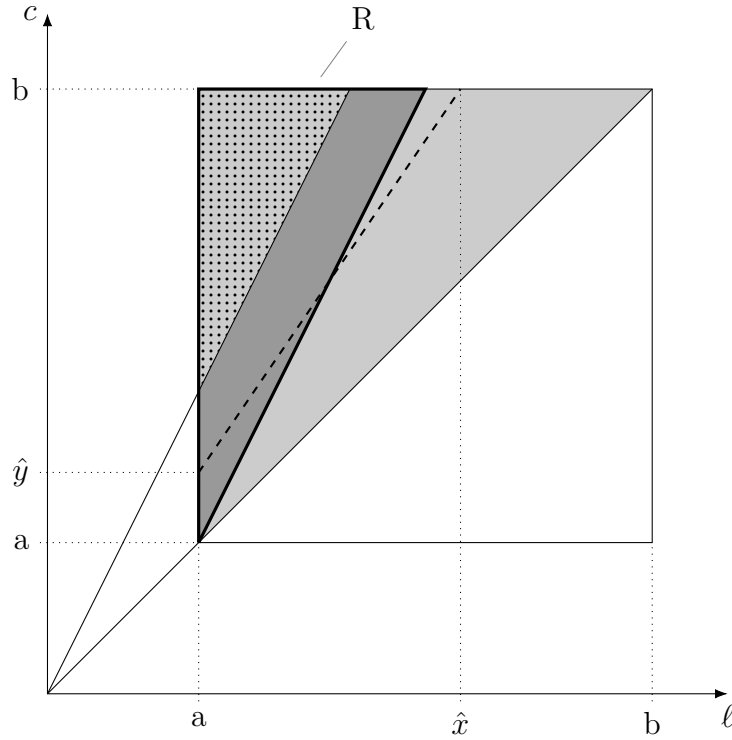
Figure 3: The creditors prefers to have the borrower disclose whether (ℓ, c) is inside the region bounded by the triangle with vertices $\{(a, \hat{y}), (a, b), (\hat{x}, b)\}$, with the dashed line as the boundary. This region is smaller than the borrower's preferred disclosure of whether $(\ell, c) \in R$. This greater probability of waste is offset for the creditors by higher expected payoffs in liquidation and in continuation.

4 Effects of risk on (non)disclosure

The objectives of borrowers and creditors are aligned whenever creditors would roll their debts over if the borrower discloses nothing. In this section, we investigate the effects of the fundamental risk associated with the payoff state's distribution ψ on whether nondisclosure is optimal. We focus on nondisclosure in this setting because it achieves first-best, avoiding any concerns over whether optimality is taken from the borrower's or creditors' viewpoint.

The notion of fundamental risk is nuanced with multiple dimensions, as is well-known (see,

e.g. Kihlstrom and Mirman 1974, 1981). To avoid the subtleties that arise in discussing the risk associated with a joint distribution ψ, we make a small but crucial modification to the economic environment. Specifically, we now assume that ex ante, both $\widetilde{\ell}$ and $\widetilde{c}$ are independently and identically distributed according to some unidimensional cumulative distribution function F with support $[a, b]$. This allows for the possibility, again ex ante, that the borrower could become insolvent.

Our focus remains entirely on inefficient liquidation, which can occur only if the realization of $\widetilde{c}$ is at least as large as that of $\widetilde{\ell}$. Otherwise, liquidation is efficient, the firm declares bankruptcy, and the game ends.

We can now associate risk with the ex ante distribution F. Given that the basic game G is reached, the creditors update their beliefs about the distributions of $\widetilde{\ell}$ and of $\widetilde{c}$, even before the borrower reveals any information. We write the cumulative distribution functions (cdfs) and probability density functions (pdfs) of $\widetilde{\ell}$ and $\widetilde{c}$ respectively as

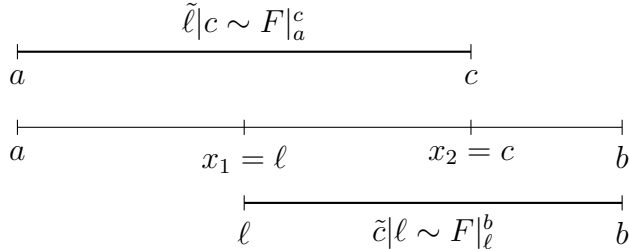$$F_L(x) = 1 - [1 - F(x)]^2 \qquad\qquad f_L(x) = 2[1 - F(x)]f(x) \qquad\qquad (7)$$

$$F_C(x) = F^2(x) \qquad\qquad f_C(x) = 2F(x)f(x) \qquad\qquad (8)$$

As noted in Section 2, this additional assumption still means that, absent a bankruptcy declaration, the creditors know that the value of $\widetilde{c}$ must be at least as high as that of $\widetilde{\ell}$. Nevertheless, this added assumption is not innocuous, because now $\widetilde{\ell}$ is the lower of two draws (the first sample order statistic), and $\widetilde{c}$ is the higher (the second sample order statistic). The creditors cannot treat the two as independent, and the borrower knows this when designing the information structure.

In what follows, we refer to the ex ante distribution F as the *implicit risk distribution*. Figure 4 illustrates the information that c provides about $\widetilde{\ell}$ and that ℓ provides about $\widetilde{c}$.

The fact that the liquidation value and continuation value are, in the current setting, informative about each other requires us to revisit the implications of disclosing either value while

Figure 4: The distributions of the continuation and liquidation values.

$$\tilde{\ell}|c \sim F|_a^c$$

$$a \qquad\qquad\qquad\qquad c$$

$$a \qquad\qquad x_1 = \ell \qquad\qquad x_2 = c \qquad b$$

$$\ell \qquad\qquad \tilde{c}|\ell \sim F|_\ell^b \qquad b$$

leaving the other undisclosed. By disclosing c, the borrower implicitly is now also telling the creditors that $\tilde{\ell} \leq c$, and this additional information content could change our assessment about whether it is ever optimal for the borrower to make a single-coordinate full disclosure. We show below that this concern turns out not to matter, i.e., that it remains the case that it is optimal to disclose either c or ℓ (but not both) only if nondisclosure is also optimal, and that the converse is false in each case. Thus, our earlier assessment that disclosing c or ℓ is either superfluous or harmful is robust to this modification of our assumptions.

We begin by showing that nondisclosure can be optimal. Theorem 3 characterizes the optimality of nondisclosure in terms of the underlying distribution. The result comes from the fact that, given the absence of a bankruptcy declaration, the borrower's continuation value is the higher of two sample order statistics.

Theorem 3 *Let μ be the mean of implicit risk distribution $F(\cdot)$. Then it is optimal for the borrower to disclose nothing if and only if*

$$E[\tilde{c}] \geq \frac{4}{3}\mu \tag{9}$$

Theorem 3 suggests that strategic risk is vanishes once fundamental risk is sufficiently high. We now show that this is the case: greater risk, in the sense of second-order stochastic dominance, is associated with making nondisclosure optimal. Lemma 1:

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Lemma 1 *Suppose $\widetilde{x}_1, \widetilde{x}_2$ are iid, $\widetilde{y}_1, \widetilde{y}_2$ are iid with mean zero and with a nondegenerate distribution, and the $\widetilde{x}_i$ and $\widetilde{y}_i$ are mutually independent. Then $E[\max\{\widetilde{x}_1 + \widetilde{y}_1, \widetilde{x}_2 + \widetilde{y}_2\}] > E[\max\{\widetilde{x}_1, \widetilde{x}_2\}]$.*

Corollary 1 *Suppose F is an implicit risk distribution under which nondisclosure achieves first-best, i.e. for which Inequality (9) holds. Let H be a mean-preserving spread of F. Then nondisclosure achieves first-best under H. The converse does not necessarily hold.*

The intuition of Corollary 1 is that risk is an argument for nondisclosure, not for increased disclosure. A similar result holds with increasing Value-at-Risk. We restrict attention here to normally distributed payoffs, as our purpose here is only to provide an illustration. In this special case, the result follows from a technical proposition

Proposition 2 *Suppose an underlying risk distribution F is a normal distribution with mean μ and variance σ^2. Then distribution $\hat{F}$ represents a higher Value-at-Risk (i.e., the negative portion of the distribution is more negative at every quantile under $\hat{F}$ than under F) if and only if the variance under $\hat{F}$ is also σ^2 and the mean under $\hat{F}$ is below μ. In this case, if nondisclosure is optimal in the sense of Theorem 3, then nondisclosure is also optimal under $\hat{F}$.*

Lastly, we show that selective full disclosure of either the liquidation value ℓ or the continuation value c is either superfluous or harmful. It is superfluous in some, but not all, cases in which nondisclosure is optimal, and harmful otherwise.

The conditions under which disclosing ℓ alone are harmless turn out to be quite strong: the cdf F must have infinite variance. A comparison with Theorem 3 helps clarify the intuition: nondisclosure is optimal if risk is high, but does not require anything nearly as strong as infinite variance.

Lemma 2 *Suppose that, for all values of $\ell \in [a, b), E[\widetilde{c}|\ell] = 2\ell$. Then F is a Pareto distribution with scale parameter $a \in \mathbb{R}_{++}$ and shape parameter $\alpha = 2$:*

$$(\forall x \in [a, \infty)) \; F(x) = 1 - \left(\frac{a}{x}\right)^2 \tag{10}$$

If $E[\widetilde{c}|\ell] \geq 2\ell$ for all ℓ with strict inequality for some ℓ, then F is more heavily tailed than the Pareto distribution with scale parameter 2. Together, these imply that either there are some values of ℓ for which $E[\widetilde{c}|\ell] < 2\ell$, or F is an infinite variance distribution.

From Lemma 2, we can now prove that disclosure of ℓ never improves on nondisclosure.

Proposition 3 *Suppose $E[\widetilde{c}|\ell] \geq 2\ell$ for all $\ell \in [a, b)$. Then nondisclosure of both c and l prevents rollover risk. That is, if disclosure of ℓ achieves first-best, then so does nondisclosure.*

The conditions under which disclosure of the continuation value alone is superfluous rather than harmful are not quite as strong. Nevertheless, we show that they are sufficient but not necessary for nondisclosure to be optimal. The important conditions for disclosing c alone to be harmless are that F is weakly concave and that $a \leq 0$. The intuition for this last requirement is as follows: suppose the creditors' claims are backed by collateral with a value that is bounded away from zero. Then disclosure of a sufficiently low realization of c is a sufficient statistic for (W, W) being risk dominant. In other words, if the collateral has value bounded away from 0, then this bound prevents the borrower from pooling information when the overall economic conditions are weak.

Lemma 3 *Suppose that, for all values of $c \in (a, b], E[\widetilde{\ell}|c] = c/2$. Then F is a uniform distribution with lower bound at 0. If $E[\widetilde{\ell}|c] \leq c/2$ for all c with strict inequality for some c, then F is weakly concave and bounded, with a nonpositive lower bound.*

Comparing Lemma 3 and Theorem 3, we see that the optimality of disclosing c depends on the conditional expectation of $\widetilde{\ell}$, and the optimality of nondisclosure depends on the

unconditional properties of F. The following result says that any bounded distribution satisfying the conditions of Lemma 3 necessarily satisfies Equation (9), but that there are distributions F for which (9) holds but Lemma 3 does not.

Proposition 4 *Assume $a > -\infty$, so that the worst possible loss a is finite. If disclosure of c makes it risk dominant to roll the debt over, then so does nondisclosure. The converse is false.*

5   Conclusion

Our main findings show that borrower disclosures concerning rollover risk optimally do not enable creditors to separate their potential gains from keeping the borrower in business from those of forcing the borrower into liquidation and recovering the value of their collateral (Theorem 1). These optimal disclosures do not require any external form of commitment, as they do not unravel into full disclosure (Proposition 1). If creditors can impose disclosure requirements, they may do so in a way that increases the likelihood of inefficient liquidation in order to assure that, when public information leads to inefficient liquidation, the cost is not too high (Theorem 2). However, this conflict of interest occurs only if the credit risk of the borrower is low. Once the fundamental risk becomes sufficiently high, all parties prefer that borrowers disclose nothing beyond the fact that they still have going concern value (Theorem 3).

These results run counter to our intuition about the conflict between creditors and equity holders (Jensen and Meckling 1976, Myers 1977). In a rollover risk context, it is the owners of the firm who care about default probability, and the creditors who care about expected net present values.

What may be more surprising is that creditors want more fundamental risk when they face strategic risk, and if their borrowers say nothing about how these fundamental risks are

playing out, then creditors are happy about it. To some extent, this is driven by a force similar to option value: higher fundamental risk increases the probability that there is a large gap between the potential gains from rolling debts over and those from seizing collateral and cutting one's losses.

The other important force is that coordination is, in principle, similar to collusion. Controlling rollover risk, from an information design viewpoint, is a problem of constructing information sets that foster collusion, as in an information sharing problem among oligopolists. This is inherently different from the informational issues we consider in competitive debt markets, in which the social optimum typically involves preventing collusion.

A Proofs

Proof of Theorem 1.   If the prior ψ satisfies $E[\widetilde{c}] \geq 2E[\widetilde{\ell}]$, then the borrower has nothing to gain from disclosure. In this case, (4) is replaced with prior expectations, as the signal t is uninformative about the payoff states. Rolling the debts over is risk dominant in this case, and achieves first-best.

It remains to show that the optimal partition does not in general reveal the values of c or ℓ. We show this for the case in which ψ is a uniform distribution over the triangle with vertices $\{(a,a),(a,b),(b,b)\}$, as extensions to other cases with atomlesses distributions that are absolutely continuous with respect to Lebesgue measure are straightforward. Let M be this triangle, i.e. $M = \ \text{supp} \ \psi$. We assume $0 < 2a < b \leq 4a$. This is unimportant and is only for technical convenience, except in that it guarantees that some inefficient liquidation is avoidable.

The borrower's objective is choose the largest possible subset R of M with a centroid on or above the $c = 2\ell$ line. Letting Pr be the probability measure associated with ψ, the

borrower's objective is

$$\max_{R \subseteq M} Pr(R) \text{ s. t. } \left(E[\tilde{c}|R] \geq 2E[\tilde{\ell}|R] \right) \tag{11}$$

The centroid of M, denoted $m := (m_\ell, m_c)$, is defined as follows:

$$m_\ell = \frac{\int_a^b (b-x)x\,dx}{\int_a^b (b-x)\,dx} = \frac{2a+b}{3} \tag{12}$$

$$m_c = \frac{\int_a^b \frac{(b^2-x^2)}{2}\,dx}{\int_a^b (b-x)\,dx} = \frac{a+2b}{3} \tag{13}$$

The ratio of m_c/m_ℓ is $(2b+a)/(2a+b)$, which is less than 2 because $a > 0$. It is therefore clear that the borrower would not choose nondisclosure in this case, as it leads to inefficient liquidation.

Because ψ is uniform, a straightforward argument (available from the authors on request) shows that the optimal partition boundary is linear. For some $x^*, y^* \in [a, b]$, consider the line segment that passes through (a, y^*) and (x^*, b), and let R be the triangle with vertices $\{(a, y^*), (a, b), (x^*, b)\}$. Our goal in this first part of the proof is to find the optimal values of (x^*, y^*) from the borrower's viewpoint. Analogous to (12–13), the centroid of R, denoted $r = (r_\ell, r_c)$, is

$$r_\ell = \frac{2a + x^*}{3} \tag{14}$$

$$r_c = \frac{y^* + 2b}{3} \tag{15}$$

Because the centroid of M is below the $c = 2\ell$ line, the constraint in (11) binds, i.e., $r_c = 2r_\ell$. Consequently,

$$y^* + 2b = 4a + 2x^*$$

$$\Rightarrow y^* = 4a - 2b + 2x^* \tag{16}$$

Under the joint uniformity assumption, the probability of R given that $\ell < c$ is the ratio of the area of R to the area of M:

$$p = \frac{(b - y^*)(x^* - a)}{(b - a)^2} \tag{17}$$

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Substituting (16) into (17),

$$p = \frac{(b - 4a + 2b - 2x^*)(x^* - a)}{(b - a)^2}$$
$$= \frac{(-2x^* + 3b - 4a)(x^* - a)}{(b - a)^2}$$
$$= \frac{-2(x^*)^2 + (3b - 2a)x^* - 3ab + 4a^2}{(b - a)^2} \tag{18}$$

The first-order condition is

$$\frac{-4x^* + 3b - 2a}{(b - a)^2} = 0 \tag{19}$$

The second derivative is -4, so the solution to (19) gives a unique global maximum. Solving for x^*,

$$x^* = \frac{3b - 2a}{4} \tag{20}$$

and therefore

$$y^* = 3a - \frac{b}{2} \tag{21}$$

The assumption that $b \leq 4a$ guarantees that y^* is not below the lower vertex $(a, 2a)$ of the full disclosure region. Other than affecting the shape of R (making it a right triangle rather than the union of a right triangle and a trapezoid), this assumption has no effect, and the optimal boundary is still parallel to the $c = 2\ell$ line. The points $(a, 3a - b/2)$ and $((3b - 2a)/4, b)$ determine the hypotenuse of the region R as the line $c = 2\ell + a - b/2$, as shown in Figure 2.

Observe that this boundary is neither a horizontal line (i.e., determined by a cutoff value of c nor a vertical line (i.e., determined by a cutoff value of ℓ). ∎

Proof of Theorem 2. From the creditors' viewpoint, the objective is to choose $\hat{R}$ to maximize the expected payoff. As shown in (6), the creditors receive the continuation value if $(\ell, c) \in \hat{R}$ and the liquidation value otherwise. Define a line that passes through $(a, \hat{y})$ and $(\hat{x}, b)$ for some values $\hat{x}$ and $\hat{y}$. Let $\hat{R}$ be the triangle with vertices $\{a, \hat{y}), (a, b), (\hat{x}, b)\}$. Our goal in this second part of the proof is to find the optimal values of $(\hat{x}, \hat{y})$ from the creditors'

viewpoint. The centroid of $\hat{R}$, denoted $\hat{r} = (\hat{r}_\ell, \hat{r}_c)$ is

$$\hat{r}_\ell = \frac{2a + \hat{x}}{3} \tag{22}$$

$$\hat{r}_c = \frac{\hat{y} + 2b}{3} \tag{23}$$

The expectation of $\tilde{c}$ given $\hat{R}$ is $\hat{r}_c = (2b + \hat{y})/3$. Because $\hat{r}$ lies on the $c = 2\ell$ line, we can substitute for $\hat{y}$ and obtain $\hat{r}_c = 2\hat{r}_\ell = (4a + 2\hat{x})/3$. Let $\hat{p}$ be the probability of $\hat{R}$. The centroid $\hat{s}$ of $\hat{S} := M \backslash \hat{R}$ is

$$\hat{s} = \left(\frac{1}{1 - \hat{p}} \right) (m - \hat{p}\hat{r}) \tag{24}$$

so that the expectation of $\tilde{\ell}$ given $\hat{S}$ is

$$\hat{s}_\ell = \left(\frac{1}{1 - \hat{p}} \right) \left(\frac{2a + b}{3} - \hat{p}\frac{2a + \hat{x}}{3} \right) \tag{25}$$

The creditors' objective is therefore to maximize

$$\begin{aligned} E[\pi] &= \hat{p}\frac{4a + 2\hat{x}}{3} + (1 - \hat{p}) \left[\left(\frac{1}{1 - \hat{p}} \right) \left(\frac{2a + b}{3} - \hat{p}\frac{2a + \hat{x}}{3} \right) \right] \\ &= \frac{\hat{p}(2a + \hat{x}) + 2a + b}{3} \\ &= \frac{(-2\hat{x} + 3b - 4a)(\hat{x} - a)(2a + \hat{x})}{3(b - a)^2} + \frac{2a + b}{3} \end{aligned} \tag{26}$$

The first-order condition is
$$\frac{-2\hat{x}^2 + 2(b - 2a)\hat{x} + ab}{(b - a)^2} = 0 \tag{27}$$

The negative root is below zero and extraneous. The positive root is

$$\hat{x} = \frac{b - 2a + \sqrt{4a^2 - 2ab + b^2}}{2} \tag{28}$$

The second order sufficient condition is

$$\frac{-4\hat{x} + 2(b - 2a)}{(b - a)^2} < 0$$

$$\hat{x} > \frac{(b - 2a)}{2} \tag{29}$$

which holds for the positive root but not for the negative root.

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Comparing the creditors' preferred value $\hat{x}$ with the borrower's preferred value x^* given in (20), we can find conditions under which they coincide:

$$\frac{b - 2a + \sqrt{4a^2 - 2ab + b^2}}{2} = \frac{3b - 2a}{4}$$

$$\Rightarrow 2b - 4a + 2\sqrt{4a^2 - 2ab + b^2} = 3b - 2a$$

$$\Rightarrow 2\sqrt{4a^2 - 2ab + b^2} = b + 2a$$

$$\Rightarrow 4a^2 - 2ab + b^2 = \frac{(b + 2a)^2}{4} = \frac{b^2}{4} + ab + a^2$$

$$\Rightarrow 3a^2 - 3ab + \frac{3}{4}b^2 = 0$$

$$\Rightarrow \left(a - \frac{b}{2}\right)^2 = 0$$

$$\Rightarrow b = 2a \tag{30}$$

By hypothesis, $b > 2a$, contradicting Equation (30), which can hold only if the probability of efficient continuation is zero. ∎

Proof of Proposition 1. First observe that both the borrower's optimal signal structure and the creditor's optimal signal structure are supersets of the full disclosure region. With either of these structures, the borrower has nothing to gain from revealing additional information after the creditors observe the public signal t. If t indicates that (ℓ, c) belongs to the region for which $\sigma(\cdot)$ recommends both creditors roll their debts over, then additional disclosures are superfluous if $c \geq 2\ell$ and destructive if $c < 2\ell$. If t indicates that (ℓ, c) belongs to a region for which $\sigma(\cdot)$ recommends both creditors force liquidation, then necessarily $c < 2\ell$, and there is no supplemental information that could alter this decision. ∎

Proof of Theorem 3. Recall that nondisclosure is optimal if and only if $E[\tilde{c}] \geq 2E[\tilde{\ell}]$. The pdfs of $\tilde{c}$ and $\tilde{\ell}$ are

$$f_C(x) = 2F(x)f(x)$$

$$f_L(x) = 2[1 - F(x)]f(x) \tag{31}$$

This means

$$E[\widetilde{c}] \geq 2E[\widetilde{\ell}] \text{ iff}$$

$$\int_a^b 2xF(x)f(x)dx \geq 2 \cdot \int_a^b 2x[1 - F(x)]f(x)dx$$

$$\Leftrightarrow 2\int_a^b 2xF(x)f(x)dx \geq 4\int_a^b xf(x)dx - 4\int_a^b xF(x)f(x)dx$$

$$\Leftrightarrow 6\int_a^b xF(x)f(x)dx \geq 4\mu_x$$

$$\Leftrightarrow E[\widetilde{c}] = 2\int_a^b xF(x)f(x)dx \geq \frac{4}{3}\mu_x \tag{32}$$

∎

Proof of Lemma 1. Let

$$(\widetilde{x}^{\#}, \widetilde{y}^{\#}, \widetilde{x}^{*}, \widetilde{y}^{*}) = \begin{cases} (\widetilde{x}_1, \widetilde{y}_1, \widetilde{x}_2, \widetilde{y}_2), & \widetilde{x}_1 \geq \widetilde{x}_2 \\ (\widetilde{x}_2, \widetilde{y}_2, \widetilde{x}_1, \widetilde{y}_1), & \widetilde{x}_2 > \widetilde{x}_1 \end{cases}$$

Then $\widetilde{x}^{\#} = \max\{\widetilde{x}_1, \widetilde{x}_2\}$.

Then

$$E[\widetilde{y}^{\#}] = E[\widetilde{y}_1 | \widetilde{x}_1 \geq \widetilde{x}_2]P(\widetilde{x}_1 \geq \widetilde{x}_2) + E[\widetilde{y}_2 | \widetilde{x}_1 < \widetilde{x}_2]P(\widetilde{x}_1 < \widetilde{x}_2)$$

Because the $\widetilde{y}_i$ are iid and independent of the $\widetilde{x}_i$,

$$E[\widetilde{y}^{\#}] = E[\widetilde{y}_1 | \widetilde{x}_1 \geq \widetilde{x}_2]P(\widetilde{x}_1 \geq \widetilde{x}_2) + E[\widetilde{y}_1 | \widetilde{x}_1 < \widetilde{x}_2]P(\widetilde{x}_1 < \widetilde{x}_2)$$

$$= E[\widetilde{y}_1] = 0$$

$$\therefore, E[\max\{\widetilde{x}_1 + \widetilde{y}_1, \widetilde{x}_2 + \widetilde{y}_2\}] = E[\max\{\widetilde{x}^{\#} + \widetilde{y}^{\#}, \widetilde{x}^{*} + \widetilde{y}^{*}\}]$$

$$= E[\max\{\widetilde{x}^{\#} + \widetilde{y}^{\#}, \widetilde{x}^{*} + \widetilde{y}^{*} + \widetilde{y}^{\#} - \widetilde{y}^{\#}\}]$$

$$= E[\max\{\widetilde{x}^{\#}, \widetilde{x}^{*} + \widetilde{y}^{*} - \widetilde{y}^{\#}\}] + E[\widetilde{y}^{\#}]$$

$$= E[\max\{\widetilde{x}^{\#}, \widetilde{x}^{*} + \widetilde{y}^{*} - \widetilde{y}^{\#}\}]$$

$$\geq E[\widetilde{x}^{\#}] = E[\max\{\widetilde{x}_1, \widetilde{x}_2\}] \tag{33}$$

Furthermore, (33) holds strictly provided

$$P(\widetilde{x}^* + \widetilde{y}^* - \widetilde{y}^{\#} > \widetilde{x}^{\#}) > 0$$

Now

$$\widetilde{x}^* + \widetilde{y}^* - \widetilde{y}^{\#} > \widetilde{x}^{\#} \Leftrightarrow$$

$$\widetilde{y}^* - \widetilde{y}^{\#} > \widetilde{x}^{\#} - \widetilde{x}^* \geq 0 \tag{34}$$

Suppose first that the distribution of X has some atom x in its support, i.e., so that

$$P(X = x) = p > 0$$

Then

$$P(\widetilde{x}^{\#} = x, \widetilde{x}^* = x) = p^2 > 0$$

Since the distribution of the $\widetilde{y}_i$ is nondegenerate, there is positive probability that $\widetilde{y}^* > 0$ and $\widetilde{y}^{\#} < 0$. In this case, (34) holds with positive probability, so that the inequality (33) is strict.

Next, suppose that the distribution of the $\widetilde{x}_i$ is purely continuous. Then the proof of the strictness of inequality (33) has two steps:

1. For every $\varepsilon > 0$, there is positive probability that $\varepsilon > \widetilde{x}^{\#} - \widetilde{x}^* \geq 0$.

2. $\exists \varepsilon > 0$ such that there is positive probability that $\widetilde{y}^{\#} - \widetilde{y}^* > \varepsilon$.

To prove the first step, let $\varepsilon > 0$ be given. Let $[m, M]$ be an interval with $m < M$ with positive X-probability. For

$$k \in \left\{ 0, \ldots, \left\lceil \frac{2(M - m)}{\varepsilon} \right\rceil \right\},$$

let

$$d_k = m + \frac{\varepsilon k}{2}$$

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The intervals (d_k, d_{k+1}) are disjoint, and at least one has positive probability. Then there is positive probability that $\widetilde{x}^{\#}, \widetilde{x}^{*}$ are in the same interval (d_k, d_{k+1}), and hence that

$$\varepsilon > \widetilde{x}^{\#} - \widetilde{x}^{*} > 0$$

To show the second step, suppose to the contrary that $\forall \varepsilon > 0$, $P(\widetilde{y}^{*} - \widetilde{y}^{\#} > \varepsilon) = 0$. Then $P(\widetilde{y}^{*} - \widetilde{y}^{\#} = 0) = 1$, violating nondegeneracy.

Therefore, if X is purely continuous, then with positive probability, (34) holds. Consequently, (33) holds with strict inequality. ■

Proof of Corollary 1. Immediate corollary of Lemma 1. ■

Proof of Proposition 2. We first observe that, if the Value-at-Risk is higher under $\hat{F}$ at every quantile if and only if F first-order stochastically dominates $\hat{F}$. That is, given normality, a shift of the left tail of the distribution implies a shift of the entire distribution.

Next, we remark that, for a normally distributed random variable with mean μ and variance σ^2, the expectation of the higher of two independent draws is $\mu + \sigma/\sqrt{\pi}$, and the expectation of the lower of two independent draws is $\mu - \sigma/\sqrt{\pi}$. This is a known result from the theory of order statistics. It follows from (4) that rolling over is a risk-dominant equilibrium if and only if $\mu \leq 3\sigma/\sqrt{\pi}$.

It is therefore enough to show that if a normal distribution becomes worse in the sense of first-order stochastic dominance, the mean decreases and the variance remains unchanged. In order to demonstrate this, assume that F and $\hat{F}$ have means μ and $\hat{\mu}$, with $\hat{\mu} < \mu$. At $\alpha = 1/2$, we know that the cdf F is to the right of the cdf $\hat{F}$.

It remains to show that if F and $\hat{F}$ do not cross, then the variance under F, denoted σ^2, must equal the variance under $\hat{F}$, denoted $\hat{\sigma}^2$. If, to the contrary, the two cdfs cross at some

quantile α^*, then it must be the case that

$$\mu + \sigma\Phi^{-1}(\alpha^*) = \hat{\mu} + \hat{\sigma}\Phi^{-1}(\alpha^*) \tag{35}$$

$$\Rightarrow \mu - \hat{\mu} = \Phi^{-1}(\alpha^*) \cdot (\hat{\sigma} - \sigma) \tag{36}$$

If $\sigma = \hat{\sigma}$, then no such α^* can exist, because the left-hand side in (36) is positive and the right-hand side is zero. Otherwise, there is a crossing point at $\alpha^* = \Phi((\mu - \hat{\mu})/(\hat{\sigma} - \sigma))$.

Consequently, F first-order stochastically dominates $\hat{F}$ if and only if the mean under F is higher than the mean under $\hat{F}$ and the variances are equal. As Value-at-Risk increases at all quantiles of a normal distribution if and only if the distribution is shifted leftward, we have an increase in Value-at-Risk is equivalent to a leftward shift in the mean. It follows that the ratio of the expected continuation value to the expected liquidation value increases in Value-at-Risk.

∎

Proof of Lemma 2. Expand $E[\tilde{c}|\ell]$ as

$$E[\tilde{c}|\ell] = \frac{1}{1 - F(\ell)} \int_\ell^b cf(c)dc \tag{37}$$

so that

$$\frac{dE[\tilde{c}|\ell]}{d\ell} = \frac{f(\ell)}{[1 - F(\ell)]^2} \int_\ell^b cf(c)dc - \frac{1}{1 - F(\ell)} \cdot \ell f(\ell)$$

$$= \frac{f(\ell)}{1 - F(\ell)} (E[\tilde{c}|\ell] - \ell) \tag{38}$$

Let $h(\ell) = E[\tilde{c}|\ell] - 2\ell$. Then

$$\frac{dh}{d\ell} = \frac{f(\ell)}{1 - F(\ell)} (E[\tilde{c}|\ell] - \ell) - 2 \tag{39}$$

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Assume that, for some value ℓ^* of ℓ, we have $E[\tilde{c}|\ell^*] = 2\ell^*$. We then require

$$\left.\frac{dh}{dl}\right|_{\ell=\ell^*} = \frac{f(\ell^*)}{1-F(\ell^*)}\left(E[\tilde{c}|\ell^*] - \ell^*\right) \geq 2$$

$$\Rightarrow \frac{f(\ell^*)}{1-F(\ell^*)} \geq \frac{2}{\ell^*}$$

$$\Rightarrow -\frac{d\ln[1-F(\ell^*)]}{d\ell^*} \geq \frac{2}{\ell^*}$$

$$\Rightarrow \ln\left(\frac{1}{1-F(\ell^*)}\right) + \kappa \geq 2\ln(\ell^*) = \ln((\ell^*)^2) \tag{40}$$

for some arbitrary constant κ. Exponentiating both sides of (40) and redefining the arbitrary constant gives

$$\frac{1}{1-F(\ell^*)} \geq e^{-\kappa}(\ell^*)^2 := \left(\frac{\ell^*}{a}\right)^2$$

$$\Rightarrow F(\ell^*) \leq 1 - \left(\frac{a}{\ell^*}\right)^2 \tag{41}$$

If $E[\tilde{c}|\ell] = 2\ell$ everywhere, then (41) must bind, i.e., it is necessary that F is a Pareto distribution with shape parameter $\alpha = 2$ for the continuation value to be exactly double the liquidation value on all of the support of F.

To see that this is also sufficient, suppose $F(x) = 1 - (a/x)^2$. Then $f(x) = 2a^2/(x^3)$. From (37),

$$E[\tilde{c}|\ell] = \frac{\ell^2}{a^2}\int_\ell^\infty \frac{2a^2}{x^2}dx$$

$$= 2\ell^2 \int_\ell^\infty \frac{1}{x^2}dx$$

$$= 2\ell^2 \left(\frac{-1}{x}\right)\Big|_\ell^\infty = 2\ell \tag{42}$$

Otherwise, the weak inequality (41) must hold whenever $E[\tilde{c}|\ell] = 2\ell$ and must not bind at all such ℓ. That is, either $E[\tilde{c}|\ell] > 2\ell$ on all of the support of F, or there is at least one ℓ^* with $E[\tilde{c}|\ell] = 2\ell$ for which (41) does not bind. In either case, it is immediate that F tails off faster than in the case where (41) binds.

Finally, note that for a Pareto distribution with shape parameter at most 2, the variance

is infinite. Thus, if reporting ℓ always rolling over a risk-dominant equilibrium in the basic game G, then the underlying distribution must have infinite variance. ∎

Proof of Proposition 3. We restrict attention to the case where F is a Pareto distribution with scale a and shape $\alpha \leq 2$. For $\alpha \leq 1$, the result is trivial because $E[\tilde{c}] = \infty$ and the mean of the distribution is also infinite, so we focus only on the finite mean case where $\alpha \in (1, 2]$. Then

$$\int_a^\infty x f(x) dx = \int_a^\infty \frac{\alpha a^\alpha}{x^\alpha} dx = \frac{\alpha a}{\alpha - 1} \tag{43}$$

From Theorem 3, (R, R) is a risk-dominant equilibrium in basic game G if and only if

$$\int_a^\infty x F(x) f(x) dx \geq \left(\frac{2}{3}\right) \left(\frac{\alpha a}{\alpha - 1}\right) \tag{44}$$

Expanding (44), we obtain the following necessary and sufficient condition:

$$\int_a^\infty x \left(1 - \left(\frac{a}{x}\right)^\alpha\right) \left(\frac{\alpha a^\alpha}{x^{\alpha+1}}\right) dx \geq \frac{2\alpha a}{3(\alpha - 1)}$$

$$\Rightarrow \frac{\alpha a}{\alpha - 1} - \int_a^\infty \frac{\alpha a^{2\alpha}}{x^{2\alpha}} dx \geq \frac{2\alpha a}{3(\alpha - 1)}$$

$$\Rightarrow \frac{\alpha a}{\alpha - 1} + \frac{\alpha a^{2\alpha}}{(2\alpha - 1) x^{2\alpha-1}} \Big|_a^\infty \geq \frac{2\alpha a}{3(\alpha - 1)}$$

$$\Rightarrow \frac{\alpha a}{\alpha - 1} - \frac{\alpha a}{2\alpha - 1} \geq \frac{2\alpha a}{3(\alpha - 1)}$$

$$\Rightarrow \frac{\alpha a}{3(\alpha - 1)} \geq \frac{\alpha a}{2\alpha - 1}$$

$$\Rightarrow 2\alpha - 1 \geq 3\alpha - 3$$

$$\Rightarrow \alpha \leq 2 \tag{45}$$

which is exactly the condition under which disclosing $\tilde{\ell}$ makes rolling over risk-dominant. ∎

Proof of Lemma 3. Let $k(c) = E[\tilde{\ell}|c] - c/2$. We require that $k(b) = 0$ and $dk/dc = 0$ on (a, b). We have

$$\frac{dk(c)}{dc} = -\frac{f(c)}{F^2(c)} \int_a^c x f(x) dx + \frac{cf(c)}{F(c)} - \frac{1}{2}$$

$$= \frac{f(c)}{F(c)} \left(c - E[\tilde{\ell}|c]\right) - \frac{1}{2} \tag{46}$$

Assume that, for some $c^* \in (a, b]$, we have $E[\tilde{\ell}|c^*] = c^*/2$. We then require

$$
\begin{aligned}
\left.\frac{dk}{dl}\right|_{c=c*} &= \frac{f(c^*)}{F(c^*)}\left(c^* - E[\tilde{\ell}|c^*]\right) \geq \frac{1}{2} \\
&\Rightarrow \frac{f(c^*)}{F(c^*)}\left(\frac{c^*}{2}\right) \geq \frac{1}{2} \\
&\Rightarrow \frac{f(c^*)}{F(c^*)} \geq \frac{1}{c^*} \\
&\Rightarrow \frac{d\ln F(c^*)}{dc^*} \geq \frac{1}{c^*} \\
&\Rightarrow F(c^*) \leq \gamma c^*
\end{aligned}
\tag{47}
$$

for some positive constant γ. If the inequality binds, then the fact that $F(b) = 1$ requires us to impose $\gamma = 1/b$, giving $F(c^*) = c^*/b$. If the inequality binds on all of $(a, b]$, we therefore have that F is a uniform distribution with lower bound 0. ∎

Proof of Proposition 4. We first show that nondisclosure can be optimal when disclosure of c is not. To that end, suppose F is a shifted χ_2^2 distribution:

$$
(\tilde{x} - 1) \sim \chi_2^2
$$

i.e., for $x \geq 1$, $F(x) = 1 - e^{-(x-1)/2}$ Then $E[\tilde{x}] = 3$, as $\tilde{x}$ is a constant of 1 added to a χ_2^2 random variable. Moreover,

$$
\int_1^\infty xF(x)f(x)dx = 2
$$

Therefore, (9) holds, so that rolling over is a risk-dominant equilibrium given that the borrower releases no information. By Lemma 3, since $a > 0$, disclosure of c makes liquidating the unique risk-dominant equilibrium. Therefore, it is possible for nondisclosure to prevent coordination failure but for mandatory disclosure of c to foster coordination failure.

Next, we show that if mandatory disclosure of c prevents coordination failure, then so does nondisclosure. We limit the proof here to the case in which $a = 0$ and $b = 1$, as other cases are analogous.

Under the assumption that disclosure of c necessarily prevents coordination failure, we require F to satisfy

$$(\forall c \in (0,1)) \quad \frac{1}{F(c)} \int_0^c x f(x) dx \leq \frac{c}{2}$$

This implies (by the Fundamental Theorem of Calculus)

$$(\forall c \in (0,1)) \quad cf(c) \leq F(c) \tag{48}$$

The problem then reduces to one in the calculus of variations: we need to show that we cannot find a differentiable nondecreasing function F satisfying $F(0) = 0, F(1) = 1$ for which (48) holds and for which

$$\int_0^1 x f(x) \left[F(x) - \frac{2}{3} \right] dx < 0 \tag{49}$$

(where (49) comes from (9)).

Differentiating the ratio $F(x)/x$ and using (48),

$$\left(\frac{F(x)}{x} \right)' = \frac{x f(x) - F(x)}{x^2} \leq 0$$

so $F(x)/x$ is nonincreasing.

Introduce the change of variables $u = F(x)$, so that $x = F^{-1}(u)$, for $u \in (0,1)$. Then the ratio

$$H(u) := \frac{F^{-1}(u)}{u} \tag{50}$$

is nondecreasing.

If we expand the product in the integrand in (49), the first term is

$$\int_0^1 x f(x) F(x) dx = \frac{1}{2} \int_0^1 x d(F^2(x))$$

Substituting $x = F^{-1}(u)$ and $u = F(x)$, this integral becomes

$$\frac{1}{2} \int_0^1 F^{-1}(u) d(u^2) = \frac{1}{3} \int_0^1 \frac{F^{-1}(u)}{u} d(u^3) \tag{51}$$

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Consider a random variable $\widetilde{z}_1$ with cdf $F_{Z_1}(u) = u^3$. Then from (51),

$$\int_0^1 xf(x)F(x)dx = \frac{1}{3}E[H(\widetilde{z}_1)]$$

The second term (ignoring the minus sign) in the product in (49) is

$$\frac{2}{3}\int_0^1 xf(x)dx = \frac{2}{3}\int_0^1 F^{-1}(u)du$$
$$= \frac{1}{3}\int_0^1 \frac{F^{-1}(u)}{u}d(u^2)$$

Let $\widetilde{z}_2$ be a random variable with cdf $F_{Z_2}(u) = u^2$. Then

$$\frac{2}{3}\int_0^1 xf(x)dx = \frac{1}{3}E[H(\widetilde{z}_2)]$$

As $\widetilde{z}_1 \geq \widetilde{z}_2$ almost surely and H is nondecreasing, we have

$$\frac{1}{3}E[H(\widetilde{z}_1)] \geq \frac{1}{3}E[H(\widetilde{z}_2)]$$

from which we obtain

$$\int_0^1 xf(x)\left[F(x) - \frac{2}{3}\right]dx \geq 0.$$

Therefore, there is no cdf F satisfying (49) and also satisfying (48). This means that there cannot be an implicit risk distribution for which disclosure of c is optimal and nondisclosure is not. ∎

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