Monte-Carlo Simulation of Radon Equilibrium Under Varying Conditions.

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Introduction

At the core of this investigation is the development of Monte-Carlo simulations of the radon, ^{222}Rn, (and thoron, ^{220}Rn) decay chains. Currently, these preliminary simulations assume an hypothetical closed cubic metre of atmosphere and the simulations are time-stepped at constant intervals. At each time-step, there is a radon (or thoron) influx and each nucleus in the decay chains decays probabilistically, with nuclei and activity being aggregated and tabulated at the end of each time-step.

The preliminary simulations have been coded in two open-source interpreted mathematical software packages:

- Scilab – user-friendly environment;
- Yorick – fast, good numerical precision.

The ^{222}Rn Decay Chain

The full version of the ^{222}Rn decay chain, including five very low probability decay paths, was modelled. This is shown in Figure 1.

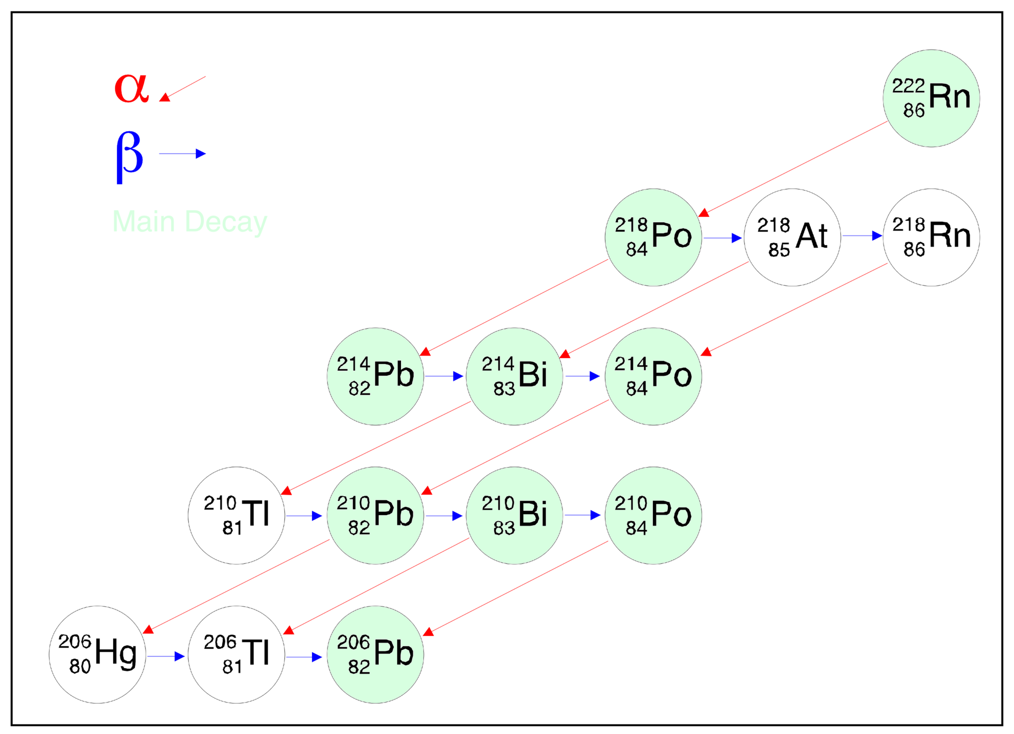

Fig 1. The ^{222}Rn Decay Chain (main decay shaded).

The Simulation – ^{222}Rn

The simulation is a time-stepped Monte-Carlo simulation. Each daughter, from ^{222}Rn (beginning) to ^{206}Pb (end, stable) is represented by a loop which determines the random decay according to the probability of decay per time-step (calculated from tabulated half-life data).

Currently, the decays in the chain are processed sequentially. Thus, at each time-step, the daughters are processed 'upstream', i.e. from ^{206}Pb to ^{222}Rn so that each daughter is 'decayed' according to the previous state before being updated.

Schematically, at each time-step, for each daughter in the decay chain:

```
P(decay) = P(daughter_timestep_decay)
activity = 0
n_nucleus = n_nucleus_old
for counter = 1 to n_nucleus_old
    P(nucleus_decay) = random_number    [0,1]
    if P(nucleus_decay) ≤ P(decay)
        n_nucleus = n_nucleus − 1
        n_next_nucleus = n_next_nucleus + 1
        activity = activity + 1
    end_if
end_for
n_nucleus_old = n_nucleus
→ plus downstream checks/updates
```

The decay probabilities are listed in Table 1.

Decay	Decay Probability 5-minute timestep	Decay	Decay Probability 5-minute timestep
^{222}Rn α	6.29×10^{-04}	^{214}Pb β	0.121
^{218}Po α	0.673	^{210}Po α	1.74×10^{-05}
^{218}Po β	1.35×10^{-04}	^{210}Bi α	6.24×10^{-10}
^{218}At α	0.999	^{210}Bi β	4.80×10^{-04}
^{218}At β	0.001	^{210}Pb α	5.61×10^{-15}
^{218}Rn α	1.000	^{210}Pb β	2.95×10^{-07}
^{214}Po α	1.000	^{210}Tl β	0.930
^{214}Bi α	3.20×10^{-05}	^{206}Tl β	0.562
^{214}Bi β	0.160	^{206}Hg β	0.346

Tab 1. Particles Decay Probabilities, 5-minute time-step.

The 'Experiments'.

1. Cyclically Varying Radon Influx.

The main investigation so far has been into cyclically varying radon influx. The phase relationships between radon influx, radon concentration and the ^{222}Rn, ^{218}Po and ^{214}Po α-particle activities have been investigated for 12h, 24h and 48h sinusoidal cycles in radon influx.

2. Radon Equilibrium Factor.

The effect of different metallic-daughter plate-out probabilities on the equilibrium of ^{222}Rn and its α-emitting daughters has also been investigated. This allows calculation of the radon equilibrium factor, *F*.

Results: Cyclic Lags.

The cyclic relationships are shown in Figure 2, for 12h and 24h cycles. The lags for 12h, 24h and 48h cycles are shown in Table 2.

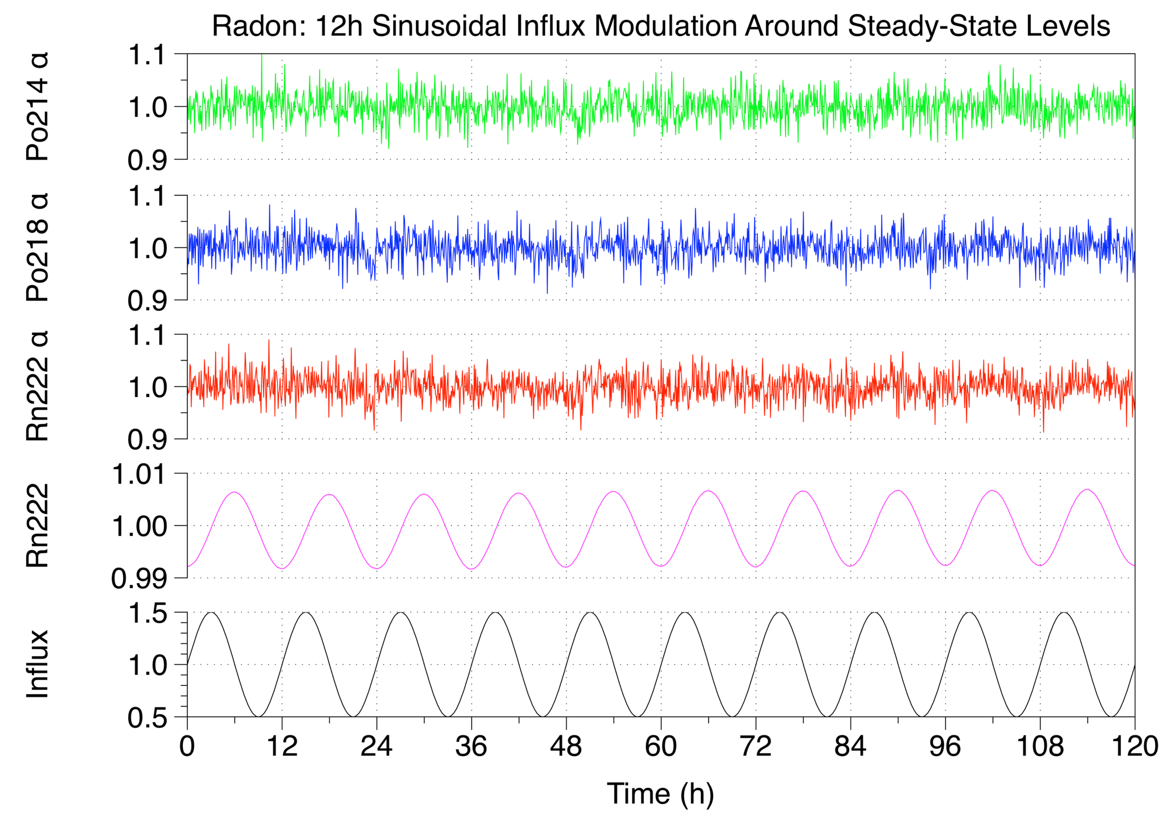

Fig 2a. 12h Sinusoidal Variation in Radon Influx.

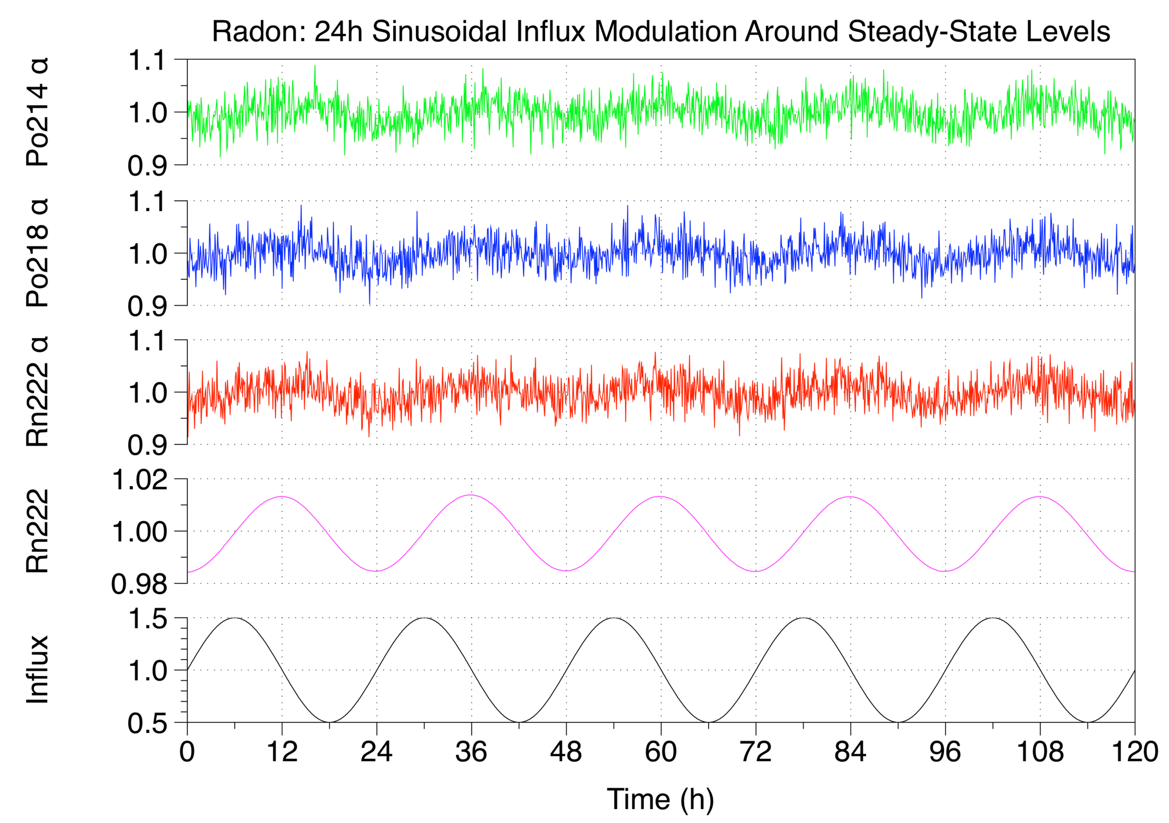

Fig 2b. 24h Sinusoidal Variation in Radon Influx.

		Cycle Period		
		48h	24h	12h
Lag of ^{222}Rn concentration behind ^{222}Rn influx (% of period)		11h 31m (24.0%)	5h 51m (24.4%)	2h 56m (24.4%)
Lag behind ^{222}Rn concentration	^{222}Rn α	2 m	0 m	7 m
	^{218}Po α	9 m	5 m	14 m
	^{214}Po α	85 m	88 m	94 m

Tab 2. Lags: 48h, 24h and 12h Cycles (mean, 5 runs).

Note the ~1/4 cycle lag of ^{222}Rn behind influx and the ~90m lag of ^{214}Po α-activity behind ^{222}Rn.

Results: Radon Equilibrium Factor.

The radon equilibrium factor, *F*, was calculated for plate-out rates as shown in Figure 3, according to the formula cited by the Wise Uranium Project.

$$F = \frac{0.106 n_{^{218}Po} + 0.514 n_{^{214}Pb} + 0.380 n_{^{214}Bi}}{n_{^{222}Rn}}$$

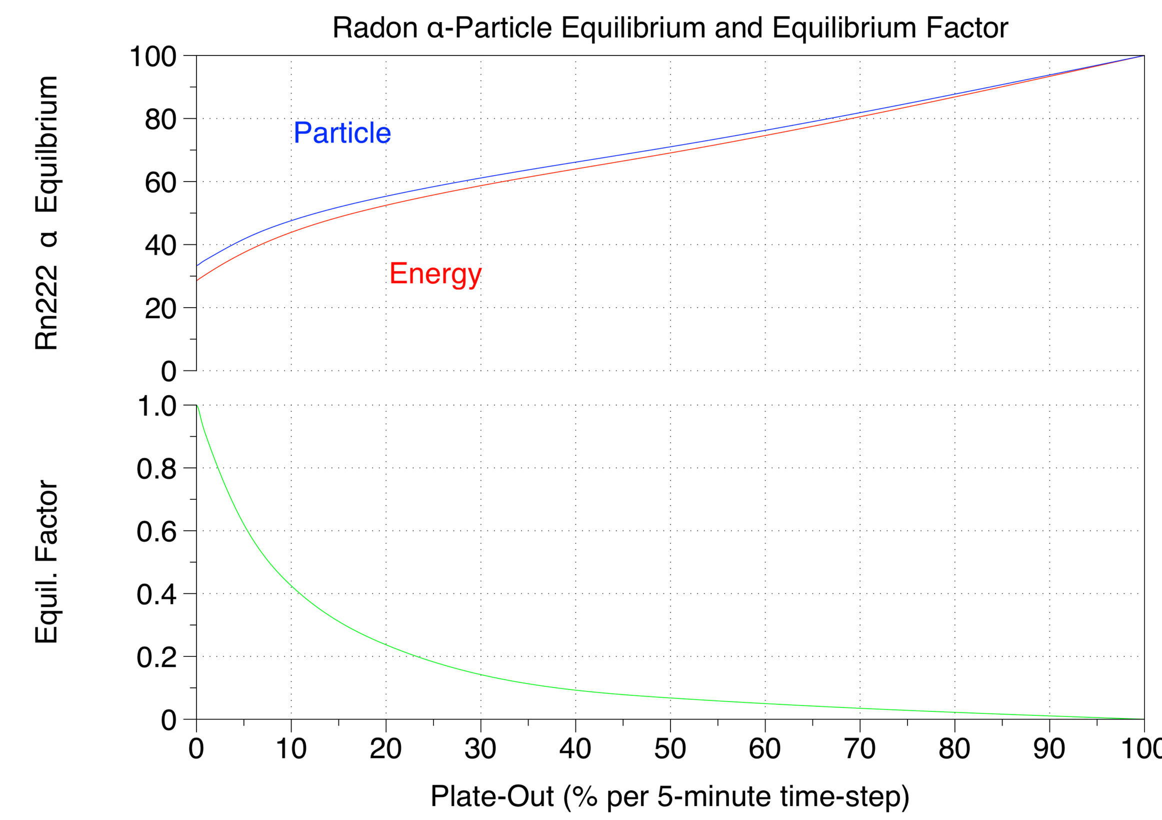

Fig 3. Radon α-Particle Equilibrium.

Conclusion

These preliminary results, obtained using a coarse time-step, start to illuminate the behaviours of the radon-daughter system when the influx is cyclically varying: it is evident that the shorter the cyclic period, the more damped the response of the daughters to the variation.

However, it is clear that:

i) ^{222}Rn concentration lags the influx by ~¼ cycle

ii) ^{214}Po α-activity lags ^{222}Rn and ^{218}Po α-activity by ~90m

It is intended to recode the simulation to run on a multi-core HPC cluster such that daughters can be decayed in parallel rather than sequentially, to reduce the run-time and enable finer time-stepping.

The thoron, ^{220}Rn, decay-chain will also be simulated: this will assist with understanding the equilibrium and cyclic behaviours in situations where both radon isotopes are present.

References

1. Wise-Uranium, http://www.wise-uranium.org/rdcrnh.html .
2. Scilab (http://www.scilab.org)
   Yorick (http://yorick.sourceforge.net).