

On the plate-like and layer-like response of slab foundations to ground-borne vibration

G. Sanitate, J.P. Talbot

Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

Abstract

The increasing urban population is leading to the exploitation of building sites, close to surface or underground railways, with considerable levels of ground-borne vibration. An important design consideration regards the levels of perceptible vibration and/or re-radiated noise in the completed buildings. A fundamental question concerns to what extent the mass and stiffness of a building foundation influences these levels. This paper explores this question in relation to a concrete slab foundation.

Previous research has explored the influence of the coupling between a thin, flexural plate and an elastic half-space on the free-surface displacements arising from surface Rayleigh waves. Here, a numerical, wave-based approach is used to model the slab foundation as an elastic layer of finite thickness, overlying the half-space. The latter is subjected to incident waves in the form of Rayleigh, P- and SV-waves. It is found that thin-plate theory alone is insufficient for modelling the slab over the full frequency range of interest, and that the assumed soil-slab boundary condition plays a significant role. Design plots are presented in order to summarise the influence of the salient dimensionless parameters, and to help guide the design of a slab foundation to achieve a specific reduction in ground vibration level.

Keywords: foundation, ground-borne vibration, soil-structure interaction, railway

1. Introduction

2 Ground-borne vibration generated by railways is an increasing concern for building designers. Vibration
3 generated at the wheel-rail interface propagates through the track and supporting structures, into the un-
4 derlying ground, from where it may propagate to the foundations of nearby buildings. Depending on the
5 mass, stiffness and damping distribution within a building and its foundation, the resulting levels of per-

Email addresses: gs528@cam.ac.uk (G. Sanitate), jpt1000@cam.ac.uk (J.P. Talbot)

6 ceptible vibration and re-radiated noise may lead to the annoyance of occupants or, in the case of specialist
7 manufacturing or research facilities, the malfunctioning of sensitive equipment.

8 A number of strategies are available for mitigating ground-borne vibration in buildings, by addressing
9 the source [1, 2], the transmission path [3, 4] or the receiver (building) [5]. One example of mitigation
10 at the receiver is the introduction of a thick slab foundation to help suppress the vibration, primarily by
11 stiffening the base of the building. This approach is usually employed for specialist manufacturing facilities,
12 such as silicon wafer fabs [6], where particularly stringent vibration criteria must be met. It has also been
13 suggested as a straightforward approach for commercial and residential buildings, since it may simply
14 involve constructing a thicker slab than pure structural considerations dictate. However, the extent to which
15 a slab influences the ground vibration field is not clear, and there is little or no guidance for designers
16 wishing to achieve a specific reduction in ground vibration level.

17 The behaviour of a slab foundation resting on a soil deposit subject to ground-borne vibration represents
18 a particular case of soil-structure interaction (SSI). Significant literature exists on the SSI associated with
19 both flexible and rigid foundations [7, 8, 9, 10]. However, the main focus has been earthquake-related
20 problems, which involve low frequencies and long wavelengths, and therefore allow the foundations to
21 be treated as rigid. In the case of ground-borne vibration due to railways, the frequency range of interest
22 extends to much higher frequencies, typically from approximately 25 Hz to 250 Hz [11], and therefore
23 involves relatively short wavelengths that are comparable with the dimensions of a typical foundation. In
24 this case, the flexibility of a foundation is one of the dominant factors governing its response.

25 In the context of ground-borne vibration, Auersch [12] considers the response of thin, flexural plates
26 (representing the foundation slab) resting on an elastic half-space (the soil deposit) to surface Rayleigh wave
27 excitation. Both finite and infinitely-long plates are considered, using a combined finite-element boundary-
28 element method, and a semi-analytical method in the frequency-wavenumber domain. Valuable results from
29 a parametric study are presented that considers the influence of mass, stiffness and soil layering on the soil-
30 slab interaction and the extent to which a slab attenuates ground vibration levels. In particular, Auersch
31 concludes that the slab thickness is the dominant parameter governing the level of attenuation, since this
32 essentially governs the frequency above which attenuation occurs.

33 The work presented here begins by reproducing some of the results of Auersch, this time using the dy-
34 namic stiffness method (DSM) implemented within the Elastodynamics Toolbox [13, 14] in MATLAB [15].
35 The foundation slab is now modelled as an elastic layer, of infinite horizontal extent but finite thickness,

36 overlying an elastic half-space. The assumption of infinite horizontal extent is unsuitable for modelling
37 the detailed response of a particular foundation to a particular vibration source. In particular, it fails to
38 account for any modal behaviour of the slab due to the absence of wave reflections from the slab bound-
39 ary. However, these modes are often heavily damped, due to the radiation damping provided by the soil,
40 and are of secondary interest to the fundamental, underlying SSI. Auersch demonstrates that, for the case
41 of Rayleigh wave excitation and for a relaxed boundary condition at the soil-foundation interface, the SSI
42 associated with a finite plate is approximated well by that of an infinitely long one. An infinite slab is
43 therefore adopted here to investigate further some of the fundamental behaviour: to enable the identification
44 of the salient non-dimensional groups; and to investigate the influence of the assumed boundary conditions
45 at the soil-slab interface, as well as the effect of the plate-like assumption for the slab. The assumption of
46 linear-elastic soil behaviour is justified on the basis of the low strain levels associated with ground-borne
47 vibration, which usually lie within the elastic regime for most soils (see, for example, Connolly et al. [16]).

48 Having considered surface Rayleigh wave excitation, the case of a buried source is investigated by con-
49 sidering incident P- and SV-waves, before considering the overall implications of the study for foundation
50 design.

51 **2. Overview of the problem**

52 Our interest is the particular SSI associated with a flexible, concrete slab foundation, that is, the influence
53 of such a slab on the free-surface vibration field, expressed in terms of the ratio of the vibration amplitudes
54 before and after the construction of the slab. As a first approximation, the slab may be assumed to be of
55 infinite horizontal extent. This assumption is supported by Auersch, who demonstrates that, for the case of
56 Rayleigh wave excitation, the SSI associated with a finite plate may be approximated by that of an infinitely
57 long one. Here, the slab is treated as an elastic layer of thickness h and infinite extent in both the x and y
58 directions, with shear modulus G_c , Poisson's ratio ν_c and mass density ρ_c (see Figure 1). The underlying
59 soil is modelled as a homogeneous, isotropic, elastic half-space, with shear modulus G_s , Poisson's ratio ν_s
60 and mass density ρ_s . Damping in both the slab and the soil is assumed to be hysteretic, as described by a
61 frequency-independent loss factor in shear, with no associated damping in dilatation (see Hunt [17]). Both
62 surface and body, plane-wave excitation is considered: a Rayleigh wave travelling in the x -direction with
63 speed V_R , and either an incident P- or SV-wave travelling respectively with speed V_P or V_S at an incidence
64 angle θ_P or θ_S . Such excitation may be regarded as being broadly representative of that from either a surface

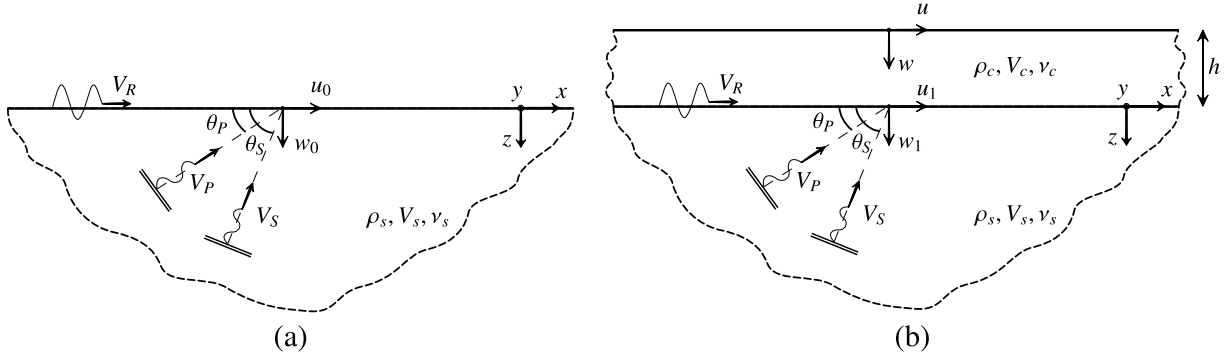


Figure 1: (a) The free-field displacement amplitude \mathbf{u}_0 on the surface of an elastic half-space subjected to surface and body plane-wave excitation. A Rayleigh wave travelling in the x -direction with speed V_R , and a P- and SV-wave travelling with speeds V_P and V_S at incidence angles θ_P and θ_S are considered; (b) the corresponding response following the construction of a slab foundation, expressed in terms of the interface displacement \mathbf{u}_1 and the displacement \mathbf{u} on the free-surface of the slab.

65 or underground railway.

66 The wave excitation is assumed to be plane and therefore invariant in the y -direction. The problem
 67 can then be examined in the $x - z$ plane with reference to the vertical and horizontal displacement ampli-
 68 tudes. The free-field displacement \mathbf{u}_0 of the half-space, the interface displacement \mathbf{u}_1 and the free-surface
 69 displacement \mathbf{u} of the slab can be expressed in vector form as:

$$\mathbf{u}_0 = \mathbf{A}_0 \exp\{i(\omega t - k_x x)\}; \quad \mathbf{u}_1 = \mathbf{A}_1 \exp\{i(\omega t - k_x x)\}; \quad \mathbf{u} = \mathbf{A} \exp\{i(\omega t - k_x x)\} \quad (1)$$

70 where ω and k_x are the angular frequency and horizontal wavenumber of the vibration field. The relative
 71 amplitudes of the vectors $\mathbf{A}_0 = [u_0, w_0]^T$, $\mathbf{A}_1 = [u_1, w_1]^T$ and $\mathbf{A} = [u, w]^T$, which are independent of the x
 72 coordinate due to the infinite extent of the slab, describe the influence of the slab foundation. Depending on
 73 the type of wave excitation considered, the horizontal wavenumber k_x can be written as:

$$k_x = k_R; \quad k_x = k_P \cos \theta_P; \quad k_x = k_S \cos \theta_S \quad (2)$$

74 where k_R, k_P and k_S are the wavenumbers of the Rayleigh, P- and SV-waves respectively.

75 The influence of the slab foundation on the free-field displacement \mathbf{u}_0 relates to a general result in the
 76 SSI literature [7]. So-called weak coupling is assumed, in which only the coupling between the soil and the
 77 slab is accounted for; any coupling with the original source of the vibration is assumed to be negligible. By
 78 ensuring equilibrium and compatibility at the soil-foundation interface, the displacement \mathbf{u}_1 at the interface

79 can be expressed as:

$$\mathbf{u}_1 = \left[\mathbf{I} + \mathbf{H}_s \mathbf{H}_f^{-1} \right]^{-1} \mathbf{u}_0 \quad (3)$$

80 where \mathbf{H}_f and \mathbf{H}_s are the frequency-response function (FRF) matrices of the foundation and soil respectively,
81 which can be found by inverting the respective dynamic stiffness matrices [18]. Equation 3 is also found
82 in the ground-borne vibration literature in the context of soil-building interaction [5]. In the case of finite
83 slabs, Equation 3 may be solved directly; in the case of infinite slabs, the solution may be obtained via a
84 frequency-wavenumber formulation. Auersch used the latter approach for an infinitely-long slab subjected
85 to Rayleigh waves, and assumed a relaxed boundary condition between the soil and the slab in which only
86 the vertical displacements at the interface are coupled [12]. In this case, Equation 3 reduces to the following
87 scalar equation for the vertical displacement ratio at the interface:

$$\frac{w_1}{w_0} = \frac{1}{\left(1 + \frac{H_{sz}}{H_{fz}} \right)} \quad (4)$$

88 where H_{fz} and H_{sz} are the vertical driving-point FRFs of the slab and the soil respectively, formally in
89 the frequency-wavenumber domain. However, since the wavenumber is a function of frequency and the
90 Rayleigh wave speed, this ratio of the displacement amplitudes is a function of frequency alone, and inde-
91 pendent of position due to the infinite length of the plate.

92 Focussing on the vertical displacements is common practice when dealing with ground-borne vibration
93 in buildings, often based on the assumption that the flexibility of a building in the horizontal direction
94 provides sufficient decoupling from any horizontal vibration. However, there is theoretical evidence that
95 suggests otherwise, with all coupling degrees-of-freedom (vertical, horizontal and rotational) between a
96 building structure and its foundation being potentially significant [19]. Furthermore, given the small strains
97 associated with ground-borne vibration, the influence of friction at the soil-slab interface may lead to a
98 fully-coupled boundary condition that includes coupling of the horizontal displacements. In this case, the
99 full matrix form of Equation 3 must be solved, although this does reduce to a scalar equation, similar to
100 Equation 4, for the horizontal u_1/u_0 and vertical w_1/w_0 displacement ratios in the case of normally-incident
101 SV- and P-waves respectively.

102 The fully-coupled condition calls into question the assumption of plate-like behaviour for the slab. By
103 modelling the slab as a thin plate, the through-thickness deformation is assumed to be negligible, such that
104 $w = w_1$, and there is no consideration given to the in-plane horizontal displacement. The aim of the current
105 study is therefore to investigate how the SSI associated with a slab foundation is influenced by the slab-soil

106 boundary condition and the plate-like assumption, and how this varies for the different incident wave fields.

107 Before presenting the results, it is helpful to review the dimensionality of the problem. By dimensional
108 analysis, the displacement ratios w/w_0 , w_1/w_0 , etc. can each be expressed as a function of the following
109 form:

$$\frac{w}{w_0} = \Psi\left(\frac{V_s}{V_c}, \frac{\rho_s}{\rho_c}, \nu_s, \nu_c, \theta, \frac{\omega h}{V_s}\right) \quad (5)$$

110 Assuming typical values for the Poisson's ratios ν_s and ν_c (Table A.1), the dimensionless groups V_s/V_c
111 and ρ_s/ρ_c enable the influence of the stiffness and density of the slab relative to the soil to be investigated
112 for a given wave incidence angle θ and non-dimensional frequency $a_0 = \omega h/V_s$. The latter enables the
113 thickness of the slab to be described relative to the shear wavelength in the soil. Given typical ranges for
114 the shear wave speed in the soil $V_s = 150 - 300$ m/s, the slab thickness $h = 0.5 - 1.5$ m and the frequency
115 range of ground-borne vibration $f = 25 - 250$ Hz, the corresponding range of interest for the dimensionless
116 frequency a_0 lies between approximately 0.2 and 15.

117 3. A slab foundation subjected to Rayleigh wave excitation

118 This section explores the SSI associated with a slab foundation subjected to Rayleigh wave excitation. Be-
119 fore considering the slab as an elastic layer, the case of an infinitely long, strip foundation is first reviewed,
120 following the approach of Auersch [12]. Having reproduced Auersch's results, the slab is then modelled as
121 the elastic layer illustrated in Figure 1, enabling the influence of the slab-soil boundary condition and the
122 slab's finite thickness to be investigated.

123 3.1. The slab as an infinitely long, strip foundation

124 The infinitely long, strip foundation is modelled as an elastic plate of width b and thickness h . The vertical
125 dynamic stiffness of the plate in the frequency-wavenumber domain is given by [20]:

$$K_{fz}(\omega, k_x) = Bbk_x^4 - \rho_c b h \omega^2 \quad (6)$$

126 where $B = E_c h^3 / 12(1 - \nu_c^2)$ is the bending stiffness. The vertical driving-point FRF is obtained as $H_{fz} =$
127 $1/K_{fz}$.

128 The vertical driving-point FRF of the elastic half-space H_{sz} refers to a 2.5-D problem because of the
129 finite width b of the plate in the y -direction. Auersch solved this by assuming plane-strain conditions and
130 using a numerical integration approach in the wavenumber k_y , regarding the plate as infinitely flexible along

131 the y-axis by assuming a constant stress distribution across the width of the plate. According to the relaxed-
 132 boundary condition, the vertical displacement ratio w_1/w_0 is then obtained from Equation 4.

133 Figure 2 plots the ratio w_1/w_0 as a function of frequency for the parameter values of the benchmark prob-
 134 lem considered by Auersch (see Table A.1 in Appendix A). Several values of the foundation width b are
 135 considered. Irrespective of the width, at a particular frequency known as the ‘‘coincidence frequency’’ (f_{co}),
 the bending stiffness and inertia of the plate are such that a unit value of w_1/w_0 is obtained. The coinci-

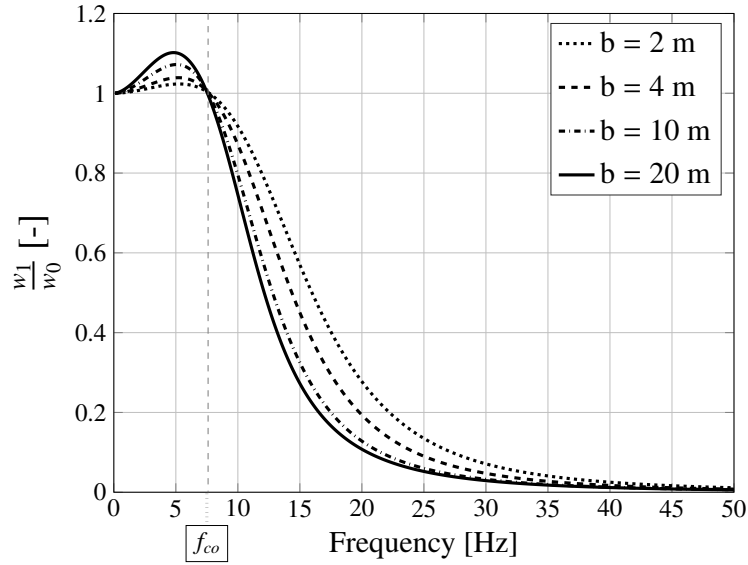


Figure 2: The influence of the width of a strip foundation b on the soil-foundation interaction under Rayleigh wave excitation, as described by the ratio of the vertical displacement amplitudes at the soil-foundation interface, before (w_0) and after (w_1) the construction of the foundation. Benchmark parameter values (Table A.1).

136
 137 dence point represents the scenario in which the free-flexural wavelength λ_f of the plate coincides with the
 138 horizontal wavelength λ_x of the input wave-field. The corresponding frequency can be found as [20]:

$$f_{co} = \frac{V_R^2}{2\pi} \sqrt{\frac{\rho_c h}{B}} = \frac{V_R^2}{2\pi} \sqrt{\frac{\rho_c 12(1 - \nu_c^2)}{E_c h^2}} \quad (7)$$

139 The coincidence frequency is important because it governs the extent to which the plate attenuates ground
 140 vibration levels: it defines the transition from the low-frequency, mass-controlled region where amplifica-
 141 tion occurs, to the stiffness-controlled region where considerable attenuation is achieved with respect to the
 142 free-field displacement w_0 . The influence of the plate mass and bending stiffness on the value of f_{co} can be
 143 determined from Equation 7. Increasing the plate thickness h results in a reduction in both f_{co} and the ratio

144 w_1/w_0 (for frequencies $f > f_{co}$). A similar effect may be obtained by decreasing the mass density ρ_c of
 145 the plate. A comprehensive discussion of the influence of the bending stiffness, mass and width of the strip
 146 foundation, and the shear wave speed V_s of the soil, can be found in Auersch [12], also with reference to a
 147 finite plate. The results of Figure 2 also illustrate the significance of increasing the strip width b , with the
 148 limiting case being that of a slab foundation of infinite extent along the x- and y-axis ($b \rightarrow \infty$).

149 3.2. The slab as an elastic layer

150 We now assume the slab foundation to be of infinite extent in both the x and y direction, modelling it as an
 151 elastic layer of finite thickness overlying the elastic half-space. The dynamic stiffness matrices \mathbf{K}_s and \mathbf{K}_f
 152 of both the half-space and the layer are calculated by means of the dynamic stiffness method (DSM) [18],
 153 making use of the ElastoDynamics Toolbox (EDT) [13, 14] in MATLAB [15]. The FRF matrices appearing
 154 in Equation 3, \mathbf{H}_s and \mathbf{H}_f , are obtained by inverting the respective dynamic stiffness matrices.

155 In the case of the relaxed boundary (RB) condition, the vertical displacement ratio w_1/w_0 can be re-
 156 trieved from Equation 4, with the vertical driving-point FRFs, H_{sz} and H_{fz} , being extracted from the FRF
 matrices. Figure 3a plots the ratio w_1/w_0 calculated by both Auersch's strip model and the equivalent

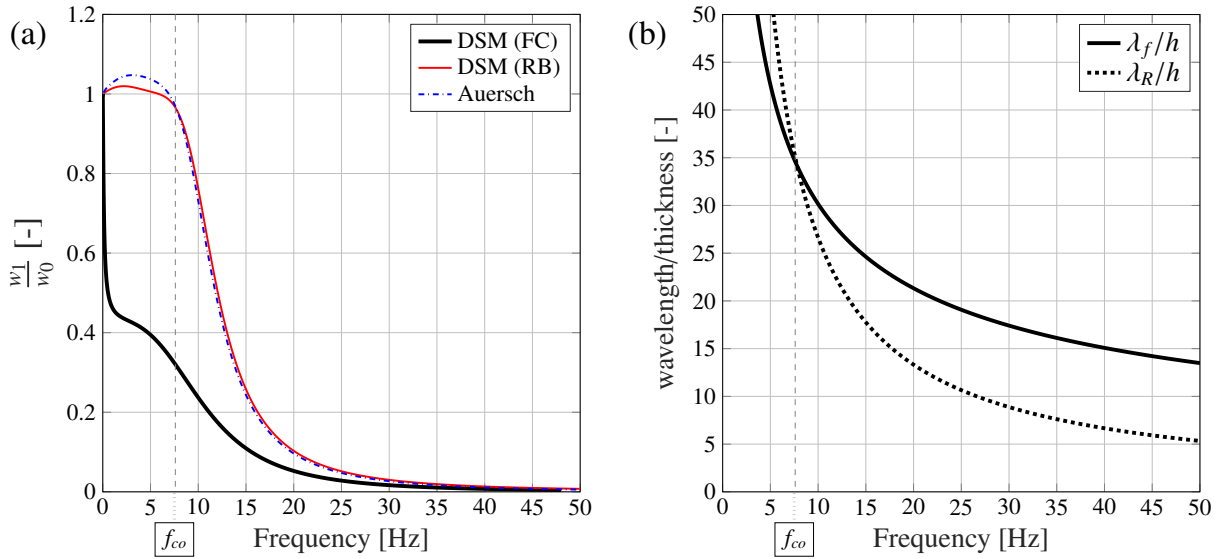


Figure 3: (a) The vertical displacement ratio at the soil-foundation interface of an infinitely long (Auersch) and infinitely large (DSM) slab foundation subjected to an incident Rayleigh wave, considering the fully-coupled (FC) and relaxed (RB) boundary conditions. (b) Comparison of the Rayleigh wave-field wavelength λ_R and the free-flexural wavelength of the strip foundation λ_f . Benchmark parameter values (Table A.1).

158 DSM (RB) model, for the same benchmark parameter values (Table A.1). It is clear that the two different
 159 methods agree well, with the strip model achieving convergent results with the DSM (RB) model for a slab
 160 width of $b = 1000$ m. Figure 3b plots the wavelength-frequency curves associated with the Rayleigh wave-
 161 field and the free-flexural response of the plate model. The coincidence point again indicates the transition
 162 region from amplification to attenuation.

163 In the fully-coupled condition (FC), the displacement ratios w_1/w_0 and u_1/u_0 are obtained by calcu-
 164 lating the interface displacement \mathbf{u}_1 using Equation 3, with reference to both the horizontal and vertical
 165 components of the Rayleigh wave [21]. The ratio w_1/w_0 obtained with this DSM (FC) model is also plot-
 166 ted in Figure 3a. In this condition, the SSI leads to significant attenuation for all frequencies, without any
 167 amplification below the coincidence frequency. The fully-coupled condition highlights the importance of
 168 considering both the horizontal and vertical components of \mathbf{u}_0 , and the cross-stiffness terms at the soil-
 169 foundation interface.

170 Figure 4a plots the horizontal displacement ratio u_1/u_0 , calculated by the DSM model for both the
 171 relaxed (RB) and fully-coupled (FC) conditions. The common feature is that both conditions result in a
 significant attenuation of the horizontal displacement. Again, the assumption of plane-wave excitation sim-

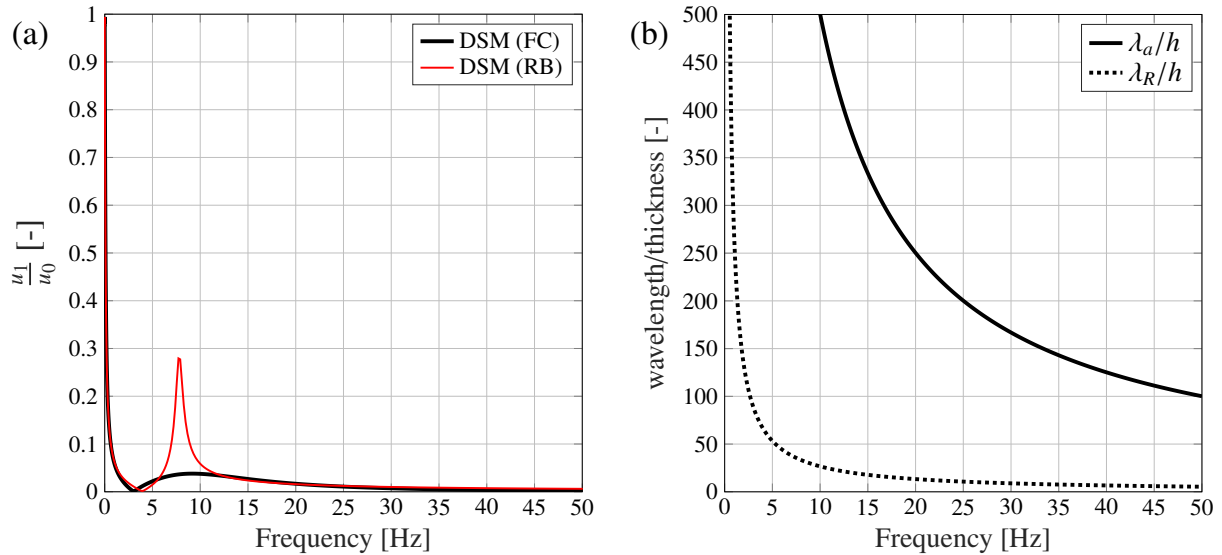


Figure 4: (a) The horizontal displacement ratio at the soil-foundation interface of an infinitely large slab foundation subjected to an incident Rayleigh wave, considering the fully-coupled (FC) and relaxed (RB) boundary conditions. (b) Comparison of the Rayleigh wave-field wavelength λ_R and the free-axial wavelength of the slab λ_a . Benchmark parameter values (Table A.1).

173 plifies the problem and enables a physical interpretation of the SSI. From the perspective of the (horizontal)
 174 slab, the incident wave-field is characterised by a horizontal wavelength $\lambda_x = \lambda_R$. This is true for the hori-
 175 zontal displacement u_1 as for the vertical displacement w_1 seen earlier. The horizontal displacement of the
 176 slab is influenced by its free-axial response along the x direction, the wavelength of which is given by:

$$\lambda_a = \frac{1}{f} \sqrt{\frac{E_c}{\rho_c (1 - \nu_c^2)}} \quad (8)$$

177 Figure 4b plots this wavelength-frequency relationship together with the curve for the Rayleigh wave-field.
 178 It is clear that λ_R is always shorter than λ_a , with no coincidence possible. It follows that the axial (in-plane)
 179 behaviour of the slab always restrains the horizontal displacement at the soil-foundation interface, resulting
 180 in attenuation for all frequencies for both the fully-coupled and relaxed boundary conditions.

181 A better understanding of the results from the different models may be obtained by expressing the
 182 displacement ratios in decibels (dB) and referring to the non-dimensional frequency a_0 on a logarithmic
 183 scale. Figure 5a re-plots in this form the results of Figure 3a for the vertical displacement ratio at the soil-
 foundation interface. Also included in Figure 5a are the results for the displacement ratio w/w_0 calculated

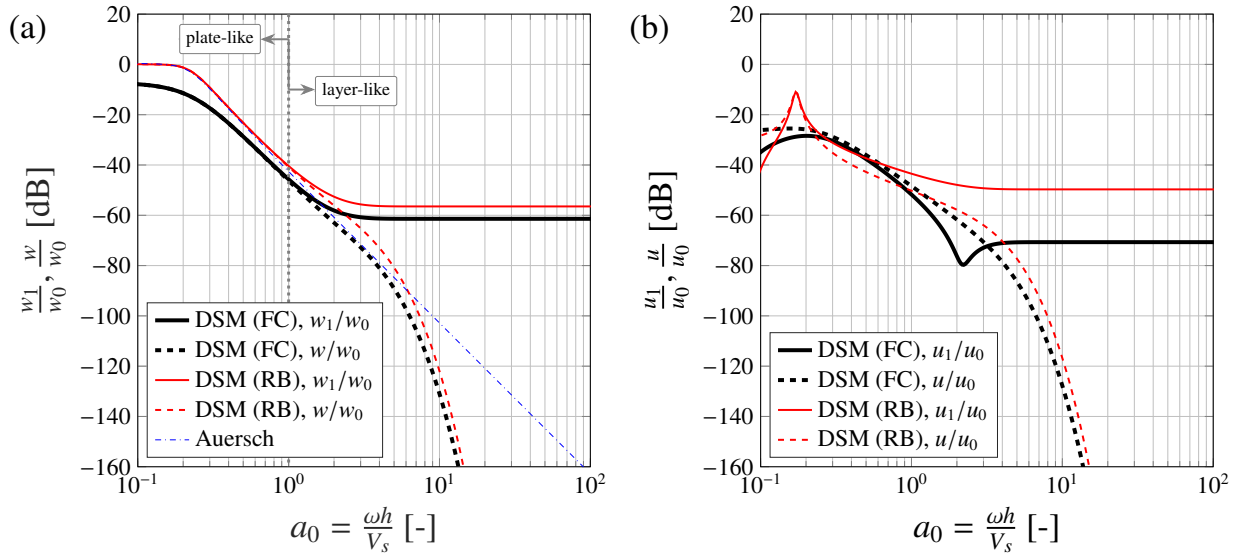


Figure 5: (a) The vertical displacement ratio at the free-surface (w/w_0) and soil-foundation interface (w_1/w_0) of an infinitely large slab foundation subjected to an incident Rayleigh wave, considering the fully-coupled (FC) and relaxed (RB) boundary conditions. Comparison is made with Auersch's strip model. (b) The corresponding horizontal displacement ratios. Benchmark parameter values (Table A.1).

185 at the free-surface - the location of most interest, since this will support any future building. The plate-
186 like behaviour of the slab at relatively low frequencies is clearly captured by the Auersch strip model,
187 which shows a good agreement with the DSM (RB) model up to $a_0 \approx 1$. The plate-like assumption is
188 therefore sufficient to capture the SSI, as observed at the soil-slab interface (ratio w_1/w_0) with the relaxed
189 boundary condition, but only up to a restricted value of h/λ_x . For shorter wavelengths, the through-thickness
190 effects become more important, with the layer-like behaviour of the slab attenuating the wave field at the
191 interface w_1 down to a plateau at high frequencies. This is a common feature for both the relaxed and
192 fully-coupled boundary conditions, although the former leads to lower attenuation because it neglects the
193 retrograde elliptical motion of particles at the soil-foundation interface, characteristic of Rayleigh waves,
194 accounting for only the vertical component. At the free surface, the high-frequency plateau is not evident.
195 Instead, the level of attenuation is observed to increase with frequency. In the case of the relaxed boundary
196 condition, the vertical displacement w is obtained simply as:

$$\frac{w}{w_1} = \frac{\tilde{H}_{fz}}{H_{fz}} \quad (9)$$

197 where \tilde{H}_{fz} is the FRF for the vertical displacement at the free-surface due to a vertical traction at the
198 interface, which can be retrieved from the FRF matrix of the layer \mathbf{H}_f . Again, it is clear from Equation 9
199 that the DSM (RB) model neglects the coupling between the horizontal and the vertical motion. The ratio
200 w/w_1 approaches unity at low frequencies, when the through-thickness effects are negligible, but introduces
201 increasing attenuation at higher frequencies.

202 Figure 5b plots the corresponding results for the horizontal displacements, this time re-plotting the re-
203 sults of Figure 4a for the displacement ratio at the interface u_1/u_0 , together with the free-surface ratio u/u_0 .
204 A similar trend to the vertical displacements is observed but with the exception of the clearly absent amplifi-
205 cation region at low frequencies for the DSM (RB) model, due to the lack of any coincidence phenomenon.

206 For both the vertical and the horizontal displacements, the results of the DSM (RB) and DSM (FC)
207 models have common features that indicate the importance of both the finite thickness of the slab, which
208 becomes increasingly significant with frequency, and the assumed soil-foundation boundary condition. The
209 influence of the latter results in at least 10 dB difference in the final free-surface displacements of the slab
210 between the two conditions, without influencing the general trend in the results.

211 4. A slab foundation subjected to incident P- and SV-waves

212 So far, the SSI associated with a slab foundation has only been investigated in the context of Rayleigh
213 waves. The case of incident P- or SV-waves can be similarly investigated by using the same DSM model for
214 the elastic layer but now with the free-field displacement components associated with these wave types [21].
215 Considering an incident P- or SV-wave at an incidence angle θ_P or θ_S (see Figure 1), the displacement field
216 \mathbf{u} of the coupled soil-foundation system is calculated by the superposition of several elastic states [22]. Only
217 the fully-coupled slab is considered here, in the belief that this better represents the actual soil-foundation
218 boundary condition at the low strain levels present in practice.

219 4.1. Influence on the vertical displacements (w_1/w_0 and w/w_0)

220 Figure 6 plots the interface displacement ratio w_1/w_0 , together with the wavelength-frequency curves for
221 incident P-waves (Figure 6a and 6c) and SV-waves (Figure 6b and 6d), for the benchmark parameter values
222 (Table A.1) and an arbitrary incidence angle $\theta_P = \theta_S = 3\pi/8$. The results in this particular case can be
223 qualitatively divided into the following frequency regions:

- 224 1. an initial low-frequency region of either moderate attenuation (for P-waves) or amplification (for
225 SV-waves), up to a frequency $f_1 < f_{co}$;
- 226 2. a second region, with increased values of w_1/w_0 , which are always greater than unity for SV-waves
227 but not necessarily for P-waves. A maximum value is obtained at a frequency $f_2 > f_{co}$;
- 228 3. a third region of sharply increasing attenuation, up to a frequency f_3 , beyond which w_1/w_0 tends to a
229 limiting value.

230 The wavelength-frequency curves (Figure 6c and 6d) are again helpful for interpreting the SSI. The coinci-
231 dence frequency f_{co} lies between f_1 and f_2 , and does not correspond to a distinct feature in the w_1/w_0 curve.
232 This is because the free-flexural wavelength λ_f of the slab is calculated based on the thin-plate assumption;
233 in practice, shear deformation is likely to contribute to higher values of λ_f , resulting in lower values of f_{co} .

234 As seen before, a better understanding of the results is obtained by using logarithmic scales. Figure 7
235 shows such a representation for the results in Figure 6 (a) and (b). The trend in the ratio w_1/w_0 described
236 before can be observed more clearly with reference to the local minimum and maximum at $a_0^{(1)}$ and $a_0^{(2)}$,
237 and the plateau beyond $a_0^{(3)}$. Additionally, the free-surface displacement ratio w/w_0 is plotted. Analogous
238 to what is observed for Rayleigh wave excitation, additional attenuation is obtained for the free-surface

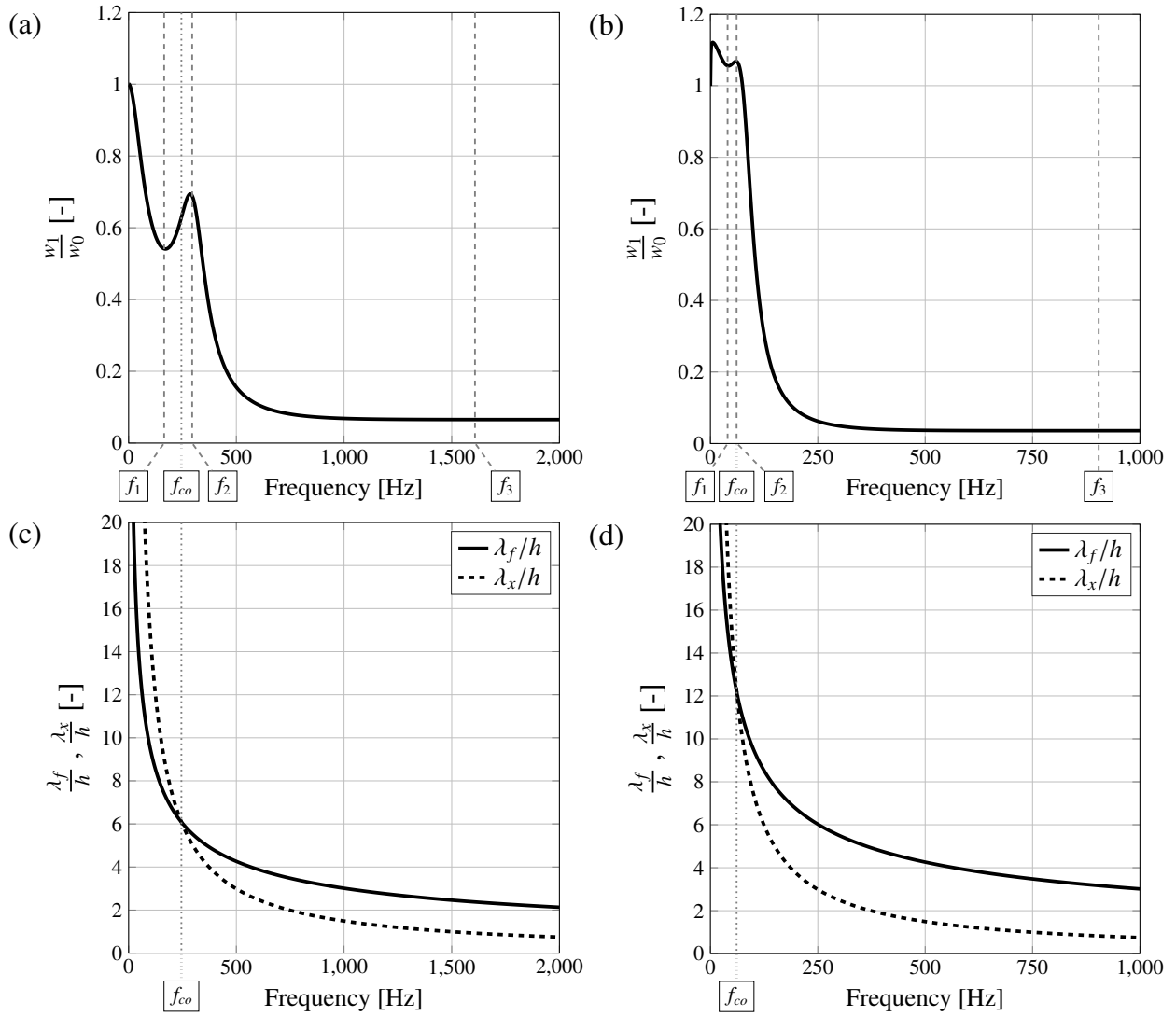


Figure 6: The vertical displacement ratio w_1/w_0 at soil-foundation interface of an infinitely large slab foundation subjected to (a) an incident P-wave at an angle $\theta_p = 3\pi/8$ and (b) an incident SV-wave at an angle $\theta_s = 3\pi/8$. The wavelength-frequency curves are plotted below for the two cases ((c) P- and (d) SV-waves) with reference to the coincidence frequency f_{co} based on the thin-plate assumption. Benchmark parameter values (Table A.1).

239 displacement w , compared to the interface displacement w_1 , as a result of through-thickness effects at
 240 relatively short wavelengths. The results obtained using Auersch's thin-plate assumption are also included
 241 to highlight the influence of the relaxed boundary condition and the thin-plate assumptions.

242 Under Rayleigh wave excitation, investigated in Section 3, the attenuation provided by the fully-coupled

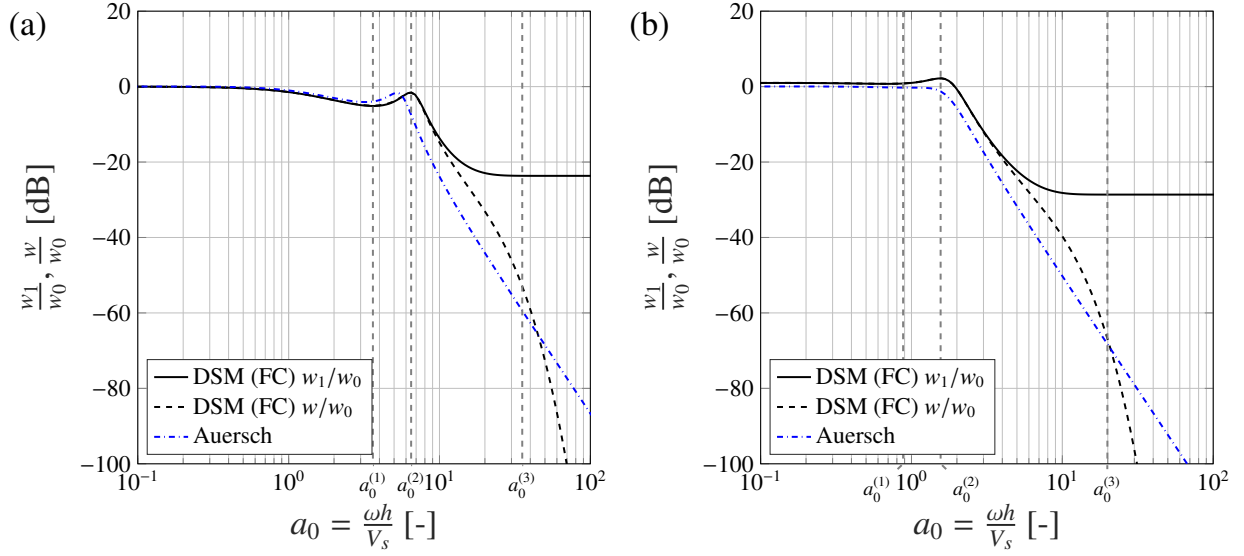


Figure 7: The vertical displacement ratio at the free-surface (w/w_0) and soil-foundation interface (w_1/w_0) of an infinitely large slab foundation subjected to (a) an incident P-wave at an angle $\theta_p = 3\pi/8$ and (b) an incident SV-wave at an angle $\theta_s = 3\pi/8$. Results obtained using Auersch's thin-plate assumption are included for comparison. Benchmark parameter values (Table A.1).

243 slab is considerable for all frequencies, and significantly greater than that provided to P- and SV-waves.
 244 The horizontal wavelength λ_x associated with an incident P- or SV-wave is always longer than that of a
 245 Rayleigh wave of the same frequency (see Figure 6c and 6d). Consequently, the coincidence frequencies
 246 f_{co} associated with P- and SV-waves are always higher than that of the Rayleigh wave, and the region where
 247 strong attenuation is to be expected from the slab shifts to relatively high frequencies. The attenuation
 248 provided by the slab to Rayleigh waves therefore represents a limiting case.

249 Another limiting case is that of normally incident P-waves ($\theta_p = \pi/2$). In this case $k_x = 0$ and the
 250 restraining effect of the slab foundation is maximum at the free-axial natural frequencies f_n of the elastic
 251 layer in the vertical direction:

$$f_n = \frac{V_{Pc}}{4h}(2n - 1) \quad (10)$$

252 where V_{Pc} is the compressional wave speed in the slab. At these frequencies, the slab behaves like a dynamic
 253 vibration absorber. However, given the application, and the relatively high P-wave speed V_{Pc} , f_n lies well
 254 beyond the frequency range of interest ($f_{n=1} \approx 1270$ Hz for the parameters in Table A.1), leading to only
 255 moderate attenuation in the frequency range of interest.

256 4.2. Influence on the horizontal displacements (u_1/u_0 and u/u_0)

257 A similar trend is observed in the horizontal displacements. This is illustrated in Figure 8 for incident P- and SV-waves at an arbitrary incidence angle $\theta = 3\pi/8$. Although, by physical interpretation, no coincidence

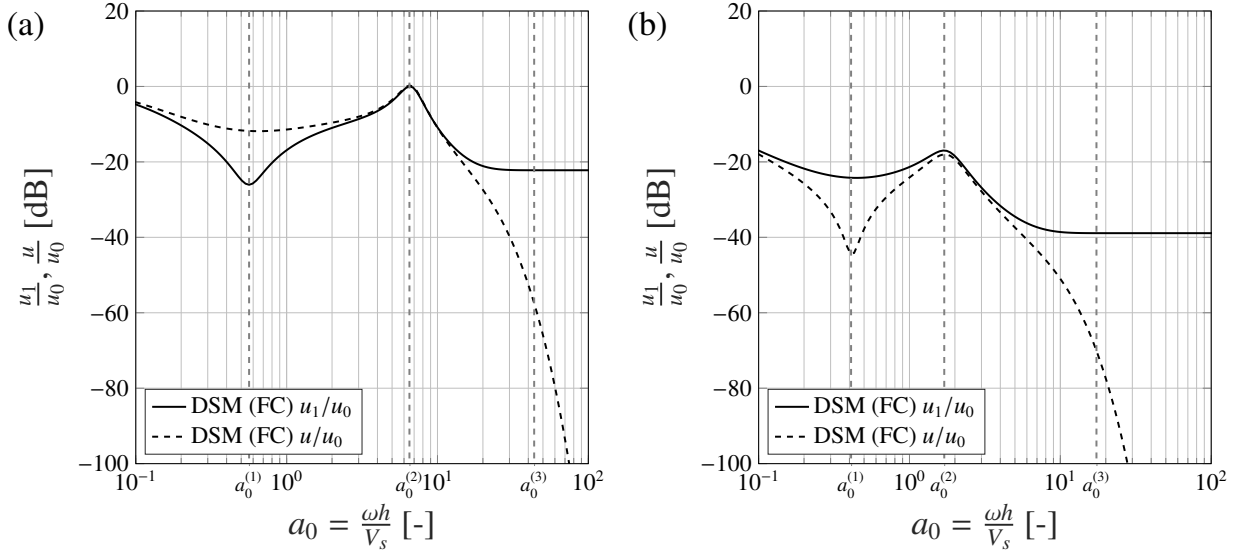


Figure 8: The horizontal displacement ratio at the free-surface (u/u_0) and soil-foundation interface (u_1/u_0) of an infinitely large slab foundation subjected to (a) an incident P-wave at an angle $\theta_p = 3\pi/8$ and (b) an incident SV-wave at an angle $\theta_s = 3\pi/8$. Benchmark parameter values (Table A.1).

258
 259 phenomenon is expected in this case (see Section 3.1), local minima ($a_0^{(1)}$) and maxima ($a_0^{(2)}$) are observed
 260 for the ratios u_1/u_0 and u/u_0 . Moreover, the through-thickness effects at relatively short wavelengths result,
 261 once more, in additional attenuation of the free-surface displacement u . Following a similar argument to
 262 that used for the vertical displacements, it is clear that the Rayleigh wave and the normally incident SV-wave
 263 represent limiting cases for the horizontal displacements u_1 and u_0 .

264 4.3. The case of normally incident P- and/or SV-waves

265 The case of normally incident waves is worth exploring further, since these may be representative of the
 266 wave field from a deep source, such as an underground railway. For normal incidence, the wave propagation
 267 problem is independent of x and effectively one-dimensional. The free-surface and interface displacement
 268 amplitudes \mathbf{u} and \mathbf{u}_1 can be found, in terms of the incident wave amplitude, by considering the superposi-
 269 tion of multiple reflections and transmissions in the half-space-layer system, as illustrated schematically in

270 Figure 9. It is clear that a normally incident P- or SV-wave will induce only vertical or horizontal displace-
 271 ments respectively. With a similar discussion also being valid for the case of a normally incident SV-wave,
 the notation adopted in the following refers only to a normally incident P-wave (see Appendix B).

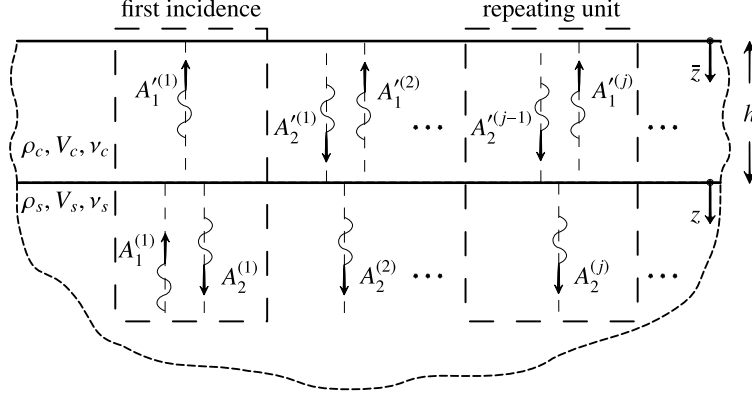


Figure 9: Schematic representation of the multiple reflected-transmitted wave amplitudes in an elastic layer overlying an elastic half-space, due to a normally incident P-wave. An infinite repetition of a reflection-transmission unit follows the reflection and transmission of the incident amplitude $A_1^{(1)}$.

272
 273 As shown in Figure 9, the multiple reflections/transmissions trace back to the incidence of the amplitude
 274 $A_1^{(1)}$ at the interface, and to the repeating reflection/transmission of the amplitude $A_2'^{(j-1)}$ that results from
 275 the reflection of $A_1'^{(j-1)}$ at the free-surface. In general, the vertical displacement in the layer can be written
 276 as:

$$w(\bar{z}) = ik'_p \left(\sum_{j=1}^{\infty} A_1'^{(j)} \exp(ik'_p \bar{z}) - \sum_{j=1}^{\infty} A_2'^{(j)} \exp(-ik'_p \bar{z}) \right) \quad (11)$$

277 By ensuring equilibrium and compatibility at the interface for the two cases, one can find:

$$\frac{A_2^{(1)}}{A_1^{(1)}} = \frac{\beta_P - 1}{\beta_P + 1} \quad \frac{A_1'^{(j)}}{A_2'^{(j-1)}} = \frac{1 - \beta_P}{\beta_P + 1} \exp(-i2k'_p h) = \zeta$$

$$\frac{A_1'^{(1)}}{A_1^{(1)}} = \frac{k_P}{k'_P} \frac{2}{\beta_P + 1} \exp(-ik'_p h) \quad \frac{A_2^{(j)}}{A_2'^{(j-1)}} = \frac{k'_P}{k_P} \frac{2\beta_P}{\beta_P + 1} \exp(-ik'_p h)$$

278
 279 where $\beta_P = (\rho_c V_{Pc})/(\rho_s V_{Ps})$. The amplitude $A_2'^{(j)} = -A_1'^{(j)}$ because of the stress-free condition at $\bar{z} = 0$. It is
 280 clear that all the amplitudes involved in the repeating unit in Figure 9 trace back to the amplitude $A_1^{(1)}$ and

281 that the latter refers to $A_1^{(1)}$. The vertical displacement at the free-surface can then be found as:

$$w(\bar{z} = 0) = ik'_p \left(\sum_{j=1}^{\infty} A_1^{(j)} - \sum_{j=1}^{\infty} A_2^{(j)} \right) = 2ik'_p A_1^{(1)} (1 - \zeta + \zeta^2 + \dots) = \frac{2ik'_p A_1^{(1)}}{1 + \zeta} \quad (12)$$

282 Substituting the terms ζ and $A_1^{(1)}$, and considering the free-field amplitude w_0 from Appendix B:

$$\frac{w}{w_0} = \frac{1}{\cos(k'_p h) + i\beta_p \sin(k'_p h)} \quad (13)$$

283 With a similar argument for the interface displacement w_1 , we can write:

$$\frac{w_1}{w_0} = \frac{1}{1 + i\beta_p \tan(k'_p h)} \quad (14)$$

284 From the latter, as anticipated from Equation 10, it is clear that the slab has a restraining effect on the
 285 interface displacement w_1 at the free-axial natural frequencies f_n of the elastic layer. This effect disappears
 286 at frequencies $2f_n$, when $w_1 = w_0$ if damping is neglected. The latter are frequencies for which the half-
 287 wavelength of the P-waves, or a multiple, matches the slab foundation thickness. In this case, the ratios
 w/w_0 and w_1/w_0 will converge to the same value, that is, unity in the undamped case. In general, for

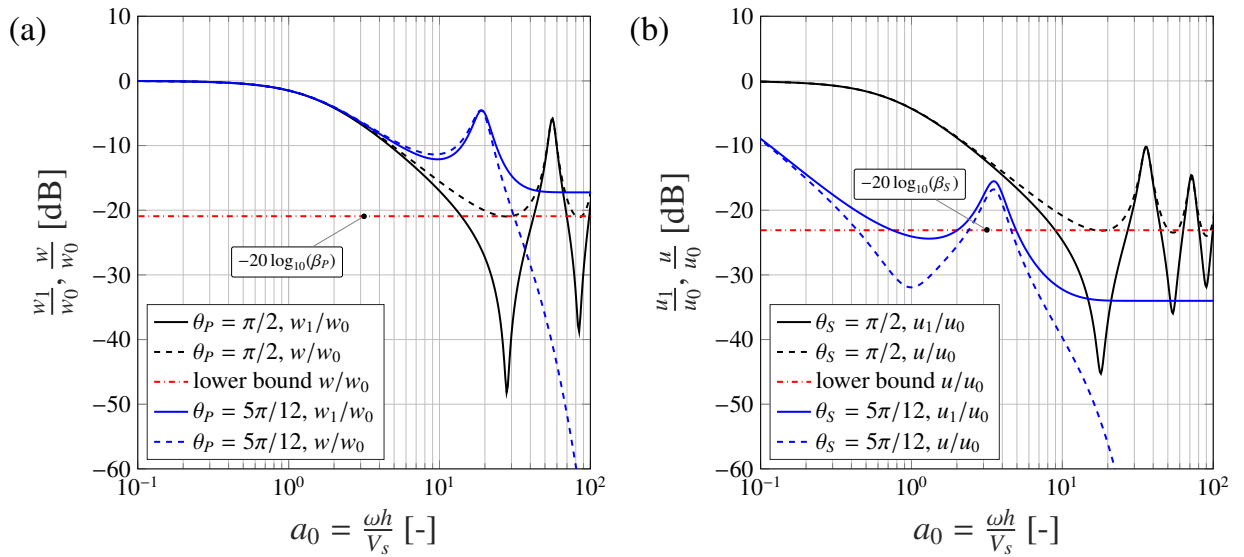


Figure 10: (a) The vertical displacement ratios at the free-surface (w/w_0) and soil-foundation interface (w_1/w_0) of an infinitely large slab foundation subjected to an incident P-wave at angles of $\pi/2$ and $5\pi/12$. (b) The equivalent horizontal displacement ratios due to an incident SV-wave. Benchmark parameter values (Table A.1).

288

289 the case of normal incidence, the free-surface displacement w is greater than, or at least equal to, the

290 interface displacement w_1 . The behaviour illustrated here depends only on the relative characteristics of
 291 the soil-slab system, that is, the impedance ratio β . This is generally greater than unity for both P-waves,
 292 $\beta_P = (\rho_c V_{Pc})/(\rho_s V_{Ps})$, and SV-waves, $\beta_S = (\rho_c V_{Sc})/(\rho_s V_s)$, so that the slab has an overall attenuating effect
 293 on the displacement amplitudes. In contrast to the interface displacement, for the free-surface displacement
 294 w such attenuation has a limiting value, again related to the impedance ratio β . This is shown in Figure 10a
 295 for normally incident P-waves and, in Figure 10b, for normally incident SV-waves. Figure 10 also shows
 296 the results for an incidence angle just off normal, that is, $\theta = 5\pi/12$.

297 It is clear that normal incidence represents a special case, when the slab behaves as a simple elastic
 298 layer, with no plate-like behaviour. For both P- and SV-waves, this ensures no amplification of the incident
 299 waves but limits the available attenuation. Once the incidence angle deviates from normal, even by a small
 300 amount, plate-like behaviour becomes significant, resulting in additional attenuation that increases with
 301 frequency.

302 5. Design of slab foundations against ground-borne vibration

303 This section considers the overall implications of this study for foundation design. In particular, it considers
 304 what guidance may be drawn regarding the design of a slab foundation to achieve a specific reduction in
 305 ground-borne vibration level. It is clear that the SSI associated with a slab foundation is complex, even
 306 for the simplified system considered here. Nevertheless, some useful guidance may be presented in the
 307 form of Figures 11, 12 and 13. These present a series of summary design plots that illustrate the influence
 308 of the dimensionless groups V_s/V_{sc} , ρ_s/ρ_c and θ , over the typical ranges associated with ground-borne
 309 vibration, for the three incident wave types (P, SV and Rayleigh). Typical values are assumed for the
 310 Poisson's ratios and damping loss factors of the slab and soil (Table A.1). The results are given in terms
 311 of the displacement ratios at the free-surface of the slab, for the fully-coupled boundary condition at the
 312 soil-foundation interface.

313 5.1. Surface Vibration Sources

314 In the case of a surface source, it is reasonable to assume that the incident vibration field will be dominated
 315 by Rayleigh waves, at least for locations remote from the source. In this case, the plots in Figure 13 sum-
 316 marise concisely the behaviour of the slab. It is clear that the density ratio has a relatively weak influence,
 317 with the level of attenuation varying by no more than 6 dB over the typical range $\rho_s/\rho_c = 0.6 - 1.2$. A
 318 relatively dense slab is therefore desirable but probably not worth pursuing actively, given the expense of

319 specialist high-density concrete. In contrast, the wave speed ratio has a strong influence, with the attenuation
320 varying by up to 25 dB over the typical range $V_s/V_{sc} = 0.05 - 0.2$. This sensitivity to the wave speed ratio
321 indicates the significance of the relative stiffness of the slab, and the importance of obtaining an estimate of
322 soil stiffness before relying on a particular slab for vibration mitigation.

323 For a given site, the controlling parameter is the slab thickness, which the designer is free to select in
324 order to provide acceptable attenuation at the lowest frequency of concern, in the knowledge that any higher
325 frequencies are attenuated further. For example, the results indicate that, for the benchmark soil properties,
326 typical of London Clay ($V_s = 200$ m/s, $\rho_s = 2000$ kg/m³), a concrete slab ($V_{sc} = 2284$ m/s, $\rho_c = 2500$ kg/m³)
327 of thickness 0.5 m would provide 24 dB of attenuation to vertical vibration at 25 Hz, increasing to 81 dB
328 at 250 Hz. The corresponding attenuation of horizontal vibration is slightly greater, ranging from 33 dB to
329 79 dB.

330 5.2. Buried Sources

331 In the case of a buried source, when body waves are more significant, the situation is more complex. As
332 with Rayleigh excitation, the attenuation provided by the slab is influenced only weakly by the density ratio
333 (again, by no more than 6 dB) but strongly by the wave speed ratio, this time by up to 38 dB. For P-waves,
334 the attenuation of vertical vibration is negligible for frequencies below $a_0 \approx 1$ (below 64 Hz for the 0.5 m
335 thick slab considered in Section 5.1) but then increases steadily with frequency. For horizontal vibration,
336 the low-frequency attenuation may be as high as 20 dB or more, but this depends strongly on the wave speed
337 ratio and incidence angle; the minimum attenuation may be close to zero.

338 For SV-waves, the most significant observation is that the slab may amplify low-frequency vibration,
339 particularly in the vertical direction, and by up to 10 dB depending on the wave speed ratio and incidence
340 angle. To avoid any amplification, the slab must be sufficiently thick to ensure a minimum non-dimensional
341 frequency of $a_0 \approx 2$. This corresponds to a slab thickness of at least 2.5 m, assuming the benchmark
342 properties and that the minimum frequency of concern is 25 Hz.

343 In general, for both P- and SV-waves, the vibration incidence angle is by far the most influential factor,
344 causing variations in attenuation of 60 dB or more. This sensitivity presents a challenge to the designer,
345 who may, at best, only estimate the incidence angle based on the approximate location of the source.

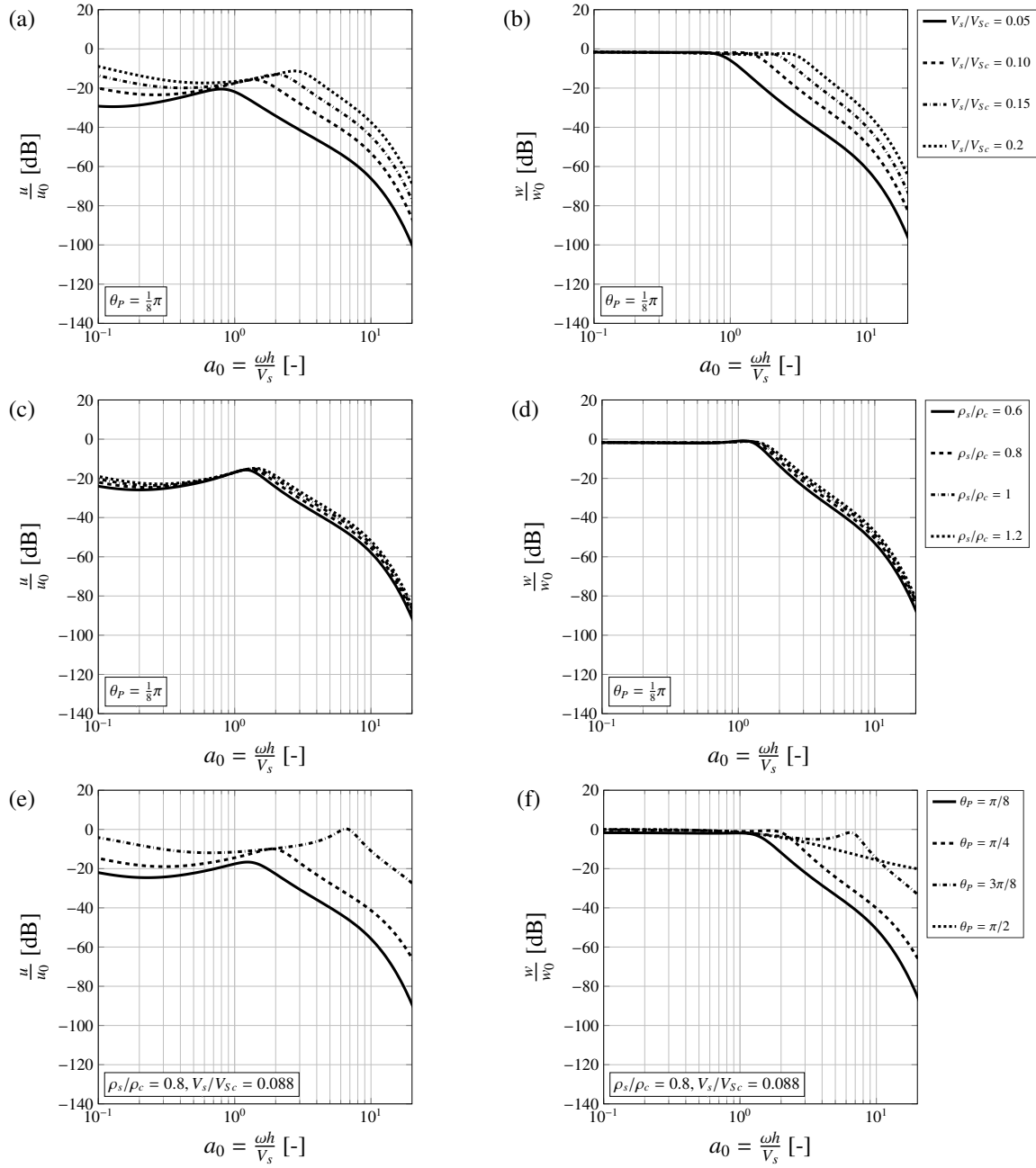


Figure 11: The horizontal (left) and vertical (right) displacement ratios at the free-surface of an infinitely large slab foundation subjected to an incident P-wave at an angle $\theta_p = \pi/8$, showing the influence of (a, b) the ratio V_s/V_{Sc} , for $\rho_s/\rho_c = 0.8$, and (c, d) the ratio ρ_s/ρ_c for $V_s/V_{Sc} = 0.088$. (e, f) The influence of the incidence angle θ_p for $V_s/V_{Sc} = 0.088$ and $\rho_s/\rho_c = 0.8$.

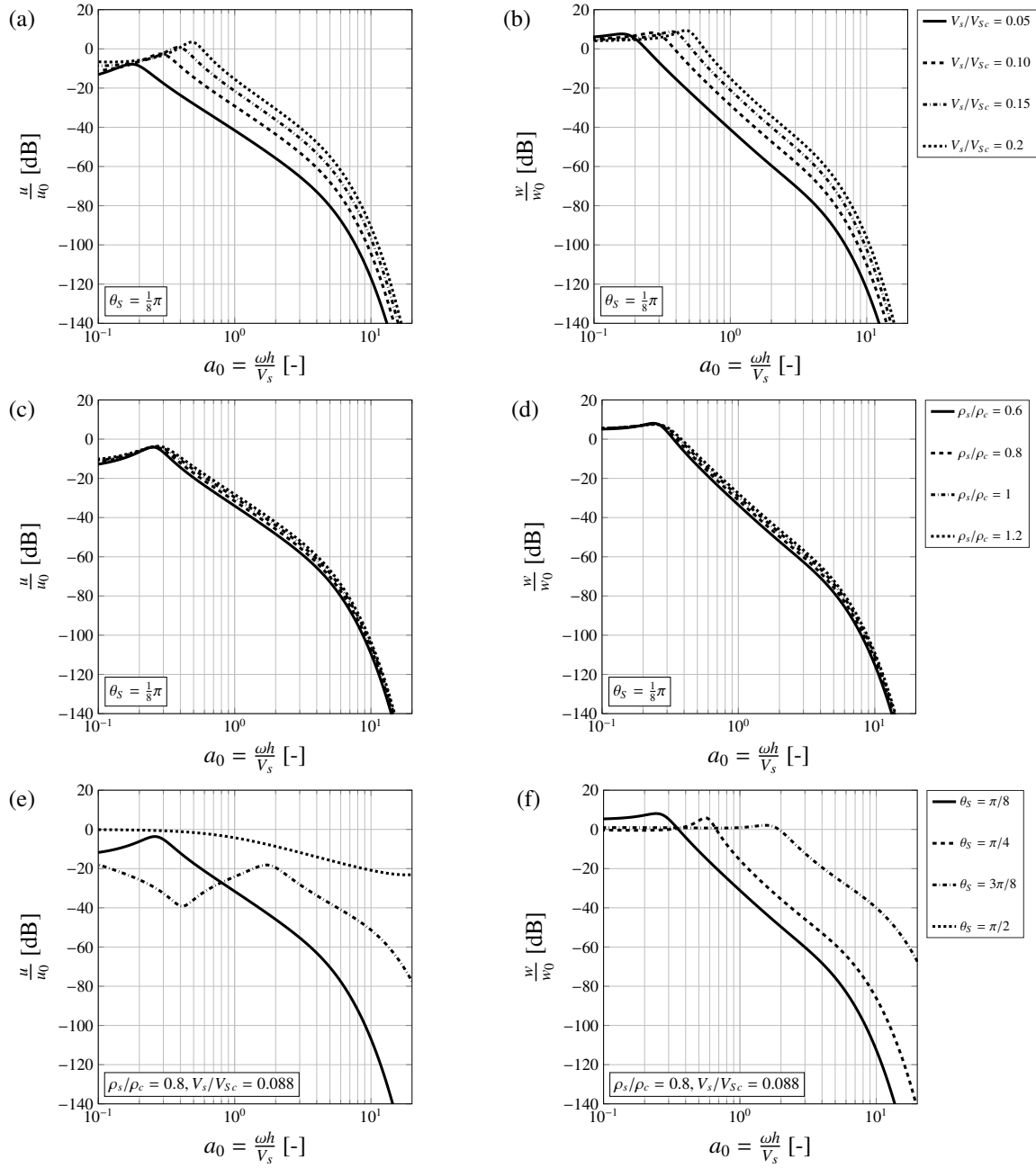


Figure 12: The horizontal (left) and vertical (right) displacement ratios at the free-surface of an infinitely large slab foundation subjected to an incident SV-wave at an angle $\theta_P = \pi/8$, showing the influence of (a, b) the ratio V_s/V_{Sc} , for $\rho_s/\rho_c = 0.8$, and (c, d) the ratio ρ_s/ρ_c for $V_s/V_{Sc} = 0.088$. (e, f) The influence of the incidence angle θ_S for $V_s/V_{Sc} = 0.088$ and $\rho_s/\rho_c = 0.8$.

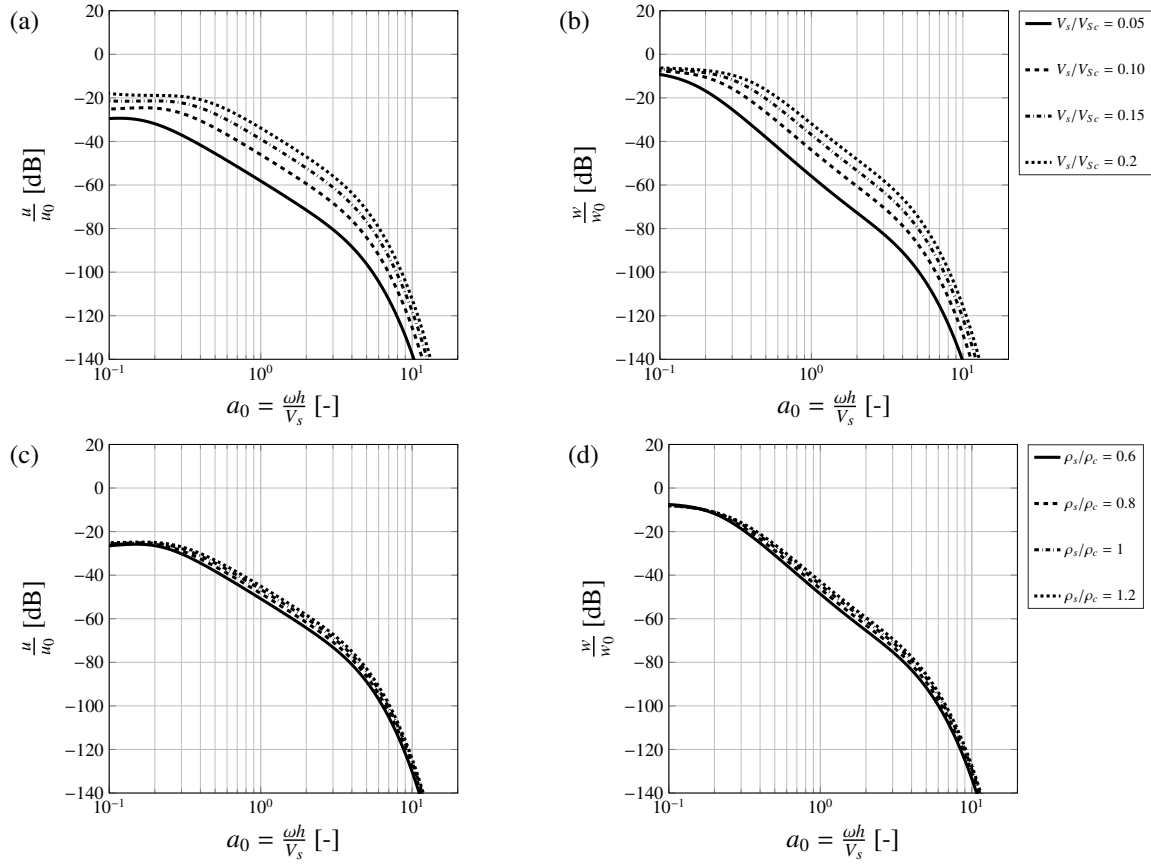


Figure 13: The horizontal (left) and vertical (right) displacement ratios at the free-surface of an infinitely large slab foundation subjected to an incident Rayleigh wave, showing the influence of (a, b) the ratio V_s/V_{Sc} , for $\rho_s/\rho_c = 0.8$, and (c, d) the ratio ρ_s/ρ_c for $V_s/V_{Sc} = 0.088$.

346 6. Conclusions

347 This paper has considered the fundamental dynamic behaviour of a concrete slab foundation excited by
 348 ground-borne vibration. By modelling the slab as an elastic layer overlying an elastic half-space, and
 349 using the corresponding dynamic stiffness matrices in a wave-based approach, the response of the slab to
 350 incident Rayleigh, P- and SV-waves has been explored. Having referred to previous work that assumes
 351 plate-like behaviour, it is clear that thin-plate theory alone is insufficient for modelling the effect of a slab
 352 foundation on an existing ground vibration field over the full frequency range of interest. Furthermore,
 353 models that assume a relaxed boundary condition at the soil-slab interface have been found to underestimate
 354 the attenuation provided by the slab by at least 10 dB in the case of Rayleigh wave excitation.

355 The study has illustrated the importance of the coincidence phenomenon, in which the free-flexural
356 wavelength of the slab coincides with the horizontal wavelength of the incident wave-field. For frequencies
357 above coincidence, considerable attenuation can be achieved, but this depends on a number of dimensionless
358 groups, as illustrated in a series of summary design plots. The least significant of these is the soil-slab
359 density ratio, which, for all three wave types, has only a weak influence on the level of attenuation provided.
360 In contrast, the relative stiffness of the slab (expressed as a ratio of wave speeds) has a strong influence. For
361 a given site, the controlling parameter available to the foundation designer is the slab thickness. In the case
362 of Rayleigh excitation, the design plots summarise concisely the behaviour of the slab, indicating that at
363 least 20 dB of attenuation might reasonably be assumed for a typical, 0.5 m thick slab (considerably more at
364 high frequencies). For P- and SV-waves, the situation is more complex because the coincidence frequencies
365 associated with these wave types are always higher than that of the Rayleigh wave, which shifts the region
366 of strong attenuation to relatively high frequencies. The overall attenuation therefore depends strongly on
367 the slab stiffness and, importantly, the incidence angle of the vibration.

368 A fundamental assumption of this study is that the ground may be represented by a homogeneous half-
369 space, but this is often not the case due to soil layering. Layering introduces additional wave reflections and
370 mode conversions, and this is likely to reduce the attenuation provided by the slab. The extent to which this
371 is the case remains the subject of future research.

372 **7. Acknowledgements**

373 The authors gratefully acknowledge the support of Prof. Geert Degrande and colleagues in KU Leuven,
374 who provided the Elastodynamics Toolbox for this work, together with the financial support of WSP and
375 EPSRC (RG75686) for the studentship of the first author.

376 **References**

- 377 [1] H. Saurenman, J. Phillips, In-service tests of the effectiveness of vibration control measures on the BART rail transit system,
378 *J. Sound Vib.* 293 (2006) 888–900.
- 379 [2] J. P. Talbot, Lift-over crossings as a solution to tram-generated ground-borne vibration and re-radiated noise, *Proc. Inst.*
380 *Mech. Eng. Part F J. Rail Rapid Transit* 228 (2014) 878–886.
- 381 [3] P. Coulier, V. Cuéllar, G. Degrande, G. Lombaert, Experimental and numerical evaluation of the effectiveness of a stiff wave
382 barrier in the soil, *Soil Dyn. Earthq. Eng.* 77 (2015) 238–253.
- 383 [4] D. J. Thompson, J. Jang, M. G. R. Toward, M. F. M. Hussein, A. Dijkmans, P. Coulier, G. Degrande, G. Lombaert,
384 Mitigation of railway-induced vibration by using subgrade stiffening, *Soil Dyn. Earthq. Eng.* 79 (2015) 89–103.

- 385 [5] J. P. Talbot, Base-isolated buildings: towards performance-based design, *Proc. Inst. Civ. Eng. - Struct. Build.* 169 (2016)
386 574–582.
- 387 [6] B. R. Barben, L. M. Hanagan, Investigation of a slab on grade supporting sensitive equipment, in: F. N. Catbas (Ed.), *Mech.*
388 *Biol. Syst. Mater. - Proc. 2013 Annu. Conf. Exp. Appl. Mech.*, Conference Proceedings of the Society for Experimental
389 *Mechanics Series*, Springer New York LLC, Orlando, 2014, pp. 53–60.
- 390 [7] E. Kausel, Early history of soil-structure interaction, *Soil Dyn. Earthq. Eng.* 30 (2010) 822–832.
- 391 [8] H. L. Wong, J. E. Luco, Dynamic response of rectangular foundations to obliquely incident seismic waves, *Earthq. Eng.*
392 *Struct. Dyn.* 6 (1978) 3–16.
- 393 [9] J. Qian, D. E. Beskos, Harmonic wave response of two 3-D rigid surface foundations, *Soil Dyn. Earthq. Eng.* 15 (1996)
394 95–110.
- 395 [10] M. Iguchi, J. E. Luco, Dynamic response of flexible rectangular foundations on an elastic half-space, *Earthq. Eng. Struct.*
396 *Dyn.* 9 (1981) 239–249.
- 397 [11] D. J. Thompson, G. Kouroussis, E. Ntotsios, Modelling, simulation and evaluation of ground vibration caused by rail vehicles,
398 *Veh. Syst. Dyn.* 0 (2019) 1–48.
- 399 [12] L. Auersch, Response to harmonic wave excitation of finite or infinite elastic plates on a homogeneous or layered half-space,
400 *Comput. Geotech.* 51 (2013) 50–59.
- 401 [13] M. Schevenels, G. Degrande, S. François, EDT: An ElastoDynamics Toolbox for MATLAB, in: *Proc. Inaug. Int. Conf. Eng.*
402 *Mech. Inst.*, Minneapolis, U.S.A, pp. 82–82.
- 403 [14] M. Schevenels, S. François, G. Degrande, EDT: An ElastoDynamics Toolbox for MATLAB, *Comput. Geosci.* 35 (2009)
404 1752–1754.
- 405 [15] The Mathworks Inc., MATLAB 2018b, Natick, Massachusetts, United States, 2018.
- 406 [16] D. Connolly, G. Kouroussis, P. Woodward, P. Alves Costa, O. Verlinden, M. Forde, Field testing and analysis of high speed
407 rail vibrations, *Soil Dyn. Earthq. Eng.* 67 (2014) 102–118.
- 408 [17] H. E. M. Hunt, Measurement and modelling of traffic-induced ground vibration, Ph.d. dissertation, University of Cambridge,
409 1988.
- 410 [18] E. Kausel, J. M. Roesset, Stiffness matrices for layered soils, *Bull. Seismol. Soc. Am.* 71 (1981) 1743–1761.
- 411 [19] J. P. Talbot, H. E. M. Hunt, Isolation of Buildings from Rail-Tunnel Vibration: a Review, *Build. Acoust.* 10 (2003) 177–192.
- 412 [20] F. Fahy, P. Gardonio, *Sound and Structural Vibration*, Elsevier, 2006.
- 413 [21] K. Graff, *Wave motion in elastic solids*, Oxford: Clarendon, 1975.
- 414 [22] M. Schevenels, S. François, G. Degrande, EDT: elastodynamics toolbox for matlab, Technical Report July, Katholieke Uni-
415 versiteit Leuven, Leuven, 2010.
- 416 [23] W. Ewing, W. Jardetzky, F. Press, *Elastic waves in layered media*, McGraw-Hill, 1957.

417 **Appendix A. Benchmark Data**

Table A.1: Benchmark Data

Soil	Shear modulus	$G_s = 8 \times 10^7 \text{ N/m}^2$	Slab	Young's Modulus	$E_c = 3 \times 10^{10} \text{ N/m}^2$
	Shear wave speed	$V_s = 200 \text{ m/s}$		Poisson ratio	$\nu_c = 0.15$
	Poisson ratio	$\nu_s = 1/3$		Mass density	$\rho_c = 2500 \text{ kg/m}^3$
	Mass density	$\rho_s = 2000 \text{ kg/m}^3$		Damping loss factor	$\eta_c = 0.1$
	Damping loss factor	$\eta_s = 0.1$		Width	$b = 2, 4, 10, 20, \infty \text{ m}$
				Thickness	$t = 0.7 \text{ m}$

418 **Appendix B. Displacement field u_0 and w_0 for incoming P- and SV-waves**

419 Consider an elastic and homogeneous half-space (Figure 1a) subjected to incident, plane P- or SV-waves
 420 at an angle θ_P or θ_S respectively. The wave equations for the P-SV problem can be written in terms of the
 421 potentials Φ and H_y [21, 23]:

$$\nabla^2 \Phi = \frac{1}{V_P^2} \frac{\partial^2 \Phi}{\partial t^2}; \quad \nabla^2 H_y = \frac{1}{V_S^2} \frac{\partial^2 H_y}{\partial t^2} \quad (\text{B.1})$$

422 The solution can be found in terms of incoming and outgoing waves of amplitude A_1, B_1 and A_2, B_2 respec-
 423 tively, with reference to the potentials Φ and H_y :

$$\Phi(x, z, t) = A_1 \exp \{i(\omega t - k_P \cos \theta_P x + k_P \sin \theta_P z)\} + A_2 \exp \{i(\omega t - k_P \cos \theta_P x - k_P \sin \theta_P z)\} \quad (\text{B.2a})$$

424

$$H_y(x, z, t) = B_1 \exp \{i(\omega t - k_S \cos \theta_S x + k_S \sin \theta_S z)\} + B_2 \exp \{i(\omega t - k_S \cos \theta_S x - k_S \sin \theta_S z)\} \quad (\text{B.2b})$$

425 The case of an incident P- or SV-wave can be explored by considering a known amplitude, A_1 or B_1 , and
 426 imposing the stress-free condition at the free surface of the half-space to obtain the amplitudes A_2 and B_2
 427 of the reflected waves:

Case of incident P-wave

$$\frac{A_2}{A_1} = \frac{\sin 2\theta_P \sin 2\theta_S - \gamma^2 \cos^2 2\theta_S}{\sin 2\theta_P \sin 2\theta_S + \gamma^2 \cos^2 2\theta_S} \quad (\text{B.3})$$

$$\frac{B_2}{A_1} = \frac{2 \sin 2\theta_P \cos 2\theta_S}{\sin 2\theta_P \sin 2\theta_S + \gamma^2 \cos^2 2\theta_S} \quad (\text{B.4})$$

428

Case of incident SV-wave

$$\frac{B_2}{B_1} = \frac{\sin 2\theta_P \sin 2\theta_S - \gamma^2 \cos^2 2\theta_S}{\sin 2\theta_P \sin 2\theta_S + \gamma^2 \cos^2 2\theta_S} \quad (\text{B.5})$$

$$\frac{A_2}{B_1} = -\frac{\gamma^2 \sin 4\theta_S}{\sin 2\theta_P \sin 2\theta_S + \gamma^2 \cos^2 2\theta_S} \quad (\text{B.6})$$

429 where $\gamma = V_P/V_S$ is the ratio between the pressure and shear wave speeds in the half-space. Finally, the
 430 displacement amplitude $\mathbf{u}_0 = [u_0 \ w_0]$ at the free surface of the half-space can be retrieved from the poten-
 431 tials as:

Case of incident P-wave

$$\frac{u_0}{-ik_P A_1} = \cos \theta_P \left(1 + \frac{A_2}{A_1}\right) - \gamma \sin \theta_S \frac{B_2}{A_1} \quad (\text{B.7})$$

$$\frac{w_0}{-ik_P A_1} = \sin \theta_P \left(\frac{A_2}{A_1} - 1\right) + \cos \theta_P \frac{B_2}{A_1} \quad (\text{B.8})$$

Case of incident SV-wave

$$\frac{u_0}{-ik_P B_1} = \cos \theta_P \frac{A_2}{B_1} + \gamma \sin \theta_S \left(1 - \frac{B_2}{B_1}\right) \quad (\text{B.9})$$

$$\frac{w_0}{-ik_P B_1} = \sin \theta_P \frac{A_2}{B_1} + \cos \theta_P \left(1 + \frac{B_2}{B_1}\right) \quad (\text{B.10})$$

433