"Nota il moto del livello dell'acqua, il quale fa a uso de'capelli, che ãnno due moti, de'quali l'uno attēde al peso del uello, l'altro al liniamento delle volte; cosi l'acqua à le sue volte revertiginose, delle quali una parte attende al inpeto del corso principale, l'altro attēde al moto incidēte e reflesso".

Leonardi da Vinci

THE BEHAVIOUR OF SIDE WEIRS

IN

PRISMATIC RECTANGULAR CHANNELS.

A Thesis

Presented for the Degree of Doctor of Philosophy

of

Glasgow University

by

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To Dr. William Hunter and his other colleagues, the author expresses his thanks for their tolerance of the occasional flooding which occurred during some of the tests (Fig. 5.7).

Mr. R. Raynor is also due thanks for his assistance in erecting and maintaining the apparatus.

CONTENTS.

Page.

7.4.2. Estimation of the Accuracy Obtainable.

LIST OF FIGURES AND TABLES.

List of Figures. To face page.

List of Figures — To face page.

List of Tables. — To face page.

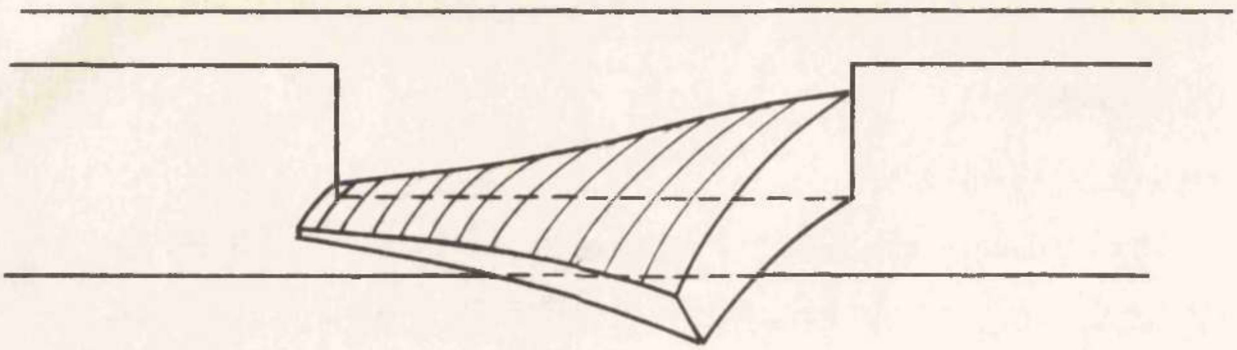


Fig. 1.1

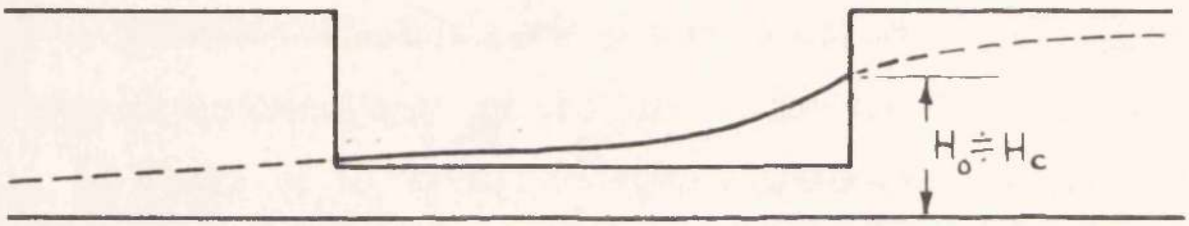


Fig. 1.2 CASE I

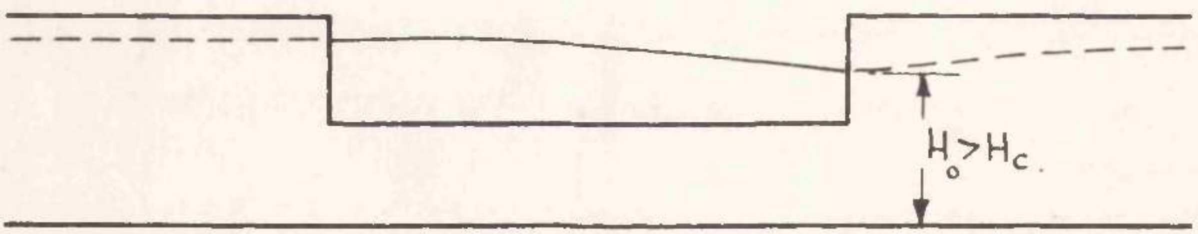


Fig. 1.3 CASE II

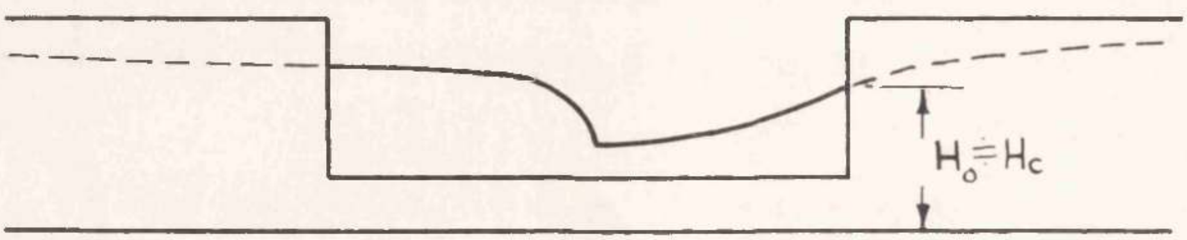


Fig. 1.4 CASE III

1. Introduction.

1.1. The Side Weir.

An important problem in flow in open channels is the removal of excess water from the channel, and a widely used method of achieving this is by means of a weir formed in the side of the channel; so that when the level of the water rises above the crest of the weir, discharge from the channel occurs. Such a device is termed a side weir (Fig. 1.1).

Variants of this device consist of a weir on both sides of the channel in order that the overall length of the construction may be cut down; alteration of the channel section downstream of the weir etc. These and other variants have been described by Lloyd-Davies and others[1].

The dangers of errors in the design of a side weir are obvious. In the case of open aqueducts, overtopping will cause flooding and may, by scour, damage the aqueduct. In sewers, where side weirs are used to a great extent, surcharging of the downstream section can, and often does, occur, and the pressures so developed may cause damage to the sewer itself.

1.2. Available Information.

An examination of the available information on side weirs reveals an abundance of formulae and a great scarcity of experimental work. The literature on the subject contains such statements as:-

"If side weirs are used and no proper throttle is provided errors may exceed 200%"[2].

No indication is given as to the nature of a "proper throttle".

[1] The numbers refer to the bibliography, page 65 .

"In using this formula, assume a reasonable value for h, depending on the permissible fluctuations in maximum water surface downstream and solve for ℓ. If too long a weir is obtained for an economic design, use a larger value for h" [3].

Study of formulae used in British and American practice shows that these formulae are purely empiric and often contradictory, having been developed before channel flow phenomena were completely understood. Continental formulae, while taking into account modern concepts of channel flow, suffer from the lack of experimental data.

For the above reasons, it was felt that the subject offered scope for study which, if successful, would yield useful results.

1.3. Preliminary Investigations.

Due to the contradictions of the various formulae, a small pilot model of a side weir in a rectangular channel was constructed in order that a qualitative study could be made of the flow, before any major experiments were carried out. The model results gave a very clear indication of the behaviour of a side weir and showed that the mode of motion is dependent on the properties of the downstream channel. In all, three distinct modes of motion were noted:-

Case I. Fig. 1.2. Rapid flow in the main channel with the depth of flow decreasing downstream along the weir length. With a mild slope in the downstream channel, a hydraulic jump occurred downstream of the weir section, its position being dependent on the downstream backwater curve. The depth of flow in the main channel at the commencement of the weir section was approximately the critical depth, and obviously this case was not encountered where the weir height was greater than the critical depth.

Case II. Fig. 1.3. Tranquil flow in the main channel with the depth of flow increasing along the weir section. This case was met with if the weir height was greater than the critical depth or if the downstream channel was dammed back and the length of the weir was relatively short.

Case III. Fig. 1.4. Rapid flow at the start of the weir section, a hydraulic jump in the weir section and tranquil flow after the jump. This case arose when the damming back of the downstream channel was not sufficient to cause Case II flow to develop.

1.4. Main Investigations.

From the experience gained with the pilot apparatus, experiments were carried out in a prismatic, rectangular channel whose width could be varied up to 9 in. The weir length and depth could be varied, thus allowing a study to be made of the effect of varying the dimensions of the weir section. The discharge and head variation were investigated over a large range and it was found that the flow again reproduced the three modes of motion described above.

From the results of these investigations, and from a theoretical analysis, semi-empirical formulae have been evolved covering each case, for rectangular channels.

These formulae provide a complete explanation of side weir phenomena and, it is felt, give a reasonably accurate basis for design purposes.

2. Historical Review.

2.1. General.

2.1.1. Introduction.

Existing formulae, giving the relationships between quantity, depths of flow and lengths for side weirs, may be divided into two distinct classes; those advanced before the concept of specific energy, coupled with the principle of equating force to rate of change of momentum [4], gave a satisfactory explanation of local phenomena in channel flow, and those based on these concepts. The former are generally empiric, or based on incorrect assumptions, while the latter, although dimensionally correct, are supported by insufficient experimental work, and the variability of experimental coefficients with the weir dimensions, and flow conditions, has not been studied.

Practically all the experimental work and theoretical analysis has been confined to rectangular, prismatic channels of mild or zero slope with horizontal weir crests.

2.1.2. Notation and Units.

The various formulae discussed have been converted into the notation of this paper (page 62) and where additional symbols are required, they are defined in the text. In addition, where possible, the formulae have been adjusted to render them independent of any one system of units. Where this was not possible the units applicable are noted.

2.2. Formulae Unrelated to Channel Flow Phenomena.

2.2.1. Fruling [5].

This formula is unsupported by experimental work, but is still quoted in text books:-

$$Q_W = 0.707\sqrt{g}\ L\ (H - D)^{3/2}$$

No indication is given as to how H is to be determined.

2.2.2. Parmley [6].

By assuming a constant velocity, V, in the main channel and a discharge formula of the Francis type for the elements of length of the weir section, Parmley arrived at the following expression:-

$$L = 0.106\sqrt{g}\quad WV\ \left(\frac{1}{(H - D)^{\frac{1}{2}}} - \frac{1}{(H_o - D)^{\frac{1}{2}}}\right)$$

The formula is dimensionally correct, but is unsupported by experimental work. As will be shown, the assumption of constant velocity is wrong and no indication is given as to how its value is to be found. L is only positive if $H < H_o$.

2.2.3. Coleman and Smith [7].

Experiments were carried out on a channel 4¾ inches wide and 6 inches deep, with a weir plate set, as far as can be ascertained, 1 inch from the channel bottom. In all cases reported the depth of flow decreased from upstream to downstream, although it was noted that, under certain conditions, a standing wave was formed downstream. Thus the experiments were confined to rapid flow (Case I) conditions in the weir section.

Three formulae were advanced, the first:-

$$Q_W = 0.671\ L^{0.72}\ (H_o - D)^{1.645}\quad \text{cusecs},$$

for a knife edge weir, being entirely empiric. It will be noted, however, that the formula is almost dimensionally correct. Experiments with rounded weir crests gave slightly varying values of the coefficients.

By the erroneous assumption that the discharge over the weir varies as the channel width, a second formula:-

$$Q_w = 1.65 \; W \; L^{0.72} \; (H_o - D)^{1.645} \text{ cusecs,}$$

was empirically deduced. This could have been obtained from the first by dividing the coefficient, 0.671 by W, and this inclusion of W renders the expression dimensionally incorrect. This is not apparent since W was constant throughout the tests.

From a theoretical analysis, a semi-empirical formula:-

$$L = 0.55 \; W \; Vo \; (H - D)^{0.13} \left\{ \frac{1}{(H - D)^{\frac{3}{2}}} - \frac{1}{(H_o - D)^{\frac{3}{2}}} \right\} \text{ feet,}$$

was obtained.

No details were given as to the values of H_o, H and V_o which should be used.

It was noted that alteration of the hydraulic gradient of the upstream and downstream channels had no effect on the weir discharge. In the light of modern knowledge this is obvious since the flow condition obtaining over the weir section is rapid, and the depth of flow at the entry section is approximately critical.

The third formula, together with that of Parmley, are those chiefly used in British practice [2], although it is known that erroneous results are obtained.

2.2.4. Engels [8]. ✗

By assuming constant specific energy of flow, Engels obtained the relationship:-

$$H - H_o = \frac{Q_o^2}{2g \; W^2 H_o^2} - \frac{Q^2}{2g \; W^2 H^2}$$

He recognised that H must be governed by the downstream channel conditions and hence, knowing Q_o, Q and H, a value of H_o could be obtained. He then assumed the discharge to be:-

$$Q_W = \tfrac{2}{3}\, C_d \sqrt{2g}\, (H - D)^y\, L^x$$

$\sqrt{2g}\,(H-D)^y L^x$

where C_d is a coefficient of discharge and:-

$$x + y = 2.5$$

in order that the expression is dimensionally correct.

From experimental analysis the equation became:-

$$Q_W = \tfrac{2}{3}\, C_d \sqrt{2g}\, \sqrt[3]{(H - D)^5\, L^{2.5}}$$

the values of $\tfrac{2}{3} C_d$ being 0.57 for a rounded weir crest and 0.49 for a sharp crest. The experimental work was carried out under conditions of tranquil flow (Case II).

Some experiments on weirs having a contracted downstream channel gave the following formula:-

$$Q_W = \tfrac{2}{3}\, C_d \sqrt{2g}\, \sqrt[3]{(H - D)^{2.7}\, L^{4.8}}$$

$\tfrac{2}{3} C_d$ having the same values as before. It is stated that neither the ratio of upstream and downstream channel widths, nor whether the transition occurs on the weir side or on the other, influence this formula.

2.2.5. Forchheimer [9].

Proceeding from the same assumption of constant specific energy, but introducing a term for frictional losses, Forchheimer obtained the following relationship between the upstream and downstream heads:-

$$H - H_o = \frac{Q_o^2 - Q^2}{2g\, A_m^2} - \left(\frac{Q_o + Q}{2A_m}\right)^2 \cdot \frac{n^2}{M^{1.4}}\, L$$

where n is a frictional coefficient, Am is the mean area of the main channel and M is the mean hydraulic mean radius. The application of the ordinary rectangular weir formula, using a mean head then gave

$$Qw = \frac{2}{3} C_d \sqrt{2g} \left(\frac{H_o + H}{2} - D\right)^{3/2} L.$$

There is no evidence of any experimental support for the formula and no value is given for C_d, the coefficient of discharge.

2.2.6 Velatta [10].

From experiments carried out in a level rectangular channel of variable dimensions (W - 12.0, 24.5 and 40.5 cm; D - 11.0, 11.3 and 16.0 cm; L - 20.7 - 155.4 cm.) Velatta, after examining various formula, found that the discharge formula due to Forchheimer (2.2.5) gave the best results. He obtained a value of $C_d = 0.64$, giving a scatter of ± 9.3% with a discard of 12% of the tests. In place of the constant specific energy relation of Forchheimer he advanced a dimensionally incorrect, empiric formula in order to complete the solution:-

$$L = 1.9 D + 0.42 \frac{H - H_o}{H + H_o + 2D} \text{ metres.}$$

He also concluded that flow with a decreasing depth of flow along the weir from upstream to downstream (Case I, Rapid Flow)

"........ cannot be other than a particular case and difficult to reproduce, obtaining under conditions which the English experimenters Coleman and Smith have not made clear".

This conclusion is not surprising since in each of his 63 tests the weir height was great compared with the depth of flow over it and tranquil flow was always obtained.

2.3. Formulae Related to Channel Flow Phenomena.

2.3.1. Favre [11].

Favre considered the general case of water entering or leaving a channel of variable cross section and, by applying the principle of equating force to rate of change of momentum between two sections an infinitesimal distance apart and making allowance for friction losses, obtained the following differential equation:-

$$-dH = + \frac{V'^2}{k^2 M^{4/3}} dL' + \frac{V'dV'}{g} + \left(1 - \frac{V^*}{V'}\right) \frac{Qdq}{gA^2}$$

where k is a frictional coefficient, M is the hydraulic mean radius and V^* is the component, in the direction of flow in the main channel, of the velocity of the quantity of water being discharged from, or received into, the main channel.

This equation can be reduced to a finite difference form:-

$$-\Delta H' = \frac{V_m^2}{k^2 M^{4/3}} \Delta L' + \frac{V_2^2 - V_1^2}{2g} + \left(1 - \frac{V^*}{V_m}\right) \frac{Q_2^2 - Q_1^2}{2g \ A_m^2}$$

where $\Delta H'$ is the increase in depth of flow between sections 1 and 2, distance $\Delta L'$ apart, and where Vm and Am are the mean velocity and mean area of flow between sections 1 and 2.

It will be shown later that, due to a supposedly second order approximation, the formula is only applicable to prismatic channels.

It may be noted that, in the case of water being discharged in such a manner that $V^* =$ Vm, the formula reduces to one similar to that advanced by Forchheimer (2.2.5). Again by dropping the frictional term, a reduction to the assumption of constant specific energy made by Engels (2.2.4) results.

The equation can not be solved unless the quantity entering or leaving the main channel in the length $\Delta L'$ and its velocity component in the direction of flow in the main channel are known. If these are independent variables, as in the case of a collecting channel being supplied in such a manner that the supplying flow is not controlled by the main channel, the solution is straight forward. If they are dependent variables, as in the case of discharge from the main channel, by holes, siphons or side weirs, or in the case of an interfering supplying flow, another expression relating quantity, head and length must be sought in order to obtain a solution. This is not made clear. The complexity of the equation prevents a ready discussion of the possibilities of flow.

The formula was verified on several occasions for collectors, notably on studies of the Boulder Dam spillways [12], and Favre and Brandle [13] carried out a further series of tests to check on the frictional term in the formula. The experimental work was carried out in a level rectangular channel, 20.06 cm. in width. Weirs 200 cm. long were let into the walls of the main channel on both sides their crests being semicircular of 4 cm. radius and at a height of 21.96 cm. from the channel bottom. No variation was made in these dimensions throughout the tests.

Preliminary tests consisted of determining the relationship between the frictional coefficient and the velocity in the main channel and in determining the relationship between the head over the weirs and the discharge. This latter was accomplished by allowing discharge into the main channel over the weirs from supply channels, which were of such large dimensions that a constant head was obtained over the length of the weirs. Both weirs gave the same calibration curve.

A series of 9 tests was carried out with the main channel acting as a collector, being supplied by the large channels via the weirs. In this case the quantity of water entering the channel per unit length was constant, being given by the calibration tests and a straight forward solution for the finite difference equation was obtained. Theoretical and experimental results agreed fairly well.

A second series of 9 tests was carried out with the weirs functioning as side weirs, discharge taking place over both weirs, and a third series of 10 tests, with discharge taking place over one weir only. In both cases the flow in the main channel was tranquil. In calculating the theoretical curves a trial and error method had to be adopted, taking the discharge for a length $\Delta L'$ from the mean head over the weir and the calibration curve.

In the side weir tests, theoretical and experimental results agreed very well, the inclusion of the frictional term making very little difference.

2.3.2. De Marchi [14, 15]

In a theoretical analysis, De Marchi assumed that the specific energy remained constant along the length of a side weir and that the discharge per unit length of the weir at any section was proportional to the head over the weir at that section raised to the power 3/2. He discussed the implications of this in prismatic channels and arrived at the following equations governing the behaviour of the weir.

$$Qw = S \sqrt{\{2g \ (E_o - 2H)\}}$$

$$\frac{d\,Qw}{d\,L} = \frac{2}{3}\,Cd\,\sqrt{2g}\ (H - D)^{3/2}$$

where E_o is the specific energy of flow at the entrance to the weir section:-

$$E_o = \frac{V_o^2}{2g} + H_o.$$

The mode of motion was shown to be dependent on the downstream channel properties and on the weir height, but the full implications were not discussed, three modes of motion being shown to be possible:-

A. Tranquil flow in the upstream and downstream channels with

$$D > \tfrac{2}{3} E_o \text{ and } H > H_o > H_c$$

B. Rapid flow in the upstream and downstream channels with

$$H_c > H_o > H$$

C. Tranquil flow in the upstream channel, critical flow at the entry section and a hydraulic jump in the downstream channel with

$$D < \tfrac{2}{3} E_o \text{ and } H_o = H_c > H$$

A graphical method of obtaining a step by step solution was indicated, and for the case of a rectangular channel an analytical solution of the problem was obtained giving:-

$$L = \frac{W}{3 C_d} \left\{ \phi\left(\frac{H}{E_o}\right) - \phi\left(\frac{H_o}{E_o}\right) \right\}$$

where

$$\phi\left(\frac{H}{E_o}\right) = \frac{2E - 3D}{E - D} \sqrt{\left(\frac{E_o - H}{H - D}\right)} - 3 \text{ arcsin } \sqrt{\left(\frac{E_o - H}{E_o - D}\right)}$$

in obtaining this formula C_d was assumed to be constant. In the case of tranquil flow $\phi\left(\frac{H}{E_o}\right)$ changes very slowly as H tends to E_o and, therefore, large errors are likely under these conditions in determining L .

This theory was investigated by Gentilini [16] in a series of 12 tests carried out in a rectangular level channel of 21.4 cm. width. The weir crest was knife edged and set at varying heights from 5.09 cm. to 25.23 cms. above the channel bottom. Two lengths of weir were used, 106.8 cm. and 49.9 cm. Of the tests 8 gave tranquil flow, and 4 gave rapid flow conditions..

The theory gave fair agreement with the results for the tranquil flow tests, taking C_d as 0.60 but in the rapid flow tests no such agreement was obtained and Gentilini's conclusion was that the theory applied when:-

$$\frac{H}{E_o} > 0.9 \quad \text{and} \quad \frac{D}{E_o} > 0.75$$

Citrini [17] pointed out that an approximation could be applied which simplified calculation, and Ruggiero [18] showed that, for tranquil flow, a closer agreement could be obtained (with Gentilini's results) if C_d was assumed to vary in accordance with the Rehbock formula. Ruggiero also attempted to investigate the rapid flow results, but since there are only four tests, his theory, besides being based on a wrong assumption, is of no great importance.

FIGURE 3.1. SIDE WEIR APPARATUS

3. Experimental Apparatus.

3.1. Pilot Apparatus.

3.1.1. General Arrangement.

The apparatus consisted of a level, aluminium-lined, rectangular, wooden channel 3 in. wide by 3 in. deep and 12 ft. long. In the centre section, the wood of one side was cut away and a weir formed in the exposed aluminium wall, with its crest at a height of 3/4 in. above the channel bottom. The vertical sides and the weir crest were worked to a knife edge form. The initial tests were carried out on a weir length of 15 in., and the length was then extended by cutting away the aluminium to lengths of 22½ in. and 30 in. An aluminium faced wooden batten 10 ft. long provided the means of varying the channel width. The water discharging over the weir was passed to a collecting channel and thence to the sump of the laboratory supply system.

3.1.2. Measurement of Depths of Flow.

A straight-edge, carrying a depth gauge, was supported over the main channel in such a way that depths of flow could be measured at any point.

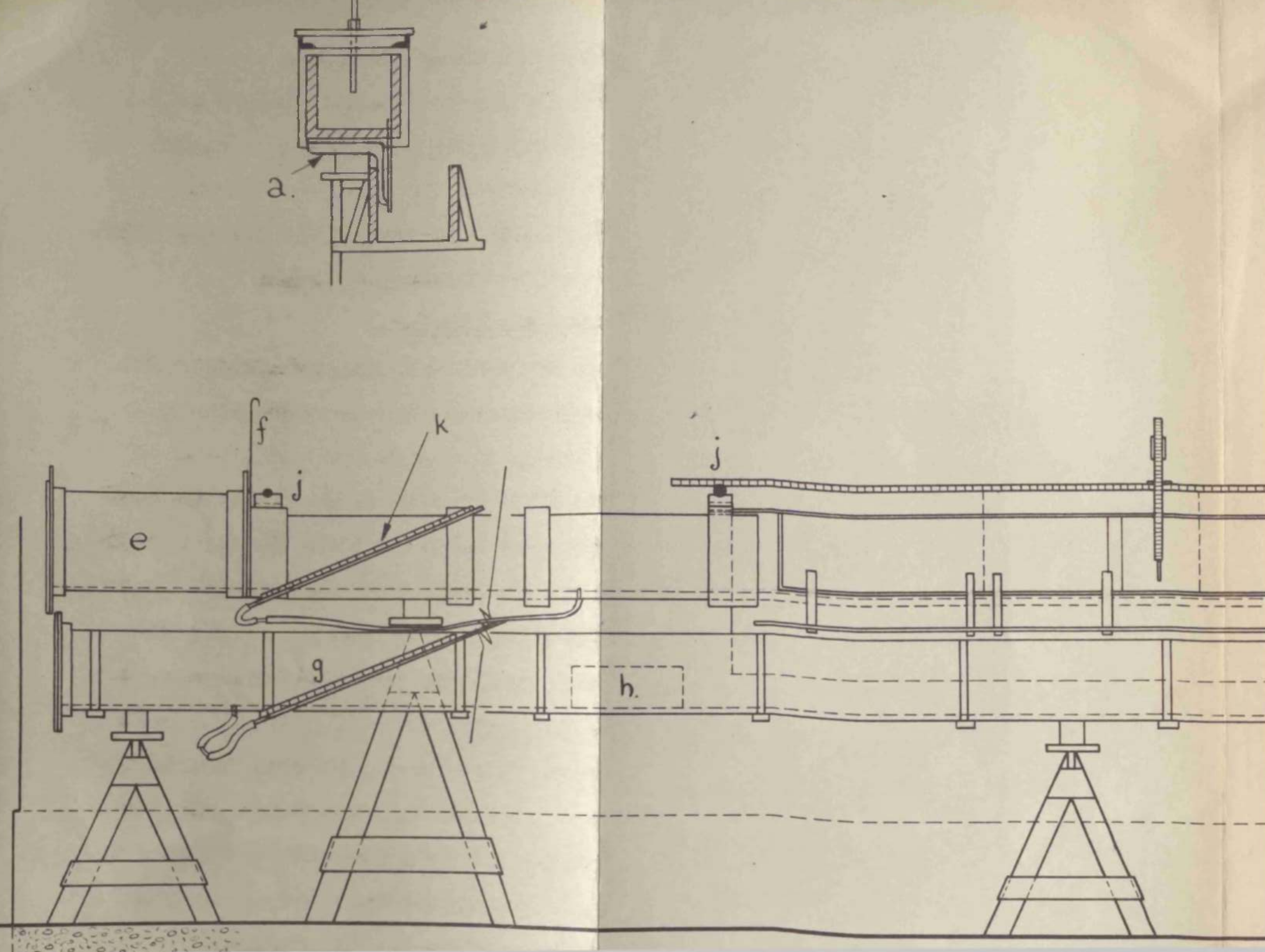
3.1.3. Water supply and Calibration.

Water was supplied from the laboratory system via a stilling tank, and calibrated orifice tanks were used to measure the flow in the main channel downstream of the weir, and the discharge over the weir.

3.2. The Main Apparatus.

3.2.1. General Arrangement. (Figs. 3.1, 3.2.)

The arrangement finally adopted was similar to the pilot



a.
f
k
j
e
j
g
h.

apparatus and is shown in the photograph, Fig. 3.1. The rectangular channel was 9 in. wide by 9 in. deep and 20 ft. long, the weir length being variable in steps of 1½ in. to 5 ft., and means being provided for setting the weir crest at any height above the channel floor, and for varying the width of the channel. Details of the arrangement are shown in the sketch, Fig. 3.2.

3.2.2. The Main Channel. (Fig. 3.2.)

The channel was constructed of three, 6 ft. long, wooden sections, the centre length being L shaped in cross-section and fitted with angle brackets, a, which carried the weir plate. A metal tank, b, was connected to the entry of the channel and fitted with a perforated plate, c, and vanes, d, for the purpose of smoothing the flow. The channel terminated in a 2 ft. long, metal section, e, incorporating a sluice gate, f, at the junction to the wooden channel for the purpose of controlling the downstream discharge and water level. The width was varied by the insertion of a 12 ft. length of framed hard-board in the channel, the transitions to the 9 in. width being made with flexible hard-board extensions. The arrangement was supported on wooden trestles with the bottom of the channel set level, no provision being made for slope variation.

3.2.3. The Weir Plate. (Figs. 3.2, 3.3, 3.4, 3.5)

The weir was formed from a 3/16 in. thick steel plate, 6 ft. long by 2 ft. deep, having a section, 5 ft. long and 9 in. deep, cut out from it. The upstream vertical end and the crest were of knife-edge section, and holes were drilled for the purpose of attaching the plate to the brackets on the main channel (a, Fig. 3.2),

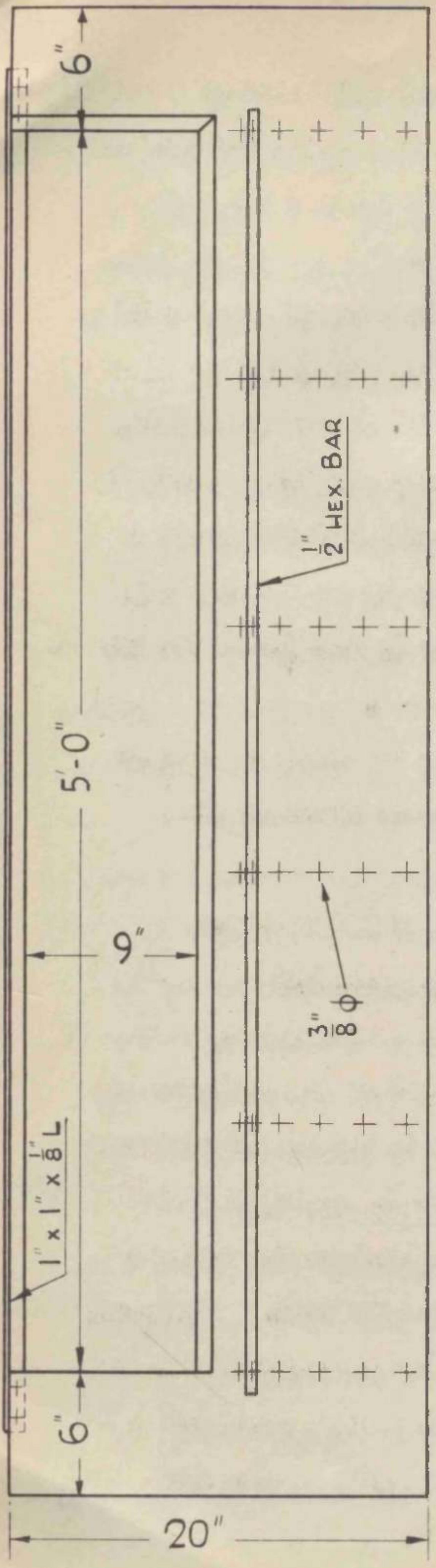


FIG. 33. WEIR PLATE

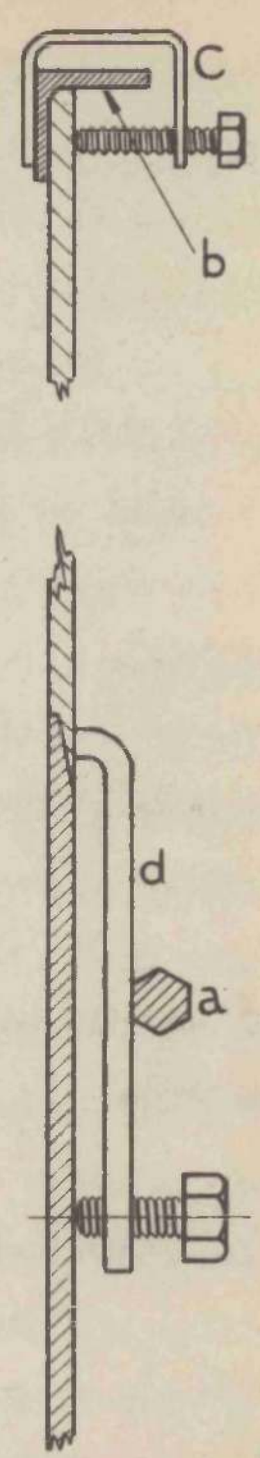


FIG. 34.

ATTACHMENT OF PLATES TO WEIR PLATE

these brackets being slotted to allow vertical adjustment of the weir crest. A hexagon rod, a, Fig. 3.4, was bolted to the front of the weir, 3 in. below the crest, and a 1 in. by 1 in. by ⅛ in. angle, b Fig. 3.4, was fitted to the top of the plate.

The length of the weir was varied by means of a series of metal plates of lengths 2 ft; 1 ft. (2 thus); 6 in: 3 in: and 1½ in: a special end plate 1⅛ in. long being provided to give a vertical, knife-edge termination to the weir. These plates bore against the angle, b, Fig. 3.4, at the top, and were prepared at the bottom to fit the crest of the weir, being held in position by steel U clips, c, Fig. 3.4 at the top and by cantilever clips, d, Fig. 3.4 at the bottom. The photograph, Fig. 3.5, shows the arrangement.

No means were provided for altering the slope of the weir crest, this being set level and parallel to the bottom of the channel.

All steel parts were protected by cadmium plating.

3.2.4. The Collecting Channel. (Figs. 3.1, 3.2)

The collecting channel consisted of a rectangular, level channel, 9 in. wide by 9 in. deep, constructed of framed hard-board and set parallel to the main channel, so as to receive the discharge from the side weir. It terminated in a brass, rectangular weir provided with an inclined manometer, g, connected to the bottom of the channel at a distance of 2 ft. 4 in. from the weir. The manometer provided a means of measuring the head in the collecting channel and the rubber tubing connection was fitted with a short length of capilliary tubing as a damper. Smoothing vanes, h, were also provided.

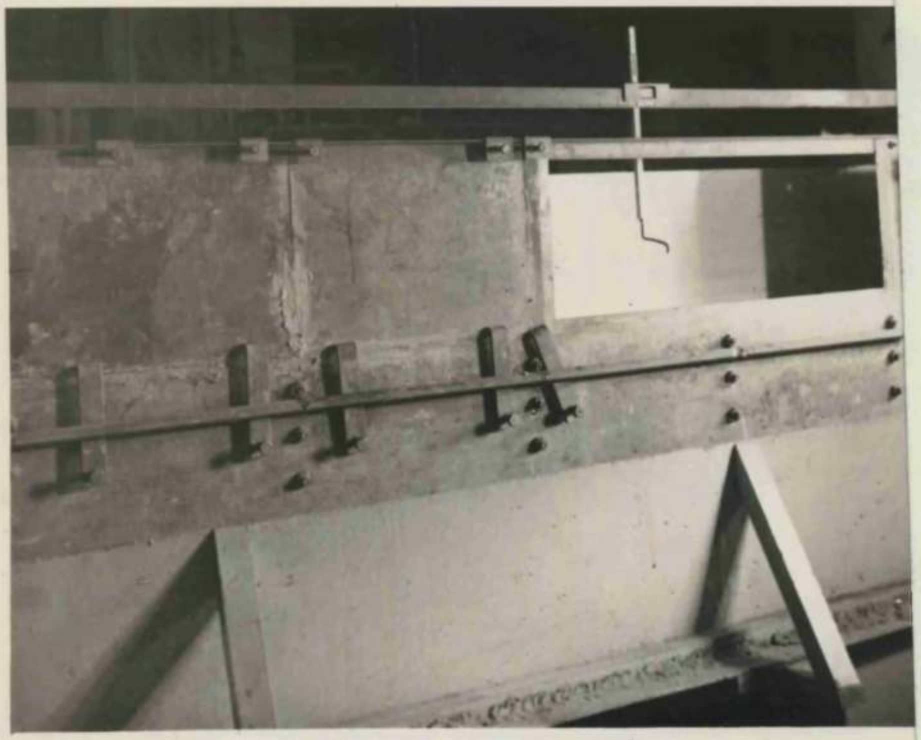


FIGURE 3.5. DEPTH GAUGE.

3.2.5. Measurement of Depths of Flow. (Figs. 3.2, 3.5)

The depth gauge consisted of a $\frac{3}{8}$ in. square, aluminium rod graduated to 0.01 ft., held by springs in a brass mounting, and sliding freely in this mounting under a slight pressure of the finger. A $\frac{1}{4}$ in. diameter, brass rod, with a conical point, about 3 in. long, was screwed to the bottom of the aluminium rod. The assembly was carried on a 1 in. square, copper bar, 6 ft. 6 in. long and engraved at 0.1 ft. intervals. A view of the assembly is shown in the photograph, Fig. 3.5. Four 1 in. diameter steel bars, j, Fig. 3.2, were set at intervals along the top of the main channel and adjusted to lie in the same horizontal plane. On these the copper bar could be mounted.

It was found that the reading accuracy of this relatively simple system was of the order of ± 0.001 ft. with a still water surface, and this was deemed sufficient.

Under Case II and III conditions, it was not found possible to measure depths of flow accurately at the end of the weir section, and for this reason, a manometer, k, Fig. 3.2, was fitted at a distance of 7' 6" from the entry to the weir section. Fitted with a length of capilliary tube as a damper, this afforded a measure of the static pressure in the downstream channel and since tranquil flow obtains in these cases, the effect of this static pressure being measured at a varying distance from the end of the weir section is small since the pressure will change little with length.

3.2.6. Water supply.

The laboratory is equipped with a centrifugal pump of about 1 cusec. capacity supplying a tank, 2 ft. square, and 4 ft. deep, which is fitted with a long, overflow weir to provide a constant head. A triangular weir is fitted to the tank. From this tank a supply was taken via a valve and a 5 in. diameter, flexible, rubber pipe to the inlet tank of the main channel. After passing through the apparatus the water was returned to the pump sump.

Owing to the laboratory layout, the maximum available head between the supply tank and the entry tank was about 2 ft. 6 in., and it was impossible to insert a critical section, such as an orifice or a weir, between the entry tank and the apparatus and provide adequate flow. Even with the straight-through connection, a maximum discharge of 0.5 cusecs was all that was obtained. This meant that the long, overflow weir could not be used since the experimental programme demanded a constant flow for varying conditions in the apparatus. Fortunately, the pump delivery is very constant and the supply valve of the pump was used directly as the flow controller.

3.2.7. Calibration of the Apparatus.

The triangular weir on the supply tank has been calibrated by direct volume measurement, and the weir on the collecting channel was calibrated from it. Water was allowed to flow over the weir of the supply tank until steady conditions obtained. The gate valve was then opened and the flow circulated via the main channel and the side weir to the rectangular weir on the collecting channel, the sluice gate on the main channel having been closed. A calibration curve of quantity against the reading on the inclined manometer was this obtained.

A weir could not be mounted on the main channel because of the lack of space to provide a sufficiently long channel after the sluice gate.

Test Series	Width inches	Upstream Quantity cusecs.	Number of Tests		
			Case I	Case II	Case III
A.1.	6	0.172	7	-	-
A.2.	6	0.210	7	-	-
A.3.	6	0.240	7	-	-
A.4.	6	0.262	7	-	-
A.5.	6	0.283	7	-	-
A.6.	6	0.309	7	-	-
A.7.	6	0.448	8	-	-
B.1.	3	0.307	7	Discarded	
B.2.	3		Discarded		
B.3.	3	0.130	6	Discarded	
B.4.	3	0.199	8	Discarded	
B.5.	3	0.241	8	Discarded	
B.6.	3	0.317	8	Discarded	
C.1.	9	0.260	8	16	7
C.2.	9	0.318	8	17	11
C.3.	9	0.387	8	10	7
C.4.	9	0.447	8	8	8
1.	6	0.430	-	-	16
2.	6	0.345	-	-	12
3.	3	0.249	-	-	11
4.	3	0.173	-	-	6
Favre & Braendle	7.91	Various	-	8	-
Gentilini	8.40	Various	4	8	-
		Totals	124	67	78

Table 4.1. Summary of Tests.

4. Experimental Procedure.

4.1. General.

For each test series, the quantity flowing in the upstream channel was kept constant, the weir length being varied by means of the detachable plates, and the downstream channel characteristics being varied by means of the sluice gate.

Tests were carried out with channel widths of 3 in., 6 in., and 9 in., respectively, the weir height being kept constant at 0.055 ft. Table 4.1. gives a summary of the tests carried out and includes those of Favre & Braendle and of Gentilini and in the tables in the appendix detailed results of each test may be found.

4.2. Measurement of Quantity Flowing in the Upstream Channel.

For each test series, the pump valve was set to give the required quantity, and the water allowed to flow over the triangular weir on the main supply tank, the valve supplying the side weir apparatus being kept closed. When steady conditions were obtained, the quantity was measured.

The valve supplying the side weir apparatus was now fully opened, and the total flow allowed to pass through the apparatus, via the side weir, to the sump; the sluice gate on the downstream channel being kept closed.

When steady conditions were obtained, a reading of the head over the weir in the collecting channel was made in order to obtain a check. At intervals throughout the test series the whole flow was passed over this weir as a check that there was no change in the total quantity.

4.3. Case I. Flow - Measurement of Surface Profile and Quantities.

The sluice gate in the downstream channel was now opened fully and the side weir length increased until any further increase would have resulted in a clinging nappe occurring on the side weir. Measurements of depths of flow at the side weir crest, at the centre of the channel and at the back of the channel were then taken along the weir at 3 in. intervals. The quantity flowing over the side weir was then measured by means of the weir on the collecting channel.

The length of the side weir was then reduced in 3 in. steps, the quantity flowing over the side weir being measured for each length and measurements being made of the depth of flow at the beginning and at the end of the weir section. It may be mentioned that the profile remained unaltered in the weir section as the length was reduced.

4.4. Case II and III Flow. Measurement of Depths of Flow and Quantities.

Under these conditions of flow, turbulence was very great and in Case III flow it was accompanied by a surging, or oscillation of the position of the jump. For this reason measurements of depths of flow at the downstream end were very inaccurate and the static pressure was measured at a fixed point in the downstream channel, using the manometer described in Section 3.2.5.

The side weir was lengthened to its initial length and the sluice gate set to a position to give Case II or Case III flow, this procedure being repeated to give a number of tests for the chosen length.

With Case II flow, the quantity flowing over the side weir, the depths of flow at the commencement of the weir section, and the manometer reading in the downstream channel were recorded.

With Case III flow, the length along the weir at which the jump occurred was also noted. The estimation of this length was very difficult owing to the unstable nature of the flow, in some cases the range of surging was of the order of 9 in. Various methods of damping were employed such as screens, floats and constrictions, and average values of the length thus obtained.

The above procedure was then repeated for various lengths of side weir.

5. General Theory.

5.1. Dimensional Analysis.

5.1.1. General.

In any problem of fluid motion, the various factors affecting the flow fall into three main classes:- (a) the geometric shape of the flowing fluid, (b) the flow characteristics such as velocity and pressure, and (c) the physical properties of the fluid. The theory of dimensional analysis affords a method of arranging these factors into significant dimensionless groups, in order that the effect of each on the flow may be studied. The usual method of presenting the result, for the dimensionless groups can be arranged in several ways, is as follows:-

$$\phi\left(\frac{a}{b} \cdot \frac{a}{c} \cdot \frac{a}{d} \ldots\ldots \frac{\rho V^2}{p} \cdot \frac{\rho V^2}{\gamma a} \cdot \frac{\rho V a}{\mu} \cdot \frac{\rho V^2 a}{\sigma}\ V\sqrt{\rho/e}\right) = 0$$

where a, b, c, d .... are lineal dimensions defining the geometric shape of the flowing liquid.

V is a velocity at some part of the flowing liquid.

p is a pressure at some part of the flowing liquid.

ρ is the density of the fluid.

μ is the viscosity of the fluid.

σ is the surface tension of the fluid.

e is the modulus of elasticity of the fluid.

The relationship contains a series of dimensionless parameters which are purely geometric and define the geometric boundaries of flow a/b. a/c. a/d ....; a dimensionless parameter $\frac{\rho V^2}{p}$ involving the flow characteristics; and a series of dimensionless parameters, $\frac{\rho V^2}{\gamma a}$, $\frac{\rho Va}{\mu}$, $\frac{\rho V^2 a}{\sigma}$ and $V\sqrt{\rho/e}$ which involve the physical properties of density, specific weight, viscosity, surface tension and elasticity. These last parameters are known as the Froude, Reynolds, Weber and Mach numbers respectively and the expression is usually written as:-

$$\frac{\rho V^2}{p} = \phi^1\left(\frac{a}{b}, \frac{a}{c}, \frac{a}{d} \ldots\ldots F.R.W.M.\right) \ldots\ldots\ldots\ldots\ldots\ldots 5.1.$$

5.1.2. Elimination of Variables.

In the particular case of the side weir it is possible to eliminate several of the variables from the expression by studying results which have been obtained in other cases and which, although not entirely similar, nevertheless have parameters of the same order as those of the side weir. In particular, the surface tension and elasticity of the fluid will have little effect on the flow so long as certain precautions are taken.

Surface tension is of importance only in cases where the curvature is appreciable, that is at low heads over the weir and in cases of clinging nappes. If this condition is avoided in model studies the Weber number may be dropped from equation 5.1. In a similar way the Mach number, which represents the ratio of the velocity of flow to the velocity of an elastic wave, will not affect the flow unless velocities are extremely high and hence can be dropped.

A third parameter, the Reynolds number, requires further consideration. The Reynolds number is indicative of the rate of dissipation of energy and if this is low compared with the total energy of flow, the effect of viscosity can be ignored. A side weir is generally incorporated in a long channel of mild slope and forms a

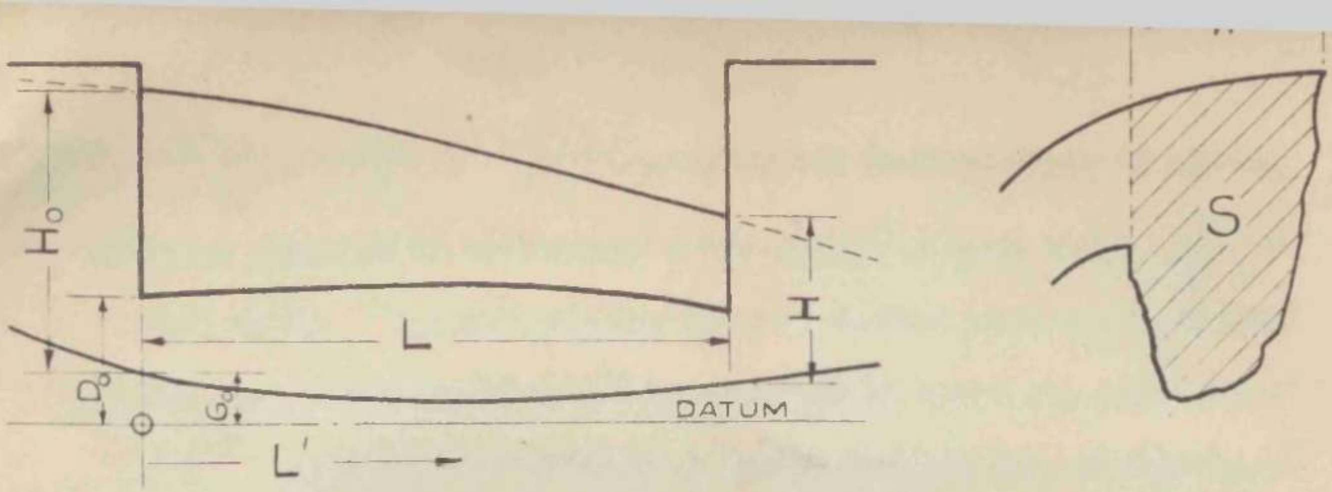
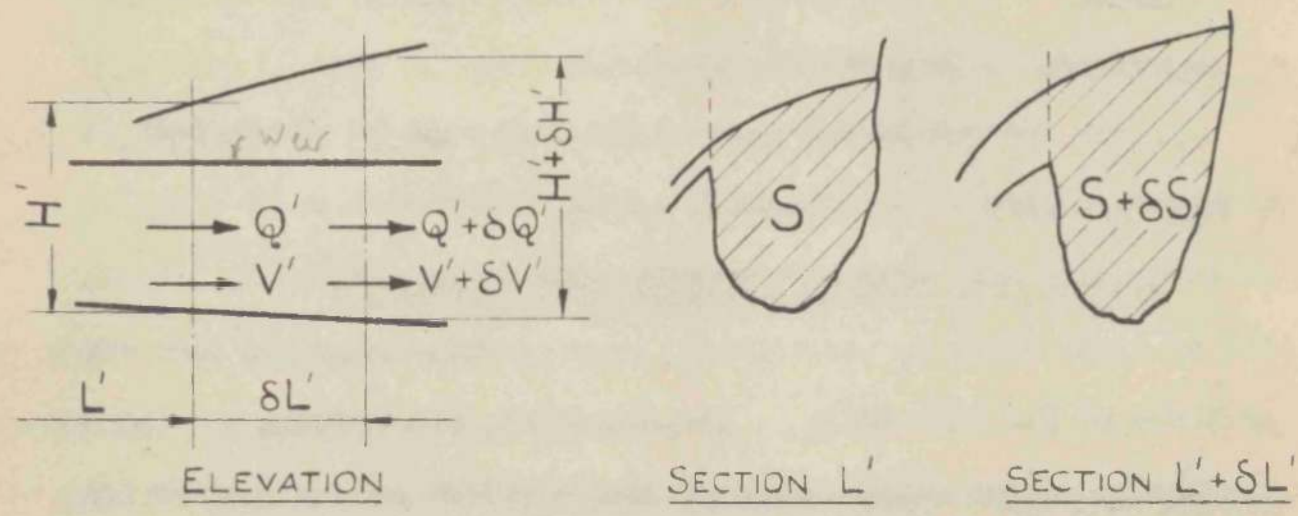
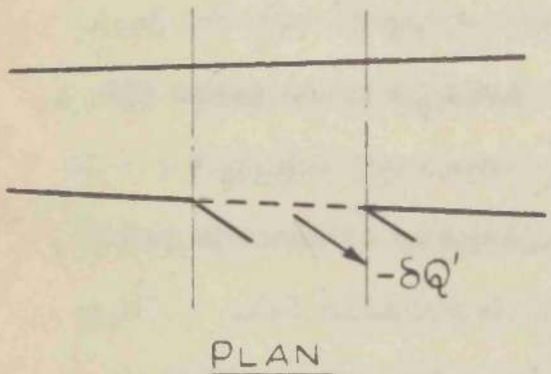


FIGURE 5.1





DIMENSIONS - GENERAL CASE

FIGURE. 5.2.

THE TRANSFORMED SECTION

FIGURE 5.3.

relatively small part of the whole system. The velocities are of the same order as the channel velocities, even in cases where rapid flow is involved, hence the rate if energy dissipation will also be of the same order and unlikely to be significant over the weir section. In addition, experimental analysis of similar local phenomena, such as the hydraulic jump and recent studies on high velocity flow in open channels [19], has shown that the Reynolds number may be ignored in such cases. Despite this it is possible, as will be seen later, to determine the approximate magnitude of the effect of viscosity and friction and make an allowance for it.

5.1.3. Application to the Side Weir - General.

Having reduced equation 5.1 by the elimination of these variables, it may now be applied to the problem under consideration. Referring to Fig. 5.1, and assuming that the direction of mean velocity in the main channel is along the L' axis, the zero of the L' axis is taken at the commencement of the weir section. Let G, the elevation of the lowest stream filament in the main channel above the datum, be expressible as a function of L' and Go, the value at the commencement of the section. Similarly, let D, the elevation of the weir crest above the datum be a function of D_o and L' ; W, the width of the flow surface between the far boundary and the vertical through the crest of the weir, be a function of W_o and L'. Finally let S, the area of flow in the main channel bounded by the wetted perimeter, the flow surface and the vertical through the weir crest, be expressible as a function of L' and W_o. It is assumed that the crest of the weir is parallel to the L' axis in the vertical plane. If this is not so another parameter must be introduced. Let V_o and H_o be the mean velocity in the main channel and the mean height of the flow surface above the lowest stream filament at the commencement of the weir section. Let V and H be the mean velocity in the main channel and the mean height of the flow surface above the lowest stream filament at the end of the weir section.

Finally let p be the pressure intensity of the lowest stream filament in the main channel at the end of the weir section.

Considering the flow as a whole, equation 5.1 may now be written as:-

$$\frac{\rho V_o^2}{p} = \phi_1 \left(\frac{H_o}{D_o} . \frac{H_o}{W_o} . \frac{H_o}{G_o} . \frac{H_o}{L} . \frac{\rho V_o^2}{\gamma H_o} \right) \quad \ldots\ldots\ldots\ldots 5.2$$

where L is the length of the weir section.

If the form of this function is known a value of p can be obtained corresponding to the entry conditions.

If the mean velocity in the main channel at the end of the weir section is now taken in place of V_o, equation 5.1 now becomes:-

$$\frac{\rho V^2}{p} = \phi_2 \left(\frac{H_o}{D_o} . \frac{H_o}{W_o} . \frac{H_o}{G_o} . \frac{H_o}{L} . \frac{H_o}{H} . \frac{\rho V^2}{\gamma H_o} . \frac{\rho V_o^2}{\gamma H_o} \right) \quad \ldots\ldots\ldots\ldots 5.3$$

The additional Froude number, $\frac{\rho V^2}{\gamma H_o}$, which defines conditions in the channel downstream of the weir, must be included since the flow is now divided and the continuity equation of constant quantity does not hold.

5.2. Conventional Analysis - Correlation of Quantity and Depth of Flow.

5.2.1. General.

The dimensional relationships developed above do not indicate how the variables will be combined or the form of the function and this must be investigated by experimental analysis. Attempting to analyse the experimental data where five or six variables are concerned is a formidable task, and an analysis along more classic lines will be of assistance in indicating the possible form of the functions or a significant grouping of one or more variables.

The flow conditions at the weir are very complex and any treatment must of necessity make certain assumptions. These assumptions are of such a nature that a detailed mathematical analysis is unjustified and inevitably experimental coefficients must be introduced. For this reason, the approach to the problem has been to obtain simple formulae, adjust the formulae by experimental coefficients and investigate the variation in the coefficients with the variation of the significant dimensionless variables obtained in the dimensional analysis.

It may be noted, at this point, that the following analysis was carried out before the work of Favre and De Marchi was brought to the author's attention.

5.2.2. General Case.

Consider an element of length of side weir as shown in Fig. 5.2. Q' is the quantity of water flowing in the main channel, at distance L' from the commencement of the weir section. The main channel is defined, as before, as the area S, bounded by the vertical through the crest of the weir, the water surface and the wetted perimeter. The area of flow in the main channel is a function of L' and of a depth of flow, H', which is taken as the average depth of flow in the main channel and which is itself a function of L'.

The assumptions which must be made before starting the analysis are as follows:-

1. The pressure head at any point is equal to the depth of that point below the flow surface. This assumption is usually made in channel flow analysis.
2. The velocity, V', is uniform at any section in the main channel. This assumption is the usual one made in channel flow problems. A correction factor can be applied but considering the other assumptions, no gain in accuracy is likely to be made.
3. Frictional losses are ignored between section L' and $L' + \delta L'$. This is equivalent to assuming that flow conditions are independent of the Reynolds Number.
4. The component of the velocity, parallel to the main channel, of the quantity $-\delta Q'$ passing over the weir is equal to the velocity V' in the main channel.
5. After the quantity $-\delta Q'$ passes through the vertical through the weir crest, there is no pressure at any point.
6. The lowest streamline is horizontal or of such a small slope that it can be assumed horizontal.

Equating force to rate of change of momentum between sections L' and $L' + \delta L'$ and ignoring the frictional resistance and the component of weight of the fluid, gives:-

$$SZ + \frac{Q'V'}{g} = \frac{Q'(V' + \delta V')}{g} + (S + \delta S)(Z + \delta Z)$$

where Z is the depth of the centroid of the transformed section, which is defined, Fig. 5.3, as the section having the same area, and the same bedding depths across the flow surface but with this surface horizontal. The mean height H' is, of course, the same for both sections.

On simplification the above equation becomes:-

$$\frac{V'dV'}{g} = -\frac{1}{S}d(SZ)$$

which may be expanded as:-

$$\frac{V'dV'}{g} = -\frac{1}{S}\ \frac{\partial(SZ)}{\partial H'}dH' - \frac{1}{S}\ \frac{\partial(SZ)}{\partial L'}dL'$$

Now it may be shown that:-

$$\frac{\partial(SZ)}{\partial H'} = S$$

and the expression now becomes:-

$$\frac{V'dV'}{g} = -dH' - \frac{1}{S}\cdot\frac{\partial(SZ)}{\partial L'}dL' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5.4$$

This expression is not of much use in its present form, and before integration can be carried out, H' must be expressible as a function of L'. In addition, another equation, connecting quantity and length, is required before a solution can be obtained.

5.2.3. Prismatic Channels.

Considering a channel where S and Z are functions of H' alone, that is a prismatic channel, equation 5.4 now becomes:-

$$\frac{V'dV'}{g} = -dH'$$

Integrating and inserting the boundary conditions that at the commencement of the weir section $V' = V_0$ and $H' = H_0$ gives:-

$$\frac{V'^2}{2g} + H' = \frac{V_o^2}{2g} + Ho = E_o \quad \text{.......} \quad 5.5$$

where E_o is the specific energy at the commencement of the weir section.

This equation is no more than a statement that the specific energy of flow, $\frac{V'^2}{2g} + H'$ is constant; that is, over the weir length there is no energy exchange between the water in the main channel and that flowing over the weir.

In the review of Favre's theory (paragraph 2.3.1 page 8) it was noted that by dropping the frictional term in this equation, it reduced to a statement of constant specific energy for the side weir case. It is obvious from the above that this is only correct for prismatic channels where the term $\frac{1}{S} \cdot \frac{\partial (SZ)}{\partial L'} \cdot dL'$ vanishes.

5.2.4. Possible Modes of Motion in Prismatic Channels.

The result obtained above for prismatic channels, that the specific energy of flow remains constant, leads to the fact that, for conventional channel sections (rectangular, circular, etc.) there are two possible depths of flow for a given entry quantity and given entry specific energy, these depths corresponding to rapid and tranquil flow respectively.

In the majority of cases the flow in the upstream channel will be tranquil, so that the depth of flow at or near the entry to the weir will never fall below the critical depth. Hence in these cases three modes of motion may occur in the weir section, and these have been observed to occur.

FIGURE 5.6. CASE III FLOW, UNDULAR JUMP.

FIGURE 5.7. CASE III FLOW, SURFACE ROLLER.

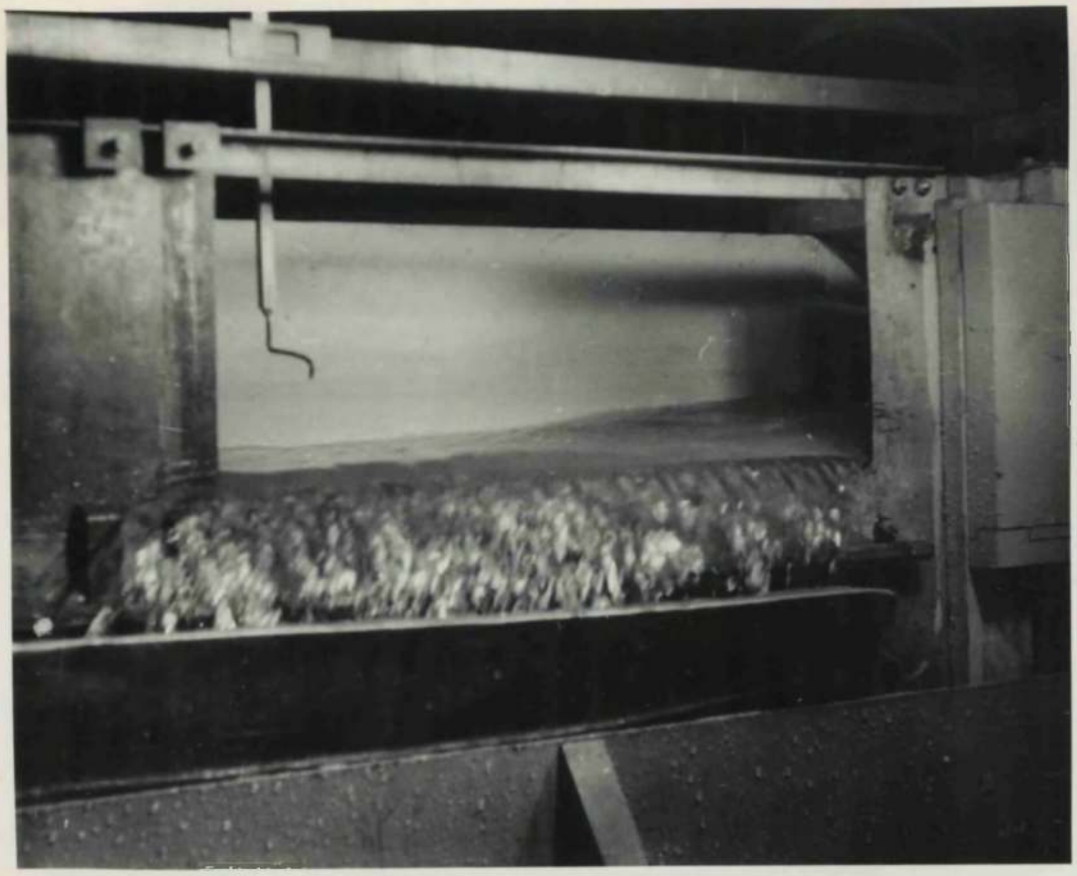


FIGURE 5.4. CASE I FLOW.

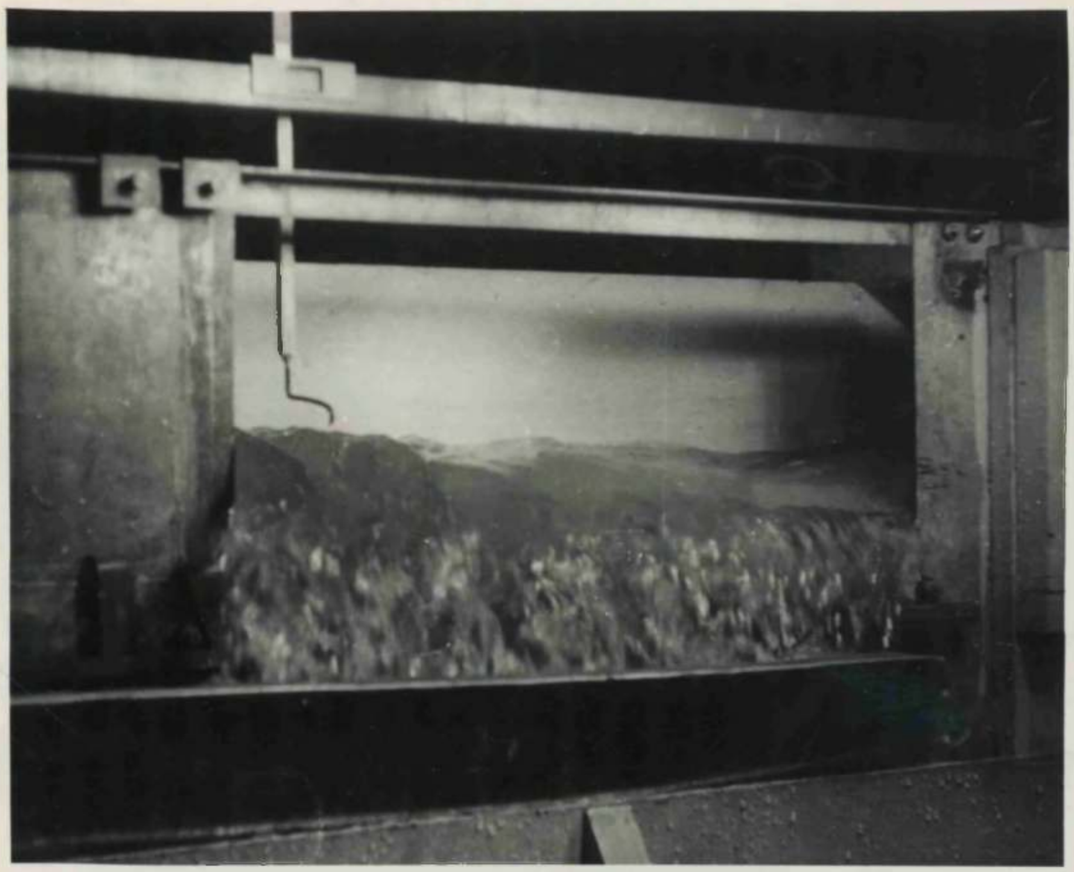


FIGURE 5.5. CASE II FLOW.

Case I. Critical conditions at or near the entry with rapid flow in the weir section, the depth of flow decreasing along the weir. (Fig. 5.4)

Case II. Depth of flow greater than critical at the entry with tranquil flow in the weir section, the depth of flow increasing along the weir section. (Fig. 5.5)

Case III. Case I. flow at the beginning of the weir section with a hydraulic jump occurring in the weir section and Case II. type of flow after the jump at a lower specific energy level due to jump losses. (Fig. 5.6, 5.7)

If rapid flow is possible in the upstream channel, other two modes of motion may occur.

Case IV. Depth of flow less than critical at the entry with rapid flow in the weir section, the depth of flow decreasing along the weir section.

Case V. Case IV. flow at the beginning of the weir section with a hydraulic jump occurring in the weir section and Case II. flow after the jump at a lower specific energy level due to jump losses.

So far cases IV. and V. have not been observed or studied.

Of these five cases, only three are noted by De Marchi as consequent on the assumption of constant specific energy; Case 1. corresponding to Case C, Case II. corresponding to Case B and Case IV. corresponding to Case A. In addition, the limits assigned by him to Case II. i.e., $D > \frac{2}{3}E_o$ are incorrect, as will be shown.

5.3. Conventional Analysis - Correlation of Quantity and Length.

5.3.1. General.

The effect of the side weir on the general flow in the channel is to superimpose a velocity at right angles to the weir on that portion of the fluid above the weir. This means that the angle the resultant velocity makes with the weir will vary, and that in cases of rapid flow the angle will be small and the velocity of approach high compared with velocities of approach for normal weirs. This is the reason why application of the normal rectangular weir formulae, even when modified to allow for the falling head in the rapid case, has not been successful.

On the other hand, with tranquil flow the resultant velocity makes a greater angle with the weir and conditions are nearer to the normal rectangular weir. Hence formulae based on the normal rectangular weir formula are more likely to give successful results.

Various attempts at a theoretical analysis were made, but in application to the results, no great success was met with and these were abandoned in favour of simple semi-empirical curve fitting.

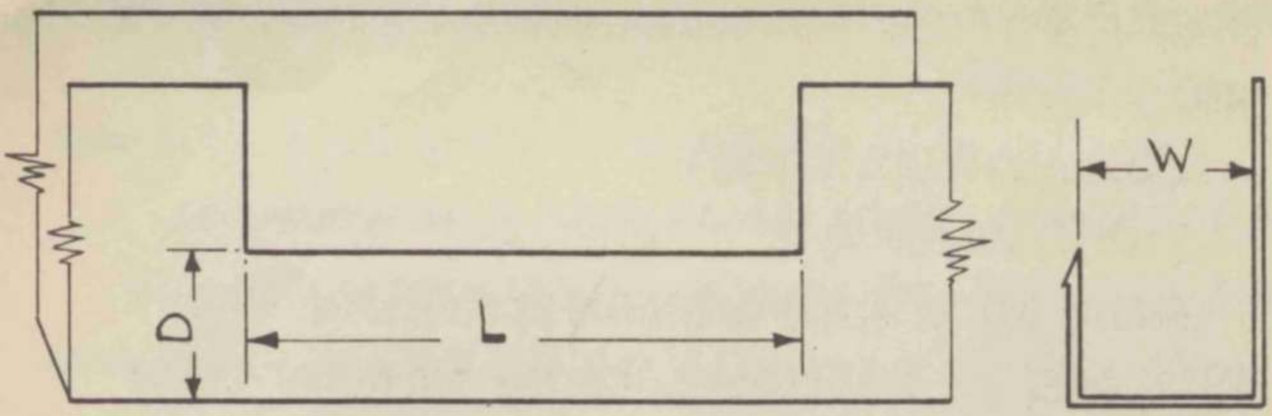


FIG. 6.1. - WEIR SECTION; PRISMATIC, RECTANGULAR CHANNEL

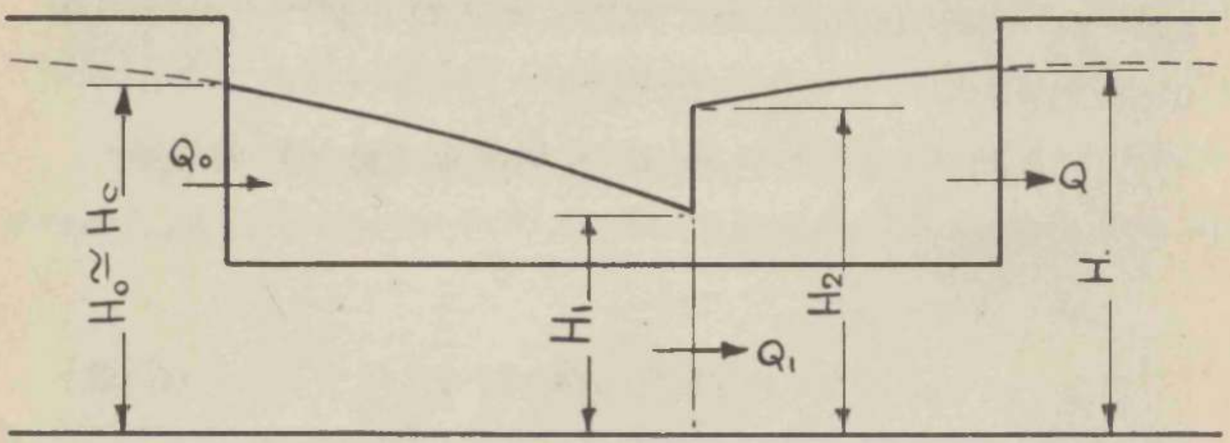


FIG. 6.2. - CASE III; IDEALIZED CONDITIONS

6. Prismatic Rectangular Channels - Analysis and Examination of Results.

6.1. Analysis.

6.1.1. Dimensional Analysis.

The dimensional equations 5.2 and 5.3 are of little use as they stand and will be further simplified on application to any particular case. Considering a prismatic rectangular channel with the lowest stream filaments horizontal at the weir section, the geometric parameters simplify and, referring to Fig. 6.1, equation 5.2. now becomes

$$\frac{\rho V_o^2}{p} = \phi_1 \left(\frac{H_o}{D} \cdot \frac{H_o}{W} \cdot \frac{H_o}{L} \cdot \frac{\rho V_o^2}{\gamma H_o} \right)$$

and equation 5.3 becomes

$$\frac{\rho V^2}{p} = \phi_2 \left(\frac{H_o}{D} \cdot \frac{H_o}{W} \cdot \frac{H_o}{L} \cdot \frac{H_o}{H} \cdot \frac{\rho V^2}{\gamma H_o} \cdot \frac{\rho V_o^2}{\gamma H_o} \right)$$

where D and W are constants and independent of L', the datum being taken as the bottom of the channel.

Now if Q_o is the quantity flowing in the main channel upstream of the weir section;

$$Q_o^2 = W^2 g H_c^3$$

where g is the acceleration due to gravity and H_c is the critical height in the main channel upstream of the weir.

Also, if Q is the quantity flowing in the main channel downstream of the weir section,

$$Q^2 = q^2 Q_o^2 = q^2 W^2 g H_c^3$$

where $q = \frac{Q}{Q_o}$ is the proportional discharge in the downstream channel. Equations 5.2 and 5.3 may then be further simplified and appear in the following form:-

$$\frac{H^3_c \gamma}{H^2_o P} = \phi_1 \left(\frac{H_o}{D} . \frac{H_o}{W} . \frac{H_o}{L} . \frac{H^3_c}{H^3_o} \right)$$

and

$$\frac{q \, H^3_c \gamma}{H^2 \, P} = \phi_2 \left(\frac{H_o}{D} . \frac{H_o}{W} . \frac{H_o}{L} . \frac{H_o}{H} . \frac{q \, H^3_c}{H^2 \, H_o} . \frac{H^3_c}{H^3_o} \right)$$

Dividing each lineal dimension by H_c then gives, with simplification:-

$$\frac{P}{\gamma H_c} = \phi_3 \left(d . w . \ell . h_o \right) \qquad \text{where } d = \frac{D}{H_c}$$

and

$$w = \frac{W}{H_c}$$

$$\frac{q}{P/\gamma H_c} = \phi_4 \left(d . w . \ell . h_o . h . \right) \qquad \ell = \frac{L}{H_c}$$

$$h = \frac{H}{H_c}$$

$$h_o = \frac{H_o}{H_c}$$

In Case I. it was not possible to measure the pressure intensity to any degree of accuracy and similarity in Cases II and III the downstream depth of flow could not be measured, and a mean pressure intensity was measured. In effect the experimental results are deficient of one significant factor, either the depth of flow or the pressure intensity, and the only way of resolving the difficulty is by making the assumption of hydrostatic pressure distribution at all sections of the flow. Again reference to studies of similar phenomena [19] shows that no serious error will be introduced. Making this assumption the equations may be written in their final form:-

$$h = \phi_3 (d.\ w.\ \ell.\ h_o) \quad \ldots\ldots \quad 6.1$$

and

$$q = \phi_4 (d.\ w.\ \ell.\ h_o.\ h) \quad \ldots\ldots \quad 6.2$$

These two equations may be combined, giving

$$q = \phi_5 (d.\ w.\ \ell.\ h_o) \quad \ldots\ldots \quad 6.3$$

6.1.2. - Conventional Analysis - Correlation of Quantity and Depth. Cases I and II.

In the case of a prismatic rectangular channel as studied in the experiments, if the proportional discharge $q = \frac{Q}{Q_o}$ and the proprotionate depth of flow at the end of the weir section, $h = \frac{H}{H_c}$ are introduced, equation 5.5 now becomes

$$\frac{q^2}{h^2} = 2h_o + \frac{1}{h^2_o} - 2h$$

or

$$q = h \sqrt{(A - 2h)} \quad \ldots\ldots \quad 6.5$$

where $A = 2h_o + \frac{1}{h^2_o}$ and can be interpreted as twice the specific energy of flow in the main channel at the commencement of the weir divided by the critical depth at this section.

On comparing this equation with the dimensionless equation 6.2 it reveals that, according to the above analysis, the discharge can be written generally as:-

$$q = \phi (h_o, h)$$

and that it is independent of the other weir dimensions. Further it is probable that this function will be of the form

$$q = h \sqrt{(M - Nh)} \quad \ldots\ldots \quad 6.6$$

where M and N are functions of h_o or constants. If the assumptions are correct,

$$M = A \text{ and } N = 2.$$

On considering equation 6.5 it will be seen that corresponding to each value of q there are two values of h. These are, of course, the rapid flow state including the case described before as Case I and the tranquil flow state described as Case II. Case III will arise where a jump occurs from rapid flow to tranquil flow in the weir section.

6.1.3. Conventional Analysis - Correlation of Quantity and Depth of Flow. Case III.

This case may be regarded as a combination of Cases I and II, but since a hydraulic jump is always associated with a loss of energy, the tranquil flow, Case II, after the jump will take place at constant specific energy which, however, will be lower than the specific energy of the rapid flow state. In order to obtain a value for this new specific energy level, it is necessary to make the assumption that the rise in level of the water surface from a rapid state to a tranquil state occurs abruptly. It is known that this is not so in ordinary channel flow and that the transition length, during which expansion of the stream takes place, is of the order of five times the height of the jump. However, in the side weir case it was noted that the rise although not abrupt, took place in a shorter length. This is probably due to the effect of the flow toward the weir tending to remove the usual roller.

Again in the case of the Froude number having the value between 1 and 2, it is well known that the jump is undular in form, but that the mean value of the depth of flow is given accurately by the usual formula.

Fig. 6.2 represents the idealised conditions of Case III. The usual jump formula derived from the force rate of change of momentum relationship gives,

$$h_2 = \frac{h_1}{2}\left\{\sqrt{\left(\frac{8q_1^2}{h_1^3} + 1\right)} - 1\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.7$$

where $h_2 = \frac{H_2}{H_c}$; $h_1 = \frac{H_1}{H_c}$; and $q_1 = \frac{Q_1}{Q_o}$, H_1 and H_2 being the depths of flow before and after the jump respectively and Q_1 the quantity flowing in the main channel at the jump section.

Treating the flow after the jump as Case II flow, the equation relating quantity and depth now becomes:-

$$q = h\sqrt{(B - 2h)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.8$$

where $B = 2h_2 + \frac{q^2_1}{h^2_2}$ and $q = \frac{Q}{Q_o}$

B could, of course, be expressed as a function of h_1 and A but no simplification results, and also analysis of the experimental data may entail modification of the equations derived above.

6.2. Examination of Results - Case I. Rapid Flow.

6.2.1. General.

As already noted, the depths of flow were measured along the weir crest, along the centre line and along the back of the weir.

The choice of an average depth required certain consideration and after several trials, it was decided to use Simpsons rule and take as the mean proportionate depth of flow,

$$h = \frac{h_b + 4\,h_c + h_w}{6}$$

where h_b, h_c and h_w refer to the proportionate depths of flow at the back of the channel, the centre line of the channel and at the weir crest respectively.

On examination of the results several important points were noted.

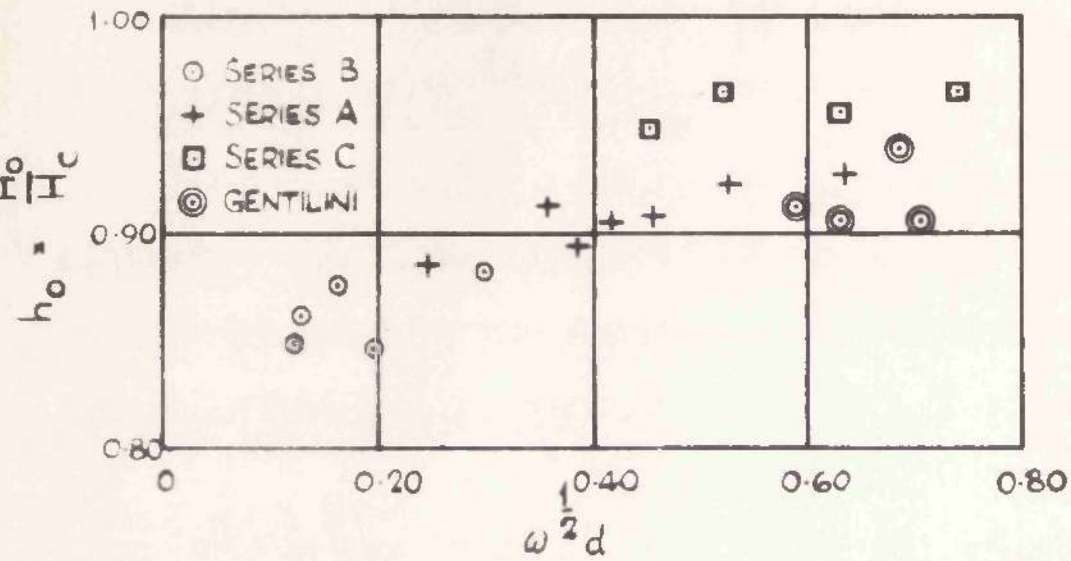
Firstly, over the whole range of results, the depth of flow at the entry to the weir section is approximately equal to the critical depth.

Actually h_o varies from 0.848 to 0.966 as d, the proportionate weir height varies from 0.154 to 0.39 and w, the proportionate width varies from 0.7 to 4.22.

This result is not surprising, since in every case considered, the flow in the upstream channel is tranquil and a transition to rapid flow involves a passage through a critical section near the disturbing element, in this case the side weir. The present investigation is not concerned with cases of rapid flow in the upstream channel, since this would involve either a steep slope or some other factor, which would not be met with in the main applications of the side weir.

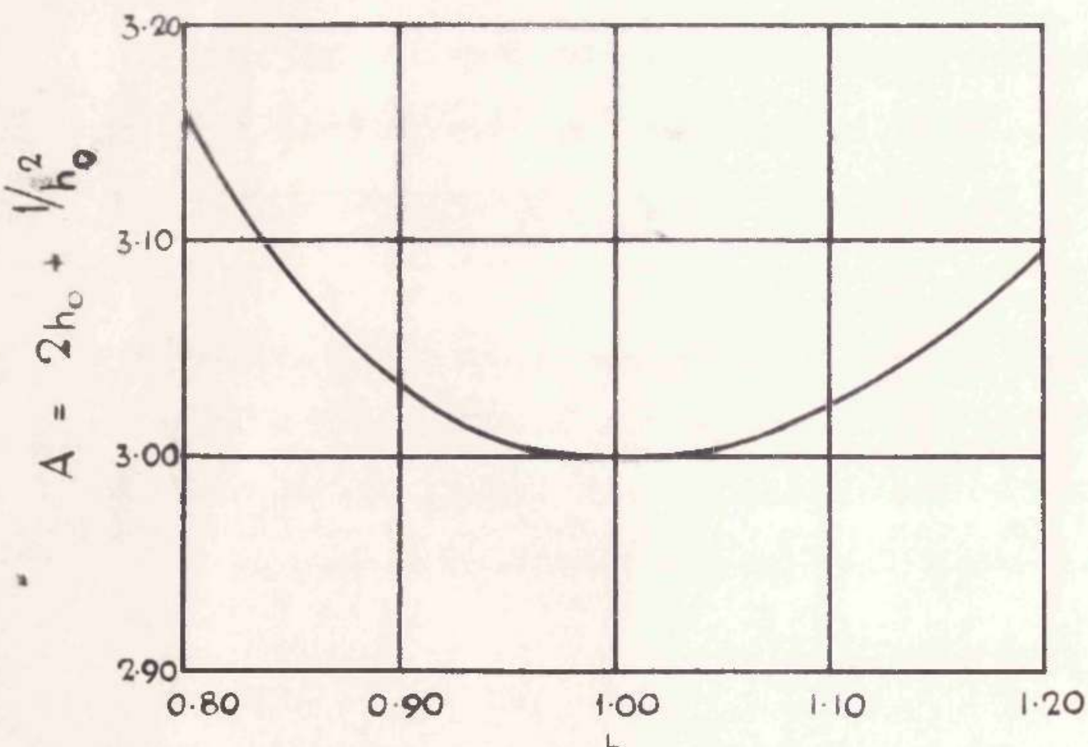
An investigation of the relationship between h_o, d and w indicated that over the range covered,

$$h_o = F\ (w^{\frac{1}{2}} d)$$



1.00
0.90
0.80
0
0.20
0.40
0.60
0.80
SERIES B
SERIES A
SERIES C
GENTILINI
$h_o = \frac{H_o}{H_c}$
$\omega^{\frac{1}{2}} d$

FIG. 6.3



3.20
3.10
3.00
2.90
0.80
0.90
1.00
1.10
1.20
$A = 2h_o + 1/h_o^2$
h_o

FIG. 6.4

and that this function is approximately linear as will be seen from Fig. 6.3. The matter was not gone into in further detail, since h_o appears in the form $2h_o + \frac{1}{h^2_o}$ in the various expressions, and over the experimental range, this quantity is practically stationary as will be seen from Fig. 6.4. Secondly, the experimental q - h curves show a tendency to vary with d and w but the scatter is small and a mean curve may be fitted which will serve for all practical purposes.

Finally, the profile in Case I is independent of downstream conditions.

This result is to be expected. Careful measurements of the profiles showed that there was no change as the conditions downstream of any section were varied (either by altering the flow in the downstream channel or altering the length of the weir) so long as rapid flow was maintained. An exception to this, is that a slight rise in surface level occurs just before the beginning of the jump.

6.2.2 Correlation of Quantity and Depth of Flow - Case I.

Assuming that $H_o = H_c$, that is assuming the critical section occurs at the entry to the weir section, the value of A in equation 6.5 becomes 3 and this equation can be written as

$$\frac{q^2}{2h^2} + h = 1.5$$

the left-hand side of this expression being the specific energy of flow divided by H_c.

In the appendix, Figs. 11.1 to 11.16, values of

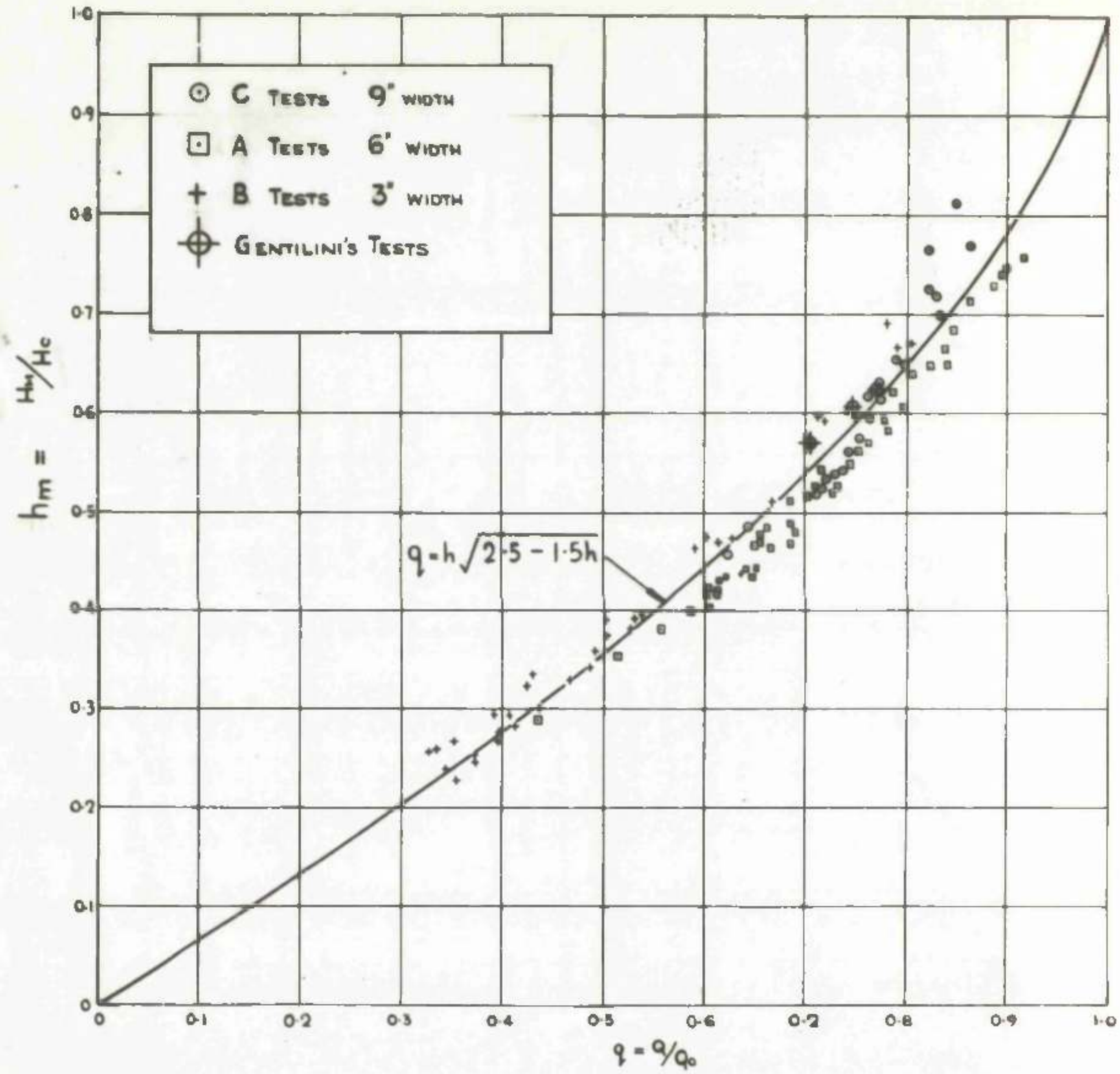


FIGURE 6.5 CORRELATION OF QUANTITY AND DEPTH OF FLOW

CASE I - SHOOTING FLOW

$\frac{q^2}{2h^2} + h$ have been plotted against h for each test series and although there is a definite variation with the weir parameters a reasonably close approximation is given by,

$$\frac{q^2}{2h^2} + h = 1.25 + 0.25h$$

which can be written as,

$$q = h \sqrt{(2.5 - 1.5h)} \quad \ldots\ldots\ldots\ldots \quad 6.9$$

One of the reasons for the losses in specific energy is probably the assumption of hydrostatic pressure which is not obtained in curvilinear flow.

In Fig. 6.5, equation 6.9 is shown and all experimental points, including those of Gentilini, plotted thereon. It was decided to adopt this relation since the advantage of a single curve for all cases of rapid flow more than outweighs the loss of accuracy of ignoring the variation with w and d. Again, considering Case III, where the deviation from the empirical curve is greatest, the Froude number is in the region of unity and any jump occurring in this section will be undular in form and errors due to measuring the depth will probably be greater than the effects of variation with w and d.

6.2.3. Correlation of Quantity and Length - Case I.

From the beginning it was realised that the normal rectangular weir formula, even when corrected for a variation in the depth of flow, would not apply and a few trials were sufficient to confirm this. After several attempts at a theoretical solution a purely empirical approach was adopted which met with

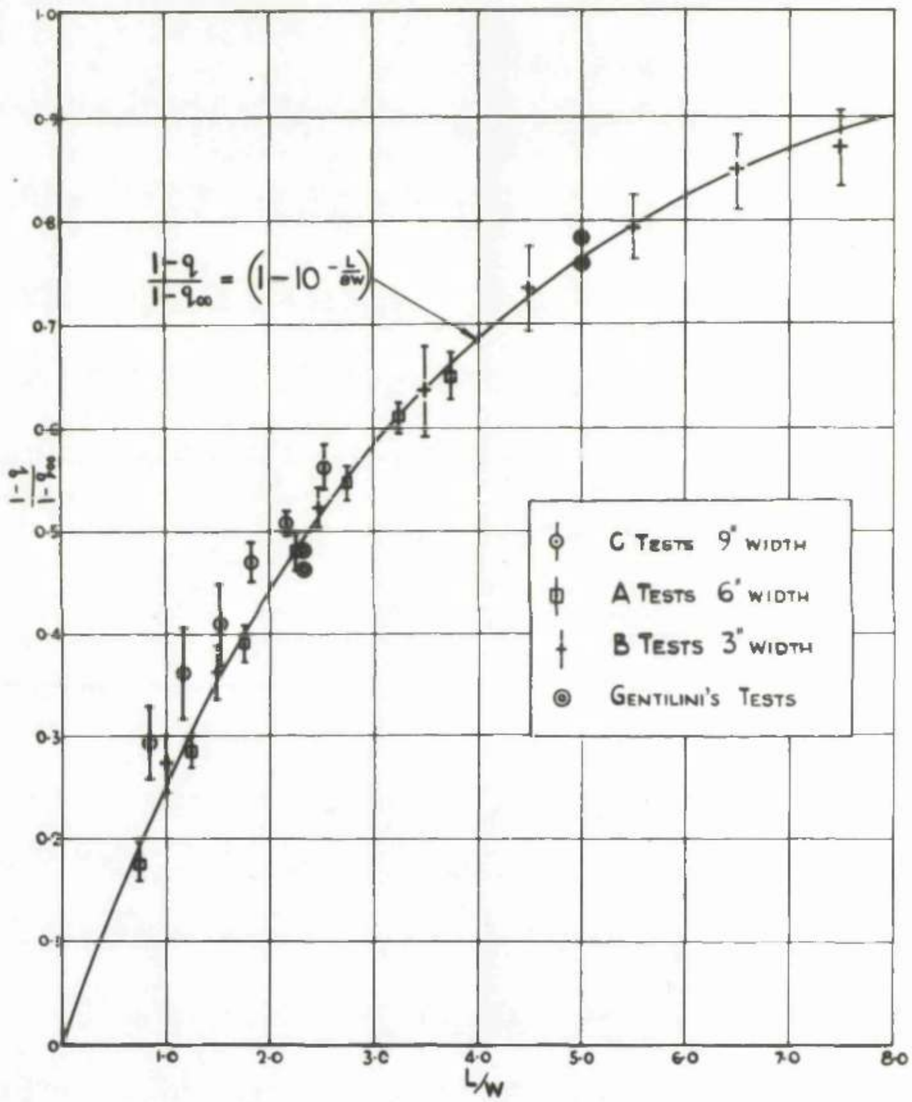


FIGURE 6.6. CORRELATION OF QUANTITY AND LENGTH

CASE I - RAPID FLOW

success.

If the length of the weir is infinite, the discharge in the downstream channel will be

$$q_{\infty} = d \sqrt{(2.5 - 1.5\, d)}$$

where d is the height of the weir.

The ratio $\frac{1 - q}{1 - q_{\infty}}$ then represents the ratio of discharge over the weir in length ℓ to the discharge over a weir of the same proportionate height whose length is infinite.

In Fig. 6.6 the mean and range of this ratio has been plotted for each value of the ratio $\frac{L}{W}$ and, as will be seen, the experimental points conform fairly well to the equation

$$\frac{1 - q}{1 - q_{\infty}} = 1 - 10^{-\frac{L}{8W}} \quad \text{.................................} \quad 6.10$$

In order to examine each test the theoretical q and h curves have been calculated and the experimental points plotted in Figs. 11.17 to 11.32 and these show the close agreement obtained. In Figs. 11.33 to 11.36 the test results obtained by Gentilini have been treated in the same manner, extending the range over which the relation holds.

6.3. Examination of Results - Case II . Tranquil Flow.

6.3.1. General.

An important fact was noted in this case, the flow is a function of conditions in the downstream channel.

In other words, any change in these conditions such as a change of slope alters the discharge and the depths of flow in the weir section. This is to be expected since in tranquil flow; the

TEST SERIES	No OF EXPERIMENTS	MEAN OF $\frac{q^2}{h^2A} + \frac{2h}{A}$	STANDARD DEVIATION	RANGE OF WEIR. PARAMETERS				
				$1 - q$	ℓ	h_o	d	ω
GENTILINI.	7	1·000	0·009	1·000-0·216	6·38 - 15·25	1·69 - 3·43	1·32 - 3·04	2·58 - 3·05
FAVRE + BRAENDLE	8	0·991	0·010	1·000-0·250	17·1 - 31·5	2·07 - 3·68	1·88 - 3·47	1·72 - 3·16
C. 1.	16	0·989	0·012	1·000-0·265	1·61 - 7·26	1·14 - 3·46	0·355	4·84
C. 2.	17	0·982	0·024	1·000-0·311	1·40 - 6·34	1·19 - 3·38	0·309	4·22
C. 3.	10	0·980	0·015	1·000-0·184	1·23 - 4·30	0·99 - 2·00	0·271	3·69
C. 4.	8	0·971	0·011	0·725-0·399	1·68 - 3·92	1·26 - 2·08	0·247	2·47

WEIGHTED MEAN. OF $\frac{q^2}{h^2A} + \frac{2h}{A} = 0{\cdot}990$

STANDARD DEVIATION = 0·018

TABLE 6.1. CASE II CORRELATION OF q + h

velocity of a disturbance of the gravity type exceeds the velocity of flow and hence will be propagated upstream. This means that a knowledge of the downstream channel properties is necessary before any solution of this case can be obtained.

6.3.2. Correlation of Quantity and Depth of Flow - Case II.

As in Case I the average proportionate depth of flow was obtained from Simpsons rule. It was considered that the pressures in this case would be closer to hydrostatic since the flow is tranquil and hence that the results would conform more closely to the theoretical curve. Equation 6.5 was rewritten in the form,

$$\frac{q^2}{h^2A} + \frac{2h}{A} = 1$$

and the values of the left hand side of the equation calculated. This quantity is simply the ratio of the specific energy at the end of the weir section to the specific energy at the entry of the weir section, $^E/E_o$. Details for all tests, including those of Gentilini and of Favre and Braendle are given in the appendix Figs. 11.37 to 11.42 and a summary of the results is given in table 6.1 opposite.

It will be noted that $^E/E_o$ varies from 0.971 to 1.000, the weighted mean being 0.990 and the standard deviation from the weighted mean being 0.018. No trend with any of the weir parameters was found and hence the equation governing quantity and depth of flow becomes by experiment,

$$\frac{q^2}{h^2A} + \frac{2h}{A} = 0.99$$

or

$$q = h\sqrt{(0.99A - 2h_o)} \quad \text{.............................} \quad 6.11$$

6.3.3. Correlation of Quantity and Length - Case II.

In this case there is less variation in the mean depth of flow along the length of the weir than in Case I, at the most, the theoretical maximum rise will be $H = 1.5 H_o$, when $H_o = H_c$. The velocity is not so high and the head over the weir is relatively greater. In addition, it was observed that the angle the stream makes with the weir is greater and does not vary so much over the length and that the drawdown at the weir is fairly constant over the length.

For these reasons it was decided to adopt the conventional rectangular weir formula,

$$Q_o - Q = Cd \frac{2\sqrt{2g}}{3} (\bar{H} - D)^{3/2} L$$

where $\bar{H}$ is a mean depth of flow in the main channel. Inserting proportional values gives

$$1 - q = C \ (\bar{h} - d)^{3/2} x \quad \text{..........................} \quad 6.12$$

where $C = \frac{2\sqrt{2}}{3}$ Cd and $x = \frac{L}{W}$

By a consideration of the shape of the rising flow $\bar{h}$ was taken as $\frac{2h + h_o}{3}$, h being the depth of flow at the end of the weir.

The value of C was then calculated from each test including those of Gentilini and ranged from 0.61 to 0.40.

The variation of C was then studied as a function of the weir dimensions and it was found that over the experimental range

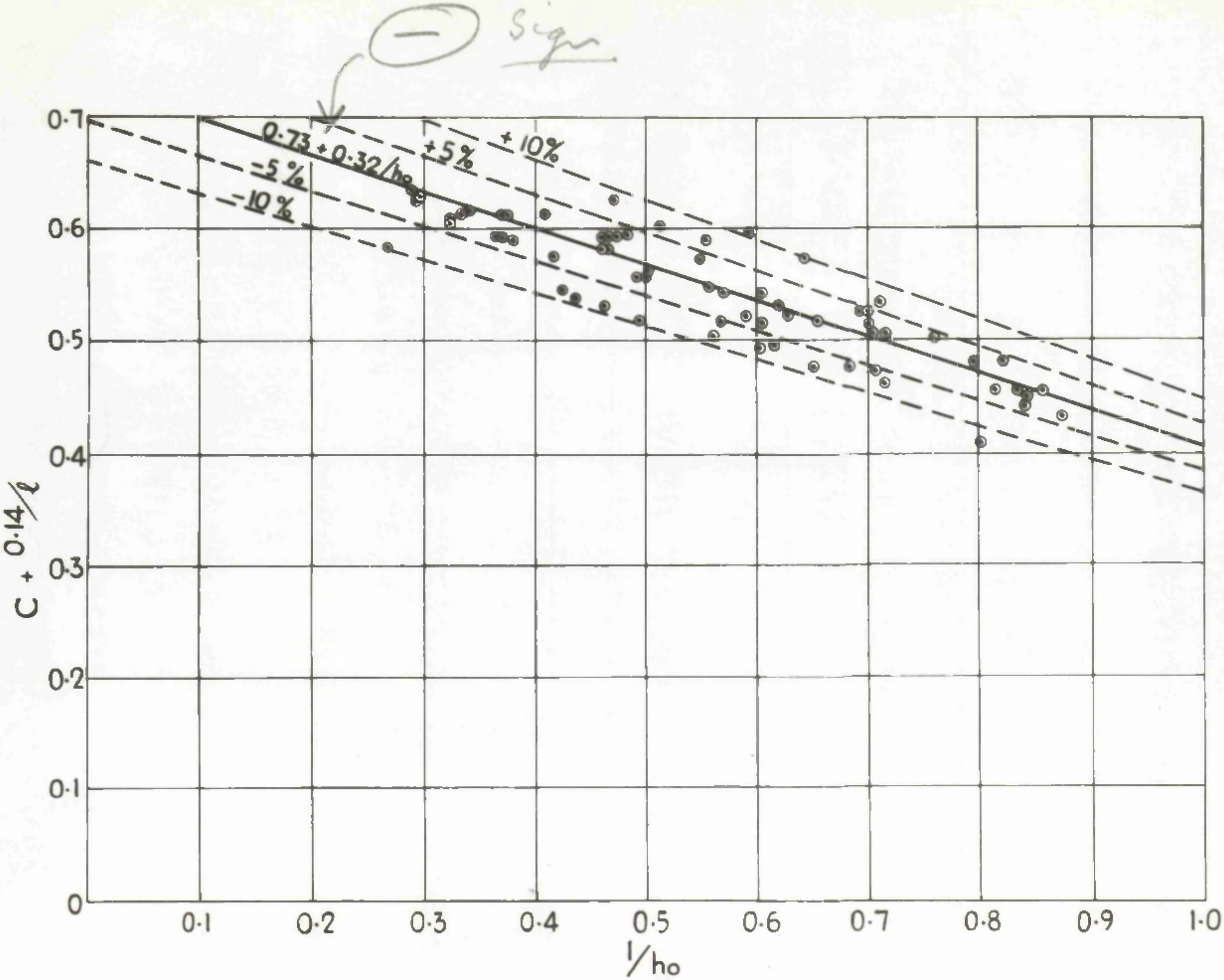


FIG. 6.7 CASE II VARIATION OF COEFFICIENT OF DISCHARGE

$$C = 0.73 - \frac{0.32}{h_o} - \frac{0.14}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.13$$

In Fig. 6.7 all values of $C + \frac{0.32}{h_o}$ are shown as a function of $\frac{1}{\ell}$ and in the appendix, Figs. 11.37 to 11.42, the relationship is shown for each test series.

6.4. Examination of Results - Case III Hydraulic Jump in the Weir Section.

6.4.1. General.

This case was most interesting, but on the whole the most difficult, from an experimental point of view and in the analysis of the results. Again the flow is conditioned by the downstream channel characteristics and the position of the jump could be altered, within limits, by altering these characteristics by means of the sluice.

In practically all cases there was violent surging, making it extremely difficult to obtain accurate values of the measurements. Attempts were made to reduce the surging by means of screens, obstructions and floats in the downstream channel. Although some success was met with by these devices, it was impossible to stop surging completely, and as noted before, some experiments were so affected as to be completely useless.

As with a hydraulic jump in a normal open channel, two types of jump were noted, the undular jump and the surface roller type. With the latter type, an angle was made by the jump front with the axis of the channel. No measurements were made of this angle, but from observation there appears to be a similarity between

Test Series.		3.	4.
Number of Tests		11	6
Range of Weir Parameters.	q_1	0.478 - 0.765	0.615 - 0.710
	q	0.072 - 0.280	0.081 - 0.243
	h	0.90 - 1.21	0.94 - 1.11
	x	2.50 - 7.50	3.50 - 5.50
	x_2	0.86 - 1.90	1.14 - 1.50
	W	0.80	1.02
	d	0.180	0.233
Mean of $q^2/h^2B + 2h/B$ $q > 0.700$		0.944	0.835
Standard Deviation		1 Test	2 Tests
Mean of $q^2/h^2B + 2h/B$ $q < 0.700$		0.856	0.831
Standard Deviation		0.039	0.064
Mean of $q_1 - q/(h-d)^{3/2}x_2 = C_{III}$ $q > 0.600$		0.533	0.515
Standard Deviation		0.009	0.103

Weighted Mean of $q^2/h^2B + \frac{2h}{B}$, $q < 0.700 = 0.872$

Standard Deviation 0.055

Weighted Mean of $\frac{q_1 - q}{(h-d)^{3/2}x_2}$, $q > 0.600 = 0.523$

Standard Deviation 0.074

Table 6.3. Case III. Summary Tests 3 and 4.

Test Series.		C.1.	C.2.	C.3.	C.4.	1.	2.
Number of Tests		7	11	7	8	16	12
Range of Weir Parameters.	q_1	0.680-0.790	0.708-0.822	0.700-0.794	0.700-0.786	0.478-0.760	0.581-0.761
	q	0.331-0.624	0.268-0.604	0.212-0.506	0.266-0.515	0.105-0.256	0.061-0.265
	h	1.185-1.325	1.230-1.405	1.255-1.345	1.245-1.342	0.90-1.24	1.10-1.41
	x	2.167-2.500	1.500-2.500	1.833-2.500	1.500-2.500	2.25-5.75	2.25-4.75
	x_2	0.140-0.858	0.182-0.875	0.233-0.847	0.182-0.740	0.61-1.11	0.59-1.25
	w	4.83	4.21	3.70	3.36	1.76	2.56
	d	0.355	0.318	0.271	0.247	0.194	0.280
Mean of $q^2/h^2B + {}^{2h}/B$, $q > 0.700$		1.003	1.027	1.027	1.008	0.930	1.030
Standard Deviation.		0.023	0.016	0.024	0.021	2 Tests	0.015
Mean of $q^2/h^2B + {}^{2h}/B$, $q < 0.700$		0.984	-	-	-	0.906	0.953
Standard Deviation.		1 Test	-	-	-	0.036	0.049
Mean of $q_1 - q/(h-d)^{3/2}\,0.18\omega\,x_2$, $q > 0.600$		1.02	0.97	1.02	1.01	1.02	1.01
Standard Deviation.		0.17	0.05	0.09	0.07	0.10	0.04

Table 6.2. Case III. Summary of Tests C.1 C.2 C.3 C.4 1 and 2.

this phenomenon and wave propagation in rapid flow as studied by Ippen (reference 19 p. 1290 et seq.)

By careful measurement and observation it was confirmed that the presence of a jump in the weir section had no effect on the flow before the jump. Hence the laws governing this part of the flow are those of Case I, leaving only the flow after the jump to be investigated.

6.4.2. Correlation of Quantity and Depth of Flow after the Jump - Case III.

From the mean values of the length of weir discharging under Case I conditions, the proportionate quantities q_1 flowing in the main channel, at the point where the jump occurred, were obtained using the quantity-length curves for Case I flow. The corresponding values of $B = 2h_2 + \frac{q_1^2}{h_2^2}$ were then computed, h_2 being, of course, the conjugate depth calculated by equation 6.7.

The theoretical equation 6.8 was written as,

$$\frac{q^2}{h^2 B} + \frac{2h}{B} = 1$$

and values of $\frac{q^2}{h^2 B} + \frac{2h}{B}$ were calculated for each test and the results are shown in Tables 11.1 to 11.9 in the appendix. A summary of the results is given in Tables 6.2 and 6.3.

The value of h was obtained in Cases C1, C2, C3 and C4 from Simpsons rule as before, but in Cases 1, 2, 3 and 4 the centre line depth was taken. Actually there is no significant difference.

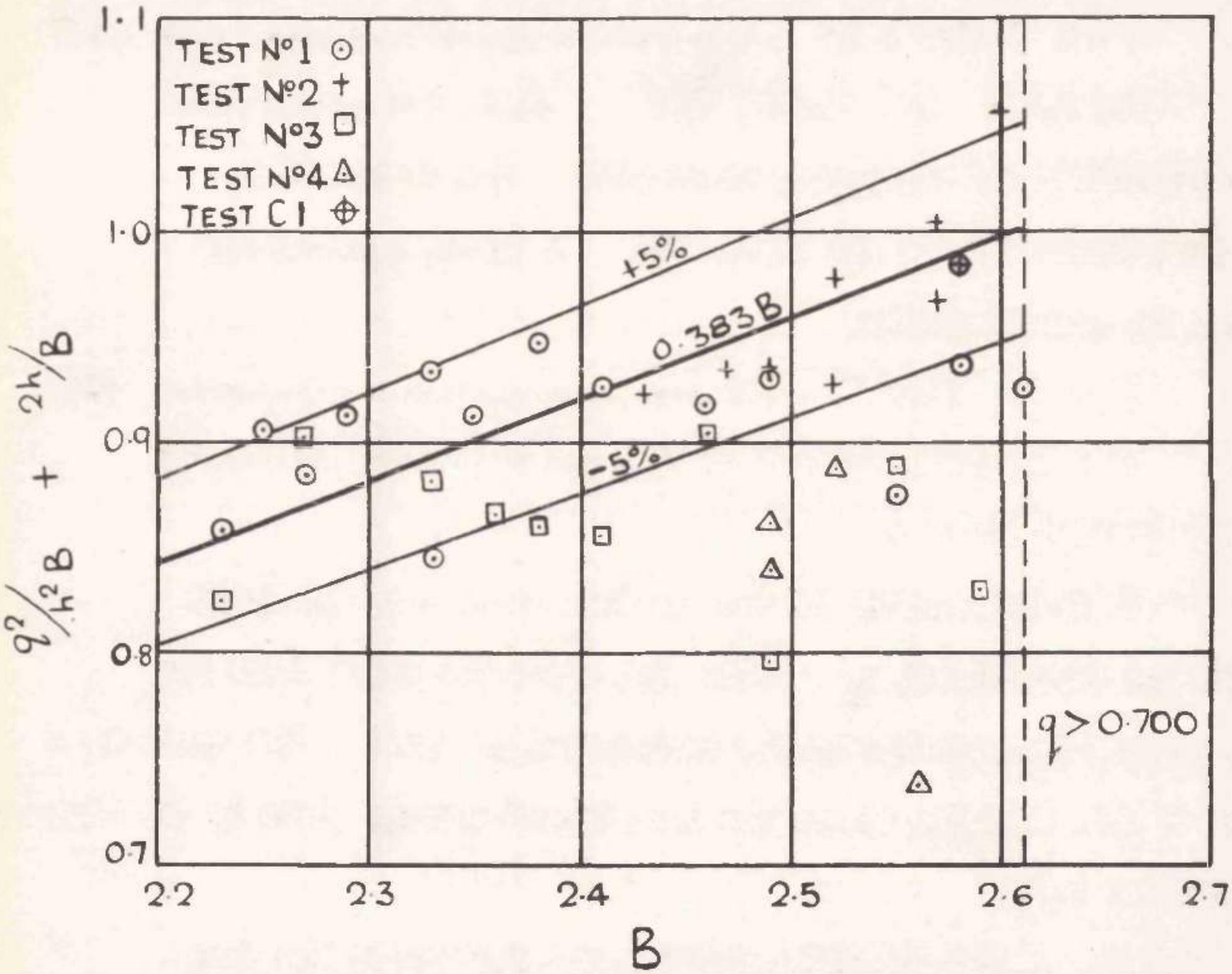


FIG. 6. 8 CASE III CORRELATION OF q & h

$q < 0.700$

The quantity $\frac{q^2}{h^2B} + \frac{2h}{B}$ is not constant over the experimental range and varies with B and with W. No single expression could be obtained for the relationship and the results are, therefore, given for various ranges as follows:-

For $0.882 > q_1 > 0.700$; $4.83 > \omega > 2.56$

$$q = h\sqrt{(1.01B - 2h)} \quad \text{.......... 6.14}$$

For $0.700 > q_1 > 0.478$; $2.56 > \omega > 0.80$, the results are very scattered, and although a trend with ω can be observed, a complex relationship is not warranted. A linear relationship with B was assumed giving,

$$q = h\sqrt{(0.38 B^2 - 2h)} \quad \text{.......... 6.15}$$

This equation is shown in Fig. 6.8 with all experimental results plotted thereon.

It is interesting to note at this stage that the limit of the two expressions, $q_1 = 0.700$ also marks the point where the jump changes from undular to the surface roller type. The quantity - depth of flow relation before the jump is, of course, given by the Case I equation, 6.9.

6.4.3. Correlation of Quantity and Length after the Jump.

Treating the flow after the jump as Case II, equation 6.12 becomes

$$q_1 - q = CIII\ (\bar{h} - d)^{3/2}\ x_2 \quad \text{.......... 6.16}$$

where x_2 is the proportionate length of weir discharging after the jump. The value of x_1 the proportionate length discharging as Case I flow before the jump is, of course, given by equation 6.10.

In the calculations, the mean depth of flow $\bar{h}$ was taken as h, the depth of flow at the end of the weir section.

CIII was calculated for all tests and an investigation of the variation with the weir parameters was carried out. Here again no single relation could be obtained covering all cases and the experimental laws governing the various ranges are given below.

For $0.882 > q_1 > 0.600$; $4.83 > \omega > 1.76$ it was found that CIII was a function of x_2 and W and that equation 6.16 reduced to

$$\frac{q_1 - q}{0.55\,(h - d)^{3/2}\,x_2^{0.18\omega}} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.17$$

The left hand side of this relation was calculated for all relevant tests, and the results are given in Tables 11.1 to 11.5 and table 11.7 in the appendix and are summarised in Table 6.2. page 45. The weighted mean for all tests is 1.00 and the standard deviation about the weighted mean is 0.09.

It is interesting to note that distribution of the residuals is not normal, 81% of them lying between $+\sigma$ and $-\sigma$ as against 68% for a normal distribution. The average error is 0.05.

For $0.765 > q_1 > 0.600$ and $1.02 > \omega > 0.80$, CIII is constant, having a mean of 0.523 and a standard deviation of 0.074.

$$\text{CIII} = 0.523 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.18$$

These results are not so constant, the surging in the two relevant tests being considerable. The values for the relevant tests are given in Tables 11.8 and 11.9 in the appendix and are summarised in Table 6.3. page 45.

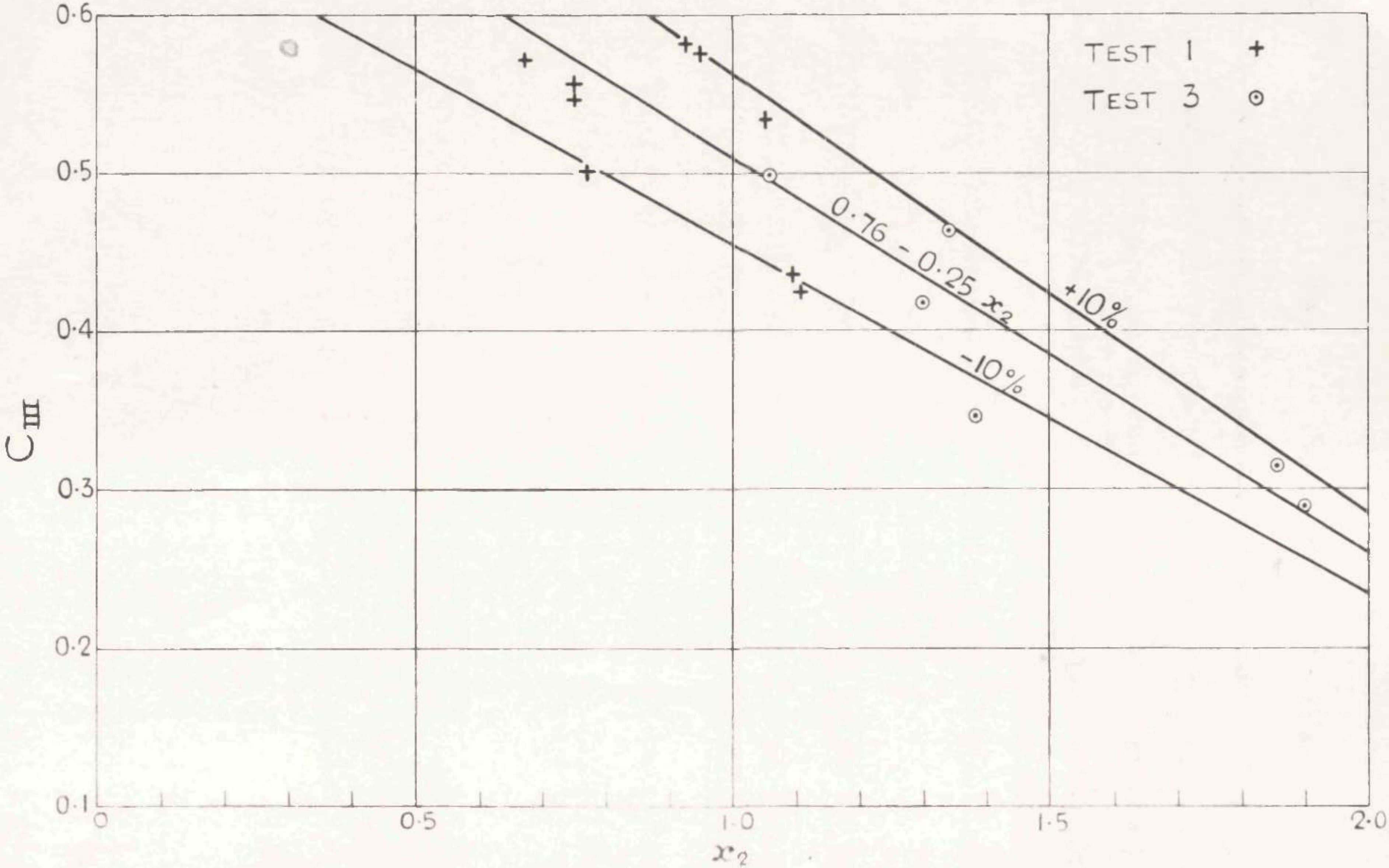


FIG 6.9. CASE III CORRELATION OF q AND x_2

$0·600 > q > 0·478\ ;\quad 1·76 > \omega > 0·80$

For $0.600 > q_1 > 0.478$; $1.76 > w > 0.80$

$$CIII = 0.72 - 0.26\,x_2 \quad \text{.............................} \quad 6.19$$

Again the results are not very good but the trend is indicated. The relevant values are shown in Tables 11.6 and 11.8 in the appendix and the equation is shown in Fig. 6.9 with the experimental points plotted thereon.

7. Discussion of Results and Conclusions.

7.1. Summary of Results.

7.1.1. Quantity - Depth of Flow Relationships.

Some of the experimental coefficients have been modified slightly at this stage in order to eliminate discontinuities between the various cases or ranges. These modifications are small, not more than 1%, and where this has been done it is denoted by an asterisk beside the equation reference.

The results are summarised below-

Case I $q = h\sqrt{(1.25 - 1.5\,h)}$ 6.9

Case II $q = h\sqrt{(A - 2h)}$ 6.11*

where $A = 2h_o + \dfrac{1}{h_o^2}$

Case III $0.882 > q_1 > 0.700; \quad 4.83 > \omega > 2.56$

$q = h\sqrt{(B - 2h)}$ 6.14

$0.700 > q_1 > 0.478; \quad 2.56 > \omega > 0.80$

$q = h\sqrt{(0.38\,B^2 - 2h)}$ 6.15

where $B = 2h_2 + \dfrac{q_1^2}{h_2^2}$

7.1.2. Diagrammatic Representation of Quantity - Depth of Flow Relationship.

The four relations given above can be very conveniently shown in diagrammatic form and this is given in Fig. 7.1. The plotting of Case I presents no difficulty. In plotting Case II equations, a value of A was chosen and proportionate quantities calculated for various proportionate depths.

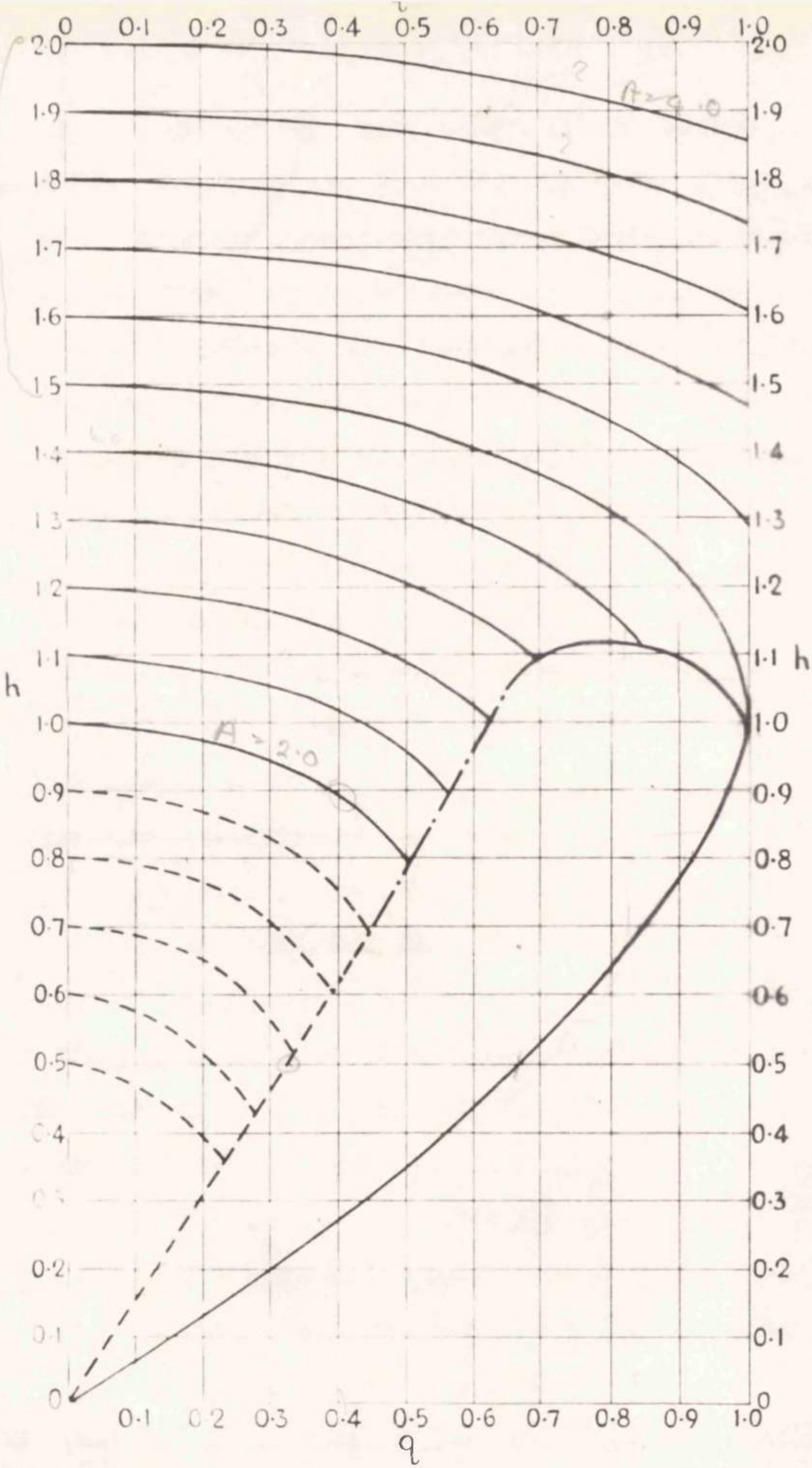


Fig. 7.1. q - h Relationship, All Cases

Case III presents slightly more complications. The method adopted was to take a value of B and calculate proportionate quantities for various proportionate depths of flow, the curve being plotted until the proportionate quantity corresponding to the selected B value was reached. The proportionate depth of flow at this point is the proportionate conjugate depth for this proportionate quantity which is, of course, q_1 and flow at greater proportionate quantities must take place under Case I conditions. It is to be noted that with equation 6.15 the proportionate depth of flow at this point is not that given by the conventional conjugate depth formulae.

The chain dotted line on the curve represents the proportionate depth of flow immediately after the jump. The diagram cannot, of course, allow for the fact that in the normal hydraulic jump there is a certain distance in which expansion of the flow occurs, and cases where the jump occurs near the end of the weir are likely to be inaccurate. In addition the use of formula 6.15 leads to complications at low values of B, giving a double solution. Obviously this means that the formula is not applicable. The region where this condition occurs has not been covered in the experimental work. However, it is to be expected that curves similar to those shown dotted would be obtained. These have sketched in merely to complete the diagram.

As will be seen later, this diagram is of extreme importance in explaining side weir phenomena and in the design of such weirs.

7.1.3. Quantity - Length Relationship.

The formulae obtained by experiment are summarised below.

Case 1 $$\frac{1 - q}{1 - q_{\infty}} = 1 - 10^{-\frac{L}{8W}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.10$$

This formula illustrated the inefficiency of Case I flow. If L = 8W 90% of the maximum discharge can occur and if L = 16W 99% can occur.

Case II $$1 - q = C(\bar{h} - d)^{3/2} x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.12$$

where $$C = 0.73 - \frac{0.32}{h_o} - \frac{0.14}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.13$$

and $$\bar{h} = \frac{2h + h_o}{3}$$

Case III. For the discharge before the jump, the relation for Case I, equation 6.10 holds.

After the jump:-

For $0.882 > q_1 > 0.600$; $4.83 > \omega > 1.76$

$$\frac{q_1 - q}{0.55\,(h - d)^{3/2}\, x_2^{\;0.18\omega}} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6.17$$

For $0.765 > q_1 > 0.600$; $1.02 > \omega > 0.80$

$$q_1 - q = 0.523\,(h - d)^{3/2}\, x_2 \quad \ldots\ldots\ldots\ldots \quad 6.16,\ 6.18$$

For $0.600 > q_1 > 0.478$; $1.76 > \omega > 0.80$

$$q_1 - q = (0.72 - 0.26\, x_2)\,(h - d)^{3/2} x_2 \quad \ldots\ldots \quad 6.16,\ 6.19$$

It is not possible with these relations to obtain a diagrammatic solution but, as will be seen, there is not the same need for such a solution.

7.2. The Behaviour of Side Weirs in Prismatic Rectangular Channels.

7.2.1. The Influence of the Downstream Channel.

Assuming flow in the downstream channel to be tranquil, there will be to each depth of flow, a unique quantity flowing at a given point in this channel. This relation is governed by the characteristics of the channel below the given point, and can be obtained by calculating backwater curves from the first critical section downstream of the point. If the channel is long and of uniform slope, this relation will be given, of course, by the normal depths for each quantity. It is, therefore, possible to construct a diagram showing this depth - quantity relationship for the downstream channel at the end of the weir section. Such a diagram will be called the "Acceptance Curve" for the downstream channel, and it is this curve which determines the mode of motion in the weir section, since conditions at the end of the weir section must conform to it.

The behaviour of the side weir can now be studied by use of the proportionate quantity proportionate depth of flow diagram and the acceptance curve suitably modified. The simplest method of study is to consider a fixed quantity Q_o in the upstream channel, a fixed weir height D, an acceptance curve for the downstream channel, and to consider the effect if varying the length of the weir.

If the depth of flow ordinates of the acceptance curve are divided by H_c, the critical depth corresponding to Q_o, and the

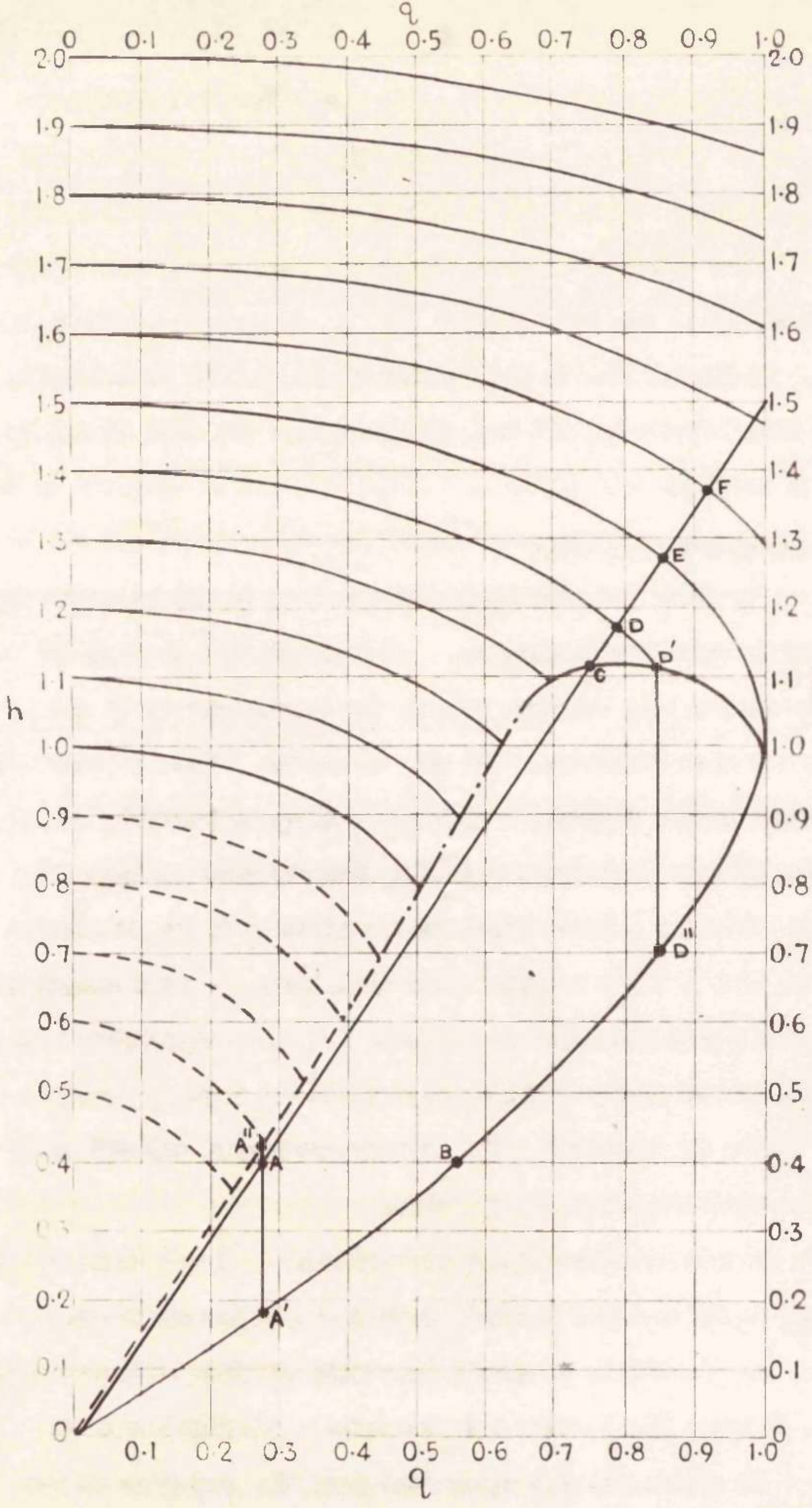


Fig. 7.2. Behaviour of Side Weir.

quantity ordinates divided by Q_o, the proportionate acceptance curve can be superimposed on the weir proportionate diagram and thus a graphical solution of a complex relation can be obtained.

Fig. 7.2 shows such a superimposition, chosen to explain the behaviour of the side weir. It is assumed that when Q_o is flowing in the downstream channel the depth of flow is 1.5 times the critical depth H_c, and that the height of the weir is 0.4 H_c i.e. d = 0.4.

7.2.2. Case I Flow.

Consider the weir to be infinitely long and hence giving the maximum possible discharge. At a proportionate depth of flow = 0.4, that is the weir height, the acceptance curve shows that the proportionate quantity will be 0.26. This is indicated by point A on the diagram. The q - h diagram, however, indicates that for this proportionate quantity, flow is only possible if h = 0.18, point A', under rapid flow conditions or greater than 0.42, point A'', under tranquil flow conditions. Such conditions hold until point B is reached and this point, corresponding to a proportionate quantity of 0.55, represents the maximum possible discharge of the weir under the assumed conditions and will only occur if the weir length is infinite.

Flow in the downstream channel will be rapid, continuing so until conditions are reached, such that a jump can occur.

The condition of Case I flow will continue with decreasing length of weir, until point C is reached. At this point the depth of flow given by the acceptance curve is conjugate to the

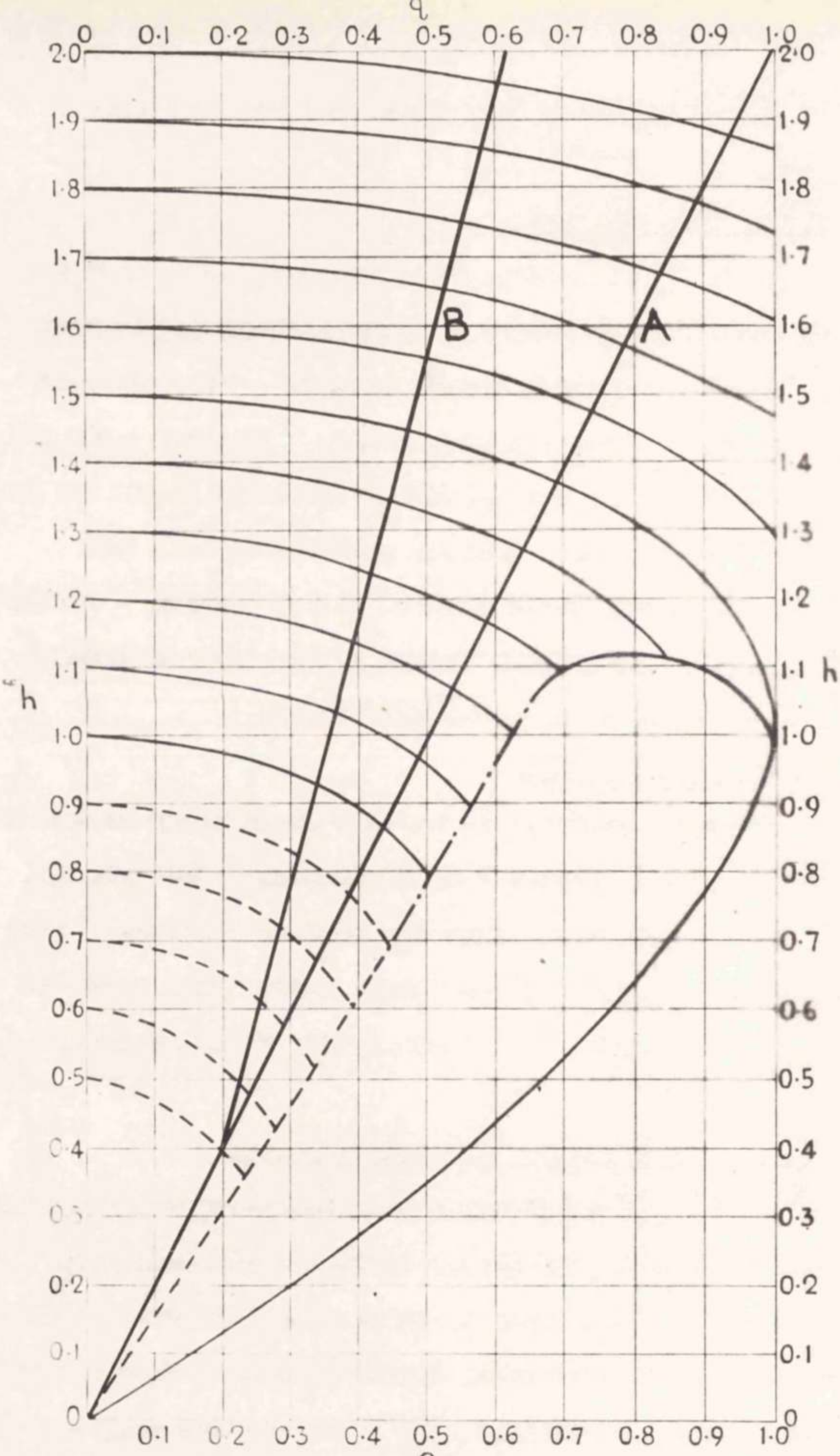


FIG 7.3. BEHAVIOUR OF SIDE WEIR.

depth of flow at the end of the weir and the jump will occur at this point.

7.2.3. Case III Flow.

Shortening the weir length further will bring conditions to those represented by point D. Here the total proportionate quantity flowing in the main channel is 0.78. The jump occurs at point D^1 on the diagram where $q_1 = 0.84$. In other words 0.06 is discharged after the jump and 0.16 is discharged before the jump under Case I conditions. Case III conditions persist with shortening of the weir length until point E is reached. At this point the flow in the upstream channel is drawn down to critical depth and expansion occurs in the weir section.

7.2.4. Case II Flow.

A further shortening of the weir length brings conditions to those represented by point F on the diagram. Tranquil flow now occurs, the head rising along the length of the weir. Such conditions persist until the weir length becomes zero, when flow occurs with a quantity Q_o at a depth of flow of 1.5 H_c, point G on the diagram. Not show

7.2.5. Other Possible Sequences of Events.

In Fig. 7.3 two different proportionate acceptance curves are shown, Curve A, one similar to the one discussed above but of milder slope and Curve B, one in which a throttle or baffle is introduced in the downstream channel arranged so as to operate when the depth of flow becomes greater than the weir height. It will be noted that with both these curves, Case I flow does not occur and that the efficiency of the weir is increased, greatly so when a throttle is introduced.

7.3. Design of Side Weirs in Prismatic Rectangular Channels.

7.3.1. Data Required.

The ideal requirement for an overflow is that the quantity accepted by the downstream channel shall be constant when the overflow is discharging and shall be independent of the quantity delivered to the overflow by the upstream channel. A side weir cannot satisfy this requirement. As the quantity delivered by the upstream channel increases, so does the quantity accepted by the downstream channel.

A decision, therefore, must be made as to,

(a) the quantity to be accepted by the downstream channel without discharge from the side weir, Q_H.

(b) the maximum quantity which can be accepted safely by the downstream channel, Q.

In order to complete the design the additional data required is:-

(c) the downstream channel acceptance curve.

(d) the maximum discharge of the upstream channel Q_o.

7.3.2. Design Procedure.

The steps in the design calculations are outlined below,

(a) Obtain the critical height, H_c, for the maximum discharge of the upstream channel Q_o.

(b) Fit the proportionate acceptance curve to the proportionate quantity - depth of flow diagram, Fig. 7.1, as explained in section 7.2.1.

(c) Obtain the head H_N corresponding to the quantity, Q_N, at which discharge from the side weir is required to commence, then D, the weir height, is equal to H_N and $d = \frac{H_N}{H_c}$

(d) The conditions to give maximum quantity, Q, to be accepted by the downstream channel are then given on the combined diagram by point $q = \frac{Q}{Q_o}$ on the proportionate acceptance curve.

(e) If this point represents a possible mode of motion the length of weir can then be calculated by the use of equations 6.10; 6.12; 6.13; 6.17; 6.16, 6.18; or 6.16, 6.19; according to the mode of motion predicted.

(f) If the point, q, represents an impossible mode of motion, the design data must be amended until a possible solution is found. This can be done in a number of ways. If the quantities Q and Q_N must not be altered, the characteristics of the downstream channel must be altered by adjusting the slope or installing a throttling device.

7.3.3. The Effect of Channel Friction.

Several formulae, notably that of Favre [13], include a term in the relationship between quantity and head to allow for frictional losses. The inclusion of this term means that the length variable and a frictional coefficient are introduced into the expression. A simple relationship between quantity and head, independent of dimensions, is thus no longer possible and study of the weir phenomena and design calculations to obtain the best solution of a particular problem are exceedingly complicated.

It is considered that the frictional losses would be very small in the application of side weirs to channels of mild slope and the disadvantages of ignoring frictional losses are more than outweighed by the simplicity of the calculations.

Despite this it is possible to make allowance in design, for these losses, in the following manner. The weir depth and length are obtained by the analysis outlined above. The mean velocity in the weir section is then calculated and the loss of head by friction assessed by any of the usual formulae. This loss of head is then expressed as a slope and the bottom of the channel over the weir length and the weir plate set to this slope.

7.4. Accuracy of the Experiments.

7.4.1. Possible Sources of Error.

Owing to the laboratory layout, it was not possible to calibrate the weir on the collecting channel by direct volumetric measurement. In addition it was not possible to permit a greqt depth of water in the collecting channel, which resulted in the sill of the weir being set at a lower level than was desirable. Hence at large flows the depth of flow was near critical, with the constant instability associated with this type of flow.

Under conditions of Case III flow, the surging which occurred, despite all efforts to reduce it, made the obtaining of accurate experimental data very difficult. Some tests, as already noted, had to be rejected for this reason.

7.4.2. Estimation of the Accuracy Obtainable.

With the large number of variables to contend with, any assessment of accuracy is apt to be approximate but the following analysis will show that results have been obtained within the limits of the apparatus.

Assuming the error of volumetric measurement in the calibration of the main weir supplying the apparatus to be negligible, the standard deviation of a measurement of quantity by the collecting channel is given by the expression,

$$\left(\frac{\sigma_{Q_W}}{Q_W}\right)^2 = \left(5\frac{\sigma_{Y_T}}{Y_T}\right)^2 + \left(3\frac{\sigma_Y}{Y}\right)^2$$

where, Q_W is the quantity measured, σ_{Q_W} its standard deviation.

Y_T is the head over the main weir, σ_{Y_T} its standard deviation, Y is the head over the collecting channel weir, σ_Y its standard deviation.

In the deduction of the formula, account has been taken that there are two measurements associated with each weir, calibration and the actual quantity measurement.

The head over the main weir could be read to an accuracy of ± 0.0005 ft., giving an approximate standard deviation of ± 0.00025 ft. Under conditions of no surging, the head over the weir on the collecting channel could be read to an accuracy of ± 0.001 ft., giving a standard deviation of ± 0.0005 ft. Under surging conditions, however, the accuracy was halfed giving a standard deviation of about ± 0.001 ft.

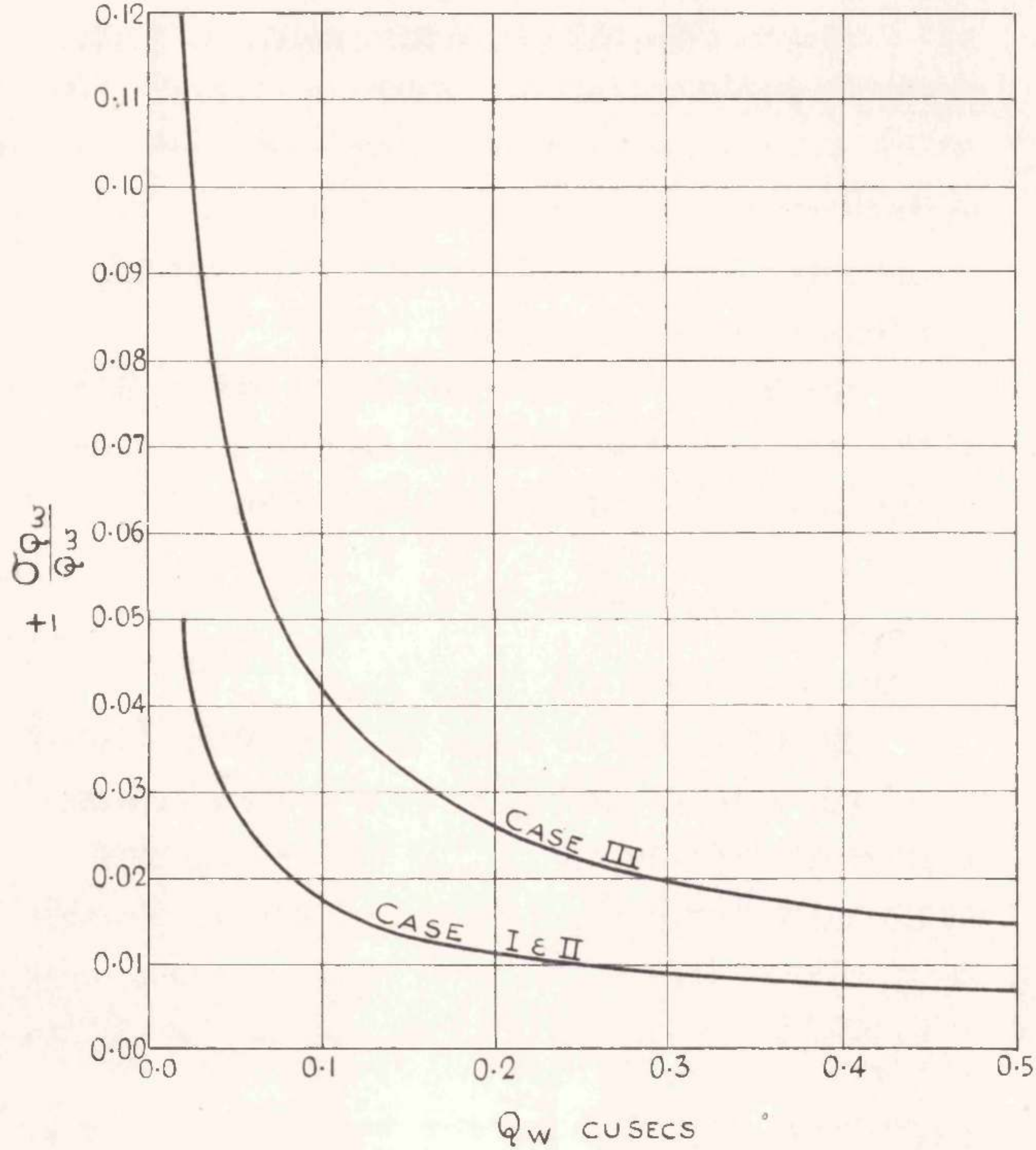


FIG. 7. 4. PROBABLE ACCURACY OF RESULTS DUE TO ERRORS IN WEIR MEASUREMENTS.

In Cases I and II the quantities used in the calculations were obtained by subtraction so that the expression for standard deviation becomes

$$\left(\frac{\sigma_{Q_W}}{Q_w}\right)^2 = 2\left\{\left(5\frac{\sigma_{Y_T}}{Y_T}\right)^2 + \left(3\frac{\sigma_Y}{Y}\right)^2\right\}.$$

In these cases there is no surging and substituting the values gives the relation shown in Fig. 7.4.

With Case III, a further subtraction is necessary in order to obtain the amounts discharging before and after the jump and the expression for standard deviation, therefore, becomes

$$\left(\frac{\sigma_{Q_W}}{Q_W}\right)^2 = 3\left\{\left(5\frac{\sigma_{Y_T}}{Y_T}\right)^2 + \left(3\frac{\sigma_Y}{Y}\right)^2\right\}$$

This relationship is also shown in Fig. 7.4.

The deviations of the results from the empirical equations are, of course, somewhat greater as would be expected, particularly so in Case III due to the surging. Of course, the empirical formulae ignore the noted variations with the other weir dimensions such as width and height of weir and the unsteady, turbulent nature of the flow, which can be seen in the Figs. 5.4; 5.5; 5.6; and 5.7.

8. Conclusions.

8.1. General.

The theory evolved in the thesis shows, and the experimental work confirms, for rectangular channels, that the mode of motion of water flowing over a side weir in a channel is dependent on, the geometric parameters of the weir section, the downstream channel characteristics and, if flow in the upstream channel is rapid, the upstream channel characteristics.

Two equations are required in order to define the flow, the first being obtained from equating force to rate of change of momentum and the second by considering the discharge over the weir.

The theory, confirmed by the experimental work, explains clearly the functioning of a side weir, in particular the three predicted modes of motion for tranquil flow in the upstream channel do occur in prismatic rectangular channels.

The results obtained have enabled a rational design procedure to be evolved, which has been lacking in the past.

8.2. Correlation of Quantity and Depth of Flow.

For prismatic and rectangular channels, there exists a relationship between quantity and depth of flow which is independent of the length of the weir and practically independent of the height of the weir. This experimental relationship, shown diagrammatically in Fig. 7.1., involves some modification to the theoretical, probably due to the assumption of hydrostatic pressure distribution since the modification is required where the flow is decidedly curvilinear.

8.3. Correlation of Quantity and Length.

It is found that, in Case II flow, and also in the tranquil flow conditions after the jump in Case III, the ordinary rectangular weir formula can be applied over the experimental range by expressing the coefficient of discharge as a variable of the weir parameters. The conditions of Case I flow, and of the similar rapid flow before the jump in Case III, are, however, so complex that the rectangular weir formula is not applicable and an empirical formula, expressed in terms of the weir parameters has been adopted, which fits the results over the experimental range.

9. Notation.

The symbols defined below are those which appear throughout the text with reference to weirs in prismatic rectangular channels; other symbols are fully defined in the particular section in which they occur.

In general, upper case letters refer to actual values and lower case letters to proportionate values.

Weir Dimensions.

Actual Value.	Proportionate Value = Actual Value. $\div H_c$	$\div W$	
D	d	-	Height of weir crest above bottom channel.
L	ℓ	x	Total length of weir.
L'	-	-	Length from start of weir to any section.
-	-	x_1	Length of weir discharging before jump, Case III flow.
-	-	x_2	Length of weir discharging after jump, Case III flow.
W	w	-	Channel width.

Depths of Flow.

Actual Value.	Proportionate Value = Actual Value $\div$ Hc	
Hc	1	Critical depth of flow in upstream channel.
H'	h'	Mean depth at any section.
H_o	h_o	Mean depth at entry to weir section

$\bar{H}$	$\bar{h}$	Mean depth after jump, Case III flow.
H	h	Mean depth at end of weir section.
H_1	h_1	Mean depth just prior to jump, Case III flow.
H_2	h_2	Mean depth just after jump, Case III flow.

Quantities.

Actual Value.	Proportionate Value = Actual Value ÷ Q_o	
Q_o	1	In main channel at entry to weir section.
Q'	q'	In main channel at any section.
Q	q	In main channel at end of weir section.
Q_1	q_1	In main channel at jump section.
-	q_{∞}	In main channel at end of weir section in Case I flow with infinite weir length.
Q_w	-	Total quantity flowing over side weir.

Miscellaneous.

A, Ratio of twice the specific energy at beginning of weir section to critical ~~specific energy~~ in upstream channel. depth. X

B, Ratio of twice the specific energy just after the jump to critical specific energy in upstream channel. No such thing exists X

C, Coefficient of Discharge, Case II flow.

Cd, Coefficient of Discharge, general.

CIII, Coefficient of Discharge, Case III flow after jump.

E, Specific Energy of flow at end of weir section.

Eo, Specific Energy of flow at commencement of weir section

g, Acceleration due to gravity.

M,N, Constants

S, Area of flow in main channel at distance L' from entry to weir section.

V_o, Average velocity in main channel at entry to weir section.

V', Average velocity in main channel at distance L' from entry to weir section.

V, Average velocity in main channel at end of weir section.

Z, Depth of centroid of transformed section from surface.

ϕ, A function of

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11. APPENDIX, DETAILED RESULTS.

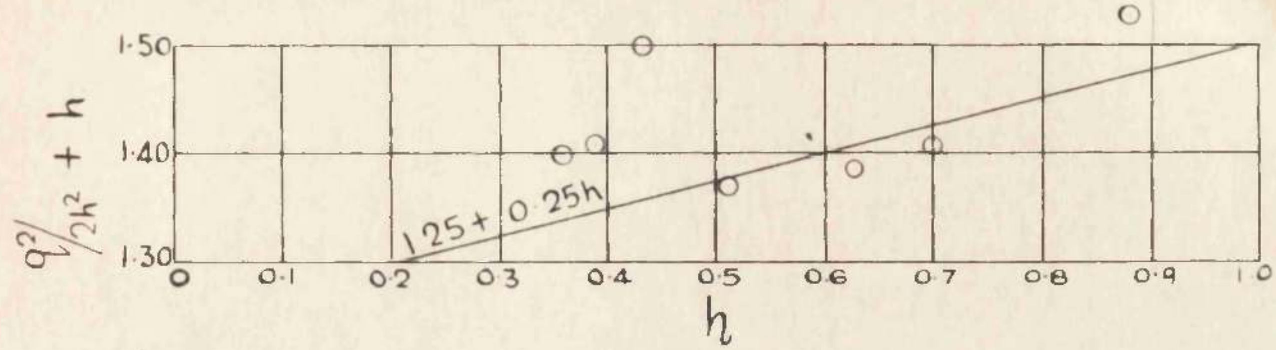


FIG 11.1 TEST B3. Q_o = 0·130 cs. ; H_e = 0·204 ft.

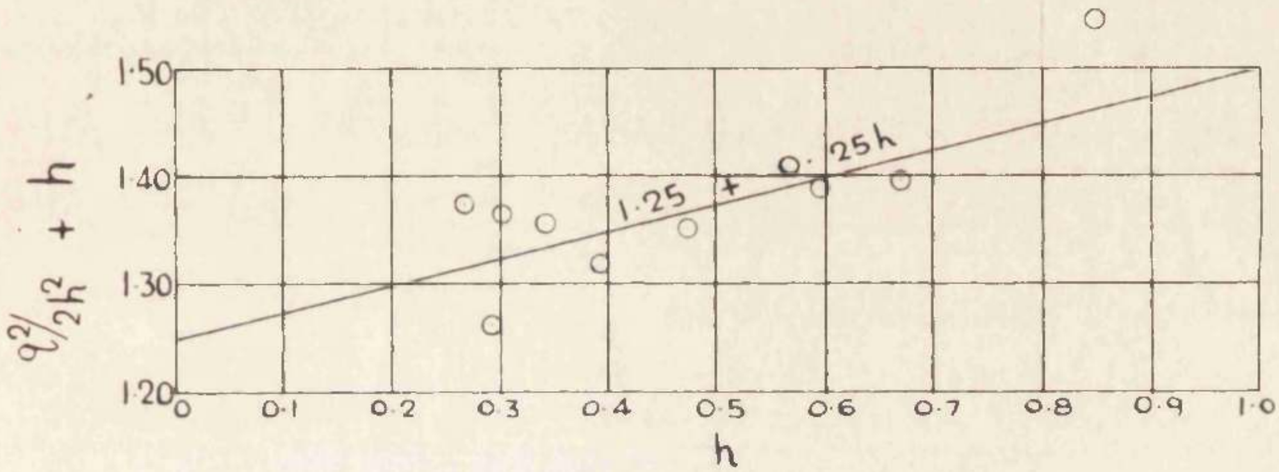


FIG 11.2 TEST B4. Q_o = 0·199 cs ; H_c = 0·270 ft.

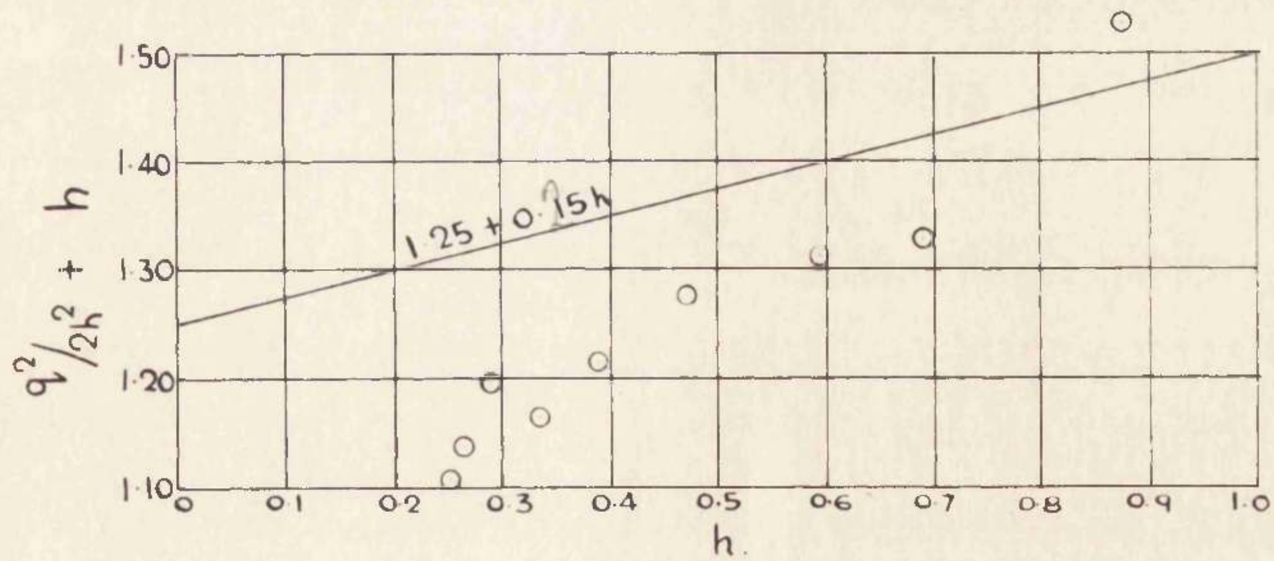


FIG 11.3 TEST B.5. Q_o = 0·241 cs ; H_c 0·307 ft

CASE I RAPID FLOW W = 3 in

VARIATION OF SPECIFIC ENERGY

WITH DEPTH OF FLOW

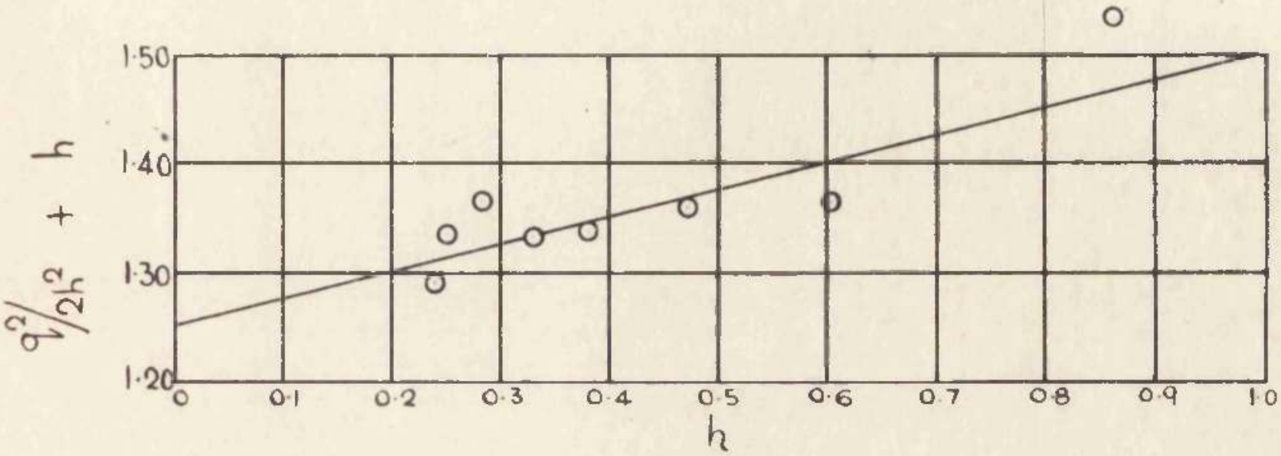


FIG. 11.4 TEST B1 Q_0 = 0·307cs ; H_c = 0·357ft.

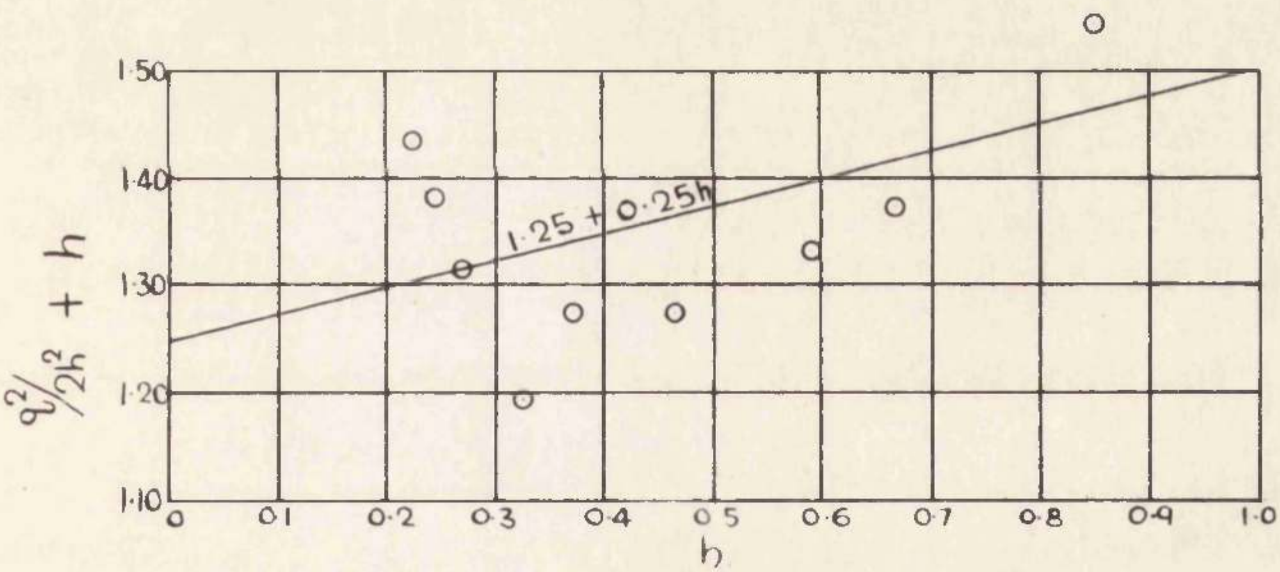


FIG. 11.5 TEST B.6 Q_0=0·317cs; H_c=0·369 ft.

CASE I RAPID FLOW W = 3 in.

VARIATION OF SPECIFIC ENERGY

WITH DEPTH OF FLOW.

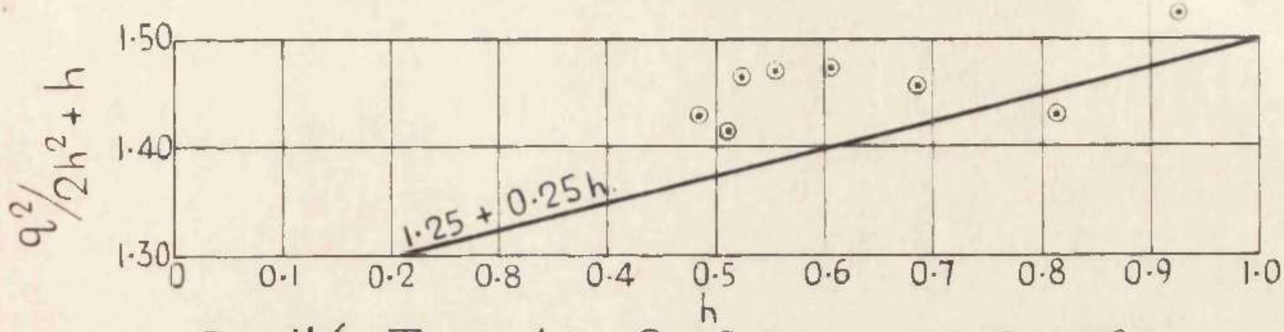


FIG. II.6 TEST A.1. $Q_o = 0{\cdot}172$ cs ; $H_c = 0{\cdot}155$ ft.

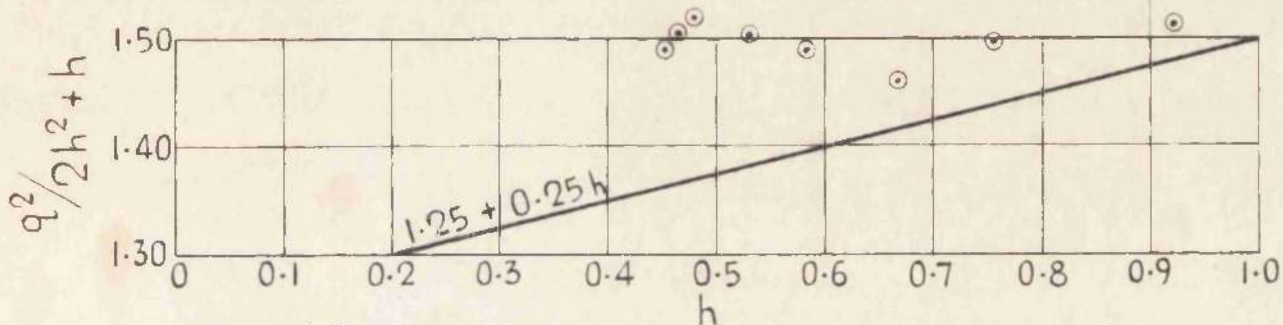


FIG. II.7 TEST A2 $Q_o = 0{\cdot}210$ cs ; $H_c = 0{\cdot}177$ ft.

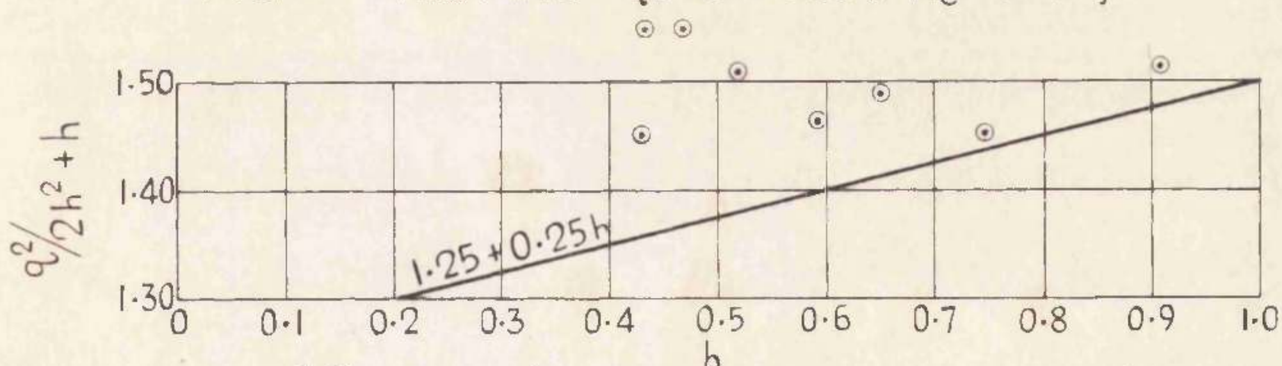


FIG. II.8 TEST A3 $Q_o = 0{\cdot}240$ cs ; $H_c = 0{\cdot}193$ ft

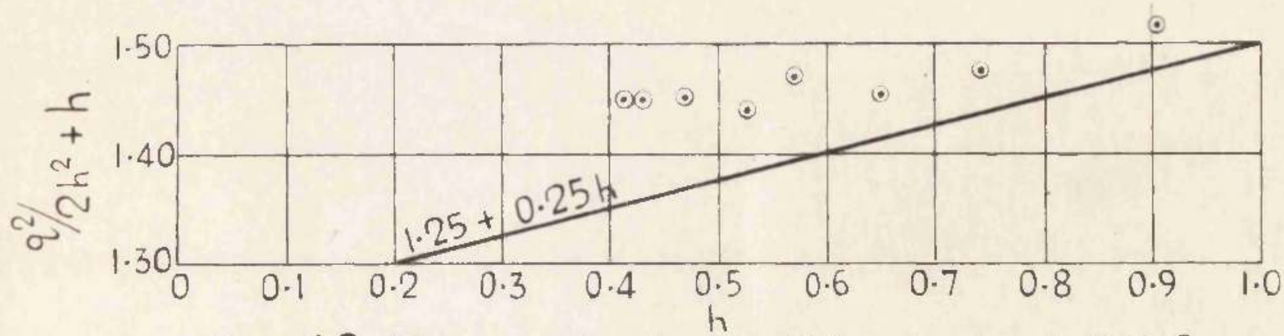


FIG. II.9 TEST A4 $Q_o = 0{\cdot}262$ cs ; $H_c = 0{\cdot}205$ ft

CASE I. RAPID FLOW. W = 6 in.

VARIATION OF SPECIFIC ENERGY WITH DEPTH OF FLOW

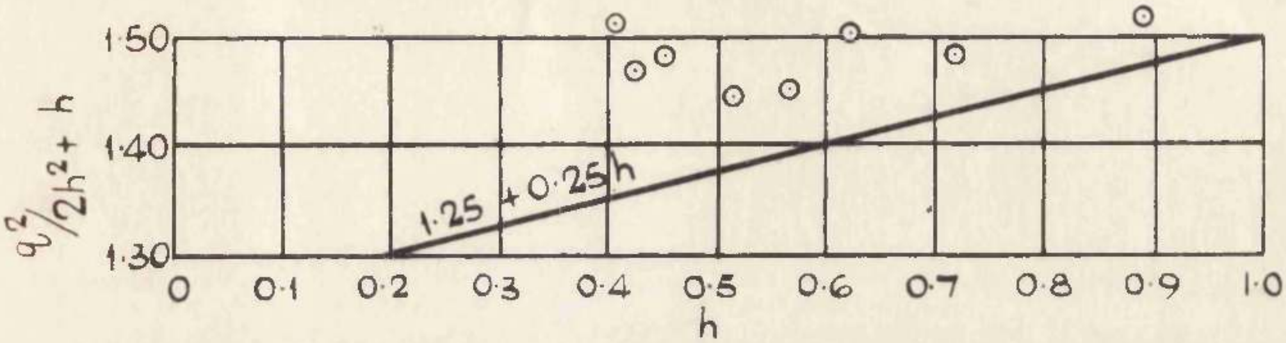


FIG. 11·10 TEST A5 Q_o = 0·283cs; Hc = 0·216 ft

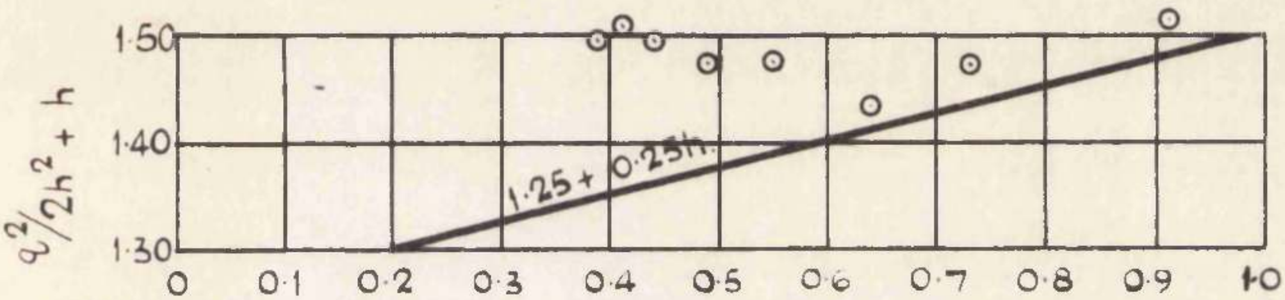


FIG. 11.11 TEST A6. Q_o = 0·309cs; Hc = 0·228 ft

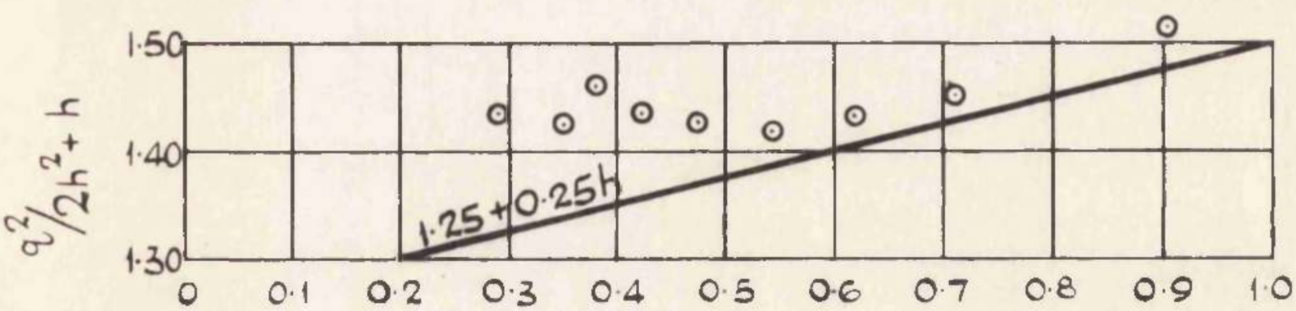


FIG. 11.12 TEST A.7 Q_o = 0·448cs; Hc = 0·293

CASE I RAPID FLOW W = 6 IN

VARIATION OF SPECIFIC ENERGY

WITH DEPTH OF FLOW

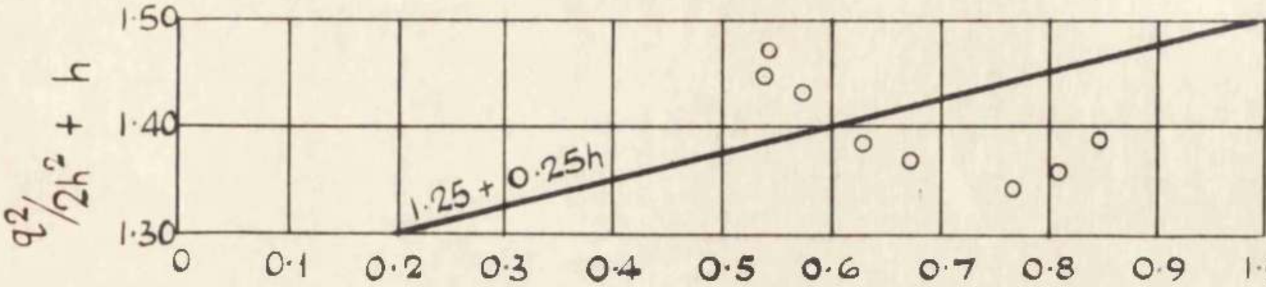


FIG. 11.13 TEST C1 Q_o = 0·260 cs; H_c = 0·155 FT

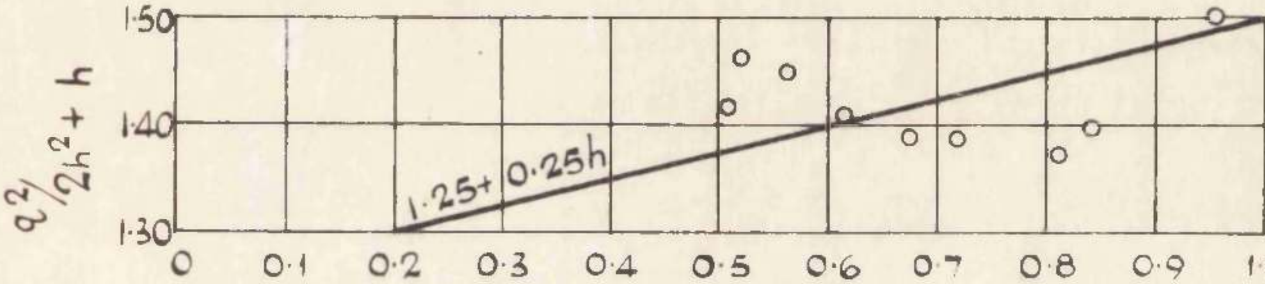


FIG. 11.14 TEST C.2. Q_o = 0·318 c.s; H_c = 0·178 FT

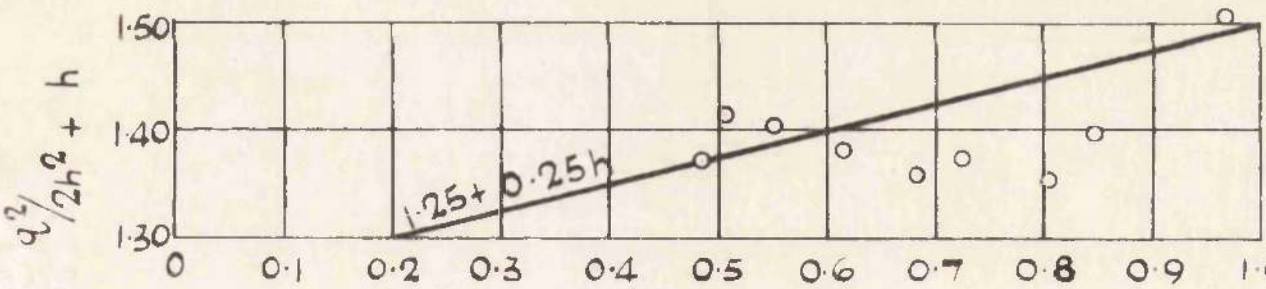


FIG. 11.15 TEST C.3. Q_o = 0·387 cs; H_c = 0·203 FT

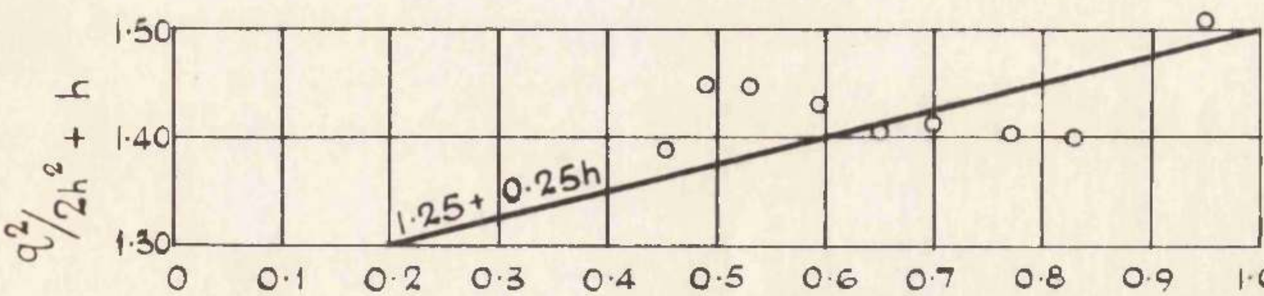


FIG. 11.16 TEST C4. Q_o = 0·447 cs; H_c 0·223 FT

CASE I RAPID FLOW W = 9 IN

VARIATION OF SPECIFIC ENERGY

WITH DEPTH OF FLOW.

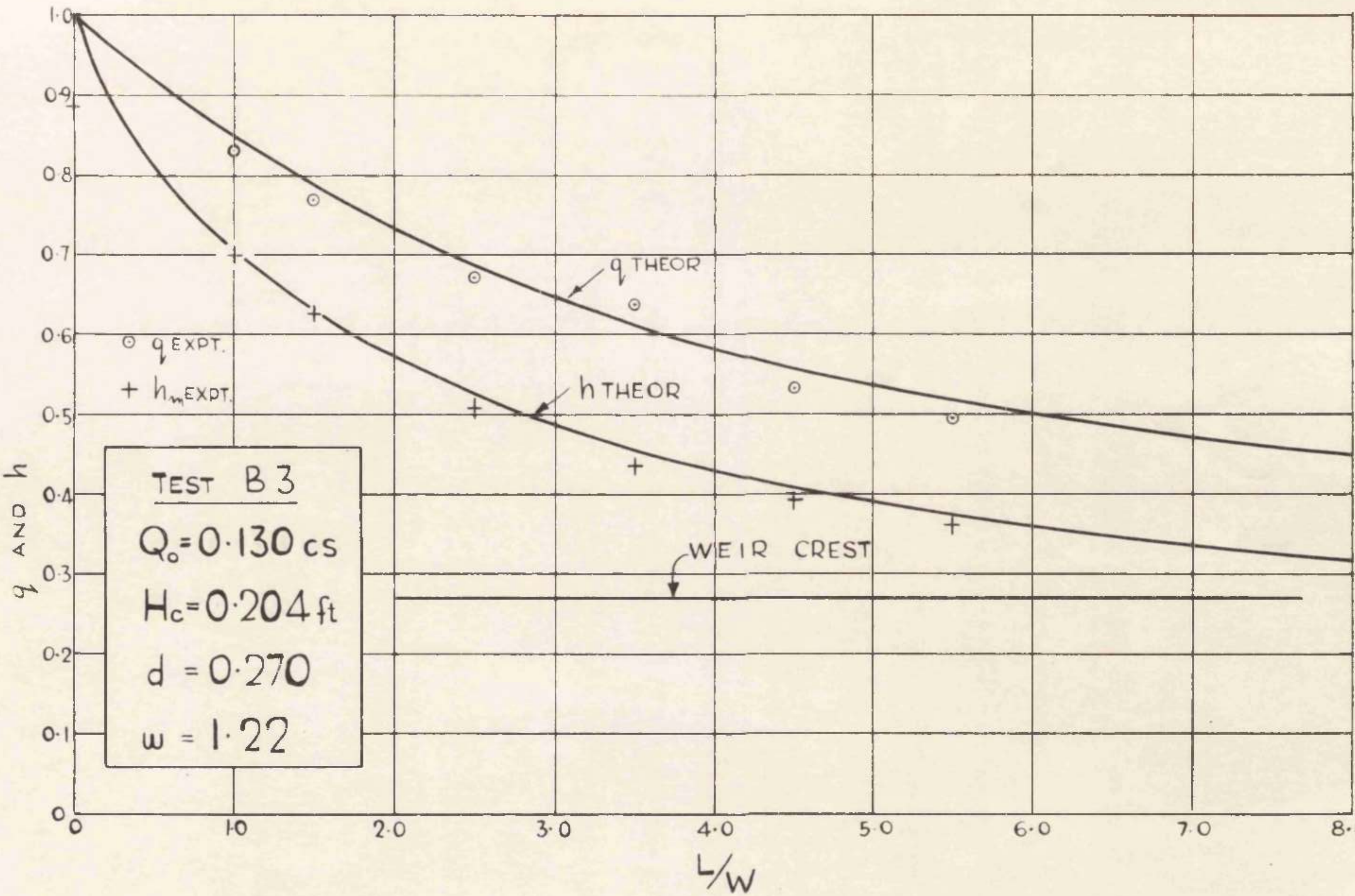

q THEOR
h THEOR
q EXPT.
h_m EXPT.
WEIR CREST
TEST B 3
Q₀=0·130 cs
Hc=0·204 ft
d = 0·270
w = 1·22
q AND h
L/W
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0

FIGURE II.17.

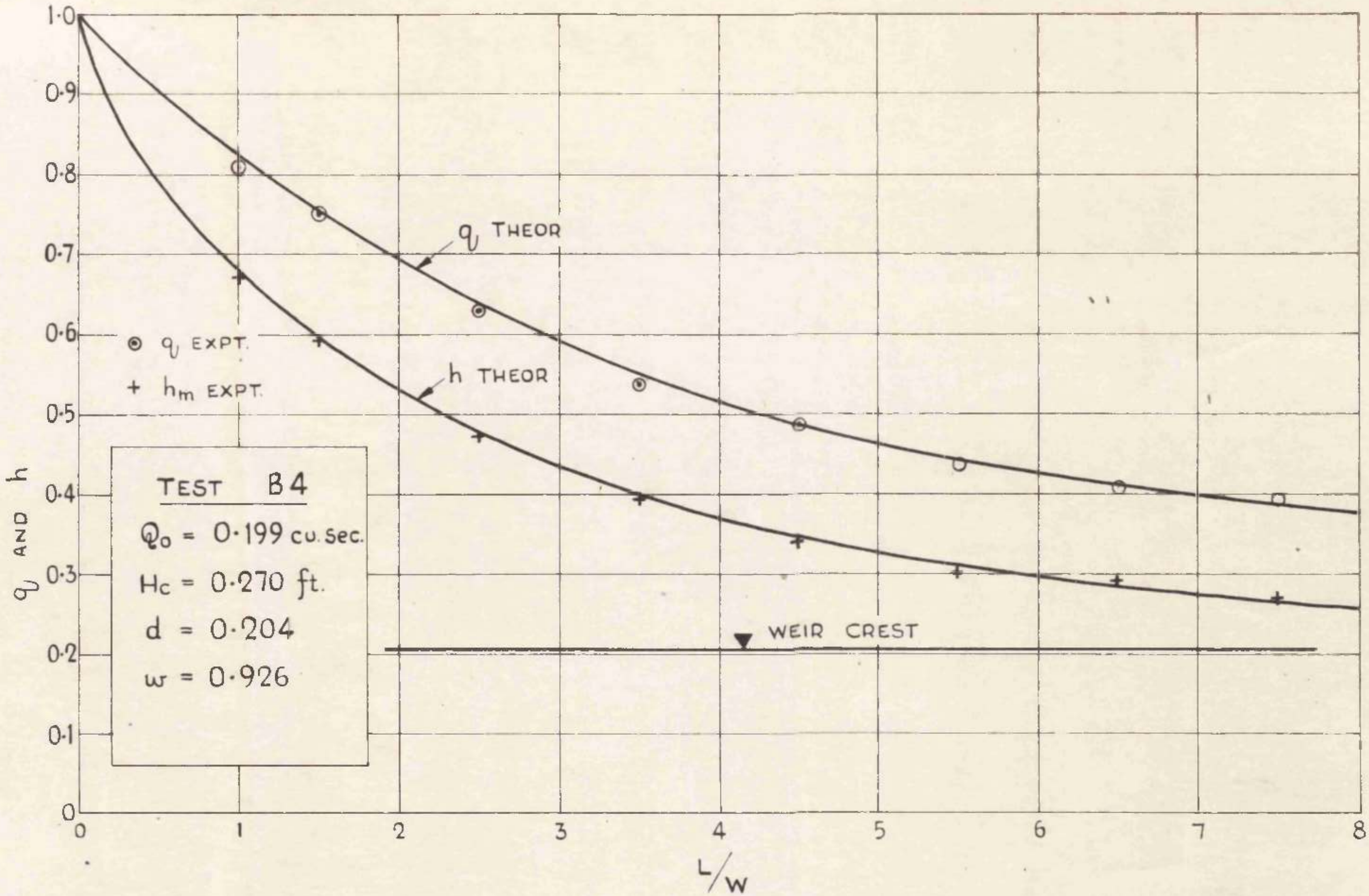

1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
q AND h
0
1
2
3
4
5
6
7
8
L/W
q THEOR
h THEOR
⊙ q EXPT.
+ h_m EXPT.
TEST B4
Q₀ = 0·199 cu. sec.
Hc = 0·270 ft.
d = 0·204
w = 0·926
WEIR CREST

FIGURE II.18.

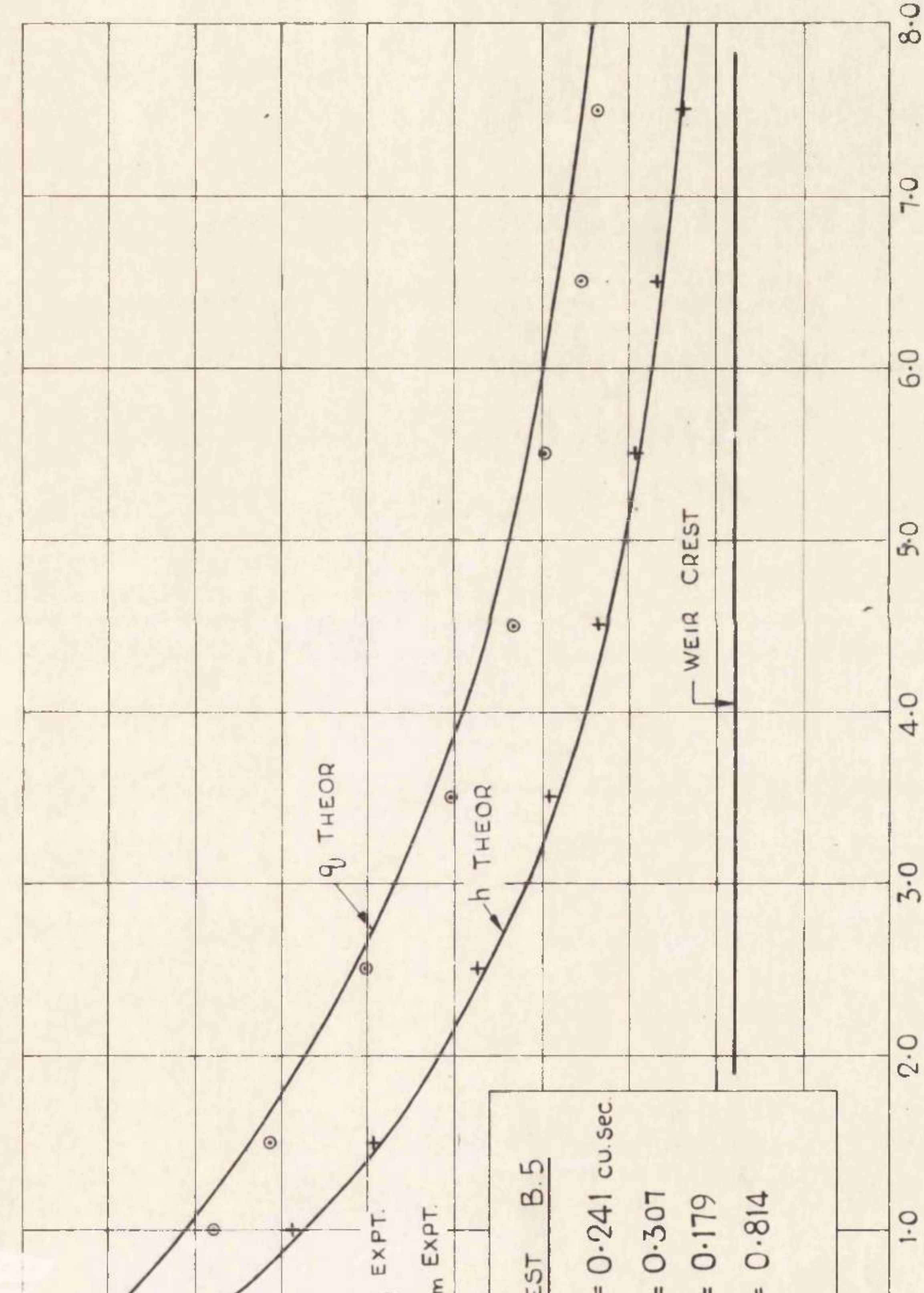

TEST B.5
Q_o = 0·241 cu.sec.
H_c = 0·307
d = 0·179
ω = 0·814
⊙ q EXPT.
+ h_m EXPT.
q THEOR
h THEOR
WEIR CREST
L/W
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0

FIGURE II.19.

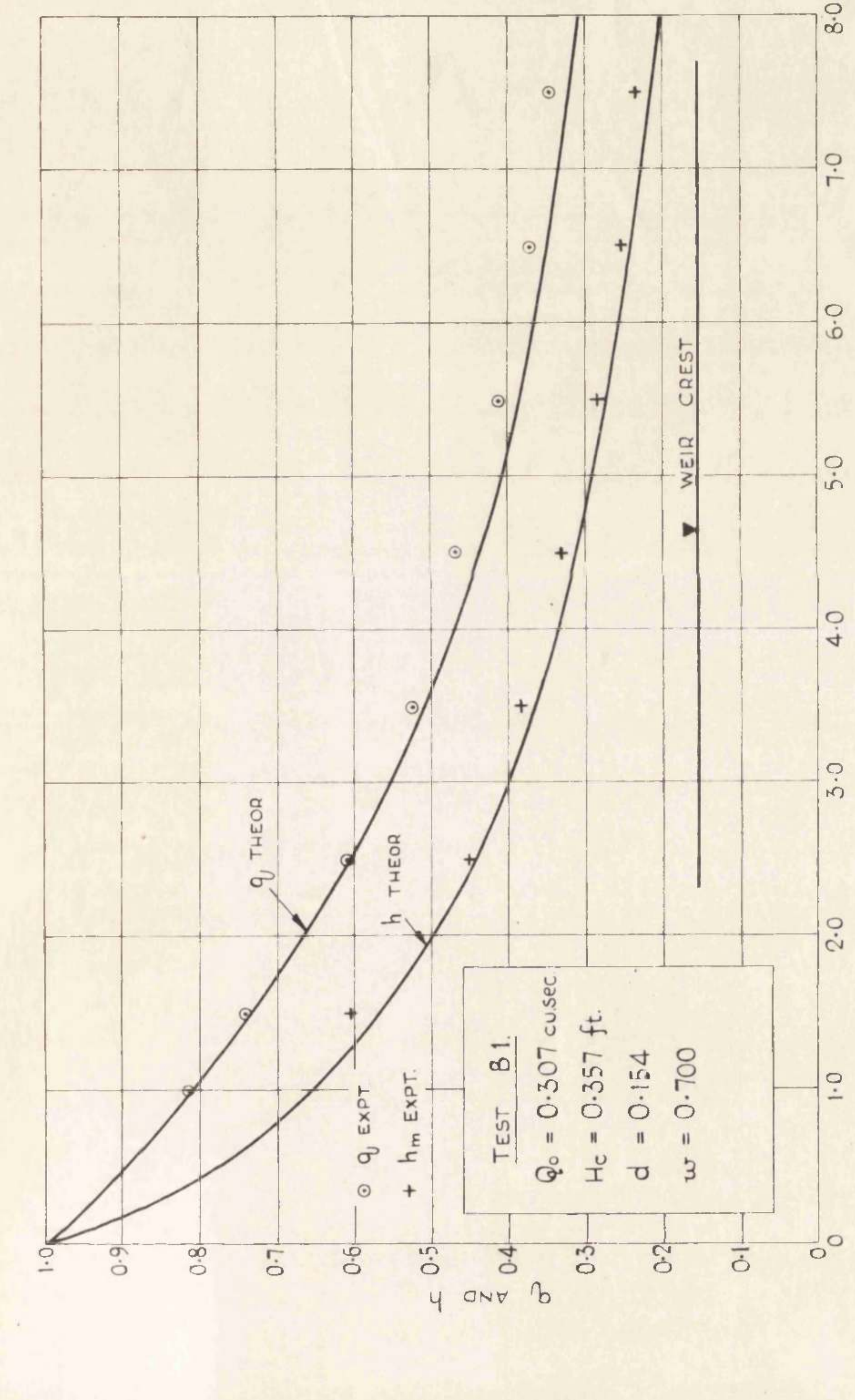
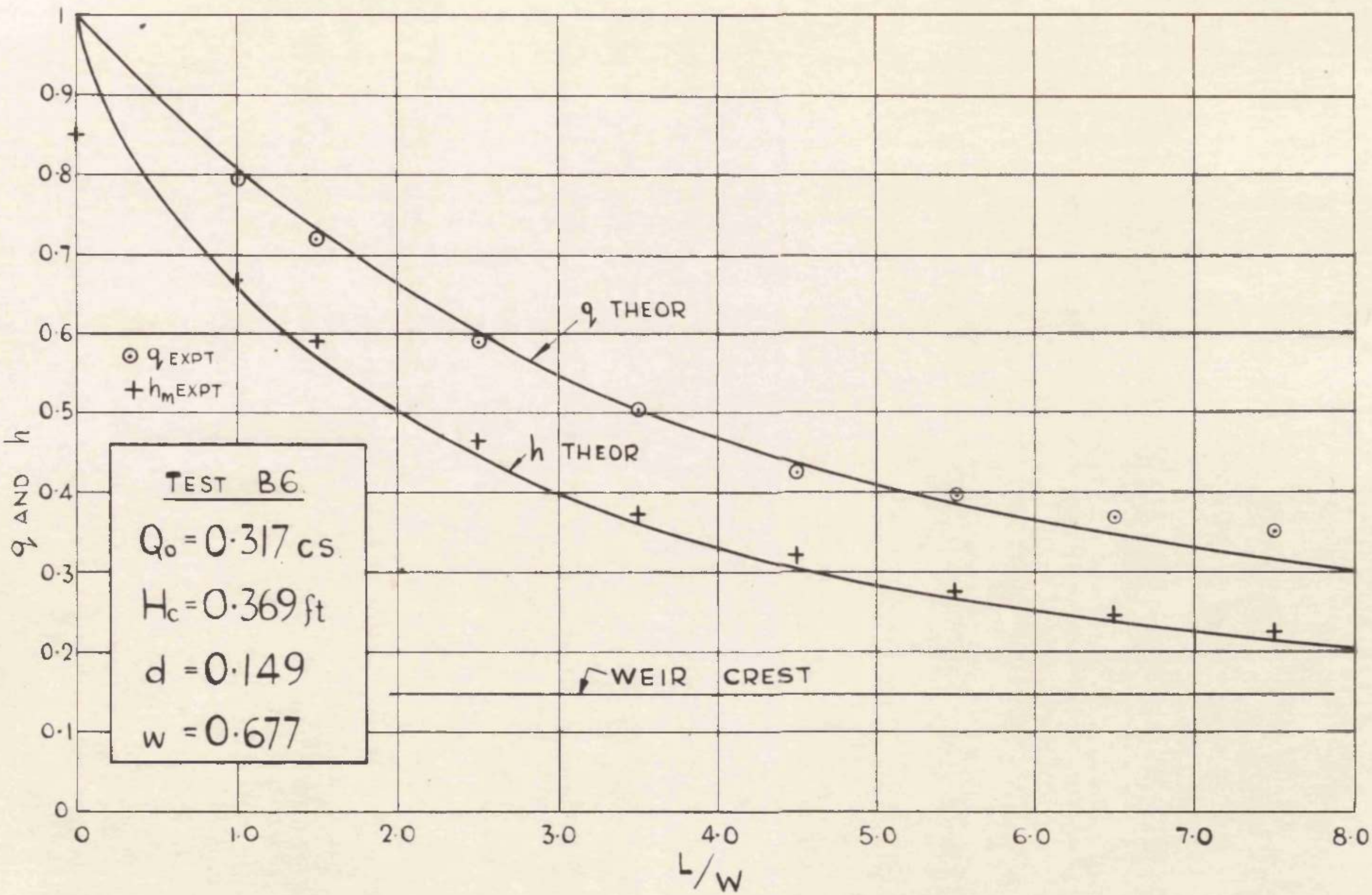

q AND h
L/w
⊙ q EXPT
+ h_m EXPT.
q THEOR
h THEOR
TEST B1.
Q_o = 0·307 cusec.
H_c = 0·357 ft.
d = 0·154
ω = 0·700
WEIR CREST

FIGURE 11.20



TEST B6
Q_o = 0·317 cs
H_c = 0·369 ft
d = 0·149
w = 0·677
⊙ q EXPT
+ h_m EXPT
q THEOR
h THEOR
WEIR CREST
q AND h
L/W
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0

FIGURE 11.21.

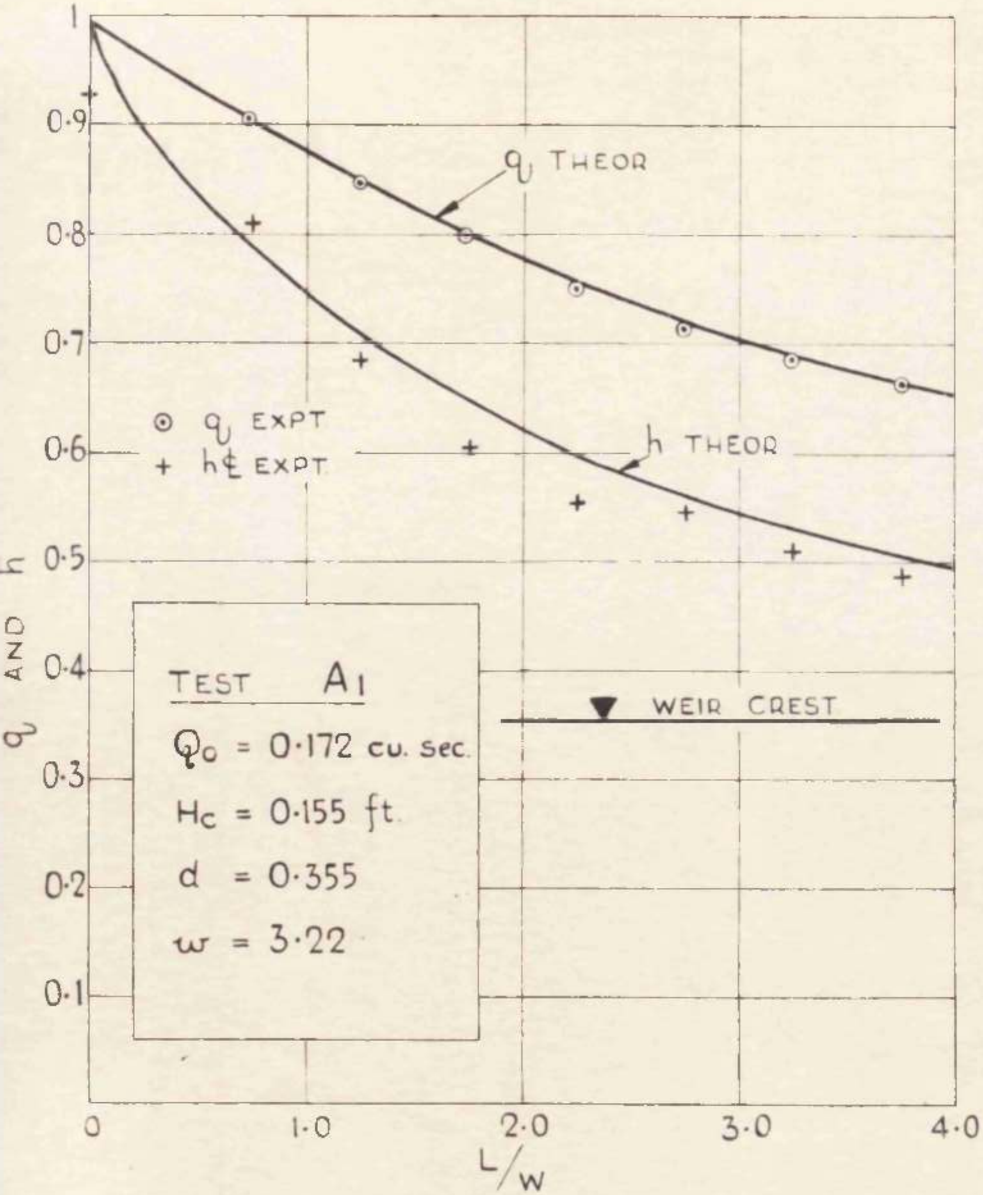


FIG. 11.22.

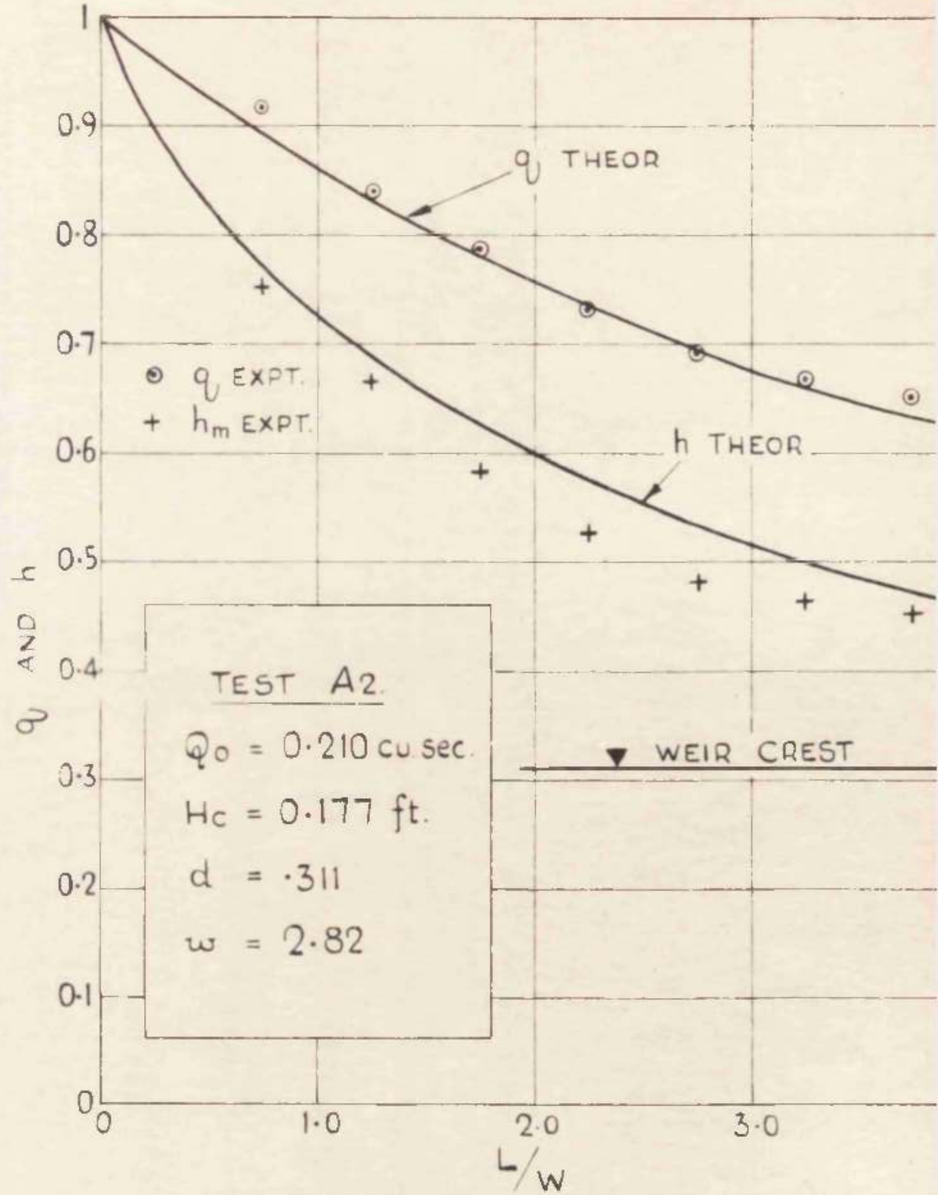


FIG. 11.23

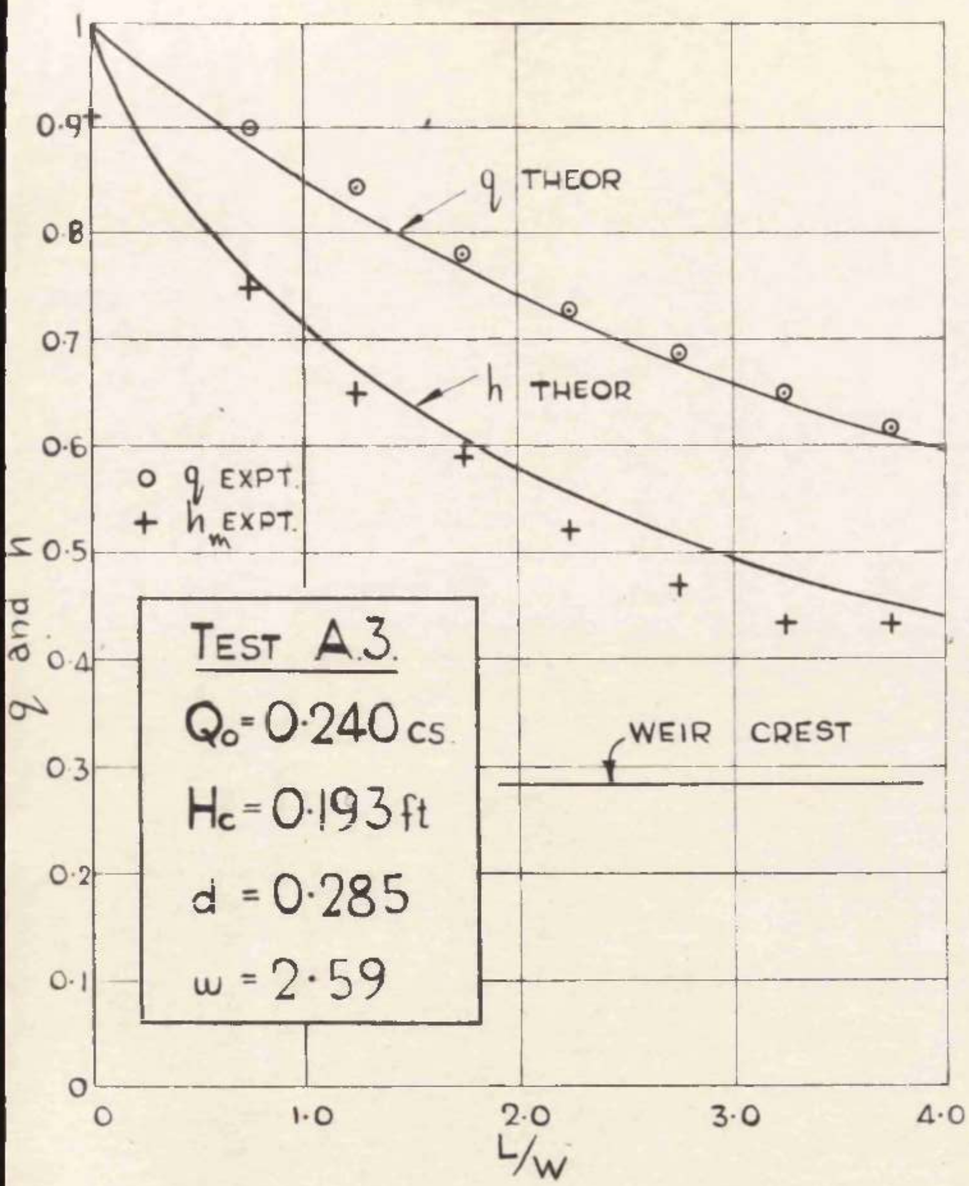


FIGURE 11.24.

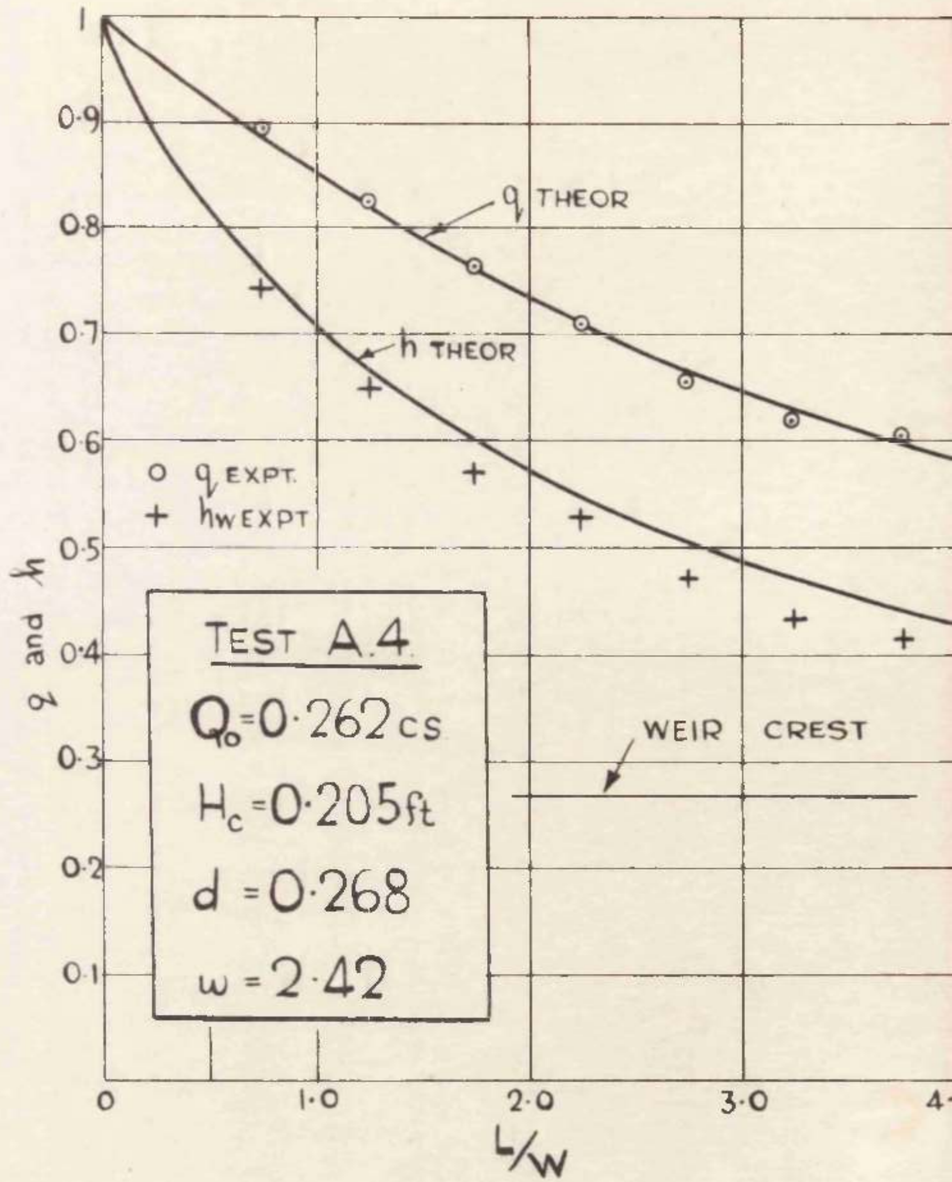


FIGURE 11.25.

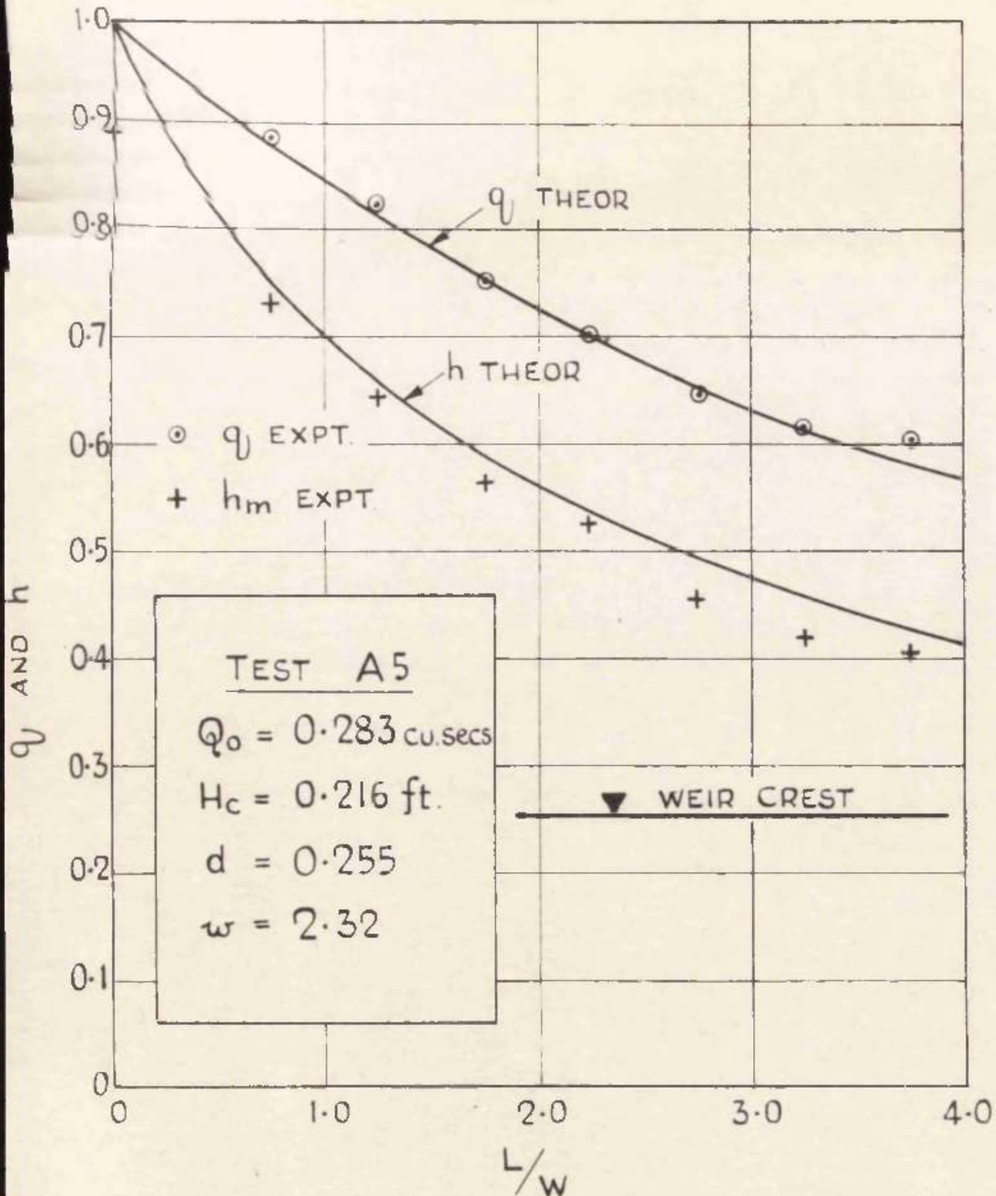


FIGURE 11.26

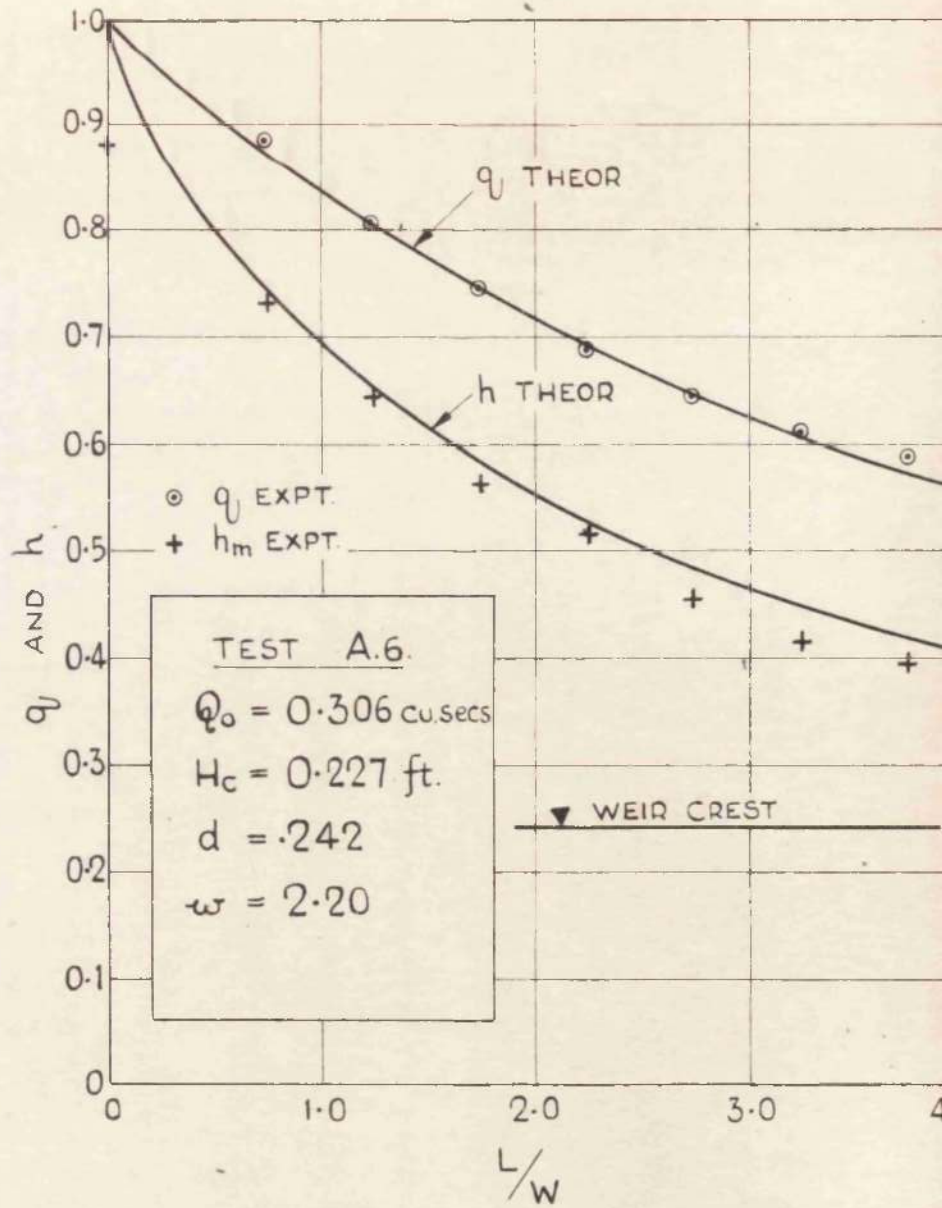


FIGURE 11.27

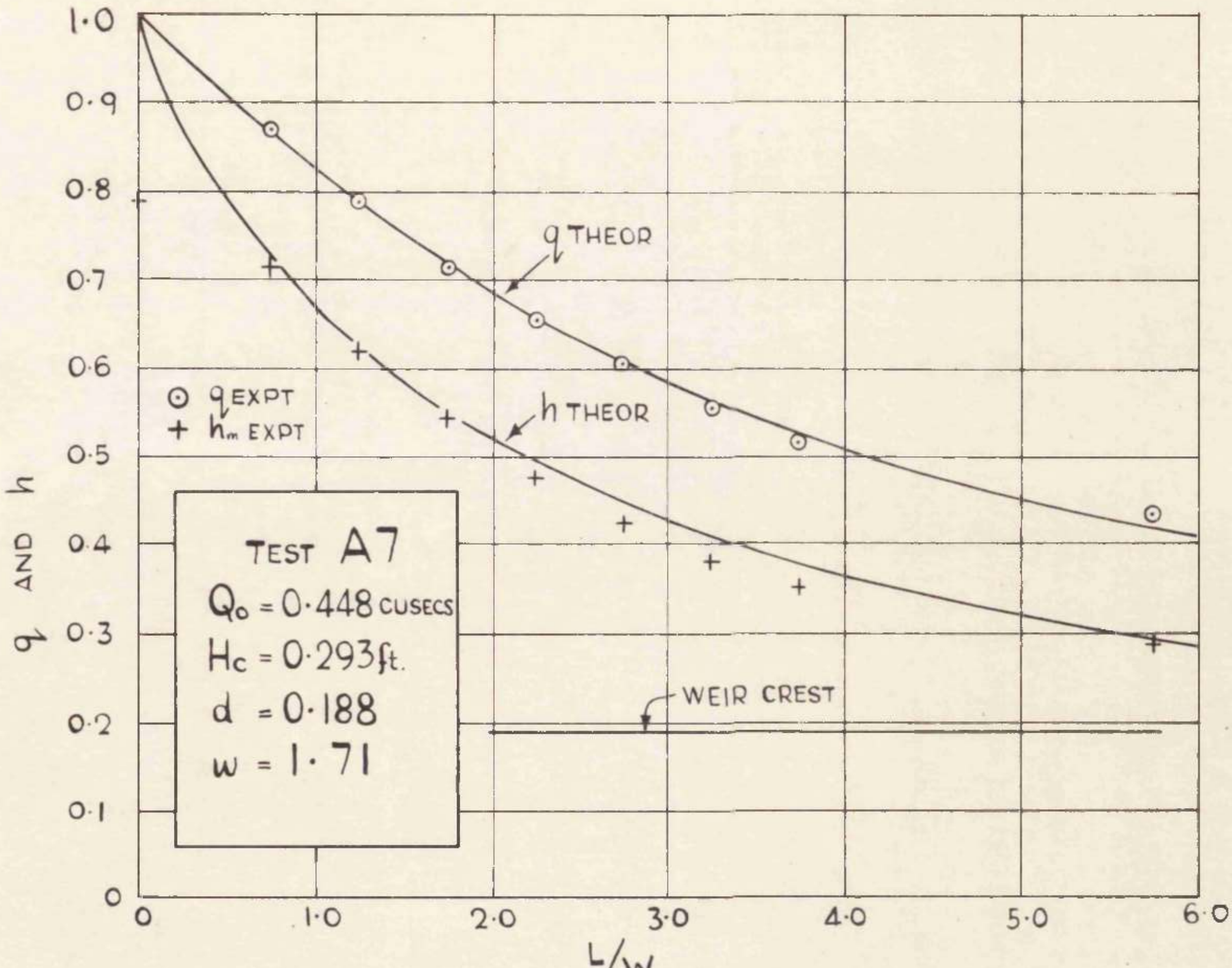

1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
q AND h
0
1·0
2·0
3·0
4·0
5·0
6·0
L/W
q THEOR
h THEOR
⊙ q EXPT
+ hm EXPT
TEST A7
Qo = 0·448 CUSECS
Hc = 0·293 ft.
d = 0·188
w = 1·71
WEIR CREST

FIGURE 11.28.

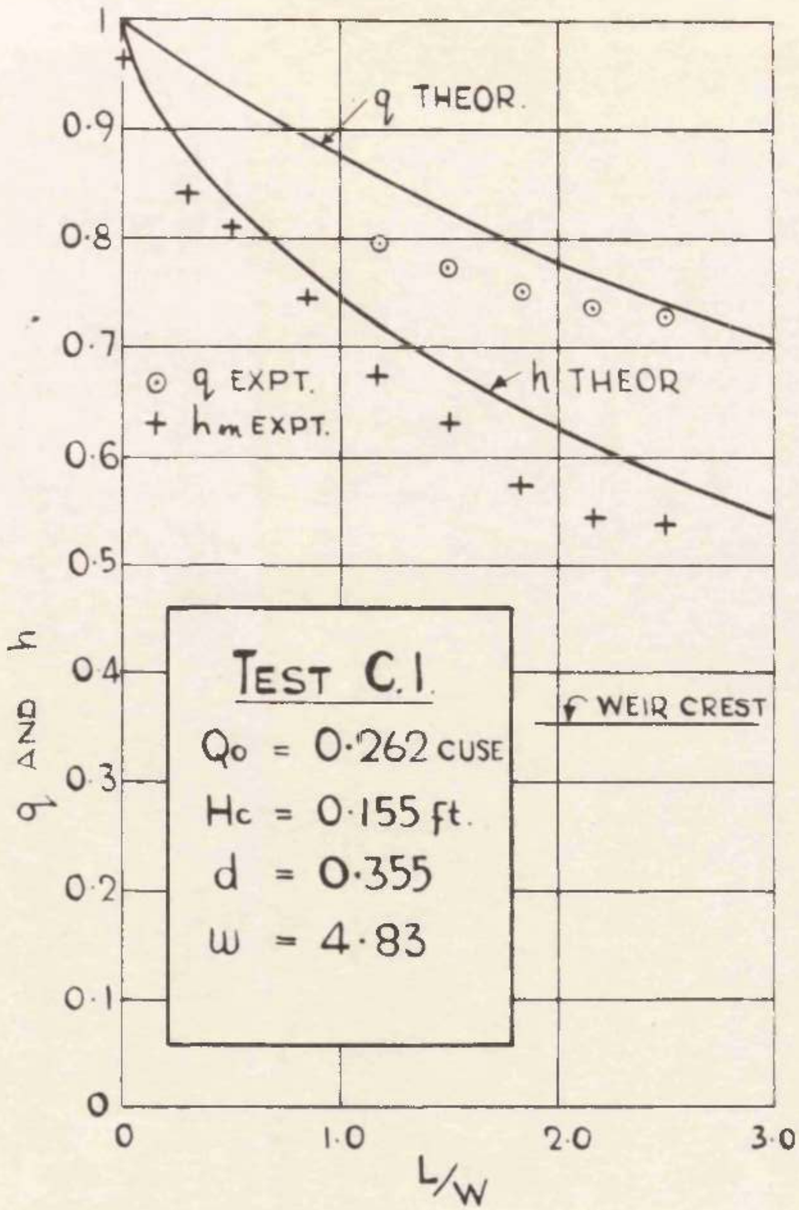


FIGURE II.29

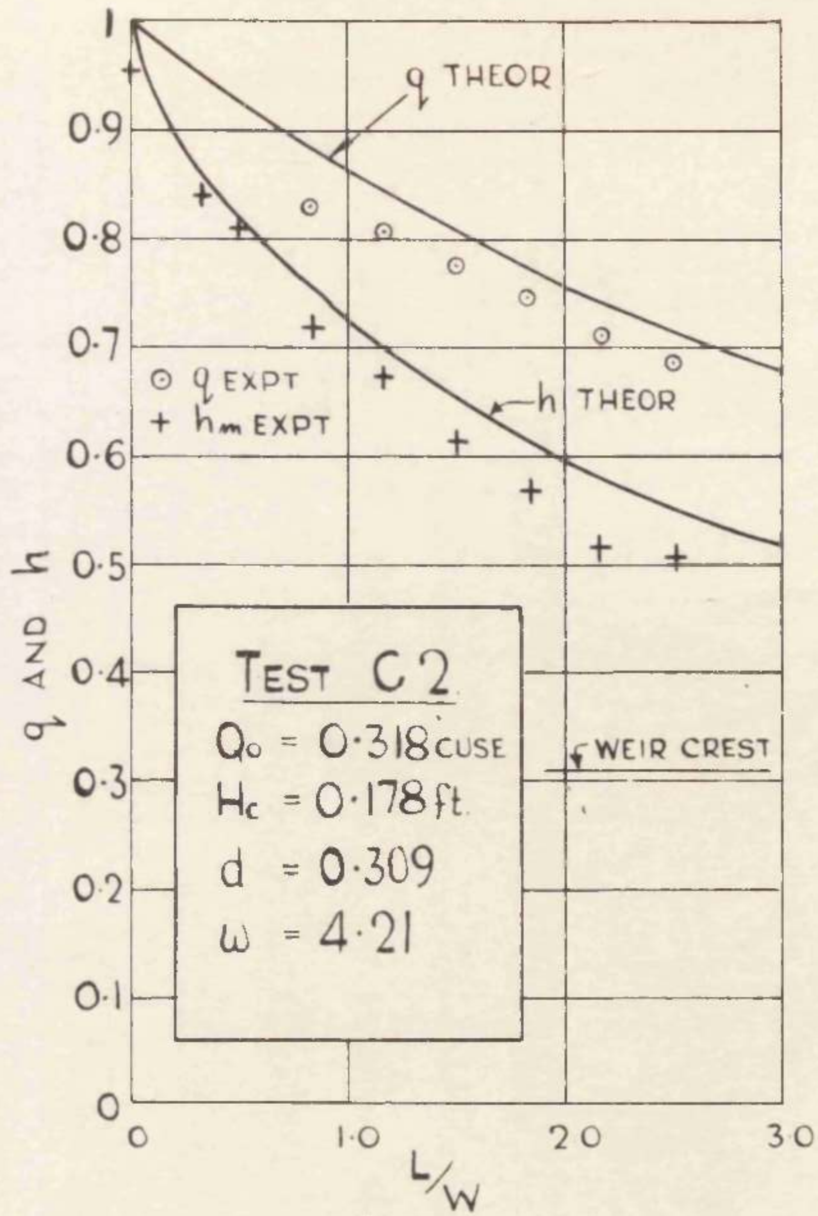


FIGURE II.30

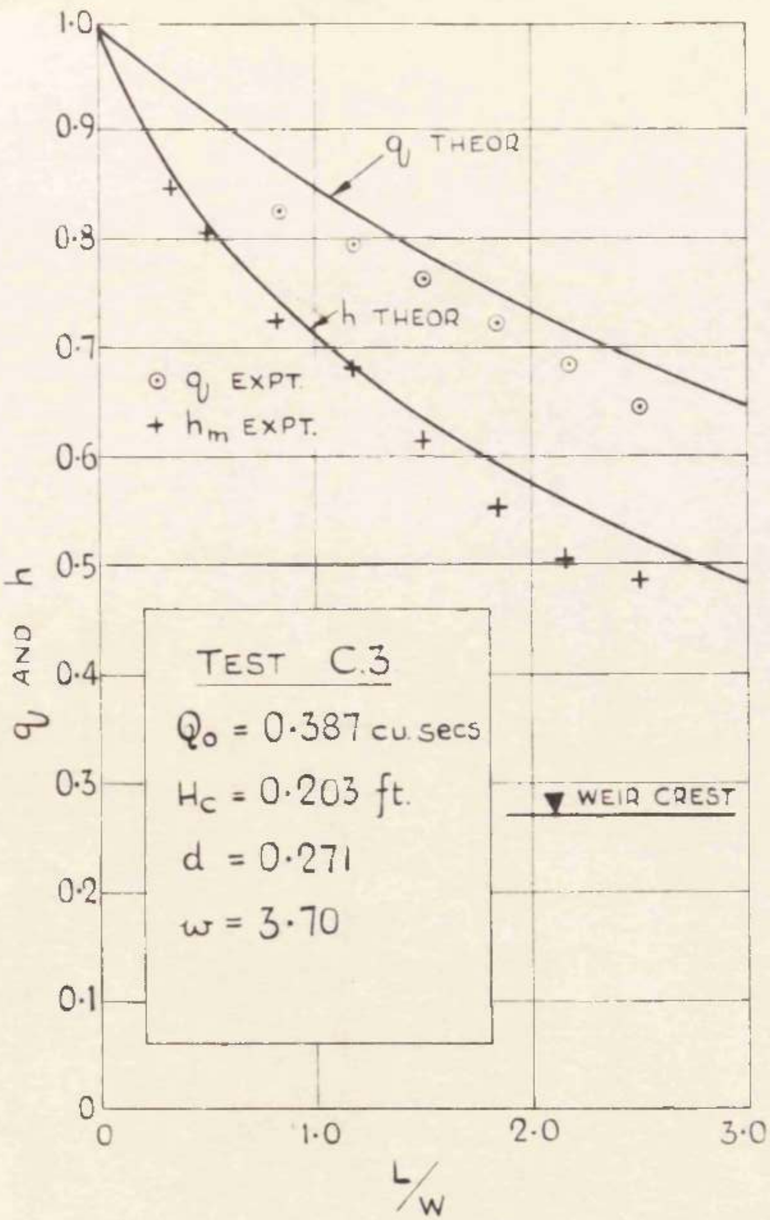


FIGURE 11.31.

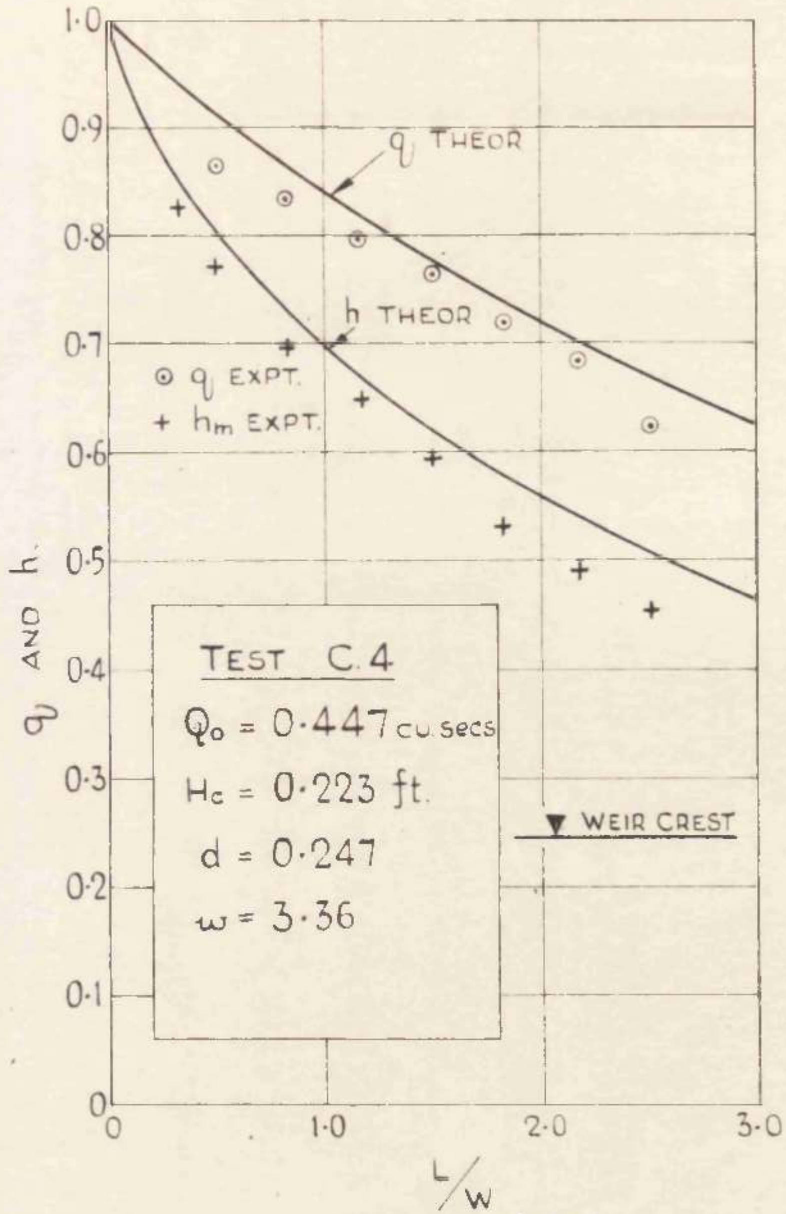
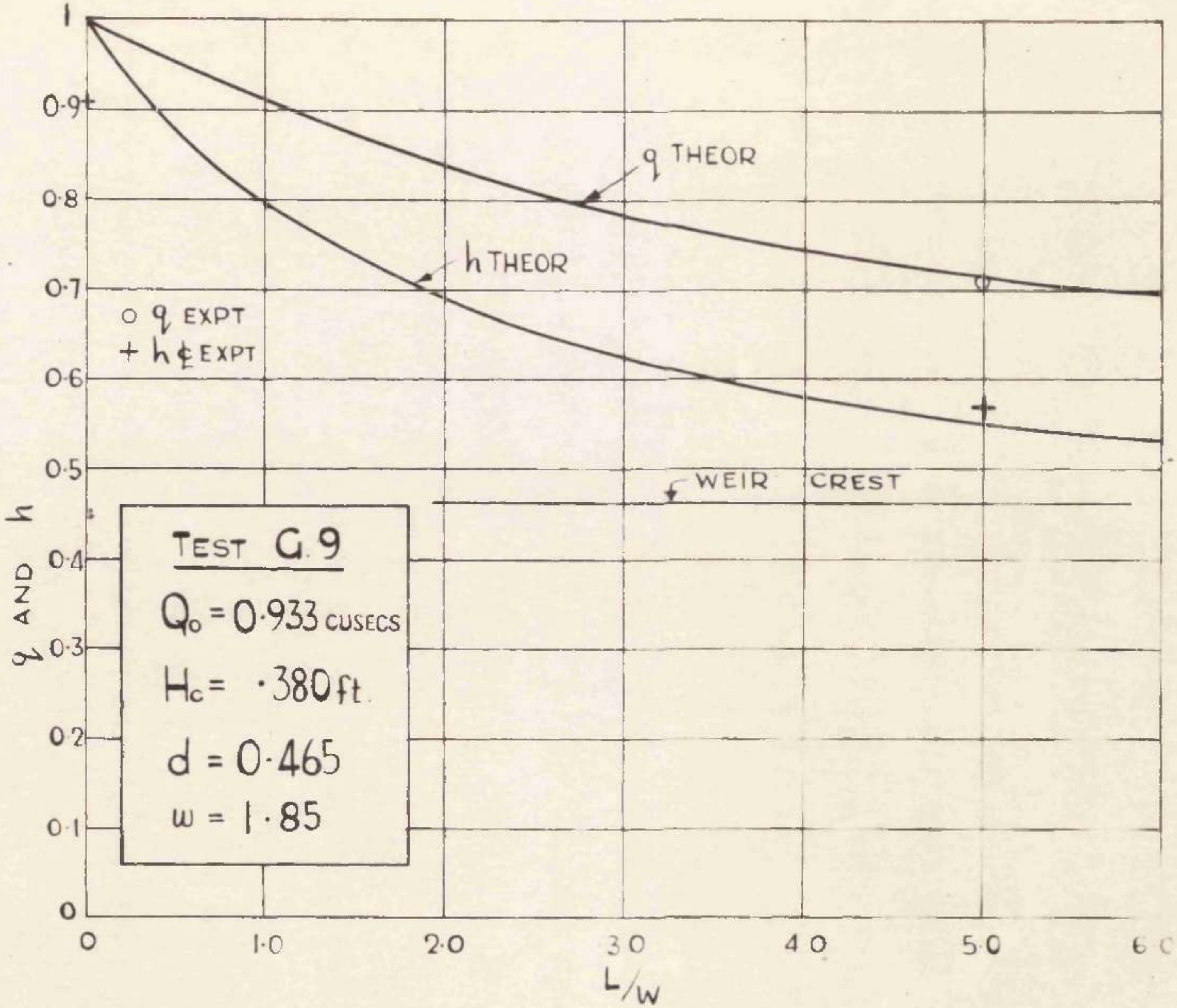


FIGURE 11.32



q THEOR
h THEOR
○ q EXPT
+ h ℄ EXPT
WEIR CREST
TEST G.9
Q₀ = 0·933 CUSECS
H꜀ = ·380 ft
d = 0·465
w = 1·85
q AND h
L/W
1
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
1·0
2·0
3·0
4·0
5·0
6·0

FIGURE II.33

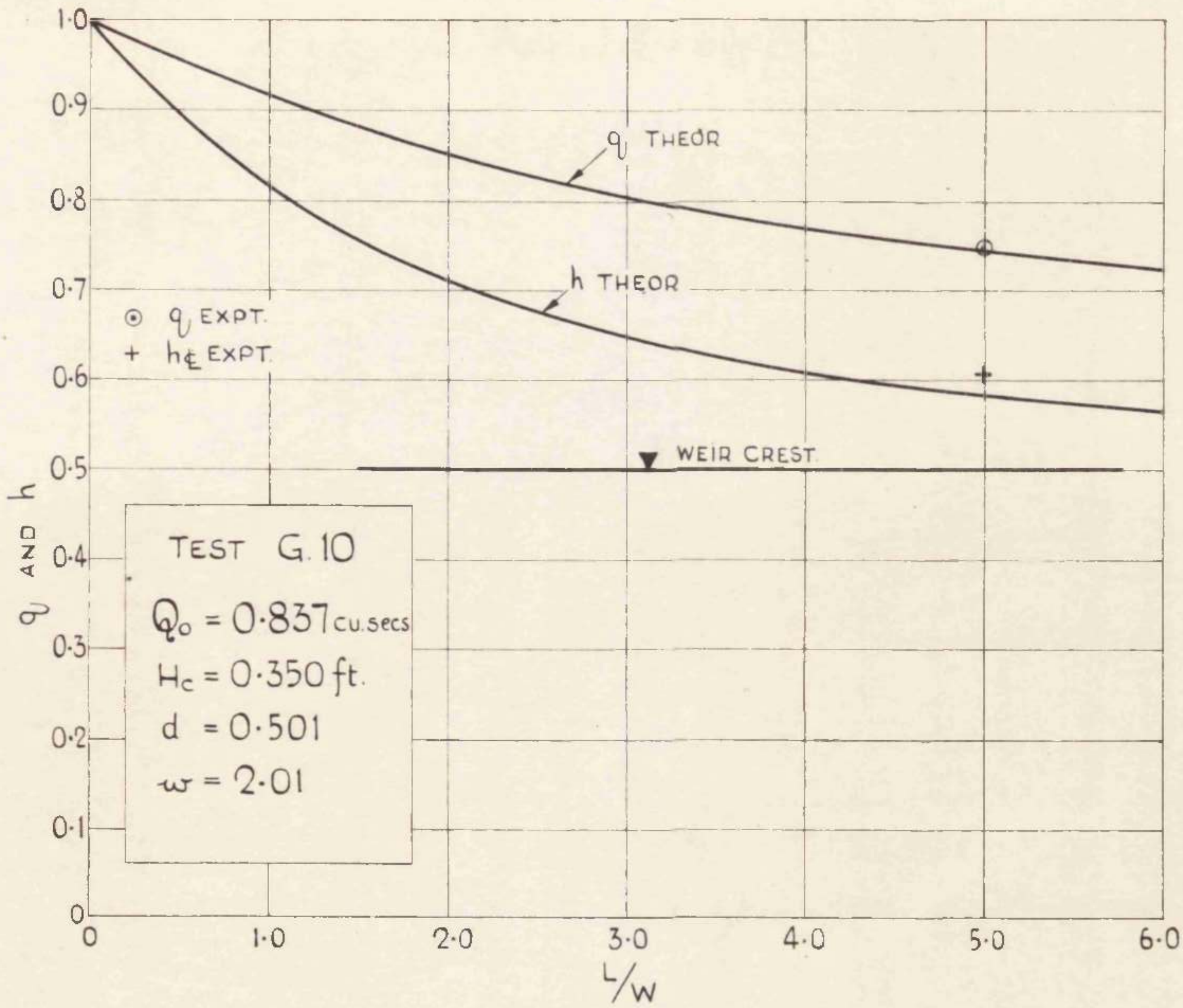

q THEOR
h THEOR
⊙ q EXPT.
+ h℄ EXPT.
WEIR CREST
TEST G.10
Q_o = 0·837 cu.secs
H_c = 0·350 ft.
d = 0·501
ϖ = 2·01
q AND h
L/W
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
1·0
2·0
3·0
4·0
5·0
6·0

FIGURE 11.34

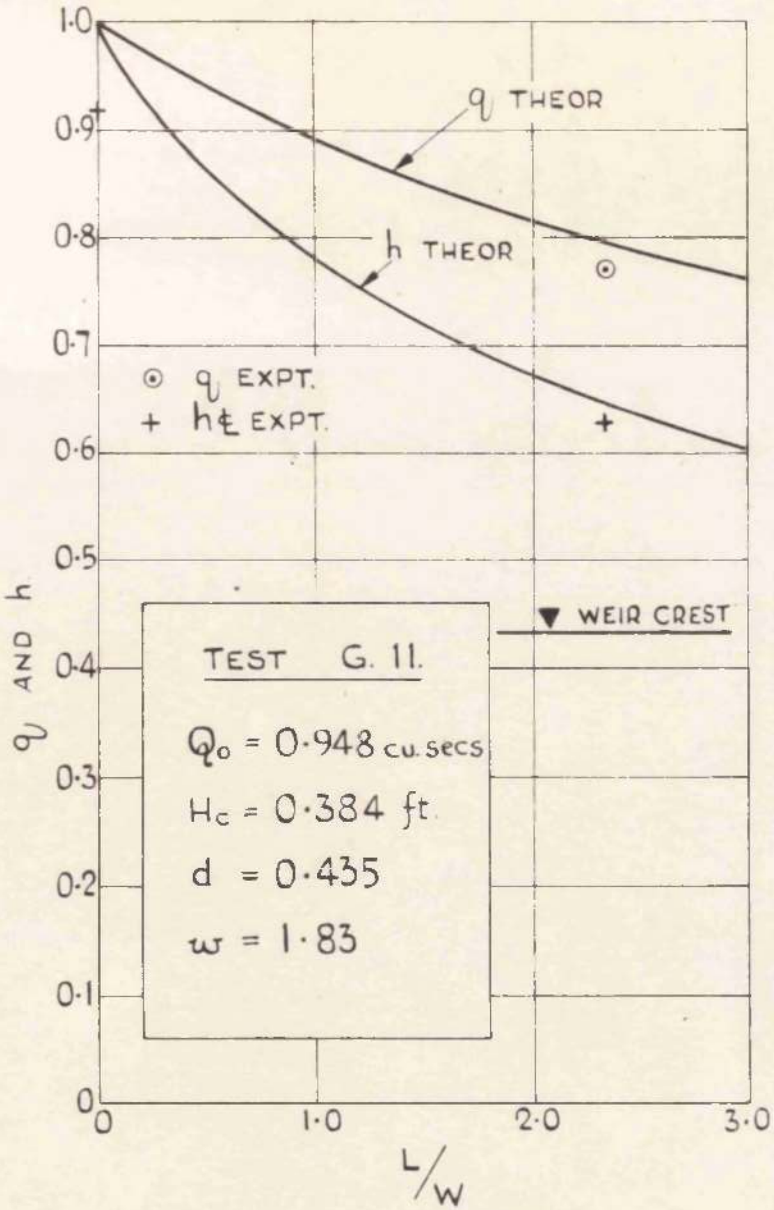


FIGURE 11.35.

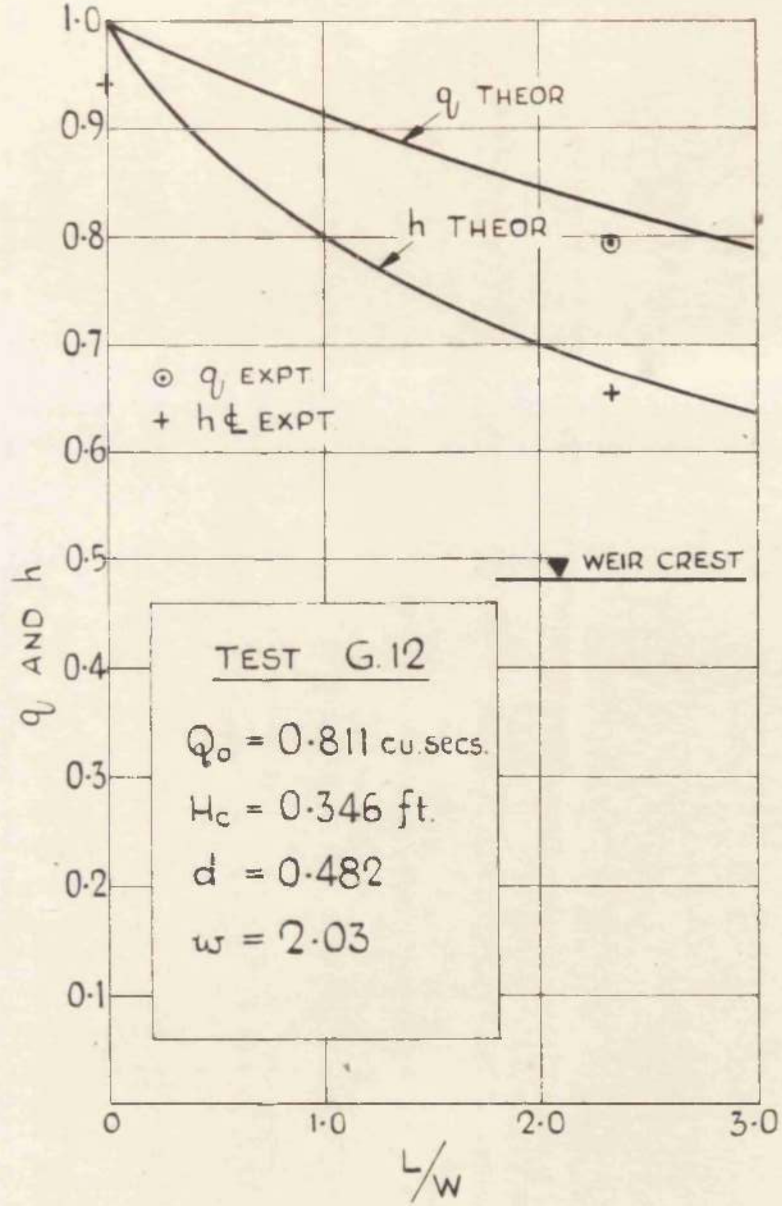


FIGURE 11.36.

TEST C.I. $Q_o = 0{\cdot}260$ c.s ; $H_c = 0{\cdot}155$ ft ; $d = 0{\cdot}355$; $\omega = 4{\cdot}84$								
ℓ	1·61	1·61	1·61	1·61	2·42	2·42	2·42	2·42
$1-q$.	1·000	0·576	0·400	0·265	1·000	0·600	0·400	0·273
h_o	3·46	2·64	2·16	1·76	2·75	2·17	1·77	1·47
h	3·47	2·63	2·23	1·83	2·78	2·24	1·86	1·58
C	0·547	0·503	0·477	0·457	0·535	0·472	0·449	0·421
$q^2/h^2A + 2h/A$	0·998	0·967	1·001	0·990	0·988	0·993	0·991	0·990
ℓ	4·03	4·03	4·03	5·65	5·65	5·65	7·26	7·26
$1-q$.	1·000	0·646	0·476	1·000	0·654	0·491	1·000	0·684
h_o	2·04	1·67	1·42	1·66	1·40	1·14	1·45	1·20
h	2·14	1·80	1·61	1·79	1·61	1·48	1·60	1·48
C	0·524	0·469	0·439	0·521	0·437	0·415	0·510	0·438
$q^2/h^2A + 2h/A$	0·966	0·984	1·000	0·972	0·986	1·007	0·944	0·970

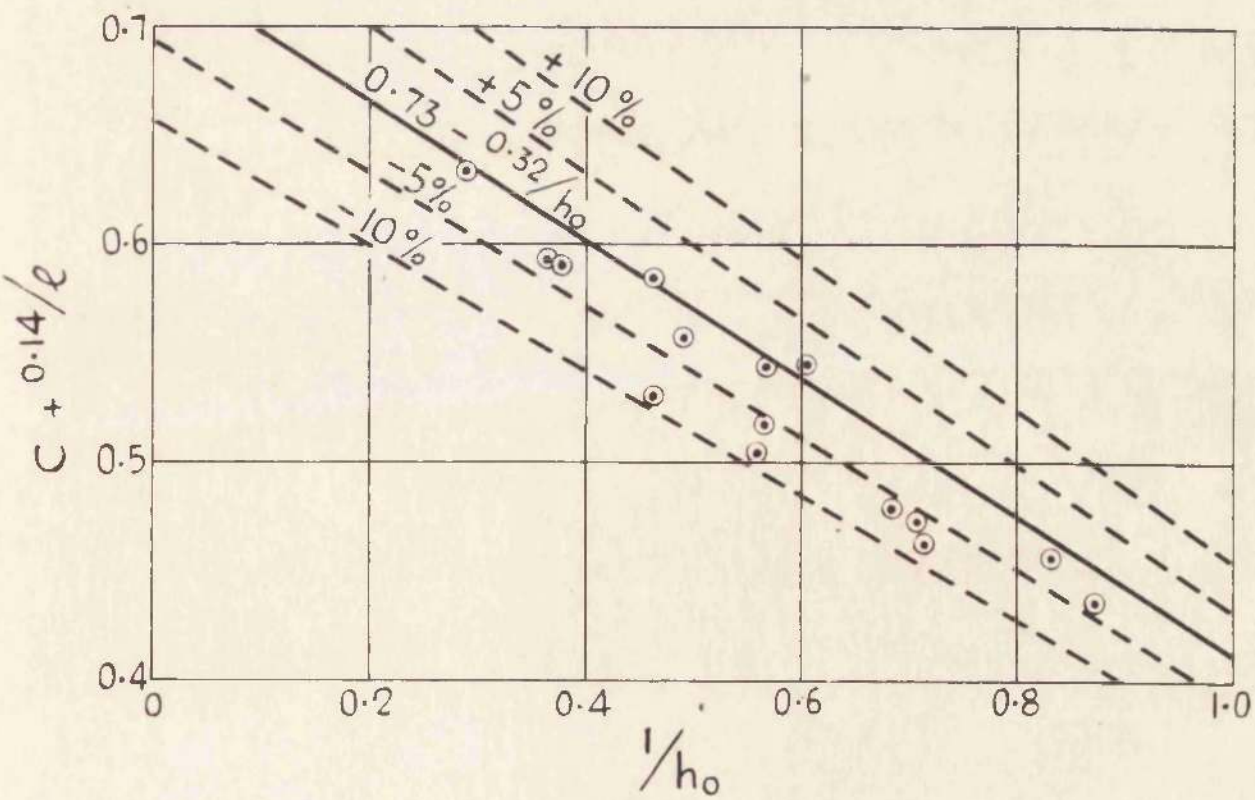


FIG. II.37 CASE II RESULTS – TEST. C.I.

TEST C.2. $Q_o = 0.318$ cs; $H_c = 0.178$ ft; $d = 0.309$; $\omega = 4.21$								
ℓ	1.40	1.40	1.40	1.40	2.12	2.12	2.12	2.12
$1 - q$	1.000	0.629	0.437	0.311	1.000	0.686	0.471	0.339
h_o	3.38	2.68	2.18	1.83	2.72	2.18	1.80	1.53
h	3.54	2.71	2.25	1.90	2.76	2.26	1.91	1.67
c	0.529	0.511	0.493	0.475	0.526	0.516	0.484	0.452
$q^2/h^2A + 2h/A$	1.037	0.989	1.000	0.993	0.991	0.994	0.998	1.002

ℓ	3.52	3.52	3.52	3.52	4.94	4.94	4.94	6.34	6.34
$1 - q$	1.000	0.689	0.529	0.390	1.000	0.704	0.541	1.000	0.692
h_o	2.01	1.66	1.54	1.23	1.63	1.40	1.17	1.43	1.19
h	2.09	1.80	1.62	1.47	1.75	1.52	1.45	1.58	1.46
c	0.519	0.478	0.438	0.418	0.520	0.480	0.429	0.495	0.422
$q^2/h^2A + 2h/A$	0.977	0.984	0.949	0.998	0.962	0.931	0.980	0.947	0.961

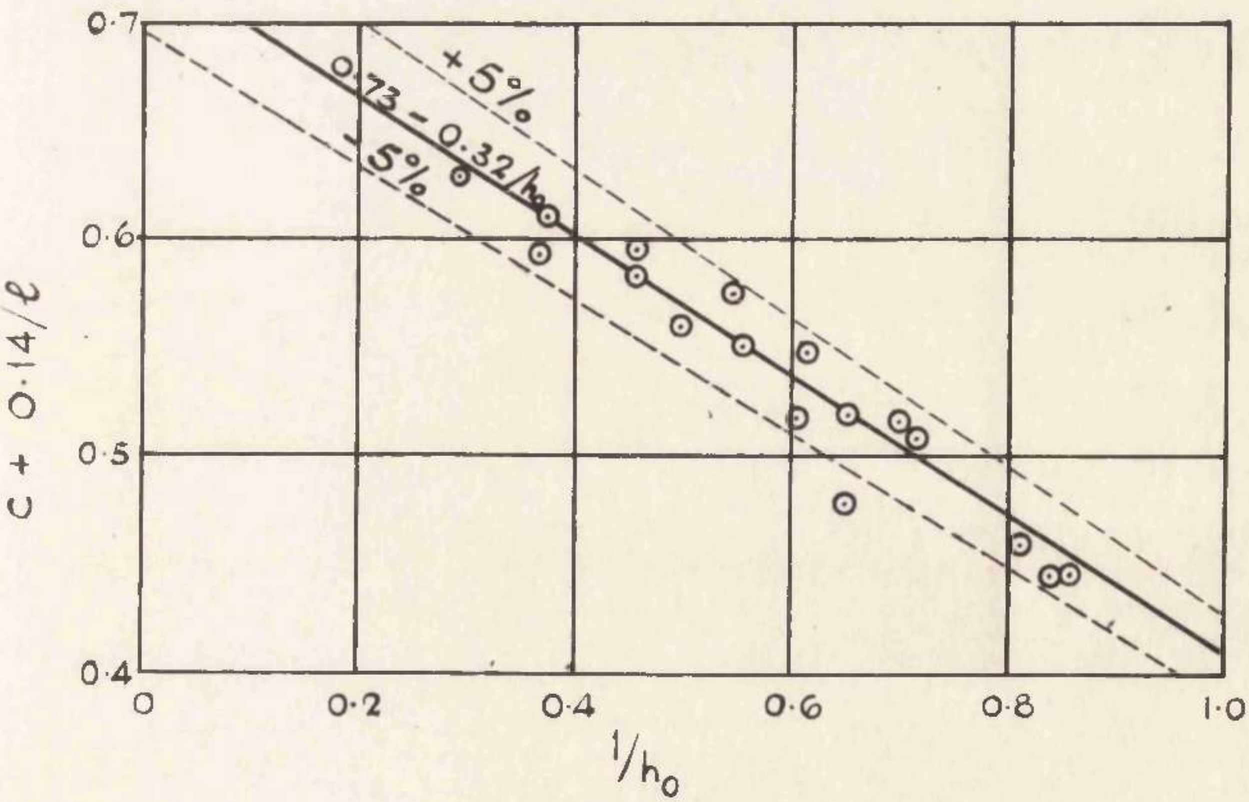


FIG. II.38. CASE II RESULTS – TEST C.2.

TEST C.3. $Q_o = 0{\cdot}387$ cs ; $H_c = 0{\cdot}203$ ft ; $d = 0{\cdot}271$; $\omega = 3{\cdot}69$										
ℓ	1·23	1·23	1·85	1·85	3·07	3·07	3·07	4·30	4·30	4·30
$1 - q$	0·370	0·184	0·408	0·196	1·000	0·741	0·226	1·000	0·785	0·561
h_o	1·96	1·41	1·66	1·19	2·00	1·70	0·99	1·62	1·42	1·25
h	2·02	1·50	1·75	1·35	2·04	1·84	1·25	1·74	1·59	1·53
C	0·489	0·422	0·470	0·376	0·515	0·476	0·319	0·500	0·476	0·379
$q^2/h^2A + 2h/A$	0·987	0·990	0·983	0·991	0·960	0·987	0·964	0·962	0·963	0·998

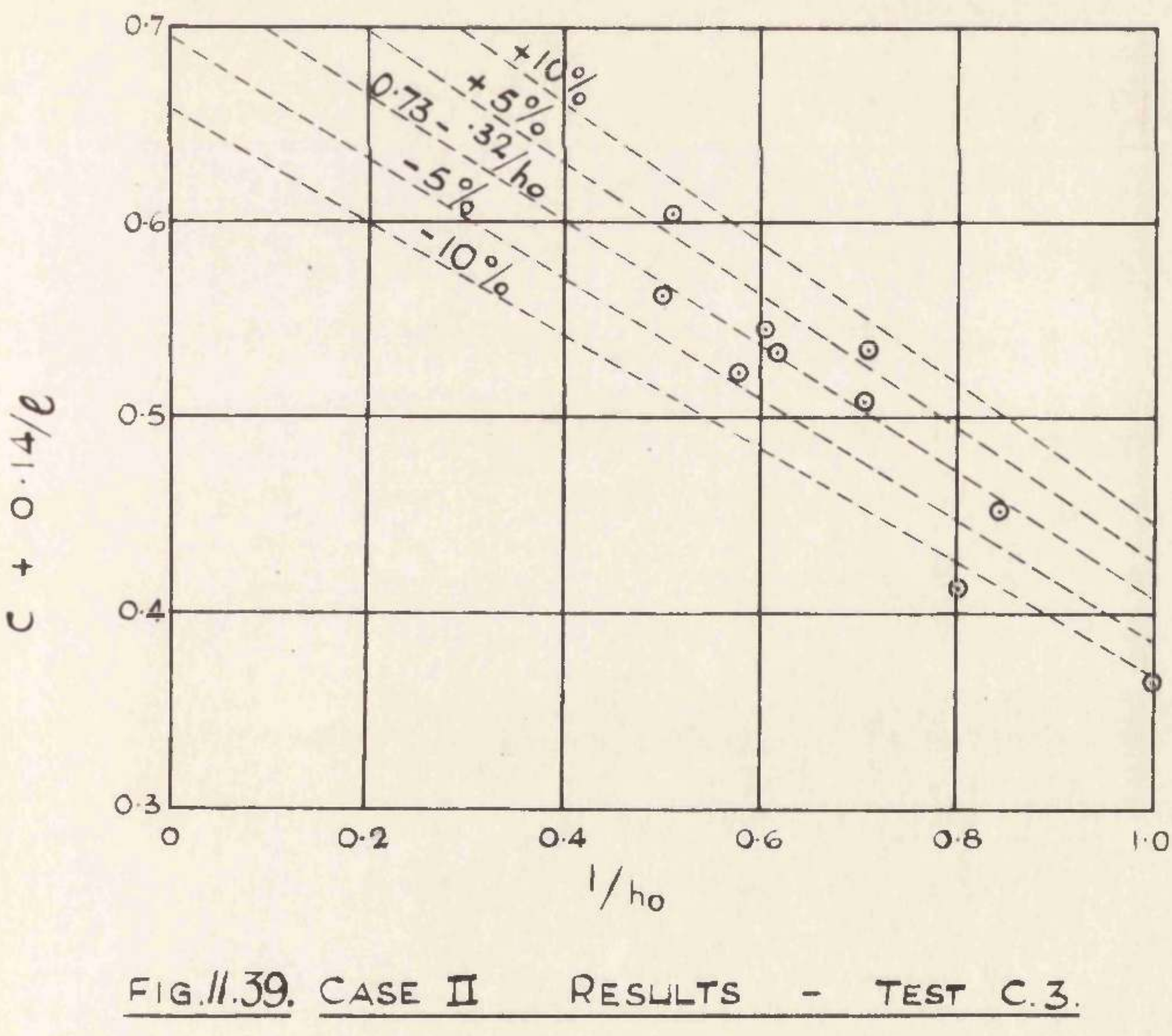


FIG. II.39. CASE II RESULTS - TEST C.3.

TEST C.4. $Q_0 = 0.447$ cs., $H_c = 0.223$ ft; $d = 0.247$; $\omega = 2.47$								
ℓ	1·68	1·68	1·68	2·80	2·80	2·80	3·92	3·92
$1-q$	0·654	0·508	0·399	0·694	0·573	0·45	0·725	0·604
h_o	2·08	1·81	1·56	1·60	1·43	1·26	1·32	1·22
h	2·14	1·86	1·66	1·73	1·59	1·48	1·53	1·45
C	0·513	0·508	0·492	0·482	0·472	0·432	0·467	0·436
$q^2/h^2A + 2h/A$	0·978	0·974	0·979	0·973	0·972	0·984	0·962	0·947

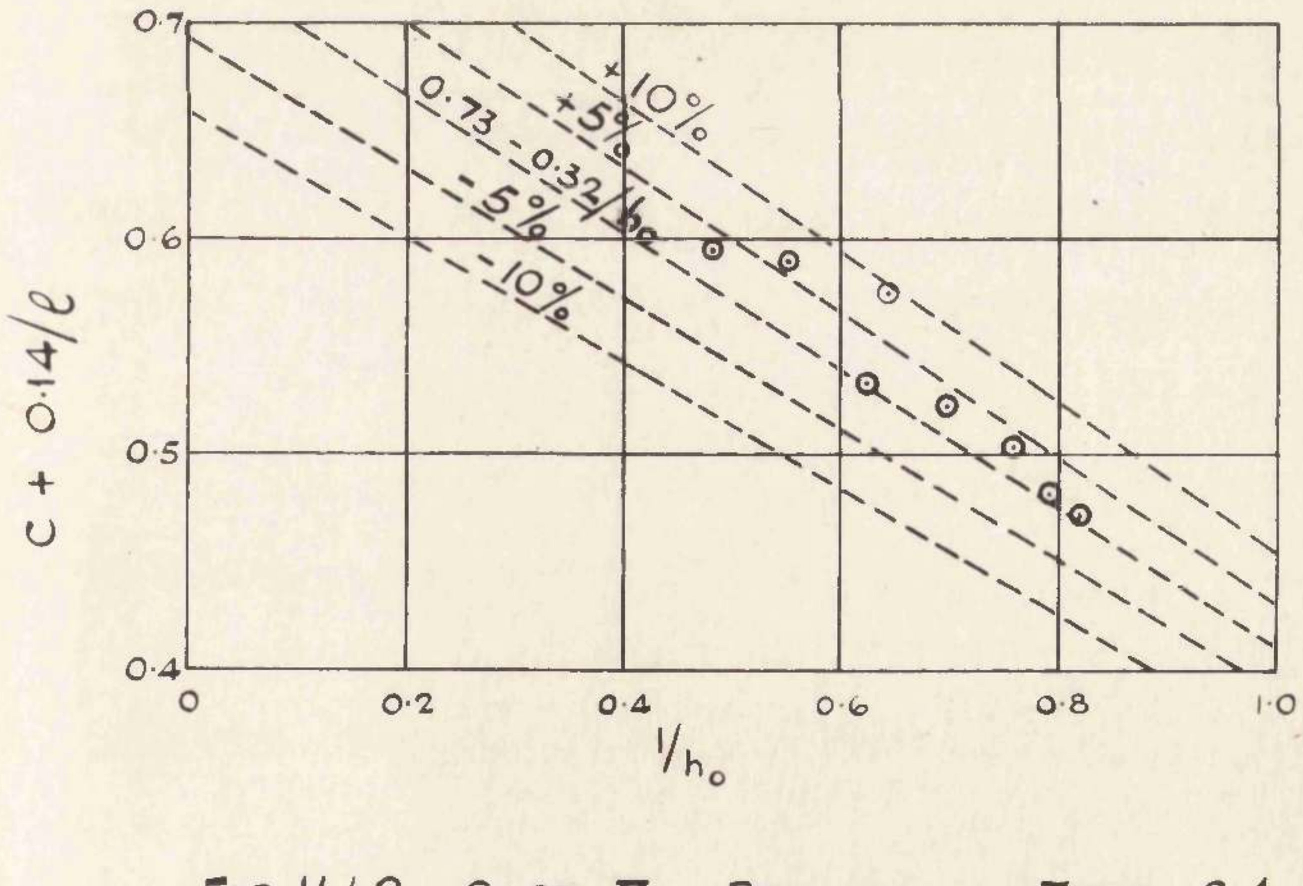


FIG. 11.40. CASE II RESULTS - TEST C4.

TEST Nº	c1	c2	c3	c4	c5	c6	c7	c8
Q_o cm³/sec	25,000	25,000	20,000	20,000	20,000	20,000	15,000	10,000
H_c cm	11·68	11·68	10·06	10·06	10·06	10·06	8·30	6·34
ℓ	17·1	17·1	19·9	19·9	19·9	19·9	24·1	31·5
w	1·72	1·72	2·00	2·00	2·00	2·00	2·42	3·16
d	1·88	1·88	2·18	2·18	2·18	2·18	2·64	3·46
$1 - q$	0·400	1·000	0·250	0·500	0·750	1·000	1·000	1·000
h'_o	2·03	2·13	2·30	2·36	2·40	2·45	2·93	3·68
h'	2·07	2·20	2·32	2·40	2·46	2·50	2·96	3·78
C	0·510	0·619	0·539	0·546	0·576	0·614	0·609	0·582
$q^2/h^2A + 2h/A$	0·983	0·981	0·992	0·989	0·990	0·988	0·989	1·014

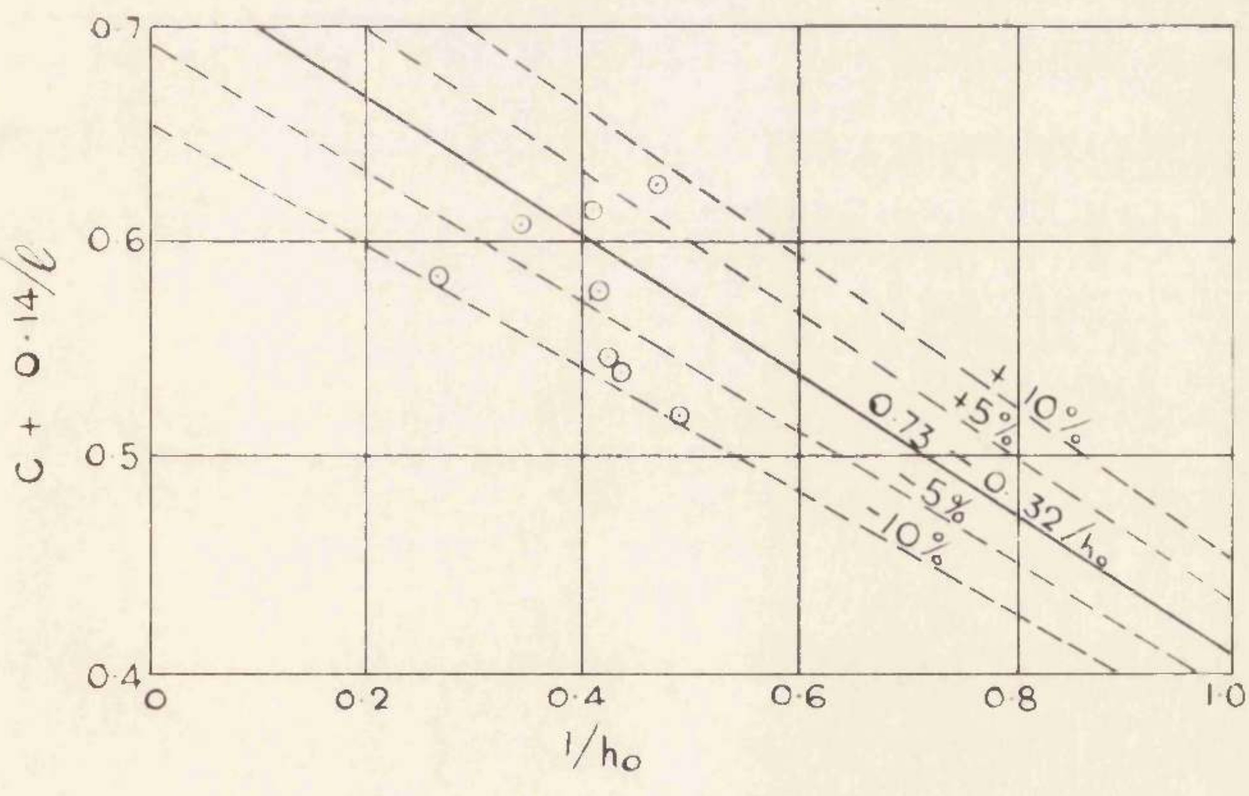


FIG II.41. CASE II RESULTS – FAVRE AND BRAENDLE

TEST Nº	1	2	3	4	5	6	7	8
Q_0 cm ^{3}/sec	21650	18850	16050	21650	17720	12450	18250	14650
H_c cm	10·13	9·23	8·31	10·13	8·87	7·01	7·52	7·82
l	10·50	11·57	12·85	10·50	12·04	15·25	6·64	6·38
w	2·04	2·32	2·58	2·04	2·42	3·06	2·85	2·74
d	2·49	2·74	3·04	2·49	2·84	1·32	1·75	1·69
$1 - q$	1	0·748	0·813	0·355	0·216	0·872	0·271	0·478
h_0	2·94	3·11	3·43	2·70	3·00	1·69	2·11	2·16
h	2·98	3·14	3·46	2·75	3·01	1·81	2·20	2·21
C	0·604	0·593	0·615	0·598	0·600	0·590	0·593	0·594
$q^2/h^2A + 2h/A$	0·997	0·998	1·003	0·997	0·997	0·974	1·017	0·987

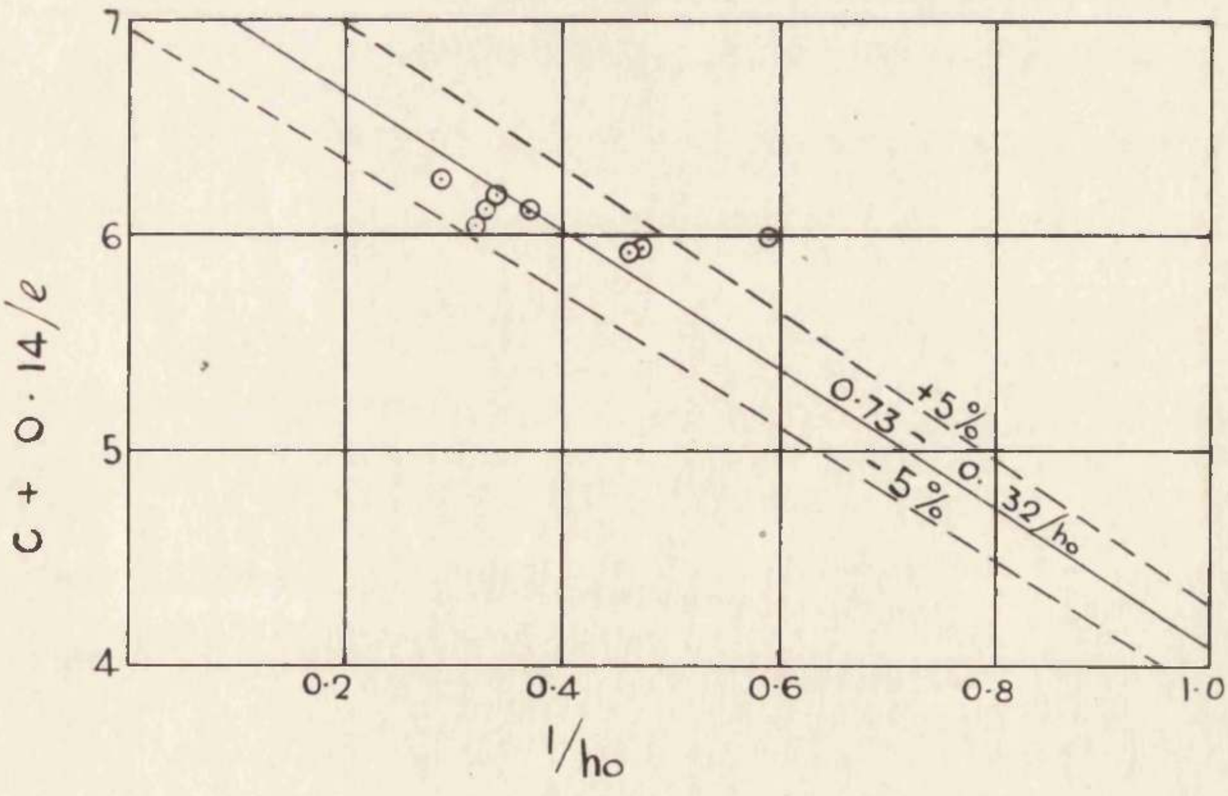


FIG. 11.42.- CASE II RESULTS - GENTILINI.

Test C.1. Qo = 0.262 cs, Hc = 0.155 ft., d = 0.355, w = 4.83

q_1	0.790	0.772	0.750	0.775	0.750	0.735	0.680
q	0.331	0.492	0.624	0.331	0.481	0.624	0.483
h	1.325	1.291	1.240	1.290	1.280	1.185	1.269
B	2.74	2.71	2.68	2.72	2.68	2.66	2.58
$q^2/h^2B + 2h/B$	0.992	1.007	1.043	0.972	1.010	0.997	0.984
x	2.167	2.167	2.167	2.500	2.500	2.500	2.500
x	0.858	0.607	0.192	1.031	0.522	0.140	0.300
$q_1 - q / (h-d)^{3/2} x_2^{0.18w}$	1.00	0.86	1.15	0.87	0.92	1.34	1.02

Table 11.1 Case III Results. Test C.1.

	Test C.2. Qo = 0.318 cs. Hc = 0.178 ft., d = 0.309, w = 4.21.										
q_1	0.822	0.792	0.815	0.785	0.755	0.786	0.760	0.734	0.764	0.728	0.708
q	0.453	0.604	0.308	0.446	0.589	0.277	0.434	0.579	0.268	0.427	0.566
h	1.405	1.355	1.392	1.365	1.290	1.360	1.329	1.240	1.300	1.292	1.230
B	2.78	2.74	2.77	2.73	2.69	2.73	2.70	2.66	2.70	2.65	2.62
$q^2/h^2B + 2h/B$	1.048	1.063	1.022	1.039	1.037	1.021	1.035	1.016	0.980	1.016	1.017
χ	1.500	1.500	1.833	1.833	1.833	2.167	2.167	2.167	2.50	2.50	2.50
χ_2	0.540	0.220	0.847	0.527	0.182	0.834	0.527	0.220	0.875	0.435	0.220
$q_1^{-1}/(h-d)^{3/2} \chi_2^{0.18\omega}$	0.93	1.01	0.92	0.92	1.01	0.98	0.93	0.98	1.01	1.03	0.92

Table 11.2 Case III Results Test C.2.

	Test C.3. Qo = 387 cs. Hc = 0.203 ft. d = 0.271. w = 3.70						
q_1	0.794	0.754	0.782	0.744	0.732	0.734	0.700
q	0.375	0.506	0.212	0.359	0.491	0.209	0.354
h	1.39	1.345	1.402	1.350	1.300	1.305	1.255
B	2.74	2.69	2.72	2.67	2.64	2.66	2.61
$q^2/h^2B + 2h/B$	1.040	1.052	1.036	1.038	1.040	0.992	0.994
x	1.833	1.833	2.167	2.167	2.167	2.500	2.500
x_2.	0.621	0.233	0.847	0.501	0.327	0.767	0.500
$q_1 - q / (h-d)^{3/2}\ x_2^{0.18} w$.	0.86	1.06	0.96	0.96	0.85	1.08	1.01

Table 11.3 Case III Results Test C.3.

Test C.4.	Q_o = 0.477 cs., Hc 0.233 ft., d = 0.247, W = 3.36							
q_1	0.786	0.762	0.746	0.755	0.724	0.704	0.732	0.700
q	0.515	0.396	0.512	0.266	0.396	0.507	0.268	0.397
h	1.342	1.320	1.290	1.308	1.280	1.245	1.270	1.258
B	2.71	2.70	2.68	2.69	2.65	2.62	2.66	2.61
$q^2/h^2B + 2h/B$	1.042	1.011	1.022	0.988	1.003	1.014	0.971	1.010
x	1.500	1.833	1.833	2.167	2.167	2.167	2.500	2.500
x_2	0.274	0.380	0.205	0.620	0.341	0.182	0.740	0.470
$q_1 - q/(h-d)^{3/2}\, 0.18\, w\, x_2$	0.94	1.07	1.04	1.08	1.09	1.00	0.98	0.88

Table 11.4 Case III Results. Test C.4.

Test 1. $q > 0.600$, $Qo = 0.430$ cs. $Hc = 0.284$ ft., $d = 0.194$, $y = 1.76$

q_1	0.655	0.615	0.695	0.675	0.760	0.738
q	0.105	0.209	0.132	0.233	0.132	0.240
h	1.110	1.140	1.205	1.190	1.240	1.230
B	2.55	2.49	2.61	2.58	2.70	2.67
$q^2/h^2B + 2h/B$	0.876	0.930	0.925	0.938	0.924	0.936
x	3.25	3.25	2.75	2.75	2.25	2.25
x_2	1.01	0.66	0.81	0.65	0.81	0.63
$q_1-q/(h-d)^{3/2}\ x_2^{0.18\omega}$	1.13	0.90	1.07	0.92	1.13	0.98

Table 11.5 Case III Results Test 1 ($q > 0.600$)

	Test 1. $q < 0.600$, $Q_o = 0.430$ cs. He 0.284 ft. $d = 0.194$, $v = 1.76$									
q_1	0.481	0.478	0.498	0.488	0.527	0.511	0.556	0.536	0.598	0.570
q	0.119	0.256	0.105	0.237	0.132	0.220	0.132	0.209	0.105	0.209
h	0.95	0.90	1.00	0.99	1.08	1.02	1.12	1.05	1.11	1.10
B	2.23	2.33	2.27	2.25	2.33	2.29	2.38	2.35	2.46	2.41
$q^2/h^2B + 2h/B$	0.859	0.846	0.886	0.906	0.934	0.911	0.948	0.912	0.918	0.927
x	5.75	5.75	5.25	5.25	4.75	4.75	4.25	4.25	3.75	3.75
x_2	0.95	0.65	0.93	0.61	1.11	0.77	1.09	0.75	1.05	0.75
$q_1 - q/(h-d)^{3/2} x_2 = C_{III}$	0.576	0.571	0.581	0.576	0.424	0.501	0.434	0.548	0.533	0.556

Table 11.6 Case III Results Test 1 ($q < 0.600$)

Test 2. Qo = 0.245 cs., Hc = 0.195 ft., d = 0.280, v = 2.56												
q_1	0.605	0.581	0.635	0.615	0.671	0.640	0.692	0.678	0.720	0.710	0.761	0.731
q	0.082	0.216	0.061	0.171	0.074	0.265	0.082	0.171	0.098	0.171	0.151	0.252
h	1.15	1.10	1.23	1.15	1.29	1.14	1.37	1.24	1.33	1.34	1.41	1.36
B	2.47	2.43	2.52	2.49	2.57	2.52	2.60	2.58	2.64	2.63	2.70	2.65
$q^2/h^2B + 2h/B$	0.934	0.922	0.978	0.933	1.004	0.927	1.057	0.967	1.010	1.026	1.044	1.041
x	4.75	4.75	4.25	4.25	3.75	3.75	3.25	3.25	2.75	2.75	2.25	2.25
x_2	1.21	0.77	1.21	0.89	1.25	0.77	1.07	0.83	0.95	0.81	0.91	0.59
$q_1-q/(h-d)^{3/2}x_2^{0.18w.}$	1.08	1.00	1.03	1.04	0.97	0.96	0.95	1.06	1.07	0.99	0.96	0.99

Table 11.7 Case III Results Test 2.

Test 3. $Q_0 = 0.249$ cs. $H_c = 0.313$ ft., $d = 0.180$, $v = 0.80$

q_1	0.496	0.478	0.535	0.570	0.556	0.545	0.620	0.599	0.685	0.652	0.765
q	0.072	0.177	0.076	0.136	0.076	0.145	0.076	0.136	0.108	0.280	0.136
h	1.02	0.90	1.03	0.96	1.02	1.01	0.99	1.10	1.07	1.10	1.21
B	2.27	2.23	2.34	2.41	2.38	2.36	2.49	2.46	2.59	2.55	2.70
$q^2/h^2B + 2h/B$	0.902	0.825	0.884	0.806	0.860	0.866	0.797	0.901	0.830	0.688	0.944
x	7.50	7.50	6.50	6.50	5.50	5.50	4.50	4.50	3.50	3.50	2.50
x_2	1.90	1.38	1.86	1.30	1.34	1.06	1.42	1.06	1.26	0.86	1.14
$q_1 - q/(h-d)^{3/2} x_2$	0.290	0.358	0.316	0.418	0.465	0.498	0.527	0.526	0.547	0.535	0.528

Table 11.8 Case III Results Test No. 3.

Test 4. $Q_o = 0.173$ cs. $H_c = 0.246$ ft., $d = 0.233$, $\mu = 1.02$						
q_1	0.617	0.615	0.662	0.640	0.710	0.700
q	0.098	0.225	0.081	0.243	0.081	0.243
h	1.07	1.02	0.94	1.09	1.05	1.11
B	2.49	2.49	2.56	2.52	2.63	2.61
$q^2/h^2B + 2h/B$	0.861	0.840	0.738	0.886	0.800	0.870
x	5.50	5.50	4.50	4.50	3.50	3.50
x_2	1.66	1.26	1.50	1.14	1.30	1.10
$q_1 - q\ /(h-d)^{3/2} x_2$	0.402	0.437	0.647	0.472	0.637	0.495

Table 11.9 Case III Results. Test No.4.

THE BEHAVIOUR OF SIDE WEIRS IN PRISMATIC, RECTANGULAR CHANNELS

Summary of a Thesis

presented for the Degree of Doctor of Philosophy of Glasgow University

by WILLIAM FRAZER, B.Sc.

The subject of the thesis is the investigation of the flow over a weir, set in one side of a prismatic, rectangular channel of mild or zero slope, with its crest parallel to the bottom of the channel, discharge occurring over the weir as the water in the channel rises above the crest. Such a device, termed a side weir, is used, mainly in sewerage practice, to remove excess water from the channel.

A review of the literature on the subject reveals a lack of experimental data and an abundance of formulae, often contradictory. The present investigations were, therefore, undertaken to accumulate data over a wide range of parameters and, from these data to obtain a rational method of design of such weirs.

Preliminary, qualitative experiments showed that, with channels of mild slope, three modes of motion are possible at the weir section :-

Case I. Rapid flow in the main channel, with the depth of flow decreasing downstream, and being approximately equal to the critical depth at the commencement of the weir section.

Case II. Tranquil flow in the main channel, with the depth of flow increasing downstream.

Case III. Rapid flow, similar to Case I, at the start of the weir section; a hydraulic jump in the weir section, and tranquil flow, similar to Case II, after the jump.

Case III, so far as is known, has been previously briefly noted but has not been investigated.

It was also noted that the mode of motion obtaining at the weir section is influenced not only by the weir parameters, but also by the flow characteristics of the downstream channel. In other words the quantity and depth of flow at the end of the weir section must be acceptable to the downstream channel . On the other hand, the flow characteristics of the upstream channel have no influence on the mode of motion at the weir section, so long as tranquil flow is maintained in the upstream channel.

The main experiments, carried out in a channel of 9 in. by 9 in., maximum cross-section, with a maximum weir length of 5 ft. were so designed to cover these three cases of motion, and confirmed the conclusio of the preliminary experiments.

A theoretical study is given of the general case of a weir set in a channel of varying cross-section. Dimensional analysis, besides revealing the significant parameters governing the flow, indicates that tw independent equations are required for a solution; while the more classical approach, using the principle of equating force to rate of ch of momentum in the main channel and ignoring frictional losses, indicat the probable combination of these parameters in one of these equations. the case of prismatic channels, this equation reduces to a statement that the specific energy of flow is constant in the main channel over the weir section for Case I and II flow; in Case III flow, however, account must be taken of the loss of energy at the jump, tranquil flow after the jump occurring with constant specific energy less than that before the jump.

Constant specific energy of flow has been assumed, or is implicit, in formulae advanced by other investigators, but in no case, so far as is known to the author, has it been shown that this applies only to prismatic channels.

Analysis of the experimental data shows that the specific energy of flow is sensibly constant in the case of tranquil flow (Case II) but that the more complex flow conditions of Case I and III requires extended treatment. In these cases flow is more curvilinear and the assumption of hydrostatic pressure distribution does not hold. In addition, in Case III, conditions after the jump are very turbulent. There is, in consequence, a loss of specific energy of flow along the weir. It has been found possible to express this loss in terms of one of the weir parameters and thus to obtain suitable formulae covering those cases.

The second equation required to complete the solution is obtained from a consideration of the discharge over the weir. It is found that in Case II flow, and also in the tranquil flow conditions after the jump in Case III, the ordinary rectangular weir formula can be applied over the experimental range by expressing the coefficient of discharge as a variable of the weir parameters. The conditions of Case I flow, and of the similar rapid flow before the jump in Case III, are, however, so complex that the rectangular weir formula is not applicable and an empirical formula, expressed in terms of the weir parameters has been adopted which fits the results over the experimental range.

A graphical method of presenting the first equation in dimensionless form is given which explains the behaviour of side weirs and which shows the dependency of the flow on the downstream channel characteristics, and an outline of the suggested design procedure, using this graphical presentation, is given, together with a method of allowing for frictional losses in the weir section.