Essays in Credit Risk Management

by

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Abstract

Credit risk management is becoming more and more important in recent years. Credit risk refers to the risk that an obligor fails to make payments on any type of debt at the time of maturity. Credit risk models are statistical tools to infer the future default probabilities and loss distribution of values of a portfolio of debts. This doctoral thesis focus on the application of credit risk management in different areas.

To better understand the credit risk management, in the first chapter, we introduce the basic ideas in credit risk management and review the models developed in the last decades. To empirical test the performance of models reviewed in the first chapter, in the second chapter, we compare the reduce-form model with the structural model based on the China's stock market. It turns out that both models contribute to explaining the default risk of listed firms, however, reduce-form model outperformances the structural model. The empirical results from the second chapter suggests that reduce-form model can better predict the firm's default risk, but the correlated default risk between firms has not been answered yet. So therefore in the third chapter, we investigate the correlated default risk using copula theory which has been introduced in the first chapter. Based on the insurances firms and other financial firms in the US market, both short-term and long-term default dynamic correlations are found. Another interesting finding from the third chapter is that insurance firms which were considered to be stable actually have higher default risk. This motive us to further explore the determinants of default risk of insurance firms in the fourth chapter and new risk factors (macroeconomic and insurance-specific variables) are found.

The first chapter of this thesis provides a comprehensive review of recent developments of credit risk models and important empirical research in the large and growing finance and economics literature. This chapter first briefly introduces the definition and properties of

credit risk models as well as several related concepts. Then reviews model selection tests for credit risk models in the literature. At last, interesting topics for further research is suggested.

The remaining three chapters investigate applications of credit risk modelling in three topics: credit risk of public listed firms, systemic risk within insurance and financial sectors and credit risk determinants for general insurance industry.

Chapter two investigates the credit risk of public listed firms in China. There are few papers on the credit risk for public listed firms due to the lack of default data in China. For this reason, we construct the unique default dataset of Chinese public firms from 1999 to 2013. We explore the determinants of firm failure using both firm-specific and macroeconomic covariates by the multinomial logit model. We find a distinct difference in firm-specific variables between post-default and tranquil times. The results show that both binomial and multivariate logit models outperform the KMV-Merton structural model. Our result is surprisingly different from the US market, and it turns out that some traditional determinants of credit risk, for example, profitability, are not significant in China's market. Unlike the US market, firms who are holding more liquid assets, are more likely to default in good market condition (low interest rate and high index return). In addition, we find the special treatment (ST) which is commonly used in previous literature may not be good default proxy for China's market. We also investigate if a high PD leads a firm to be delisted out of the stock market. The results show no clear difference of default probability between active firms and delist firms in recent years.

Chapter three investigates the linkage between the insurance and financial sectors based on the default probability for individual firms. Time-varying symmetric correlations between these two sectors are found for both short-term and long-term horizons. We investigate the joint default risk between insurance firms and other financial institutions using student *t* copula with the generalized autoregressive score model (GAS) with skewed *t* and empirical distribution function (EDF) marginal distribution. We find the existence of a time-varying correlation between these two sectors which indicating a dynamic and symmetric dependence. And this correlation varies widely in different time horizons, the negative correlation is found in the 1-month horizon. Short-term correlations are relatively lower but more fluctuating while the long-term correlations suggest that insurance and financial firms are positively correlated. We find economic drivers of the movement of insurance and finance industries by regressing the copula correlations with macroeconomic variables. By using the cupula correlations, we can also simulate the joint default probability for the two sectors. In addition, we find the joint PD between these two sectors are very sensitive to global financial market events.

Chapter four applies reduced form model to access the credit risk of general insurance (non-life) firms in the United Kingdom. This paper extends previous research of credit risk of insurance industry in UK market by using a much larger database and more risk factors. New risk factors, both macroeconomic and firm-specific variables, are found in assessing the credit risk of general insurance firms. In addition, we firstly consider both insolvency and other exit like transferring business when modelling the default process for insurance firms to avoid censoring bias. Our research firstly applies 30 years' regulation data of 515 firms to assess credit risk of UK insurance industry. With the support by Bank of England, we study and identify insolvent events from different data sources. Though business concentration is not the determinant of insolvency, our analysis of firms' PD with different business lines shows significant differences, especially when natural disasters happened. Also we firstly analyse system risk of general insurance industry in UK using more than 300 individual firms' PD and find slight joint default risk. At last, we further study the relationship between reinsurance assumed and credit risk of GI firms. The empirical results may provide implications to regulators of the GI firms' supervision under the coming Solvency II.

Chapter Five concludes with recommendations for further study.

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Dedication

This dissertation is dedicated to

my wife Ajiao Fan and my parents

Weijun Zhang and Li Chen.

Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signature:

Printed name: Xuan Zhang

Chapter 1

Credit Risk Management in Finance and Economics: A Comprehensive Survey

Credit risk refers to the probability of loss due to an obligor's failure to make payments on any type of debt at maturity. The global financial crisis, the credit crunch and the European debt crisis that followed put credit risk management into the regulatory spotlight. The purpose of this chapter is to provide a comprehensive review of recent developments in credit risk models and some important applications in systemic risk management by the Copula theory. First, we briefly introduce the credit scoring model, which is an accounting-based model. Second, we review the development of widely used structural models and discuss their advantages and limitations. Third, we provide a comprehensive review of recently developed reduced-form models in multi-period forecasting. Finally, we discuss copula-based models in joint default risk modelling.

1. Introduction

The importance of credit risk management cannot be overstated. Credit risk refers to the risk of an obligor failing to make payments on any type of debt at the time of maturity. Credit risk models are statistical tools to infer the future default probabilities and loss distribution of values of a portfolio of debts.

2. Credit Scoring Models

Beaver (1966, 1968) and Altman (1968) study corporate bankruptcy prediction models which relied on accounting-based measures as variables. Recent accounting-based models use composite measures that statistically combine several accounting variables, such as Altman (1968) Z-Score and Ohlson (1980) O-Score.

The widely used Z-score indicates the probability of a company entering bankruptcy within the next two years. The higher score, the lower the probability of firm's bankruptcy. A score above 3 indicates bankruptcy is unlikely; below 1.8 means bankruptcy is probable. Altman's Z-Score for US firms calculated in the following:

$$\begin{aligned}\text{Altman's Z} - \text{Score} \\ &= 1.2 * (\text{Working Capital/Total Assets}) + 1.4 \\ &* (\text{Retained Earnings/Total Assets}) + 3.3 \\ &* (\text{EBIT/Total Assets}) + 0.6 \\ &* (\text{Market Value of Euquity/Total Liabilities}) \\ &+ (\text{Sales/Total Assets}).\end{aligned} \tag{1.1}$$

There are some clear drawbacks of credit scoring models: for example, the historical data used in credit scoring models doesn't necessarily contain enough information to estimate future performance, and the fixed value of firm's asserts and debts could affect the accuracy when measuring the threshold of default. Hillegeist et al. (2004) argue that there are several problems when modelling probability of default based on accounting data. Firstly, the estimates of probability of default are the likelihood of future events, but financial statements are designed to measure past performance of a firm. As a result, the accounting data may not be indicative of the future status of the firm. Secondly, the predictive ability of these models is limited by their design. Financial statements are designed based on the going-concern assumption, which assumes the firm will not default. Thirdly, due to the conservatism principle, fixed assets and intangibles are sometimes undervalued relative to their market prices. Downward-biased asset valuations will cause accounting-based leverage measures to be overstated.

In addition, asset volatility is ignored in accounting-based default prediction models. Volatility is a critical factor in asset pricing models, and price of assets are crucially important when estimating the default probability, however it is omitted in both the Altman (1968) and Ohlson (1980) corporate default prediction models. Campbell et al. (2001) argue that since firms exhibit considerable cross-sectional variation in volatility, the omission of volatility measurement in the accounting-based models may lead to a substantial reduction in their prediction accuracy. Without incorporating volatility into the model, the model will fail to capture the likelihood of whether the firm's assets' value will below its debts in a dynamic way. High volatility will increase the probability of default. For example, two identical firms with same financial statements ratios can have very different probability of default depending on their asset volatilities. Obviously, credit scoring models are easy to implement but have lower prediction accuracy compared to other models. But volatility of assets and debts should be incorporated into the models and market-risk factors should be introduced to the models, which may substantially improve the estimation accuracy.

3. Structural Models

Structural models are also called market-based models that provide an alternative and potentially superior source of firm's information compared to credit scoring models. Structural models are widely used to model firm's credit risk. The information contained in the financial statements may not be enough to estimate future default probabilities accurately. Besides the financial statements, structural models add stock prices which aggregate information from the market into the models. Beaver (1968) has recognized the potential default-related information contained in the market variables. Cheung (1991) and Dhaliwal et al (1992) argue that structural models provide a natural starting point to extract the probability of default related information from market prices.

Since the 1970s, many structural models have been developed to forecast and price credit risk. The methodology of these models, which is based on Black and Scholes (1973) and Merton (1974), is to consider corporate liabilities as contingent claims on the assets of the firms. Merton (1974) first applied option theory to derive the value of a firm's liabilities in the presence of default, considering only one single type of debt. As a result, structural models are also called Merton models. Merton also adds some restrictions on corporate debt, such as the firm cannot issue any new debt and bondholders will not receive any cash dividend from the firm. The probability of default is embedded in the structural models. Both volatility of assets and total liabilities are key variables in the option-pricing models. The firm's equity value can be viewed as a call option on the value of the firm's assets. Default will occur if the asset value which is the value of the firm is not enough to cover the firm's liabilities. When the asset value falls behind the liabilities value, there is no value of holding equity. If the value of the equity is zero or negative, the firm cannot fully cover creditors' claims. As a result, the firm is in default.

The interest rate in structural models used to be non-stochastic. Interest rates are treated as non-stochastic processes in the first generation structural-form models (see Black and Cox, 1976; Geske, 1977; Leland and Toft, 1996). It's a very strong assumption and the fixed interest rates cannot be hold in reality. As a result, a stochastic process of the interest rates has been introduced many researchers (see Nielsen et al. 1993; Longstaff and Schwartz, 1995). Hillegeist et al (2004) suggest that the main advantages of using structural models in default prediction are that the models provide guidance about the theoretical determinants of default risk and they supply the structure to extract default-related information from market prices. However, in practice, many simple assumptions of the models cannot be hold. Any violations of the assumptions will introduce errors and biases into the resulting probability of default estimates. Another shortcoming of these option-based models is that the stock price may not efficiently include all publicly-available default-related information. Sloan (1996) argues exactly this, that the market does not accurately include all of the information in financial statements. Thus, Hillegeist et al (2004) propose that whether the market-based default models or the accounting-based default models can achieve better estimates is ultimately an empirical question

3.1 First Generation Structural Models

The first generation structural models are based on Merton (1974). The Merton models follow Black and Scholes (1973) option pricing theory. The default occurs if the value of the firms' assets falls below a critical value *L* which is calculated based on the firm's short-term and long-term liabilities.

The models consider firms as a call option, and total value of asset *A* is financed from the market with value of equity *E* and a zero-coupon bond with notional value *F* and maturity at time *T*[1].

Whether a firm will default or not will be assessed only at end of time *T*, which means the default can only happen at maturity time *T*. A firm will go bankrupt when the value of assets is below the value of its liabilities *L* at the maturity.

The key assumption of the structural-form model is that the stock price follows an independently log normal distribution with annual volatility σ and the following parameters:

$$\mathrm{InA_t} \sim \mathrm{N}\left(\mathrm{InA_t} + \left(\mu - \frac{\sigma^2}{2}\right)(\mathrm{T}-\mathrm{t}), \sigma^2(\mathrm{T}-\mathrm{t})\right), \tag{1.2}$$

If the asset value *A* falls below the liability value *L*, the value of equity *E* will be zero, the payoff to equity holders described as:

$$\mathrm{E_t} = \max(0, \mathrm{A_T} - \mathrm{L}), \tag{1.3}$$

The equity value *E* can be calculated using the standard Black–Scholes European call option formula:

$$\mathrm{E_{t=}A_t} \cdot \emptyset(\mathrm{d_1}) - \mathrm{Le^{-r(T-t)}}\emptyset(\mathrm{d_2}), \tag{1.4}$$

where:

$$\mathrm{d_1} = \frac{\ln\left(\frac{\mathrm{A_t}}{\mathrm{L}}\right) + \left(\mathrm{r} + \frac{\sigma^2}{2}\right)(\mathrm{T}-\mathrm{t})}{\sigma\sqrt{\mathrm{T}-\mathrm{t}}} \text{ And } \mathrm{d_2} = \mathrm{d_1} - \sigma\sqrt{\mathrm{T}-\mathrm{t}}, \tag{1.5}$$

After values of *L*, $\mathrm{A_t}$ μ,and σ^2 are calculated, the probability of default can be estimated by:

$$\Pr(\text{Default}) = \emptyset\left(\frac{\mathrm{InL} - \mathrm{LnA_t} - (\mu - \frac{\sigma^2}{2})(\mathrm{T}-\mathrm{t})}{\sigma\sqrt{\mathrm{T}-\mathrm{t}}}\right), \tag{1.6}$$

[1] There will be no payments until the maturity time *T*. There will also be no dividends payment.

The three the unknown parameters will be estimated, which are μ (i.e. the annual expected asset returns), σ (i.e. the annual asset volatility) and A_t (i.e. market value of assets at time t).

The distance to default (DTD) which is the deviation of asset value from the default point *L* can be calculated in the following:

$$\mathrm{DTD} = \frac{\mathrm{In}A_t + \left(\mu - \frac{\sigma^2}{2}\right)(T-t) - \mathrm{In}L}{\sigma\sqrt{T-t}}, \quad (1.7)$$

The Merton's model considers the firm as a European call option and this method bear the same assumptions by Black and Scholes (1973)[2]. But most assumptions of the Black-Scholes option pricing formula may not be held in reality. Black and Cox (1976), Geske (1977), and Vasicek (1984) develop the original Merton model by removing some unrealistic assumptions. In the original Merton models, the first unrealistic assumption is that default will only occur at the end of the maturity. The possibility of reorganization of the firm before default happens is not taken into consideration in Merton's model. Black and Cox (1976) add safety covenants which entitles debt holders to force the firm to reorganize when the value of a firm falls below a threshold, and receive a discounted value of the debt's principal amount[3] into the indenture. Whether the firm has reorganized before the maturity will be

[2] The structural models apply assumptions of the Black-Scholes (1973) option pricing formula which are:
1) Constant return and volatility
2) No transaction costs
3) No dividends payments
4) No riskless arbitrage
5) Continuous trading
6) Risk free rate is constant for all maturities
7) Short selling proceeds is permitted

[3] The safety covenant is a contractual provision that lets debt holders to claim the ownership of a firm's assets which provide a floor value of the bond.

taken into account when calculating the value of the firm at the maturity time by using Cox and Ross (1975).

The second unrealistic assumption in original Merton's model is that it assumes only one single type debt of the firm. The tranche structure[4] should be allowed in the firms' debt. When the threshold of safety covenant provided in the bond is lower, subordinated bonds will not benefit from this in the first place, and no payments will be received for the subordinated bonds until all payments for the senior bonds have been made. The seniority structures of firm's debts are very complex and empirical evidence shows that violation of this seniority rule is very likely in practice. Tranche structure in bonds is added into the model by Black and Cox (1976).

Third, the term structure of debt which has a significant effect of default probability is also ignored in Merton's models. Short-term debt and long-term debt should be given different weights when measuring the default probability. It's complicated when tranche structure and term structure are both taken into consideration. Vasicek (1984) finds the distinction between short-term and long-term liabilities. He analyses three different cases and finds that not only a firm's mark-to-market asset value, but also a firm's maturing debts and higher priority debt, would affect the expected loss for the firm.

Fourth, interest payment should be added into the original Merton's model. Black and Cox (1976) assume that interest payments are continuous and not a closed form solution for discrete payments. However, Geske (1977) develops an approach to value the bonds with an arbitrary number of discrete payments. Geske (1977) develops the original Merton models by considering the debt structure as a coupon bond. The models allow bonds to have discrete interest payment by viewing each payment as a compound option by Geske (1966).

[4] Tranching allow for the senior bonds obtaining full payment before junior bonds.

Though Merton's models have been developed in recent years, the model still has some drawbacks. The first generation structural models have few inputs that could not capture enough information on the firm. Expected returns of assets, risk aversion, accounting ratios and macroeconomic covariates are not taken into consideration. In these models the term structure of interest rates is assumed to be constant and flat and no stochastic process is applied into the models. However, the term structure of interest rate has a significant effect of value of firm's debts. The using stochastic interest rates will allow correlations between asset values which would improve the Merton models' performance. Further studies by Nielsen (1993) and Longstaff & Schwarts (1995) consider stochastic interest rate in the models.

3.2 Second Generation Structural–form Models

The original Merton's model has been improved in many ways, but still retains some drawbacks. One unrealistic assumption held in the first generation structural-form models is that default happens only at the time of maturity. In the real world of finance, a firm will default any time between issuance time and maturity when it fails to pay interests or coupons. Another unrealistic assumption held in the first generation structural-form models is that a risk-free rate is constant and flat not following any stochastic process. The original structural models apply flat term-structure of the interest rates which is unlikely to hold in reality.

Kim et al. (1993) develop Merton's original model by allowing default to happen anytime in the entire life cycle of the bond and introducing a stochastic process for the evolution of the short-term rates. They point out that net cash flow is one of the key factors when measuring a firm's default risk. One assumption of their model is that the bondholders and shareholders receive net cash flow from the firm and the assets of the firm cannot be sold. As a result, when a firm fail to have enough net cash to make a coupon payment then the firm is in default even through the asset value of the firm may higher than the liability value of the firm. In addition, since most corporate bonds are callable, the study of yield spread between callable corporate bonds and treasury bonds is important. Kim et al. (1993) find that the call policy is more sensitive to interest rates than to the firm's value[5]. And because the interaction between the call provision and the default risk, the stochastic interest rates have a significant effect of the yield difference between a callable treasury bond and a callable corporate bond. Flat term-structure of interest rates is one of the assumptions in original Merton's model. While incorporating stochastic processes in interest rate will affect capital structure of a firm. And the uncertainty of interest rate may have a significant effect of credit spread. But the study from Kim et al. (1993) shows that the effect is quite limited. However, the difference between shore-term rate and long-term rate plays a more important role in pricing the credit spread.

The relationship between firms' assets and interest rate is important. Empirical research by Longstaff and Schwartz (1995) suggests that firms with same default risk may have different credit spreads due to the different degree of correlations between firms' asset and the interest rates. The interest rate in their model follows the Vasicek (1977) process[6] in order to capture the macroeconomic factors of interest rates. A closed-form solution of the valuation of a risky floating-rate bond is derived by Longstaff and Schwartz (1995) which summing the values of the fixed-rate coupons and the value of principal payment at the time

[5] When a firm calls a bond, it requires an instantaneous cash outflow while reducing high coupon payments.

[6] The Vasicek (1977) process is a mathematical model describing the evolution of interest rates. It is a one-factor model, and it describes interest rate movements as driven by only one source of market risk. It is used in the valuation of interest rate derivatives, and has also been adapted for credit markets. The models consider the interest rates following stochastic movements:

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

Where $a(b - r_t)$ is the drift movement. W_t is a Wiener process modelling the random market risk. The models capture the mean-reverting property of the interest rate. And b is the long-term average value, r_t is the short-term interest rate at time t.

of maturity to calculate the value of the risky floating-rate coupon bond. They run regressions of the changes in credit spreads with two key factors: an asset value factor and an interest rate factor. It turns out a negative correlation between interest rate risk and credit spread, and the volatility of credit spread mostly come from the interest risk. They conclude that both interest rate risk and default risk are key explanatory variables when modelling the credit spread.

Despite the fact that most recent credit risk research focus on reduced-form models, structural models are still popular among researchers. For example, Camara et al. (2012) present a comparative study of the probability of default using structural models for global financial firms with traded options in the US. Due to complicated capital structures of financial firms, there is a lack of research in default probability in financial firms, and most recent research focus on estimating the default risk of non-financial firms. Camara et al. (2012) present empirical evidence on default risk models for global financial firms. Their model for estimating default risk can be easily applied in other markets or other financial areas.

They modify the original Merton model and derive the implied probability of default from the model which has a better forecasting accuracy compared to the original Merton model. They compare the performance of their model with EDF (expected default frequencies) based on the Moody's KMV model and credit ratings from agency during the subprime mortgage crisis period by CAP (cumulative accuracy profile) and the ROC (receiver operating characteristic) methods. The results show their model gets a better accuracy of default probability prediction. In their model, the probability of default is defined as the probability that shareholders will lose all their money in the stock during the life of the option. And their ex-ante default probability is embedded in the market price (i.e. option price) of each firm. Instead of using hedging arguments in Black and Scholes (1973), they derive the option prices by using equilibrium arguments developed by Rubinstein (1976), Brennan (1979), Stapleton and Subrahmanyam (1984), Amin and Ng (1993) and Camara (2003) [7] .Valuable information on financial distress, bankruptcy reorganization and liquidation, and capital structure, and risk management can be interpreted from the default probability which implied by the firm's stock price and firm's option price. They propose that stock prices and option prices of a firm contain enough information to measure default probability and as a result, the value of asset and value of debt won't be incorporated into the model.

To sum up, there are still two main problems need to be solved in the second-generation structural models. First, the firm's value needed to be estimated in the models in some certain ways when the deterministic parameters cannot be observed directly. As a result, the estimation of firms' market value leads to poor empirical performance when using the structural models. Second, credit rating contains valuable information of the firms' financial health, but the credit rating cannot be incorporated into structural models. In addition, some credit derivatives which payoffs depend on credit ratings cannot be properly priced in structural models.

4. Reduced-Form Models

The second-generation of structural model still faces the difficulties of estimating the firm's market value and the credit rating which can reveal the financial health of a firm cannot be incorporated in the structural models. The standard structural models from Black and Scholes (1973), Merton (1974), Fisher et al. (1989), and Leland (1994) treat the asset process to be a geometric Brownian motion. The covariates of the models are market prices

[7] Schroder (2004) derives general conditions for option price using equilibrium arguments.

of stocks and accounting data (i.e. short-term and long-term liabilities) from the financial statements. Crosbie and Bohn (2002) and Kealhofer (2003) find that the structural models have been developed and widely used in industry practice like Moody's KMV. Duffie and Lando (2001) argue that distance to default estimated from the structural models is a crucial covariate for default estimation, but if the distance to default cannot be measured accurately or a firm's financial health may have multiple influences over time from firm-specific, cross-sector and macroeconomic factors, adding other covariates may reveal additional information about the default probability of the firm.

In recent years, the reduced-form models which use an exogenous Poisson random variable to determine the default probability is becoming more and more popular[8]. Reduced-form models treat default as an unpredicted event given by hazard process. The exogenous variable which is not dependent on the value of firm's asset makes reduced-form models more tractable than structural models.

The reduced-form models assume an exogenous Poisson random variable that drives the default events. A firm will default when the exogenous random variable shifts in its level over any time interval. As a result, the default event, which, treated as unpredictable Poisson events, will not be forecasted on the basis of the value of the firm's assets. The stochastic process in the models does not directly link to firm's assets' value which makes the models more tractable. The Poisson process is one of the most important stochastic processes. It is a jump process that the variable jumps instantaneously from one value to another at random times. In reduced-form models, the event in a Poisson process is "the default". The stochastic intensity parameter $\lambda_{(s)}$ called "the default intensity" or the "frequency of jump" and that describes the likelihood that the numbers of events $N_{(t)}$ occurs in the time interval$[0, t]$.

[8] Papers like Shumway (2001), Chava and Jarrow (2004), Campbell, et al (2008) develop statistical models such as logistic models and hazard models. In these papers, firms that disappear for reasons other than defaults are left untreated, which will result in censoring biases.

λ refers to the rate at which event occur. Before the maturity time, when $N_{(t)}$is nonzero, defaults happen. In the case of a constant λ, the probability distribution is described by:

$$\mathrm{P}\big(N_{(t)} = k\big) = \exp(-\lambda t)\frac{(\lambda t)^k}{k!}, \qquad k = 0,1,2,\ldots. \tag{1.8}$$

Jarrow and Turnbull (1995) and Duffie and Singleton (1999) first propose the reduced-form model for credit risk modelling. The assets value of the firm does not determine the default probability in the reduced-form models. The reduced-form models are based on assumptions that both probability of default and recovery rate are dynamic. Recovery rate is exogenous and independent from probability of default. Whenever default occurs, the recovery will be paid only at the time of maturity. And they incorporate the credit ratings of issued debts that will cover information about the financial status of the firms without considering the firms' assets value. But the model in Jarrow and Turnbull (1995) only describes two possible states: survival and default. There are a lot of movements from one credit rating to another credit rating with different probabilities published periodically by rating agencies. Jarrow et al. (1997) incorporate credit ratings into the valuation of a contingent claim model. They extend the model introduced by Jarrow and Turnbull (1995) by studying the term structure of credit spread in a model with the default process following a Markov chain in credit rating. They assume the recovery rate is exogenous and the recovery can only be received at the time of maturity through the default occurs prior to maturity[9]. Another assumption in their model is that the stochastic processes of risk-free rate and recovery rate are dependent. However, the assumption in the model that recovery payment will only be obtained at the time of maturity is unrealistic. In addition, the recovery rate is described as an exogenously specified percentage of risk-free bonds. As a result, the

[9] Jarrow et al. (1997) assume that the bondholders will receive certain recovery money at the maturity, if default happens prior to maturity.

recovery payment may exceed the actual recovery amount at the default time. The actual recovery payment that depends on the liquidation value of the firm is fluctuating over time.

A new method is proposed by Duffie and Singleton (1999) to model the valuation of contingent default claim. They point out that the value of a default claim is the present value of the promised payoff discounted by the adjusted short rate. They propose that default probability distribution of a firm follows a translated single-factor square root diffusion process[10]. Also, they develop the model by allowing the recovery payment to be made at any time. The model keeps the recovery rate to be a fixed fractional of the non-default bond price when default occurs. As a result, the recovery amount will fluctuate that depending on the firm's liquidation value at the time of default.

4.1 Multi-Period Default Prediction

Recent multi-period credit default prediction models focus on modelling the time-varying covariates. For instance, in the context of assessing corporate default probability, a typical approach is to use time-varying covariates like balance sheet or stock price as regressors. If a model needs to cover multiple prediction periods, the unknown future evolution of covariates needs to be estimated.

Studies in the default prediction literature often deal with a fixed and often short-term prediction horizon. For example, Chava and Jarrow, 2004; Hillegeist et al., 2004; Shumway, 2001 propose an approach to estimate a discrete-time hazard model with yearly data directly yielding one-year default probabilities. Time horizon for more than one year is not available in these models since time-varying evolution of the covariates is unknown.

[10] Variables like default risk, liquidity difference and state taxes etc. Duffie and Singleton (1999) take these factors into consideration when measuring the stochastic default process.

Hamerle et al. (2007) reduce the dimension problem by forecasting the credit score[11]. A Monte Carlo simulation of the credit scores is used to calculate the default probabilities with different horizons. A Poisson intensity approach (i.e. Cox doubly stochastic process) is proposed by Duffie et al. (2007), and further developed by Duffie et al. (2009), and Duan et al. (2012) to model corporate defaults where default risk is measured by common risk factors and firm-specific attributes. The model links individual default events along with economic predictors as a dynamic panel data. Common risk factors can be observable, for example, Duffie et al. (2009) use four covariates: firm's distance to default; firm's trailing one-year stock return; three-month Treasury bill rate; trailing one-year return on the S&P 500 index. Parameter uncertainty of the models arises since any errors and biases in the forecasting of the evolution of the covariates will impact the models' default prediction accuracy. Duffie et al. (2007) propose the model with time-varying covariates and then forecasting the evolution of covariates process using Gaussian panel vector autoregressions. And Duan et al. (2012) apply a forward intensity approach to estimate multi-periods PD.

4.1.1 A Doubly Stochastic Poisson Process with Time-varying Covariates

Duffie, et al (2007) propose the time-series dynamics of the explanatory covariates in order to estimate the likelihood of default over different future periods[12]. Considering the benefits of parsimony and the need to model the joint time-series development of all the covariates, they adopt a relatively small set of firm-specific and macroeconomic covariates. Duffie et al. (2007) suggest that a firm's distance to default, US interest rates and stock-market returns have significant dependence of the level and shape of the term structure of conditional future default probabilities, among other covariates.

[11] The credit score here is the inner product of the estimated parameter vector and the covariate vector.
[12] They estimate the model for US-listed firms, using firm-months of data over 20 years.

A doubly-stochastic formulation of the point process for default[13] is proposed by Duffie et al. (2007), where the conditional probability of default within s years is

$$q(X_t, s) = E\left(\int_t^{t+s} e^{-\int_t^z (\lambda(u)+\varphi(u))du}\, \lambda(z)dz \middle| X_t\right) \tag{1.9}$$

X_t is a Markov state vector of firm-specific and macroeconomic covariates which intertemporal variation in a firm's default intensity $\lambda_t = \Lambda(X_t)$. λ_t is the conditional mean arrival rate of default measured in events per year. The firm may exit for other reasons like merger or acquisition, the intensity is defined as $\varphi_t = A(X_t)$. Thus the total exit intensity is $\varphi_t + \lambda_t$. The doubly-stochastic model allows to combine two decouple estimators β and γ to obtain the maximum likelihood estimator of the default probability $\mathrm{q}(\mathrm{X_t}, \mathrm{s})$. In the default estimation models, given the path of state-vector *X*, the merger or acquisition and default times of the firm are conditionally independent.

$(\Omega, \mathcal{F}, \mathrm{P})$ is a fixing probability space with an information filtration $\{\mathcal{G}_t: t \geq 0\}$ satisfying the usual conditions. Let $\mathrm{X} = \{X_t: t \geq 0\}$ be a time-homogeneous Markov process in $\mathcal{R}^d$, for some integer $\mathrm{d} \geq 1$. The X_t is a state vector that a covariate for a given firm's exit intensities. Let (*M*, *N*) be a doubly-stochastic nonexplosive two-dimensional counting process driven by *X*, with intensities $\varphi = \{\varphi_t = A(X_t): t \in [0, \infty)\}$ for *M* and $\lambda = \{\lambda_t = \Lambda(X_t): t \geq 0\}$ for *N*, for some non-negative real valued measurable functions $A(\cdot)$ and$\Lambda(\cdot)$ on $\mathcal{R}^d$[14].

A given firm exits at $\tau = \inf\{t: M_t + N_t > 0\}$, which is the earlier of the first event time of *N* (i.e. default) and the first event time of *M* (i.e. exits for other reasons). They also allow the state vector X_t to include firm-specific default covariates that cease to be observable when the firm exits at τ. And they suppose that $X_t = (U_t, Y_t)$, where U_t is firm-specific

[13] The doubly-stochastic assumption is that conditional on the path of the underlying state process *X* which determining default and other exit, exit times are the first event times of independent Poisson process.
[14] This means that, conditional on the path of *X*, the counting processes *M* and *N* are independent Poisson processes with conditionally deterministic time-varying intensities, α and λ respectively.

and Y_t is macroeconomic. They consider conditioning by an observer whose information is given by the smaller filtration $\{\mathcal{F}_t: t \geq 0\}$[15], The firm's default time is the stopping time $T = \inf\{t: N_t > 0, M_t = 0\}$.

4.2.1 A Doubly Stochastic Poisson Process without Time-varying Covariates

A more convenient approach to calculate the default risk is developed by Duan et al. (2012) which applying the forward intensity rate.

The key point is to derive the forward intensity rate at time τ.

First, we define $F_t(\tau)$ to be the time conditional distribution function of the combined exit time.

$$\psi_t(\tau) = -\frac{ln\big(1 - F_t(\tau)\big)}{\tau} = -\frac{lnE_t\left[exp\left(-\int_t^{t+\tau}(\lambda_u + \varphi_u)dz\right)\right]}{\tau} \tag{1.10}$$

Then, $exp[-\psi_t(s)s]$ becomes the survival probability at time interval (t, t+τ).

And the forward combined exit intensity is defined as

$$g_t(\tau) \equiv \frac{F'_t(\tau)}{1 - F_t(\tau)} = \psi_t(\tau) + \psi'_t(\tau)\,\tau. \tag{1.11}$$

Also the forward default intensity censored by other forms of exit

$$\begin{aligned} f_t(\tau) &\equiv e^{\psi_t(\tau)\tau} \lim_{\Delta t \to 0} \frac{P_t(t + \tau < \tau_D = \tau_c \leq t + \tau + \Delta t)}{\Delta t} \\ &= e^{\psi_t(\tau)\tau} \lim_{\Delta t \to 0} \frac{E_t\left[\int_{t+\tau}^{t+\tau+\Delta t} exp\big(-\int_t^z(\lambda_u + \varphi_u)du\big)\lambda_z dz\right]}{\Delta t}, \end{aligned} \tag{1.12}$$

So the default intensity between (t, t+s) will be $\int_0^{\tau} -e^{\psi_t(\tau)\tau} f_t(s)ds$.

[15] $\mathcal{F}_t$ is the σ-algebra generated by $\{(U_s, M_s, N_s): s \leq min(t,\tau)\} \cup \{Y_s: s \leq t\}$.

And since X_t is the state variables contain both firm-specific and macroeconomic factors.

$$f_t(\tau) = exp\big(\alpha_0(\tau) + \alpha_1(\tau)X_{t,1} + \alpha_2(\tau)X_{t,2} + \cdots + \alpha_k(\tau)X_{t,k}\big), \tag{1.13}$$

$$g_t(\tau) = f_t(\tau) + exp\big(\beta_0(\tau) + \beta_1(\tau)X_{t,1} + \beta_2(\tau)X_{t,2} + \cdots + \beta_k(\tau)X_{t,k}\big). \tag{1.14}$$

The pseudo-likelihood function for the prediction time τ defined as

$$\mathcal{L}_\tau(\alpha, \beta; \tau_C, \tau_D, X) = \prod_{i=1}^{N}\prod_{t=0}^{T-1} \mathcal{L}_{\tau,i,t}(\alpha, \beta), \tag{1.15}$$

And the likelihood function $\mathcal{L}_{\tau,i,t}(\alpha, \beta)$ for firm *i* consists of five situations: the first is the firm *i* survives in the prediction time period (i.e. no default or any other exit events happen), the second is the firm *i* defaults in the prediction period, the third is the firm *i* exits for other reasons, the fourth is the firm *i* exits after this prediction time period and the last is the firm *i* exits before the start of this time interval:

$$\mathcal{L}_{\tau,i,t}(\alpha, \beta) = 1_{\{t_{0i} \le t, \tau_{Ci} \ge t+\tau\}} P_t(\tau_{Ci} > t + \tau) + 1_{\{t_{0i} \le t, \tau_{Di} = \tau_{Ci} \le t+\tau\}} P_t(\tau_{Di} = \tau_{Ci} \le t + \tau) + 1_{\{t_{0i} \le t, \tau_{Di} \ne \tau_{Ci}, \tau_{Ci} \le t+\tau\}} P_t(\tau_{Di} \ne \tau_{Ci}, \tau_{Ci} \le t + \tau) + 1_{\{t_{0i} > t\}} + 1_{\{t_{Ci} < t\}}. \tag{1.16}$$

where this pseudo-likelihood function can be decomposed into two process α and β:

$$\mathcal{L}_\tau(\alpha(s)) = \prod_{i=1}^{N}\prod_{t=0}^{T-s-1} \mathcal{L}_{i,t}(\alpha(s)), s = 0,1, \dots, \tau - 1$$

$$\mathcal{L}_\tau(\beta(s)) = \prod_{i=1}^{N}\prod_{t=0}^{T-s-1} \mathcal{L}_{i,t}(\beta(s)), s = 0,1, \dots, \tau - 1.$$

4.2.3 Other Studies about Reduced-Form Models

Orth (2013) present simple alternatives for multi-period prediction that avoid modelling the covariate evolution process. This model delivers high out-of-sample predictive accuracy. Orth (2013) propose a multi-period credit risk model. The model observes obligor i, i= 1,…N, for T_i periods and record its default history and a vector of time-varying covariates X_{it}. For each period t, t=1,…T_i, the model define Y_{it} to be the lifetime (i.e. the time to be default history) of obligor i starting in period t. since there is no information about the lifetime starting in the last period, the model observes the T_{i-1} for each obligor instead and define additionally the corresponding censoring indicator variable C_{it} which is zero in the case of no censoring (i.e. the lifetime ends with a default event and one for censored lifetimes). Orth (2013) build models in terms of the continuous-time hazard rate[16]:

$$\lambda(y) = \lim_{\Delta y \longrightarrow 0} \frac{P(y \leq Y < y + \Delta y | Y \geq y)}{\Delta y}. \tag{1.17}$$

Orth (2013) suppose that at a point in time t they want to predict the default probabilities for the next H periods using the current information[17]. A simple solution is to specify the hazard rate in period t+s, $\lambda(t+s)$ as a function of the covariates in period t, X_{it}, and the forecast time s. For example, within the proportional hazard framework a possible specification would be

$$\lambda(t+s, X_{it}) = \lambda_0(s) \exp(\beta' X_{it}). \tag{1.18}$$

$\lambda_0(s)$ is the baseline hazard rate. $\lambda_0(s)$ captures the variation of the hazard rate over the forecast time and may be referred as a kind of duration dependence[18]. There, the hazard rate can be freely fluctuating for different s due to the repeated estimation of the model. Orth (2013) imposes a structure on the evolution of the hazard rate over the forecast time by

[16] The hazard rate defines the instantaneous risk of default.
[17] The covariates, we have at t.
[18] The forecast time s is the analogue to the lag length in the approach of Campbell et al. (2008).

integrating s as an argument into the functional form of the model. The default probabilities are calculated in closed form:

$$\mathrm{P}(\mathrm{Y}_{\mathrm{it}} \leq \mathrm{H}) = 1 - \exp\left(-\int_{0}^{\mathrm{H}} \lambda(\mathrm{t}+\mathrm{s}, \mathrm{X}_{\mathrm{it}})\mathrm{ds}\right). \tag{1.19}$$

An advantage of the covariate forecasting approach like Duffie, et al (2007), as compared to Orth (2013), is that it can analyse the portfolio credit risk as the covariate processes provide a model for the dependence of defaults. Orth (2013) focuses on single-obligor credit risk and present models designed for the purposes of rating and probability of default estimation with a multi-period prediction horizon.

5. Default Prediction Accuracy

Two methods are commonly used to assess the default prediction accuracy. One is the cumulative accuracy profiles and another is the receiver operating characteristic.

5.1 Cumulative Accuracy Profiles (CAP)

First we will rank firms by their default probabilities from highest to lowest that predicted by the model.

Then, we define α as an integer between 0 and 100 which represent α percent of the total number of sorted firms (i.e. firms rank from highest predicted default risk to lowest predicted default risk), we record the corresponding number of defaulted companies being captured as a percentage (*z%*) of total number of actual default firms.

And obviously $f(\alpha = 0) = 0, and\ f(\alpha = 100) = 1$

where

$$f(\alpha) = \frac{\text{number of default firms captureed at } \alpha\%}{\text{number of total actual default firms}}$$

The CAP curve can then be traced out by varying α from 0 to 100 and plotting the corresponding values of *x* and *y* along and *x-axis* and *y-axis* respectively.

A good model will result in a majority of the default firms having relatively high default probability estimates and the percentage of default firms being captured will increase quickly as α increases.

[INSERT FIGURE 1.1 ABOUT HERE]

If the model has zero information of the future default probability, for example, the default probability will be ranked randomly for a large number of times, and then $f(\alpha)$ would be equal to α. As a result, the CAP curve will correspond to the 45-degree line (the red curve in Figure1.1). And a good model will be more like the blue line in Figure1.1.

Finally, the accuracy ratio by CAP curve= (the area under a model's CAP)/ (the area under the ideal CAP)

5.2 Receiver operating characteristic (ROC)

Receiver operating characteristic (ROC), or simply ROC curve, is a graphical plot which measures the performance of a binary classifier system as its threshold is varied. It is created by plotting the percentage of true positives out of the total actual positives (TPR) vs. the percentage of false positives out of the total actual negatives (FPR) at different cut-off probability.

When we use the ROC curve to measure the performance of default prediction, the threshold is every cut-off probability, the ROC curve defines the "true positive rate" (percentage of defaults that the model correctly predicts as default) on the *y-axis* as a function of the corresponding "false positive rate" (percentage of non-defaults that are mistakenly predicted as default or other exits) on the *x-axis*.

First, in order to construct the ROC curve, all firms are ordered by their default probabilities from highest to lowest.

Then at each default probability γ we calculate a set of two fractions, the first one is the percentage of defaults that the model correctly predicts as default, where the correct prediction means the default probability of the firm is equal or greater than γ.

$$\mathrm{f}^{\mathrm{y}}(\gamma) = \frac{\text{number of correctly predicted as default at the threshold } \gamma}{\text{number of total correctly predict default}},$$

The second one is the percentage of non-defaults that are mistakenly predicted as default, also here the firm has a default probability that equal or greater than γ.

$$\mathrm{f}^{\mathrm{x}}(\gamma) = \frac{\text{number of mistakenly predicted as default at the threshold } \gamma}{\text{number of total mistakenly predict default}}.$$

[INSERT FIGURE 1.2 ABOUT HERE]

If the model contains no default information, then the ROC curve (the green line in Figure1.2) will correspond to the 45-degree line. However, a perfect model (the blue line in Figure1.2) will have a ROC curve that goes straight up from (0, 0) to (0, 1) and then across to (1, 1). And a good model will be more like the red line in Figure 1.2.

6. Correlated default

A good measure of correlated default risk in financial system is crucial for credit risk management given the fact that financial institutions are engaged more closely in many ways. Das et al (2006) point out that both default correlations and default probabilities vary through the different economic environment. Time-series variation can be observed in default probabilities and the series show high volatility when to shift from the different economic environment. For example, default probabilities are more than double in high default risk than low default risk time. These are also well captured by the correlated default probability curves from our model in different time horizons. Dembo et al. (2004) suggest that macroeconomic variables are significant in explaining the default probabilities. Other studies show common risk factors such as GDP growth rate, market volatility, and interest rates have a direct effect on joint default risk. Duffie et al. (2007) suggest that common risk factors are not the only reason of correlated default risk. Both contagion factor and latent frailty factors play a significant role in explaining joint default risk. Here the contagion effect means one firm default will cause the change of default risk of other firms. Default risk could spread quickly during financial firms through credit derivatives like CDS and CDOs. For example, Jorion & Zhang (2007) and Stulz (2010) find the counterparty risk is transferred through financial products like the CDS protection seller will be exposed to higher default risk caused by the protection buyer's other counterparties. And the frailty factors are the remaining sources of the joint default risk which are not explained by the common factors and contagion factors.

The modelling of correlated default risk is well developed. The models include intensity-based models, barrier-based firm's value models, and copula-based models. Traditional reduced-form model of portfolio applies a bottom-up model which portfolio intensity is an aggregate of individual intensities or a top-down model which portfolio intensity is calculated without including individual intensities. Schönbucher (2003) points out that the drawback of intensity-based models is the computational complexities may be very time-consuming. For the barrier-based models, it's difficult to calibrate and implement the model.

The copula-based models have some natural advantages because of the mathematical properties of correlated default modelling. The copula allows great flexibility in choosing a marginal distribution for each sector. Random variables with different marginal distributions can be easily linked with the copula function. In our case, when modelling the joint default risk, the marginal distributions of firm's default probability can be transformed to the portfolio default probability by the copula. Gaussian copula was first introduced to default risk to simulate the dependence structure of the times until default by Li (2000). And this Gaussian assumption is further studied by Andersen and Sidenius, 2004; Das et al., 2007; Glasserman and Li, 2005; Giesecke, 2004; Hull and White, 2004. Since there is no tail dependence of the Gaussian copula, but empirical data always shows 'fat tails' in their distribution, while obviously, Gaussian copula cannot capture the extreme increase in default probability when the correlated default happens. Also, empirical study by Wolfgang et al. (2003) turns out student *t* copula is better than Gaussian copula. Meneguzzo and Vecchiato (2004) find that due to the ability to capture the tail dependence, the Student's *t* copula outperformance Gaussian copula when pricing the CDO and basket CDS. In addition, Christoffersen et al. (2016) suggest that default dependence between reference entities is non-Gaussian and time-varying.

7. Conclusion

The credit risk modelling has developed rapidly in recent years, and become an important component in financial risk management. This chapter discusses credit risk models developed in last decades. This chapter presents a number of different approaches to measure the probability of default of a firm.

The accounting-based credit scoring model is first proposed to model the credit risk for firms. Then structural models and reduced form model are developed rapidly for modelling the credit risk. Structural models consider firm's liabilities as contingent claims on the value of firm's assets. The models assume that default occurs when the assets' value fall below a certain threshold, whereas reduced form models use a hazard rate framework to measure default risk. The structural models are further developed in many ways, such as including stochastic process of interest rate, considering term structure of debt and credit spread etc. Reduced-form models define the default process as an exogenous Poisson random process. The models could incorporate additional information like credit ratings. The exogenous variable which is not dependent on the value of firm's asset makes reduce-form models more tractable than structural models. However, the academic literature in both types of models has also been lacking concerned of the term structure of default probabilities. The prediction accuracy of time-varying covariates, the availability of large number of default data which including other exits like merger and acquisition and also some covariates selection problem etc. will be great changelings of the multi-period default prediction.

The modelling of correlated default risk is well developed. The joint default risk models include intensity-based models, barrier-based firm's value models, and copula-based models. Among all the models, the copula-based models have some natural advantages because of the mathematical properties in correlated default modelling. The copula allows the great flexibility of choosing a marginal distribution for each sector. Random variables with different marginal distributions can be easily linked with the copula function. As a result, copula-based models have been well developed and widely used in recent years.

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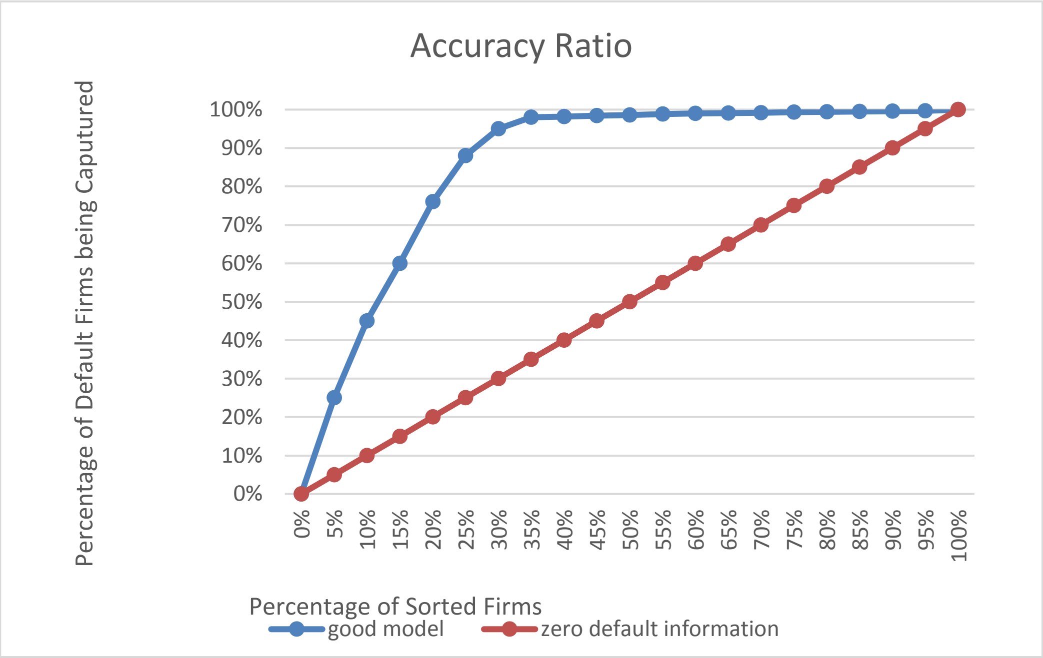

Figure 1.1 CAP of estimated default probability

Notes: Figure1.1 plots the CAP of the estimated default probability. If the model has zero information of the future default probability, for example, the default probability will be ranked randomly for a large number of times, and then $f(\alpha)$ would be equal to α (i.e., α % of the default firms with about α % of the observations). As a result, the CAP curve will correspond to the 45-degree line (the red curve). And a good model will be more like the blue line.

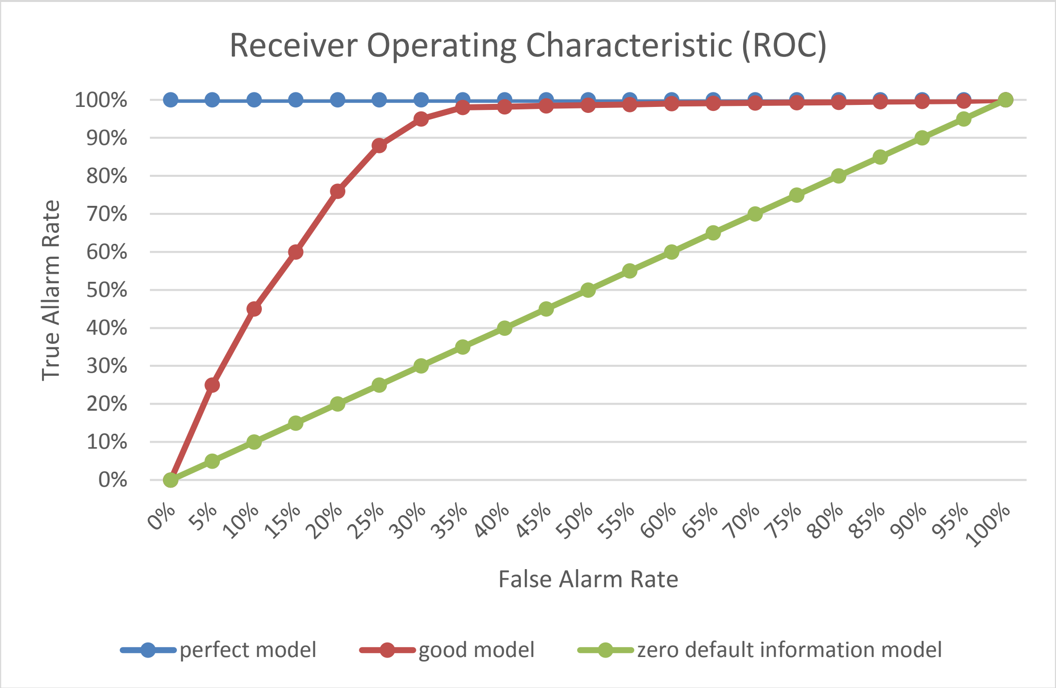

Figure 1.2 ROC of estimated default probability

Notes: Figure1.2 plots the receiver operating characteristic of the estimated default probability. If the model contains no default information, then the ROC curve (the green line) will be corresponds to the 45-degree line, however a perfect model (the blue line) will have a ROC curve that goes straight up from (0, 0) to (0, 1) and then across to (1, 1). And a good model will be more like the red line.

Chapter 2

Revisit to Corporate Default Risk in China

There are few papers on the credit risk for public listed firms due to the lack of default data in China. In this paper, we construct a large default dataset of Chinese public firms from 1999 to 2013 and revisit the corporate credit risk in China. First, the results show that both binomial and multivariate logit models outperform the KMV-Merton structural model. Second, unlike the US market, some traditional determinants of credit risk, like profitability and M/B are not significant in China's market. Third, we find useful economic information for default prediction from the post-default firms which were discarded by previous literature. Fourth, we find the special treatment (ST) and delisted firms which are commonly used in previous literature are not reliable good default proxies for actual default event in China's market. Overall, our study makes a turning point in investigating the corporate credit risk of China's public listed firms.

This chapter is the early version of the working paper "Cerrato, Mario., Kim, Minjoo., Zhang, Bing., and Zhang, Xuan. (2016). Revisit to Corporate Default Risk in China". I would like to thank Tang Shengrui from Huatai Securities for providing the data, and helpful comments from Risk Management Institute of National University of Singapore, and the business school of Nanjing University.

1. Introduction

Most of previous literature study the credit risk of Chinese public firms using a default proxy due to the lack of actual default dataset. Most frequently used proxy is the Special Treatment (hereafter ST)[19], a delist warning sign designated by the regulators (Chen, 2012; Yang ,2010; Zhang et.al, 2010; Huang et.al, 2010; Zeng and Wang, 2013; Peng, 2012). However, they suffer from the suspicion on the reliability of ST as the good proxy for actual default. Orth (2013) uses default ratings (D or SD) from Standard & Poor's for US firms to avoid that limitation, but there are also many problems with this data when it comes to listed companies in China, given that fact that most listed firms in China are not rated by the rating agency. For these reasons, we endeavour to construct actual default data of Chinese public firms and revisit the corporate credit risk using the actual default and bankruptcy data which are obtained from the Credit Research Initiative (CRI) database maintained by Risk Management Institute, National University of Singapore. [20] The CRI database covers credit events from Bloomberg, Compustat, CRSP, Moody's report, exchange websites and news sources.

The most fundamental question is whether ST is the good proxy for the actual default or not. According to the definition of ST, firms being marked as 'ST' mostly face profitability problems. Nonetheless, ST is not directly related to firm's actual default. Thus assessing the credit risk of Chinese public firms using ST is subject to its reliability as the good proxy. To investigate this question, we incorporate ST as a dummy variable into our firm's default model and test how well ST is able to predict the actual default.

[19] With ST or *ST before the stock code. When a stock is marked as *ST, its trading is suspended for one accounting year. More details of ST regulations can be found in ST firm's analysis section.

[20] Thanks to the default data provided by Risk Management Institute (RMI) of National University of Singapore (NUS). The actual default events include: bankruptcy filing-restructuring, bankruptcy-liquidation, bankruptcy-subsidiary bankrupt, bankruptcy-sued by creditor, default corporate action-loan payment, default corporate action-debt restructuring, default corporate-principal payment, and default resolution.

Next research interest directs to accessing the determinants of firm's credit risk with the actual default data. Some previous studies (Carling et al., 2007; Jacobson et al., 2008) show that macroeconomic variables are not important in the U.S. firms. On the contrary, Duan et al. (2012) show that macroeconomic variables such as interest rate and stock index return significantly influence firm's credit risk along with form specific factors like leverage, liquidity, profitability and volatility. Following Duan et al. (2012), in our paper, both level of covariates and the trend of covariates is taken into consideration[21]. For example, consider two firms holding the same amount of cash but with different profiles of cash holding in the past; e.g., the average of cash holding over the last 12 months in our paper. Suppose that one firm's cash holding has been reduced while the other has been increased over the last 12 months. Then, ceteris paribus, two firms would face different default probabilities. By including the average of covariates in the last period, the model could catch the overall trend of firms.

Next, we are interest in firm's behaviour during post-default time. We observe that most default events are not hard default (i.e. firms survive after defaults happened) in our actual default data. For this reason, we prefer to retain these firms to avoid the selection bias. Then we consider firms in three states (i.e., default, post-default or tranquil) rather than a binary state (i.e., default or no default).

We find several new features and implications from the empirical analysis with actual default data. First, when we include the ST dummy variable in the model, it is insignificant. This indicates that ST is not the good predictor of the actual default event. Further, when we use the ST dummy variable as the dependent variable, the estimation results are quite different from those obtained from the actual default data. Both analysis suggests that previous studies using ST may mislead their conclusion due to incorrect default information.

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[21] The ‘‘level’’ denotes the current value, ‘‘trend’’ denotes the difference between current value and its previous 12-month average.

Our analysis challenges previous studies relying on ST as it could contain incorrect default information. Second, we find that the determinants of credit risk of Chinese public firms are quite different from the US firms. The usual default determinants like profitability and market-to-book value do not have the significant influence on the default of Chinese public firms. Another interesting finding is that firms holding more cash are very likely to default. Furthermore, firms have higher default probability when the market is in good conditions such as low interest rate and high stock index returns. This may be caused by the misallocation of finance (see Whited and Zhao, 2016) and the financial constraints faced by small and medium-sized firms. Third, given the fact that most default firms did not exit the market, it's not appropriate to discard the post-default data (i.e. firms with binary states) when modelling the PD. As a result, three states of default event (default, post-default, and tranquil) which containing more information provide the better estimation accuracy than binary states (default and non-default).

Our study makes four contributions to literature and finance policy in China. First, we construct the actual default data of China's public listed firms. This work resolves the limitation of using the default proxy (i.e. ST) for the corporate credit risk research. We believe that our actual default dataset will contribute to further studies on the credit risk problem of Chinese public firms. Second, we find the importance of post-default information in the credit risk analysis of Chinese public firms. Firms show heterogeneous behaviour during the post-default and tranquil time. When the model considers the binary states (default and tranquil), it has the lower power of default prediction than the model considering the three states (default, post-default and tranquil). Third, we extend previous studies greatly by constructing the large actual default dataset. More than 800 active firms and 700 default events are included in our sample, which has never been studied in the previous literature. Lastly, we find that small and medium-sized enterprises (SMEs) suffer from the high default risk in spite of good financial market condition. This finding could draw some implications for policy-makers that they should pay attention to SMEs and push further finance reform.

The remainder of the paper is structured as follows. In section 2, we discuss the previous empirical study which mostly about US market and the credit risk studies in China market. Section 3 first presents a briefly review of credit risk modelling in recent years and then introduces the model for distance to default and for default probability. Section 4 presents the empirical research results of the binomial logit and multinomial logit Model. We further investigate the default probability of ST and delist firms based on the estimation of multinomial logit model. Section 5 makes conclude.

2. Literature Review

2.1 Credit Risk of U.S. Firms

Traditional firm-specific variables for default studies come from the credit scoring models that are proposed by Beaver (1966, 1968) and Altman (1968). They study corporate bankruptcy prediction models that rely on accounting-based variables based on actual manufacturers that filed a bankruptcy petition. Their results show that the liquidity ratio (working capital/total assets) has a positive relationship with firm's survival probability. Firms with high profitability (i.e. retained earnings/total assets ratio and earnings before interest and taxes/total assets ratio in the paper) and the capital-turnover ratio (i.e. sales/total assets) will have lower default probability.

Beside firm-specific variables, the relationship between macroeconomic variables and default risk has been found in many studies. For example, McDonald and Van de Gucht (1999) use quarterly industrial production growth in the US as a covariate to model high-yield bond default. Fons (1991), Blume and Keim (1991), and Jonsson and Fridson (1996) suggest that aggregate default rates tend to increase in the downturn of business cycles. Keenan, Sobehart, and Hamilton (1999) and Helwege and Kleiman (1997) study the default rate of US corporate bonds using many macroeconomic variables which including industrial production, interest rates, aggregate corporate earnings, and indicators for recession that may affect the default rate.

Recent studies on default risk modelling usually estimate the model based on large sample with actual default data. Campbell et al. (2008) proposed a multiple logit model to predict bankruptcy for different time horizons. Their default research is based on the monthly 800 bankruptcy and 1,600 failure data provided by Kamakura Risk Information Services. The monthly data span from 1963 to 1998. Duffie et al. (2007) study the default probability of US industrial firms, based on 2,770 firms that covering 390,000 firm-months of data spanning 1980 to 2004. The model is calibrated by actual default data from Moody's Default Risk Service, which provide detailed issue and issuer information on ration, default or bankruptcy, data and type of default (such as bankruptcy, distressed exchange, or missed interest payment). Duan et al. (2012) propose a forward intensity approach to model the default probability of a much larger sample (comparing to Duffie et al., 2007) of the US industrial and financial firms dating from 1991 to 2011 on a monthly basis. They include 12,268 firms with 1,104,963 firm-month observations in the sample. The default and bankruptcy data are obtained from the Credit Research Initiative (CRI) database maintained by Risk Management Institute, National University of Singapore.

Most firm-specific and macroeconomic variables are found to be significant and with the expected sign in the US market, based on the results from Campbell et al. (2008), Duffie et al. (2007), and Duan et al. (2012). For example, CASH/Total Assets, NI/Total Assets, firm's size, DTD, three-month Treasury rate have negative correlations with firm's default probability. It's natural that US firms holding more liquidity assets, having good earning ability, with big size, and high DTD will have a lower default probability. Interest rate, the market-to-book ratio, and index return are found to have a negative correlation with default intensity.

Their results suggest that the default probability is estimated to be significantly declining in short-term interest rates. Though, firms will bear more interest expenses facing higher rates, the sign of the coefficient for the short rate may because of the fact that the US Federal Reserve often rise short rates in order to “cool down” business expansions.

On the other hand, their results show the negative correlation between stock index return and default probability. It suggests that a bull market would be bad news for firms’ default risk. Duffie et al. (2007) argue that this could be due to correlation between individual stock returns and index returns, and perhaps due to the trailing nature of the returns and business-cycle dynamics.

Campbell et al. (2008) suggest that the market-to-book ratio of bankrupt firms is slightly higher than non-bankrupt firms in US market. Given the fact that, investors often expect the default risk and may result in lowering the market value of equity. Default firms have often experienced losses that have depreciated the book value of their equity, rising up the market-to-book ratio. on the other hand, investors often expect that default risk may lower the market value of the equity. The stock market often anticipates the bankruptcy or future losses, which drives down the market value of equity and the market-to-book ratio.

2.2 Credit Risk of Chinese Firms

Unlike the studies of U.S. firms, the previous studies on the credit risk of Chinese public firms are few and limited. Normally the sample size is small and most research focus on a single sector. Thus, their results are less reliable and hard to be generalized. In addition, they intend to apply the structural KMV-Merton model to calculating a default probability (hereafter PD) due to the lack of actual default data of Chinese public firms. ST firms are used to be default proxy for both model estimation and comparing analysis purpose. Chen (2012) investigates the threshold of default for 4 major Chinese insurance companies; Yang (2010) conducts an empirical analysis for 40 public listed firms; Zhang et al. (2010) study the credit risk of 10 logistics firms during the financial crisis; Zeng and Wang (2013) study the credit risk of 42 manufacturing public listed firms; Peng (2012) studies the credit risk of 11 small and medium size listed firms; All those researches use the KMV-Merton model. Given the limited literature in this area, one of the main contributions of our study is the large actual default dataset that we constructed.

2.3 Credit Risk Models

Previous accounting-based models use composite measures that statistically combine several accounting variables, such as Altman's (1968) Z-Score and Ohlson's (1980) O-Score. Hillegeist et al. (2004) argue that there are several problems when modelling the default probability based on accounting data. For example, the accounting data is rarely informative about firm's future status; fixed assets and intangibles are sometimes undervalued relative to their market prices due to the conservatism principle; and an asset volatility is ignored in accounting-based default prediction models. Although it is obviously easy to implement credit scoring models, they have poorer prediction accuracy compared to other models.

Obviously, it is easy to implement the credit scoring models but they have lower prediction accuracy compared to other models. The structural models (also called the market-based models) provide an alternative and potentially superior source of information compared to the credit scoring models. The information contained in the financial statements may not enough to accurately estimate future default probability. The structural models thus add stock prices, which aggregate information from the market in addition to the financial statements, into the models. However, the main concern is that the structural model may not perform well due to the inclusion of market prices which may lower the estimation accuracy for the market value of the asset. Another shortcoming is that the stock price may not efficiently include all publicly-available default-related information. In addition, Sloan (1966) argues that the market does not accurately include all of the information in financial statements, while the structure model estimation is based on stock price. Thus it is ultimately an empirical question if the market-based default models or the accounting based default models can achieve more accurate estimates (Hillegeist et al., 2004).

Most recent papers apply a reduced-form model (logit model or the Poisson model) with some exogenous variables which are assumed to capture traditional risk factors and DTD as determinants of firm's default probability. For example, Chava and Jarrow (2004), Hillegeist et al., (2004), Shumway (2001) estimate a discrete-time hazard model with yearly data and directly predict one-year default probabilities. Chava and Jarrow (2004), and Campbell et al. (2008) estimate default probability using the logistic regression models. Duffie, et al (2007), Duffie et al. (2009) and Duan et al. (2012) develop a Poisson intensity approach with common risk factors and firm-specific attributes to estimate corporate defaults.

Though previous research about China's public firms mainly applies KMV model (i.e. this model is widely used due to the lack of default data), having the actual default data, reduced-form model which could incorporate credit scoring factors and DTD is our first choice[22]. Since firms in our sample is in two state: default or non-default, the logit model seems to fit well the characteristics of our dataset well. As a result, the binomial multivariate logit model is the benchmark model in our paper. But given the fact that most firms in China's market will survive after a default (41 firms delisted in our sample period, but 656

[22] The reduced-form model can also incorporate credit scoring factors and DTD.

defaults happened in total), the binomial model which discards the post-default data will lose considerable useful information. In our paper, we use a multinomial logit model that allows the dependent variable to take three states: 1) the default (default); 2) 12 months after default has taken place (post-default); 3) the no-default regime (tranquil), which includes the remaining observations.

3. Methodology

Duffie and Lando (2001) argue that a distance to default (DTD) is a crucial covariate for the default estimation. However, if DTD cannot be measured accurately or firm's financial health may have multiple influences over time from firm-specific, cross-sector and macroeconomic risk factors, adding other covariates would reveal additional information about the default probability of the firm. The structural models still face the difficulties in estimating firm's market value or incorporating the credit rating which reveals firm's financial health. As a result, DTD is an important variable but not the only one in our default estimation. In the following, by applying multinomial logit model, we consider both firm-specific and macroeconomic variables.

3.1 Distance-to-Default

Previous research suggest that the market-based model is very effective in assessing the credit risk for China's public listed firms. So the distance-to-default is one of the key variables in our model.

The Merton's model applies the European call option developed by Black and Scholes (1973). The structural models consider that the default occurs if the asset value falls below a threshold associated with the firm's liabilities; that is, the default will occur if the asset value, i.e. the firm value, is not enough to cover the firm's liabilities. When the asset value falls behind the liabilities, which means the value of equity is zero, and there is no reason of holding equity. Thus the firm cannot fully cover creditors' claims, thereby the firm is in default.

In the structural form model, the stock price follows an independently log normal distribution with annual volatility σ and the following parameters:

$$\mathrm{InA_t} \sim \mathrm{N}\left(\mathrm{InA_t} + \left(\mu - \frac{\sigma^2}{2}\right)(\mathrm{T}-\mathrm{t}), \sigma^2(\mathrm{T}-\mathrm{t})\right), \tag{2.1}$$

The probability of default of $\mathrm{InA_t}$ falls below the threshold L will be given by $\emptyset(\frac{\mathrm{L}-\mathrm{E(x)}}{\sigma(\mathrm{x})})$, and $\emptyset$ is the cumulative standard normal distribution.

If the asset value falls below the liability value, the value of equity will be zero, the payoff to equity holders described as:

$$\mathrm{E_t} = \max(0, \mathrm{A_T} - \mathrm{L}), \tag{2.2}$$

where the liabilities value is: $L = short - term\ debt + 0.5 * long - term\ debt + \delta * other\ liabilities$. The haircut δ is estimated by the transformed-data MLE method. When $\delta = 0$, the model collapses to traditional KMV model. Duan and Wang's (2012) results suggest that when considering other liabilities, the impact is big for financial firms but for other firms like industrial firms only a minor difference from KMV method. Generally speaking, the haircut δ is relatively small for non-financial firms and the size of other liabilities is much smaller relative to its market capitalization.

The equity value can be calculated using the standard Black–Scholes European call option formula:

$$\mathrm{E_{t=}A_t} \cdot \emptyset(\mathrm{d_1}) - \mathrm{Le^{-r(T-t)}}\emptyset(\mathrm{d_2}), \tag{2.3}$$

where:

$$d_1 = \frac{\ln\left(\frac{A_t}{L}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \text{ And } d_2 = d_1 - \sigma\sqrt{T-t}$$

After values of L, A_t μ,and σ^2 are calculated, the probability of default can be measured:

$$\Pr(\text{Default}) = \emptyset\left(\frac{\text{InL}-\text{LnA}_t-\left(\mu-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right), \tag{2.4}$$

While three the unknown parameters will be estimated, which are μ (i.e. the annual expected asset returns), σ (i.e. the annual asset volatility) and A_t (i.e. market value of assets at time t).

The distance to default (DTD) which is the deviation of asset value from the default point L will be calculated:

$$\text{DTD} = \frac{\text{InA}_t+\left(\mu-\frac{\sigma^2}{2}\right)(T-t)-\text{InL}}{\sigma\sqrt{T-t}}, \tag{2.5}$$

The DTD estimated from RMI follows an alternative form that could reduce sampling errors:

$$\text{DTD} = \frac{\text{InA}_t-\text{InL}}{\sigma\sqrt{T-t}}. \tag{2.6}$$

3.2 Multinomial Logit Model

A multinomial logit model is used to estimate firm’s default probability by considering default, post-default, and tranquil states. This makes our study substantially different from the others. As explanatory variables, both firm-specific and macroeconomic variables are included.

For each firm $i = 1, \dots, n$ can be one of three states at each month: tranquil period ($j = 0$), first month of default event ($j = 1$) and 12 months after the default event ($j = 2$). The probability that a firms is in state *j* is:

Firm i would stay at the one of ternary states at the month t:

$$y_{i,t} = \begin{cases} 0 & no-default\ period \\ 1 & default \\ 2 & post-deault \end{cases} \quad (2.7)$$

where the firm is defined as the post-default until 12 months[23] after the default. The probability that the firm stays at the state *j* conditional on the feasible information set is given by

$$P\left(y_{i,t} = j \middle| \boldsymbol{x}_{i,t}\right) = \frac{exp\left(\boldsymbol{\beta'}_j \boldsymbol{x}_{i,t}\right)}{1 + \sum_{h=1}^{2} exp\left(\boldsymbol{\beta'}_h \boldsymbol{x}_{i,t}\right)}, \qquad j = 0,1,2 \quad (2.8)$$

where $\boldsymbol{x}_{i,t}$ is the vector of unit, macroeconomic variables and frim-specific factors.

The multinomial model usually adopts $\boldsymbol{\beta}_0 = \boldsymbol{0}$ to remove an indeterminacy in the model. Therefore, the probabilities are

$$P\left(y_{i,t} = j \middle| \boldsymbol{x}_{i,t}\right) = \frac{exp\left(\boldsymbol{\beta'}_j \boldsymbol{x}_{i,t}\right)}{1 + \sum_{h=1}^{2} exp\left(\boldsymbol{\beta'}_h \boldsymbol{x}_{i,t}\right)}, \qquad j = 1,2 \quad (2.9)$$

$$\Pr\left(y_{i,t} = 0 \middle| \boldsymbol{x}_{i,t}\right) = \frac{1}{1 + \sum_{h=1}^{2} exp\left(\boldsymbol{\beta'}_h \boldsymbol{x}_{i,t}\right)}. \quad (2.10)$$

However, the partial effect of multinomial model is complicated. For continuous $x_{i,k,t}$ which is the *k*th element of $\boldsymbol{x}_{i,t}$, we can write

$$\frac{\partial\, P\left(y_{i,t} = j \middle| \boldsymbol{x}_{i,t}\right)}{\partial x_{i,k,t}} = P\left(y_{i,t} = j|\boldsymbol{x}_{i,t}\right)\left[\beta_{j,k} - \sum_{h=1}^{2} \beta_{h,k} P\left(y_{i,t} = h|\boldsymbol{x}_{i,t}\right)\right], \quad (2.11)$$

[23] We test of difference in mean between post-default and tranquil firms considering the post default time from 1 months to 12 months. The 12 months' post default is the most significant one.

$$\frac{\partial P(y_{i,t} = 0|\boldsymbol{x}_{i,t})}{\partial x_{i,k,t}} = -P(y_{i,t} = 0|\boldsymbol{x}_{i,t}) \sum_{h=1}^{2} \beta_{h,k} P(y_{i,t} = h|\boldsymbol{x}_{i,t}). \qquad (2.12)$$

where $\beta_{j,k}$ is the kth element of $\boldsymbol{\beta}_j$. Alternatively, we often use the log-odds for a simple interpretation of $\beta_{j,k}$:

$$log\left[\frac{P(y_{i,t} = j|\boldsymbol{x}_{i,t})}{P(y_{i,t} = 0|\boldsymbol{x}_{i,t})}\right] = \boldsymbol{\beta}'_j \boldsymbol{x}_{i,t}, \qquad (2.13)$$

$$log\left[\frac{P(y_{i,t} = j|\boldsymbol{x}_{i,t})}{P(y_{i,t} = h|\boldsymbol{x}_{i,t})}\right] = (\boldsymbol{\beta}_j - \boldsymbol{\beta}_h)' \boldsymbol{x}_{i,t}. \qquad (2.14)$$

To estimate the coefficients in the equation (2), we maximize the log likelihood

$$L(\boldsymbol{\beta}) = \sum_{i=1}^{T} \sum_{i=1}^{n} \sum_{j=0}^{2} logP(y_{i,t} = j|\boldsymbol{x}_{i,t}; \boldsymbol{\beta}_j) \qquad (2.15)$$

where $s_{i,j,t} = 1$ if firm i is at the state j at the time t. As usual, we estimate parameters by maximizing the log likelihood (2.15). McFadden (1974) shows that the equation (2.15) is globally concave; thereby the maximization problem is straightforward.

4. Empirical Analysis

4.1 Firm-specific Data

Our dataset contains 981 public listed firms from China's *Shanghai Exchange A Share* which includes 940 currently active public listed firms along with 41 delisted firms during the time period of 1998-2013. For the firm-specific variables, we use quarterly accounting data collected from the WIND database. Individual stock prices, interest rates and stock index are collected from the Bloomberg. Data such as merger and acquisition (hereafter M&A) are hand collected from the Shanghai exchange official database. Default events are provided by Risk Management Institute (RMI) of National University of Singapore (NUS). The default events of China's public listed firms in our sample include: bankruptcy filing-restructuring, bankruptcy-liquidation, bankruptcy-subsidiary bankrupt, bankruptcy-sued by creditor, default corporate action-loan payment, default corporate action-debt restructuring, default corporate-principal payment, and default resolution. DTD data are also from RMI, NUS where the calculation method follows Duan and Wang (2012). Instead of using KMV method (i.e. commonly used in literature of modelling China's listed firms), they introduce 'other liability' fit high-leverage firms such as banks and insurance firms better. While traditional KMV models will cause serious distortion to the credit analysis of financial firms, as a result, most papers exclude financial firms in their sample.

We use common factors and firm-specific factors which are most used in the previous literature like Duffie et al. (2007) and Duan et al. (2012). We find that both trend and level of some firm-specific variables (e.g. CASH/TA and DTD) is significant of determining the default risk. Duan et al. (2012) find out that for two firms with the same DTD are very likely to have different default likelihoods if one firm's DTD has been decreasing in the past few months. The variables used in our study are the followings:

1. Index return: Shanghai A-share index return.
2. Interest rates: Three months trading treasury rates.
3. Cash: Cash to total assets ratio. It measures the liquidity in terms of immediately available cash by firms.
4. Cash trend: The difference between current liquidity and its previous 12-month average
5. Net income: Net income to total asset ratio. It measures firm's profitability.

6. Net income trend: The difference between current net income and its previous 12-month average.
7. MB: The market-to-book value ratio.
8. MB trend: The difference between current MB and its previous 12-month average.
9. Firm size: The logarithm of firm's market equity value to the market index value.
10. Distance-to-default (DTD): Most studies calculate DTD using the KMV formula (Merton, 1974) in which the default point is set by the sum of short-term debt and half of long-term debt. We however adopt Duan and Wang's (2012) method considering firm's other liabilities to the calculation of default threshold.
11. DTD trend: The difference between current DTD and its previous 12-month average.
12. Dummy variable ST[24]: firms once marked as ST will be '1' and exit ST will be '0'.

4.2 Default data

[INSERT FIGURE 2.1 ABOUT HERE]

Figure 2.1 plots the monthly actual default events from 1999 to 2013. The maximum number of defaults is 19 and it's around the year of 2004. Most default events happened from 2001 to 2011.

[INSERT TABLE 2.1 ABOUT HERE]

Table 2.1 reports the total number of active firms and defaults each year. Through the sample, the average active firm is 872 and the size of active firms is expanding year by year from 666 to 947. On average, the default events are 44 corresponding to 5% default rate. The default rate increases between 1999 and 2005 and decreases afterwards. Default rates are

[24] ST policy starts from 22/04/1998.

extremely high, 12.01% in 2004 and 10.28% in 2005. The high default rates are mainly derived by two major facts: the policy-bankruptcy was approaching its deadline and the bankruptcy of many state owned enterprise (SOEs) had spill-over effect during this period[25]. According to People.cn[26] (May 31, 2004), the State Council approved the merger and bankruptcy of the 725 SOEs, and the central government allocated total bankruptcy subsidies amounted 62.7 billion yuan. In 2004, the central government spent 19.947 billion yuan to implement the policy-bankruptcy, and a total of 537.5 thousand people were laid off. In 2005, the central government spent 22 billion yuan on the 115 state-owned enterprises to implement the policy-bankruptcy, more than 590 thousand employees were involved. After the peak of 2004, the default rates started to decline (less than 4% in the last 4 years).

[INSERT TABLE 2.2 ABOUT HERE]

Table 2.2 reports the descriptive statistics of our selected variables. The average of cash is 0.5268, suggesting that firms hold a good amount of liquid assets. The negative cash trend indicates that firms possess less liquid assets compared to the past cash holdings. In terms of profitability, the average of net income is 0.0193 but its trend is negative, indicating that firms are less profitable than the past. The average of MB is 3.7665 and its standard deviation is high. Firm size does not vary a lot: the average is 0.0466, the maximum is 0.0586 and the minimum is 0.0409. The average of DTD is 4.3048 implying a low default risk. The lowest DTD is negative (-0.7085) which indicates a definite default by the KMV structure model.

[INSERT TABLE 2.3 ABOUT HERE]

[25] Policy bankruptcy (the policy code: Guo Fa No. 199459) is part of SOEs reform in China. It started from October, 1994 and ended by 2008. It was a policy carried out by the central government to force SOEs to dissolve since SOEs with bad performance could not easily go bankruptcy.

[26] People.cn is a large-scale news platform built by People's Daily, one of the top ten newspapers in the world.

Table 2.3 shows the average values of independent variables. In our paper, firms are within three states: default, post-default and tranquil. Traditionally, when assessing the credit risk, only default and non-default are taken into consideration, and all the post-default data will be discarded. It means a lot of useful information is lost especially in our case where most firms have survived after the default. For better modelling the credit risk of China's public firms, it's better to include post-default data but also distinguish the data from tranquil time to avoid bias problem. As a result, we further study the default year data and tranquil year data, and the test shows a significant difference in mean value between these two states across all independent variables.

[INSERT TABLE 2.4 ABOUT HERE]

The table 2.4 is correlation matrix of independent variables. Most variables are not highly correlated with each other. The cash has little correlations with most variables but is positively correlated with the DTD trend (0.101), indicating that the firm holding more cash is more likely to have a growing default probability; i.e. there is a big difference between current DTD and its previous values over the past 12 months. The trend value of cash is positively correlated with DTD (0.228). This suggests that firms with higher liquid assets may have a higher default probability. Both the level and trend of the net income and MB have very low correlations with both the level and trend of DTD. This suggests that these two traditional firm-specific variables may not be relevant with for measuring the default risk of public listed firms in China.

4.3 Empirical results

4.3.1 Model selection

Since public listed firms in China are more likely to survive after a default event, we cannot ignore the post-default event. For this reason, the multinomial model should fit our data better than the binomial logit model.

We estimate the multinomial model with macroeconomic variables and firm-specific factors. For comparison, we also estimate the binomial logit model. The pseudo-R^2 and ROC both suggest that multinomial model outperformance binominal model. For the binominal model, the P pseudo-R^2 is 0.0182 while for the multinomial model, has a much larger pseudo-R^2 that is 0.0419.

In addition, we compare the two models using the receiver operating characteristic (hereafter ROC). The ROC is constructed by varying threshold probability. The ROC curve is plotted by the true positive rate (percentage of default firms that the model correctly classifies as defaults) on the y-axis as a function of the corresponding false positive rate (percentage of non-default firms that are mistakenly classified as defaults) on the x-axis. If the model has zero information of the default probability, for example, an entirely random prediction model, the ROC curve will correspond to the 45-degree line. While a perfect model will have a curve that goes straight up from (0,0) to (0,1) and then across to (1,1). In the following, we report the AUC (the area under the curve for the computed values of X and Y) of the two models. The maximum AUC is 1, which corresponds to a perfect default classifier. Larger AUC values indicate better classifier performance.

[INSERT FIGURE 2.2 ABOUT HERE]

Figure 2.2 shows the ROC curve for the three models. The solid line (multinomial) outperformance the dash curve (binominal). The multinomial model is above the binomial logit model in most of the range. We also calculate the AUCs of these models. The AUC of multinomial model (0.6794) is bigger than that of the binomial logit model (0.6263). Therefore, the multinomial model with three states outperform the binomial logit model with binary states; that is, the post-default data contains useful information for predicting the default event.

We also investigate how much we are able to improve the performance by using the actual default data. Figure 2.2 also plots the ROC of KMV-Merton's structural model (the dot line). It is below the ROCs of the two logit models and almost overlaps with the 45-degree line. Its AUC is 0.5062, which is much smaller than any of the two logit models. Therefore, the logit model with the actual default data dominantly outperforms the KMV-Merton's structural model with the market data.

Overall, the results suggest that we should use the multinomial model with the actual default data to avoid the loss of information from ignoring the post-default events. The estimations result of multinomial logit model and binominal logit model can be found in table 2.5 and table 2.6 respectively.

[INSERT TABLE 2.5 ABOUT HERE]

[INSERT TABLE 2.6 ABOUT HERE]

As a result, in the following, the paper analyse the credit risk based on the multinomial logit model. The 'default' column denotes the relative default probability in the default state compared to the default probability in the benchmark state, 'tranquil'. Analogously, the 'post-default' column denotes the relative default probability in the post-default state compared to the default probability in the 'tranquil' state.

From table 2.5, the default month column shows that interest rate, index return, cash/total asset current and trend value (from binomial model outputs in table 2.6, cash/total asset current value is not significant), firm's size, DTD both current and trend value are significant of assessing the credit risk of China's public firms. Not surprisingly, large firms are less likely to default and firms with larger DTD and DTD trend are safer.

There are two interesting findings. First, firms face higher default risk when they hold more liquid assets (cash). A possible scenario is that firms have neither good investment opportunities nor intention to expand their business in the time of distress; thereby they hold more cash in a bad financial condition. Another possible scenario is that the increase of cash flow risk is connected to the increase of idiosyncratic risk[27]. Therefore, if the idiosyncratic risk declines, firms may eventually reduce their cash holdings. The third scenario is that, with regard to default type, this result indicates that the most default events in China's market were triggered by insolvency problems (i.e. when firm's the value of assets is less than the value of its liability) but not liquidity problems (i.e. not enough cash to make payment). In addition, previous studies indicate that distance-to-default (leveraged adjusted market value of asset) is the most powerful factor determining the default risk. So when firms have less financing constraints (i.e. access external financing more easily), the impact of illiquidity will be weak.

Second, firms face higher default risk when the interest rate is low. In China, new or non-state-owned enterprises usually experience unreasonable loan covenants since they do not have well-established relationships with banks. García-Herrero et al. (2009) point out that to some extent, Chinese banks are subject to government intervention. In many cases the credit provided by banks is directly or indirectly controlled by the central and/or the local governments and banks are not free to set interest rates. Therefore, the good relationships with banks are actually advantageous to enjoy lower interest rate in China. People's Bank of

[27] Bates et al. (2009) point out that this may explain why US firms are hold more cash and the average cash ratio peaked in 2004 in their sample.

China also often raises interest rates to cool down business expansion. Hence, it is not surprising that firm's default probability is low in the high interest rate regime.

Our results show that the coefficient on the index return is positive at 5% significance level; that is, a bull market is not a good news for managing firm's default risk. The finding is in line with the results of US firms. Duan et al. (2012) suggest that this result may be due to the correlation between the index return and firm-specific variables. In addition, Duffie et al. (2009) pointed out that DTD could overstate firm's financial health after a long increase of the stock market; thus there is the positive relationship.

In terms of firm size, Buera et al. (2011) find that large scale industries such as a manufacturing industry have more external financial dependence than small scale industries such as a service industry. This abnormality could be caused by credit constraints in China. Chan et al. (2012) find that large firms face no credit constraints while small firms have significant credit constraints. Asymmetric information and 'political pecking order' are key in China, and large state-controlled banks dominate the Chinese financial system. For these reasons, small firms have difficulties to access the credit market; thereby they are more dependent on internal funds for investments.

To sum up, our empirical results suggest that investors and regulators should be more cautious to potential default events in China. First, 'Too Big to Fail' also applies to the Chinese financial market; especially, small firms are more likely to default than large firms. Second, firms holding more cash will have lower liquidity risk, but their default risk could be higher due to the high idiosyncratic risk or slow potential growth. Third, firms with smaller DTD show that the market value of their assets is low and are more likely to default.

Looking at the empirical results (both 'default' and 'post-default'), only ST shows inconsistent results. In the default state, ST is not significant, confirming that ST may not be directly related to actual default. It becomes significant in the post-default state but the negative sign indicates that ST firms are less likely to enter the post-default state. This is in line with the previous results showing that ST is not associated with the actual default event.

In the following, we calculate default probability of individual firms using parameters estimated by the multinomial model. First, we look at the overall default probability (PD) of China's public firms.

[INSERT FIGURE 2.3 ABOUT HERE]

Figure 2.3 shows the overall default probability of China's public listed firms from 2001 to 2014. Overall, China's public listed firms have relatively high default risk during early 2000's and the PD shows a clear downtrend after the year of 2006. The default probability is increasing since 2001 and peaking around 2006. The PD keeps decreasing afterwards and rise up again right after 2008 (i.e. when a global financial crisis happens) to 2009. Then PD is declining after the year of 2009.

[INSERT FIGURE 2.4 ABOUT HERE]

Then we further look at the default probability of firms in default, post-default months and tranquil months. Figure 2.4 shows the default probability of China's public firms in default months (red line), post default year (green line) and tranquil months (dash line) respectively. In most times, tranquil months have the lowest PD and post-default months are slightly higher. Not surprisingly, the default months get highest PD. The huge gap is found around the year of 2007 when the whole market was turning better but more default events actually happened at that time, this is consistent with our model results, which suggests that more defaults are expected when the market is in good condition.

4.3.2 ST Firms Analysis

In this section, we further investigate ST[28] firms to see the reliability of ST as a proxy for the default event. Especially, we use ST as a dependent variable in the binomial logit model.

Instead of complying with market-based policies, the regulatory framework for public listed firms in China is more administrative. To keep the good financial condition of listed firms in the stock exchange and also to protect investors' interests, the Chinese Securities Regulation Commission (CSRC) designs the 'Special Treatment (ST)' regulation for firms that are in risk to be delisted. Once a firm is identified as the 'ST' firm by regulators, a designation of 'ST' will be placed in the beginning of its stock ticker. The 'ST' is regarded as a warning signal to investors in the market. The firm will be on probationary status and the change of stock pricing will be limited to +-5% daily (i.e. once reach the caps, the stock's trading will be suspended for the rest of the day).

The firms being designated of Special Treatment for different reasons, but mainly are bad performance according to the financial statement and auditor decisions (being ST because of a negative audit option). According to the regulations of the Shanghai stock exchange, a firm will be designated as ST firm in the following 4 reasons:

1. A firm has negative earnings for two consecutive years;
2. The shareholder equity is lower than registered capital in the last fiscal year;
3. Auditors issue negative opinions about the firm;
4. A firm has stopped or intends to cease operations or business activities for at least three months.

[28] If not specified, ST represent both ST and *ST firms.

ST mechanism is mainly used as a warning sign for investors and sometimes helped to protect investors from 'government control' Pistor and Xu (2005) propose that the ST delisting mechanism can be used as a substitute to restrain mismanagement of firms. The ST delisting will lead to the lower quota for future IPOs from that region. Though ST is designed to protect investors from the danger of delisting, ST may not be a good warning sign for underperforming firms and be used as a default proxy to study credit risk. Jiang and Wang (2008) suggest that the ST delisting mechanism is not perfect because firms will intend to engage in earnings manipulation to avoid accounting losses. And on the hand, some viable firms will be delisted due to temporarily poor performance. In addition, the ST designation is only a warning signal to investors without revealing more useful information to the public. In China's stock market, lots of mergers and acquisitions happen within ST firms. Firms are looking for reverse merger opportunities from ST firms to help them skip the long waiting list of IPO. Based on 66 ST firms' data from 1998-2000, Bai et al. (2004) find that more than half of the largest shareholders of these ST firms are changed. And 36% of these firms changed their main business. In terms of firm's performance, surprisingly, the 66 ST firms outperformed the market by 31.8% after 24 months being designated by ST. A firm will be designated by ST anytime through the year when bad news is discovered and regulators identify problems of the firm, but another problem of this ST mechanism is that due to the criteria relating to firms' annual financial statement, ST or *ST decisions will be made during April and May and a cluster ST will be observed from the market.

In our database, we have the 940 active firms from *Shanghai A Shares*. There are 216 firms which are 22.98% of all the A Shares firms are marked or used to be marked as ST firms. And the first *ST firm appears in 2003 in the sample. Furthermore, 182 firms have been marked as '*ST' between 2003 and 2013. There are two periods when an increasing number of new firms were marked '*ST': 26 firms in 2003 and 62 firms between 2006 and 2007.

[INSERT FIGURE 2.5 ABOUT HERE]

From figure 2.5, it shows the ST rate (ST firms / active firms) increases since 2000 and peaks around 2008 with 10%. After 2012, the ST rate starts to fall and back to the same level at the year of 2004. Apparently, the convex shape of the ST rate is not well fit with the default probability lines of default months and post-default months in figure 2.4. Also, the ST rate shares no common trend with actual default events from figure 2.1. From the data, it seems ST rate and default rate differs greatly and ST may not be a good default proxy.

[INSERT TABLE 2.7 ABOUT HERE]

Before comparing the credit risk between ST firms and other firms, we estimate the binomial logit model using ST as a dependent variable. From table 2.7, the results show some clear difference from the model with actual default data and support our discussions above. Firstly, unlike actual default firms, the ST firms are not affected by the market condition. The index return has a positive sign but not significant due to a large standard error. Secondly, cash is insignificant while the net income trend becomes a crucial factor. The positive coefficient on the net income trend indicates that the ST firms are more likely to have an increasing profit compared to the previous 12 months. The profit of ST firms is increasing but it should stay low or even negative before the firms eventually marked with ST by regulators. Thirdly, DTD seems to be less important compared to the multinomial model with actual default data. Only DTD is significant at 10% while both level and trend of DTD are highly significant at 1% in the multinomial model.

[INSERT FIGURE 2.6 ABOUT HERE]

[INSERT FIGURE 2.7 ABOUT HERE]

Figure 2.6 plots the median default probability for ST and non-ST firms. The results estimated from our model show that PD of ST firms is much higher than PD of the non-ST firm during the early 2000s. Hugh default spread is found from 2002 to 2006 from figure 2.6. It suggests that ST firms being designated by ST during the early 2000s are mainly because of insolvency problems and at that times ST could be a good default proxy. But after 2006, the PDs of ST firms and non-ST firms seem to move together. And from figure 2.7, even non-ST firms have higher default probability than ST firms around the year of 2006, 2007 and 2009.

Overall, the empirical results confirm that ST is not a reliable default proxy for the actual default event.

4.3.3 Delist Firms Analysis

The variable *ST contains more relevant information than ST and we use it as a warning signal that firms will be delisted by regulators. One could consider delisted firms as default proxy, but it is not clear if firms are delisted due to only an insolvency problem. If a firm is delisted due to the insolvency (default) problem, its default probability should be higher than other firms. On the other hand, if it is delisted due to other problems ('merger and acquisition' (M&A) or 'privatization'), its default probability would be similar to or even lower than the other firms.

Our sample contains 41 delisted firms[29]. We find that 19 firms exited the market due to M&A and 4 firms were marked with ST (one of the four is *ST) out of 19 firms. The other 22 firms exited due to insolvency. Out of the 22 firms, 21 were marked with ST (18 were marked with *ST). Most delisted ST firms suffer insolvency problems, but this is a small part of the total (216 ST firms in our sample). Thus, our sample shows that delisted firms

[29] 40 firms with PD data in the following analysis.

would not be the good proxy for default events. It shows that there is a significant change of median default probability before and after 2007.

[INSERT FIGURE 2.8 ABOUT HERE]

Figure 2.8 shows predicted default probabilities of individual firms using the parameters estimated by the multinomial logit mode. We report the median default probability of three different groups (default, delist or active firms) for each month.

First, delisted firms show a higher default probability than default firms before 2007. We find that all the 21 delisted firms exited due to insolvency problems during this period. After 2007, delisted firms show a lower default probability than default firms. This is because all the 19 delisted firms exited due to M&A during this period. Second, delisted firms show a higher default probability than active firms before 2007. Since all the delisted firms exited due to insolvency problems, this result is expected. After 2007, delisted firms show a lower default probability than active firms. This is because they exited due to M&A.

Overall, the results suggest that default risk of delisted firms will be subject to the reasons of being delisted (i.e. insolvency problems or M&A) from the exchange. Furthermore, this is not a complete proxy for the default events.

5. Conclusion

The empirical literature on default of public listed firms in China is very limited. This paper addresses this limitation and constructs a unique default dataset for public listed firms in China between 1999 and 2013. This is a significant contribution on corporate credit risk study in China.

We explore the determinants of firms' default using both firm-specific and macroeconomic covariates. Since most firms in China survive after a default, we consider a model with ternary states (default, post-default and tranquil). The test statistic shows a significant difference between firm-specific data during post-default and tranquil times.

We compare the multinomial logit model with the binomial logit model. Both the pseudo-R^2 and ROC (Receiver Operating Characteristic) suggest that the multinomial logit model outperforms the binominal logit model; that is, the multinomial logit model shows better prediction accuracy than the binomial logit model. The estimation results also show that both binomial and multinomial logit models outperform the KMV-Merton structural model.

Our results are different from the results presented for the US firms. Some traditional determinants like profitability and market-to-book value are insignificant in China. In addition, unlike the US firms, firms holding more liquid assets are more likely to default. This is possibly because small and medium size firms cannot obtain sufficient credit during distress times. Holding more cash may indicate that they are uncertain about the future or that they do not have good investment opportunities.

In addition to this, we also find that the spread of default probability between ST firms and non-ST firms is larger before 2006 but it narrows afterwards. This result supports our view that ST may not be a reliable default proxy. We also investigate if delisted firms can be used as a proxy for the default event. The results show that high default probabilities could cause a delisting but not vice-versa; i.e. the default event is not the unique reason for delisting a firm.

Our results have noticeable policy implications: Firstly, regulators in China should focus more on the financial health of small-medium-enterprises (SMEs) during periods when the market is not distressed because SMEs are more likely to default when the stock market performs well and the interest rates are low. Secondly, the ST mechanism is used to be very effective as a default warning sign to investors for the public listed firms in the early period, but it has been less effective and the criteria should be updated now. The spread of default probability between ST and non-ST firms has become smaller in recent years. Finally, given the fact that SMEs outperform the large state-owned-enterprises (SIEs) (Luo and Park, 2001; Park et al., 2006) but they face more financing constraints than SOEs, more significant reforms in terms of financial liberalization should be implemented.

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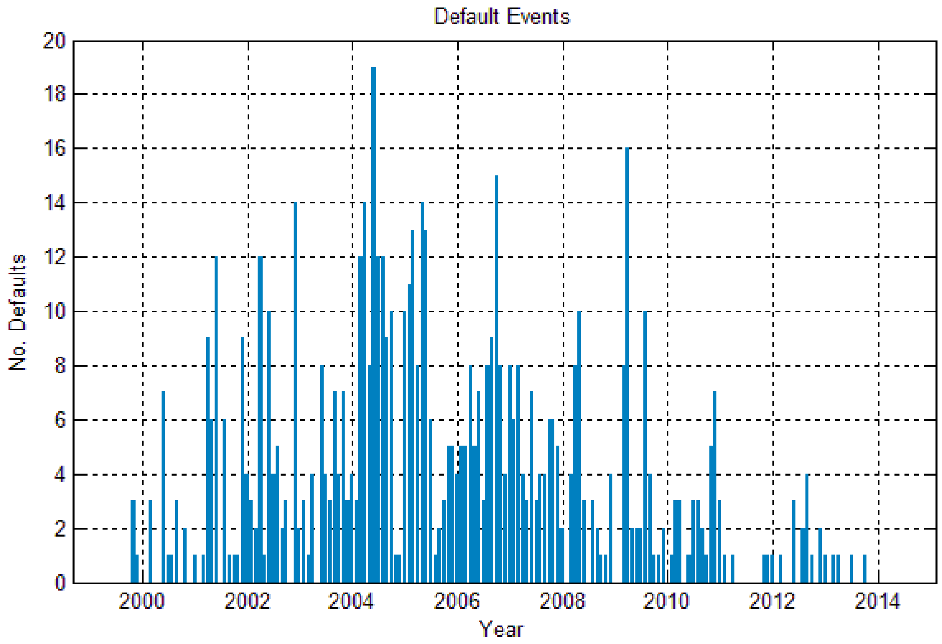
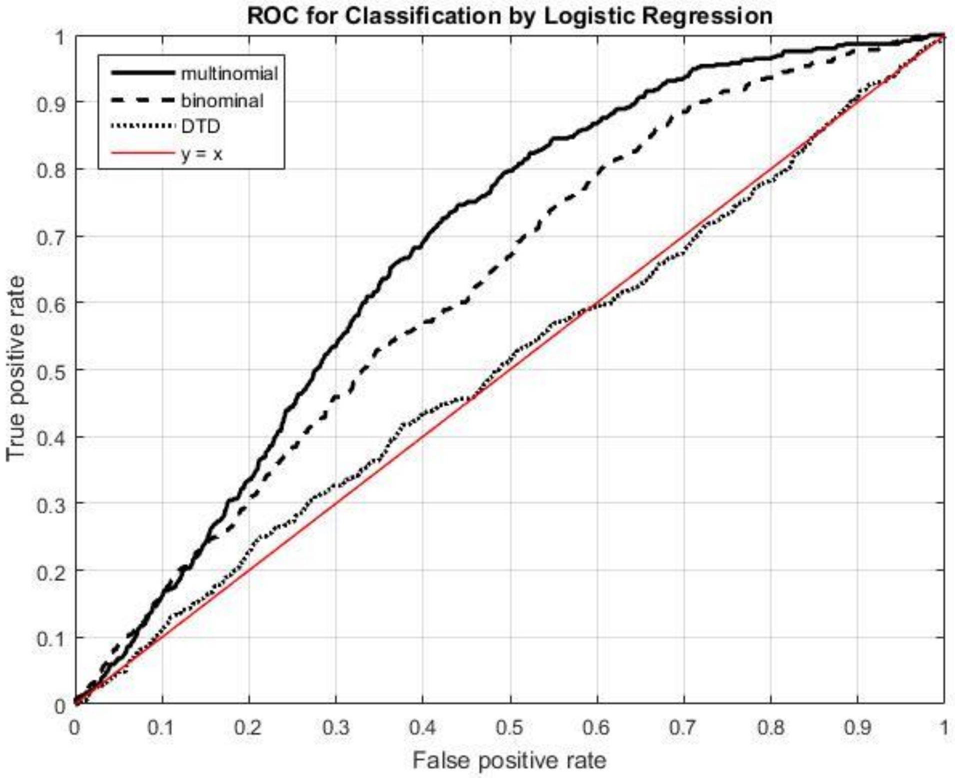


Figure 2.1 Default events by month

Notes: this figure plots the number of actual default events in each month over the sample period 1999 to 2013.

Figure 2.2 ROC of three models



Note: this figure plots the ROCs for three default risk models: the multinomial model (solid), the binomial logit model (dash) and the KMV-Merton's structural model (dot). We estimate three models using the full sample. If the model has zero information, ROC corresponds to the 45-degree line (y=x). On the other hand, the perfect model would have ROC that goes straight up from (0,0) to (0,1) and then across to (1,1).

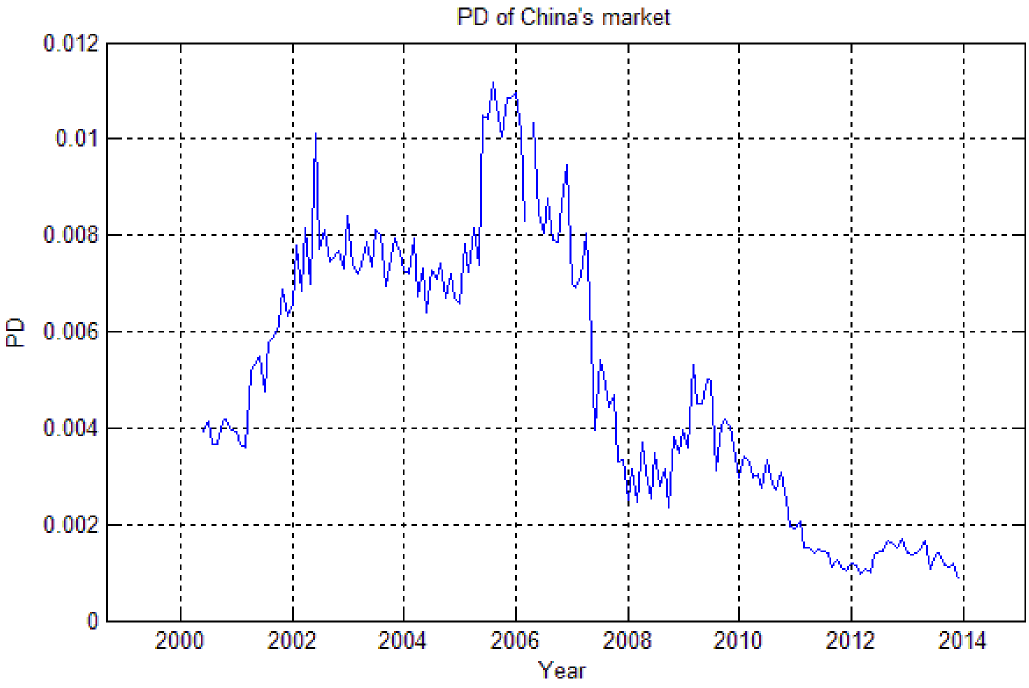


Figure 2.3 Median default probability of active firms

Note: this figures plots the median default probability of active firms from 2000 to 2013. We predict the default probability of individual firm based on the parameters estimated by the multinomial model, and take the median default probability of all active firms for each month.

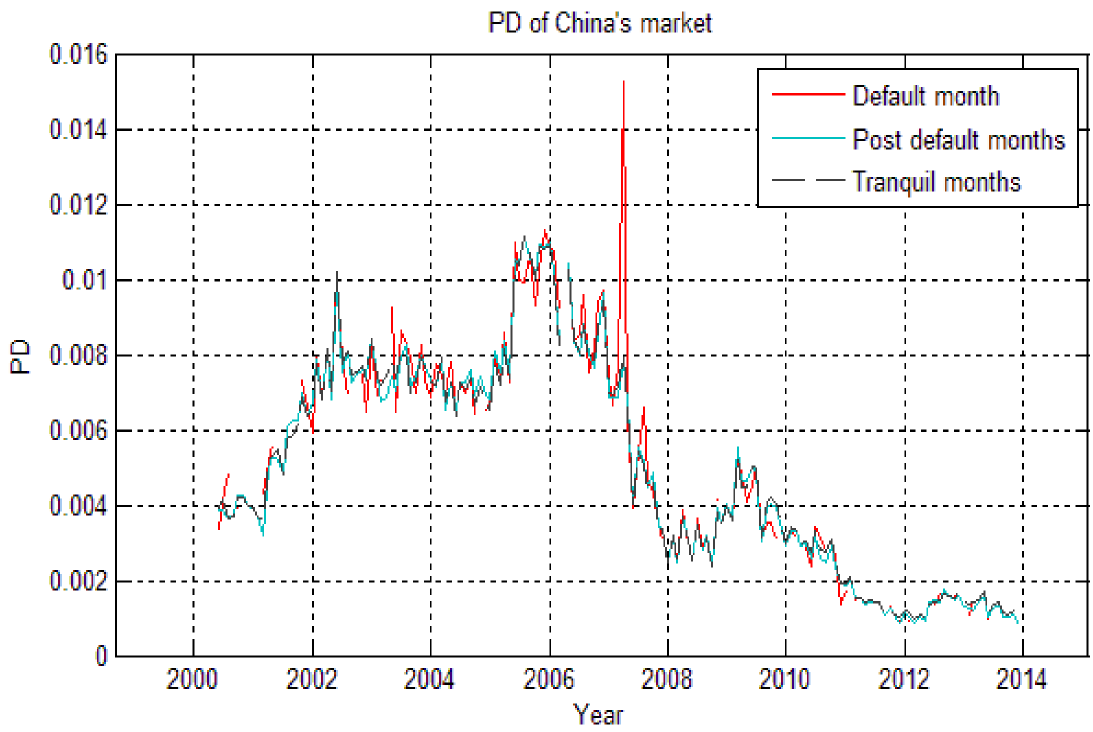


Figure 2.4 Median default probability of active firms in different states

Note: this figure plots the median default probability in the three different states from 2000 to 2013: Default (red), post-default (green) and tranquil (dot black). We first predict the default probability of individual firm based on the parameters estimated by the multinomial model. Then we take the median default probability given a state for each month.

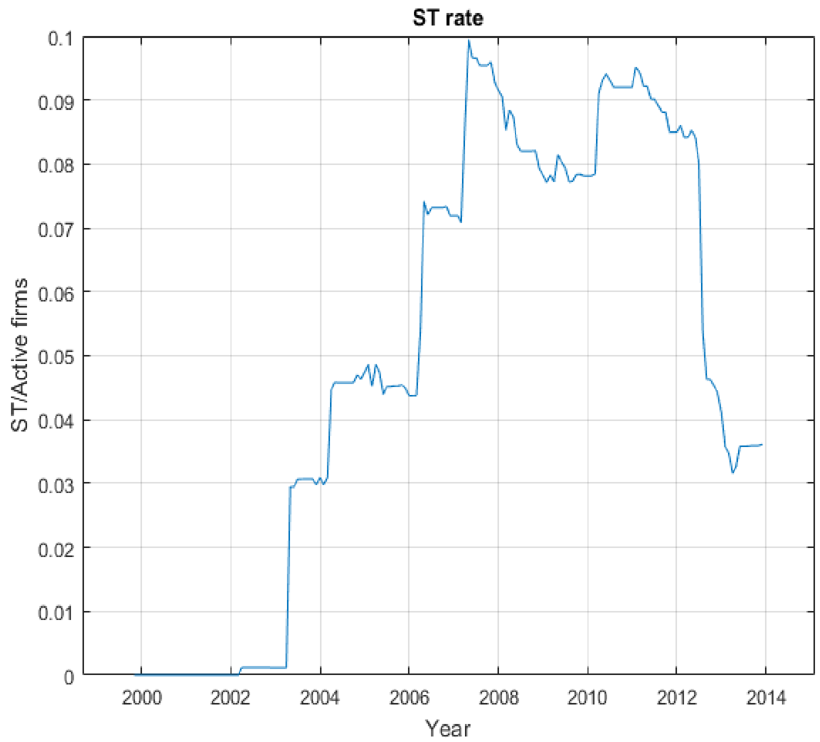


Figure 2.5 ST rate at each month (ST firms / active firms)

Note: this figure plots the ST rate at each month from 1999 to 2014. The ST rate is calculated by number of ST firms divided by the number of active firms.

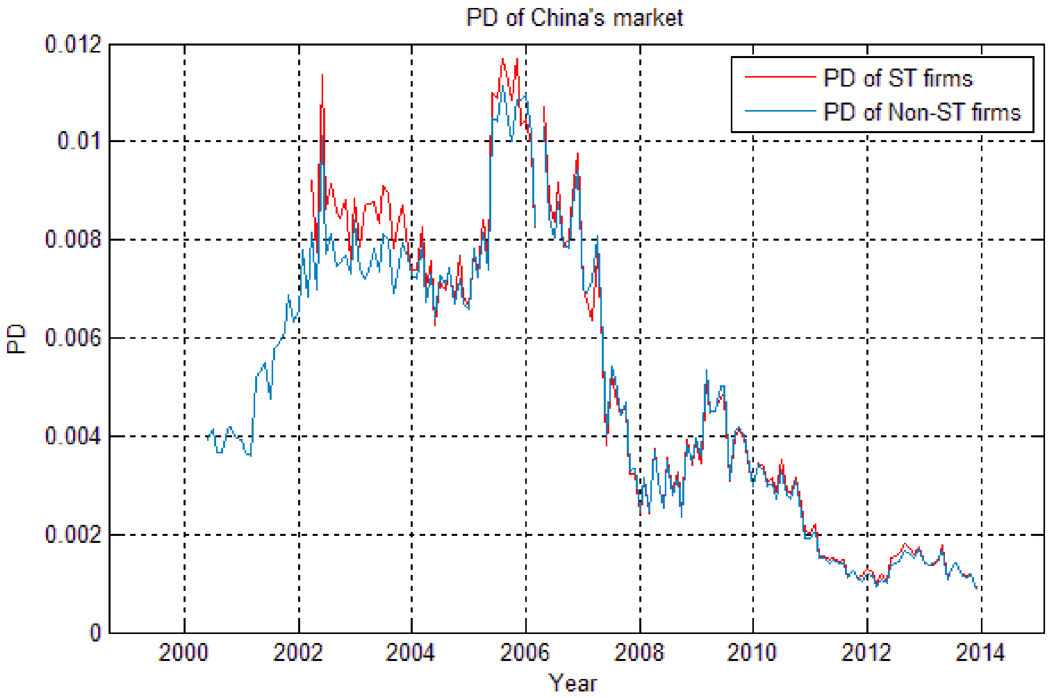


Figure 2.6 Default probability (median) of ST and Non-ST firms

Note: this figure plots the median default probability of ST (red) and non-ST (blue) firms. We first predict the default probability of individual firm based on the parameters estimated by the multinomial model. Then we take the median default probability given a state for each month.

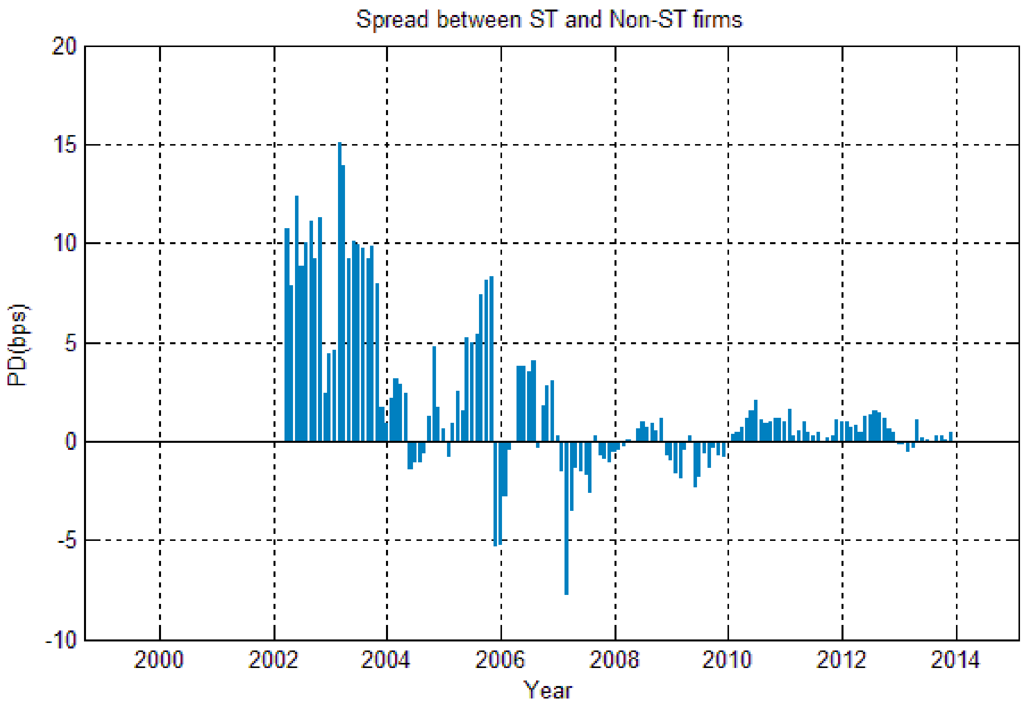


Figure 2.7 PD spread between ST and Non-ST firms

Note: this figure plots the default probability spread between ST firms and non-ST firms by using the median default probability. We first predict the default probability of individual firm based on the parameters estimated by the multinomial model. Then we take the median default probability given a state for each month.

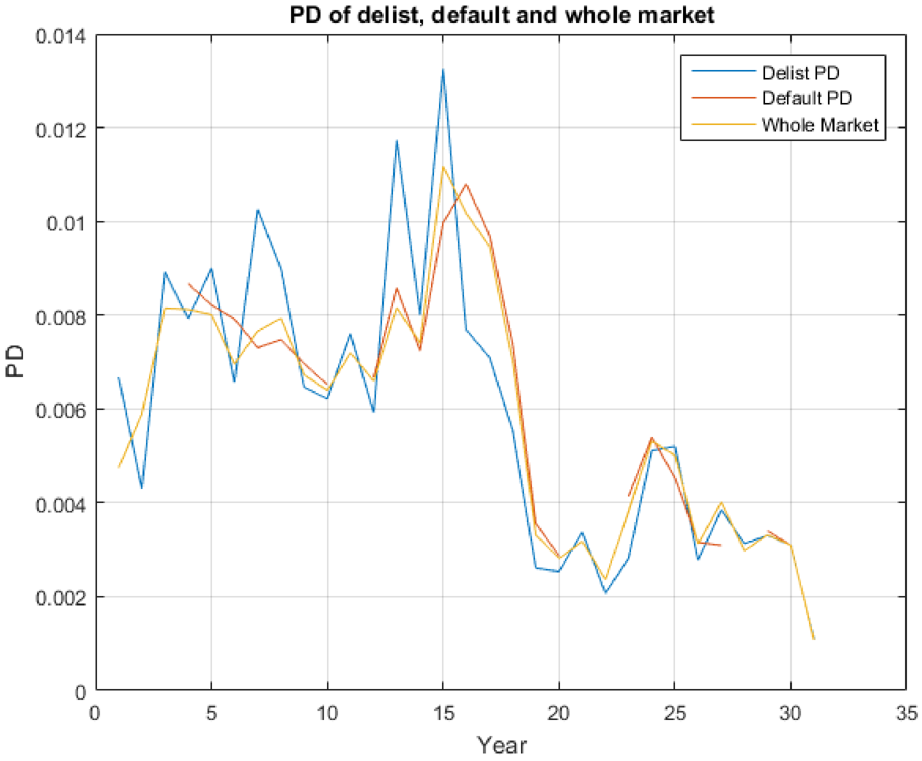


Figure 2.8 PD of delist, default and whole market

Note: this figure plots the median default probability in the three different states from 2000 to 2013: Default (orange), delist (blue) and active (yellow). We first predict the default probability of individual firm based on the parameters estimated by the multinomial model. Then we take the median default probability given a state for each month.

Table 2.1 Default rate

Year	Active firms	Defaults	Default rate
1999	666	4	0.60%
2000	707	17	2.40%
2001	780	47	6.02%
2002	831	60	7.22%
2003	850	46	5.41%
2004	874	105	12.01%
2005	885	91	10.28%
2006	890	81	9.10%
2007	904	64	7.08%
2008	930	38	4.09%
2009	958	48	5.01%
2010	957	29	3.03%
2011	954	7	0.73%
2012	951	14	1.47%
2013	947	5	0.53%

Note: This table reports the number of active firm and the number of default for each year over the sample period 1999-2013. The number of active firm is calculated by averaging the number of active firms across all months given the year.

Table 2.2 Summary Statistics

	Max	Min	Mean	Median	Std.
CASH/TA	1.0000	0.0022	0.5268	0.5333	0.2129
CASH/TAtrend	0.7220	-0.8776	-0.0661	-0.0348	0.3282
NI/TA	2.4627	-16.1121	0.0193	0.0159	0.1319
NI/TAtrend	4.0777	-2.1403	-0.0159	-0.0150	0.0653
M/B	1912.1912	-981.9954	3.7665	2.6607	21.8181
M/Btrend	5620.4129	-1365.5878	-0.6681	-0.2099	29.8502
Size	0.0586	0.0409	0.0466	0.0464	0.0024
DTD	31.1876	-0.7085	4.3048	3.9119	2.1451
DTDtrend	7.9164	-22.0075	-2.3970	-2.1945	2.2793

Note: This table reports the maximum (Max), minimum (Min), average (Mean), median (Med) and standard deviation (Std.) of firm-specific variables including cash/total asset (Cash), net income/total asset (Net income), market value/book value (MB), log of total asset (Firm size), distance to default (DTD). All variables excepting Firm size also include their trend value which is the difference between current value and its previous 12-month average.

Table 2.3 Average of independent variables

Variable	Full sample	Default month	12 months after default + tranquil times	12 months after default (1)	Tranquil times (2)	Test for difference in mean (1) vs (2)
No. of Observations	95826	453	95373	5717	89656	
CASH/TA	0.5268	0.5248	0.5268	0.5203	0.5272	2.36**
CASH/TAtrend	-0.0661	0.0789	-0.0668	0.0592	-0.0748	-30.07***
NI/TA	0.0193	0.0183	0.0193	0.0159	0.0195	2.00**
NI/TAtrend	-0.0159	-0.0137	-0.0159	-0.0132	-0.0161	-3.32***
M/B	3.7665	3.0296	3.7700	3.1804	3.8076	2.10**
M/Btrend	-0.6681	0.6137	-0.6742	0.2587	-0.7336	-2.43**
Size	0.0466	0.0468	0.0466	0.0467	0.0466	-2.52**
DTD	4.3048	4.3565	4.3045	4.1865	4.3121	4.29***
DTDtrend	-2.3970	-2.7639	-2.3953	-2.6143	-2.3813	7.50***

***Denotes significance at 1%.

**Denotes significance at 5%.

*Denotes significance at 10%.

Note: this table displays the average values of independent variables which including cash/total asset (current and trend value), net income/total asset (current and trend value), market value/book value (current and trend value), firm's size, distance to default (current and trend value) in full sample, default months, post default months and tranquil months respectively. The last column presents the test for difference in mean from post-default and tranquil months.

Table 2.4 Correlation Matrix

	CASH/TA	CASH/TA trend	NI/TA	NI/TA trend	M/B	M/B trend	Size	DTD	DTD trend
CASH/TA	1.000	-0.624	0.013	-0.012	0.005	-0.002	-0.084	-0.099	0.101
$CASH/TA_{trend}$	-0.624	1.000	-0.018	0.006	-0.009	0.022	0.323	0.228	-0.210
NI/TA	0.013	-0.018	1.000	-0.516	0.003	0.003	0.090	0.083	-0.065
NI/TA_{trend}	-0.012	0.006	-0.516	1.000	0.006	-0.001	-0.195	-0.180	0.139
M/B	0.005	-0.009	0.003	0.006	1.000	-0.211	0.006	0.039	0.008
M/B_{trend}	-0.002	0.022	0.003	-0.001	-0.211	1.000	0.009	-0.023	0.011
Size	-0.084	0.323	0.090	-0.195	0.006	0.009	1.000	0.267	-0.109
DTD	-0.099	0.228	0.083	-0.180	0.039	-0.023	0.267	1.000	-0.630
DTD_{trend}	0.101	-0.210	-0.065	0.139	0.008	0.011	-0.109	-0.630	1.000

Note: this table shows the correlation matrix of cash/total asset (current and trend value), net income/total asset (current and trend value), market value/book value (current and trend value), firm's size, distance to default (current and trend value) in full sample

Table 2.5 The multinomial logit model

The multinomial logit model estimated is a discrete dependent variable taking value 0,1 and 2 for tranquil, default month and 12 months after default respectively.

$$\Pr(Y_{it} = 1,2) = \alpha + \beta_1 Interest\ Rate + \beta_2 Index\ Return + \beta_3\, CASH/TA + \beta_4\, CASH/TA\, trend + \beta_5\, NI/TA + \beta_6\, NI/TA\ \ trend + \beta_7\, M/B + \beta_8\, M/B\ \ trend + \beta_9 Size + \beta_{10} DTD + \beta_{11} DTD\ trend + \beta_{12} ST.$$

Variables	Default month	12 months after default month
Constant	-3.0126***	0.1473
	(1.1481)	(0.3317)
Interest Rate	-0.2549***	-0.1648***
	(0.0585)	(0.0161)
Index Return	1.4406**	0.1734
	(0.6129)	(0.1717)
CASH/TA	2.0389***	1.6979***
	(0.3336)	(0.0956)
CASH/TA trend	2.2586***	2.0651***
	(0.2705)	(0.0767)
NI/TA	0.3654	0.0445
	(0.7561)	(0.1232)
NI/TA trend	0.1378	-0.0792
	(0.6976)	(0.2223)
M/B	-0.0009	-0.0004
	(0.0024)	(0.0007)
M/B trend	0.0006	0.0004
	(0.0008)	(0.0003)
Size	-51.4603**	-62.8342***
	(24.7862)	(7.1722)
DTD	-0.1019***	-0.1236***
	(0.0304)	(0.0091)
DTD trend	-0.0804***	-0.0676***
	(0.0271)	(0.0081)
ST	-0.0732	-0.1570**
	(0.2654)	(0.0771)
Number of observation	95,826	

***Denotes significance at 1%.

**Denotes significance at 5%.

*Denotes significance at 10%.

Table 2.6 The binomial logit model (post default data excluded)

The binomial logit model estimated is a discrete dependent variable taking value 0 and 1 for tranquil and default month respectively.

$$\Pr(Y_{it} = 1) = \alpha + \beta_1 Interest\ Rate + \beta_2 Index\ Return + \beta_3\ CASH/TA + \beta_4\ CASH/TA\ trend + \beta_5\ NI/TA + \beta_6\ NI/TA\ \ trend + \beta_7\ M/B + \beta_8\ M/B\ \ trend + \beta_9 Size + \beta_{10} DTD + \beta_{11} DTD\ trend + \beta_{12} ST.$$

Variables	
Constant	0.5558
	(1.2038)
Interest Rate	-0.2371***
	(0.0595)
Index Return	1.8421***
	(0.6241)
CASH/TA	0.3133
	(0.3359)
CASH/TA trend	0.5676**
	(0.2721)
NI/TA	1.0687
	(1.1890)
NI/TA trend	-0.2496
	(0.8756)
M/B	0.0001
	(0.0021)
M/B trend	0.0046
	(0.0032)
Size	-91.5022***
	(25.9405)
DTD	-0.1482***
	(0.0304)
DTD trend	-0.0982***
	(0.0263)
ST	-0.1624
	(0.2736)
Number of observation	44,497

***Denotes significance at 1%.

**Denotes significance at 5%.

*Denotes significance at 10%.

Table 2.7 The binomial logit model (post ST data excluded)

The binomial logit model estimated is a discrete dependent variable taking value 0 and 1 for tranquil and ST month respectively.

$\Pr(Y_{it} = 1) = \alpha + \beta_1 Interest\ Rate + \beta_2 Index\ Return + \beta_3\, CASH/TA + \beta_4\, CASH/TA\, trend + \beta_5\, NI/TA + \beta_6\, NI/TA\ \ trend + \beta_7\, M/B + \beta_8\, M/B\ \ trend + \beta_9 Size + \beta_{10} DTD + \beta_{11} DTD\ trend.$

Variables	
Constant	16.886
	(3.1597)
Interest Rate	-0.2329**
	(0.1044)
Index Return	1.1912
	(1.1815)
CASH/TA	-0.5982
	(0.7047)
CASH/TA trend	1.6498***
	(0.5478)
NI/TA	0.1727
	(0.1950)
NI/TA trend	1.3599***
	(0.3171)
M/B	0.0018
	(0.0027)
M/B trend	-0.0082
	(0.0064)
Size	-478.4383***
	(71.4416)
DTD	-0.1723*
	(0.0883)
DTD trend	0.0711
	(0.0767)
Number of observation	87,327

***Denotes significance at 1%.

**Denotes significance at 5%.

*Denotes significance at 10%.

Chapter 3

Dynamic Default Correlations in Financial System

In recent years, financial institutions have become increasingly closely interconnected in many ways. The systemic risk among these financial entities is a great concern for both central banks and investors. This paper studies the joint default risk between insurance firms and other financial institutions. Basing our research on publicly listed firms in US market, this paper finds dynamic symmetric default correlations between these two sectors in different time horizons by using student's t copula with the generalized autoregressive score model (GAS). Economic drivers are found for short-term and long-term correlations. In addition, we find the joint default risk between insurance firms and financial institutions are very sensitive to global financial market events.

I would like to thank my friend Yang Zhao for his help in learning copula theory and workshop discussants at the school of economics, Zhejiang University, the business school of Nanjing University, and the University of Glasgow for helpful comments.

1. Introduction

Recently, the systemic risk within financial industry is becoming a hot topic and attracts great attentions of both investors and regulators. Great concerns about the systematic risk among financial institutions have increased after the global financial crisis and EU sovereign debt crisis. Financial institutions like insurance and banks (including other small credit unions) are normally supervised by regulators separately. For regulators and policymakers, the recent financial crisis evokes their interest in systemic risk within different financial institutions.

Across different institutions, insurance firms are the special ones, it is not only because they provide insurance for other financial institutions, but also they are regarded the most stable ones. Insurance firms, like banks, are playing a more and more important role in the financial system. Janina and Gregor (2015) argue that insurance firms are becoming more similar to banks and contribute to the systemic risk of the financial system. Additionally, the study by Geneva Association (2010) indicates that the insurance industry would raise the systemic risk of the financial sector if insurers heavily engage in derivatives trading which is off balance sheet or they mismanaged short-term financing activities. In the 2008 financial crisis, after the failure of Bear Stearns and Lehman Brothers, the collapse and near-failure of insurance American International Group (AIG) was a major event in the recent financial crisis. AIG lost 99.2 billion US dollar and the Federal Reserve Bank of New York provide an 85 billion US dollar to save it. Not surprisingly, AIG is engaged with financial derivatives, the credit default swap which is the financial instrument that links all entities together causes AIG losing 30 billion US dollar.

Recent studies suggest that the financial institutions are becoming more and more closely correlated in recent years by strong business ties between them. First, the linkage between insurance firms and banks cannot be ignored and Insurance firms play a crucial role in the financial markets. For example, Main (1982) shows that banks can benefit from good correlation with insurance firms. By working with insurance firms, banks can avoid some bad debts. This is because the information of obligors provided by insurance firms helps banks to understand individual and corporate risk exposures better. In addition to that, by mitigating the losses during natural disasters, insurance firms help to reduce the probability of default of some investors (policyholders of insurance firms, both individual and corporate) who, sometimes, are the same obligors of banks (Lee C.-C. et al, 2016). Lehmann and Hofmann (2010) show that banks are more likely to transfer part or all of their risks to other financial sectors like insurance firms to avoid high correlation among assets and to reduce default probability. Trichet (2005) shows that insurance industry and banking industry is linked by ownership and the associations of credit exposure. As a result, insurance firms are playing a central role in the financial markets and this may directly affect banks and other financial institutions. Second, from the perspective of insurance firms, Billio et al. (2012) suggest that in recent years, insurers have become more likely to invest into a non-core business like insuring financial products, credit-default swaps, derivatives trading, and investments management. And this new business for insurance firms competes directly with financial institutions. Traditionally, insurance firms provide credit to hedge funds and hedge funds buy principal protections on their funds from insurers. But now insurance firms have their own proprietary trading desks and hedge funds offer insurers capital-market-intermediated insurance like catastrophe bonds.

Previous papers show that insurance firms and other financial institutions (referred as ‘financial firms’ hereinafter) are highly interconnected in many ways. Insurance firms are acting more like financial firms and they are becoming more and more closely correlated. The systemic risk within the financial system arises from the highly interconnected institutions which have business relationships and exposed to common factors so that one firm’s failure may cause a series firms’ failure. This paper investigates the correlated default risk between insurance and financial sectors based on US stock market. In our paper, we include firms from the financial category of Bloomberg which include banks, insurance firms, broker/dealers, hedge funds etc. We study the question that whether there are time-varying default correlations between insurance and financial sectors. We propose the dynamic copula model to capture the connectedness between insurance and financial sectors. In addition, will the dynamic default correlations vary with the default term structures is discussed. At last, we try to find out whether the time-varying correlations come from common factors or the interactions between firms by regressing the correlations with macroeconomic factors. At last, the simulated joint default probability clearly shows the co-movement of insurance and financial sectors in different time horizons.

In this paper, we use the individual default probability of public listed financial firms in US market from CRI database Risk Management Institute, National University of Singapore. Our results show there are time-varying symmetric correlations between insurance and financial sectors in different time horizons (1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month in our paper). Then we investigate the correlations between insurance firms and financial institutions using t copula with the generalized autoregressive score model (GAS) with skewed t and empirical distribution function (EDF) marginal respectively. The skewed t marginal distribution outperformance the EDF, so we study the copula correlations based on skewed t marginal. The copula correlations vary widely in different time horizons, and the only negative correlation is found in the 1-month horizon. Then we regress the copula correlations with economic drivers and find that the adjust R^2 is low which indicating some latent factors such as the interactive between firms contribute the joint movement greatly. At last, we simulate the joint default risk of insurance and financial sectors.

Our empirical study has the following contributions. Firstly, we find the existence of dynamic and symmetric dependence between insurance and financial firms in both short-term and long-term horizons in the US market. Billio et al. (2012) study the correlation between hedge fund, banks, brokers, and insurance firms based on principal components analysis and Granger-causality tests. Our study of term-structure of the correlations are new and interesting. The results show that the correlations vary widely in different time horizons. In general, short-term correlations are low but volatile while long-term correlations are high but stable. Secondly, we find economic drivers of the movement of insurance and financial sectors in different horizons by regressing the copula correlations with macroeconomic variables. Clear differences are found between short-term and long-term default correlations. Our results suggest that short-term correlations (from 1-month to 12-month horizon) are driven by economic factors like credit index, VIX, interest rate level, yield curve slope, TED spread, crude oil, and index return. Long-term correlations (from 24-month to 60-month) are less sensitive to the market conditions. Thirdly, by using the correlations estimated from the student's t GAS copula, we can simulate the joint default probability for the two sectors. Comparing to existing literature, we extend the systemic risk study within financial sector. While previous study focus on systemic risk among different counties (Lucas et al., 2014) or different banks (Chan et al., 2016). We first construct the joint PD of insurance and other financial firms across all time horizons which show a similar moving trend in our 20 years' time period. The longer time horizon is, the higher joint default probability can be found. In addition, we find the joint PD between these two sectors are very sensitive to global financial market events.

The remainder of the paper is structured as follows. In section 2, we briefly review the development of individual default risk modelling and correlated default risk modelling. Section 3 is the methodology, we present the modelling of individual firm's default probability, the way to model the marginal distribution of PD, the Student's t copula with

GAS model, and the joint default probability calculation. Section 4 presents the summary statistics of the data. Section 5 is the dependence analysis which includes the tests of asymmetric dependence and the time-varying dependence. Section 6 presents our empirical results of estimated time-varying copula correlations, the results of regression with economic drivers, and the joint default probability across different time horizons. Section 7 makes a conclusion.

2. Literature review

2.1 Individual Firm's Default Risk

Default risk modelling has a fast developing in recent years from credit scoring models to structural models and reduced form models now. Beaver (1966, 1968) and Altman (1968) first propose scoring models that calculate firm's default probability by using accounting-based variables. The structural model first used by Merton (1974) by applying option theory to derivate the value of a firm's liabilities in the presence of default, the probability of default is embedded in the option-pricing models and in his paper Merton only considering one single type debt of the firm. The firm's market value of assets can be derived by viewing the firm as a call option. When firm's assets' value falls below a threshold which calculated by the structure of liabilities of the firm, the firm will be considered as default. There are several issues in credit scoring and structural models above. When modelling probability of default based on accounting data, the estimate of probability of default is aimed to measure an event in the future, but financial statements are designed to capture past performance of the firm so as a result, the accounting data may not have a strong prediction power about the future status of the firm. Also, Hillegeist et al (2004) find that due to the conservatism principle, fixed assets and intangibles are sometimes undervalued relative to their market prices which will cause accounting-based leverage measures to be overstated. As for the structural model, the value of firm's assets is estimated by market prices. However, market prices may not all publicly-available default-related information of the firm. Also, the term structure, other liabilities, and off-balance liabilities are not well specified in structural models when calculating default threshold of the firm, when may lead to an inaccurate estimation of default probability.

As a result, this paper uses individual PDs from RMI-NUS database where PD is modelled by reduced-form model. Reduced-form models are becoming very popular in recent years for individual firm's probability default estimation. Jarrow and Turnbull (1995) first introduce the reduced form model and developed by Duffie and Singleton (1999). The reduced-form model assumes exogenous Poisson random variables drive the default probability of the firm. A firm will default when the exogenous variables shift from their levels. The stochastic process in the models does not directly link to firm's assets' value which makes the models more tractable. Duffie, et al. (2007) first propose doubly stochastic Poisson model with time-varying covariates and then forecasting the evolution of covariates using Gaussian panel vector autoregressions. The model is further developed by Duan, et al (2012) which applies the pseudo-likelihood to derive the forward intensity rate of the doubly stochastic Poisson processes with different time horizon.

2.2 Correlated Default Risk

A good measure of correlated default risk between insurance and financial institutions is crucial for credit risk management given the fact that insurers and banks are engaged more closely in many ways. Default risk could spread quickly during financial firms through credit derivatives like CDS and CDOs. For example, Jorion & Zhang (2007) and Stulz (2010) find the counterparty risk is transferred through financial products like the CDS protection seller will be exposed to higher default risk caused by the protection buyer's other counterparties.

Das et al (2006) point out that both default correlations and default probabilities vary through the different economic environment. Time-series variation can be observed in default probabilities and the series show high volatility when to shift from the different economic environment. For example, default probabilities are more than double in high default risk time than low default risk time. These are also well captured by the correlated default probability curves from our model in different time horizons. Merton's structural model indicates that both firm volatility and debt are significant in the movement of default probability.

Dembo et al. (2004) find out that macroeconomic variables are significant in explaining the default probabilities. Other studies show these common risk factors such as GDP growth rate, market volatility, and interest rates have a direct effect on joint default risk. Duffie et al. (2007) suggest that common risk factors are not the only reason of correlated default risk. Both contagion factor and latent frailty factors play a significant role in explaining joint default risk. Here the contagion effect means one firm default will cause the change of default risk of other firms. And the frailty factors are the remaining sources of the joint default risk which are not explained by the common factors and contagion factors.

Recent papers model the systemic risk between banks and other financial institutions are more or less relying on the stock price. For example, Demirer et al. (2015) use stock returns volatility, Acharya et al. (2012) model financial entities' capital shortfall, and Adrian and Brunnermeier (2014) apply a CoVar method. Chan et al. (2016) suggest that these constructions imply the risk indirectly comparing to model the correlation by PD. Stock prices may not contain all the information of firm's potential failure while the probability of default is the likelihood that a firm fail to pay its obligations in the future. The high PD indicates a firm is more likely to default over certain horizons and it may trigger a series default events in the system through varies contagion channels. As a result, in our paper, instead of stock price, we use individual PD of firm directly. Before using copula to estimate correlated default probability of insurance and finance industries, individual default probability needs to be modelled. This paper uses individual PDs from RMI-NUS database where PD is modelled by the reduced-form model proposed by Duan et al. (2012).

The modelling of correlated default risk is well developed. Traditional reduced-form model of portfolio applies a bottom-up model which portfolio intensity is an aggregate of individual intensities or a top-down model which portfolio intensity is calculated without including individual intensities. Schönbucher (2003) points out that the drawback of intensity-based models is the computational complexities may be very time-consuming. The copula-based models have some natural advantages because of the mathematical properties in correlated default modelling. The copula allows the great flexibility of choosing a marginal distribution for each sector. Random variables with different marginal distributions can be easily linked with the copula function. In our case, when modelling the joint default risk, the marginal distributions of firm's default probability can be transformed to the portfolio default probability by the copula. Gaussian copula was first introduced to default risk to simulate the dependence structure of the times until default by Li (2000). And this Gaussian assumption is further studied by Andersen and Sidenius, 2004; Giesecke, 2004; Hull and White, 2004; Glasserman and Li, 2005; Das et al., 2007. Since there is no tail dependence of the Gaussian copula, but empirical data always shows 'fat tails' in their distribution, while obviously Gaussian copula cannot capture the extreme increase in default probability when the correlated default happens. An empirical study by Wolfgang et al. (2003) turns out student t copula is better than Gaussian copula. Other papers such as Meneguzzo and Vecchiato (2004) find that the Student's t copula well capture the tail dependence between different underlying assets when pricing the CDO and basket CDS. Their results show that the Student's t copula outperformance Gaussian copula. In addition, Christoffersen et al. (2016) study dependence in corporate credit and equity returns for 215 firms and find that the default dependence is non-Gaussian and highly variable. As a result, in this paper, Student t copula is used to calculate the joint default probability. Traditional way uses copula theory to estimate the parameters of risk factors in credit risk management. This paper applies time-varying t copula to capture varying correlation between two sectors.

The linkage between insurance and financial sectors is highly dynamic, especially as both sectors are changing greatly over the last decade. The time-varying correlation results from connectedness among financial institutions' holdings and the sensitive to market condition. The modelling of this dynamic default correlation is playing a very important role in estimating joint default probability. This dynamic dependence is captured by the GAS model, which is pioneered by Creal et al. (2013). Unlike parameter-driven models like the stochastic volatility model and stochastic intensity models (Shephard, 2005; Bauwens and Hautsch, 2006; Koopman et al., 2008), the GAS model is an observation-driven model based on the score function. The time variation of the parameter is driven by the scaled score. The GAS model is widely used in recent empirical research, for example, Creal et al. (2014a) forecast macro, credit, and loss given default risk conditions for U.S. Moody's-rated firms. Janus et al (2014) develop a model for volatility and time-varying dependence in daily financial return series that are subject to long memory dynamics and heavy-tailed densities. Lucas et al. (2014) study the joint and conditional sovereign default risk in EU from observed CDS prices. Salvatierra and Patton (2015) develop new dynamic copula models for daily asset returns to extract information from high-frequency data.

3. Methodology

In this paper, there are four steps to estimate the correlated default risk between insurance and financial sectors. First, the individual default probability for each firm in the insurance and financial sectors obtained from RMI-NUS database which is calculated following Duan et al. (2012) model should be standardized. Second, we construct a time series of sector's PD by using the median value of all active firms' PDs within each sector. Third, we estimate the dynamic correlations between these two sectors using the t copula with the generalized autoregressive score model. Fourth, we estimate the joint default probability of the insurance and financial sectors by the dynamic copula correlations.

3.1 Individual Firm's Probability of Default

We construct the time-series PD for each sector by using median PD of all active firms in that sector. The PD of individual firms is obtained from the National University of Singapore, Risk Management Institute, CRI database.

The model used to calculate default probability based on a doubly stochastic process which first presented by Duffie et al. (2007), and further developed by Duan et al. (2012). Duan et al. (2012) provide a forward intensity model for multi-period default perdition without covariates forecasting. The overlapped pseudo-likelihood function is used and the pseudo-likelihood function can be decomposed to different forward times. The estimation of parameters with different time horizon is independent of each other.

A doubly-stochastic formulation of the point process for default is proposed by Duffie et al. (2007), where the conditional probability of default within τ years is:

$$q(X_t, \tau) = E\left(\int_t^{t+\tau} e^{-\int_t^z (\lambda(u)+\varphi(u))du}\, \lambda(z)dz \middle| X_t\right) \tag{3.1}$$

X_t is the Markov state vector of firm-specific and macroeconomic covariates. λ_t (i.e. the conditional mean arrival rate of default measured in events per year) is a firm's default intensity. The firm may exit for other reasons like merger or acquisition, the intensity is defined as φ_t. Thus the total exit intensity is $\varphi_t + \lambda_t$.

The doubly-stochastic model allows to combine two decouple estimators β and γ to obtain the maximum likelihood estimator of the default probability $q(X_t, \tau)$. In the default estimation models, given the path of state-vector X, the merger or acquisition and default times of the firm are conditionally independent. The detail log-likelihood function and forward intensity can be found in the appendix.

3.2 Marginal distribution of PD

First, since the forward default intensity in Duan et al. (2012) is exponentials of linear combinations of frim-specific and macroeconomic variables and the PD is equal to one minus the exponential of multiplication of default intensity and the time horizon (in our paper, from 1-month to 60-month). We make some transformations of the PDs, the transformation in the following makes PDs back to have linear correlation with default attributes (see Duan and Miao, 2016).and then calculate the first difference of the PD:

$$p_t = \ln\{-\ln[1 - Pt(1)]\}, \tag{3.2}$$

$$p_t = logP_t - logP_{t-1}. \tag{3.3}$$

Before modelling the PD dependence of the two sectors, we base the transformed PD of each sector on the following structure:

$$p_{it} = \mu_i(Z_{t-1}) + \sigma_i(Z_{t-1})z_{it}, i = 1,2 \tag{3.4}$$

where the estimated standardized residuals $\widehat{\varepsilon_{it}} \equiv p_{it} - \mu_i(Z_{t-1})/\sigma_i(Z_{t-1})$.

Then each PD series have potentially time-varying conditional mean and variance. We use ARMA (m, n) model up to order (5,5) to model the conditional mean and GJR-GARCH (1, 1, 1) (Glosten et al., 1993) to model the conditional variance and z_it is the standardized PD.

$$p_{i,t} = c_i + \varepsilon_{i,t} + \sum_{k=1}^{m} \varphi_{i,k} p_{i,t-k} + \sum_{k=1}^{n} \theta_{i,k} \varepsilon_{i,t-k} \quad (3.5)$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \varepsilon_{i,t-1}^2 I_{i,t-1} \quad (3.6)$$

where $I_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$ and $I_{i,t-1} = 0$ if $\varepsilon_{i,t-1} > 0$

We consider both parametric and nonparametric marginal distributions for each PD series.

After taking conditional mean and variance, we consider both parametric and nonparametric marginal distributions for each PD series.

For the parametric marginal, z_it follows Hansen (1994) skewed Student's t distribution.

$$z_{i,t} \sim F_{skew-t,i}(v_i, \lambda_i),\ v_i \in (2, \infty], \lambda_i \in (-1,1). \quad (3.7)$$

where v_i is the degree of freedom and λ_i is the skewness parameter.

For the nonparametric marginal, the cumulative distribution function (EDF) F_i:

$$\widehat{F_i}\ (Z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1 \left\{ \widehat{z_{i,t}} \leq z \right\}. \quad (3.8)$$

3.3 Student's *t* copula with GAS model

The Sklar (1959) theorem shows that the conditional joint distribution can be decomposed into marginal distributions and a copula. This paper calculated the joint default probability and conditional default probability based on *t* copula with skew-*t* and empirical distribution on a monthly basis. The marginal distribution of individual firm's PD is defined as $\mathrm{F_i}, i \geq 2$. The joint default probability of each industry can be calculated by applying a copula function C :

$$F_{joint} = C[F_1(u_1), F_2(u_2), \ldots, F_d(u_d)]. \quad (3.9)$$

The copula $C\ [0,1]^d \to [0,1]$ above is the one and only one when the marginal distributions are continuous. And in our case, the unique copula (*t* copula) is given by (The *t* Copula and Related Copulas):

$$C_{v,\rho}^{t}(u) = \int_{-\infty}^{t_v^{-1}(u_1)} \dots \int_{-\infty}^{t_v^{-1}(u_d)} \frac{\Gamma(\frac{v+d}{2})}{\Gamma(\frac{v}{2})\sqrt{(\pi v)^d|\rho|}} (1 + \frac{x\prime\rho^{-1}x}{v})^{-\frac{v+d}{2}} dx. \quad (3.10)$$

where t_v^{-1} is the quantile function of a standard univariate t_v distribution and ρ is the correlation matrix.

The correlation matrix ρ in our paper is modelled by GAS model of Creal, et al. (2013). The time-varying copula parameter which in our case is the correlation parameter δ_t is a function of its lagged values and a standardized score of the copula log-likelihood. In order to make sure the parameter to be (-1, 1), we follow Patton (2012) method to transform the parameter:

$$f_t = h(\delta_t) \Leftrightarrow \delta_t = h^{-1}(f_t) \quad (3.11)$$

where $\delta_t = (1 - \exp\{-f_t\})/(1 + \exp\{-f_t\})$

$$f_{t+1} = \omega + \beta f_t + \alpha I_t^{-1/2} s_t \quad (3.12)$$

where $s_t \equiv \frac{\partial}{\partial\delta} log\boldsymbol{c}(U_{1t}, U_{2t};\ \delta_t)$; $I_t \equiv E_{t-1}[s_t s_t'] = I(\delta_t)$

The time-varying parameter is a function of constant ω, current value f_t, and score of copula-likelihood $I_t^{-1/2} s_t$.

3.4 Joint default probability

The joint default probability $P_{i,j,t}$ for insurance sector and financial sector can be defined by:

$$P_{i,j,t} = \text{Prob}\,(z_{i,t} > F_{i,t}^{-1}(1 - p_{i,t}), z_{j,t} > F_{j,t}^{-1}(1 - p_{j,t})), \tag{3.13}$$

where $z_{i,t}$ is the default intensity of the sector.

4. Data

The individual default probability data are provided by the CRI database, Risk Management Institute, National University of Singapore. We analyse the insurance and financial public listed firms with PDs in US market over a time span of 20 years. 43 public listed insurance firms 35 public listed financial firms which have monthly data from 1996 to 2015 (240 data points in total without missing) are used in our sample. For each firm, we have a monthly default probability of 1 month's, 3 months', 6 months', 12 months',24 months', 36 months', and 60 months' time horizon. Market data such as Chicago Fed National Financial Conditions Credit Subindex, VIX index, 10-year treasury rate, 3-month treasury bill, The TED spread, Crude Oil Price, CPI, and NASDAQ Composite Index returns are collected from Federal Reserve Bank of St. Louis.

4.1 Individual default probability

[INSERT TABLE 3.1 ABOUT HERE]

The transformed PD which is the default intensity of insurance and financial firms has the linear relationship with both macroeconomic and firm-specific variables. Table 3.1 shows the log returns of transformed PD of insurance and financial sectors in different horizons. The insurance sector is believed to be relatively stable but Table 3.1 suggests that the average PD returns of insurance firms is higher than financial firms in most time horizons. In the longer time horizons (36 months and 60 months in our case), insurance and financial firms have the same moving trend. The variation of Insurance and financial sectors PD returns is almost the same except for the 1-month horizon where insurance firms have bigger standard derivation. Quantile analysis indicates non-zero skewness across different time horizons of both insurance and financial sectors. The non-zero skewness motivate us to use skewed t and empirical distributions to model marginal distributions for both sectors.

5. Dependence analysis

5.1 Asymmetric dependence

The summary statistics gives us a quick review of marginal distributions of insurance and financial firms. In the next step, we will test the asymmetric dependence between two sectors.

For a symmetric dependence we have:

$$\lambda^q = \lambda^{1-q} \ \forall \ q \in [0,1] \tag{3.14}$$

where λ^q is the quantile dependence and it is defined as:

$$\lambda^q = \begin{cases} \Pr[U_{1t} \leq q | U_{2t} \leq q] \,, 0 < q \leq 1/2 \\ \Pr[U_{2t} > q | U_{2t} > q], 1/2 < q < 1 \end{cases}.$$

Follow the method from (Patton, 2012b), we test the asymmetric dependence between insurance and financial sectors:

$$\hat{\lambda} \equiv [\lambda^{q1}, \lambda^{q2}, \dots, \lambda^{2p}]' \tag{3.15}$$

where

$q_{p+j} = 1 - q_j$, for j $= 1,2, \dots,$ p and q $\in \{0.025, 0.05, 0.10, 0.975, 0.95, 0.90\}$

we test:

$$H_0 \colon R\lambda = 0 \ vs. H_\alpha \colon R\lambda \neq 0 \tag{3.16}$$

where R $\equiv [I_p \ \vdots \ -I_p]$

The results show that the test fails to reject the null that dependence between two sectors is symmetric in all time horizons (1-month, 3-month, 6-month, 12-month, 24-monthm, 36-month and 60-month) with a p-value of 0.9023, 0.9837, 0.9853, 0.9929, 0.9437, 0.9206 and 0.9945 respectively. For insurance and financial sectors, there is no evidence of asymmetric dependence. As a result, in the next section, we will model the time-varying dependence by t-copula with GAS model.

5.2 Time-varying dependence

[INSERT FIGURE 3.1 ABOUT HERE]

Patton (2012b) suggests that the conditional dependence structure may vary through time like conditional volatility. A simple way to look at time-varying dependence is to calculate the rolling rank correlation between two series. Figure 3.1 shows the rolling 60-day rank correlation for 1-month, 3-month, 6-month, 12-month, 24-monthm, 36-month and 60-month PD returns. Clear time-varying correlations can be found in each time horizon and this motivates us to apply the time-varying copula estimation in the next session.

Further, we test the time-varying dependence based on the 'ARCH LM' test proposed by Engle (1982). The test use autocorrelation as a measure of dependence which captures by the autoregressive model. From table2, the evidence by the p-value suggests that non-zero autocorrelations for lags 5 and 10 across all time horizons but no evidence at lag 1.

[INSERT TABLE 3.2 ABOUT HERE]

From table 3.2, we have 'ARCH LM' test results for time-varying dependence. The test applies an autoregressive model and regard the autocorrelation as a measure of dependence. The p-values of AR (5) is 0.0150, 0.0090, 0.0020, 0.0060, 0.0400, 0.0670, and 0.5710 and AR (10) is 0.0190, 0.0200, 0.0160, 0.0190, 0.0230, 0.0310, and 0.1780. It suggests that time-varying dependence exists between insurance and finance sectors in 1-month, 3-month, 6-month, 12-month, 24-monthm and 36-month. 60-month horizon is an exception and our results consist with this that 60-month correlation is the flattest one.

6. Empirical analysis

6.1 Estimation results

[INSERT TABLE 3.3 ABOUT HERE]

Table3.3 shows the estimated parameters of the student's t GAS copula model for the insurance and financial sectors with 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon. We estimate the student's t GAS copula by both skewed t distribution and empirical distribution (EDF). The log-likelihood suggests that parametric model outperformance semiparametric model across different time horizons. So in the next session, we analyze the time-varying Copula correlations based on skewed t distribution for insurance and financial sectors.

6.2 Time-varying correlations

[INSERT TABLE 3.4 ABOUT HERE]

Table 3.4 shows the average, standard derivation, median, maximum, and minimum of time-varying Copula correlations between insurance and financial sectors for 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon. Generally speaking, for a longer time horizon, the correlation between the insurance and financial sectors is higher but with smaller fluctuation comparing to short-time horizons. It's not surprising that the highest volatility of correlations is found in the 1-month horizon with a value of 0.1738 which is more than 20 times larger than that of 60-month horizon. Also the largest gap between maximum and minimum correlations across all time horizons can be found in the 1-month horizon.

In addition, the only negative correlation exists in the 1-month horizon, with a value of -0.1928. The lowest average Copula correlation 0.3521 is found in the 1-month horizon. For the 3-month, 6-month, 12-month, 24-month, and 36-month time horizon, the average Copula correlations are around 0.4 and the correlation rises dramatically to nearly 0.5 for 60-month time horizon. Results from table 3.4 suggest that insurance firms and financial firms have time-varying correlations both in short-term and long-term horizons. But the correlations in the short-term horizon are lower than in long-term horizons but are more volatile.

[INSERT FIGURE 3.2 ABOUT HERE]

Figure 3.2 shows the time-varying copula correlations for insurance and financial sectors for 1-month (rhot1), 3-month (rhot3), 6-month (rhot6), 12-month (rhot12), 24-month (rhot24), 36-month (rhot36), and 60-month (rhot60) horizon. The correlations are estimated by Student's t GAS copula from 1996 to 2015.

Figure 3.2 shows that 1-month correlation is clearly the most volatile one and the 60-month correlation is more like a flat line with average values around 0.35 and 0.5 respectively. The negative correlations are found within 1-month correlation in three different periods. The most sudden increases across all the correlations can be found during the early 2000s when Dotcom bubbles happened, around the year of 2008 when the global financial crisis began, and from 2010 to 2012 when European debt crisis started and later S&P downgraded US sovereign ratings. Our results suggest that insurance and financial sectors are positively high correlated in long-term horizon, but the correlations fluctuate more in the short-term horizon. In addition, the correlation between these two sectors is more likely to increase during global financial events.

6.3 Economic drivers of the correlations

The different behaviours from short-term and long-term correlations between insurance and financial sectors motivate us to investigate the economic drivers behind them. In order to understand the economic drivers of the copula correlations in different time horizons, we run regressions of the time-varying correlations with macroeconomic factors. This also motived by whether the time-varying movements of dependence between insurance and financial sectors come from the economy condition or the interactions among firms. We select the following factors based on the paper of Christoffersen et al., (2016):

1. The log of Chicago Fed National Financial Conditions Credit Subindex is used to measure the credit conditions in the market. The credit subindex is composed of measures of credit conditions. Increasing risk, tighter credit conditions and declining leverage suggest tightening financial conditions. Therefore, a positive value indicates the corresponding credit conditions is tighter than on average, while negative values indicate the opposite.

2. The log of the VIX index to model equity market risk. VIX measures market expectation of near term volatility conveyed by stock index option prices.

3. The spread between 10-year Treasury rate and 3-month Treasury bill is used as a slope variable to model the term structure and 3-month Treasury bill as a level variable.

4. The TED spread which is the difference between the Libor rate and short-term government debt rate is used to capture the liquidity in fixed income markets. The Series is calculated as the spread between 3-Month LIBOR based on US dollars and 3-Month Treasury Bill.

5. The log of Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma.

6. The log of CPI to measure the inflation.

7. The log return of NASDAQ Composite Index.

[INSERT TABLE 3.5 ABOUT HERE]

Table 3.5 presents the regression results of 1-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. The interest rate, yield curve slope, and TED spread are significant at 1%. In addition, crude oil and index return are significant at 10%.

[INSERT TABLE 3.6 ABOUT HERE]

Table 3.6 presents the regression results of 3-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. The interest rate, yield curve slope, and TED spread are significant at 1%. Credit index is significant at 5%. VIX and index return are significant at 10%.

[INSERT TABLE 3.7 ABOUT HERE]

Table 3.7 presents the regression results of 6-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. The interest rate, yield curve slope, and TED spread are significant at 1%. Index return is significant at 5%. And VIX is significant at 10%.

[INSERT TABLE 3.8 ABOUT HERE]

Table 3.8 presents the regression results of 12-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. The interest rate, yield curve slope, and TED spread are significant at 1%. Credit index and VIX are significant at 10%.

[INSERT TABLE 3.9 ABOUT HERE]

Table 3.9 presents the regression results of 24-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. Interest rate and yield curve slope are significant at 1%. VIX are significant at 5%.

[INSERT TABLE 3.10 ABOUT HERE]

Table 3.10 presents the regression results of 36-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. Interest rate and yield curve slope are significant at 1%. VIX are significant at 10%.

[INSERT TABLE 3.11 ABOUT HERE]

Table 3.11 presents the regression results of 60-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. Interest rate and yield curve slope are significant at 1%.

Regression results of 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return can be found from table 3.5, table 3.6, table 3.7, table 3.8, table 3.9, table 3.10 and table 3.11 respectively. The adjusted R^2 of 1-month correlation is 0.177 and it continue to rise with the time horizon increases. It peaks at 6-month with the highest adjusted R^2 0.358. Then the adjusted R^2 of 12-month correlation drops to 0.212. For 24-month and 36-month correlations, the economic drivers in our sample have very weak explanation power with the adjusted R^2 around 0.01. Finally, the adjusted R^2 of 60-month correlation increases to 0.134.

Across all time horizons, interest rate level and yield curve slope are the only two variables remain significant (significant at 1%) and inflation is the only variable remain not significant. In general, short-term correlations (from 1-month to 12-month horizon) are more likely driven by economic factors, such as credit index, VIX, interest rate level, yield curve slope, TED spread, crude oil, and index return. When the financial condition (credit and liquidity) is tighter than average, equity market risk is high along with the high return, and crude oil price rises, insurance firms, and financial firms become more correlated. On the contrary, the two sectors are less correlated when interest rate level and yield curve slope is high.

Long-term correlations (from 24-month to 60-month), they are less sensitive to the market condition. Our results show they are only affected by interest rate level, yield curve slope, and the market volatility (except for 60-month). Consistent with short-term horizons, insurance firms and financial firms are less correlated when interest rate level and yield curve slope is high while being more correlated when equity market risk is high.

6.4 Joint default probability

[INSERT FIGURE 3.3 ABOUT HERE]

Figure 3.3 shows the estimated time-varying joint default probabilities of insurance and financial sectors from 1996 to 2015 by Student's t GAS copula. The joint default probabilities are estimated based on 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon respectively.

The arrows indicate time points of several major events in the global financial market. The joint default probabilities across all time horizons show a similar moving trend in our 20 years' time period. The longer time horizon is, the higher joint default probability can be found.

In the previous section, we find economic drivers of the time-varying correlations across different horizons. In addition, we find the joint default probabilities between insurance and financial sectors which are simulated by the Copula correlations are very sensitive to global financial market. From 1997 to 1998, the PDs slightly rise when Asia crisis began. Later, a big increase can be found when the Argentine and Russian financial crises happened and further the joint PDs peak around 2000 when Dotcom bubble bust. At the year of 2008, when the global financial crisis started, the joint PDs grow dramatically and then decline slightly afterwards until the end of 2009. But two upward slopes can be found in the next two years when European debt crisis happened and the S&P downgrading of US sovereign debt ratings.

7. Conclusion

Previous research shows that insurance firms are acting more closely with financial institutions in many ways. Our paper investigates the linkage between the insurance and financial sectors from the prospective of dynamic copula correlations and joint default probability. Our database includes individual's default probability of public listed insurance firms and financial firms in US market from the National University of Singapore, Risk Management Institute, CRI database. Evidence of time-varying symmetric correlations between these two sectors are found for both short-term and long-term horizons. We estimate the time-varying symmetric correlations and the joint default risk by student's *t* GAS copula correlations with skewed *t* and empirical distribution.

Our results show the existence of time-varying correlations between these two sectors which indicating dynamic and symmetric dependencies. And the correlations vary widely in different time horizons, the only negative correlation is found in the 1-month horizon. Short-term correlations are relatively lower but more fluctuating while the long-term correlations suggest that insurance and financial firms are positively correlated. We find economic drivers of the movement of insurance and financial sectors by regressing the copula correlations with macroeconomic variables. Across all time horizons, interest rate level and yield curve slope always remain significant (significant at 1%) and inflation is the only variable remain not significant. In general, short-term correlations (from 1-month to 12-month horizon) are more likely driven by economic factors like credit index, VIX, interest rate level, yield curve slope, TED spread, crude oil, and index return. Long-term correlations (from 24-month to 60-month) are less sensitive to the market condition. Our results suggest they are only affected by interest rate level, yield curve slope, and the market volatility (except for 60-month). Low adjust R^2 indicates that the joint movement may come from the interactions between insurance firms and financial firms. By using the cupula correlations, we can also simulate the joint default probability for the two sectors. The joint PD of all time horizons show similar moving trend in our 20 years' time period. The longer the time horizon is, the higher the joint default probability that can be found. In addition, we find the joint PD between these two sectors are very sensitive to global financial market events, such as 1997 Asia crisis, 1998 Argentine and Russian financial crisis, 2000 Dotcom bubble, 2008 global financial crisis, 2010 European debt crisis, and the 2011 S&P downgrading US of sovereign debt ratings.

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Appendix

In the following, it's a brief introduction of forward intensity from in Duan et al. (2012) model.

Likelihood Function

Instead of modelling the time-varying covariates to calculate the default probability, Duan *et al.* (2012) use the forward intensity rate to calculate the default rate. The model needs to derive the forward intensity rate at time τ.

The pseudo-likelihood function for the prediction time τ defined as

$$\mathcal{L}_{\tau}(\alpha, \beta; \tau_C, \tau_D, X) = \prod_{i=1}^{N} \prod_{t=0}^{T-1} \mathcal{L}_{\tau,i,t}(\alpha, \beta), \tag{1}$$

And the likelihood function $\mathcal{L}_{\tau,i,t}(\alpha, \beta)$ for firm *i* consists of five situations: the first is the firm *i* survives in the prediction time period, the second is the firm *i* defaults in the prediction period, the third is the firm *i* exits for other reasons (i.e. like merger and acquisition), the fourth is the firm *i* exits after this prediction time period and the last is the firm *i* exits before the start of this time interval:

$$\begin{aligned}&\mathcal{L}_{\tau,i,t}(\alpha, \beta)\\ &= 1_{\{t_{0i} \le t, \tau_{Ci} \ge t+\tau\}} P_t(\tau_{Ci} > t+\tau) + 1_{\{t_{0i} \le t, \tau_{Di} = \tau_{Ci} \le t+\tau\}} P_t(\tau_{Di} = \tau_{Ci} \le t+\tau) + 1_{\{t_{0i} \le t, \tau_{Di} \neq \tau_{Ci}, \tau_{Ci} \le t+\tau\}} P_t(\tau_{Di} \neq \tau_{Ci}, \tau_{Ci} \le t+\tau) + 1_{\{t_{0i} > t\}} + 1_{\{t_{Ci} < t\}}.\end{aligned} \tag{2}$$

Forward Intensity of Individual Firm's PD

First, we define $F_t(\tau)$ to be the time conditional distribution function of the combined exit time.

$$\psi_t(\tau) = -\frac{ln\left(1 - F_t(\tau)\right)}{\tau} = -\frac{lnE_t\left[exp\left(-\int_t^{t+\tau}(\lambda_u + \varphi_u)dz\right)\right]}{\tau} \tag{3}$$

Then, $\exp[-\psi_t(s)s]$ becomes the survival probability at time interval (t, t+τ).

And the forward combined exit intensity is defined as:

$$g_t(\tau) \equiv \frac{F'_t(\tau)}{1 - F_t(\tau)} = \psi_t(\tau) + \psi'_t(\tau)\,\tau. \tag{4}$$

Also, the forward default intensity censored by other forms of exit:

$$\begin{aligned} f_t(\tau) &\equiv e^{\psi_t(\tau)\tau} \lim_{\Delta t \to 0} \frac{P_t(t + \tau < \tau_D = \tau_c \leq t + \tau + \Delta t)}{\Delta t} \\ &= e^{\psi_t(\tau)\tau} \lim_{\Delta t \to 0} \frac{E_t\left[\int_{t+\tau}^{t+\tau+\Delta t} exp\left(-\int_t^z (\lambda_u + \varphi_u)du\right)\lambda_z dz\right]}{\Delta t}, \end{aligned} \tag{5}$$

So the default intensity between (t, t+s) will be $\int_0^{\tau} -e^{\psi_t(\tau)\tau} f_t(s)ds$.

And since X_t is the state variables contain both firm-specific and macroeconomic factors.

$$f_t(\tau) = exp\left(\alpha_0(\tau) + \alpha_1(\tau)X_{t,1} + \alpha_2(\tau)X_{t,2} + \cdots + \alpha_k(\tau)X_{t,k}\right), \tag{6}$$

$$g_t(\tau) = f_t(\tau) + exp\left(\beta_0(\tau) + \beta_1(\tau)X_{t,1} + \beta_2(\tau)X_{t,2} + \cdots + \beta_k(\tau)X_{t,k}\right) \tag{7}$$

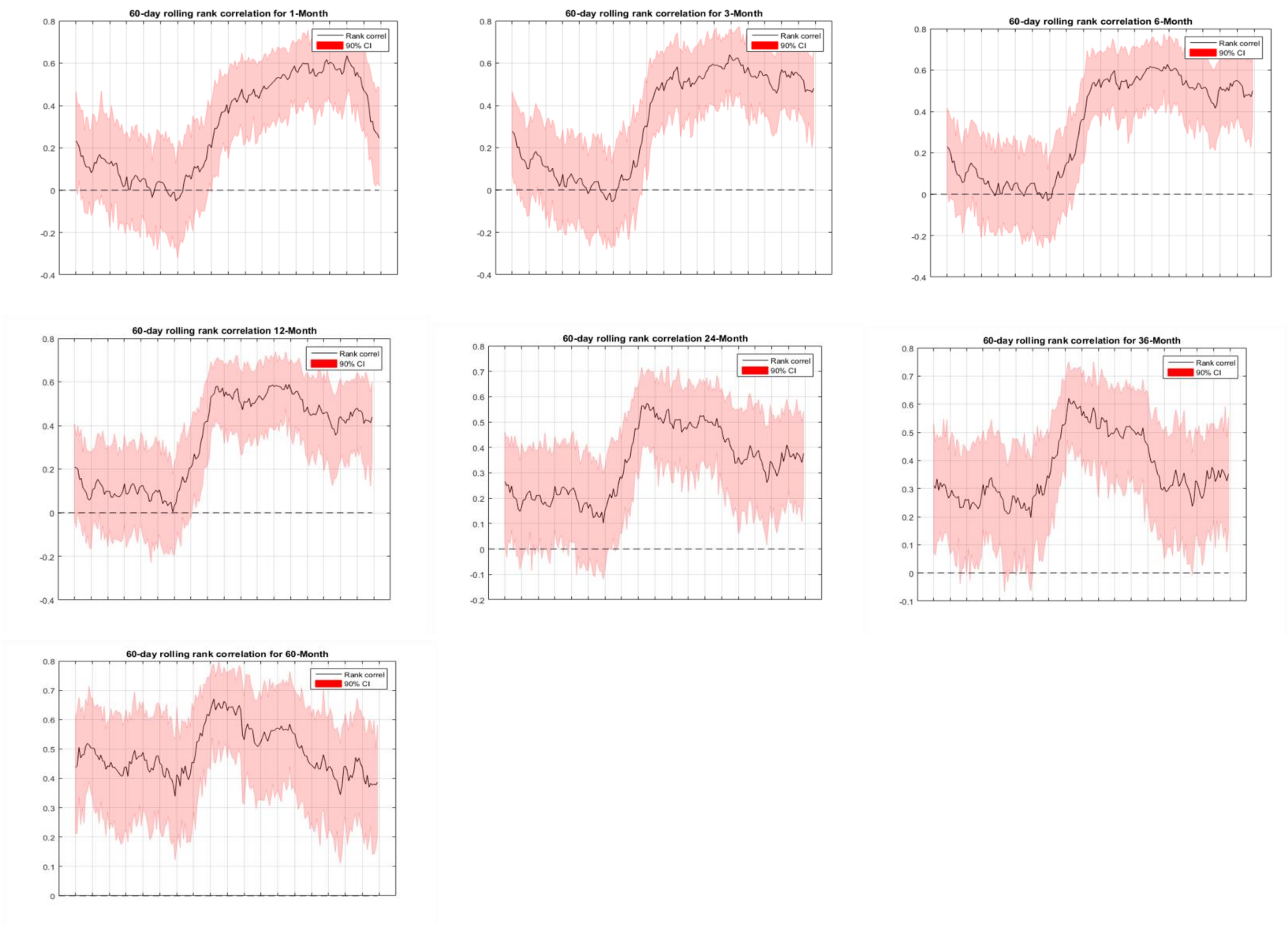

Figure 3.1 60-day rolling rank correlation for insurance and finance sector.

Note: Figure3.1 plots the 60-day rolling rank correlation for insurance and finance sector for the 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon respectively.

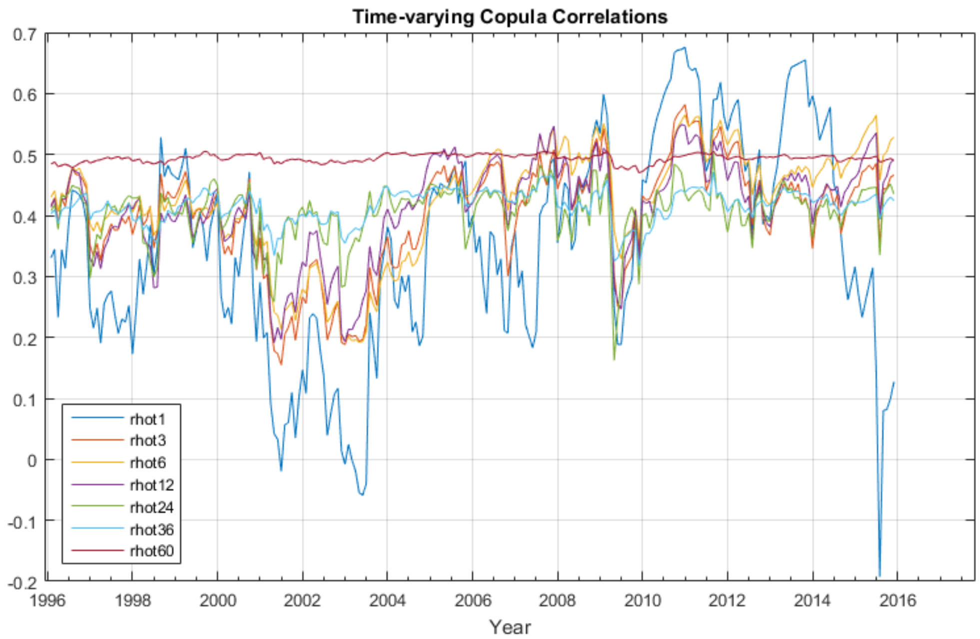

Figure 3.2 Time-varying copula correlations for insurance and financial sectors

Notes: Figure3.2 plots the estimated time-varying correlation between insurance and financial sectors from 1996 to 2015 by Student's t GAS copula. The time-varying correlations are estimated based on 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon respectively.

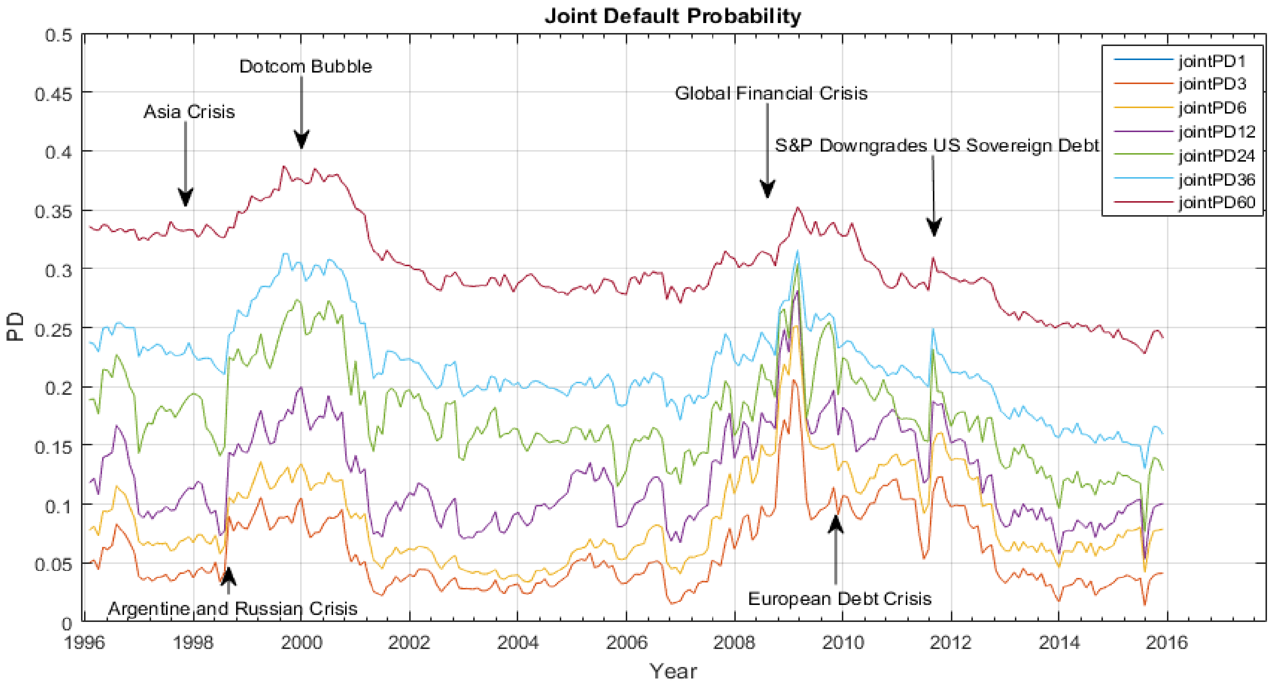

Figure 3.3 Joint default probability for insurance and finance industry

Notes: Figure3.3 plots the estimated time-varying joint default probabilities of insurance and financial sectors from 1996 to 2015 by Student's t GAS copula. The joint default probabilities are estimated based on 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon respectively. The arrows indicate time points of several major events in the global financial market.

Table 3.1 Summary statistics of transformed PD log returns

Month	Sector	mean	std	$q_{5\%}$	$q_{25\%}$	$q_{50\%}$	$q_{75\%}$	$q_{95\%}$
1	Insurance	0.0005	0.0421	-0.0572	-0.0147	0.0000	0.0181	0.0548
	Finance	0.0003	0.0356	-0.0515	-0.0181	0.0000	0.0206	0.0515
3	Insurance	0.0006	0.0359	-0.0611	-0.0196	0.0017	0.0229	0.0544
	Finance	0.0003	0.0359	-0.0494	-0.0195	0.0018	0.0228	0.0526
6	Insurance	0.0007	0.0361	-0.0671	-0.0197	0.0027	0.0228	0.0523
	Finance	0.0004	0.0373	-0.0473	-0.0205	0.0036	0.0228	0.0516
12	Insurance	0.0008	0.0360	-0.0649	-0.0195	0.0026	0.0224	0.0573
	Finance	0.0006	0.0365	-0.0497	-0.0218	0.0036	0.0227	0.0539
24	Insurance	0.0011	0.0328	-0.0567	-0.0169	0.0032	0.0189	0.0516
	Finance	0.0010	0.0326	-0.0472	-0.0174	0.0042	0.0198	0.0488
36	Insurance	0.0013	0.0298	-0.0486	-0.0177	0.0039	0.0182	0.0478
	Finance	0.0013	0.0294	-0.0465	-0.0161	0.0026	0.0198	0.0451
60	Insurance	0.0015	0.0277	-0.0451	-0.0150	0.0033	0.0175	0.0419
	Finance	0.0015	0.0275	-0.0417	-0.0127	0.0034	0.0164	0.0458

Note: the table3.1 shows the summary statistics of transformed PD log returns from insurance and finance sectors in 1-month, 3-month, 6-month, 12-month, 24-monthm, 36-month, and 60-month horizons. The original historical individual PDs are from RMI-CRI database with the sample period from January 1996 to December 2014.'std' stand for standard derivation and '$q_{\alpha\%}$' represents the α quantile.

Table 3.2 Tests for time-varying dependence

Month	AR(1)	AR(5)	AR(10)
1	0.7290	0.0150	0.0190
3	0.9190	0.0090	0.0200
6	0.9110	0.0020	0.0160
12	1.0000	0.0060	0.0190
24	0.9510	0.0400	0.0230
36	0.9550	0.0670	0.0310
60	0.9600	0.5710	0.1780

Note: this table shows 'ARCH LM' test results for time-varying dependence. P-values of AR (1), AR (5), and AR (10) are presented here with 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon respectively.

Table 3.3 Student's t GAS Copula Parameter Estimation

		1-month	3-month	6-month	12-month	24-month	36-month	60-month
Skewed t	$\widehat{\omega}$	0.0538	0.0741	0.0387	0.0851	0.3061	0.1452	0.1579
		0.3994	0.0303	0.0156	0.0492	0.0543	0.0397	0.1967
	$\hat{\alpha}$	0.1531	0.0880	0.0633	0.0890	0.0869	0.0380	0.0089
		5.4661	0.0236	0.0140	0.0854	0.0064	0.0798	0.0085
	$\hat{\beta}$	0.9283	0.9134	0.9597	0.9062	0.6459	0.8368	0.8543
		0.3669	0.0422	0.0174	0.0483	0.0598	0.0000	0.1965
	$\hat{\eta}^{-1}$	0.1067	0.1351	0.1443	0.1093	0.0295	0.0303	0.0850
		0.1750	0.0501	0.0998	0.0920	0.0034	0.0897	0.0906
	$log\mathcal{L}$	21.8124	28.0295	29.6078	25.4414	23.0667	21.9689	34.0670
EDF	$\widehat{\omega}$	0.0528	0.0515	0.0384	0.0417	0.9187	0.1064	0.1406
		0.0245	0.0535	0.0097	0.0074	0.0715	0.0511	0.0995
	$\hat{\alpha}$	0.1593	0.0745	0.0721	0.0448	0.1552	0.0350	0.0080
		0.0809	0.1116	0.0148	0.0067	0.0178	0.0089	0.0040
	$\hat{\beta}$	0.9293	0.9414	0.9606	0.9558	0.0000	0.8855	0.8735
		0.0000	0.0506	0.0138	0.0000	0.0000	0.0519	0.1030
	$\hat{\eta}^{-1}$	0.1110	0.1671	0.1417	0.0691	0.0105	0.0313	0.1290
		0.0510	0.1150	0.0790	0.0153	0.0004	0.0042	0.0748
	$log\mathcal{L}$	19.7963	26.4328	27.8666	23.4586	22.5222	22.4378	34.4077

Note: this table presents the estimated parameters of the student's *t* GAScopula model for the insurance and financial sectors with 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon. The marginal distributions are estimated using a skewed *t* distribution or empirical distribution (EDF). Standard errors and log-likelihood for both parametric and semiparametric models are reported.

Table 3.4 Time-varying Copula correlations

Month	Mean	std	Median	Max	Min
1	0.3521	0.1738	0.3604	0.6757	-0.1928
3	0.4022	0.0891	0.4227	0.5820	0.1545
6	0.4221	0.0911	0.4368	0.5650	0.1921
12	0.4132	0.0794	0.4246	0.5492	0.1911
24	0.4064	0.0453	0.4154	0.5020	0.1621
36	0.4155	0.0266	0.4206	0.4717	0.3177
60	0.4938	0.0068	0.4946	0.5055	0.4699

Note: this table presents the average, standard derivation, median, maximum, and minimum of time-varying Copula correlations for 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month horizon.

Table 3.5 Regression of 1-month correlation

	1	2	3	4	5	6
Constant	0.352***	0.352***	0.400***	0.564***	0.521***	0.520***
Credit Index	0.013*	0.013*	0.012*	0.009	0.008	0.010
VIX		0.026	0.049	0.049	0.016	0.132
Interest rate level			-0.02***	-0.045***	-0.049***	-0.053***
Yield curve slope				-0.059***	-0.055***	-0.058***
TED					0.093***	0.112***
Crude oil						0.247*
Inflation						1.822
Index return						0.417*
Adjusted R^2	0.012	0.144	0.068	0.115	0.153	0.177

Note: this table shows the regression results of 1-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.6 Regression of 3-month correlation

	1	2	3	4	5	6
Constant	0.402***	0.402***	0.413***	0.553***	0.529***	0.524***
Credit Index	0.009**	0.009**	0.008**	0.006**	0.006*	0.007**
VIX		0.032	0.037	0.037	0.018	0.078*
Interest rate level			-0.005*	-0.026***	-0.028***	-0.03***
Yield curve slope				-0.05***	-0.048***	-0.049***
TED					0.054***	0.064***
Crude oil						0.069
Inflation						2.121
Index return						0.235**
AdjustedR^2	0.023	0.022	0.032	0.167	0.218	0.240

Note: this table shows the regression results of 3-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.7 Regression of 6-month correlation

	1	2	3	4	5	6
Constant	0.422***	0.422***	0.437***	0.627***	0.598***	0.592***
Credit Index	0.008**	0.008**	0.007**	0.004	0.003	0.004
VIX		0.035	0.042	0.043	0.020	0.077*
Interest rate level			-0.006**	-0.035***	-0.038***	-0.039***
Yield curve slope				-0.068***	-0.065***	-0.066***
TED					0.064***	0.072***
Crude oil						0.030
Inflation						2.093
Index return						0.237**
Adjusted R^2	0.016	0.015	0.034	0.275	0.343	0.358

Note: this table shows the regression results of 6-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.8 Regression of 12-month correlation

	1	2	3	4	5	6
Constant	0.413***	0.413***	0.427***	0.548***	0.531***	0.526***
Credit Index	0.007**	0.007**	0.006**	0.005*	0.004	0.005*
VIX		0.03	0.036	0.036	0.023	0.064*
Interest rate level			-0.006**	-0.024***	-0.026***	-0.027***
Yield curve slope				-0.044***	-0.042***	-0.043***
TED					0.037***	0.046***
Crude oil						0.047
Inflation						2.533
Index return						0.163
Adjusted R^2	0.017	0.016	0.037	0.166	0.194	0.212

Note: this table shows the regression results of 12-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.9 Regression of 24-month correlation

	1	2	3	4	5	6
Constant	0.406***	0.406***	0.409***	0.434***	0.433***	0.431***
Credit Index	0.001	0.001	0.001	0.001	0.001	0.001
VIX		0.033*	0.034*	0.034*	0.033*	0.052**
Interest rate level			-0.001	-0.005**	-0.005**	-0.005***
Yield curve slope				-0.009**	-0.009*	-0.009***
TED					0.002	0.004
Crude oil						-0.011
Inflation						0.604
Index return						0.084
Adjusted R^2	-0.002	0.006	0.006	0.018	0.014	0.010

Note: this table shows the regression results of 24-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.10 Regression of 36-month correlation

	1	2	3	4	5	6
Constant	0.415***	0.415***	0.415***	0.445***	0.444***	0.443***
Credit Index	0.002	0.002	0.002	0.001	0.001	0.001
VIX		0.020*	0.020*	0.020*	0.019*	0.026*
Interest rate level			0.000	-0.004***	-0.004***	-0.005***
Yield curve slope				-0.011***	-0.011***	-0.011***
TED					0.003	0.005
Crude oil						0.013
Inflation						0.622
Index return						0.028
Adjusted R^2	0.005	0.014	0.010	0.077	0.075	0.075

Note: this table shows the regression results of 36-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Table 3.11 Regression of 60-month correlation

	1	2	3	4	5	6
Constant	0.494***	0.494***	0.493***	0.504***	0.503***	0.503***
Credit Index	0.000	0.000	0.000	0.000	0.000	0
VIX		0.003	0.003	0.003	0.003	0.001
Interest rate level			0.000	-0.001***	-0.001***	-0.001***
Yield curve slope				-0.004***	-0.004***	-0.004***
TED					0.001	0.002
Crude oil						0.002
Inflation						0.229
Index return						-0.008
Adjusted R^2	-0.001	0.001	0.002	0.129	0.131	0.134

Note: this table shows the regression results of 60-month correlation on credit risk index, VIX, interest rate level yield curve slope, TED spread, crude oil, inflation, NASDAQ, and Composite index return. *Significant at 10%, ** Significant at 5%, *** Significant at 1%.

Chapter 4

Analysing the Determinants of Credit Risk for General Insurance Firms in the UK

This paper estimates a reduced-form model to assess the credit risk of General Insurance (GI) non-life firms in the UK. Compared to earlier studies, it uses a much larger sample including 30 years of data for 515 firms, and also considers a much wider set of possible determinants of credit risk. The empirical results suggest that macroeconomic and firm-specific factors both play important roles. Other key findings are the following: credit risk varies across firms depending on their business lines; there is default clustering in the GI industry; different reinsurance levels also affect the credit risk of insurance firms. The implications of these findings for regulators of GI firms under the coming Solvency II are discussed.

This chapter is the early version of the working paper "Caporale, Guglielmo Maria., Cerrato, Mario., and Zhang, Xuan. (2016). Analysing the Determinants of Credit Risk for General Insurance Firms in the UK." I am grateful to Standard & Poor's and the Bank of England for data support. I have benefited from comments from Papachristou Dimitris, Claus Stefan, Khaleghy Jennifer, Winter Richard, David Simmons, Nylesh Shah and others from the General Insurance Division of the Bank of England. I would like to thank the conference discussant at 3rd annual Young Finance Scholars' Conference and Workshop in Brighton, The 48th Money, Macro and Finance Research Group Annual Conference in Bath, and School of Economics Zhejiang University, AFR seminar in Hangzhou for helpful discussions and comments.

1. Introduction

The UK's non-life insurance industry is worth £60bn and is the largest in Europe and the third largest in the world (after the US and Japan). It comprises more than 300 active firms (both domestically- and foreign-owned); [30] in addition, 94 Lloyds's syndicates also underwrite non-life business (Lloyd's Annual Report 2014). In total, it currently generates approximately £48.217bn in gross written premium income (International Underwriting Association, 2015).

Compared to the banking sector, the insurance industry is relatively stable and less subject to risk: unlike banks, insurers do not accept deposits from customers, and therefore they do not face the risk of a sudden shortage in liquidity which may cause bank runs; also, they often hold more long-term than short-term liabilities. Bell and Keller (2009) find that insurers are less interconnected than banks and there is less contagion among them. However, Janina and Gregor (2015) argue that insurance firms are becoming more similar to banks and contribute to the systemic risk of the financial sector. Further, a study by the Geneva Association (2010) indicates that the insurance industry increases systemic risk if insurers engage heavily in trading derivatives off the balance sheet or mismanage short-term financing activities.

It is very difficult to access data on the credit risk of insurance firms because very few have become "insolvent" - the majority choose instead to transfer their business to other insurance firms or just stop underwriting new business. As an alternative, third-party rating agencies may provide a good overview of their financial condition. The main problem with external rating agencies is that not all insurance firms are rated and the ratings normally stay

[30] In addition, more than 500 non-life insurance firms that are not regulated by the UK government are licensed by the European Economic Area to conduct business in the UK (Financial Services Authority, 2013).

the same for many years. Also, different rating agencies such as *A M Best, Standard & Poor's, Moody's* and *Fitch* have different rating methodologies and labelling systems.

Under *Solvency II*, credit risk is defined as the risk of loss (or of adverse change) in the financial situation of a company which results from fluctuations in the credit standing of issuers of securities, counterparties and any debtors to which a *Solvency II* undertaking is exposed, in the form of counterparty default risk, spread risk, or market risk concentration.[31]

In this paper, we assume that the credit risk of insurance firms is made up of three components. The first is the credit quality of their investment portfolio, whose performance we measure using investment returns. The second is the counter-party risk through reinsurance activity and the purchasing of derivative contracts. A high reinsurance ratio and holding derivative contracts increase the credit risk exposure of firms. The reinsurance ratio and a dummy variable for the use of derivative contracts are therefore used in our study to capture this second component. The third is the direct default risk of insurers when their liabilities are less than their assets and therefore they might become insolvent. The financial health of firms is measured here using the leverage, profitability, solvency and liquidity ratios. Size, growth and claims volatility are also taken into account.

We consider different exit situations for firms, including insolvency and transferring business to a third party. Ours is the first study to use a very large dataset consisting of 30 years of data for 515 firms to analyse the credit risk of General Insurance (GI) firms in the UK. We show that other risk factors (macroeconomic and firm-specific factors), in addition to the standard ones considered by the literature (i.e. interest rate, whole sale price change, credit supply change, usage of financial derivatives and combined ratios) affect insurers' insolvency. When assessing their profitability, we take into account profit from both the traditional underwriting business and investment activities. We estimate both the individual

[31] Art. 13(32) of the *Solvency II Directive*

probability of default (PD) for all available firms and the joint one using pair correlations. Ours is the first study to analyse the systemic risk of insurance firms in the UK on the basis of their individual PD. Our results show high dependence between insurance firms when their individual PDs are high; this suggests that the joint probability of default is higher during distress times. Finally, we examine the relationship between reinsurance and change in the credit risk of insurance firms. Previous studies show that primary insurers can benefit from reinsurance contracts in many ways (e.g. they can hedge against risk, and hold more capital to underwrite new business). We show that reinsurers may also benefit from reinsurance activities by reducing their credit risk.

The layout of the paper is as follows. Section 2 briefly reviews the relevant literature. Section 3 discusses the data and the various determinants of risk considered. Section 4 outlines the modelling approach. Section 5 discusses the empirical results. Section 6 summarises the main findings and offers some concluding remarks.

2. Literature Review

Insurance firms play a very important role in the economy. Most studies (Carmichael, Pomerleano, 2002, Das et al., 2003, Lee, 2013 and Lee and Chang, 2015) suggests that they can enhance financial stability by transferring risk to multiple parties through insurance and reinsurance activities. Rothstein (2011) shows that a healthy and well-developed insurance industry will improve the stability of financial markets. In addition, insurance firms protect individuals and corporations from losses arising from natural disasters such as floods etc. (Adams et al., 2003; Faure and Heine, 2011; Kugler and Ofoghi, 2005; Ward and Zurbruegg, 2000).

The insurance industry is thought to be less exposed to turbulence in financial markets than other industries such as the banking one. There are several possible reasons for this difference. Harrington (2009) argues that insurance firms have to comply with more rigorous capital requirements than other financial institutions, and as a result credit events in the insurance industry have a small effect on the stability of the financial system as a whole. Das et al. (2003) suggest that the insurance industry is more stable because insurers do not suffer from bank runs, and the cancellation process for insurance policies takes longer than closing a bank account. Furthermore, because of larger premia, policy holders would suffer a loss if the policy were cancelled.

However, recent developments have made the insurance industry less stable; in particular, the fast growth of financial derivatives has meant that insurance firms have become more engaged with banks.[32] Schinasi (2006) and Rule (2001) find that more insurance firms are buying credit default swaps to hedge their credit risk and using alternative risk transfer (ART) tools such as catastrophe bonds to transfer the catastrophe risk to other investors. Also, investment in asset-backed securities has increased. As a result, as pointed out by Baluch et al. (2011), insurance firms have become more vulnerable during crises. This is also shown by studies such as those by Das et al., (2003), who find that linkages through reinsurance activities may cause several primary insurance firms to fail at the same time, and by Acharya et al. (2015), who suggest that large insurance firms are more likely to invest in high-risk assets because they are correlated with different financial institutions. One of the contributions of this paper is to assess the vulnerability of the UK insurance industry; to the best of our knowledge, this is the first time this issue is addressed in the case of the UK.

[32] Insurance firms are more engaged with banks through trading financial derivatives and other investment activities.

The linkage between insurance firms and banking crises should not be ignored. Insurance firms play a crucial role in financial markets. For example, Main (1982) shows that banks can avoid some bad debts by working with them. This is because the information on obligors provided by insurance firms helps banks to understand individual and corporate risk exposures better. In addition, by mitigating the losses during natural disasters, insurance firms help to reduce the probability of default of some investors (policy holders, both individual and corporate) who, sometimes, are the same obligors as those of banks (Lee et al, 2016). Lehmann and Hofmann (2010) show that banks are more likely to transfer part or all of their risks to other financial sectors to avoid high correlation between assets that may lead to a higher default probability. Trichet (2005) shows that the insurance and banking industries are linked by ownership and the associations of credit exposure. As a result, insurance firms are playing a central role in financial markets and this may directly affect banks.

One of the most recent papers on the UK non-life insurance industry is by Shiu (2011), who uses data from 1985 to 2002 to investigate the relationship between reinsurance and capital structure. He shows that insurers with higher leverage tend to purchase more reinsurance, and those with higher reinsurance dependence tend to have a higher level of debt. Using the same database, Shiu (2007) shows that an insurer’s size, liquidity, interest rate risk exposure, line of business concentration and organizational form are important factors associated with the decision to employ financial derivatives. Adam et al. (2003) explore the determinants of credit ratings in the UK insurance industry analysing a sample of 65 firms over the period from 1993 to 1997. They find that mutual insurers are generally given higher ratings than non-mutual ones. Also, liquidity and profitability have a significant positive effect on ratings.

Most previous studies on credit risk focused on banking crises. For example, early warning systems are well developed for banks (e.g., Kaminsky and Reinhart, 1999; Borio and Drehmann, 2009; Drehmann and Juselius, 2014; Demirgüç-Kunt and Detragiache, 1998; Barell et al., 2010 and Schularick and Taylor, 2012). Given the fact that insurance firms play a central role in financial markets and are becoming more engaged with banks, analysing their credit risk is clearly important. In the present study, we employ a reduced-form model to assess it in the case of the UK.

3. Data and Covariates

Firm-specific variables are collected from *SynThesys Non-Life*.[33] This consists of FSA data (now regulated under the Prudential Regulation Authority, Bank of England) of non-life annual return regulatory data. This database allows quick access to FSA return data for the current year and past years back to 1985. Over 400 companies are covered by the current *SynThesys Non-Life* system and the data include statements of solvency, components of capital resources, statements of net assets, calculations of capital requirement, analysis of admissible assets, liabilities, the profit and loss account, analysis of derivative contracts, summary of business carried on, technical account, analysis of premiums, analysis of claims, analysis of expenses and analysis of technical provisions etc. Also, there are approximately 180 ratios currently included, with all the underlying calculations being done by *SynThesys*.

Since most general insurance firms are small and non-public, there is no specific default list. Further, in the UK market, instead of becoming insolvent, most general insurance firms go into 'Run-Off' (i.e. stop underwriting new business and wait for their financial condition to improve or transfer their business to others). All the credit events in our paper have been

[33] From *Standard & Poor's*

collected by hand from *Appendix D: Company Changes, Transfers, Mergers of SynThesys Non-Life UserGuide version 10.1*[34], *PwC - Insurance insolvency*[35] *and Financial Services Compensation Scheme*[36], and the final list has been further discussed with technical specialists and senior supervisors from the Insurance Division at the Bank of England. Macroeconomic data have been obtained from the *World Bank*. Therefore, ours is a unique dataset for the insurance industry in the UK.

Before analysing the data, we remove firms without at least one-year balance data for all the variables. This is because these firms cannot be used to calibrate the model. We also remove firms for reasons such as merger & acquisitions, insolvency or simply running off and disappearing. If the model fails to take other types exit (i.e. except insolvency) into consideration, which means active firms will only have two states within the prediction period either insolvent or survival. As a result, firms exited due to other reasons will be treated as survival firms which will affect the estimation accuracy. This of course will affect the calibration of the model. More discussion of the different types of firms' exit can be found in the model section. In the end, we are left with 363 firms with 14 firm-specific variables and 6 macroeconomic variables in our dataset spanning from 1986 to 2014. These include 35 firms that became insolvent during that period and 45 firms that exited owing to other reasons such as transferring their business to other firms.

To lessen the effect of outliers, we use the following strategy: for example, we cap the reinsurance ratio and leverage ratio at the 95 percentile value and remove the lower 5 percentile value. Also we cap the liquidity ratio, profitability ratio, growth premium written change, claim change and excess capital ratio at the 99 percentile value and remove the lower

[34] Updated on November 2014
[35] http://www.pwc.co.uk/services/business-recovery/insights/insurance-insolvency-case-updates-pwc-uk.html
[36] http://www.fscs.org.uk/what-we-cover/products/insurance/insurance-insolvencies/

1 percentile value. The summary statistics and correlation matrix of firm-specific and macroeconomic variables respectively are reported in Table 4.1, 4.2 and 4.3.

3.1 Covariates

Following the literature, [37] we choose the following firm-specific variables: leverage (Net Technical Provisions /Adjust Liquid Assets, *SynThesys* Appendix K: Ratio Definitions R12), profitability (underwriting profit to Total Assets), growth (the change in the natural logarithm of total admissible assets), firm size (the natural logarithm of total admitted assets), reinsurance (the ratio of Reinsurance Premiums Ceded to Gross Premium written), claims change (the change in net claims incurred), capital (the change in excess capital resources to cover general business CRR), liquidity (Cash/Total Asset: the ratio of the sum of cash and short-term investments to total assets), gross premium written (the annual change in gross premium written), combined ratio (Incurred Claims + Management Expense) / Gross Premium Written), Line-of-Business Concentration (Herfindahl index), organizational form (mutual or non-mutual firm) and a derivative dummy variable (Derivative dummy defined based on *SynThesys* form 17, i.e. whether the sum of form 17 is zero or not). In addition, we also include UK macroeconomic variables, namely GDP growth, the change in the wholesale price index (2010=100), the change in foreign direct investment, net inflows, the real interest rate, the real effective exchange rate index (2010=100) and the change in the credit provided by financial institutions (% of GDP) from *the World Bank.*

Leverage

[37] Brotman (1989), Adams (1995), Pottier (1997, 1998), Adams et al. (2003) and Shiu (2011) etc.

Higher leverage may have an adverse effect on insurance firms by affecting their underwriting performance and making an insurer's capital more vulnerable to economic shocks. Further, Adams et al. (2003) find that insurers with lower financial leverage are more likely to be given a higher credit rating. Previous studies such as Brotman (1989) and Pottier (1997, 1998) also report a negative relationship between financial leverage and the capital structure of insurance firms.

Profitability

Profitability indicates the ability of insurance firms to generate a surplus to develop their current business and generate new business. A higher profitability ratio means that the insurance firm can manage expenses effectively and set competitive premium rates. Titman and Wessels (1988) and Frank and Goyal (2009) also suggest that highly profitable firms have a lower debt ratio and hence a lower credit risk.

Firm Growth

Normally, a positive growth ratio signals a good financial condition for a firm. But for issuers with significant new business growth could be achieved by poor underwriting standards and mispricing strategy (Adams et al., 2003). Borde et al. (1994) and Pottier (1997) find that this will lead to greater uncertainty about the capital reserve risk for insurance firms. Further, Frank and Goyal (2009) conclude that firms with a high growth ratio face more debt-related agency issues and higher associated cost.

Firm Size

Prior studies such as Bouzouita and Young (1998) find that large insurers are less likely to become insolvent; they normally benefit from economies of scale, and given their sizeable market shares and higher ratings have lower financing costs than small insurers (Adams et al., 2003).

Reinsurance

Berger et al. (1992) point out that there are two types of traditional reinsurance activities involving a direct insurer ceding all or part of its assumed underwritings to another insurance company. Insurance firm transfer part of their risk to third parties by reinsurance, which result in lower uncertainty concerning their future losses and enables them to reduce their capital reserves. Adams (1996) suggests that reinsurance improves the ability of the primary insurer to survive an external economic shock. On the other hand, the financial health of a heavily reinsured firm will be adversely affected by the insolvency of reinsurance firms.

Claims Growth[38]

In the insurance industry, incurred claims are the amount of outstanding liabilities for policies over a given valuation period. A significant increase in net claims may generate liquidity risk for an insurer, which will eventually become insolvent if it cannot raise enough capital.

Capital

When measuring the default risk of insurance firms, it is natural to include the Excess (deficiency) of capital resources to cover general business CRR (Capital Requirements Regulation). Insurance firms should hold enough capital to cover the policies they underwrite.

Liquidity

We use the cash ratio as a measure of a firm's liquidity. For insurance firms, a high liquidity ratio indicates good claim-paying ability. Previous studies such as Carson and Scott (1997)

[38] Difference of annual incurred claims

and Bouzouita and Young (1998) show a negative correlation between the liquidity risk and the credit rating of insurance firms.

Gross Premium Written Growth

Generally speaking, an increase in gross premia written[39] indicates that the insurance firm is in a good financial condition. Incorporating this variable into the model will automatically exclude 'run-off' firms from the sample; these are very common in the insurance industry (insurance firms can stop underwriting business but still exist for many years).

Derivative Dummy

Shiu (2011) notes that insurers use derivatives to hedge risk, which may also increase their exposure to counterparty risk. Following his study, we obtain label 1 for a derivative user by looking for nonzero values from Form 17 of the PRA returns.

Organizational Form

Adams (1995) argued that the organizational form can partly affect the decision-making of insurance firms. A mutual insurance firm is an organisation that supplies insurance services, and that is owned by its customers, or members, which means that there are no shareholders to pay dividends to or account to. Such a firm can concentrate entirely on delivering products and services that best meet the needs of its customers. In our analysis we separate mutual and non-mutual firms.

Combined Ratio

[39] Gross premia written are the total premia written, which include both direct and assumed premia written, before any reinsurance.

We include the combined ratio defined as Incurred Claims + Management Expense) / Gross Premium Written) in the model to capture any mispricing by insurers.

Line-of-Business Concentration

Insurers with a high line-of-business concentration may have a higher earning risk. We follow Shiu (2011) in using the Herfindahl index to proxy the line-of-business concentration (a higher number indicates a lower level of business mix, the max value is:

$$H = \sum_{i=1}^{N} S_i^2 \tag{4.1}$$

where s_i is the premium written for one business line / [Gross Premium Written (Total Primary direct & fac. Business) – Gross Premium Written (Total Treaty reinsurance accepted business)].

GDP Growth

The annual percentage growth rate of GDP at market prices is calculated using values in 2005 US dollars.

Change of Wholesale Price Index (2010=100)

The wholesale price index includes a mix of agricultural and industrial goods at various stages of production and distribution, including import duties.

Change of Foreign Direct Investment, Net Inflows

Foreign Direct Investment is defined as direct investment equity flows in the reporting economy. It is the sum of equity capital, reinvestment of earnings, and other capital. Direct investment is a category of cross-border investment associated with a resident in one economy having control or a significant degree of influence on the management of an enterprise that is resident in another economy.[40]

Real Interest Rate

The real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator. The terms and conditions attached to lending rates differ by country, however, which limits their comparability.

Real Effective Exchange Rate Index (2010=100)

The real effective exchange rate is the nominal effective exchange rate (which measures the value of a currency against a weighted average of several foreign currencies) divided by a price deflator or index of costs.

Change of Credit provided by Financial Institutions (% of GDP)

Domestic credit provided by the financial sector includes all credit to various sectors on a gross basis, with the exception of credit to the central government, which is net. The financial sector includes monetary authorities and deposit money banks, as well as other financial corporations when data are available (including corporations that do not accept transferable deposits but incur such liabilities as time and savings deposits).[41]

3.2 Summary Statistics

[40] According to the World Bank, ownership of 10 percent or more of the ordinary shares of voting stock is the criterion for determining the existence of a direct investment relationship.
[41] Examples of other financial corporations are finance and leasing companies, money lenders, insurance corporations, pension funds, and foreign exchange companies.

As already pointed out, the insurance industry is relatively more stable than other industries. This is confirmed by Table 4.1, which shows that the standard deviation (Std) of underwriting profitability (PT) and investment return (InvR) is very small.[42]

[INSERT TABLE 4.1 ABOUT HERE]

[INSERT TABLE 4.2 ABOUT HERE]

Insurance firms, on average, have a higher investment profit than the traditional underwriting profit.[43] This could be driven by derivative trading (Schinasi, 2006). On average, general insurance firms in the UK hold more than 17 % (excess) capital than last year, the reason being that they have to comply with more rigorous capital requirements than financial institutions (Harrington 2009). The size of firms changes a lot over the sample period that we are considering, suggesting that, although the industry as a whole is relatively stable, many insurance firms transfer their business to others before they become insolvent.[44] This is also why we consider the transfer of the business to other companies as a possible form of exit. The high volatility of the combined ratio (Combined) could be due to mispricing problem. The Herfindahl index is 0.71 on average with a small standard deviation. This further supports the view that insurance firms are stable and their business is not very diverse.

Table 4.2 shows that, compared to the full sample, on average default firms have negative both underwriting profit (PT) and gross premium written change (GPW %), smaller firm's size (Size), lower cash ratio (CA), smaller incurred claim increase (Claim %), negative firm's growth (Growth), smaller excess capital increase (Excess %) and lower investment return (InvR). This may indicate that they lose money from their main business and their

[42] Compared to banks, insurers do not face the risk of a'bank run' and claiming payments from them normally takes longer than withdrawing cash from banks.
[43] Investing in high-risk portfolios may yield higher returns than the normal underwriting business.
[44] To protect the interests of policy-holders, insurance firms are more likely to stop writing new business or transfer their business to other firms rather than becoming insolvent.

investment performance is not as good as in the case of other firms. They are relatively small size firms with slow business growth, and holding less capital makes them more vulnerable. They also have higher leverage, reinsurance ratio, combined ratio and Herfindahl index (H-index), which suggests that they are more exposed to interest risk, credit risk and market risk. Overall, the evidence in Table 4.2 confirms the previous findings of the literature (see, e.g., Adams et al., 2003 and Shiu, 2011).

[INSERT TABLE 4.3 ABOUT HERE]

Table 4.3 shows the correlation matrix of the 12 firm-specific variables. Underwriting profitability (PT) is negatively correlated to leverage (Lev) and the reinsurance ratio. This suggests that financing activities through debt and reinsurance may reduce profits. Firm size has a positive relationship with leverage (Lev) but a negative one with the cash ratio (CA), which indicates that large firms are relatively more leveraged and hold less cash. Finally, firm size and the Herfindahl index (H-index) are negatively correlated, which suggests that large firms have a relatively concentrated business.

4. The Model

Default risk modelling has developed considerably in recent years. Beaver (1966, 1968) and Altman (1968) first proposed credit scoring models that calculate the default probability for a firm using accounting-based variables. The structural model, first used by Merton (1974), applies option theory to derive the value of a firm's liabilities in the event of default.

There are several issues arising in the context of such models. Estimating the probability of default on the basis of accounting data amounts to trying to predict a future event using financial statements designed to capture the past performance of a firm; therefore, the obtained estimates might not have strong predictive power about the future status of the firm.

Also, Hillegeist et al. (2004) find that, owing to the conservatism principle, fixed assets and intangibles are sometimes undervalued relative to their market prices causing accounting-based leverage measures to be overstated. As for the structural model, the value of a firm's assets is estimated at market prices; however, these may not contain all publicly available default-related information on the firm. Also, the term structure, off-balance and other liabilities are not well specified in structural models when calculating the default threshold of the firm, which may lead to inaccurate estimates of the default probability.

For these reasons, in this paper instead we estimate default probabilities using reduced-form models that have become increasingly popular for individual firms in recent years. Jarrow and Turnbull (1995) first introduced this type of models, which were then extended by Duffie and Singleton (1999). They assume that exogenous Poisson random variables drive the default probability of a firm. A firm will default when the exogenous variables shift from their normal levels. The stochastic process in the model is not directly linked to the firm's assets value. This makes the models more tractable. Duffie et al. (2007) first proposed a doubly stochastic Poisson model with time-varying covariates and then forecast the evolution of covariate processes using Gaussian panel vector autoregressions. The model was further developed by Duan et al. (2012), who applied a pseudo-likelihood method to derive the forward intensity rate of the doubly stochastic Poisson processes at different time horizons.

The Poisson process with stochastic intensities has been widely applied to model default events. The specification adopted in this paper assumes that the stochastic intensity has a linear relationship with macroeconomic and firm-specific variables. A doubly-stochastic formulation of the point process for default is proposed by Duffie et al. (2007), with the conditional probability of default within τ years being given by:

$$q(X_t, \tau) = E\left(\int_t^{t+\tau} e^{-\int_t^z (\lambda(u)+\varphi(u))du}\, \lambda(z)dz \middle| X_t\right) \quad (4.2)$$

where X_t is the Markov state vector of firm-specific and macroeconomic covariates, and λ_t (i.e. the conditional mean arrival rate of default measured in events per year) is a firm's default intensity. The firm may exit for other reasons, such as merger and acquisition or transfer of business to other firms, in which case the intensity is defined as φ_t. Thus the total exit intensity is $\varphi_t + \lambda_t$.

The forward default intensity is given by:

$$f_t(\tau) = exp\big(\alpha_0(\tau) + \alpha_1(\tau)X_{t,1} + \alpha_2(\tau)X_{t,2} + \cdots + \alpha_k(\tau)X_{t,k}\big) \quad (4.3)$$

and the forward combined exit intensity is defined as:

$$g_t(\tau) = f_t(\tau) + exp\big(\beta_0(\tau) + \beta_1(\tau)X_{t,1} + \beta_2(\tau)X_{t,2} + \cdots + \beta_k(\tau)X_{t,k}\big) \quad (4.4)$$

We use the pseudo-likelihood function derived by Duan et al. (2012) to estimate the forward default intensity. The details of the derivation of its large sample properties can be found Appendix A in Duan's (2012) paper. In short, the pseudo-likelihood function for the prediction time τ is defined as:

$$\mathcal{L}_\tau(\alpha, \beta; \tau_C, \tau_D, X) = \prod_{i=1}^N \prod_{t=0}^{T-1} \mathcal{L}_{\tau,i,t}(\alpha, \beta), \quad (4.5)$$

Our sample period goes from 0 to T and the frequency is annual. Firm i first appears in the sample at t_{0i} and τ_{Di} is the default time while τ_{Ci} is the combined exit time. During the sample period, if firm i exits because of default, then $\tau_{Di} = \tau_{Ci}$, otherwise $\tau_{Ci} < \tau_{Di}$. As previously explained, X_{it} are the covariates including common factors and firm-specific variables. The prediction horizon τ is measured in years with $\Delta t = 1$, and α and β are the model parameters for default and other exit processes respectively.

According to the double stochastic assumption (also known as the conditional independence assumption), firms' default probabilities only depend on common factors and firm-specific variables and are independent from each other, i.e. the default of one firm will not influence other firms' exit probabilities.

The likelihood function $\mathcal{L}_{\tau,\mathrm{i},\mathrm{t}}(\alpha,\beta)$ allows for five possible cases for firm i: in the prediction time period it can survive, default,45 exit for other reasons (which in our sample means that the insurance firm transferred its business to other firms); it can also exit after or before the prediction time period:

$$\mathcal{L}_{\tau,i,t}(\alpha,\beta)$$

$$= 1_{\{t_{0i}\le t,\tau_{Ci}\ge t+\tau\}}P_t(\tau_{Ci}>t+\tau) + 1_{\{t_{0i}\le t,\tau_{Di}=\tau_{Ci}\le t+\tau\}}P_t(\tau_{Ci};\tau_{Di}=\tau_{Ci}\le t+\tau) + 1_{\{t_{0i}\le t,\tau_{Di}\ne\tau_{Ci},\tau_{Ci}\le t+\tau\}}P_t(\tau_{Ci};\tau_{Di}\ne\tau_{Ci},\&\,\tau_{Ci}\le t+\tau) + 1_{\{t_{0i}>t\}} + 1_{\{t_{Ci}<t\}} \tag{4.6}$$

where

$$P_t(\tau_{Ci}>t+\tau) = exp\left[-\sum_{s=0}^{\tau-1} g_{it}(s)\Delta t\right]$$

$$P_t(\tau_{Ci};\tau_{Di}=\tau_{Ci}\le t+\tau)$$

$$=\left\{\begin{matrix} 1-\exp[-f_{it}(0)\Delta t]\,, when\ \tau_{Ci}=t+1 \\ \exp[-\sum_{s=0}^{\tau_{Ci}-t-2} g_{it}(s)\Delta t] * \{\exp[-f_{it}(C_{it}-t-1)\Delta t] - \exp[-g_{it}(\tau_{Ci}-t-1)\Delta t]\}, \\ when\ t+1<\tau_{Ci}\le t+\tau \end{matrix}\right\}$$

$$P_t(\tau_{Ci};\tau_{Di}\ne\tau_{Ci},\&\,\tau_{Ci}\le t+\tau)$$

$$=\left\{\begin{matrix} \exp[-f_{it}(0)\Delta t]-\exp[-g_{it}(0)\Delta t]\,, when\ \tau_{Ci}=t+1 \\ \exp[-\sum_{s=0}^{\tau_{Ci}-t-2} g_{it}(s)\Delta t] * \{\exp[-f_{it}(\tau_{Ci}-t-1)\Delta t] - exp[-g_{it}(\tau_{Ci}-t-1)\Delta t]\}, \\ when\ t+1<\tau_{Ci}\le t+\tau \end{matrix}\right\}$$

45 Default events are collected from SynThesys Non-Life and include insolvent, in liquidation, placed in administration and dissolved.

The pseudo-likelihood function $\mathcal{L}_{\tau,i,t}(\alpha,\beta)$ can be maximized numerically to obtain the estimated parameters $\hat{\alpha}$ and $\widehat{\beta}$. Owing to the overlapping nature of this function, the inference is not immediately clear. For example, at time t_5 and the prediction horizon $\tau = 2$, firm A's default over the 2-year period starting 1 year ahead (t_4 to t_6) will be correlated with firm's B default in the next time period (t_6 in this case).

In addition, the pseudo-likelihood function can be decomposed into two processes α and β, and each process can be further decomposed into different prediction horizon τ. As a result, estimates of $\hat{\alpha}$ and $\widehat{\beta}$ can be obtained at the same time.

$$\mathcal{L}_{\tau}(\alpha(s)) = \prod_{i=1}^{N}\prod_{t=0}^{T-s-1}\mathcal{L}_{i,t}(\alpha(s)), s = 0,1,\dots,\tau-1 \quad (4.7)$$

$$\mathcal{L}_{\tau}(\beta(s)) = \prod_{i=1}^{N}\prod_{t=0}^{T-s-1}\mathcal{L}_{i,t}(\beta(s)), s = 0,1,\dots,\tau-1. \quad (4.8)$$

In order to test the consistency of the model, we split the sample by using data up to 2010, 2011, 2012, 2013 and 2014, and then we calibrate the model by using the four different samples.

5. Empirical Results

5.1 Estimations Results

In our model, the logarithm of forward default intensity has a linear relationship with the covariates:

$$\lambda_t(\tau) = \alpha_0(\tau) + \alpha_1(\tau)X_{t,1} + \alpha_2(\tau)X_{t,2} + \cdots + \alpha_k(\tau)X_{t,k}. \quad (4.9)$$

$X_{t,k}$ includes GDP growth, the real interest rate, the real exchange rate, FDI, wholesale price change, credit by financial change, underwriting profit, leverage, firm's size, cash ratio, gross premium written change, reinsurance, incurred claimed change, firm's growth, excess capital change, investment return, combined ratio, Herfindahl index, derivative dummy and organizational form.

Table 4.4 below shows the main estimation results based on four different samples. Other exit (i.e. transferring business to others) outputs can be found in Appendix B

[INSERT TABLE 4.4 ABOUT HERE]

Table 4.4 shows the results for the sample periods 1985 to 2010, 2011, 2012, 2013 and 2014 respectively. We split the whole sample (1985 to 2014) into five subsamples to check if variables are consistently significant.

Overall, most our results are in line with those of Adam et al. (2003). Leverage, profit, reinsurance and organizational form[46] are significant factors both for assessing the credit risk of insurance firms and determining the quality of credit rating. On the other hand, firm size and growth are not significant[47]. Further, we find that macroeconomic and firm-specific factors (the change of credit provided by financial institutions, whole sale price change, investment profitability, combined ratio, and the usage of financial derivatives) are also important, and therefore should not be neglected. As far as we are aware ours is the first study documenting their key role in determining the credit risk of insurance firms.

In general, the real interest rate and the change in wholesale prices have a positive effect on default intensity which could lead to a higher default probability (PD). Writing Profitability and investment profitability are negatively correlated with the default intensity. This suggests that high profitable firms are less likely to become insolvent. Our results are consistent with those of Titman and Wessels (1988) and Frank and Goyal (2009), and also

[46] 'Business Activitiy' in Adam et al. (2003)

[47] This could be due to the difference in the dataset. In our paper, we have 363 general insurance firms and data from 1985 to 2014. Adam et al., (2003) paper instead, use only 40 firms rated by A.M. Best plus 25 firms rated by S&P and they apply both general insurance and life insurance firms into the model.

with the findings of Carson and Scott (1997) and Bouzouita and Young (1998), which show that higher liquidity is consistent with higher credit rating. Therefore, firms holding more capital tend to have higher liquidity and a lower default probability. In general, large firms typically have a good reputation and therefore it is easier for them to obtain credit in the market.

Bouzouita and Young (1998) find that large insurers are less likely to default. Our results suggest that firm size is not significant in determining the solvency of insurance firms. Firms with larger gross premium written will not only have cash inflows in the short term but also potential claims in the long term. The one-year PD shows that writing more premiums will lower an insurer's default probability.

Highly leveraged firms are less likely to survive during recessions and therefore normally have a higher PD. Ad hoc structured reinsurance may reduce the credit risk exposure - for example, Adams (1996) shows that reinsurance improves the ability of the primary insurer to survive an external economic shock. However, we find a positive relationship between reinsurance and PD, which suggests that heavily reinsured firms are more likely to default. Insurers transfer part or all of their risk to other insurers through reinsurance and release capital reserves; in this way they have resources to write new business. Our results show that the effect of negative perspective of reinsurers cannot be ignored. That is, insurers are exposed to the counterparty risk of reinsures (e.g. when reinsurers are insolvent or run-off, insurers will have to pay the claims to the policy-holders) and their fast-growing business may lead to higher potential losses in the future. One important question is if using derivatives increases the counterparty risk or if it is only useful for hedging. For example, derivatives could be used for hedging risk, but Shiu (2011) shows that this could also increase the exposure to counterparty risk. Our findings indicate that the use of derivatives may increase the probability of a firm becoming insolvent. This has important implications for assessing risk in this industry.

5.2 Overall Probability of Default for the General Insurance Industry

In this section we estimate the default probabilities for all active firms as well as the survival firms. This is in fact the first study using historical default risk for all active firms based on their individual PD. We also consider the performance of insurance firms during natural disasters and financial distress times (that is, at times when insurers are vulnerable and more likely to default). Figure 4.1 below shows the probability of default for the General Insurance industry from 1986 to 2014. The PDs are calculated using the parameters estimated using the full sample.

[INSERT FIGURE 4.1 ABOUT HERE]

[INSERT FIGURE 4.2 ABOUT HERE]

Figure 4.1 and Figure 4.2 show that the PD of insolvent firms is much higher and fluctuates more compared to the whole GI industry (despite the different sample size of solvent and insolvent firms, the median PD reported in Appendix C shows the same trend as in Figure 4.1 and Figure 4.2). The highest PD of insolvent firms is about 0.09 which is almost double that of the whole industry. For the GI industry, the PD peaks around the early 90s and then decreases until 2000. The PD is relatively low but increased sharply in 2008, at the time of the global financial crisis. The PD of insolvent firms peaks around 1990 and then decreases until 1998. There are two spikes in 2000 and 2003, before the 2008 global financial crisis. The average PD for default firms is 317 bps and 124 bps for the whole industry (112 bps for survival firms). The standard deviation for default firms is 0.0276, which is much higher than for the whole industry (0.0119) and survival firms (0.0101). In general, default firms are more risky compared to the whole GI industry.

[INSERT FIGURE 4.3 ABOUT HERE]

Figure 4.3 shows a large PD spread between insolvent firms and the whole GI industry during the early 90s and a sharp rise in 2000 and 2003. It is generally positive before 2009. During the financial crisis, it increases rapidly, which indicates that all insolvent firms faced a worse financial situation and are more vulnerable than the whole GI industry. After the 2008 financial crisis, it is much smaller or even negative.

To sum up, the default probability of the General Insurance industry varies over time and the PD of insolvent firms is more volatile than that of the survival firms. High PDs are usually found when there are disasters such as floods; this indicates that, unlike other financial institutions, insurance firms are relatively stable but very sensitive to natural disasters. The high PDs around the time of the 2008 financial crisis suggest that the insurance and banking industries are closely correlated. Regulators should consider these interactions and be aware of the contagion effect in distress times.

5.3 Probability of Default for Different Business Lines

In general, insurance firms have business in different sectors and firms may change their main business line over time. In addition to business concentration, the change in the credit risk in different sectors has also important implication for the regulators' supervision and policy-making decisions. This crucial issue has been completely overlooked in the literature. Here we extend the credit risk analysis to the insurance firms' business line.

On the basis of the gross premium written by each firm for different business lines, we classify insurance firms into 7 groups: 1. Accident & Health; 2. Motor; 3. Marine, Aviation

& Goods in Transit; 4. Third-party Liability; 5. Financial Loss; 6. Household& Domestic All risks and Property; 7. Miscellaneous.

[INSERT FIGURE 4.4 ABOUT HERE]

[INSERT FIGURE 4.5 ABOUT HERE]

Before 1995, Household & Domestic All Risk and Property (group 6) has the highest default probability, with Marine, Aviation & Goods in Transit (group 3) having the second highest. Third-party Liability (group5) has the third highest, and Accident & Health (group 1) the lowest. After 1995, the PDs of group 6 and group 3 are decreasing, while those of group 5 and group 1 are fluctuating around their average. The PDs of most groups exhibit an upward trend during the 2008 financial crisis, the exceptions being Marine, Aviation & Goods in Transit as well as Miscellaneous, which are relatively flat. After the financial crisis, Financial Loss and Household & Domestic All Risk & property become the riskiest groups.

Insurance firms are generally very vulnerable when natural disasters happen. Decomposing the PD of the insurance industry into different business lines reveals clear differences around the early 90s. Household, Property, Motor, Transportation and Third-party Liability are the riskiest businesses because they are more likely to be exposed to catastrophes such as floods, earthquakes etc. However, the PDs of all business lines have the same upward trend in distress times. These findings suggest that regulators might want to consider the varying composition of the premium written by insurance firms when setting capital requirements. This forward-looking PD could give supervisors at the central bank a warning sign of a risky business, and the under Solvency II the central bank could take action, for instance requiring firms to provide an additional buffer before they breach their MCR (minimum capital requirement) and SCR (solvency capital requirement)[48].

[48] The SCR and MCR act as trigger points in the 'supervisory ladder of intervention' introduced by Solvency II.

5.4 Default Clustering and Systemic Risk

Since most insurance firms are non-public firms with short history, very few of them are rated by credit rating agencies (e.g. Moody's, S&P, Fitch and A&M Best). Adam et al. (2003) analyse 65 non-life and life firms rated by A.M. Best and S&P, while our sample includes more than 300 GI firms. Using the estimation results from our model (see Table 4.4), we calculate firms' PD for all available firms (see equation 4.2). We can then extend the analysis to investigate the joint default risk. We compute pair correlations of firms for different quantiles, and use the average value to obtain the default correlation. Previews studies like Das et al. (2007) find strong default correlations among corporate obligors. It is interesting to establish whether insurance firms are likely to default jointly when their individual PDs are high. Also, high PD correlations may suggest insurers are exposed to common factors besides firm-specific factors.

[INSERT FIGURE 4.6 ABOUT HERE]

Figure 4.6 shows the average pair PD correlations of all firms within different groups (from low risky 0%-20% to high risky 80%-100%) based on data for the period 1985-2014. The highest PD correlations are found at the 0%-20% quantile, and the second highest in the riskiest group 80%-100%.

Figure 4.6 shows that the highest correlation is that between insurers with the lowest PD. This suggests that when insurance firms are less exposed to risk there is a more important role of common factors, in our case macroeconomic factors such as credit supply and wholesale price changes. Insurers within the highest 80%-100% quantile have the second highest PD correlation, which indicates that their credit risk is affected by both common and firm-specific factors. The trend of PD correlations among different quantiles supports our choice of considering macroeconomic factors as well as firm-specific factors. Overall, our empirical results are in line with those of Bell and Keller (2009), who show that insurers are less interconnected than banks and there is a lower contagion effect among them. The average PD correlation across firms is 0.1069. The PDs for the 20% quantile has the highest correlation (0.2311). The lowest correlation is 0.0072, while the PD correlation for the group '40% - 60%' is much higher (0.0675), and the PD correlation for the group '20% - 40%' is slightly higher (0.0914). The '80%-100%' group has the second largest PD correlation (0.1375); this is not every different from the industry average, and points to some default clustering. The highest PD correlation is found for the group '0%-20%', which suggests that in safe times most insurance firms are in a good financial situation.

5.5 Reinsurance and Default Risk

[INSERT FIGURE 4.7 ABOUT HERE]

Reinsurance is a pure hedging contract that enables primary insurers to transfer risks to third parties (i.e. reinsurers receive a share of annual premiums written from primary insurers to compensate potential loss events). Previous papers show that corporate hedging decisions such as reinsurance affect the strategic performance of firms (Harris and Raviv, 1991; Adam et al., 2007). Aunon-Nerin and Ehling (2008) show that indemnity contracts such as reinsurance contracts are pure hedging instruments. Harrington and Niehaus (2003) argue that reinsurance is important because solvency risk matters to both policy holders and regulators. Upreti and Adam (2015) find that reinsurance enables primary insurers to have sufficient risk capacity for planning and pricing new business lines. Therefore, insurers can be exposed to new risks through risk financing as well as reinsuring actives.

As we discussed in previous section of estimate results, high reinsurance ratio will lead primary insurers high PD. So the counter-party risk of reinsurance will increase the insolvency risk of primary insurers, and not surprisingly, under *Solvency II*, reinsurance assets are listed separately from cash and financial assets in one insurance firm's balance sheet. And the technical provision (i.e. provisions for expected future claims) of reinsurance has been incorporated into *Solvency II* as part of liabilities. The reinsurance actives link different insurers together acting as a contagious channel in the system in distress times, so the performance of reinsurance firms (firms buying reinsurance) is very important to the whole general insurance industry.

Next, we investigate the performance of firms when they use the reinsurance market. We first classify them into different groups based on the percentage of reinsurance they accept relative to their total written gross premium, and then we analyse the credit spread of firms who accepted reinsurance across the different groups.

The maximum spreads for each group are 61 bps, 194 bps, 945 bps, 386 bps, 109 bps, and 62 bps respectively, and their standard deviations 0.0064, 0.0073, 0.0198, 0.0104, 0.0042 and 0.0027. For each group, we calculate the credit spread between firms accepting reinsurance and firms not accepting it.

Figure 4.7 shows that the lowest 20% firms have a negative spread, especially during the early 90s, with respect to the firms which do not use the reinsurance market. This may indicate that firms that are less involved with the reinsurance market often have good creditworthiness and take reinsurance as part of their business plan.[49] By contrast, firms accepting 20% to 40% reinsurance can have either positive or negative spreads at different points in time in the sample period. During the financial crisis, their spread vis-à-vis firms not accepting reinsurance is negative. This may reflect their risk management strategy aimed at reducing potential losses during distress times. For the group of firms which accept between 40% to 60% and 60% to 80% reinsurance, the spread is negative most of the time,

[49] Through reinsurance, firms will have less liability, fewer reserves requirement, but release more capital to write new business or investment in other products.

except during the 90s.[50] Finally, for the group accepting more than 80% reinsurance the spread peaked during the early 90s,[51] and during the financial crisis becomes relatively small and sometimes negative.

In terms of firm size, the percentage of firms for each group is changing over years. The total percentage of firms accepting reinsurance peaked in 1988 and decreases afterwards. Most firms accept less than 20% reinsurance, and the percentage of firms is decreasing since the year of 1987. Less than 50% of firms accept 20% to 80% reinsurance, most of the time, in the last 30 years. There are more than 15% of firms in the upper 20% group between 1996 and 2005, and the number of firm peaked in 2005.

The average spread between firms accepting reinsurance and those not accepting is only -1 bps, which indicates only a slightly smaller credit risk for the former. In our sample period there are 12 years with a positive spread, and 17 years with a negative one. The results imply that firms accepting reinsurance have a lower default probability, especially during bad times (i.e. early 90s Burns' Day Storm and 2008 financial crisis). This is a new and important result. Firms taking reinsurance may choose a more determined risk management strategy and therefore buy good-quality reinsurance to help reduce risk in distress times. Doherty and Tinic (1981) find that reinsurance contracts make primary insurers manage cash flow volatility more effectively, and result in better future underwriting capacity, and lower insolvency probability. Our results show that, on the other hand, reinsurers also benefit from indemnity contracts resulting in lower default probabilities.

6. Conclusions

[50] The Burns' Day Storm occurred on 25–26 January 1990 over north-western Europe and is one of the strongest European windstorms on record. Winds of up to 100 mph kill 97 people and cause £3.37 billion worth of damage, the most costly weather event for insurers in British history.
[51] Affect by the Burns' Day Storm occurred in 1990.

This paper analyses the credit risk of general insurance (GI) firms in the UK using a unique dataset. We use a reduced form model to estimate the credit risk of GI firms by considering both insolvency and other exits such as transferring business. Our results show that most classic risk factors (for example, profitability, leverage and reinsurance etc.) are significant for assessing insurers' credit risk. In addition, we find that insurance firms are exposed to some common factors and new firm-specific risk factors such as usage of financial derivatives and investment profitability. However, in contrast to other studies, we also show that macroeconomic factors (GDP growth, interest rate, whole sale price and credit provided by financial institutions) and firm-specific factors (underwriting profit, leverage, growth premium written, reinsurance, incurred claims, excess capital, combined ratio, investment profit, usage of derivatives and organizational form) are also crucial for assessing the credit risk of General Insurance firms. This represent new and interesting evidence.

Further, we investigate the credit risk of firms with different business lines. We show that in the early 90s, owing to natural disasters, the group Household & Domestic All Risk has the highest credit risk, while after the financial crisis Third-party Liability becomes the most risky sector. Our time-varying estimates of PD could be used as an early warning for risky sectors to which regulators might want to apply more stringent minimum capital requirements. We also show the default correlation between different insurance firms is low, but there is a default clustering for GI firms. The estimated default correlations suggest insurance firms may be exposed to common risk factors.

Our findings indicate that different reinsurance levels affect the credit risk of insurance firms. Reinsurers can benefit from reinsurance contracts that have a lower default probability. This has implications for regulators of GI firms under the coming Solvency II which incorporates reinsurance into the technical provision calculations.

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Appendix

Appendix A - Insolvency cases:

AA Mutual Intl Ins

Andrew Weir Ins

Anglo American

Atlantic Mutual Intl

BAI (Run-Off)

BlackSea&Baltic

Bryanston Ins

Chester St Emp

City Intl Ins

Drake Ins

Exchange Ins

FolksamIntl UK

Highlands Ins UK

HIH Cas&Gen Ins

Independent Ins

Island Cap Europe

London Auths Mut

Millburn Ins

Municipal General

North Atlantic Ins

OIC Run-Off

Paramount Ins

Scan RE

SovereignMar&Gen

UIC Ins

Baloise Ins Ukbr

East West Ins

Fuji Intl Ins

Hiscox Ins

Metropolitan RE

Moorgate Ins

Nippon InsCo Europe

Polygon Ins UK

Swiss RE (UK)

Tower Ins Ukbr

Appendix B - Multi-Period Default Estimation Outputs

	1	2	3	4	5
C	-26.959***	-37.507	-36.182	-49.547	-18.585
	-8.411	-125.922	-61.624	-110.445	-93.697
GDP_growth	-0.013	0.045	0.079	0.449	0.015
	-0.098	-0.87	-0.373	-0.694	-0.432
Real_IR	0.109**	0.217	0.152	0.265	0.189
	-0.043	-0.422	-0.322	-0.77	-0.598
Real_EXrate	0.007	0.011	0.004	0.019	-0.003
	-0.013	-0.275	-0.145	-0.238	-0.22
FDI %	-0.073	0.022	-0.148	-0.095	-0.022
	-0.072	-0.865	-0.546	-0.566	-0.183
Wholesale Price %	22.777***	25.717	24.151	38.491	6.943
	-7.452	-95.525	-49.076	-88.161	-77.494
Credit by Financial %	-4.323***	-0.286	0.584	-2.292	4.415***
	-1.586	-1.295	-0.878	-1.739	-0.801
PT	-4.895***	-3.455	1.09	1.096	3.884
	-1.284	-38.286	-34.568	-53.766	-66.484
Lev	1.972***	2.134	1.857	1.585	2.288
	-0.323	-5.682	-4.868	-7.259	-9.352
Size	0.022	0.127	0.12	0.062	-0.146
	-0.062	-0.607	-0.31	-0.41	-0.472
CA	-0.21	0.21	-0.001	-2.414	-2.698
	-0.562	-0.757	-1.692	-5.414	-1.921
GPW%	-0.414***	-0.143	0.098	-0.164	0.016
	-0.111	-0.506	-0.28	-0.456	-0.432
Rein	1.123***	1.898	2.034	1.783	1.078
	-0.304	-9.542	-6.758	-9.141	-8.315
Claim%	0.052***	-0.012	-0.003	0.04	0.047
	-0.015	-0.331	-0.079	-0.217	-0.164
Growth	0.059	0.111	0.283	0.187	0.251
	-0.118	-1.817	-1.759	-2.643	-2.181
Eecess%	-0.229*	-0.261	-0.187	0.027	-0.101
	-0.123	-0.671	-0.356	-0.471	-0.598
InvR	-24.849***	-14.78	-4.048	-9.868	-3.836
	-7.364	-301.181	-205.288	-303.345	-303.642
Combined Ratio	0.026***	0.013	0.025	0.017	0.065

	-0.01	-0.092	-0.036	-0.148	-0.049
Herfindahl index	0.429	1.021	1.683*	1.994	1.512
	-0.377	-2.248	-0.978	-2.022	-1.518
Derivative Dummy	0.654**	0.298	0.521	0.528	0.967
	-0.275	-0.757	-0.39	-0.359	-0.779
Organizational Form	0.642**	0.604	0.577	0.225	0.07
	-0.303	-2.535	-1.484	-1.723	-1.926

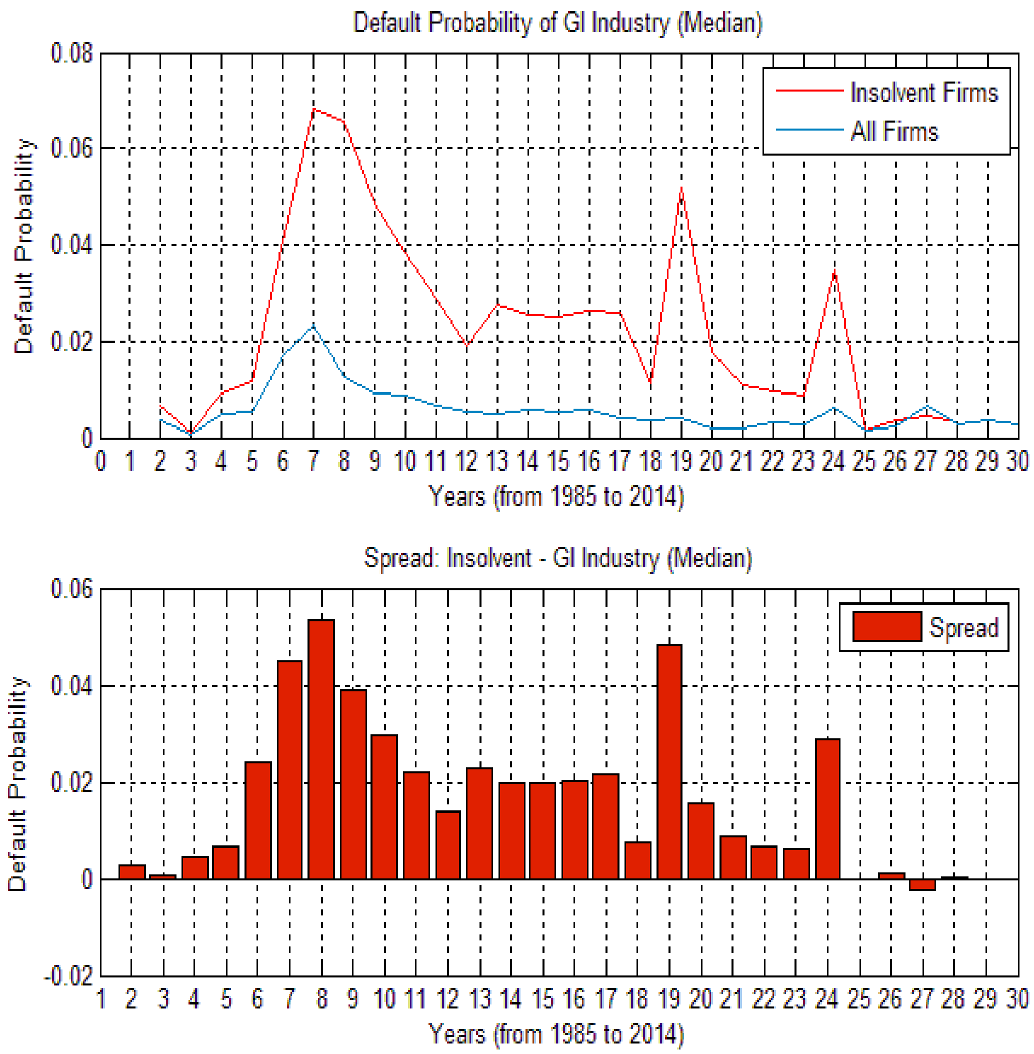
Note: this table shows the multi-period default estimation results in 1,2,3,4 and 5 years' time horizon.

Appendix C - Multi-Period Other Exit Estimation Outputs

	1	2	3	4	5
C	-2.25	-5.996	-3.812	-10.554	-6.543
	-29.285	-29.742	-18.084	-22.116	-29.397
GDP_growth	-0.046	-0.017	0.01	0.073	0.046
	-0.093	-0.045	-0.053	-0.054	-0.074
Real_IR	-0.01	0.049	0.048	0.011	-0.024
	-0.242	-0.261	-0.17	-0.216	-0.295
Real_EXrate	0.014	0.005	0.002	0.003	0.009
	-0.061	-0.058	-0.032	-0.039	-0.048
FDI %	0.002	0.011	-0.053	-0.191	0.006
	-0.132	-0.092	-0.109	-0.22	-0.092
Wholesale Price %	-4.248	-0.067	-2.454	5.55	3.514
	-23.218	-23.994	-14.397	-17.451	-26.661
Credit by Financial %	-0.535	-1.482	-1.109	-3.343	-6.793***
	-1.131	-1.07	-1.495	-2.921	-2.103
PT	-2.119	-0.118	-1.599	-2.416	-2.672
	-14.92	-16.619	-10.447	-14.4	-15.724
Lev	0.456	0.41	0.126	-0.095	-0.697
	-2.591	-2.863	-1.83	-2.345	-2.58
Size	-0.002	0.09	0.154	0.246	0.343
	-0.272	-0.288	-0.182	-0.216	-0.225
CA	-1.479*	-1.214**	-0.859**	-0.372	-0.953*
	-0.769	-0.521	-0.372	-0.414	-0.566
GPW%	-0.029	0	-0.018	0.035	0.038
	-0.158	-0.18	-0.122	-0.157	-0.154
Rein	0.56	0.718	0.655	0.622	0.744
	-2.674	-3.045	-2.04	-2.925	-4.098
Claim%	0.056	0.063***	0.043***	-0.088***	-0.114**
	-0.037	-0.022	-0.01	-0.03	-0.047
Growth	-0.315	-0.401	-0.447	-0.483	-0.505
	-0.756	-0.887	-0.602	-0.846	-1.132
Eecess%	-0.08	-0.014	-0.035	-0.055	-0.07
	-0.141	-0.158	-0.12	-0.191	-0.192
InvR	7.119	5.56	2.541	-0.625	7.667
	-120.96	-126.475	-82.1	-112.554	-134.414
Combined Ratio	0.097***	0.096***	0.101***	0.104***	0.043***
	-0.009	-0.01	-0.009	-0.006	-0.008
Herfindahl index	1.078**	1.496***	1.237***	1.227***	1.342***
	-0.435	-0.286	-0.253	-0.357	-0.271
Derivative Dummy	1.059**	0.636	0.349	0.316	0.454
	-0.53	-0.533	-0.327	-0.426	-0.489
Organizational Form	-0.1	-0.173	-0.131	-0.031	-0.181
	-1.23	-1.15	-0.725	-0.937	-1.021

Note: this table presents the multi-period estimation results of other exit in 1,2,3,4 and 5 years' time horizon.

Appendix D – Default Probability of GI Industry (Median)



Note: the figure plots one-year median default probability of all GI firms (blue) and insolvent firms (red) from 1985 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the median default probability given a state (active firms or insolvent firms) for each year. And the spread is calculated by the median PD of insolvent firms minus that of all firms.

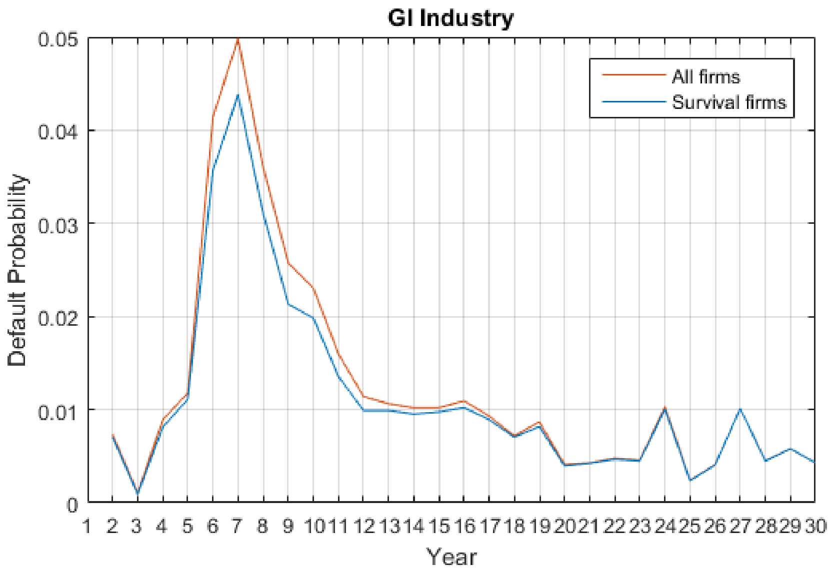


Figure 4.1 PD of all firms

Note: this figure plots one-year default probability of all GI firms (in red line) and survival firms (in blue line) from 1986 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the average default probability given a state (active firms or survival firms) for each year.

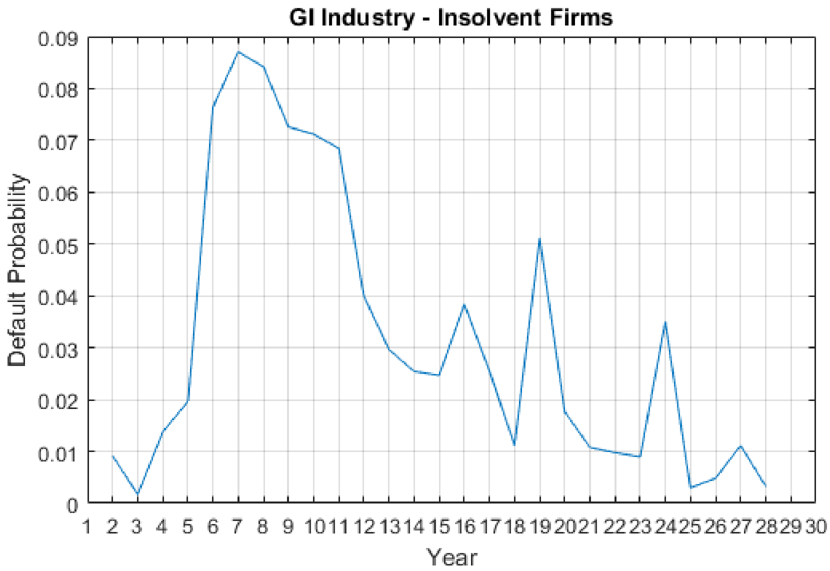


Figure 4.2 PD of all firms and insolvent firms

Note: this figure plots one-year default probability of GI firms from 1986 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the average default probability of all insolvent firms for each year.

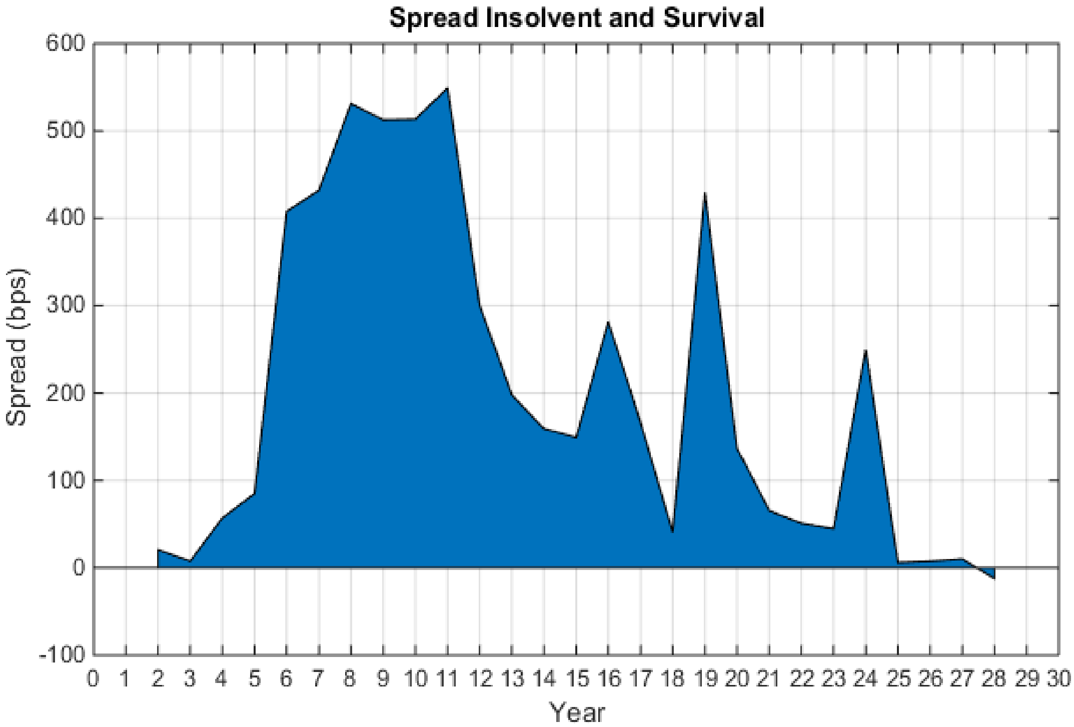


Figure 4.3 PD of Survival firms and spread between default and survival firms

Note: this figure plots one-year default probability of survival firms and the spread between survival and default firms from 1986 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the average default probability given a state (insolvent firms or survival firms) for each year. And the spread is calculated by the median PD of insolvent firms minus that of survival firms.

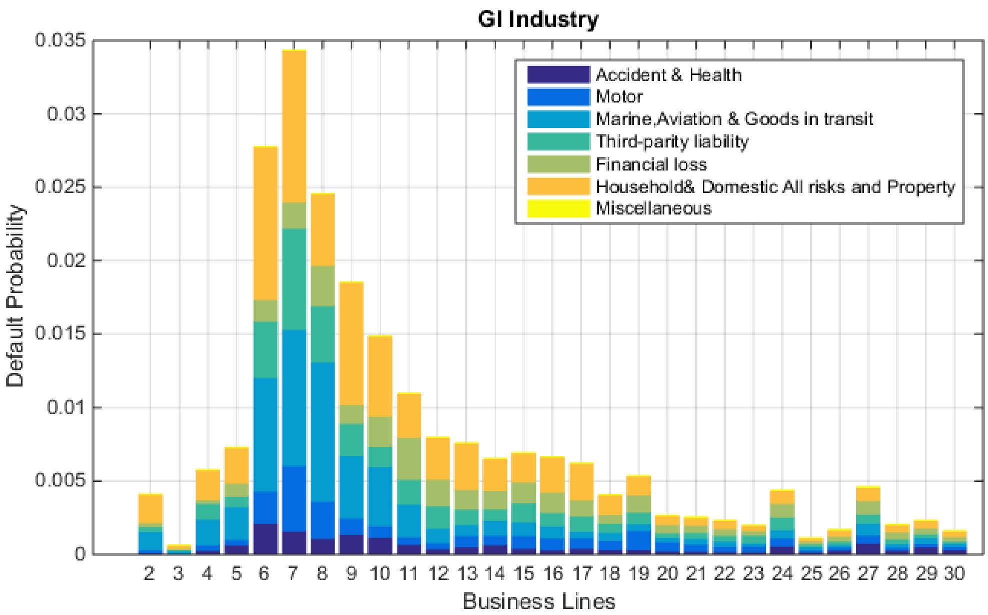


Figure 4.4 PD of 7 Groups

Note: this figure plots one-year default probability which is decomposed to 7 groups' PD from 1986 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the average default probability given a business line (accident & health; motor; marine, aviation & goods in transit; third-party liability; financial loss; household & domestic all risks and property; miscellaneous.) for each year.

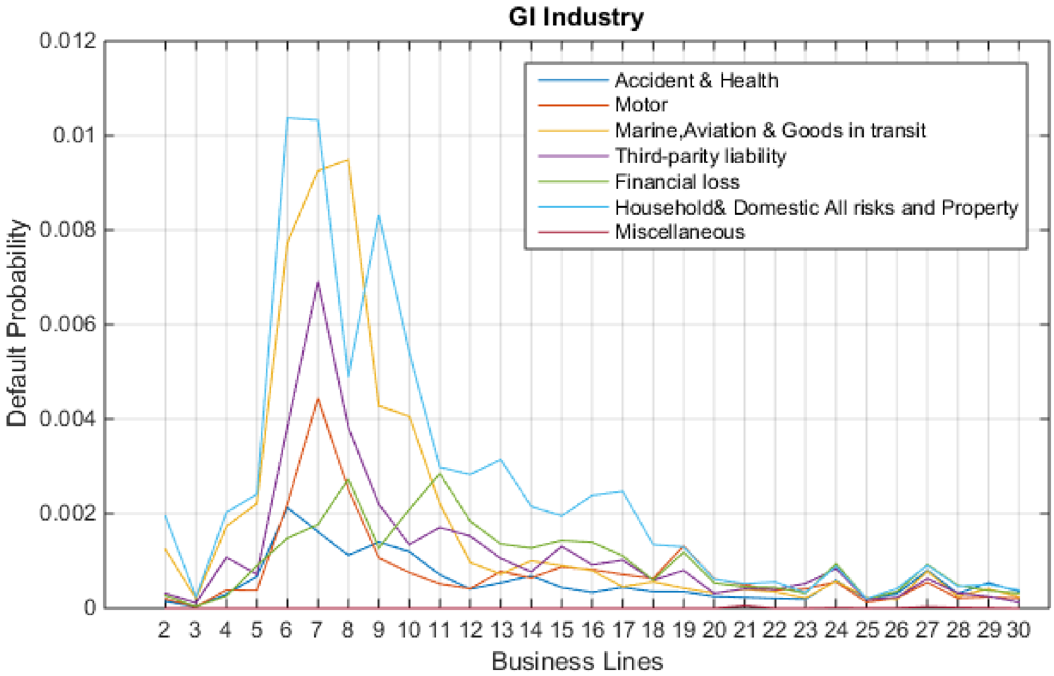


Figure 4.5 PD of 7 Groups

Note: this figure presents the one-year default probability of 7 groups: Accident & Health, Motor, Marine, Aviation & Goods in transit, Third-party liability, Financial loss, Household & Domestic all risks and property, and Miscellaneous from 1986 to 2014. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we take the average default probability given a business line for each year.

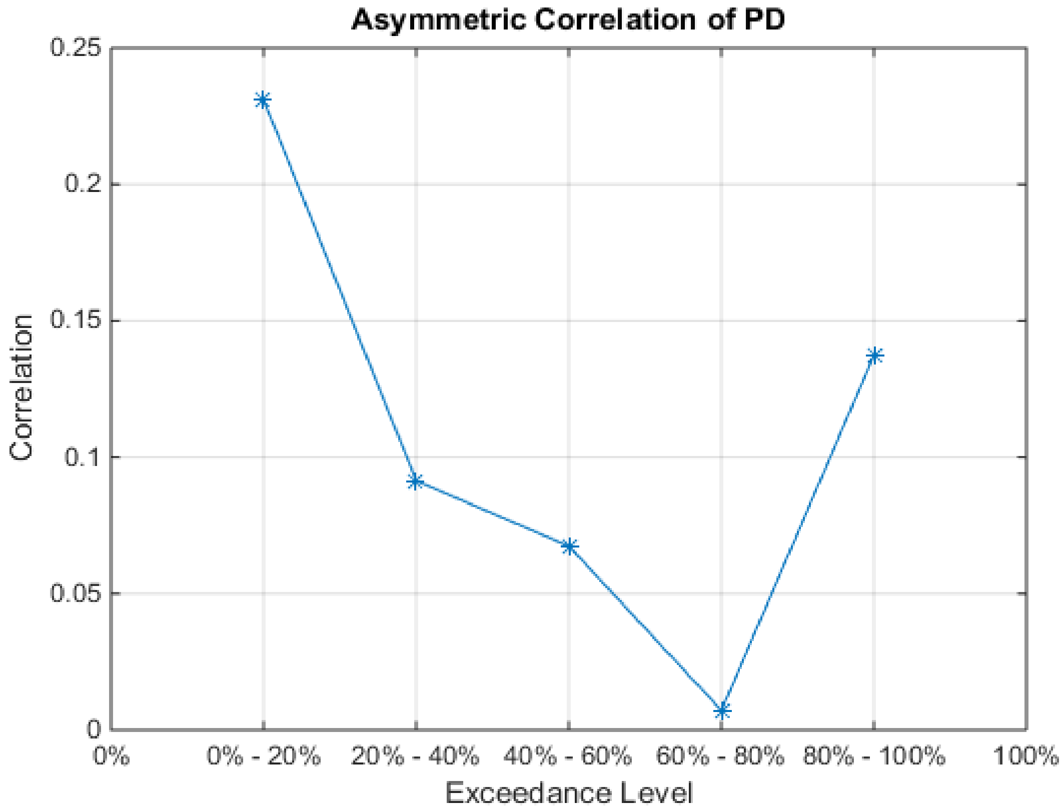


Figure 4.6 Asymmetric Correlation of PDs

Note: this figure plots correlation of PDs for the different PD quantiles in our study. We fist predict the default probability of individual firm based on the parameters estimated by the doubly stochastic Poisson model. Then we rank the PD of the whole GI industry (average PD of all firms at the year) from the lowest to highest. And we find exceedance levels according the GI industry PD. The lowest quantile 0%-20% indicates the least risky years and the highest 80%-100% quantile mean the most risky years of GI industry. And then we calculate average correlations of each pairs in the same group.

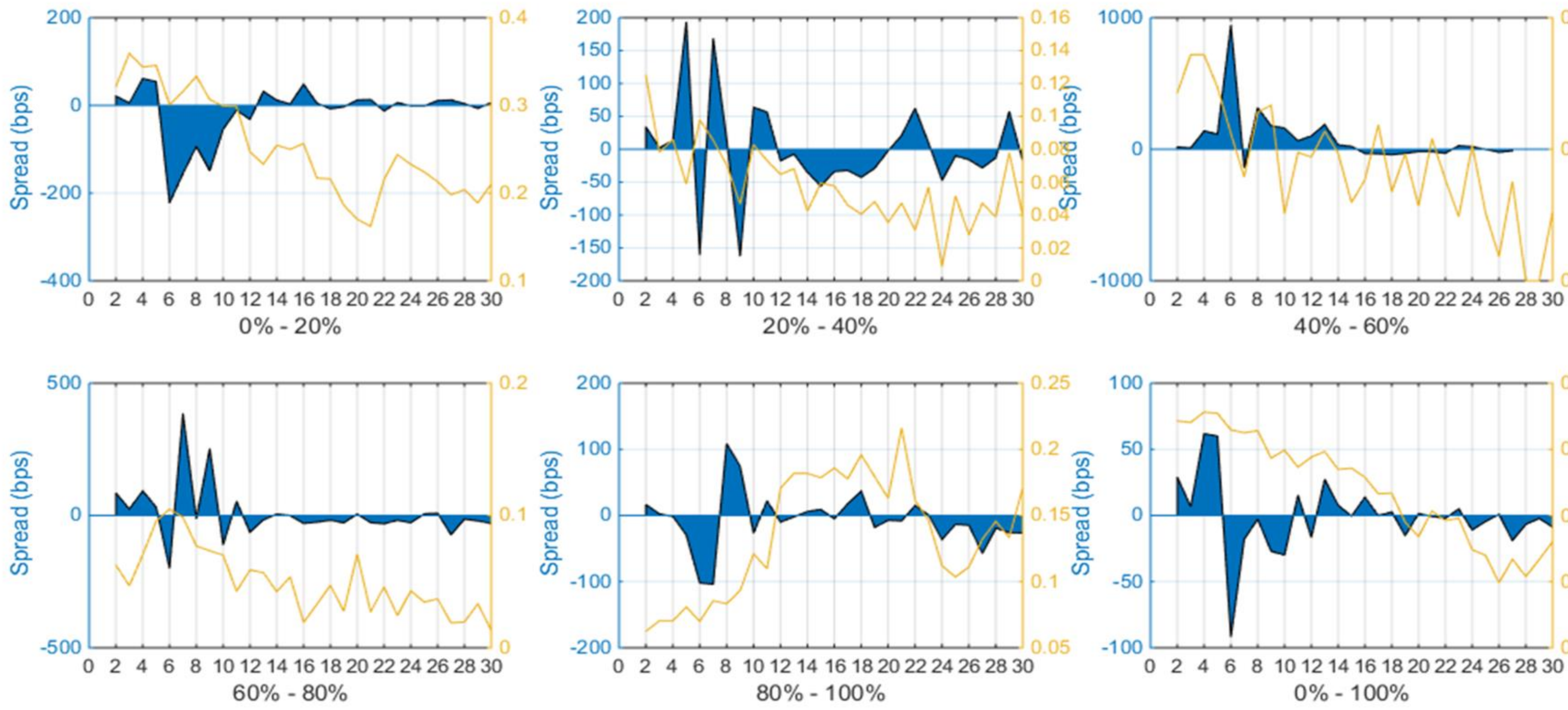


Figure 4.7 Credit spread when taking Reinsurance

Note: this figure shows the credit spread between firms accepting reinsurance and firms not accepting reinsurance at different levels (full sample calibration). Orange line: percentage of firms with reinsurance accepted at certain level; blue line: credit spread.

Table 4.1 Summary Statistics - Full Sample

	Max	Min	Median	Average	Std
PT	0.31	-0.30	-0.01	-0.01	0.06
Lev	1.36	0.02	0.60	0.59	0.32
Size	17.83	1.29	11.31	11.34	2.18
CA	0.99	0.00	0.11	0.21	0.23
GPW %	11.21	-3.50	0.03	0.08	0.93
Rein	1.00	0.00	0.24	0.32	0.30
Claim %	25.89	-6.89	0.03	0.23	1.96
Growth	6.47	-9.45	0.01	0.02	0.41
Excess %	8.74	-4.22	0.05	0.17	0.96
Combined	24.12	-1.00	0.44	1.03	2.71
InvR	0.11	-0.04	0.03	0.03	0.02
H-Index	1.00	0.00	0.76	0.71	0.28

Note: this table shows the maximum, minimum, median, average and standard deviation of firm-specific variables which including underwriting profit, leverage ratio, firm size, cash ratio, change of gross premium written, reinsurance ratio, change of incurred claims, growth ratio, change of excess capital, combined ratio, investment return and Herfindahl index of the whole industry.

Table 4.2 Summary Statistics - Default Firms

	Max	Min	Median	Average	Std
PT	0.29	-0.24	-0.02	-0.03	0.06
Lev	1.35	0.02	0.76	0.73	0.30
Size	15.25	5.82	11.46	11.33	1.66
CA	0.97	0.00	0.12	0.19	0.20
GPW %	8.03	-3.39	-0.07	-0.10	0.95
Rein	1.00	0.00	0.44	0.45	0.27
Claim %	25.07	-6.04	0.01	0.09	1.76
Growth	1.59	-1.42	-0.02	-0.01	0.28
Excess %	6.97	-3.43	0.01	0.09	1.01
Combined	24.06	-0.98	0.48	1.75	3.47
InvR	0.09	-0.03	0.02	0.02	0.02
H-Index	1.00	0.19	0.78	0.73	0.26

Note: this table presents the maximum, minimum, median, average and standard deviation of firm-specific variables which including underwriting profit, leverage ratio, firm size, cash ratio, change of gross premium written, reinsurance ratio, change of incurred claims, growth ratio, change of excess capital, combined ratio, investment return and Herfindahl index of default firms.

Table 4.3 Correlation Matrix of Firm-specific Variables

	PT	Lev	Size	CA	GPW %	Rein	Claim %	Growth	Excess %	Combined	InvR	H-Index
PT	1.00	-0.30	-0.08	0.08	0.00	-0.07	-0.07	-0.01	0.09	-0.08	-0.10	0.13
Lev	-0.30	1.00	0.46	-0.22	0.00	-0.13	-0.01	0.05	-0.01	0.05	0.01	-0.23
Size	-0.08	0.46	1.00	-0.42	-0.01	0.00	0.00	0.15	0.05	-0.08	-0.08	-0.46
CA	0.08	-0.22	-0.42	1.00	0.05	-0.16	0.01	0.01	0.03	-0.03	0.16	0.18
GPW %	0.00	0.00	-0.01	0.05	1.00	-0.05	0.23	0.32	0.02	-0.11	0.01	-0.06
Rein	-0.07	-0.13	0.00	-0.16	-0.05	1.00	0.00	-0.04	0.00	0.05	-0.33	-0.14
Claim %	-0.07	-0.01	0.00	0.01	0.23	0.00	1.00	0.18	0.02	-0.08	0.01	0.00
Growth	-0.01	0.05	0.15	0.01	0.32	-0.04	0.18	1.00	0.25	-0.17	-0.05	-0.07
Excess %	0.09	-0.01	0.05	0.03	0.02	0.00	0.02	0.25	1.00	-0.04	0.00	0.00
Combined	-0.08	0.05	-0.08	-0.03	-0.11	0.05	-0.08	-0.17	-0.04	1.00	-0.04	0.11
InvR	-0.10	0.01	-0.08	0.16	0.01	-0.33	0.01	-0.05	0.00	-0.04	1.00	-0.06
H-Index	0.13	-0.23	-0.46	0.18	-0.06	-0.14	0.00	-0.07	0.00	0.11	-0.06	1.00

Note: this table shows the correlation matrix of underwriting profit,leverage ratio, firm size, cash ratio, change of gross premium written, reinsurance ratio, change of incurred claims, growth ratio, change of excess capital, combined ratio, investment return and Herfindahl index of the whole industry.

Table 4.4 Estimations Results

	2010	2011	2012	2013	2014
Parameters					
C	-19.364***	-25.172***	-24.116***	-25.313***	-26.959***
	(-6.143)	(-7.441)	(-7.798)	(-8.054)	(-8.411)
GDP_growth	-0.092	-0.019	-0.028	-0.010	-0.013
	(-0.062)	(-0.083)	(-0.087)	(-0.090)	(-0.098)
Real_IR	0.009	0.034	0.085**	0.098**	0.109**
	(-0.055)	(-0.045)	(-0.042)	(-0.043)	(-0.043)
Real_EXrate	-0.003	0.006	0.006	0.007	0.007
	(-0.009)	(-0.011)	(-0.012)	(-0.012)	(-0.013)
FDI %	-0.027	0.008	-0.052	-0.109	-0.073
	(-0.086)	(-0.089)	(-0.067)	(-0.075)	(-0.072)
Wholesale Price %	17.830***	23.977***	21.153***	21.988***	22.777***
	(-5.355)	(-6.754)	(-6.691)	(-6.998)	(-7.452)
Credit by Financial %	-5.103**	-6.674***	-5.011***	-4.824***	-4.323***
	(-2.001)	(-2.244)	(-1.650)	(-1.669)	(-1.586)
PT	-4.112***	-4.409***	-4.815***	-4.867***	-4.895***
	(-1.328)	(-1.143)	(-1.204)	(-1.224)	(-1.284)
Lev	1.863***	1.840***	1.966***	1.973***	1.972***
	(-0.311)	(-0.296)	(-0.311)	(-0.313)	(-0.323)
Size	0.030	0.028	0.016	0.018	0.022
	(-0.061)	(-0.059)	(-0.061)	(-0.061)	(-0.062)
CA	-0.182	-0.160	-0.228	-0.224	-0.210
	(-0.574)	(-0.549)	(-0.560)	(-0.561)	(-0.562)
GPW%	-0.180	-0.435***	-0.416***	-0.413***	-0.414***
	(-0.175)	(-0.109)	(-0.108)	(-0.109)	(-0.111)
Rein	1.109***	1.003***	1.091***	1.108***	1.123***
	(-0.275)	(-0.259)	(-0.270)	(-0.279)	(-0.304)
Claim%	0.041***	0.040***	0.055***	0.053***	0.052***

	(-0.015)	(-0.015)	(-0.015)	(-0.015)	(-0.015)
Growth	0.014	-0.010	0.055	0.060	0.059
	(-0.120)	(-0.105)	(-0.115)	(-0.116)	(-0.118)
Eecess%	-0.287**	-0.281**	-0.230*	-0.231*	-0.229*
	(-0.122)	(-0.113)	(-0.122)	(-0.123)	(-0.123)
InvR	-25.362***	-25.449***	-25.184***	-24.899***	-24.849***
	(-5.517)	(-4.585)	(-5.832)	(-6.190)	(-7.364)
Combined Ratio	0.024***	0.023**	0.025**	0.025**	0.026***
	(-0.009)	(-0.010)	(-0.010)	(-0.010)	(-0.010)
Herfindahl index	0.606	0.508	0.439	0.435	0.429
	(-0.380)	(-0.367)	(-0.373)	(-0.374)	(-0.377)
Derivative Dummy	0.607**	0.641**	0.655**	0.653**	0.654**
	(-0.278)	(-0.281)	(-0.276)	(-0.275)	(-0.275)
Organizational Form	0.509	0.528*	0.553*	0.559*	0.642**
	(-0.321)	(-0.316)	(-0.308)	(-0.310)	(-0.303)

*Significant at 10%
** Significant at 5%
*** Significant at 1%

Note: this table shows coefficients of constant, GDP growth, real interest rate, real exchange rate, foreign direct investment %, whole sale price %, credit provided by financial institutions %, underwriting profit, leverage, size, cash ratio, gross premium written %, reinsurance, incurred claims %, growth, excess capital, combined ratio, investment return, Herfindahl index, derivative dummy and organizational form based on five different samples using data from 1985 to 2010, 2011, 2012, 2013 and 2014 respectively. Results of multi-periods (5 years' horizon) estimation which based on full sample (1985-2014) can be found in Appendix B and Appendix C.

Chapter 5

Conclusions and Further Research

Credit risk management has been well developed in the last decades. From credit scoring models to structural models, and reduced-form models, it allows us to quantify the credit risk of firms based on firm-specific and macroeconomic variables. The global financial crisis and the credit crunch, European debt crisis that followed make credit risk management to play a crucial role in the central bank.

The first chapter of this thesis provides an extensive review of recent developments credit risk models and some important applications in systemic risk management by Copula theory. We briefly introduce the credit scoring model which is an accounting-based model. Second, we review the development of widely used which are structural models and reduced form model. The structural-form models are developed in many ways, such as stochastic process, term structure and credit spread etc. However, structural-form models still face difficulties in estimating a firm’s market value and credit rating, which can reveal that the financial health of a firm is not incorporated in the structural models. In recent years, the reduced-form models which use an exogenous Poisson random variable to determine the default probability is becoming more and more popular. Reduced-form models treat default as an unpredicted event given by hazard process. The exogenous variable which is not dependent on the value of firm's asset makes reduce-form models more tractable than structural models. Then, we provide a comprehensive review of recent developed reduced-form model in multi-periods forecasting. Lastly, we discuss copula-based models in joint default risk modelling.

The second chapter of this thesis is to assess the credit risk of China's public listed firms. The research is based on the unique default dataset of Chinese public firms from 1999 to 2013 which is provided by Risk Management Institute (RMI) of National University of Singapore (NUS). In order to explore the determinants of firm failure using both firm-specific and macroeconomic covariates, we apply the binomial multivariate logit model as the benchmark model which takes default and non-default into account (i.e. after a firm default, the rest data of the firm will be discarded). Since in China's market, most firms survive after the default, we consider three states (default, post-default and tranquil) of firms to include as much information as possible (post-default data will remain in the sample). The test shows a significant difference of firm-specific data between post-default and tranquil times. In the following, the multinomial logit model is selected to assessing firm's default probability. The estimation results suggest that both binomial and multivariate logit models outperform the KMV structural model. Some results are surprisingly different from the US market: for example, some traditional determinants like profitability and market / book value are not significant in China. In addition, unlike the US market, firms in China holding more liquid assets are more likely to default. In addition, we find ST may not be a good default proxy. We further analyse the PD of delisted firms and the results show no clear difference of default probability between active firms and delisted firms in recent years.

The third chapter investigates the linkage between the insurance and financial sectors based on the default probability for individual firms. Time-varying symmetric correlations between these two sectors are found for both short-term (1-month to 12-month) and long-term horizons (24-month to 60-month). We estimate the time-varying symmetric correlations by student's t GAS copula correlations with skewed t distribution. The negative correlation is found in the 1-month horizon. Short-term correlations are relatively lower but more fluctuating while the long-term correlations suggest that insurance and financial firms are positively correlated. In addition, we find economic drivers of the movement of insurance and finance industries by regressing the copula correlations with macroeconomic variables. In general, short-term correlations (from 1-month to 12-month horizon) are more likely driven by economic factors like credit index, VIX, interest rate level, yield curve slope, TED spread, crude oil, and index return. Long-term correlations (from 24-month to 60-month) are less sensitive to market conditions. Our results suggest they are only affected by interest rate level, yield curve slope, and market volatility (except for 60-month). Lastly, we simulate the joint default probability for the two sectors by using the cupula correlations. The joint PD between these two sectors are very sensitive to global financial market events, such as 1997 Asia crisis, 1998 Argentine and Russian financial crises, 2000 Dotcom bubble, 2008 global financial crisis, 2010 European debt crisis, and 2011 S&P downgrading of US sovereign debt ratings.

The forth chapter analyses the credit risk of general insurance (GI) firms in the UK using a unique dataset. By using the reduced form model to estimate the credit risk of GI firms, we consider both insolvency and other exits such as transferring business. Our results show that most classic risk factors (for example, profitability, leverage and reinsurance etc.) are significant for assessing insurers' credit risk. In addition, we find that insurance firms are exposed to some common factors and new firm-specific risk factors such as usage of financial derivatives and investment profitability. We also show that macroeconomic factors (GDP growth, interest rate, whole sale price and credit provided by financial institutions) and firm-specific factors (underwriting profit, leverage, growth premium written, reinsurance, incurred claims, excess capital, combined ratio, investment profit, usage of derivatives and organizational form) are also crucial for assessing the credit risk of General

Insurance firms. Further, we investigate the credit risk of firms with different business lines. The time-varying estimates of PD could be used as an early warning for risky sectors to which regulators might want to apply more stringent minimum capital requirements. We also look at the default correlation between different insurance firms and find the correlation is low, but there is a default clustering for GI firms. In addition, we study the relationship between reinsurance levels and the credit risk of insurance firms. The results show that reinsurers can benefit from reinsurance contracts that have a lower default probability. This has implications for regulators of GI firms under the new Solvency II regulations which incorporate reinsurance into the standard technical provision calculations.

The empirical findings in this thesis have important implications for both investors and policy-makers for the credit risk management. However, some challenges are still left for future research. First, the modelling of default probability is well studied, but the study is based on large datasets which include firm-specific variables, macroeconomic variables, and most importantly, the default data. The default probability modelling of small and new markets which lack actual default events is a very difficult but interesting topic. Second, in the thesis, we study the determinants of credit risk of China's market, and the results show that ST may not be a good default proxy. Given the fact that many mergers and acquisitions happened within ST firms, the determinants of ST firms in China's market is not very clear and it would be worthwhile to explore that in future. Third, we investigate the dynamic correlation between insurance firms and other financial institutions in the US market using the copula method. The other joint movements among financial entities is interesting: for example, the correlation between banks and hedge funds. Finally, there are many unexplored areas of credit risk management in the insurance industry. The possible extension may include the insurance firms' capital structure before and after Solvency II and how reinsurance and derivatives trading affect the creditworthiness of insurance firms.