Empirical Essays in Quantitative Risk Management

by

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M.Sc. in Finance and Investment

B.Sc. in Statistics

Submitted in fulfilment of the requirements for
the Degree of Doctor of Philosophy

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University of Glasgow

March 2016

Abstract

Copula theory is particularly useful for modeling multivariate distributions as it allows us to decompose a joint distribution into marginal distributions and a copula. Copula-based models have been widely applied in finance, insurance, macroeconomics, microeconomics and many other areas in recent years. This doctoral thesis particularly pays attention to applications of copula theory in quantitative risk management.

The first chapter of this thesis provides a comprehensive review of recent developments of copula models and some important applications in the large and growing finance and economics literature. The first part of this chapter briefly introduces the definition and properties of copulas as well as several related concepts. The second part reviews estimation and inference methods, goodness-of-fit tests and model selection tests for copula models considered in the literature. The third part provides an exhaustive review of the extensive literature of copula-based models in finance and economics. Finally, an interesting topic for further research is suggested.

The remaining three chapters investigate applications of copula theory in three topics: market risk prediction, portfolio optimization and credit risk estimation.

Chapter Two investigates the dynamic and asymmetric dependence structure between equity portfolios from the US and UK. We demonstrate the statistical significance of dynamic asymmetric copula models in modeling and forecasting market risk. First, we construct “high-minus-low" equity portfolios sorted on beta, coskewness, and cokurtosis. We find substantial evidence of dynamic and asymmetric dependence between characteristic-sorted portfolios. Second, we consider a dynamic asymmetric copula model by combining the generalized hyperbolic skewed t copula with the generalized autoregressive score (GAS) model to capture both the multivariate non-normality and the dynamic and asymmetric dependence

between equity portfolios. We demonstrate the usefulness of this model by evaluating the forecasting performance of Value-at-Risk and Expected Shortfall for high-minus-low portfolios. From backtesting, we find consistent and robust evidence that our dynamic asymmetric copula model provides the most accurate forecasts, indicating the importance of incorporating the dynamic and asymmetric dependence structure in risk management.

Chapter Three investigates the dependence between equity and currency in international financial markets and explores its economic importance in portfolio allocation. First, we find striking evidence for the existence of time-varying and asymmetric dependence between equity and currency. Second, we offer a methodological contribution. A novel time-varying skewed *t* copula (TVAC) model is proposed to accommodate non-Gaussian features in univariate time series as well as the dynamic and asymmetric dependence in multivariate time series. The multivariate asymmetry is captured by the skewed *t* copula derived from the mutlivariate skewed *t* distribution in Bauwens and Laurent (2005) and the time-varying dependence is captured by the GAS dynamics proposed by Creal et al. (2013). This model can be easily generalized from the bivariate case to the multivariate case. Third, we show that findings of dynamic and asymmetric dependence between equity and currency have important implications for risk management and asset allocation in international financial markets. Our empirical results show the statistical significance of the TVAC model in risk management and its economic values in real-time investment.

Chapter Four studies the credit risk of UK top-tier banks. We document asymmetric and time-varying features of dependence between the credit risk of UK top tier banks using a new CDS dataset. The market-implied probability of default for individual banks is derived from observed market quotes of CDS. The default dependence between banks is modeled by a novel dynamic asymmetric copula framework. We show that all the empirical features of CDS spreads, such as heavy-tailedness, skewness, time-varying volatility, multivariate asymmetries and dynamic dependence, can be captured well by our model. Given the marginal default probability and estimated copula model, we compute the joint and conditional probability of default of UK banks by applying a fast simulation algorithm. Comparing our model with traditional copula models, we find that the traditional models may underestimate the joint credit risk most of the time, especially during a crisis. Furthermore, we perform an extensive regression analysis and find solid evidence that time-varying tail dependence between

CDS spreads of UK banks contains useful information to explain and predict their joint and conditional default probabilities.

Chapter Five concludes with recommendations for further study.

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Acknowledgements

Firstly I would like to express my sincere gratitude to my first advisor Professor Mario Cerrato, for his patience, enthusiasm, wisdom and thoughtful guidance. Mario gave me the freedom to do my research during my doctoral study, continually contributing insightful feedback and helpful suggestions. I also want to express my heartfelt thanks to my second advisor Professor Christian Ewald for his immense knowledge, warm encouragement, and continuous support of my doctoral study and research. I also gratefully acknowledge Dr. Minjoo Kim, who has given me numerous good suggestions and comments, and sometimes taken the role of third advisor. Without them, this thesis would never have been written.

Besides my advisors, I would like to acknowledge the rest of my thesis committee: Professor Neil Kellard, Dr Georgios Sermpinis and Dr Serafeim Tsoukas for their time, interest, valuable suggestions and insightful comments. I would also like to thank Professor Paul Embrechts and Professor Alexander McNeil. Professor Embrechts's masterclass in Cambridge was my first official training on copula theory, and his magic in that masterclasses made copulas fascinating for me. His wisdom inspired me to start learning about copulas and exploiting their usage in quantitative finance. He also provided many very insightful answers and suggestions in answer to all my confusion about copulas after that. Professor McNeil has given me lots of free opportunities to attend trainings and industrial events on copula theory and quantitative risk management organized by the Scottish Financial Risk Academy (SFRA). They have been my primary resource for learning copula theory and its applications in finance. I am deeply thankful to Professor Carol Alexander, for her warm encouragement and insightful comments on the early version of my third chapter. In particular, I am grateful to Dr. Carlo Rosa, who is the advisor of my M.Sc. dissertation, for enlightening me in my first stage of postgraduate research.

Special thanks go to my friends and colleagues, Yang Pang, Ding Liu, Yuping Huang, Xuxin Mao, Yukun Shi, Chao Qiu, Jun Qiu, Xuan Zhang, Xiao Zhang, Shuo Cao and Jilong Chen, amongst many others. My sincere thanks also go to my great office mates and good friends, Stasinakis Charalampos and Sultan Towaim. They have made my time in Glasgow both more enjoyable and more productive.

Finally, and most importantly, I would particularly like to acknowledge my parents Dongfang Zhao and Yaling Yao, for their unconditional love, constant support, and for enormous sacrifices they have willingly made for me.

Dedication

I dedicate this dissertation to
my parents Dongfang Zhao and Yaling Yao
for their constant support and unconditional love.

Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signature:

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Chapter 1

Copula Models in Finance and Economics: A Comprehensive Survey

Copula theory is particularly useful for modeling multivariate distributions as it allows us to decompose a joint distribution into marginal distributions and a dependence function (copulas). Copula-based models have been widely used in finance, insurance, macroeconomics, microeconomics and many other areas in recent years. The purpose of this chapter is to provide a comprehensive review of recent developments of copula models and some important applications in the large and growing finance and economics literature. First, we briefly introduce the definition and properties of copulas as well as several related concepts. Second, we review estimation and inference methods, goodness-of-fit tests and model selection tests for copula models considered in the literature. Third, we provide an exhaustive review of the extensive literature of copula-based models in finance and economics.

1.1 Introduction

The concept of "copula" can be traced back to the work of Sklar (1959), but its importance in applications of finance and economics only has been realized since the late nineties. Mathematically speaking, copula is a multivariate distribution function with uniformly distributed margins. Nelsen (2013) defines copulas as "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions". It provides a convenient and flexible way of describing the dependence between different components (See Sklar, 1996 for a detailed proof of Sklar's Theorem).

There are several desirable properties of copulas. First, the flexibility of copulas allow us to specify marginal distributions separately from the dependence structure, without the requirement that they come from the same family of joint distributions. Second, a copula-based model can be used as a practical and flexible instrument to generate Monte Carlo scenarios of risk factor returns. Third, copulas express dependence on a quantile scale, which is able to describe both non-linear and tail dependence and is natural in a risk management context. Fourth, copulas allow us to combine more sophisticated marginal models with a variety of possible dependencies and to investigate the sensitivity of risk to the dependence specifications (McNeil et al., 2005; Nelsen, 2013).

Embrechts et al. (2002), which was widely circulated as a working paper in 1999, is one of the earliest papers exploiting the usage of copulas in finance. Copulas have attracted much attention and became popular in financial industry after the appearance of another influential work by Li (2000). Since then, copulas have been widely applied to handle the comovement between time series in literature, including finance (Christoffersen et al., 2012; Chollete et al., 2011; Christoffersen and Langlois, 2013; Chollete et al., 2009; Patton, 2004, 2006a,b; Rosenberg and Schuermann, 2006; Jin and De Simone, 2014, etc), actuarial science and insurance (Embrechts et al., 2002; Frees and Valdez, 2008; Rosenberg and Schuermann, 2006, etc.), and economics (Acharya et al., 2012; Bonhomme and Robin, 2009; Brendstrup et al., 2007; Engle et al., 2015, etc.), among many others. The amount of papers on copula theory and its applications in the above fields has been growing remarkably in recent decades. We provide a more detailed discussion about applications of copula models in finance and economics in Section 1.5.

There are several important surveys on copula theory and applications in the existing literature. Two fundamental and comprehensive textbooks are Joe (1997) and Nelsen (2013), which provide exhaustive introductions to copula theory and dependence modeling with mathematical proofs. These two textbooks introduce the mathematical foundations of copula modeling in detail, including definitions and basic properties of copulas, methods of constructing copulas, Archimedean copulas and dependence measures. Cherubini et al. (2004) and Cherubini et al. (2011) present comprehensive overviews of copula-based models on applications of mathematical finance, such as credit derivatives pricing and option pricing. A concise review of the early start of copula models in finance and insurance can be found in Embrechts et al. (2009). This review also contains a list of copula must-read references as well as a comment on potential future development of this field. Genest et al. (2009a) provide an bibliometric overview of the rapid development of copula models in mathematics, statistics, actuarial science and finance from 1970 to 2005. Jaworski et al. (2010) contains several surveys from different aspects of copula modeling and empirical studies investigating applications of copulas to finance and insurance. For instance, one survey in this book, Choroś et al (2010), extensively reviews both parametric and nonparametric estimations of copulas for both independent and identically distributed (hereafter *i.i.d.*) data and time series data. Manner and Reznikova (2012) review different copula models with time-varying dependence structure and compare their applicability in different cases. Patton (2009a), Patton (2012a) and Patton (2012b) provide comprehensive reviews of copula models for economic and financial time series, as well as detailed empirical studies to illustrate estimation and inference methods of various copula models. Embrechts and Hofert (2014) concisely review mathematical properties and important algorithms of copula models to show their usage in quantitative risk management. See also Alexander (2009) and Andersen et al. (2013) for succinct reviews of treatments of copulas in financial risk management. More in-depth discussions about the treatment and usage of copula-based models in quantitative risk management can be found in McNeil et al. (2005) and its updated version McNeil et al. (2015).

The structure of the remainder of this chapter is as follows. Section 1.2 introduces the definitions and basic properties of copulas. Several important related concepts are also briefly introduced in this section. In Section 1.3, we survey estimation and inference methods of copulas in literature. Section 1.4 reviews several widely used goodness-of-fit tests to evaluate copula models. In Section 1.5, we provide a comprehensive review of applications of copulas in finance and economics. In Section 1.6, we summarize and discuss several possible extensions of copula modeling. Figures are presented in the appendix.

1.2 Copulas: Concept and Properties

1.2.1 Sklar's Theorem

Theorem 1 (Sklar, 1959): Suppose a vector random variable $\mathbf{X} = [X_1, X_2, ..., X_n]'$, with joint distribution function $\mathbf{F}$ and marginal distribution functions F_n, then there exists a copula C (a distribution function on $[0,1]^n$ with uniform marginals on $[0,1]$, i.e. $C : [0,1]^n \to [0,1]$) such that

$$\mathbf{F}(x_1, ..., x_n) = \mathbf{C}(F_1(x_1), F_2(x_2), ..., F_n(x_n)), \ \forall \mathbf{x} \in \mathbb{R}^n. \tag{1.1}$$

C is uniquely defined if F_n are continuous. From equation 1.1, we know that a copula is essentially a joint distribution of probability integral transformations (PITs) of random variables.[1] A detailed analytic proof of this theorem can be found in Sklar (1996).

Given any copula C and univariate distribution functions F_n , $\mathbf{F}$ defined by Equation 1.1 is a n-variate distribution function with margins F_n. Define U_i as the PIT of X_i, i.e. $U_i \equiv F_i(X_i),\ i \in \{1, ..., n\}$, then $U_i \sim \mathbf{U}[0,1]$ and define $\mathbf{U} = [U_1, U_2, ..., U_n]'$, then the distribution function of $\mathbf{U}$ can be denoted by a copula C. Thus, Equation 1.1 can be also written as

$$C(u_1, u_2, ..., u_n) = \mathbf{F}\left(F_1^{-1}(u_1), F_2^{-1}(u_2), ..., F_n^{-1}(u_n)\right), \tag{1.2}$$

where F_i^{-1} denotes the inverse distribution function of F_i, where $X_i = F_i^{-1}(U_i)$ for $0 \leq U_i \leq 1$.[2] There are two important interpretations of Sklar's theorem. First, it allows one to decompose any multivariate distribution function into its univariate margins and a copula and thus to investigate the univariate margins and multivariate distribution, separately. Suppose the joint distribution function $\mathbf{F}$ is n-times differentiable, F_i has density f_i and copula C has

[1] Probability transformation: Let a random variable X has a continuous distribution with the cumulative distribution function F, then $F(X)$ has a uniform distribution $F(X) \sim \mathrm{U}[0,1]$.

[2] Quantile transformation: Let $U \sim \mathrm{U}[0,1]$ and F the distribution function of any random variable X, then $F^{-1}(U) \equiv X$ so that $F^{-1}(U) \sim F$.

density c, then the n^{th} cross-partial derivative of equation 1.1, i.e. the density $\mathbf{f}$ of $\mathbf{F}$, is given by

$$\begin{aligned}
\mathbf{f}(\mathbf{x}) &\equiv \frac{\partial^n}{\partial x_1 \partial x_2 \cdots \partial x_n} \mathbf{F}(\mathbf{x}) \\
&= \prod_{i=1}^{n} f_i(x_i) \cdot \frac{\partial^n}{\partial x_1 \partial x_2 \cdots \partial x_n} C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \\
&\equiv \prod_{i=1}^{n} f_i(x_i) \cdot c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)).
\end{aligned} \tag{1.3}$$

so from equation 1.3, we know that the joint density can also be decomposed into two parts: the product of its marginal densities and a corresponding copula density. The decomposition of joint density implies that the joint log-likelihood of $\mathbf{f}$ can be split into the sum of univariate log-likelihood and the likelihood of copula density:

$$\log \mathbf{f}(\mathbf{x}) = \sum_{i=1}^{n} \log f_i(x_i) + \log c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)). \tag{1.4}$$

Thus, the parameters can be estimated through standard maximum likelihood estimation (MLE). To illustrate the ability of copulas to model various dependence structures, we provide counter plots for different bivariate copula densities in Figure 1.1. More functional forms and details of copula models can be found in Joe (1997) and Nelsen (2013).

[INSERT FIGURE 1.1 ABOUT HERE]

1.2.2 Survival Copula

Let $\mathbf{x}$ be a random vector with multivariate survival functions $\mathbf{S}$, marginal distribution functions F_n and marginal survival function S_n, where $S_i = 1 - F_i$. Using Sklar's theorem, we have

$$\mathbf{S}(x_1, x_2, \ldots, x_n) = \bar{C}(S_1(x_1), S_2(x_2), \ldots, S_n(x_n)). \tag{1.5}$$

where $\bar{C}$ is the survival copula of C. For the bivariate case, the relationship between a copula and corresponding survival copula is that

$$\begin{aligned}\bar{C}(1-u_1,1-u_2) &= 1-(1-u_1)-(1-u_2)+C(1-u_1,1-u_2) \\ &= -1+u_1+u_2+C(1-u_1,1-u_2)\end{aligned} \tag{1.6}$$

If $\bar{C}=C$, C is called radially symmetric.

1.2.3 The Fréchet–Hoeffding Bounds Theorem

Theorem 2 (Fréchet–Hoeffding bounds): For any n-dimensional copula $C:[0,1]^n \rightarrow [0,1]$ and any $\mathbf{u}=[u_1,u_2,...,u_n] \in [0,1]^n$, the following bounds hold:

$$W(u_1,u_2,...,u_n) \leq C(u_1,u_2,...,u_n) \leq M(u_1,u_2,...,u_n). \tag{1.7}$$

where W denotes the lower Fréchet–Hoeffding bound and is defined as

$$W(u_1,u_2,...,u_n) = \max\left\{1-n+\sum_{i=1}^{n} u_i, 0\right\}. \tag{1.8}$$

and M denotes the upper Fréchet–Hoeffding bound and is defined as

$$M(u_1,u_2,...,u_n) = \min\{u_1,u_2,...,u_n\}. \tag{1.9}$$

W is a copula if and only if $n=2$ (the *comonotonic* model) and M is a copula for all $n \geq 2$ (the *countermonotonic* model). This theorem is important as it suggests that the marginal distribution functions constrain the joint distribution function built on them. For the bivariate case, the Fréchet–Hoeffding bounds theorem states:

$$\max\{u+v-1,0\} \leq C(u,v) \leq \min\{u,v\} \tag{1.10}$$

The Fréchet–Hoeffding bounds have important implications in risk management and portfolio optimization. Figure 1.2 shows distributions of the Fréchet–Hoeffding upper bound, lower bound and independence copula.

[INSERT FIGURE 1.2 ABOUT HERE]

1.2.4 Tail Dependence

Let X_1 and X_2 be continuously distributed random variables with distribution functions F_1 and F_2. Provided a limit $\lambda_L \in [0,1]$ exists, the lower tail dependence coefficient (LTDC) λ_L of X_1 and X_2 is defined by

$$\lambda_L = \lim_{q \to 0^+} \mathbb{P}\left(X_2 \leq F_2^{-1}(q) \mid X_1 \leq F_1^{-1}(q)\right), \tag{1.11}$$

If $\lambda_L \in [0,1]$, then X_1 and X_2 are lower tail dependent. If $\lambda_L = 0$, then X_1 and X_2 are asymptotically independent in the lower tail. Analogously, provided a limit $\lambda_U \in [0,1]$ exists, the upper tail dependence coefficient (UTDC) λ_U of X_1 and X_2 is defined by

$$\lambda_U = \lim_{q \to 1^-} \mathbb{P}\left(X_2 > F_2^{-1}(q) \mid X_1 > F_1^{-1}(q)\right), \tag{1.12}$$

If $\lambda_U \in [0,1]$, then X_1 and X_2 are upper tail dependent. If $\lambda_U = 0$, then X_1 and X_2 are asymptotically independent in the upper tail. Tail dependence is a property of a copula, since

$$\begin{aligned}
& \mathbb{P}\left(X_2 \leq F_2^{-1}(q) \mid X_1 \leq F_1^{-1}(q)\right) \\
= & \frac{\mathbb{P}\left(X_2 \leq F_2^{-1}(q), X_1 \leq F_1^{-1}(q)\right)}{\mathbb{P}\left(X_1 \leq F_1^{-1}(q)\right)} \\
= & \frac{\mathbb{P}\left(F_2\left(X_2 \leq F_2^{-1}(q)\right), F_1\left(X_1 \leq F_1^{-1}(q)\right)\right)}{\mathbb{P}\left(F_1\left(X_1 \leq F_1^{-1}(q)\right)\right)} \\
= & \frac{\mathbb{P}\left(F_2(X_2) \leq q, F_1(X_1) \leq q\right)}{\mathbb{P}\left(F_1(X_1) \leq q\right)} = \frac{C(q,q)}{q}, \; q \in (0,1)
\end{aligned} \tag{1.13}$$

thus $\lambda_L = \lim\limits_{q \to 0^+} \frac{\mathbf{C}(q,q)}{q}$. Analogously, for the upper tail dependence, we use Equation 1.6 to obtain

$$\lambda_U = \lim_{q \to 1^-} \frac{1-2q+C(q,q)}{1-q} = \lim_{q \to 1^-} \frac{\bar{C}(1-q,1-q)}{1-q} = \lim_{q \to 0^+} \frac{\bar{C}(q,q)}{q}, \; q \in (0,1) \tag{1.14}$$

where $\bar{C}$ is the survival copula of C (see the definition in Equation 1.5). For both lower tail and upper tail dependence, we have $\lambda_L, \lambda_U \in [0,1]$, since λ_L and λ_U are conditional probabilities. For radially symmetric copulas, $\lambda_L = \lambda_U$, since $\bar{C} = C$, see McNeil et al. (2015).

1.3 Model Estimation and Inference

1.3.1 One-stage Maximum Likelihood Estimation

In this section, we briefly describe some general approaches of estimation and inference for copula-based models. Theoretically, estimation of margins and a copula can be done by the one-stage maximum likelihood estimation (also known as full maximum likelihood estimation, FML). The vector of parameters can be estimated by maximizing the log-likelihood function of Equation 1.4. Although the one-step MLE estimator is efficient and asymptotically normal, it normally suffers computationally complexities as it solves the whole system simultaneously.

1.3.2 Two-stage Maximum Likelihood Estimation

Alternatively, the two-stage estimation procedure for copula modeling is more popular in practice. First, the one-stage MLE is computationally difficult especially when the number of parameters increases, whereas the two-stage MLE is computationally easier to implement without losing much asymptotic efficiency (see Joe, 1997; Patton, 2006b). Second, the two-stage procedure is relatively straightforward and convenient for the comparison of different copula candidates with the same specification of univariate margins. Third, modeling the margins and dependence separately may yield more insight and allow a more in-depth analysis of various model components (McNeil et al., 2015).

Various methods have been proposed in literature to obtain the marginal estimate $F_{i,t}$ and they can be generally classified into three types:

(1) Parametric estimation. There are many possible choices for the parametric models (distributions) for $F_{i,t}$. For the return data of financial risk factors, several widely used distributions including the Gaussian, the Student's t, the skewed t and normal inverse Gaussian (NIG), can be considered.[3] It is most important to choose an appropriate distribution from the many possible candidates and then to fit our data to it, by MLE. For modeling operational loss or insurance data, the Pareto or the lognormal distribution are normally considered.

(2) Nonparametric estimation. The nonparametric estimate for marginals is based on the empirical distribution function (EDF) $\hat{F}_i$ and the estimated probability integral transform variable $\hat{U}_{i,t}$ can be obtained via a rescaled empirical distribution function

$$\hat{F}_i(z) \equiv \frac{1}{T+1}\sum_{t=1}^{T} \mathbf{1}\left(\hat{z}_{i,t} \leq z\right), \text{ where } \hat{u}_{i,t} = \hat{F}_i\left(\hat{z}_{i,t}\right) \sim Unif\ (0,1), \tag{1.15}$$

where $z_{i,t}$ denotes the standardized residuals of risk factor returns, the denominator is different from the normal empirical distribution function by using $T+1$ instead of T. We use this to guarantee that $\hat{u}_{i,t}$ lies strictly within the unit cube.

(3) Semiparametric estimation. Some empirical studies suggest that the EDF cannot capture well tail behaviors of underlying distribution. One way to solve this problem is using a generalized Pareto distribution (GPD) from extreme value theory (EVT) to model the tail behaviors. The center of distribution can be still described by EDF. See McNeil and Frey (2000) for more detailed discussion and implementation.

Thus, the copula-based models can be estimated either parametrically or semiparametrically or nonparametrically. When both marginal distributions and the copula are estimated using parametric models, the resulting estimation is fully parametric, see Joe (1997). When the marginal distribution is estimated by the nonparametric or semiparametric models, but their dependence is characterized by parametric copula functions, the resulting estimation is semiparametric, see Genest et al. (1995), Chen and Fan (2006a) and Chen and Fan (2006b), etc. When the marginal distributions are estimated using nonparametric models, and dependence is also characterized by nonparametric copula functions, the resulting joint distribution is fully nonparametric, see Genest and Rivest (1993) and Capéraà et al. (1997), etc.

For the full parametric case, we apply MLE to obtain parameters for univariate margins in the first stage and then, holding the univariate parameters fixed from the first stage, we

[3] Many different univariate skewed t-type distributions have been proposed in literature and several widely used versions in financial time series include Hansen (1994), Fernández and Steel (1998), Jones and Faddy (2003) and Aas and Haff (2006), etc.

use MLE again to estimate copula parameters in the second stage. The likelihood of a fully parametric copula model for conditional distribution of $\mathbf{z}_t$ takes the form:

$$\begin{aligned} L(\theta) &= \prod_{t=1}^{T} \mathbf{f}(\mathbf{z}_t | \mathcal{F}_{t-1}; \theta) \\ &= \prod_{t=1}^{T} \left[c_t \left(u_{1,t}, \ldots, u_{d,t} | \mathcal{F}_{t-1}; \theta_C \right) \prod_{i=1}^{N} f_{i,t} \left(z_{i,t} | \mathcal{F}_{t-1}; \theta_i \right) \right] \end{aligned}$$

with log-likelihood

$$\mathscr{L}(\theta) = \sum_{t=1}^{T} \log \mathbf{f}(\mathbf{z}_t | \mathcal{F}_{t-1}; \theta)$$

$$= \sum_{t=1}^{T} \sum_{i=1}^{d} \log f_{i,t} \left(z_{i,t} | \mathcal{F}_{t-1}; \theta_i \right) + \sum_{t=1}^{T} \log c_t \left(F_{1,t} \left(z_{1,t} | \mathcal{F}_{t-1}; \theta_1 \right), \ldots, F_{d,t} \left(z_{d,t} | \mathcal{F}_{t-1}; \theta_d \right) | \mathcal{F}_{t-1}; \theta_C \right)$$

where θ denotes the parameter vector for the full model parameters, θ_i denotes the parameters for the ith marginals, θ_C denotes the parameters of copula model and $\mathcal{F}_{t-1}$ denotes the information set available at time $t-1$. Following the two-stage maximum likelihood estimation (also known as the Inference Method for Marginals, IFM) of Joe and Xu (1996) and Joe (1997), we first estimate the parameters of marginal models using maximum likelihood:

$$\hat{\theta}_{\mathbf{i}} = \underset{\theta_i}{\operatorname{argmax}} \sum_{t=1}^{T} \log f_{i,t} \left(z_{i,t} | \mathcal{F}_{t-1}; \theta_i \right), \ i = 1, \ldots, N$$

and then using the estimations in the first stage, we calculate $F_{i,t}$ and estimate the copula parameters via maximum likelihood:

$$\hat{\theta}_{\mathbf{C}} = \underset{\theta_C}{\operatorname{argmax}} \sum_{t=1}^{T} \log c_t \left(F_{1,t} \left(z_{1,t} | \mathcal{F}_{t-1}; \theta_1 \right), \ldots, F_{d,t} \left(z_{d,t} | \mathcal{F}_{t-1}; \theta_d \right) | \mathcal{F}_{t-1}; \theta_C \right) \tag{1.16}$$

Notice that IFM does not lead to efficient estimators but the asymptotic efficiency loss for this method is acceptable, see Joe (1997) and Joe (2005). Shih and Louis (1995) show asymptotic properties of this estimator for *i.i.d.* data and Patton (2006b) shows the conditions under which the two-stage MLE is not less asymptotically efficient than the one-stage MLE for time series data via simulations. The most attractive advantage of IMF estimation is that this method can significantly reduce the computation complexity.

In the semiparametric estimation (also known as Canonical Maximum Likelihood Estimation, CML), the univariate marginals are estimated nonparametrically using the empirical distribution function (See Equation 1.15) and the copula model is again parametrically estimated via maximum likelihood:

$$\hat{\theta}_{\mathbf{C}} = \arg\max_{\theta_C} \sum_{t=1}^{T} \log c_t \left(\hat{u}_{1,t}, \ldots, \hat{u}_{i,t} | \mathcal{F}_{t-1}; \theta_C\right) \tag{1.17}$$

This estimator is also sometimes called "pseudo-maximum likelihood" in literature, see for instance Genest et al. (1995) and Klaassen and Wellner (1997). It is noted that the semiparametric estimator $\hat{\theta}_{\mathbf{C}}$ is consistent, asymptotically normal and fully efficient at independence, see Genest et al. (1995). Shih and Louis (1995) investigate the asymptotic properties of the semiparametric estimator in the *i.i.d.* case. Chen and Fan (2006a) and Chen and Fan (2006b) also establish the asymptotic properties of the semiparametric estimator for time series data. Furthermore, some studies show the robustness of semiparametric estimation via extensive simulations. For instance, Kim et al. (2007) show that without knowing the marginal distributions, the semiparametric copula model can provide better estimations than FML and IFM in statistical computations and data analysis.

Some studies also investigate fully nonparametric estimations of copula models. For instance, Genest and Rivest (1993) studies the nonparametric estimation of Archimedean copulas for *i.i.d.* data. The nonparametric estimation of various extreme value copulas are extensively studied by many researchers, for instance Capéraà et al. (1997), Zhang et al. (2008), Genest and Segers (2009), Kojadinovic and Yan (2010), Gudendorf and Segers (2011), Gudendorf and Segers (2012), among many others. The estimation for time series data are considered by Fermanian and Scaillet (2003), Fermanian et al. (2004), Sancetta and Satchell (2004), Gagliardini and Gouriéroux (2007) and Ibragimov (2009), etc. Furthermore, a novel estimation of copula models based on a simulated method of moments (MM) has been recently proposed by Oh and Patton (2013b).

1.4 Evaluation of Copula–based Models

The evaluation of a proposed model is of particular importance in econometric modeling. Two types of tests are most commonly applied to evaluate copula-based models: goodness-of-fit tests (GoF) and model selection tests. The former test is normally used to access how well the proposed copula fits a set of specified observations and the latter is commonly used to determine which copula in a given set of candidates is able to provide the best performance in dependence measuring. In econometric application, these two types of tests are mutually complementary.

1.4.1 Goodness-of-fit Test

Various GoF tests for copulas have been proposed in recent decades, see Fermanian (2005), Scaillet (2007) and Savu and Trede (2008) for the GoF tests for *i.i.d.* data, and Breymann et al. (2003), Malevergne and Sornette (2003) and Kole et al. (2007) for the case of time series data. For the semiparametric copula model, a simulation-based approach is considered by Chen and Fan (2006a) to obtain the critical values for copula GoF tests.

Berg (2009) implements a power comparison of copula GoF tests by examining the effect of dimension, sample size and strength of dependence, and concludes that no approach always performs better than any other. Genest et al. (2009b) also provide a critical review of various copula GoF tests available in the literature. They carry out a comparative power study of the GoF tests for copula models through an extensive simulation study and find that a Cramér–von Mises test is the most powerful.

1.4.2 Model Selection

The primary goal of model selection is to determine the most parsimonious model that adequately fits specific data sample. Comparison between candidates of copula models can be accomplished via statistical criteria. The most straightforward way is using classical likelihood ratio tests, see Greene (2011) and Davidson and MacKinnon (2004). The intuition behind a likelihood ratio test is that if the restricted copula model is inadequate, the difference between the log-likelihood values of a restricted model (e.g. the Gaussian copula) and an unrestricted model (e.g. the Student's t copula) should be significantly different from zero. Given the log-likelihood values for two competing copula-based (restricted and unrestricted) models, the test statistic can be easily obtained by

$$\mathrm{LR} = 2\left(\mathscr{L}_1\left(\hat{\theta}_1\right) - \mathscr{L}_2\left(\hat{\theta}_2\right)\right),\ \mathrm{LR} \sim \chi^2(k) \tag{1.18}$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the maximum likelihood estimators of restricted and unrestricted models, respectively. LR is distributed as χ^2 with the degree of freedom k, where k is the number of restrictions. Notice that the classical likelihood ratio requires nested models, i.e. the unrestricted model can be transformed into the restricted model by imposing some restrictions. In other words, the restricted one should be a special case of the unrestricted one, as the Gaussian copula is a special case of the t copula or the t copula a special case of the GH skewed t copula.

Another more generalized likelihood ratio test proposed by Vuong (1989) and Rivers and Vuong (2002) is also considered in copula literature, for instance Patton (2012a) and Patton (2012b). The most desirable property of their test is that it allows non-nested competing models. The null hypotheses and alternative hypotheses are given by

$$\begin{aligned} H_0 &: E\left[\mathscr{L}_{1,t}\left(\hat{\theta}_1\right) - \mathscr{L}_{2,t}\left(\hat{\theta}_2\right)\right] = 0 \qquad (1.19) \\ H_1 &: E\left[\mathscr{L}_{1,t}\left(\hat{\theta}_1\right) - \mathscr{L}_{2,t}\left(\hat{\theta}_2\right)\right] > 0 \\ H_2 &: E\left[\mathscr{L}_{1,t}\left(\hat{\theta}_1\right) - \mathscr{L}_{2,t}\left(\hat{\theta}_2\right)\right] < 0 \\ \text{where } \mathscr{L}_{i,t}\left(\hat{\theta}_i\right) &\equiv \log \mathbf{f}_{i,t}\left(\mathbf{z}_t | \mathcal{F}_{t-1}; \hat{\theta}_i\right) \qquad (1.20) \end{aligned}$$

The t-statistic of the difference between the sample averages of the log-likelihoods is asymptotically normally distributed. Patton (2012a) shows that this test can be used for both constant and dynamic conditional copula models. Furthermore, Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC), which include a penalty of the number of estimated parameters, can also be applied to the model selection.

1.5 Applications of Copulas in Finance and Economics

Applications of copulas in finance and economics can be broadly categorized into eight areas.

1.5.1 Quantitative Risk Management

Copula-based models have been considered as a useful tool in quantitative risk management since early 2000 and have been widely used to estimate and forecast portfolio risk in recent years. The advantages of copula-based models in risk management are three-fold. First, copula-based models provide us with a flexible tool to handle different dependence structures between assets, since various copulas can be used to model different kinds of dependence (See Figure 1.1). Second, a bottom-up approach can be built based on the copula theory, which allows us to model margins and their dependence separately. This property is very attractive in risk management because it enable us to combine more sophisticated marginal models with various copulas in multivariate modeling. Third, copulas are easily simulated and thus it is very convenient to implement Monte Carlo simulation based on estimated copulas. This provides us a powerful method to generate a variety of scenarios for portfolios to obtain their P&L distributions (McNeil et al., 2015).

The well-publicized trading losses of several large financial institutions (e.g. Barings Bank, Long-Term Capital Management, Lehman Brothers) and the recent global financial crisis of 2007-2009 have led financial regulators and supervisory authorities to favor quantitative techniques which assess the possible loss that these institutions can incur. Value-at-Risk (VaR) and Expected Shortfall (ES) are the most prevalent measures to quantify market risk. VaR provides an estimation (quantile) of the likely losses which could arise from price changes over a pre-determined horizon at a given level. ES is a coherent risk measure and can be viewed as the expected loss when VaR is exceeded. For more details about theoretical concepts and modeling techniques of VaR and ES, see Jorion (2007) and Alexander (2009).

In the past decade, numerous studies have developed and applied a variety of copula-based models to estimate and forecast portfolio VaR or ES for different assets in different markets, see, for example Cherubini and Luciano (2001), Embrechts et al. (2002), Glasserman et al. (2002), Embrechts et al. (2003a), Christoffersen and Pelletier (2004), Embrechts and Höing (2006), Kole et al. (2007), Hsu et al. (2008), Huang et al (2009), Giacomini et al. (2009), Chollete et al. (2009), Hafner and Reznikova (2010), Hsu et al. (2012), Patton (2012a) and Cerrato et al. (2015), among many others. One copula paper of integrated risk management is Rosenberg and Schuermann (2006), which measures integrated risk by combining market, credit and operational risks together using copulas. McNeil et al. (2015) provides a comprehensive treatment of copula theory and its applications in quantitative risk management.

1.5.2 Portfolio Management

Empirical studies provide solid evidence that modeling time-varying and asymmetric dependence are very informative for portfolio decision making. Patton (2004) find economic significance of univariate and multivariate asymmetries for out-of-sample asset allocation decisions. Hong et al. (2007) demonstrate the economic importance of incorporating asymmetric characteristics into asset allocation using Clayton and mixture copulas.

Some studies apply copula-based models to study the international diversification benefit. For instance, Chollete et al. (2011) examine diversification opportunities in international markets using copula models and show international limits to diversification. Christoffersen et al. (2012) propose a dynamic asymmetric copula to capture multivariate nonnormality and asymmetries for developed and emerging market indices and find that diversification benefits have mostly disappeared for developed markets, whereas emerging markets still able to offer substantial diversification benefits.

Instead of modeling asset or index returns, Christoffersen and Langlois (2013) study the dependence structure between four important equity market factors (i.e. Fama-French three-factors and momentum factor) and show that significant economic gains are obtained when accounting for the nonlinear dependence measured by dynamic copulas. Elkamhi and Stefanova (2014) develop a copula-based model that allows for increased and asymmetric dependence between extreme asset returns. They demonstrate that taking into account dependence between extreme events in portfolio decisions can yield significant economic values. Another interesting topic is copula-based portfolio optimization, however related studies are very limited, see Boubake and Sghaier (2013) for instance.

1.5.3 Financial Derivatives

Credit derivatives, such as credit default swaps (CDS) and collateralized debt obligations (CDO), normally contain risks coming from different sources. Thus, copulas are very powerful tools to model and price multivariate risks from these derivative contracts. To the best of our knowledge, Li (2000) is the first study exploiting the usage of the Gaussian copula function in credit risk modeling. He applies this copula function to investigate default correlation and further value credit derivatives, such as CDS and first-to-default contracts. A large number of studies exploit applications of copula functions in credit risk, including Frey and McNeil (2003), Hull and White (2004), Giesecke (2004), Hull and White (2006), Hull and White (2010), Hofert and Scherer (2011), Duffie (2011), Duffie and Singleton (2012), Oh and Patton (2013a) and Christoffersen et al. (2014), among many others. In addition, some studies also investigate applications of copulas in other derivatives, including pricing multivariate contingent claims (Rosenberg, 2003), quanto FX options (Bennett and Kennedy, 2004), better-of-two-markets and worse-of-two-markets options (van den Goorbergh et al., 2005), currency options (Salmon and Schleicher, 2006; Taylor and Wang, 2010), and energy futures (Grégoire et al., 2008), among others. Cherubini et al. (2004) and Cherubini et al. (2011) provide detailed treatments of applications of copula functions in mathematical finance.

1.5.4 Asymmetric Dependence Testing and Modeling

It is a stylized fact in finance that dependence between asset returns is more correlated during market downturns than during market upturns. This phenomenon is named "asymmetric correlation/dependence" and has been investigated by a large number of empirical studies without drawing on copula theory, see for instance, Kroner and Ng (1998), Longin and Solnik (2001), Ang and Chen (2002), Ang and Bekaert (2002), Bae et al. (2003), Yuan (2005), among many others. More generally, they find overwhelming evidence against the assumption that asset returns are multivariate normally distributed. These findings indicate that the dependence between asset returns is non-Gaussian. One attractive property of copula-based models is that some can capture well non-Gaussian features between asset returns, such as multivariate asymmetry.

The amount of empirical studies on asymmetric dependence in financial markets using copulas has been increasing rapidly since 2000. A strand of studies applies copula models to verify and investigate multivariate asymmetries between assets in different markets, including equity markets (see Christoffersen et al., 2012; Jondeau and Rockinger, 2006; Christoffersen and Langlois, 2013; Cerrato et al., 2015; Hong et al., 2007; Chollete et al., 2009; Okimoto, 2008; Patton, 2004; Kang et al., 2010; Li, 2014, etc), the foreign exchange market (Bouyé and Salmon, 2009; Patton, 2006a; Li, 2011), bond markets (Garcia and Tsafack, 2011), future markets (Hsu et al., 2008) and energy markets (Aloui et al., 2014; Reboredo, 2011), among others. Another strand of studies examines asymmetric dependence between different kinds of assets, such as equity and currency (Ning, 2010), equity and commodity (Wen et al., 2012), and currency and commodity (Wu et al., 2012).

1.5.5 Time-varying Dependence Modeling

Empirical finance studies provide a wealth of evidence that the correlations and volatility of financial time series change over time, thus, many multivariate GARCH models have been proposed to capture time-varying correlations and covariance matrix, see Bollerslev et al. (1988), Bollerslev (1990), Engle (2002) and Tse and Tsui (2002), among many others. For comprehensive reviews , see Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009). Furthermore, many empirical studies also show another stylized fact. that conditional dependence structure between asset returns varies through time, see Dias and Embrechts (2004), Patton (2006a), Jondeau and Rockinger (2006), Giacomini et al. (2009) and Christoffersen et al. (2012), etc. This noteworthy phenomenon motivates the consideration of time-varying copulas, which allow correlation parameters to change dynamically. Recently, various models have been proposed in the literature to capture this feature of financial time series.

Patton (2006a) introduces the notion of time-varying copula where the copula function is constant but its parameter is allowed to vary through time as a function of an autoregressive term and transformations of the lagged data. Similar observation driven copula models also can be found from Jondeau and Rockinger (2006), Ausin and Lopes (2010), Fermanian and Wegkamp (2012). Creal et al. (2013) proposed a popular driving mechanism for time-varying copulas called the Generalized Autoregressive Score (GAS) model. This is an observation driven model which uses the scaled score of likelihood function to update the model parameters over time. They show that the GAS model is a consistent and unified framework, which encompasses some well-known models in the literature. They also illustrate the GAS framework by introducing new model specifications for time-varying copula. Based on the results of simulation and empirical evidence, they note that the driving mechanism of Patton (2006a) only captures part of the variations in the dependence coefficients. They point out that Patton's conditional copula model cannot capture the dynamics of upper and lower tail dependence simultaneously. Correspondingly, the GAS specification has better performance in capturing different types of dynamics.

Christoffersen et al. (2012) develop a dynamic asymmetric copula (DAC) model to capture trends in dependence, multivariate nonnormality, and asymmetries in a large set of developed and emerging markets. They utilize the dynamic conditional correlation (DCC) model proposed by Engle (2002) and Tse and Tsui (2002) as the driving mechanism to update dynamic copula correlations. The function of dynamic copula parameter is composed by the weighted average of a constant term, the lagged conditional correlation matrix and the lagged cross-product of the standardized copula shocks. They apply recent econometric innovations and a composite likelihood procedure proposed by Engle et al. (2008) to overcome computational complications arising from the high dimensionality that promote estimation using long samples of asset returns for a large number of countries. They also illustrate that the implementation of their DAC model is relatively straightforward and computationally efficient. Other studies, which also consider combining DCC GARCH with copula models, include Lee and Long (2009), Fengler et al. (2012), among others.

There are other three strands of literature about time-varying copula models. The first strand is termed the "Stochastic copula", which is based on the different stochastic volatility models, see Manner and Segers (2011), Hafner and Manner (2012) and Creal and Tsay (2015). The second strand of research is inspired by the regime switching model of Hamilton (1989). Some studies consider the "Regime switching copula" that allows the functional form of copula to vary for different states, see for instance Rodriguez (2007), Okimoto (2008), Chollete et al. (2009), Garcia and Tsafack (2011) and Wang et al. (2013), etc. The third strand, called the "Locally constant copula", is considered by Giacomini et al. (2009), Dias and Embrechts (2010), Guegan and Zhang (2010), Harvey (2010), Hafner and Reznikova (2010) and Busett and Harvey (2011). For a comprehensive review of time-varying copula models, see Manner and Reznikova (2012)

1.5.6 Copula-based Quantile Regression

One interesting topic of copula application is the copula-based quantile regression. Several copula-based regression models have been developed in statistical literature, see for instance Oakes and Ritz (2000), Pitt et al. (2006), etc. However, their applications in finance and economics have not been extensively investigated in literature. For instance, Chen et al. (2009a) propose a copula-based nonlinear quantile autoregression and point out its usefulness in estimation and inference about VaR in financial time series. Bouyé and Salmon (2009) propose a dynamic copula quantile regression to investigate dependency in the foreign exchange market. For a comprehensive survey of copula-based quantile regression, see Kolev and Paiva (2009).

1.5.7 High-dimensional Application

Dealing with high-dimensional data is a common task in financial practice. Problems arise when estimating high-dimensional copulas due to the "curse of dimensionality". This motivates researchers to develop various copula-based models to resolve high-dimensional issues in estimation and inference. For instance, Aas et al. (2009), Min and Czado (2010) and Acar et al. (2012) consider models which use a cascade of pair-copulas to decompose high dimension copula into bavariate cases. Similarly, Okhrin et al. (2013) and Hering et al (2010) also develop efficient techniques to model high-dimensional multivariate distributions through hierarchical Archimedean copulas. Another similar method, named "nested Archimedean copulas", is considered by Hofert and Scherer (2011).

Christoffersen et al. (2012) implement the ingenious composite likelihood method proposed by Engle et al. (2008) to solve large-scale estimation problems in their dynamic copula model for 33 equity markets. Oh and Patton (2015a) decompose dependence between high-dimensional returns into linear and nonlinear components. They utilize high frequency data to forecast linear dependence and high dimension copulas to capture nonlinear dependence. Their estimation procedure also relies on the composite likelihood method of Engle et al.

(2008). Oh and Patton (2015b) also propose a novel factor copula model based on a latent factor structure to handle the high dimensional problem. Despite the factor copula model lacking a closed-form density, they show that some properties can be still obtained analytically.

1.5.8 Other Applications

Although most of the important contributions of copula models in finance and economics have been reviewed above, there are still some conspicuous contributions that cannot be appropriately classified into any of above topics. Smith (2003) proposes a method to model sample selection using Archimedean copulas; Demarta and McNeil (2005) investigate the properties of Student's t copula and some related extensions; Granger et al. (2006) present a definition for a common factor for bivariate time series using copulas; Zimmer and Trivedi (2006) consider trivariate copulas to model sample selection and treatment effects; Bartram et al. (2007) consider a time-varying copula model to study the impact of the introduction of the Euro on the dependence between stock markets in European countries; Heinen and Rengifo (2007) propose a multivariate autoregressive conditional double Poisson model based on copulas to investigate time series of count data; Rodriguez (2007) applies copulas to measure financial contagion; Dearden et al. (2008) and Bonhomme and Robin (2009) study earnings dynamics using copulas; Lee and Long (2009) propose copula-based multivariate GARCH with uncorrelated residuals; Patton (2009b), Dudley et al. (2011) and Kang et al. (2010) investigate the dependence structures between hedge fund and different assets using copula-based models; Zimmer (2012) shows the restrictions of the Gaussian copula and explores various copula specifications to model US housing price data; Lee and Yang (2014) propose a copula-based Granger-causality test to study causal relationships between financial markets; Jin and De Simone (2014) study banking systemic vulnerabilities using a CIMDO model combined with dynamic t copula.

1.6 Conclusions and Areas for Future Research

Thus, it is very convenient to construct a multivariate density from different marginal densities via copulas. The desirable properties of copulas have been well documented in statistical literature and the usefulness of copulas have been well documented in finance and economic literature. Recently, numerous copula-based models have been proposed to model multivariate densities and some of them have been widely applied in applications of dependence modeling, aggregation of risks, and asset allocation, etc. In this chapter, we have briefly reviewed the basic definition and properties of copulas as well as some related concepts which are particularly useful to applications in financial and economic time series modeling. Then, we discussed some estimation, inference and evaluation methods widely used in existing copula literature. Finally, we provided an exhaustive review of literature on copula models in finance and economics.

High-frequency data is becoming more and more important in financial practice. In recent years, there has been a dramatic increase in the amount of high frequency data available. The theoretical and empirical literature has been growing very fast, with contributions including Engle (2000), Andersen et al. (2001), Andersen et al. (2003), Andersen et al. (2005), Xiu (2010), among others. However, the literature that considers copulas to model multivariate distribution of high-frequency financial time series is rather limited, see for instance Breymann et al. (2003), Dias and Embrechts (2010), Salvatierra and Patton (2015). Thus, using copulas to model dependence structure between high-frequency data is likely to be a promising topic for further research.

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Figure 1.1 Probability Contour Plots for Different Copulas

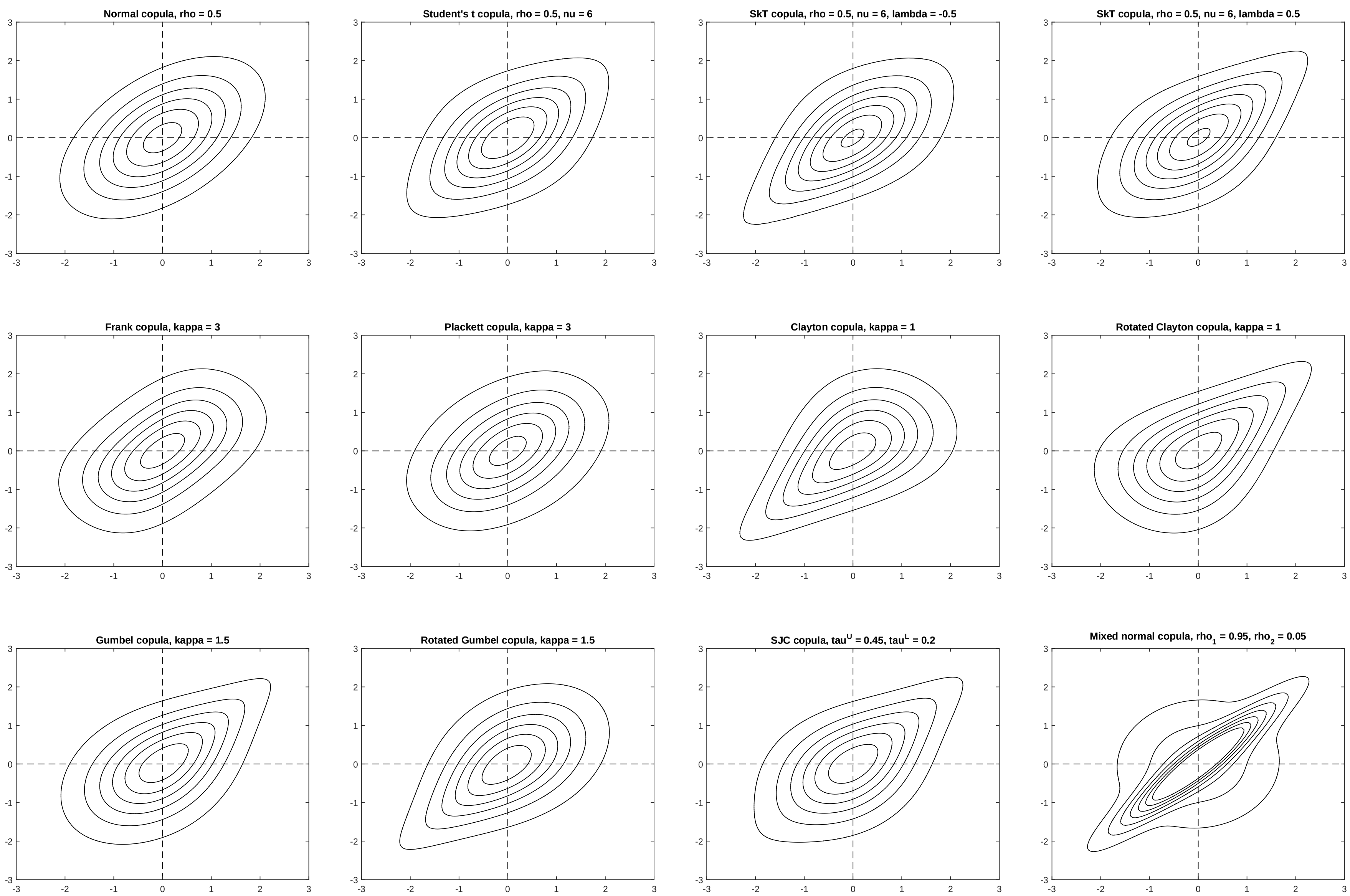
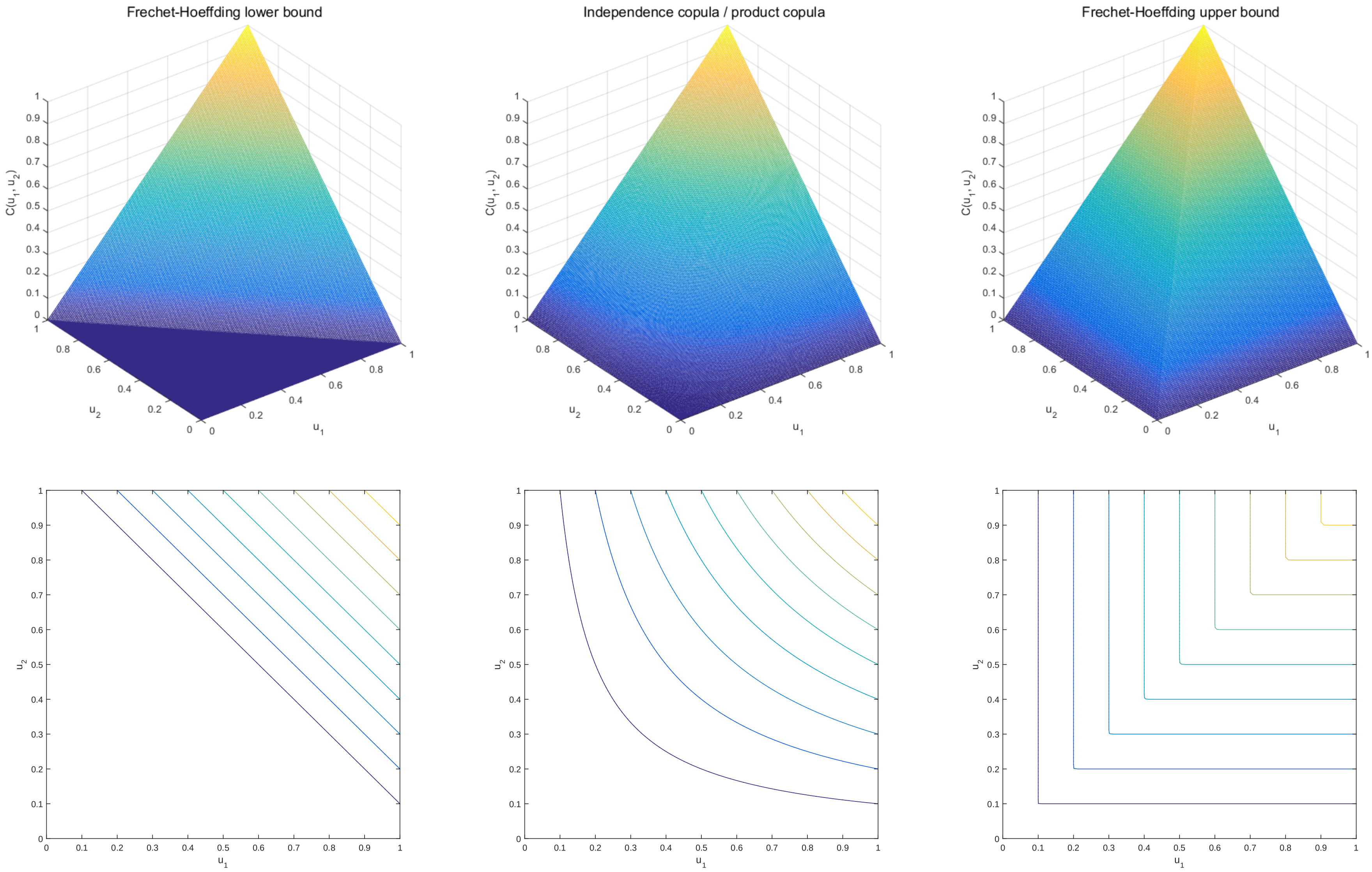


Figure 1.2 Plots Distribution of Fréchet-Hoeffding Upper Bound, Lower Bound and Product (Independence) Copula



Chapter 2

Modeling Dependence Structure and Forecasting Market Risk with Dynamic Asymmetric Copula

We investigate the dynamic and asymmetric dependence structure between equity portfolios from the US and UK. We demonstrate the statistical significance of dynamic asymmetric copula models in modelling and forecasting market risk. First, we construct “high-minus-low" equity portfolios sorted on beta, coskewness, and cokurtosis. We find substantial evidence of dynamic and asymmetric dependence between characteristic-sorted portfolios. Second, we consider a dynamic asymmetric copula model by combining the generalized hyperbolic skewed *t* copula with the generalized autoregressive score (GAS) model to capture both the multivariate non-normality and the dynamic and asymmetric dependence between equity portfolios. We demonstrate its usefulness by evaluating the forecasting performance of Value-at-Risk and Expected Shortfall for the high-minus-low portfolios. From backtesting, we find consistent and robust

[0]This chapter is the early version of the working paper "Cerrato, Marrio., Crosby, John., Kim, Minjoo., and Zhao, Yang. (2015). Modeling Dependence Structure and Forecasting Market Risk with Dynamic Asymmetric Copula. *SIRE Discussion Papers 2015*, Scottish Institute for Research in Economics (SIRE)". I would like to thank Andrew Patton, Paul Embrechts, Alexander McNeil, Drew Creal and Wolfgang Härdle for sharing their copula codes, and conference discussants at 2015 Quant Europe in London, ICBI Global Derivatives 2015 conference in Amsterdam, and workshop discussants at the University of Glasgow for helpful discussions and comments.

evidence that the dynamic asymmetric copula model provides the most accurate forecasts, indicating the importance of incorporating the dynamic and asymmetric dependence structure in risk management.

2.1 Introduction

The finance and econometrics literature provides a wealth of evidence that the conditional correlation or dependence structure between assets varies through time (see Jondeau and Rockinger, 2006; Dias and Embrechts, 2010; Longin and Solnik, 1995; Giacomini et al., 2009, etc). Moreover, asset returns also exhibit greater correlation, or more generally, greater dependence during market downturns than market upturns. One feature of the recent financial crisis is the extent to which assets that had previously behaved mostly independently suddenly moved together. This phenomenon is usually termed asymmetric dependence, see for instance Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), Poon et al. (2004), Patton (2006), Okimoto (2008) and Christoffersen and Langlois (2013). The presence of asymmetric correlations or dependence is empirically important, as it can cause serious problems in hedging effectiveness and portfolio diversification (see Hong et al., 2007). In the foreign exchange markets, Patton (2006) suggests that this asymmetry is possibly caused by the asymmetric responses of central banks to exchange rate movements. In the equity markets, although there have been many studies of asymmetric dependence, there is no consensus on the underlying economic cause. One possible cause is that risk-averse investors treat downside losses and upside gains distinctively, which is consistent with "Prospect Theory" (see Kahneman and Tversky, 1979).

Clearly, the key to portfolio risk management is therefore to recognize how quickly and dramatically the dependence structure changes. An increasingly popular method for constructing high dimensional dependence is based on copulas. Copulas are functions that connect multivariate distributions to their one-dimensional margins (Sklar, 1959). The copula approach is particularly useful in portfolio risk measurement for the following reasons. First, copulas can describe the dependence between assets under extreme circumstances, as they use a quantile scale. Second, copulas can be utilized to build a flexible bottom-up approach that can combine a variety of marginal models with a variety of possible dependence specifications (McNeil et al., 2005). Ideally, an appropriate copula for financial modeling should be capable of accommodating both positive and negative dependence, capturing both symmetric and asymmetric dependence, and allowing for possible tail dependence. The generalized hyperbolic skewed *t* copula (hereafter GHST copula) of Demarta and McNeil (2005) can be viewed as a flexible extension that contains all these desirable properties.

Further, the time-variation of dependence motivates the consideration of dynamic copula models which allow the correlation parameter to change dynamically. One such model is proposed by Patton (2006) who extended Sklar's theorem for conditional distributions and proposed an observation driven conditional copula model. This model defined the time-varying dependence parameter of a copula as a parametric function of transformations of the lagged data and an autoregressive term. Another example is the dynamic conditional correlation (DCC) model proposed by Engle (2002). Christoffersen et al. (2012) and Christoffersen and Langlois (2013) develop a dynamic asymmetric copula (DAC) model based on the DCC model to capture long-run and short-run dependence, multivariate nonnormality, and dependence asymmetries.

Creal et al. (2013) propose a class of Generalized Autoregressive Score (GAS) models, which use the scaled score of a likelihood function to update the parameters over time. The GAS model is a consistent and unified framework, which encompasses many successful econometric models including the GARCH, the autoregressive conditional duration, the autoregressive conditional intensity, and Poisson count models with time-varying mean. They illustrate the GAS framework by introducing a new model specification for a dynamic copula.[1] Based on simulation results and empirical evidence, they point out that the driving mechanism in Patton (2006) only captures some of the changes in the dependence coefficients. Specifically, it has shortcomings in tracking the upper and lower tail dependence dynamics simultaneously, since the constant mechanism applies to both types of dependence. Conversely, the GAS specification has better performance in capturing different types of dynamics. Therefore, the GAS model is becoming popular in both economics and finance applications (see Creal et al., 2014; Lucas et al., 2014; Oh and Patton, 2013, among many others). Thus, our study adopts it as the driving mechanism to update copula parameters.

Recently, Christoffersen and Langlois (2013) study how the extreme dependence structure is related to the Fama-French factors and address its role in broad areas of finance, such as asset pricing, portfolio analysis, and risk management. They also emphasize the importance of the copula modeling of the extreme dependence structure. Chung et al. (2006)

[1] Harvey (2013) proposes a similar approach for modeling time-varying parameters, which he calls a "dynamic conditional score (DCS)" model

argue that the Fama-French factors are closely related to higher-order comoments such as coskewness and cokurtosis.[2] They suggest the use of Fama-French factors as good proxies for higher-order comoments on the grounds that the latter are, in practice, difficult to accurately estimate. These interesting studies have initiated research investigating the dependence structure between portfolios sorted by higher-order comoments. Several studies show that tail dependence between portfolios has a close relationship with, not only beta, but also coskewness. For example, Garcia and Tsafack (2011), in their international bond and equity market portfolio analysis, show that a strong dependence in lower returns creates a large negative coskewness. Chabi-Yo et al. (2014) also show that a strong lower tail dependence creates a large negative coskewness. In addition they show that beta is monotonically increasing with respect to the lower tail dependence.

In line with the asset pricing literature cited above, we recognize that the Fama-French factors are closely related to higher-order comoments. Moving beyond the asset pricing literature, we argue that it is important to investigate how higher-order comoments are related to the dependence structure of equity portfolios. The latter would be a key input when investors manage the market risk from portfolios constructed using the Fama-French factors or from higher-order comoments. Hence, we use equity portfolios sorted on beta and higher-order comoments. We empirically investigate if the dynamic asymmetric copula, combining the GHST copula and GAS dynamics, significantly improves the modeling and forecasting of the market risk of the equity portfolios. We consider two popular market risk measures, Value-at-Risk (hereafter VaR) and Expected Shortfall (hereafter ES), both of which are very sensitive to the dynamics and extreme dependence structure of asset returns.

Our study makes three contributions. First, we provide a comprehensive study of the dynamic evolution of dependence in equity markets. We find striking evidence that the dependence structures between characteristic-sorted portfolios, such as the high beta portfolio and the low beta portfolio, significantly changed after the start of the global financial crisis of 2007-2009. Second, we provide new empirical evidence of asymmetric dependence in the US and UK equity markets. In general, we show that the coefficients of lower tail dependence (LTD) are greater than the coefficients of upper tail dependence (UTD) and that

[2]The importance of higher-order comoments have been demonstrated in many asset pricing literature (see Bakshi et al., 2003; Ang et al., 2006; Conrad et al., 2013; Dittmar, 2002; Guidolin and Timmermann, 2008; Harvey and Siddique, 2000).

this asymmetry is statistically significant. Third, while estimation of portfolio VaR and ES has been widely studied in the literature, there have been relatively few studies examining portfolio VaR and ES forecasting, especially forecasting through asymmetric copula. Using the characteristic-sorted portfolios, we evaluate the statistical significance of incorporating asymmetric and dynamic dependence into VaR and ES forecasts, and show ignorance of dependence asymmetry and dynamics is costly in risk management. The backtesting results provide solid evidence that the dynamic asymmetric copula model can consistently provide better VaR and ES forecasts than alternative benchmark models, especially at the 99% level. And we also find that semiparametric dynamic asymmetric copula models perform better than full parametric dynamic copula models.

The remainder of this paper is organized as follows. In Section 2.2, we detail the methods we employ for portfolio sorting, and we provide an overview of copula theory and computation methods for tail dependence coefficients. Then, we present the dynamic copula model and its estimation methodology. The data used in the paper, summary statistics and univariate model estimations are in Section 2.3. In Section 2.4, we focus on testing whether the dependence structures between characteristic-sorted portfolios are statistically dynamic and asymmetric, especially during the global financial crisis of 2007-2009 and the Euro Sovereign Debt crisis of 2010-2011, and then discuss the possible reasons for different kinds of dependence. In Section 2.5, we predict portfolio VaR and ES using dynamic copulas and benchmark models and report the comparison results of backtesting. Finally, conclusions are given in Section 2.6. All the tables and figures used in this paper are presented in the Appendix.

2.2 Model Specification

In this section, we detail the models and portfolio construction that we use in this paper.

2.2.1 Portfolio Construction

The return on an asset is defined as the first difference of the log price, $r_t = \log P_t - \log P_{t-1}$. We construct portfolios sorted on beta, coskewness and cokurtosis separately. Following the definition in Bakshi et al. (2003) and Conrad et al. (2013), the market beta, coskewness and cokurtosis are defined as:

$$BETA_{i,t} = \frac{\mathbb{E}\left[\left(r_{i,t} - \mathbb{E}\left[r_{i,t}\right]\right)\left(r_{m,t} - \mathbb{E}\left[r_{m,t}\right]\right)\right]}{Var\left(r_{m,t}\right)}, \tag{2.1}$$

$$COSK_{i,t} = \frac{\mathbb{E}\left[\left(r_{i,t} - \mathbb{E}\left[r_{i,t}\right]\right)\left(r_{m,t} - \mathbb{E}\left[r_{m,t}\right]\right)^2\right]}{\sqrt{Var\left(r_{i,t}\right)Var\left(r_{m,t}\right)}}, \tag{2.2}$$

$$COKT_{i,t} = \frac{\mathbb{E}\left[\left(r_{i,t} - \mathbb{E}\left[r_{i,t}\right]\right)\left(r_{mt} - \mathbb{E}\left[r_{m,t}\right]\right)^3\right]}{Var\left(r_{i,t}\right)Var\left(r_{m,t}\right)}. \tag{2.3}$$

All stocks are sorted on the three characteristics above and divided into five groups based on the 20th, 40th, 60th and 80th percentiles. We estimate beta, coskewness and cokurtosis each year using all the daily data within this year. Then, we form annually rebalanced portfolios, value weighted based on the capitalization of each stock.[3] We denote by BETA1 (COSK1, COKT1) the portfolio formed by stocks with the highest beta (respectively, coskewness, cokurtosis), and BETA5 (COSK5, COKT5) denotes the portfolio formed by stocks with the lowest beta (coskewness, cokurtosis).

We then take a long position in the stocks falling in the highest beta (coskewness, cokurtosis) quintile and a short position in the stocks falling in the lowest beta (coskewness, cokurtosis) quintile to construct a high-minus-low (HML) portfolio. It is not our intention to gain high excess returns from this trading strategy. We simply generate a portfolio by combining two extreme characteristics (highest and lowest) using a popular HML strategy. We expect that these two extreme characteristics could create a portfolio with a strong extreme dependence structure. We define the HML portfolio return to be

$$r_{hml,t} = r_{h,t} - r_{l,t} \tag{2.4}$$

where $r_{h,t}$ and $r_{l,t}$ denote returns from the highest beta (coskewness, cokurtosis) and the lowest beta (coskewness, cokurtosis), respectively.

[3] We compute the market capitalization of each company (stock price multiplied by the number of shares outstanding) and then use it to assign weights.

2.2.2 Modeling the Marginal Density

We allow each series ($r_{h,t}$ and $r_{l,t}$) to have time-varying conditional mean ($\mu_{i,t}$) and variance ($\sigma_{i,t}^2$), and we also assume that the standardized returns $z_{i,t} = (r_{i,t} - \mu_{i,t})/\sigma_{i,t}$ are identically distributed. We fit an AR model to the conditional mean

$$r_{i,t} = c_i + \sum_{k=1}^{p} \phi_{i,k} r_{i,t-k} + \varepsilon_{i,t}, \quad i = h, l, \text{ , where } \varepsilon_{i,t} = \sigma_{i,t} z_{i,t} \tag{2.5}$$

and an asymmetric GARCH model, namely GJR-GARCH(1,1,1) (see Glosten et al., 1993), to the conditional variance

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \varepsilon_{i,t-1}^2 I_{i,t-1} \tag{2.6}$$

where $I_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$, and $I_{i,t-1} = 0$ if $\varepsilon_{i,t-1} \geq 0$.

Let $z_{i,t}$ be a random variable with a continuous distribution F. For the parametric model, we assume that $z_{i,t}$ follows the skewed Student's t distribution of Hansen (1994):

$$z_{i,t} \sim F_{skew-t,i}(\eta_i, \lambda_i), \; u_{i,t} = F_{skew-t,i}(z_{i,t}; \eta_i, \lambda_i) \tag{2.7}$$

where $F_{skew-t,i}$ denotes the cumulative distribution function, η_i denotes the degrees of freedom, λ_i the skewness parameter, and $u_{i,t}$ the probability integral transformation. Hence, we can easily compute the probability given the estimates of parameters; $\hat{\mu}_{i,t}$, $\hat{\sigma}_{i,t}$, $\hat{\eta}_i$ and $\hat{\lambda}_i$. For the nonparametric model, we use the empirical distribution function to obtain the estimate of F_i:

$$\hat{F}_i(z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1\{\hat{z}_{i,t} \leq z\}, \; \hat{u}_{i,t} = \hat{F}_i(\hat{z}_{i,t}). \tag{2.8}$$

We estimate all parameters in (2.6) – (2.7) using the maximum likelihood estimation. Then we generate each marginal density parametrically or nonparametrically for the purpose of copula construction.

2.2.3 Copulas

In this section, we provide a brief introduction to copulas. The Sklar (1959) theorem allows us to decompose a conditional joint distribution into marginal distributions and a copula. It allows considerable flexibility in modeling the dependence structure of multivariate data. Let $\mathbf{z} = (z_1, \ldots, z_d)'$, $d \geq 2$ be a d-dimensional random vector with joint distribution function $\mathbf{F}(z_1, \ldots, z_d)$ and marginal distribution functions $F_i(z_i)$, $i = 1, ..., d$. According to Sklar's theorem, there exist a d-dimensional copula $C\,[0,1]^d \to [0,1]$ such that

$$\mathbf{F}(z_1, \ldots, z_d) = C\left(F_1(z_1), F_2(z_2), \ldots, F_d(z_d)\right), \tag{2.9}$$

and the copula $C(u_1, \ldots, u_d)$, $u_i \in (0,1)$ is unique if the marginal distributions are continuous. Let F_i^{-1} denote the generalized inverse distribution function of F_i, then $F_i^{-1}(u_i) = z_i$. The copula $C(u_1, ..., u_d)$ of a multivariate distribution $\mathbf{F}(z_1, \ldots, z_d)$ with marginals $F_i(z_i)$ is given by

$$C(u_1, ..., u_d) = \mathbf{F}\left(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_d^{-1}(u_d)\right) \tag{2.10}$$

If F_i has density f_i, the copula density c is given by

$$c(u_1, \ldots, u_d) = \frac{\mathbf{f}\left(F_1^{-1}(u_1), F_2^{-1}(u_1), \ldots, F_d^{-1}(u_d)\right)}{\prod_{i=1}^{d} f_i\left(F_i^{-1}(u_i)\right)} = \frac{\partial^n C(u_1, \ldots, u_d)}{\partial u_1 \cdots \partial u_d} \tag{2.11}$$

Sklar's theorem implies that for multivariate distribution functions, the univariate marginals and the dependence structure can be separated. In our study, we only consider the case of a bivariate copula.

2.2.4 Computation of Asymmetric Dependence

A primary goal of our paper is to investigate how the characteristic-sorted portfolio returns covary and whether their dependence structures are asymmetric. Consequently, we consider three different dependence structures: The threshold correlation; the quantile dependence; and the tail dependence.

Following Longin and Solnik (2001) and Ang and Chen (2002), the threshold correlation for probability level p is given by

$$\rho^- = Corr\left(r_{h,t}, r_{l,t} | r_{h,t} \leq r_h(p) \text{ and } r_{l,t} \leq r_l(p)\right) \text{ if } p \leq 0.5 \tag{2.12}$$

$$\rho^+ = Corr\left(r_{h,t}, r_{l,t} | r_{h,t} > r_h(p) \text{ and } r_{l,t} > r_l(p)\right) \text{ if } p > 0.5 \tag{2.13}$$

where $r(p)$ denotes the corresponding empirical percentile for asset returns $r_{h,t}$ and $r_{l,t}$. In words, we compute the correlation between two assets conditional on both of them being less (respectively, greater) than their pth percentile value when $p \leq 0.5$ (respectively, $p > 0.5$). To examine whether this asymmetry is statistically significant, we consider a model-free test proposed by Hong et al. (2007). If the null hypothesis of $\rho^+ = \rho^-$ is rejected, then there exists a linear asymmetric correlation between $r_{h,t}$ and $r_{l,t}$.

The quantile dependence provides a more precise measure of dependence structure than the threshold correlation, as it contains more detailed information. In addition, from the risk management perspective, tails are more important than the center. Following Patton (2012), the quantile dependence can be defined as

$$\lambda^q = \begin{cases} \mathbb{P}\left\{u_{h,t} \leq q | u_{l,t} \leq q\right\} = \frac{C(q,q)}{q} & \text{if } 0 < q \leq 0.5 \\ \mathbb{P}\left\{u_{h,t} > q | u_{l,t} > q\right\} = \frac{1-2q+C(q,q)}{1-q} & \text{if } 0.5 < q \leq 1 \end{cases} \tag{2.14}$$

and nonparametrically estimated by

$$\hat{\lambda}^q = \begin{cases} \frac{1}{Tq} \sum_{t=1}^{T} 1\left\{\hat{u}_{h,t} \leq q, \hat{u}_{l,t} \leq q\right\} & \text{if } 0 < q \leq 0.5 \\ \frac{1}{T(1-q)} \sum_{t=1}^{T} 1\left\{\hat{u}_{h,t} > q, \hat{u}_{l,t} > q\right\} & \text{if } 0.5 < q < 1. \end{cases}, \tag{2.15}$$

where C denotes the corresponding copula function.

The tail dependence coefficient (TDC) is a measure of the degree of dependence in the tail of a bivariate distribution (see Frahm et al., 2005; Joe et al., 2010; McNeil et al., 2005, among others). Let z_h and z_l be random variables with continuous distribution functions F_h

and F_l. Then the coefficients of upper and lower tail dependence of z_h and z_l are

$$\lambda^L = \lim_{q \to 0^+} \frac{\mathbb{P}\left\{z_h \leq F_h^{-1}(q), z_l \leq F_l^{-1}(q)\right\}}{\mathbb{P}\left\{z_l \leq F_l^{-1}(q)\right\}} = \lim_{q \to 0^+} \frac{C(q,q)}{q} \tag{2.16}$$

$$\lambda^U = \lim_{q \to 1^-} \frac{\mathbb{P}\left\{z_h > F_h^{-1}(q), z_l > F_l^{-1}(q)\right\}}{\mathbb{P}\left\{z_l > F_l^{-1}(q)\right\}} = \lim_{q \to 1^-} \frac{1-2q+C(q,q)}{1-q} \tag{2.17}$$

The coefficients can be easily calculated when the copula C has a closed form. The copula C has upper tail dependence if $\lambda^U \in (0,1]$ and no upper tail dependence if $\lambda^U = 0$. A similar conclusion holds for the lower tail dependence. If the copulas are symmetric, then $\lambda^L = \lambda^U$, otherwise, $\lambda^L \neq \lambda^U$ (see Joe, 1997). McNeil et al. (2005) state that the copula of the bivariate t distribution is asymptotically dependent in both the upper and lower tail. The rotated Gumbel copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative tail than in the positive. Both of them allow heavier negative tail dependence than the Gaussian copula and are widely used in the finance literature. We use both the Student's t copula and the rotated Gumbel copula to estimate the tail dependence coefficient between portfolios.

2.2.5 Generalized Hyperbolic Skewed *t* Copulas

In this section, we provide a brief introduction to the generalized hyperbolic (GH) skewed t distribution which we employ to capture asymmetric extreme dependence structure between equity portfolios in our study. It belongs to the class of multivariate normal variance mixtures and has the stochastic representation

$$\mathbf{X} = \mu + \gamma W + \sqrt{W}\mathbf{Z} \tag{2.18}$$

for a d-dimensional parameter vector γ. Further, W is a scalar valued random variable following an inverse gamma distribution $W \sim IG(\nu/2, \nu/2)$ and $\mathbf{Z}$ is a d- dimensional random vector following a normal distribution $\mathbf{Z} \sim N(\mathbf{0}, \Sigma)$ and is independent of W (see Demarta and McNeil, 2005).

The density function of multivariate GH skewed t distribution is given by

$$\mathbf{f}_{skt}(\mathbf{z};\gamma,\nu,\Sigma)=\frac{2^{\frac{2-(\nu+d)}{2}}K_{\frac{\nu+d}{2}}\left(\sqrt{\left(\nu+\mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^{*}\right)\gamma^{\prime}\Sigma^{-1}\gamma}\right)e^{\mathbf{z}^{*\prime}\Sigma^{-1}\gamma}}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{\frac{d}{2}}|\Sigma|^{\frac{1}{2}}\left(\nu+\mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^{*}\right)^{\frac{-\nu+d}{2}}\left(1+\frac{1}{\nu}\mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^{*}\right)^{\frac{-\nu+d}{2}}} \tag{2.19}$$

where K_λ, ν and γ denote the modified Bessel function of the third kind, the degree of freedom and skewed parameter vector, respectively. The density of multivariate GH skewed t converges to the conventional symmetric t density when γ tends to $\mathbf{0}$. For the parametric case, we define the shocks $z^*_{i,t}=F^{-1}_{skt,i}(u_{i,t})=F^{-1}_{skt,i}\left(F_{skew-t,i}(z_{i,t})\right)$ where $F^{-1}_{skt,i}(u_{i,t})$ denotes the inverse cumulative distribution function of the univariate GH skewed t distribution and it is not known in closed form but can be well approximated via simulation. $F_{skew-t,i}$ denotes the cumulative distribution function of skewed t distribution in Hansen (1994). Note that we use $z^*_{i,t}$ not the standardized return $z_{i,t}$. For the nonparametric case, we use the EDF to obtain the estimate of $u_{i,t}$. A more detailed discussion can be found in Christoffersen et al. (2012).

The probability density function of the GHST copula defined from above multivariate GH skewed t density of Equation (2.19) is given by

$$\begin{aligned}c_{skt}(\mathbf{z};\gamma,\nu,\Sigma)=&\frac{2^{\frac{(\nu-2)(d-1)}{2}}K_{\frac{\nu+d}{2}}\left(\sqrt{\left(\nu+\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^{*}\right)\gamma^{\prime}\Sigma_t^{-1}\gamma}\right)e^{\mathbf{z}^{*\prime}\Sigma_t^{-1}\gamma}}{\Gamma\left(\frac{\nu}{2}\right)|\Sigma|^{\frac{1}{2}}\left(\nu+\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^{*}\right)^{\frac{-\nu+d}{2}}\left(1+\frac{1}{\nu}\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^{*}\right)^{\frac{-\nu+d}{2}}}\\&\times\prod_{i=1}^{d}\frac{\left(\sqrt{\left(\nu+(z_i^*)^2\right)\gamma_i^2}\right)^{-\frac{\nu+1}{2}}\left(1+\frac{1}{\nu}(z_i^*)^2\right)^{\frac{\nu+1}{2}}}{K_{\frac{\nu+1}{2}}\left(\sqrt{\left(\nu+(z_i^*)^2\right)\gamma_i^2}\right)e^{z_i^*\gamma_i}}\end{aligned} \tag{2.20}$$

where Σ_t is the time-varying covariance matrix. Specifically, $\Sigma_t=\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$, where $\mathbf{D}_t$ is an identity matrix in copula modeling and $\mathbf{R}_t$ is the time-varying correlation matrix. Note that Christoffersen et al. (2012) applied the GHST copula by constraining all the margins to have the same asymmetry parameter. Different from their model, our model consider a more generalized case by allowing the copula to have the different asymmetry parameters across margins. Although our model can be used for high-dimensional copula modeling, in this paper, only the bivariate case is considered as modeling the dependence and market risk of long-short portfolio is our main task. Figure 2.6 shows the probability contours for bivariate GHST copula with different asymmetric parameters.

2.2.6 Generalized Autoregressive Score (GAS) Model

We estimate the dynamic copula model based on the Generalized Autoregressive Score (GAS) model of Creal et al. (2013). We assume that the correlation parameter δ_t [4] is dynamic and is updated as function of its own lagged value. To make sure that it always lies in a pre-determined range (*e.g.* $\delta_t \in (-1,,1)$), the GAS model utilizes a strictly increasing transformation. Following Patton (2012), the transformed correlation parameter is denoted by g_t:

$$g_t = h(\delta_t) \Leftrightarrow \delta_t = h^{-1}(g_t), \tag{2.21}$$

where $\delta_t = (1-e^{-g_t})/(1+e^{-g_t})$. Further, the updated transformed parameter g_{t+1} is a function of a constant $\bar{\omega}$, the lagged transformed parameter g_t, and the standardized score of the copula log-likelihood $Q_t^{-1/2}\mathbf{s}_t$:

$$g_{t+1} = \bar{\omega} + \eta Q_t^{-1/2}\mathbf{s}_t + \varphi g_t, \tag{2.22}$$

where

$$\mathbf{s}_t \equiv \frac{\partial \log c\left(u_{h,t}, u_{l,t}; \delta_t\right)}{\partial \delta_t} \text{ and } Q_t \equiv \mathbb{E}_{t-1}\left[\mathbf{s}_t \mathbf{s}_t'\right].$$

Since the GAS model is an observation driven model, we estimate the parameters using the maximum likelihood estimation

$$\widehat{\delta}_t = \underset{\delta_t}{\operatorname{argmax}} \sum_{t=1}^{n} \log c\left(u_{h,t}, u_{l,t}; \delta_t\right). \tag{2.23}$$

The dynamic copulas are parametrically estimated using maximum likelihood estimation. When the marginal distributions are estimated using the skewed t distribution, the resulting joint distribution is fully parametric. When the marginal distribution is estimated by the empirical distribution function, then the resulting joint distribution is semiparametric. More details can be found in the appendix.

[4] In the baivariate case, the copula correlation is a scalar and it can be obtained from the correlation matrix $\mathbf{R}_t = \begin{bmatrix} 1 & \delta_t \\ \delta_t & 1 \end{bmatrix}$ estimated in Section 2.2.5.

2.2.7 Value-at-Risk and Expected Shortfall Forecasts

We now turn to VaR and ES forecasts of the HML portfolio defined in Equation (2.4). The ex ante VaR and ES of the HML portfolio at time t and given nominal probability $\alpha \in (0,1)$, are defined as:

$$\mathbb{P}\left[r_{hml,t} \leq VaR_{hml,t}(\alpha)\right] = 1 - \alpha, \tag{2.24}$$

$$ES_{hml,t}(\alpha) = \mathbb{E}\left[r_{hml,t} \mid r_{hml,t} < VaR_{hml,t}(\alpha)\right], \tag{2.25}$$

In our study, α is assumed to be either 0.95 or 0.99, and we report results focusing on 0.99 which is the most widely used value for market risk management. Once the dynamic copula parameters have been estimated, Monte Carlo simulation is used to generate 5000 values of $r_{h,t}^{(s)}$ and $r_{l,t}^{(s)}$ and, hence, of $r_{hml,t}^{(s)}$. From the empirical distribution of $r_{hml,t}^{(s)}$, the desired quantile VaR and ES are estimated.

2.3 Data and Marginal Distribution Modeling

2.3.1 Description of Data

Stock prices are obtained from Datastream. Daily returns of the 500 stocks listed in the S&P 500 and the 100 stocks listed in FTSE 100 are used to construct portfolios. Our data, spanning the period of the global financial crisis of 2007-2009 and European sovereign debt crisis of 2010-2011, go from January 4, 2000 to December 31, 2012, resulting in 3,268 daily observations for each stock in US and 3,283 daily observations for each stock in UK.

Given the one-year estimation period, we estimate beta, coskewness and cokurtosis using daily data (250 days) for each stock.[5] We rank securities by the estimates of beta (coskewness, cokurtosis) and form into five portfolios, highest (1st) – lowest (5th). Then we calculate daily returns for each portfolio within the estimation period.[6] In this way, we construct thirty

[5]Since we estimate a factor beta on the daily return, we use a short sample period. We also consider alternative longer estimation periods (3 years and 5 years) and find consistent results with a one year estimation period.

[6]We also calculate daily reruns for the next 12-months, which are forward looking portfolio returns, and find similar forecasting results. Since we are interested in how beta and higher-order comoments are related to

different portfolios - fifteen for US equities and fifteen for the UK. The fifteen consist of one for each of the three characteristics (beta, coskewness and cokurtosis), divided into five portfolios. We annually rebalance all the portfolios and calculate 12-month daily returns.[7] The definitions of HML portfolios are presented in Table 2.1.

[INSERT TABLE 2.1 ABOUT HERE]

Summary statistics for the high and low portfolio returns are presented in Panel A of Table 2.2. We find that the portfolio constructed from high beta stocks (i.e. BETA5) tends to offer relatively lower average returns than the portfolio constructed from low beta stocks (i.e. BETA1). This anomaly is likely driven by the fact that the US and UK equity markets have fallen 2.02% and 16.13% from 2000 to 2012. Also, it can be explained by the "betting against beta" factor and the funding constraint model in Frazzini and Pedersen (2014). The skewness of the portfolio returns are non-zero while the kurtosis of the portfolio returns are significantly higher than 3 indicating that the empirical distributions of returns display heavier tails than a Gaussian distribution. Using the Ljung-Box Q-test, the null hypothesis of no autocorrelation is rejected at lag 5 and lag 10 for all the portfolios. The ARCH test of Engle (1982) indicates the significance of ARCH effects in all the series. We also find similar results for the HML portfolios in Panel B of Table 2.2. Overall, the summary statistics show the nonnormality, asymmetry, autocorrelation and heteroscedasticity of portfolio returns.

[INSERT TABLE 2.2 ABOUT HERE]

Figure 2.1 displays the scatter plots of the high and low portfolio pairs; (BETA1, BETA5), (COSK1, COSK5) and (COKT1, COKT5). Further it provides threshold correlation coefficients at the center and at both the upper and lower tails of the empirical distribution. Beta portfolios have larger correlations at both tails than the correlations at the center in both stock markets. Coskewness portfolios of the US stock market have smaller correlation at the center than the correlation at the lower tail while those of the UK stock market have larger correlations at both tails than the correlation at the center. Cokurtosis portfolios show similar patterns to coskewness portfolios. The common feature is that the lower tail correlation

the extreme dependence structure, we prefer portfolio returns calculated within the estimation period to forward looking returns.

[7] We also consider monthly rebalancing of portfolios and find results consistent with annual rebalancing.

is larger than the upper tail correlation. This stylized fact is consistent with previous research (see Patton, 2006; Christoffersen and Langlois, 2013; Hong et al., 2007, etc). Overall, the scatter plots and the threshold correlation coefficients clearly show that the correlations between the respective high and low portfolios are nonlinear and asymmetric.

[INSERT FIGURE 2.1 ABOUT HERE]

Before modeling the joint distribution of portfolio returns, it is necessary to select a suitable model for the marginal return distribution, because misspecification of the univariate model can lead to biased copula parameter estimates. To allow for autocorrelation, heteroskedasticity and asymmetry, we use the models introduced in Section 2.2 in Equation (2.5) to (2.8).

First, we use the Bayesian Information Criterion (BIC) to select the optimal order of the AR model for the conditional mean up to order 5. Second, to allow for the heteroskedasticity of each series, we consider a group of GARCH models as candidates and find that the asymmetric GARCH model of Equation (2.6) is preferred to the others based on their likelihood values. Thus, we consider the GJR-GARCH class of up to order (2,2,2) and select the optimal order by using BIC again. The model parameters are estimated by using maximum-likelihood estimation (MLE) and the results of AR and GARCH estimations are presented in Panel A of Table 2.3. For each series, the variance persistence implied by the model is close to 1. For all the series, the leverage effect parameters γ are significantly positive implying that a negative return on the series increases volatility more than a positive return with the same magnitude.

[INSERT TABLE 2.3 ABOUT HERE]

The obvious skewness and high kurtosis of returns leads us to consider the skewed Student's t distribution of Hansen (1994) for residual modeling. We report the estimation results in Table 2.3. To evaluate the goodness-of-fit for the skewed Student's t distribution, the Kolmogorov-Smirnov (KS) and Cramer-von Mises (CvM) tests are implemented and the p-values from these two tests are reported in Table 2.3.[8] Our results suggest that the skewed

[8]The p-values are obtained based on the algorithm suggested in Patton (2012)

Student's t distribution is suitable for residual modeling. Thus, in general, the diagnostics provides evidences that our marginal distribution models are well-specified and therefore, we can reliably use the combination of AR, GARCH and skewed Student's t distribution, allied to copulas to model the dependence structure.

2.4 Dependence: Dynamics and Asymmetry

This section seeks to accomplish two tasks. First, we describe the dynamic evolution of dependence between the high beta (respectively, coskewness, cokurtosis) portfolios and the low beta (coskewness, cokurtosis) portfolios, and examine whether it is statistically time-varying. If the variation of dependence between portfolio returns were not to be statistically significant, then there would be no reason to implement a dynamic model (due to its increased computational complexity). In addition, we want to test whether the dependence structure has dramatically changed after the start of the global financial crisis of 2007-2009 and after the start of the European sovereign debt crisis of 2010-2011. Second, we measure asymmetric dependence using threshold correlation, copula-based quantile dependence and tail dependence and we test whether this asymmetry is significant.

2.4.1 Time-varying Dependence

There is considerable evidence that the conditional mean and conditional volatility of financial time series are time-varying. This, possibly, suggests the reasonable inference that the conditional dependence structure may also change through time. To visualize this variation, Figure 2.2 depicts two time series plots of average rolling 250-day rank correlation between the high and low portfolios in both US market and UK market for each year. The average rolling rank correlations for all the equity portfolios increase significantly during 2000-2002, which is probably caused by the early 2000s economic recession that affected the European Union during 2000 and 2001 and the United States in 2002 and 2003, and the bursting of the dot com bubble. In general, all the rolling window rank correlations between the high and low portfolios increase from 2000 to 2012.

[INSERT FIGURE 2.2 ABOUT HERE]

We now consider three tests for time-varying dependence. The first one is a naïve test for a break in rank correlation at specified points in the sample, see Patton (2006). A noticeable limitation of this test is that the break point of dependence structure (e.g. a specified date) must be known *a priori*. The second test for time-varying dependence allows for a break in the rank correlation coefficient at some prior unspecified date, see Andrews (1993). The third test is the ARCH LM test for time-varying volatility, see Engle (1982). The critical values for the test statistic can be obtained by using a *iid* bootstrap algorithm, see Patton (2012). The results of the above tests for time-varying dependence are summarized in Table 2.4. Suppose there is no *a priori* date for the timing of a break, we first consider naïve tests for a break at three chosen points in our sample, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 10-Dec-2001, 03-Jul-2006, and 17-Jan-2011. Then we consider another test in Andrews (1993) for a dependence break of unknown timing. As can be seen from Table 2.4, for almost all the equity portfolios, the *p*-value is significant at the 5% significance level showing clear evidence against a constant rank correlation with a one-time break. To detect whether the dependence structures between the high and low portfolios significantly changed during the global financial crisis of 2007-2009 and the European sovereign debt crisis of 2010-2011, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points. We find that the dependence between BETA1 and BETA5 significantly changed around those dates, as all the *p*-values are fairly small. For other portfolio pairs, time homogeneity of the dependence structure is rejected by at least one test.

[INSERT TABLE 2.4 ABOUT HERE]

Overall, we find evidence against time homogeneity of the dependence structure between the standardized residuals of portfolios. This result shows that the standard portfolio diversification and risk management techniques based on constant correlations (or dependence) are inadequate, especially during financial crisis. Thus, the heterogeneity of dependence provides us a strong motivation to introduce a dynamic copula model for financial forecasting.

2.4.2 Asymmetric Dependence

Standard models fail to take into account a noteworthy feature during financial crisis that asset returns often become more highly correlated (in magnitude). To test for the presence of this feature, we use threshold correlations, Equation (2.12) – (2.13). Figure 2.3 shows the lower and upper threshold correlations for the high portfolio versus low portfolio. The lower threshold correlations are always greater than the upper threshold correlation indicating that portfolios are more correlated when both of them perform poorly. From a portfolio management perspective, this feature is extremely important. For instance, for UK market, the correlation between BETA1 and BETA5 is relatively low suggesting that diversification is high, but when both BETA1 and BETA5 have poor performances, their correlation can go up to more than 0.61. Therefore, the bivariate normal distribution cannot well describe the "true" dependence for the following reasons: First, the normal distribution is symmetric. Second, in the bivariate normal distribution, the threshold correlation approaches 0 when the threshold is asymptotically close to 0 or 1. To find out whether this asymmetry is statistically significant, we perform the symmetry tests of Hong et al. (2007). Table 2.5 reports the test results and shows that, as measured by threshold correlation, half of the portfolios are significantly asymmetric at the 10% level: $HML(Beta, US/UK)$ and $HML(Cokt, UK)$.

[INSERT FIGURE 2.3 AND TABLE 2.5 ABOUT HERE]

Although threshold correlation offers some insights, it is still based on (linear) correlation and, therefore, does not take into account nonlinear information. To capture nonlinear dependence, we consider copula-based quantile dependence and tail dependence. Compared with (linear) correlation, the key advantage of copulas is that they are a "pure measure" of dependence, which cannot be affected by the marginal distributions (see Nelsen, 2006).

Quantile dependence measures the probability of two variables both lying above or below a given quantile (e.g. upper or lower tail) of their univariate distributions. Examining different quantiles allows us to focus on different aspects of the relationship. In Figure 2.4, we present the quantile dependence between the high beta (coskewness, cokurtosis) portfolios and the low beta (coskewness, cokurtosis) portfolios as well as the difference in upper and lower quantile dependence. For every portfolio pair, Figure 2.4 shows the estimated quantile dependence plot, for $q \in [0.025, 0.975]$, along with 90% (pointwise) *i.i.d.* bootstrap confidence intervals. As expected, the confidence intervals are narrower in the middle of the distribution (values of q close to 1/2) and wider near the tails (values of q near 0 or 1). Figure 2.4 also shows that observations in the lower tail are slightly more dependent than observations in the upper tail, with the difference between corresponding quantile dependence probabilities being as high as 0.3. The confidence intervals show that these differences are borderline significant at the 10% significance level, with the upper bound of the confidence interval on the difference lying around zero for most values of q. From the perspective of risk management, the dynamics implied by our empirical results may be of particular importance in the lower tails, because of its relevance for the portfolio VaR and ES.

[INSERT FIGURE 2.4, 2.5 AND TABLE 2.6 ABOUT HERE]

Next, we consider the tail dependence, which is a copula-based measure of dependence between extreme events. We employ the rotated Gumbel copula and the Student's t copula to estimate the tail dependence coefficients. All the coefficients are estimated by both parametric and semiparametric copula methods (detailed in the appendix). To avoid possible model misspecification, we use the nonparametric estimation method proposed by Frahm et al. (2005) as a robustness check and the results are consistent with results generated by the parametric and semiparametric methods.

Table 2.6 reports the coefficients of lower tail dependence (LTD) and upper tail dependence (UTD) and the difference between them. The coefficients are estimated using McNeil et al. (2005). For example, the lower tail coefficient estimated by rotated Gumbel copula (respectively, Student's t copula) for BETA1 and BETA5 in the US equity market is 0.256 (respectively, 0.171) and the upper tail coefficient estimated by rotated Gumbel copula (Student's t copula) is 0.099 (respectively, 0.018). Then we find the significant difference between the upper and lower tail dependence coefficients. In the UK equity market, we also find evidence of asymmetric dependence in that all the portfolio pairs exhibit greater correlation during market downturns than market upturns. This finding about asymmetric dependence between the high beta (coskewness, cokurtosis) portfolio and the low beta (coskewness, cokurtosis) portfolios is new. It is possibly associated with the fact that investors have more uncertainty about the economy, and therefore pessimism and panic spread from one place to another more quickly during market downturns. Another possible explanation is the impact of liquidity risk. Some “uncorrelated” liquid assets suddenly become illiquid during market downturns, and, therefore, even a small trading volume can lead to huge co-movements.

The semiparametric tail dependence approach (that is nonparametric approach for the marginal distributions and and parametric for the copula estimation) and the nonparametric tail dependence approach of Frahm et al. (2005) are used as robustness checks and both of them provide similar results to the parametric approach.

We present the dynamic evolution of tail dependence coefficient (TDC) between the standardized residuals of the high and low portfolios in Figure 2.5. The dependence between portfolios, such as BETA1 and BETA5, is quite low in 2003 and has significantly increased since then. In the US equity market, the lower tail dependence (LTD) is relatively close to or even lower than the upper tail dependence before the global financial crisis of 2007-2009. However, the LTD has become greater than UTD following 2007. In the UK market, the LTD is always greater than the UTD. This phenomenon can be interpreted from behavioural finance theory that myopic loss aversion investor has a greater sensitivity to losses than to gains. In general, for US portfolio pairs, the asymmetries (differences between LTD and UTD) become more significant during the global financial crisis of 2007-2009 but the differences are relatively stable for UK portfolio pairs during the full sample period. Thus, we can reject the null hypothesis of symmetric dependence and conclude that for, most portfolio pairs, dependence is significantly asymmetric.

2.5 Forecasting Portfolio Risk with Dynamic Asymmetric Copula

In the previous section, the significance of dynamic dependence and asymmetries has been verified. In this section, we evaluate the statistical significance of the dynamic asymmetric copula model by forecasting our portfolio VaR and ES.[9]

[9]We use the HML portfolio returns to forecast VaR and ES. This is important for the following reasons: First, the HML portfolio returns have different characteristics compared to simple long or short return series; Second, the HML portfolios have recently become increasingly popular in studies of asset pricing; Third,

We consider 10 copulas including Normal, Student's *t*, GHST, Clayton, Rotated Clayton, Gumbel, Rotated Gumbel, Plackett, Frank and Symmetrized Joe-Clayton,[10] as candidates to model the dependence between BETA1 (COSK1, COKT1, OF1) and BETA5 (COSK5, COKT5, OF5). All the copula parameters are estimated by maximizing the log-likelihood function of Equation (2.35) for the parametric case, and Equation (2.38) for the semiparametric case. Standard errors are estimated using Chen and Fan (2006). Computing the log-likelihood of each copula in constant case, we find that the Student's *t* copula and the GHST copula provide the highest likelihoods over the in-sample period in most cases.[11] In addition, we also find that all the log-likelihoods of dynamic copula models are significantly higher than constant cases.

Figure 2.6 shows the probability contour plots for bivariate normal, Student's *t* and GHST copula with different asymmetric parameters. It is clear that the GHST copula exhibits great flexibility of modeling asymmetric dependence for both upper and lower tails. Thus, we employ the *t* copula and GHST copula to model the dependence and combine with GAS model to forecast our portfolio VaR and ES. A detailed algorithm for dynamic copula-based forecasting can be found in Appendix C.

In order to evaluate VaR and ES forecasts, we use a rolling window instead of the full sample period and the rolling window size is set at 250 (one trading year) for all the data sets.[12] All the models are recursively re-estimated throughout the out-of-sample period and the correlation coefficients of copulas are forecasted by the GAS model. For the purpose of comparison, we also consider the most successful univariate model, filtered historical simulation (FHS; Barone-Adesi, et al., 2002),[13] and three simulation-based multivariate GARCH

modeling the VaR and ES of the HML portfolios is of interest to practitioners as HML strategies are widely used in the financial industry.

[10]The reason that we consider so many copula candidates is because different copula could capture different dependence structure across assets. The analytical forms of Normal, Student's *t*, Clayton, Gumbel, Plackett, Frank and Symmetrized Joe-Clayton can found in (Patton, 2004). More details about GHST copula can be found in Demarta and McNeil (2005) and Christoffersen et al. (2012). See Lucas et al. (2014) for a detailed discussion of GAS dynamics for the correlation matrix of GHST copula.

[11]For the sake of simplicity, we call the Student's *t* copula and the generalized hyperbolic skewed *t* copula as *t* copula and GHST copula, respectively.

[12]The reason we use a moving window of 250 days instead of other window length or expending window is because a moving window of 250 days is the standard estimation period by the Basel accord. In practice the selection of an optimal sample size is a nontrivial issue. As the window size increases, estimation and forecasting precision generally improves. On the other hand it also raises uncertainty about the latent market regimes caused by a sequence of rare or extreme shocks hitting the market in which case it would be more desirable to select the shorter and homogeneous sample rather than longer and heterogeneous ones.

[13]We also evaluate other univariate models such as Historical Simulation, RiskMetrics, GARCH, GJR-

models, namely, BEKK-GARCH, CCC-GARCH, DCC-GARCH.

The backtesting evaluates the coverage ability and the statistical accuracy of the VaR models. The coverage ability is evaluated by the empirical coverage probability (hereafter ECP) and Basel Penalty Zone (hereafter BPZ). The statistical accuracy is evaluated by the conditional coverage test (hereafter CC test; Christoffersen, 1998) and the dynamic quantile test (hereafter DQ test; Engle and Manganelli, 2004).

We first define the failure of the VaR model as the event that a realized return r_s is not covered by the predicted VaR. We identify it by the indicator function taking the value unity in the case of failure:

$$I_s = 1\left\{r_s < \widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)\right\},\ s = 1,\ldots,N, \tag{2.26}$$

where $\widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)$ is the VaR forecast based on the information set at $s-1$, denoted by $\mathcal{F}_{s-1}$, with a nominal coverage probability α. Henceforth, we abbreviate the notation $\widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)$ to $\widehat{Var}_s\left(\alpha\right)$.

ECP is calculated by the sample average of I_s, $\hat{\alpha} = N^{-1}\sum_{s=1}^{N} I_s$ which is a consistent estimator of the coverage probability. The VaR model for which ECP is closest to its nominal coverage probability is preferred. BPZ is suggested by Basel Committee on Banking and Supervision (1996). It describes the strength of the VaR model through the test of failure rate. It records the number of failures of the 99 percent VaR in the previous 250 business days. One may expect, on average, 2.5 failures out of the previous 250 VaR forecasts given the correct forecasting model. The Basel Committee rules that up to four failures are acceptable for banks and defines the range as a "Green" zone. If the failures are five or more, the banks fall into a "Yellow" (5–9) or "Red" (10+) zone. The VaR model for which BPZ is in the "Green" zone is preferred.

Accurate VaR forecasts should satisfy the condition that the conditional expectation of the failure is the nominal coverage probability:

$$\mathbb{E}\left[I_s|\mathcal{F}_{s-1}\right] = \alpha. \tag{2.27}$$

GARCH, Extreme Value Theory model, and CAViaR. We find that FHS strongly outperforms others for all backtestings. Hence, we include only FHS as a univariate model. The results of other univariate models are available in the Internet Appendix.

Christoffersen (1998) shows that it is equivalent to testing if $I_s|\mathcal{F}_{s-1}$ follows an i.i.d. Bernoulli distribution with parameter α:

$$H_0 : I_s|\mathcal{F}_{s-1} \sim i.i.d.\ Bernoulli\,(\alpha)\,. \tag{2.28}$$

The CC test uses the LR statistic which follows the chi-squared distribution with two degrees-of-freedom under the null hypothesis, Equation (2.28). The DQ test is a general extension of the CC test allowing for more time-dependent information of $\{I_s\}_{s=1}^{N}$. The out-of-sample DQ test is given by

$$DQ = \frac{\left(\tilde{\mathbf{I}}'\mathbf{Z}\right)\left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\left(\mathbf{Z}'\tilde{\mathbf{I}}\right)}{\alpha\,(1-\alpha)} \overset{d}{\sim} \chi^2_{p+2}, \tag{2.29}$$

where $\tilde{\mathbf{I}} = \left(\tilde{I}_{p+1}, \tilde{I}_{p+2}, \ldots, \tilde{I}_N\right)'$, $\tilde{I}_s = I_s - \alpha$, $\mathbf{Z} = \left(\mathbf{z}_{p+1}, \ldots, \mathbf{z}_N\right)'$ and $\mathbf{z}_s = \left(1, \tilde{I}_{s-1}, \ldots, \tilde{I}_{s-p}, \widehat{VaR}_s\,(\alpha)\right)'$. We use the first four lags for our evaluation, i.e., $\mathbf{z}_s = \left(1, \tilde{I}_{s-1}, \ldots, \tilde{I}_{s-4}, \widehat{VaR}_s\,(\alpha)\right)'$.

Backtesting of ES is not a straightforward task because it fails to satisfy elicitability (see Gneiting, 2011). We consider a backtesting for the ES forecast given the sample of N ES forecasts,

$$\left\{\widehat{ES}_1\,(\alpha), \ldots, \widehat{ES}_N\,(\alpha)\right\},$$

where $\widehat{ES}_s\,(\alpha)$ is the ES forecast based on the information set at $s-1$. We simply evaluate the ES forecast based on a loss function which enables researchers to rank the models and specify a utility function reflecting the concern of the risk manager. We define two loss functions:

$$\text{Absolute error} := \left|r_s - \widehat{ES}_s\,(\alpha)\right| I_s, \ \text{Squared error} := \left(r_s - \widehat{ES}_s\,(\alpha)\right)^2 I_s,$$

where $I_s = 1\left\{r_s < \widehat{VaR}_s\,(\alpha)\right\}$. In order to rank the models, we compute the mean absolute error (MAE) and the mean squared error (MSE):

$$MAE = \frac{1}{N}\sum_{s=1}^{N}\left|r_s - \widehat{ES}_s\,(\alpha)\right| I_s, \tag{2.30}$$

$$MSE = \frac{1}{N}\sum_{s=1}^{N}\left(r_s - \widehat{ES}_s\,(\alpha)\right)^2 I_s. \tag{2.31}$$

This evaluation is in line with the framework proposed by Lopez (1999) for the VaR evaluation. The smaller value indicates more accurate forecast.

For the UK portfolios, we estimate the VaR and ES models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR and ES for 18 Dec. 2000. We conduct rolling forecast by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts.

2.5.1 Backtesting of Value-at-Risk

We evaluate the coverage ability by ECP and BPZ as follows: We calculate ECP for each portfolio and then report bias and Root Mean Square Error (RMSE). Bias is the average deviation of ECP from the nominal coverage probability (1% in our case). The smaller the bias is, the more accurate the VaR forecast is. RMSE is the average of the squared deviation. It shows the dispersion of ECP from the nominal coverage probability. It makes up for the defect of bias due to the offset of positive and negative deviations. Financial regulators would prefer a VaR model with, simultaneously, a small bias and small RMSE. BPZ describes the coverage ability of the VaR model through the test of failure rate. It counts the number of failure over the previous 250 business days. Figure 2.7 presents the realized returns and 99% VaR estimated by the dynamic GHST copulas.

[INSERT FIGURE 2.7 ABOUT HERE]

2.5.1.1 Empirical Coverage Probability

Table 2.7 presents the ECPs of the VaR models. First, the bias of the parametric dynamic copula models is -0.03% and the bias of the semiparametric dynamic copula models is -0.02% which are much smaller than those of the other models. It shows that the ECPs of the dynamic copula models are very close to the nominal one. In addition, the RMSEs of dynamic GHST copula are significantly smaller than the others. The bias of the static GHST copula is 0.05% which is greater than those (magnitude) of the dynamic GHST copula models, and the RMSE is more than two times the RMSE of the dynamic t copula. This is clear evidence of the superiority of the dynamic copula model. Although, the biases of dynamic t copulas and their skewed version are similar, the RMSEs of dynamic GHST copula are clearly smaller than those of dynamic t copula, implying the importance of incorporating asymmetric dependence in risk forecasting. Second, FHS shows a large positive bias (0.31%) which implies the under-forecasts of VaR. Its RMSE is also large - twice greater than the dynamic t and GHST copula models. Finally, the bias of the multivariate GARCH models range from 0.26% to 0.36%. These are greater than for the dynamic copula models. Their RMSEs are also much greater than those of the dynamic copula models.

[INSERT TABLE 2.7 ABOUT HERE]

2.5.1.2 Basel Penalty Zone

Table 2.8 presents the BPZ of the VaR models. We find that most of the models achieve 12 Green zone using the framework of Basel committee. The static t copula, static GHST copula and BEKK GARCH achieve 11 Green zone and 1 Yellow zone. This result is not surprising as the "traffic lights" backtest is not as rigorous as other statistical tests such as CC test and DQ test.

[INSERT TABLE 2.8 ABOUT HERE]

Next, we evaluate the statistical accuracy by the CC test and the DQ tests as follows: We calculate both statistics for each portfolio and test them at the 5% significance level. Then we report the number of rejected portfolios.

2.5.1.3 Conditional Coverage Test

Table 2.9 reports the CC test results. First, the dynamic t copula models are rejected for 2 (parametric) and 1 (semiparametric) portfolios whilst the static t copula is rejected for 5 portfolios. Also, the dynamic GHST copula (parmaetric and semiparametric) models are rejected for 2 portfolios whilst the static GHST copula is rejected for 5 portfolios. Second, FHS is rejected for 3 portfolios which is slightly more than the dynamic copula models.

Third, multivariate GARCH models are rejected for 3 (BEKK; DCC) and 4 (CCC) portfolios which are slightly more than the dynamic copula models.

[INSERT TABLE 2.9 ABOUT HERE]

2.5.1.4 Dynamic Quantile Test

Table 2.10 reports the results of the DQ test. Although the number of rejections increases, the results are qualitatively consistent with those of the CC test. First, the dynamic *t* copula models are rejected for 3 portfolios whilst the static *t* copula is rejected for 6 portfolios. Also, the dynamic GHST copula models are rejected for 2 (parametric) and 3 (semiparametric) portfolios whilst the static GHST copula is rejected for 6 portfolios. Second, FHS is rejected for 4 portfolios which is slightly more than the dynamic copula models. Finally, the multivariate GARCH models are more frequently rejected for the DQ test than the dynamic copula models. BEKK, CCC and DCC are rejected for 4, 8 and 4 portfolios, respectively.

[INSERT TABLE 2.10 ABOUT HERE]

2.5.2 Backtesting of Expected Shortfall

Table 2.11 reports the MAE results for ES. Dynamic copula models provide the most accurate forecasts (lowest MAEs) in 10 out of 12 portfolios. Also, the GHST copula model generates lower average MAE in general comparing with *t* copula models, as it takes into account the asymmetric dependence between portfolios. As a robustness check, the MSE results reported in Table 2.12 also confirm this conclusion. The dynamic copula models have better performance than both univariate model and multivariate models in almost all cases. In general, the semiparametric GHST copula model tends to outperform the parametric copula models, as it allows for more flexible assumptions regarding the true distribution.

[INSERT TABLE 2.11 AND 2.12 ABOUT HERE]

In sum, we have the following implications of the copula model from the backtesting results. First, the multivariate modeling of the tail dependence is more effective than the multivariate modeling of the central dependence (e.g. covariance) for the accurate extreme event forecast. Further, it outperforms the most successful univariate model, FHS.[14] Second, the modeling of the tail dependence is more important than the modeling of the central dependence for improving the extreme event forecast. Third, a copula must take into account the dynamic nature of the tail dependence. The dynamic copula models strongly outperform the static copula models. Forth, the modeling of asymmetric tail dependence can help it to improve the extreme event forecast. The dynamic GHST copula model tends to outperform the dynamic t copula models in the extreme event forecast. Finally, to check the robustness of our results, we also examine the predication performance of all the candidate models at 95% and 97.5% significance level. The consistent results confirm our conclusion and suggest that data mining are unlikely explanations.[15]

2.6 Conclusion

This paper empirically addresses three related questions to improve our understanding of the dependence structure between financial assets with different characteristics under various market conditions and shows the statistical significance of dynamic asymmetric copula-based models from a risk management perspective. Our findings are novel as we go beyond the earlier copula literature that investigates the dependence across single assets and explore dependence in a cross-sectional setting by forming characteristics-based portfolios of stocks in US and UK markets. We sort stocks listed on the S&P 500 and the FTSE 100 into portfolios based on their comoments including beta, coskewness and cokurtosis.

First, we provide empirical evidence that the dependence between characteristic-sorted portfolios is significantly time-varying. Using empirical data, spanning recent financial crises, we conclude that the returns of portfolios exhibit time-varying dependence and that the dependence has increased in recent years. Therefore, it provides strong support and motivation to apply dynamic copulas in dependence modeling.

Second, we use several tests to verify the presence of asymmetric dependence between

[14] The multivariate GARCH models could not outperform FHS. Rather, FHS outperforms the multivariate GARCH models for some cases. It implies that the multivariate modeling of the central dependence cannot help it to improve the extreme event forecast.

[15] All the robustness checks are available on request from the authors.

high beta (coskewness, cokurtosis) portfolios and low beta (coskewness, cokurtosis) portfolios. Our empirical results confirm this asymmetry and show that most portfolio pairs have stronger dependence during market downturns than during market upturns. Our conclusion strongly confirms the results in the extant literature, see Patton (2006), Okimoto (2008), Chollete et al. (2011) and many others. It has wide implications for empirical asset pricing and asset allocation as well as for risk management.

Third, we apply a dynamic asymmetric copula framework based on Demarta and McNeil (2005) and Creal et al. (2013) to predict portfolio VaR and ES. This dynamic copula model has several attractive properties for VaR forecasting. The most attractive one is that it not only takes into account common features of univariate distributions, such as heteroscedasticity, skewness, fat tails, but also captures asymmetries and time-varying dependency between time series. All the models are estimated either parametrically, with the marginal distributions and the copula specified as belonging to parametric families, or semiparametrically, where the marginal distributions are estimated nonparametrically. Several widely used univariate and multivariate VaR and ES models are also considered for comparison. Backtestings are included in the evaluation process as well. To evaluate the predictions of ES, we consider a test in line with the one proposed for backtesting VaR in Lopez (1999). Overall, our study provides new evidence that the dynamic asymmetric copula model can offer more accurate VaR and ES forecasts.

Taken together, these empirical findings indicate the statistical significance of incorporating asymmetric and dynamic dependence in risk management. They can help investors better understand the co-movement between portfolios with different characteristics, and control portfolio risk more effectively under different market conditions. Moreover, we empirically prove that the dynamic asymmetric copula-based model can provide both the Basel committee and financial institutions with a more powerful and precise tool to forecast market risk and adjust minimum capital requirements.

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Appendix

2.A Estimation of Parametric Copula Model

The log-likelihood of a fully parametric copula model for conditional distribution of $\mathbf{z}_t$ takes the form:

$$\begin{aligned} L(\theta) &= \prod_{t=1}^{T} \mathbf{f}(\mathbf{z}_t|\mathcal{F}_{t-1};\theta) \\ &= \prod_{t=1}^{T} \left[c_t\left(u_{1,t},...,u_{d,t}|\mathcal{F}_{t-1};\theta_C\right) \prod_{i=1}^{N} f_{i,t}\left(z_{i,t}|\mathcal{F}_{t-1};\theta_i\right) \right] \end{aligned} \tag{2.32}$$

with log-likelihood

$$\begin{aligned} \sum_{t=1}^{T} \log \mathbf{f}(\mathbf{z}_t|\mathcal{F}_{t-1};\theta) &= \sum_{t=1}^{T}\sum_{i=1}^{d} \log f_{i,t}\left(z_{i,t}|\mathcal{F}_{t-1};\theta_i\right) \\ &+ \sum_{t=1}^{T} \log c_t\left(F_{1,t}\left(z_{1,t}|\mathcal{F}_{t-1};\theta_1\right),\ldots,F_{d,t}\left(z_{d,t}|\mathcal{F}_{t-1};\theta_d\right)|\mathcal{F}_{t-1};\theta_C\right) \end{aligned} \tag{2.33}$$

where θ denotes the parameter vector for the full model parameters, θ_i denotes the parameters for the ith marginals, θ_C denotes the parameters of copula model and $\mathcal{F}_{t-1}$ denotes the information set at time $t-1$. Following the two-stage maximum likelihood estimation (also known as the Inference method for marginals) of Joe and Xu (1996), we first estimate the parameters of marginal models using maximum likelihood:

$$\hat{\theta}_{\mathbf{i}} = \underset{\theta_i}{\operatorname{argmax}} \sum_{t=1}^{T} \log f_{i,t}\left(z_{i,t}|\mathcal{F}_{t-1};\theta_i\right),\ i = 1,...,N \tag{2.34}$$

and then using the estimations in the first stage, we calculate $F_{i,t}$ and estimate the copula parameters via maximum likelihood:

$$\hat{\theta}_{\mathbf{C}} = \underset{\theta_C}{\operatorname{argmax}} \sum_{t=1}^{T} \log c_t \left(F_{1,t} \left(z_{1,t} | \mathcal{F}_{t-1}; \theta_1 \right), ..., F_{d,t} \left(z_{d,t} | \mathcal{F}_{t-1}; \theta_d \right) | \mathcal{F}_{t-1}; \theta_C \right) \tag{2.35}$$

2.B Estimation of Semiparametric Copula Model

In the semiparametric estimation (also known as Canonical Maximum Likelihood Estimation), the univariate marginals are estimated nonparametrically using the empirical distribution function and the copula model is again parametrically estimated via maximum likelihood.

$$\hat{F}_i(z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1 \{ \hat{z}_{i,t} \leq z \} \tag{2.36}$$

$$\hat{u}_{i,t} \equiv \hat{F}_i(z) \sim Unif \ (0,1), \ i = 1,2,...,N \tag{2.37}$$

$$\hat{\theta}_{\mathbf{C}} = \arg\max_{\theta_{\mathbf{C}}} \sum_{t=1}^{T} \log c_t \left(\hat{u}_{1,t}, ..., \hat{u}_{i,t} | \mathcal{F}_{t-1}; \theta_{\mathbf{C}} \right) \tag{2.38}$$

where $z_{i,t}$ are the standardized residuals of the marginal model and $\hat{F}_i$ is different from the standard empirical CDF by the scalar $\frac{1}{n+1}$ (in order to ensure that the transformed data cannot be on the boundary of the unit interval $[0,1]$).

2.C Algorithm for VaR and ES Forecasting Using Dynamic GHST Copula Model

Step 1: Determine the in sample and out-of-sample period for VaR and ES forecasting.

Step 2: We predict conditional mean and conditional volatility from the prespecified time series model on rolling window and do one step ahead forecasting for each margins;

Step 3: Estimate the density model to get the probabilities for each forecasted margin. We consider both parametric (univariate skewed t) and nonparametric (EDF) estimation on sliding window.

Step 4: Estimate the parameters for full parametric and semiparametric copulas using using maximum likelihood estimation (see Appendix A and B).

Step 5: Using the estimated parameters in Step 4 as initial values, we estimate time-varying dependence parameters for asymmetric (GHST) copulas based on the GAS framework, see Equation (4.9).

Step 6: With the estimated time-varying copula parameters in hand, we can apply Monte Carlo simulation to generate N samples of shocks and then portfolio returns.

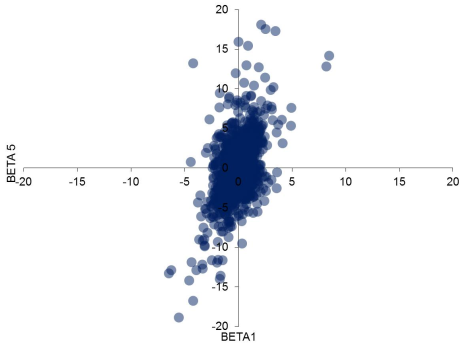
Step 7: Based on the empirical $\alpha-$quantile of forecasted portfolio return, it is straightforward to forecast corresponding VaR.

Step 8: Given the N simulated portfolio returns, we can also calculate $\alpha-$quantile expected shortfall using Equation (2.25).

Step 9: Use the realized portfolio returns to backtest VaR and ES predications.

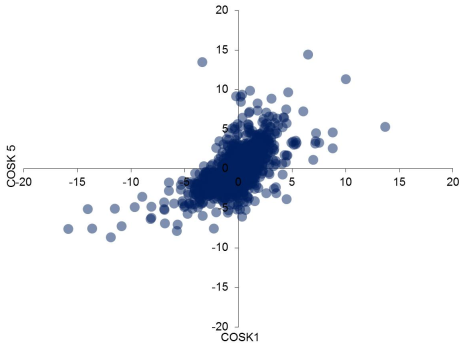
Figure 2.1 The Scatter Plots for Portfolio 1 (High) and Portfolio 5 (Low)

Panel A: US Stock Market



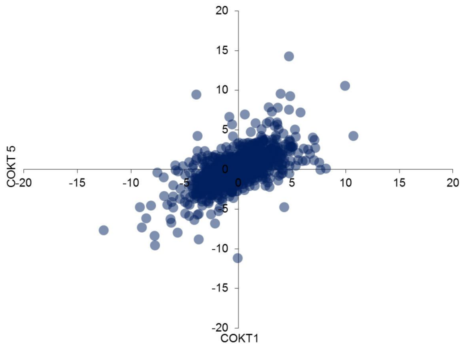
$\rho_L = 0.63,\ \rho_C = 0.42,\ \rho_U = 0.52$

Beta Portfolio



$\rho_L = 0.70,\ \rho_C = 0.63,\ \rho_U = 0.46$

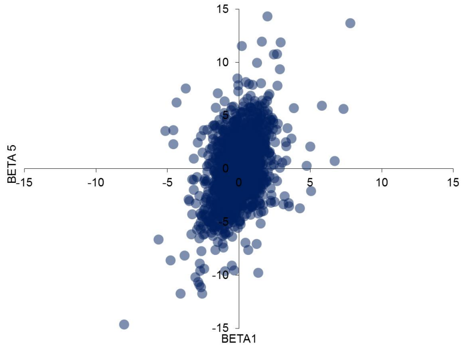
Coskewness Portfolio



$\rho_L = 0.65,\ \rho_C = 0.61,\ \rho_U = 0.41$

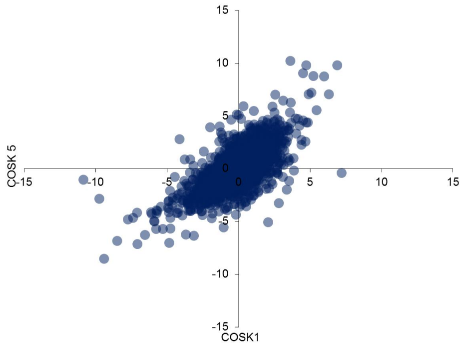
Cokurtosis Portfolio

Panel B: UK Stock Market



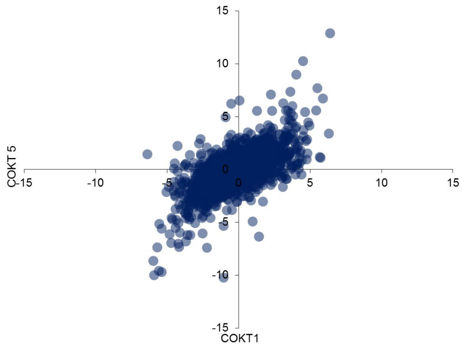
$\rho_L = 0.61,\ \rho_C = 0.29,\ \rho_U = 0.30$

Beta Portfolio



$\rho_L = 0.61,\ \rho_C = 0.46,\ \rho_U = 0.53$

Coskewness Portfolio



$\rho_L = 0.67,\ \rho_C = 0.47,\ \rho_U = 0.68$

Cokurtosis Portfolio

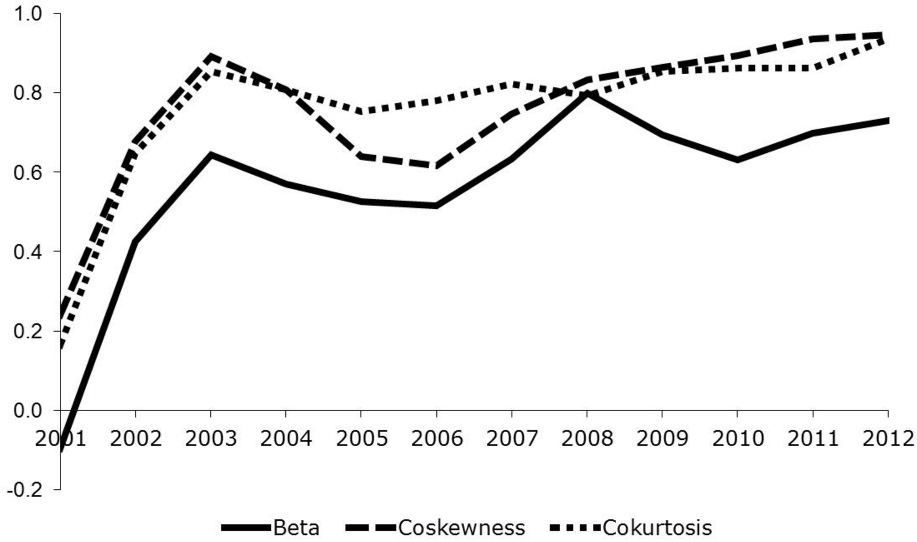
Note: This figure shows the scatter plots for different portfolio pairs, including $(BETA1, BETA5)$, $(COSK1, COSK5)$, and $(COKT1, COKT5)$. Three threshold correlation coefficients are used to demonstrate the asymmetric dependence between the portfolios:

$$\rho_L = Corr\left(r_1, r_5 | r_1 \leq F_1^{-1}(0.15), r_5 \leq F_5^{-1}(0.15)\right),$$
$$\rho_U = Corr\left(r_1, r_5 | F_1^{-1}(0.85) < r_1, F_5^{-1}(0.85) < r_5\right),$$
$$\rho_C = Corr\left(r_1, r_5 | F_1^{-1}(0.15) < r_1 \leq F_1^{-1}(0.85), F_5^{-1}(0.15) < r_5 \leq F_5^{-1}(0.85)\right),$$

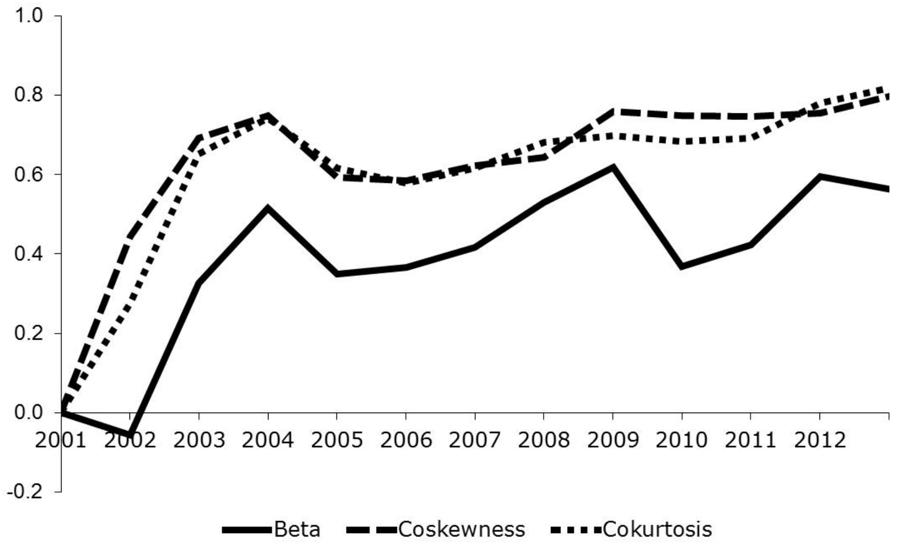
where ρ_L, ρ_U and ρ_C denote the correlation coefficients at the lower tail, upper tail and center, respectively, and F^{-1} denotes the inverse cumulative probability density function.

Figure 2.2 Time-varying Rank Correlation for High versus Low Portfolios

Panel A: US Stock Market



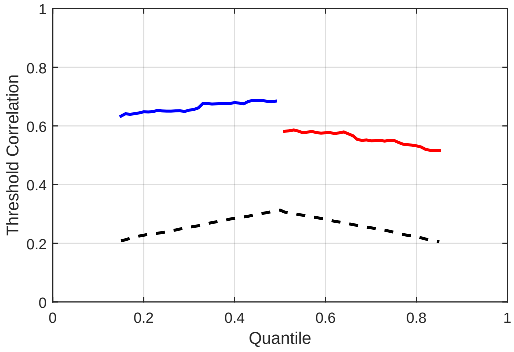
Panel B: UK Stock Market



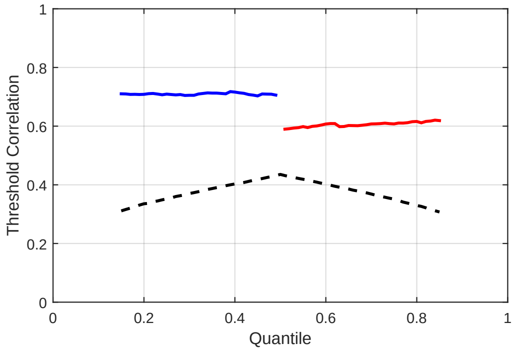
Note: This figure depicts two time series plots of average rolling 250-day rank correlation between the high and low portfolios; $(BETA1, BETA5)$, $(COSK1, COSK5)$, and $(COKT1, COKT5)$, for each year.

Figure 2.3 Threshold Correlation for High versus Low Portfolios

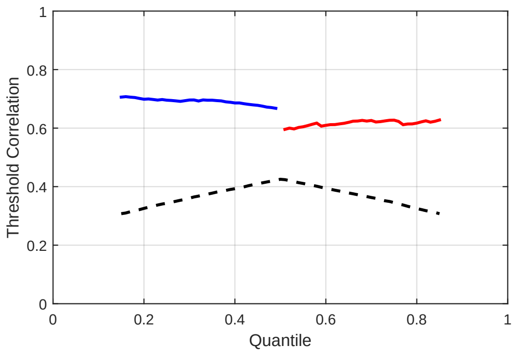
Panel A: US Stock Market



Beta Portfolio

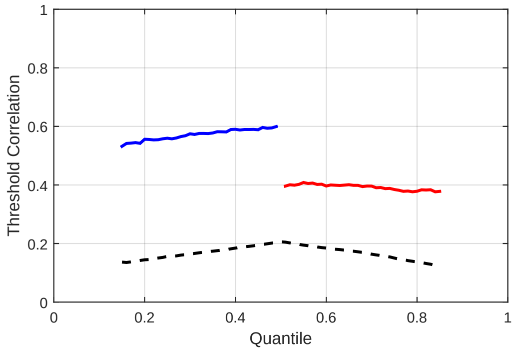


Coskewness Portfolio

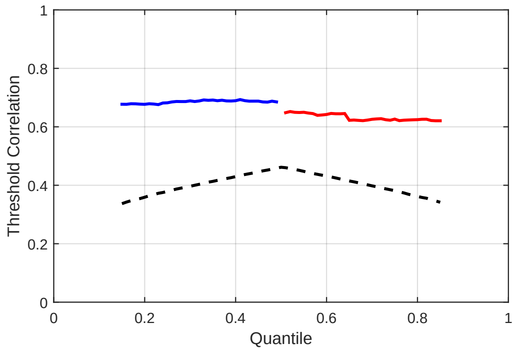


Cokurtosis Portfolio

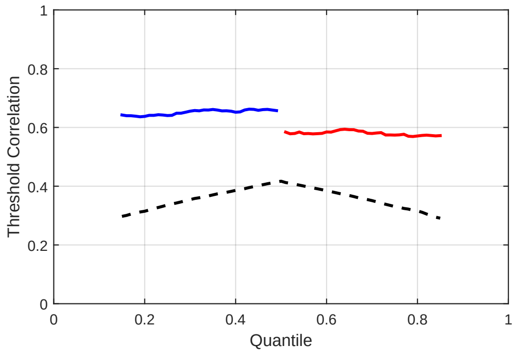
Panel B: UK Stock Market



Beta Portfolio



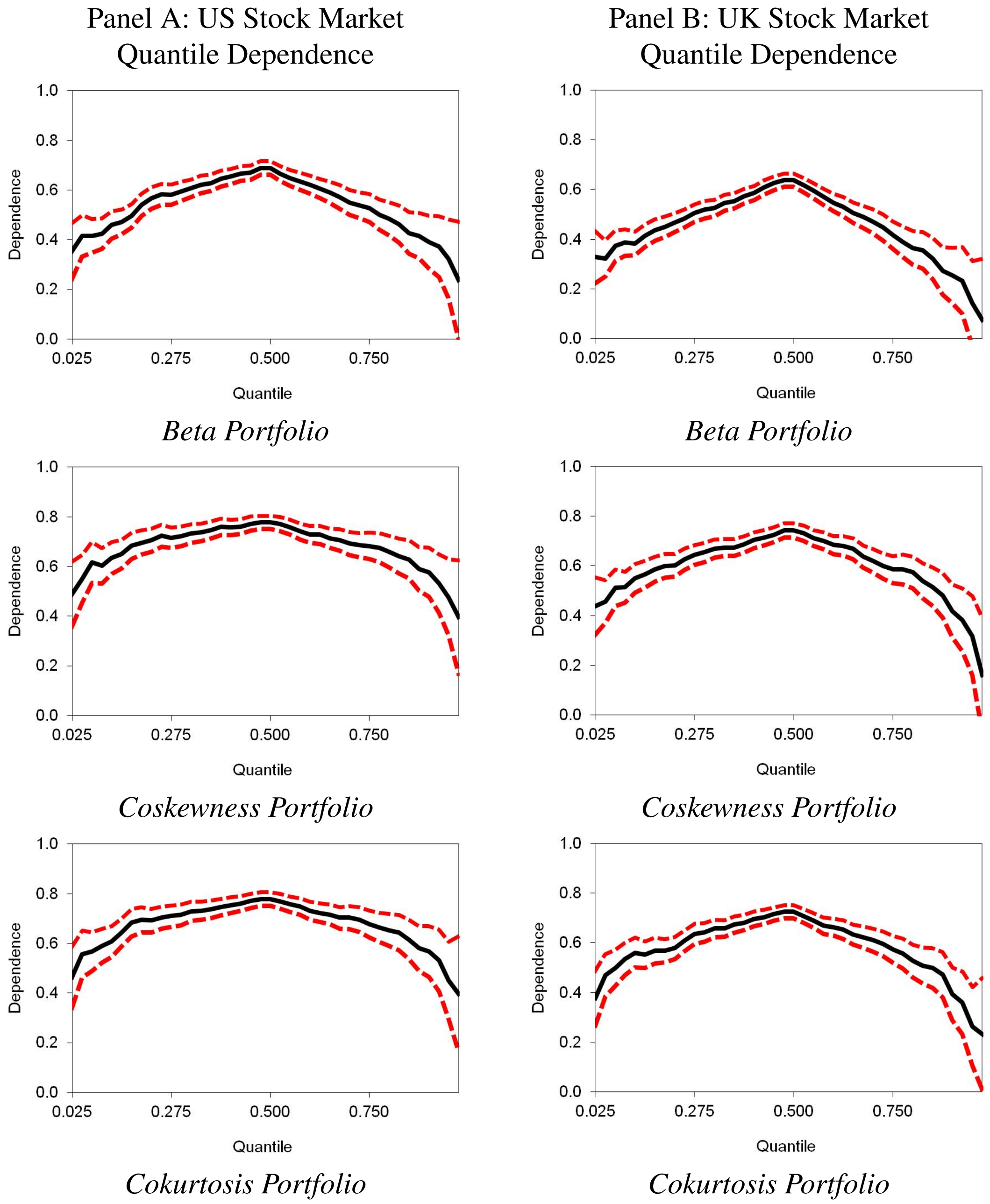
Coskewness Portfolio



Cokurtosis Portfolio

Note: This figure shows the threshold correlation (or exceedance correlation) between high beta (coskewness and cokurtosis) portfolio and low beta (coskewness and cokurtosis) portfolio. The threshold correlation measures the linear correlation between two assets when both assets increases or decreases of more than specified quantiles (see Ang and Bekaert, 2002; Ang and Chen, 2002; Longin and Solnik, 2001). A left solid line denotes (2.12) and a right solid line denotes (2.13), respectively. The dash line represents the threshold correlations implied by the bivariate normal distribution with a linear correlation rho from the data.

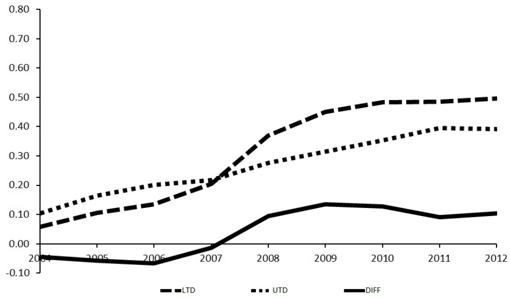
Figure 2.4 Quantile Dependence between the Standardized Residuals of High and Low Portfolios



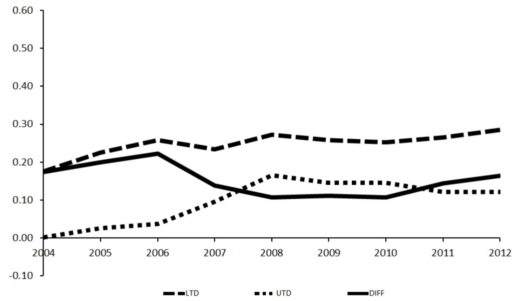
Note: This figure presents the estimated quantile dependence between the standardized residuals for high beta (coskewness and cokurtosis) portfolio and low beta (coskewness and cokurtosis), and the difference in upper and lower quantile dependence. A solid black line denotes a quantile dependence and dash lines denote 90% bootstrap confidence interval.

Figure 2.5 Time-varying Asymmetric Tail Dependence

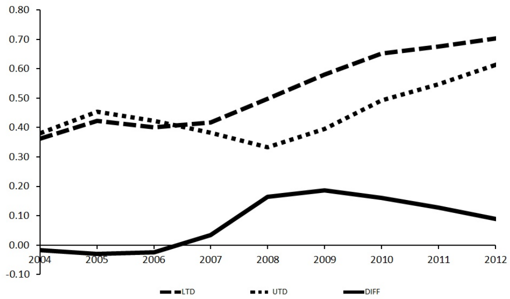
Panel A: US Stock Market | Panel B: UK Stock Market



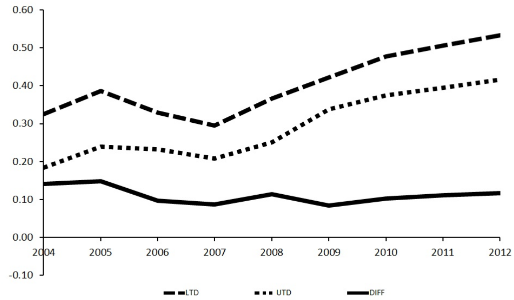
Beta Portfolio



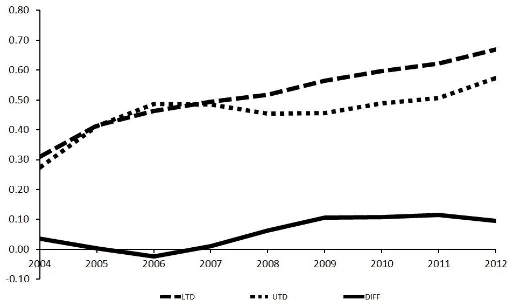
Beta Portfolio



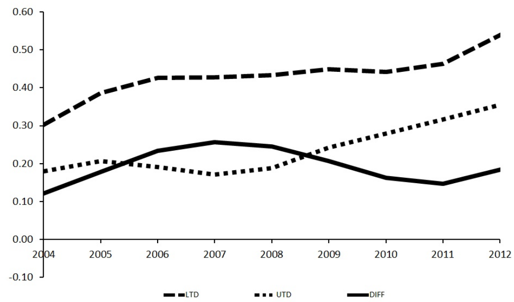
Coskewness Portfolio



Coskewness Portfolio



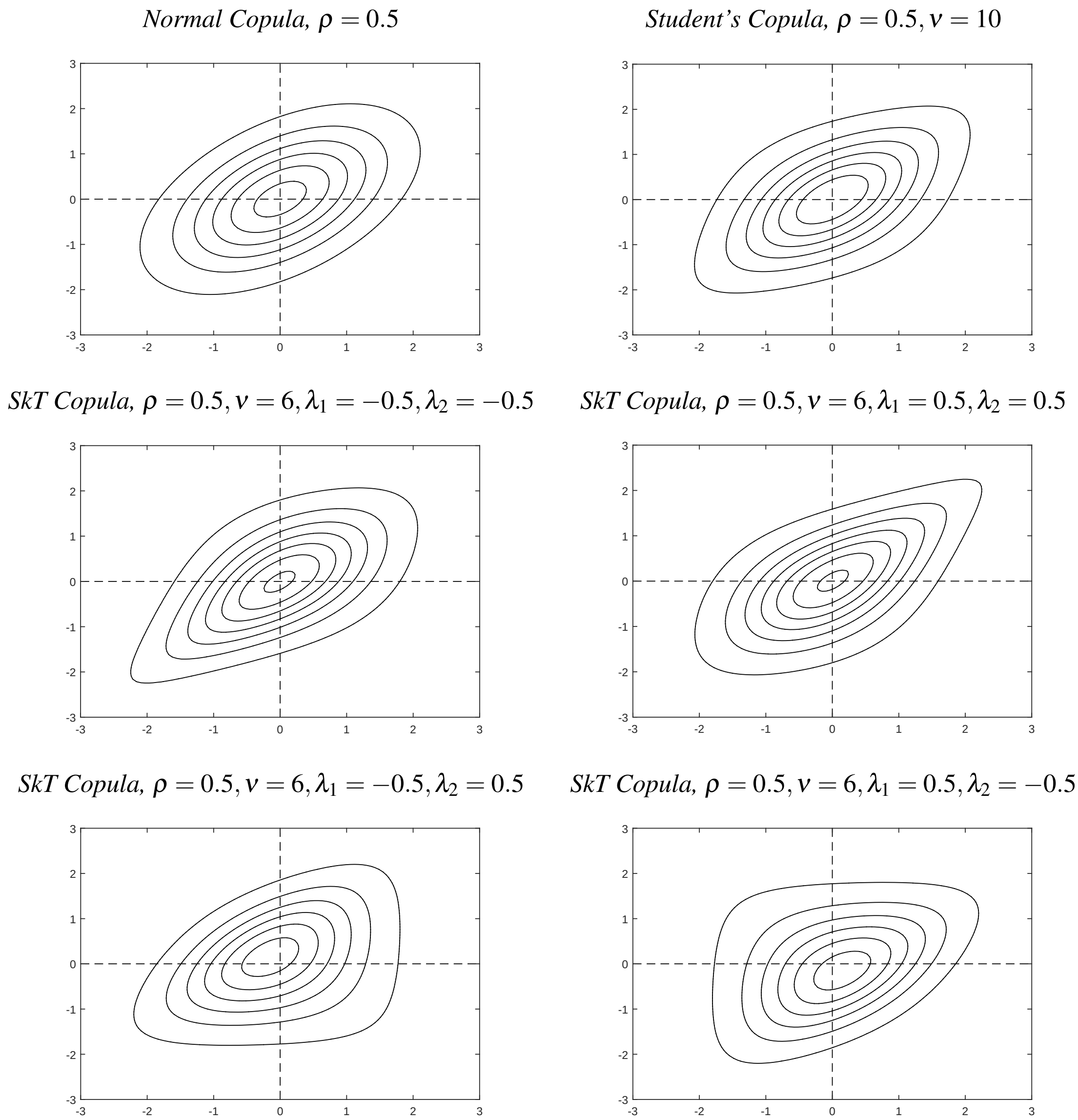
Cokurtosis Portfolio



Cokurtosis Portfolio

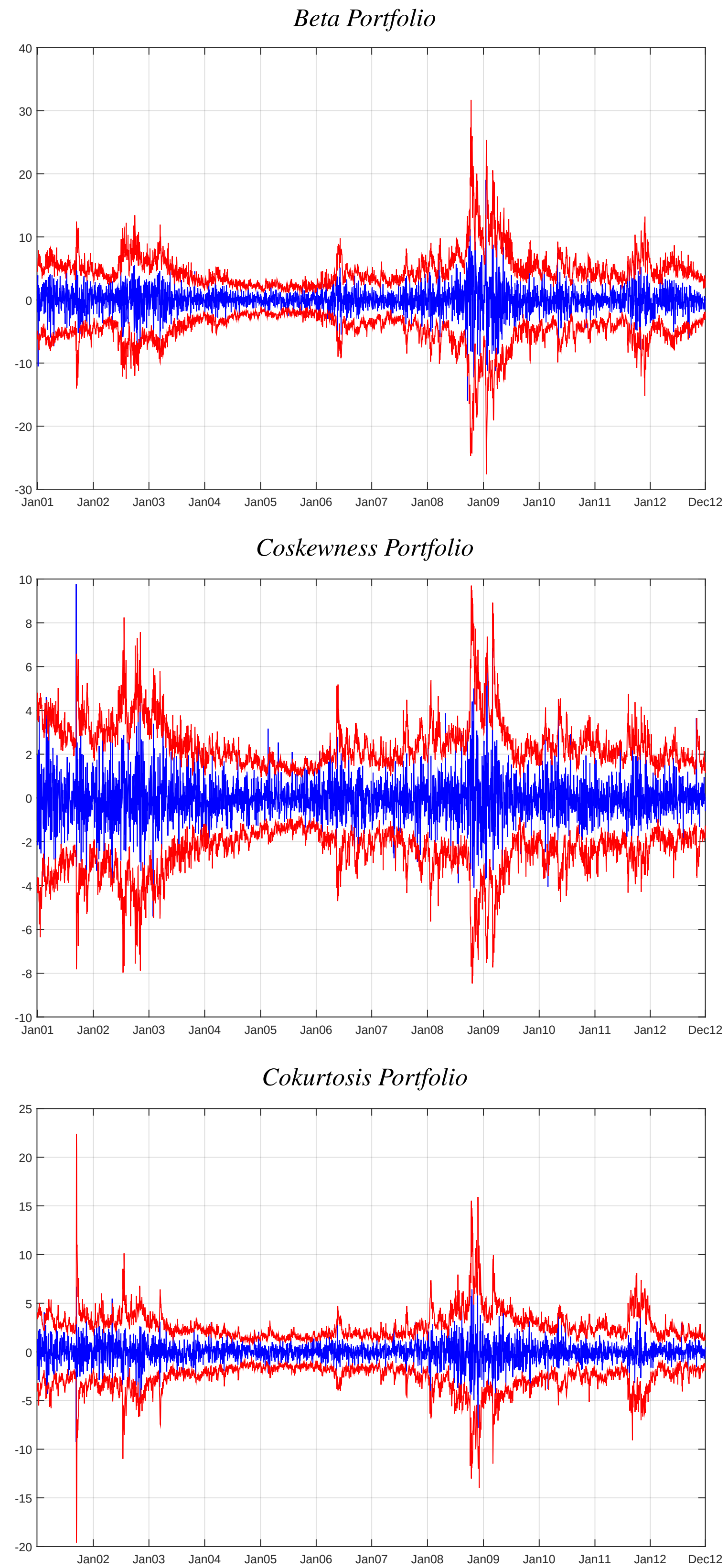
Note: This figure shows the dynamic evolution of average tail dependence coefficient (TDC). TDC is estimated by rotated Gumbel copula from rolling window with window length of 1,000 observations for all the portfolio pairs and we take the average of TDC for each year. The TDCs between portfolios generally increase over time, especially during recent financial crisis. In the US market, lower tail dependence (LTD) is relatively close to or even lower than the upper tail dependence before the financial crisis. However, the LTD has become greater than upper tail dependence (UTD) since the outbreak of the US subprime mortgage crisis in 2007. In the UK market, the LTD is always greater than UTD. Note that DIFF denotes the difference between LTD and UTD ($DIFF = LTD - UTD$).

Figure 2.6 Contour Probability Plots for Copulas



Note: This figure shows contour probability plots for the normal, Student's *t*, and GHST copulas. The probability levels for each contour are held fixed across six panels. The marginal distributions are assumed to be normally distributed. ρ denotes the correlation coefficient, ν denotes the degree of freedom, and λ denotes the asymmetric parameters of copulas.

Figure 2.7 P&L and VaR Estimated by Dynamic Asymmetric Copulas



Note: This figure shows realized returns of beta, coskewness and cokurtosis portfolios and corresponding 99% VaR estimated by dynamic asymmetric (GHST) copulas.

Table 2.1 Definitions of Portfolios

This table describes the 12 HML portfolios that we constructed for the purpose of empirical analysis in our study. Portfolios are sorted by market beta, coskewness and cokurtosis. All the portfolios are annually rebalanced.

Portfolio	Market	Description
$HML(Beta, L/S; US)$	US Stock Market	Long (short) BETA5 and short (long) BETA1
$HML(Cosk, L/S; US)$		Long (short) COSK5 and short (long) COSK1
$HML(Cokt, L/S; US)$		Long (short) COKT5 and short (long) COKT1
$HML(Beta, L/S; UK)$	UK Stock Market	Long (short) BETA5 and short (long) BETA1
$HML(Cosk, L/S; UK)$		Long (short) COSK5 and short (long) COSK1
$HML(Cokt, L/S; UK)$		Long (short) COKT5 and short (long) COKT1

Table 2.2 Descriptive Statistics for Returns on the Characteristic-sorted Portfolios

Panel A reports descriptive statistics for daily returns on the characteristic-sorted portfolios from January 4, 2000 to December 31, 2012, which correspond to a sample of 3,268 observations for US market and a sample of 3,283 observations for UK market. We sort stocks into quintiles according to market beta (coskewness, cokurtosis) and form five capitalization-weighted, annually rebalanced portfolios. BETA1 (COSK1, COKT1) contains the equities with the lowest beta (coskewness and cokurtosis), and portfolio 5 contains the equities with the highest beta (coskewness and cokurtosis). Panel B reports descriptive statistics for returns on the high minus low (HML) portfolios. Note that the means, standard deviations, minima, and maxima are reported in %. LB test lag 5 and 10 denote the p-values of the Ljung-Box Q-test for autocorrelation at lags 5 and 10, respectively. In addition, we report the p-values of Engle's Lagrange Multiplier test for the ARCH effect on the residual series.

Panel A: Characteristic-sorted Portfolio

	US Stock Market						*UK Stock Market*					
Statistics	BETA1	BETA5	COSK1	COSK5	COKT1	COKT5	BETA1	BETA5	COSK1	COSK5	COKT1	COKT5
Mean	0.023	-0.021	0.004	0.003	-0.011	0.024	0.039	-0.023	0.018	-0.002	0.009	0.006
Std dev	0.889	2.567	1.525	1.707	1.612	1.380	0.952	2.317	1.394	1.553	1.457	1.425
Skewness	0.045	-0.067	-1.181	0.733	-0.137	0.280	0.030	-0.099	-0.807	0.390	-0.018	-0.278
Kurtosis	12.413	12.447	19.667	10.315	8.763	16.443	10.804	9.144	8.785	7.349	4.768	12.906
Maximum	8.477	18.044	13.735	14.394	10.761	14.222	7.817	17.378	7.220	10.188	6.431	12.848
Minimum	-6.444	-20.238	-15.811	-8.734	-12.524	-11.224	-7.995	-16.927	10.188	-8.569	-6.356	-10.264
LB test lag 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB test lag 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Engle's test	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel B: HML Portfolio

Statistics	$HML(Beta,L;US)$	$HML(Cosk,L;US)$	$HML(Cokt,L;US)$	$HML(Beta,L;UK)$	$HML(Cosk,L;UK)$	$HML(Cokt,L;UK)$
Mean	-0.044	-0.001	0.035	-0.062	-0.020	-0.003
Std dev	2.206	1.312	1.235	2.108	1.152	1.193
Skewness	0.369	1.177	0.346	0.089	0.355	-0.041
Kurtosis	12.925	20.139	17.163	9.151	8.881	8.371
Maximum	17.389	16.719	13.292	15.970	9.767	7.769
Minimum	-18.409	-8.470	-11.182	-19.017	-7.658	-9.247
LB test lag 5	0.00	0.00	0.00	0.01	0.00	0.00
LB test lag 10	0.00	0.01	0.00	0.00	0.00	0.00
Engle's test	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.3 Parameter Estimates and Goodness of Fit Test for the Univariate Modeling

This table reports parameter estimates from AR and GJR-GARCH models for conditional mean and conditional variance of portfolio returns. We estimate all parameters using the sample from January 4, 2000 to December 31, 2012, which correspond to a sample of 3,268 observations for US market and a sample of 3,283 observations for UK market. We use * to indicate the significance of estimate at the 5% significance level. We also report the p-values of two goodness-of-fit tests for the skewed Student's t distribution. We employ Kolmogorov-Smirnov (KS) and Cramer-von Mises (CvM) tests.

	Panel A: US Stock Market						Panel B: UK Stock Market					
Portfolio	BETA1	BETA5	COSK1	COSK5	COKT1	COKT5	BETA1	BETA5	COSK1	COSK5	COKT1	COKT5
ϕ_0	0.025	-0.021	0.004	0.006	-0.011	0.029	0.039*	-0.023	0.018	-0.001	0.009	0.008
ϕ_1	-0.064*	_	_	-0.091*	_	-0.085*	_	_	_	-0.045*	_	-0.060*
ϕ_2	_	_	_	-0.085*	_	-0.061*	_	_	_	_	_	_
ω	0.012*	0.048*	0.027*	0.015*	0.021*	0.016*	0.014*	0.036*	0.021*	0.019*	0.014*	0.028*
α	0.018	0.001*	0.0235*	0.000	0.023*	0.008	0.012	0.004	0.031*	0.006	0.016	0.047*
γ	0.122*	0.133*	0.104*	0.163*	0.126*	0.179*	0.111*	0.130*	0.092*	0.135*	0.103*	0.142*
β	0.901*	0.923*	0.903*	0.916*	0.903*	0.896*	0.913*	0.924*	0.911*	0.919*	0.925*	0.867*
ν	11.507	12.805	9.404	12.331	24.329	5.637	9.593	27.910	7.384	33.361	94.667	7.475
λ	-0.111	-0.072	-0.246	0.006	-0.099	-0.099	-0.076	-0.054	-0.168	0.025	-0.062	-0.076
KS	0.61	0.17	0.43	0.11	0.14	0.97	0.77	0.96	0.53	0.53	0.35	0.36
CvM	0.33	0.10	0.42	0.10	0.16	1.00	0.87	0.92	0.30	0.40	0.45	0.27

Table 2.4 Tests for Time-varying Dependence between High and Low Portfolios

We report the p-value from tests for time-varying rank correlation between the high portfolio (e.g. BETA5) and the low portfolio (e.g. BETA1). Having no a priori dates to consider for the timing of a break, we consider naive tests for breaks at three chosen points in sample period, at t*/ T ∈{0.15, 0.50, 0.85}, which corresponds to the dates 10-Dec-2001, 03-Jul-2006, 17-Jan-2011. The 'Any' column reports the results of test for dependence break of unknown timing proposed by Andrews (1993). To detect whether the dependence structures between characteristic-sorted portfolios significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points and the 'Crisis' panel reports the results for this test. The 'AR' panel presents the results from the ARCH LM test for time-varying volatility proposed by Engle (1982). Under the null hypothesis of a constant conditional copula, we test autocorrelation in a measure of dependence (see Patton, 2012).

	Panel A: Break				Panel B: Crisis		Panel C: AR(p)		
Portfolio	0.15	0.5	0.85	Any	US	EU	AR(1)	AR(5)	AR(10)
US BETA1&5	0.00	0.00	0.04	0.00	0.00	0.07	0.00	0.00	0.00
US COSK1&5	0.00	0.03	0.82	0.04	0.22	0.38	0.00	0.00	0.00
US COKT1&5	0.02	0.30	0.67	0.25	0.92	0.42	0.18	0.75	0.09
UK BETA1&5	0.00	0.00	0.17	0.00	0.04	0.08	0.00	0.12	0.00
UK COSK1&5	0.59	0.03	0.62	0.03	0.07	0.25	0.01	0.00	0.02
UK COKT1&5	0.98	0.24	0.36	0.24	0.24	0.13	0.00	0.00	0.00

Table 2.5 Testing the Significance of the Differences of Exceedence Correlations

This table presents the statistics and p-values from a model-free symmetry test proposed by Hong et al. (2007) to examine whether the exceedance correlations between low portfolio (i.e. BETA1) and high portfolio (i.e. BETA5) are asymmetric. We report p-values in $[\cdot]$. The J statistics for testing the null hypothesis of symmetric correlation, $\rho^{+}(c) = \rho^{-}(c)$, is defined as

$$J_{\rho} = T\left(\hat{\rho}^{+} - \hat{\rho}^{-}\right)' \hat{\Omega}^{-1} \left(\hat{\rho}^{+} - \hat{\rho}^{-}\right)$$

where $\hat{\Omega} = \sum_{l=1-T}^{T-1} k(l/p)\,\hat{\gamma}_l$ and k is a kernel function that assigns a suitable weight to each lag of order l, and p is the smoothing parameter or lag truncation order (see Hong et al. (2007) for more details).

Panel A: US market			Panel B: UK market		
BETA1&5	COSK1&5	COKT1&5	BETA1&5	COSK1&5	COKT1&5
48.471	40.246	44.363	56.249	38.655	46.367
[0.06]	[0.25]	[0.13]	[0.01]	[0.31]	[0.09]

Table 2.6 Estimating Tail Dependence Using Parametric Copulas

This table reports the coefficients of lower tail dependence (LTD) and upper tail dependence (UTD) and the difference between them for all the portfolios pairs. The estimations are calculated by the parametric approach in McNeil et al. (2005). λ_L^G and λ_U^G denote the lower and upper tail dependence coefficients estimated by rotated Gumbel copula and λ_L^T and λ_U^T denote the lower and upper tail dependence coefficients estimated by t copula. The p-values of testing $\lambda_L = \lambda_U$ are computed by a bootstrapping with 500 replications and reported in $[\cdot]$.

	LTD		UTD		Difference	Difference
Portfolio	λ_L^G	λ_L^T	λ_U^G	λ_U^T	$\lambda_L^G - \lambda_U^G$	$\lambda_L^T - \lambda_U^T$
US BETA1&5	0.256	0.171	0.099	0.018	0.157 [0.00]	0.153 [0.02]
US COSK1&5	0.315	0.200	0.192	0.153	0.123 [0.01]	0.047 [0.53]
US COKT1&5	0.306	0.153	0.216	0.103	0.090 [0.12]	0.050 [0.13]
UK BETA1&5	0.165	0.104	0.024	0.018	0.141 [0.00]	0.086 [0.00]
UK COSK1&5	0.297	0.203	0.095	0.062	0.202 [0.00]	0.141 [0.01]
UK COKT1&5	0.209	0.137	0.088	0.052	0.121 [0.00]	0.085 [0.21]

Table 2.7 Backtesting of VaR: Empirical Coverage Probability

This table reports ECP for each HML portfolio and VaR model. *Bias* summarises the average deviation of 12 portfolios from the nominal coverage probability, 1%, for each VaR model, and *RMSE* (Root Mean Square Error) summarises the fluctuation of the deviation across 12 portfolios for each VaR model,

$$Bias = \frac{1}{12}\sum_{p=1}^{12}(ECP_p - 1\%)\,,\; RMSE = \sqrt{\frac{1}{12}\sum_{p=1}^{12}(ECP_p - 1\%)^2}.$$

For the UK portfolios, we estimate the VaR models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. D, (P) and (S) denote “Dynamic”, “Parametric” and “Semiparametric”, respectively.

	Student’s t Copula			GHST Copula			Univariate Models	Multivariate GARCH		
Portfolio	Static	D (P)	D(S)	Static	D (P)	D(S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	0.89%	0.93%	0.86%	0.86%	0.93%	0.86%	1.09%	1.39%	1.23%	1.46%
$HML(Cosk,L;US)$	1.09%	0.86%	1.06%	1.03%	0.80%	1.03%	1.56%	1.66%	1.49%	1.36%
$HML(Cokt,L;US)$	1.03%	0.86%	0.89%	0.99%	0.86%	0.86%	1.36%	1.29%	1.36%	1.19%
$HML(Beta,S;US)$	0.56%	1.19%	1.19%	0.56%	1.16%	1.13%	1.26%	1.36%	1.19%	1.39%
$HML(Cosk,S;US)$	1.26%	0.96%	0.99%	1.19%	0.96%	0.93%	1.23%	1.52%	1.49%	1.42%
$HML(Cokt,S;US)$	0.96%	0.86%	0.86%	1.15%	0.96%	1.03%	1.29%	1.39%	1.36%	1.29%
$HML(Beta,L;UK)$	1.19%	0.92%	0.86%	0.93%	0.92%	0.92%	1.39%	1.06%	1.26%	1.13%
$HML(Cosk,L;UK)$	1.19%	1.06%	0.92%	1.09%	1.02%	0.96%	1.16%	1.26%	1.06%	1.19%
$HML(Cokt,L;UK)$	0.99%	0.86%	0.99%	0.96%	0.92%	0.99%	1.16%	1.16%	1.03%	1.19%
$HML(Beta,S;UK)$	1.02%	1.12%	0.92%	0.99%	1.12%	0.89%	1.52%	1.39%	1.52%	1.06%
$HML(Cosk,S;UK)$	1.81%	1.19%	1.25%	1.68%	1.12%	1.22%	1.33%	1.42%	0.99%	1.26%
$HML(Cokt,S;UK)$	1.22%	0.82%	0.96%	1.12%	0.82%	0.92%	1.33%	1.42%	1.13%	1.36%
Bias	0.10%	-0.03%	-0.02%	0.05%	-0.03%	-0.02%	0.31%	0.36%	0.26%	0.28%
RMSE	0.31%	0.14%	0.13%	0.26%	0.12%	0.11%	0.34%	0.39%	0.32%	0.30%

Table 2.8 Backtesting of VaR: Basel Penalty Zone

This table reports BPZ for each HML portfolio and counts the number of portfolios for each zone. BPZ counts the number of failures of the 99 percent VaR in the previous 250 VaR forecasts. Up to four failures, on average, the portfolio falls into the range of a “Green” zone. If the failures are five or more, the portfolio falls into a “Yellow” (5–9) or “Red” (10+) zone. The VaR model of which BPZ is “Green” zone is preferred. For the UK portfolios, we estimate the VaR models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. D, (P), and (S) denote “Dynamic”, “Parametric”, and “Semiparametric”, respectively.

	Student’s t Copula			GHST Copula			Univariate Model	Multivariate GARCH		
Portfolio	Static	D(P)	D(S)	Static	D(P)	D(S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cosk,L;US)$	Green	Green	Green	Green	Green	Green	Green	Yellow	Green	Green
$HML(Cokt,L;US)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Beta,S;US)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cosk,S;US)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cokt,S;US)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Beta,L;UK)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cosk,L;UK)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cokt,L;UK)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Beta,S;UK)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
$HML(Cosk,S;UK)$	Yellow	Green	Green	Yellow	Green	Green	Green	Green	Green	Green
$HML(Cokt,S;UK)$	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
Green/Yellow/Red	11/1/0	12/0/0	12/0/0	11/1/0	12/0/0	12/0/0	12/0/0	11/1/0	12/0/0	12/0/0

Table 2.9 Backtesting of VaR: Conditional Coverage Test

This table presents the CC results. The CC test uses the LR statistic and it follows the Chi-squared distribution with two degrees-of-freedom under the null hypothesis. For the UK portfolios, we estimate the VaR models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. * indicates that the VaR model is rejected at the 5% significance level. D, (P), and (S) denote "Dynamic", "Parametric", and "Semiparametric", respectively.

	Student's t Copula			GHST Copula			Univariate Model	Multivariate GARCH		
Portfolio	Static	D (P)	D (S)	Static	D (P)	D (S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	0.84	0.69	1.06	1.06	2.79	3.44	0.99	4.43	1.96	6.91*
$HML(Cosk,L;US)$	13.07*	6.07*	8.33*	13.83*	7.43*	8.60*	8.20*	12.23*	8.19*	6.83*
$HML(Cokt,L;US)$	8.60*	11.25*	5.52	8.96*	6.07*	6.07*	3.85	2.81	9.05*	1.67
$HML(Beta,S;US)$	7.09*	1.94	4.22	8.28*	1.56	4.01	2.34	3.82	1.94	4.43
$HML(Cosk,S;US)$	8.22*	1.20	1.05	7.97*	1.20	1.42	1.96	7.35*	6.54*	6.11*
$HML(Cokt,S;US)$	0.61	1.06	1.06	0.69	1.37	1.06	3.41	5.36	5.86	2.78
$HML(Beta,L;UK)$	1.88	0.87	1.10	1.51	0.71	0.71	4.31	0.78	2.78	1.15
$HML(Cosk,L;UK)$	4.18	0.78	0.71	3.99	0.66	0.62	4.05	4.64	0.97	4.18
$HML(Cokt,L;UK)$	1.06	1.10	0.60	1.22	0.71	0.74	1.34	1.51	0.66	1.88
$HML(Beta,S;UK)$	0.66	1.21	0.71	0.60	1.21	0.87	11.46*	6.26*	11.46*	0.78
$HML(Cosk,S;UK)$	16.35*	1.88	2.78	11.85*	1.51	2.30	3.18	4.96	4.43	2.78
$HML(Cokt,S;UK)$	4.38	1.42	0.62	3.99	1.42	0.71	3.18	5.98	1.21	4.55
Number of Rejection	5	2	1	5	2	2	3	3	4	3

Table 2.10 Backtesting of VaR: Dynamic Quantile Test

This table presents the DQ test results. The DQ test uses the Wald statistic and it follows the Chi-squared distribution with 6 degrees-of-freedom under the null hypothesis (see Equation (2.29)). For the UK portfolios, we estimate the VaR models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. * indicates that the VaR model is rejected at the 5% significance level. D, (P) and (S) denote "Dynamic", "Parametric" and "Semiparametric", respectively.

	Student's t Copula			GHST Copula			Univariate Model	Multivariate GARCH		
Portfolio	Static	D (P)	D (S)	Static	D (P)	D (S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	4.93	13.32*	5.49	5.49	8.69	9.75	16.73*	44.91*	41.50*	34.03*
$HML(Cosk,L;US)$	65.11*	13.69*	33.83*	58.64*	15.82*	35.42*	12.25*	27.26*	36.14*	19.74*
$HML(Cokt,L;US)$	50.07*	36.16*	14.17*	52.44*	17.75*	15.19*	6.72	5.25	20.58*	3.64
$HML(Beta,S;US)$	6.90	3.21	10.36	7.84	2.90	10.84	4.55	5.61	3.23	6.17
$HML(Cosk,S;US)$	21.01*	4.60	2.71	22.31*	4.60	3.18	4.14	9.14	11.32	7.67
$HML(Cokt,S;US)$	1.20	3.85	1.63	1.26	4.37	1.63	4.69	10.77	15.33*	4.86
$HML(Beta,L;UK)$	4.00	3.21	1.42	3.79	2.92	1.18	24.63*	2.59	10.12	3.64
$HML(Cosk,L;UK)$	27.31*	2.54	11.84*	31.43*	2.50	11.19*	10.67	15.31*	11.93*	11.87*
$HML(Cokt,L;UK)$	12.56*	1.42	2.54	13.29*	1.18	2.54	3.18	2.72	3.61	3.24
$HML(Beta,S;UK)$	3.78	3.65	4.71	4.00	3.65	5.20	27.39*	10.54	21.48*	3.65
$HML(Cosk,S;UK)$	24.04*	3.24	4.53	15.90*	2.72	3.85	5.71*	7.88*	12.44*	9.86
$HML(Cokt,S;UK)$	10.26	1.64	1.17	9.69	1.64	1.18	5.57*	38.86*	17.98*	25.44*
Number of Rejection	6	3	3	6	2	3	4	4	8	4

Table 2.11 Backtesing of ES: Mean Absolute Error

This table reports the mean absolute error (MAE) for each HML portfolio and ES model (see Equation 2.30). For the UK portfolios, we forecast the ES models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent ES for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. The lowest MAE for each portfolio is given in bold. The average MAE and corresponding ranking are reported at the bottom of this table. D, (P), and (S) denote “Dynamic”, “Parametric’,’ and “Semiparametric”, respectively.

	Student’s t Copula		GHST Copula		Univariate Model	Multivariate Models		
Portfolio	D (P)	D (S)	D (P)	D (S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	0.0064	0.0073	**0.0061**	0.0068	0.0100	0.0138	0.0093	0.0108
$HML(Cosk,L;US)$	0.0036	0.0042	0.0037	0.0047	0.0055	0.0036	0.0032	**0.0030**
$HML(Cokt,L;US)$	**0.0052**	0.0058	0.0078	0.0084	0.0081	0.0087	0.0067	0.0056
$HML(Beta,S;US)$	0.0170	0.0172	0.0169	**0.0054**	0.0112	0.0090	0.0137	0.0149
$HML(Cosk,S;US)$	0.0118	**0.0029**	0.0103	0.0112	0.0093	0.0113	0.0096	0.0094
$HML(Cokt,S;US)$	0.0105	0.0102	0.0089	0.0099	0.0069	0.0066	**0.0049**	0.0060
$HML(Beta,L;UK)$	0.0087	0.0071	0.0084	**0.0070**	0.0127	0.0105	0.0099	0.0073
$HML(Cosk,L;UK)$	0.0043	**0.0043**	0.0045	0.0048	0.0049	0.0055	0.0044	0.0054
$HML(Cokt,L;UK)$	**0.0045**	0.0072	0.0049	0.0071	0.0108	0.0108	0.0067	0.0096
$HML(Beta,S;UK)$	0.0081	0.0062	0.0079	**0.0058**	0.0136	0.0137	0.0122	0.0099
$HML(Cosk,S;UK)$	0.0077	0.0101	**0.0074**	0.0100	0.0139	0.0174	0.0090	0.0125
$HML(Cokt,S;UK)$	0.0049	0.0050	0.0047	**0.0046**	0.0064	0.0074	0.0063	0.0067
Average MAE	0.0077	0.0073	0.0076	**0.0071**	0.0094	0.0099	0.0080	0.0084
Ranking	4	2	3	**1**	7	8	5	6

Table 2.12 Backtesting of ES: Mean Squared Error

This table reports the mean squared error (MSE) for each HML portfolio and ES model (see Equation 2.31). For the UK portfolios, we forecast the ES models using 250 business days over the period 4 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent ES for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. The lowest MSE for each portfolio is given in bold. The average MSE and corresponding ranking are reported at the bottom of this table. D, (P), and (S) denote "Dynamic", "Parametric", and "Semiparametric", respectively.

	Student's t Copula		GHST Copula		Univariate Model	Multivariate Models		
Portfolio	D (P)	D (S)	D (P)	D (S)	FHS	BEKK	CCC	DCC
$HML(Beta,L;US)$	0.0145	0.0178	**0.0140**	0.0168	0.0237	0.0422	0.0172	0.0287
$HML(Cosk,L;US)$	0.0054	0.0054	0.0051	0.0053	0.0083	0.0022	0.0020	**0.0016**
$HML(Cokt,L;US)$	**0.0116**	0.0130	0.0161	0.0176	0.0234	0.0220	0.0179	0.0169
$HML(Beta,S;US)$	0.0499	0.0514	0.0490	0.0253	0.0286	**0.0237**	0.0599	0.0497
$HML(Cosk,S;US)$	0.0289	**0.0127**	0.0242	0.0282	0.0184	0.0218	0.0190	0.0155
$HML(Cokt,S;US)$	0.0215	0.0237	0.0176	0.0219	0.0089	0.0074	**0.0053**	0.0069
$HML(Beta,L;UK)$	0.0192	0.0121	0.0166	**0.0105**	0.0257	0.0305	0.0213	0.0200
$HML(Cosk,L;UK)$	**0.0028**	0.0033	0.0031	0.0045	0.0042	0.0050	0.0034	0.0044
$HML(Cokt,L;UK)$	**0.0087**	0.0178	0.0102	0.0148	0.0550	0.0522	0.0122	0.0330
$HML(Beta,S;UK)$	0.0204	0.0122	0.0185	**0.0119**	0.0346	0.0321	0.0180	0.0155
$HML(Cosk,S;UK)$	0.0152	0.0215	**0.0149**	0.0214	0.0394	0.0580	0.0223	0.0351
$HML(Cokt,S;UK)$	0.0047	0.0050	0.0047	**0.0046**	0.0063	0.0077	0.0065	0.0068
Average MSE	0.0169	0.0163	0.0162	**0.0152**	0.0230	0.0254	0.0171	0.0195
Ranking	4	3	2	**1**	7	8	5	6

Chapter 3

Time-varying and Asymmetric Dependence of International Financial Markets: Robust Risk Management and Optimal Asset Allocation

Understanding the dependence between equity and currency returns is of particular importance in international financial markets. However, less attention has been paid to modeling dependence. This motivates us to study the dependence structure of international financial markets. We find striking evidence of time-varying and asymmetric dependence between equity and currency returns. To specify this dynamic and asymmetric dependence, we propose a new time-varying asymmetric copula (TVAC) model. We empirically demonstrate that the use of this TVAC model makes risk management more robust and asset allocation more optimal in international financial markets.

[0]The early version of this paper is formerly entitled "Time-varying Skewed *t* Copula Model with Applications". I am grateful to Carol Alexander and Sjur Westgaard, and other participants in the Young Finance Scholars' conference 2014 at the University of Sussex and workshop participants at the University of Glasgow, for helpful discussions and comments.

3.1 Introduction

The recent global financial crisis and the European sovereign debt crisis have revealed the importance of robust risk management and efficient asset allocation in international financial markets. The failure of traditional risk measures and portfolio optimization frameworks also motivates us to further investigate the intricate and dynamic relationship between financial markets and to look for a more appropriate model to accommodate their well-documented features.

Empirical finance literature has reported substantial evidence of two types of distributional asymmetries in the joint distribution of asset returns. The first type of asymmetry is from the univariate distribution of individual assets and its importance in asset pricing has been well investigated over the last four decades (see Friend and Westerfield, 1980; Harvey and Siddique, 1999, 2000; Kraus and Litzenberger, 1976). The second type comes from multivariate distribution and has attracted much attention in recent years. The evidence for asymmetric dependence between equity returns has been widely reported in empirical finance literature (see Ang and Bekaert, 2002; Ang and Chen, 2002; Ang et al., 2006; Bae et al., 2003; Christoffersen et al., 2012; Christoffersen and Langlois, 2013; Longin and Solnik, 2001; Okimoto, 2008; Patton, 2004; Poon et al., 2004). These reports find that the dependence of asset returns is greater in down markets than in up markets. This phenomenon cannot be ignored in financial modeling for two reasons. First, ignoring asymmetries across asset returns will cause substantial underestimation of portfolio tail risk. Second, traditional portfolio diversification based on the standard investment theory will be challenged if all the components of a portfolio tend to fall when the market slumps. Further, the presence of asymmetric dependence across assets may also reduce hedging effectiveness to some extent. Recent studies have revealed that asymmetric dependence is economically important in asset allocation. For instance, Patton (2004) shows that the presence of asymmetric dependence can greatly impact portfolio decisions and that significant economic gains are earned when acknowledging this asymmetry. Hong et al. (2007) show the substantial economic importance of incorporating asymmetries into portfolio selection for disappointment-averse investors. In addition, some empirical studies have paid attention to modeling the asymmetric dependence between exchange rates (see Chen and Fan, 2006b; Patton, 2006b).

Although understanding the co-movement across the equity market and the currency market is of particular importance in international investment and risk hedging, less attention has been paid to the modeling of dependence between these markets (see Ning, 2010). In our study, we focus on a country equity index and a corresponding foreign exchange rate from 6 developed markets and 6 emerging markets and consider two measures - the threshold correlation in Ang and Chen (2002) and tail dependence coefficients in McNeil et al. (2005) - to quantify the extent of asymmetric dependence between equity and currency. To verify the statistical significance of these asymmetries, we consider a model free test proposed by Hong et al. (2007) and a bootstrap method in Patton (2012a).

It is a stylized fact that the conditional covariance between financial time series appears to vary over time (see Bollerslev et al., 1988; Harvey, 1989; Ng, 1991, etc.), and this leads us to consider whether the conditional dependence between asset returns also changes over time. Empirical finance literature finds that the dependence between asset returns is regime dependent and time-varying (see Bouye and Salmon, 2009; Christoffersen et al., 2012; Christoffersen and Langlois, 2013; Giacomini et al., 2009; Patton, 2006b, etc.). Some studies show that capturing dynamic dependence is of particular importance in investment decisions. For instance, Christoffersen and Langlois (2013) find strong economic gains from modeling nonlinear and time-varying dependence between four equity market factors using an experiment of real-time investment. In this paper, we consider two types of test to see whether the dependence structure between equity and currency varies over time. The first one is a naïve test to detect change-points in rank correlation at some specified dates in the sample period (see Patton, 2012a). This test is relatively easy to implement but it requires us to have *a priori* knowledge of break-point. The second type of tests are those for structural change of unknown timing for a conditional model proposed by Andrews (1993) and Andrews and Ploberger (1994).

Christoffersen et al. (2012) propose a new dynamic asymmetric copula model based on the multivariate skewed t copula of Demarta and McNeil (2005). Their model captures the evolution of dependence by generalizing the dynamic conditional correlation (DCC) model of Engle (2002) and Tse and Tsui (2002). To capture the non-normality of univariate asset return, the multivariate asymmetry and joint dynamics across asset returns simultaneously, we develop a time-varying asymmetric copula (TVAC) model. First, our univariate model combines the autoregressive model for the time-varying conditional mean, univariate GARCH-type models for the dynamics of conditional volatility, and the univariate skewed t distribution of Fernández and Steel (1998) for the skewness of standardized residuals. Second, our proposed TVAC model is based on the combination of the multivariate skewed t copula (Bauwens and Laurent, 2005), an observation driven framework for time-varying parameters by Creal et al. (2013), and a comprehensive work on dynamic copula models for time series by Patton (2012a). Differently from Christoffersen et al. (2012) and Christoffersen and Langlois (2013), we use the version of skewed t distribution of Bauwens and Laurent (2005) and implement the Generalized Autoregressive Score (GAS) model proposed by Creal et al. (2013) as a driving mechanism to deal with the possible time variability of the model parameters. The GAS framework has become increasingly popular in recent economic and financial studies (see Creal et al., 2011, 2014; De Lira et al., 2013; Lucas et al., 2013; Oh and Patton, 2013, etc.). It is an observation-driven model based on score function and lagged information (the lagged copula parameter in our application). In this paper, we implement GAS dynamics as it has several advantages compared with other observation driven models in the literature. First, it is a unified and consistent method for modelling time-varying behaviours of parameters in nonlinear models. Second, the likelihood evaluation is computationally straightforward. Third, instead of considering the first-order moment or higher-order moments only, it investigates the complete distribution structure.

The copula function provides a flexible and convenient tool for multivariate time series modelling, and therefore it is particularly important in the econometrics of financial risk measurement, see McNeil et al. (2005). Value-at-risk (hereafter VaR) has become a universal standard in risk management, as it can provide some quantitative insight to the riskiness of asset return. Some important contributions related to the copula-based VaR model include Glasserman et al. (2002), Embrechts et al. (2003a), Embrechts et al. (2003b), Junker and May (2005), Rosenberg and Schuermann (2006), Kole et al. (2007), Perignon et al. (2010) and Hsu et al. (2012), among many others. The VaR pays attention to frequency of extreme events and can only provide the maximum loss at a specified quantile. Expected shortfall (hereafter ES, also known as CVaR or TVaR) is defined as the expectation of losses exceeding VaR and focuses on both the frequency and magnitude of losses in the case of tail events. In addition ES, as a coherent risk measure, has several desirable properties, including translation invariance, positive homogeneity, convexity, subadditivity and monotonicity, see Artzner et al. (1999). Because of these superior properties, ES has gradually gained popularity in finance and insurance. McNeil et al. (2005) and Alexander (2009) also provide comprehensive books for copulas and financial risk management. In this paper, we forecast the portfolio market risk (both VaR and ES) of hypothetical equity-currency portfolios[1] using a time-varying asymmetric copula (TVAC) approach. This approach is a combination of AR-GARCH-type models for the dynamics of univariate returns, the univariate skewed t distribution for standardized residuals, the time-varying skewed t copula model we proposed for multivariate dependence and a Monte Carlo simulation for VaR and ES forecasts. For the sake of comparison, three widely used multivariate GARCH models will also be considered. We also consider three backtests: the Empirical Coverage Probability, the Conditional Coverage test proposed by Christoffersen (1998) and the Dynamic Quantile test proposed by Engle and Manganelli (2004), to evaluate the predictive power of the TVAC model for VaR. There is no widely recognized method to backtest ES since it fails to satisfy elicitability, see Gneiting (2011). We consider the ES as a point prediction and use the backtest in Cerrato et al. (2014).

Another important application of copula-based models is portfolio optimization, which is one of the most crucial topics in financial decision-making. Finding an optimal balance between return and risk in investment is normally the main concern of investors. The classic portfolio optimization process is based on the Markowitz-inspired mean-variance framework, see Markowitz (1952). The goal of the portfolio optimization problem is to seek minimum risk for any given amount of expected excess return or to seek maximum expected excess return for any given amount of risk. Clearly, choosing appropriate risk measures is of particular importance in practice. A number of ways have been proposed to measure risk in portfolio optimization, such as standard deviation, mean-absolute deviation (MAD), VaR and ES. The prevalence of standard deviation, MAD and VaR are due to their computational simplicity. However, standard deviation and MAD are often criticized due to their symmetry, and VaR is also criticized due to its lack of subadditivity[2] and convexity as we discussed above. ES, which is the most popular coherent risk measure, has been considered as a desirable alter-

[1]For instance, we form our portfolio by buying the UK national equity index and selling British Sterling. The reason we do this will be discussed in 3.2.

[2]Failure to satisfy subadditivity means that the VaR of a diversified portfolio can be larger than the aggregate of the VaRs of its components.

native in the literature, (see Agarwal and Naik, 2004; Alexander et al., 2006; Rockafellar and Uryasev, 2000, 2002, etc.).[3] For instance, Agarwal and Naik (2004) compare mean-ES optimization with the traditional mean-variance framework and demonstrate that the mean-variance framework substantially underestimates tail losses, especially when portfolios have low volatility.

Meanwhile, traditional mean-variance method assumes that asset returns are well approximated by normal distribution. Similarly, previous literature also considers portfolio optimization with ES constraints under the normality assumption and shows that ES could be efficiently minimized using linear programming techniques and non-smooth optimization algorithms (see Rockafellar and Uryasev, 2000, 2002, etc). However, a large number of empirical studies show that normal distribution is not a reasonable assumption to make for modelling multivariate financial time series (see Affleck and McDonald, 1989; Ang and Chen, 2002; Longin and Solnik, 2001; Patton, 2006a, among many others). A comprehensive review of this topic can be found in Jondeau et al. (2007). The failure of multivariate normal distribution leads us to look for a more appropriate multivariate model. Copula can be considered as a straightforward way to extend the optimization problem from the multivariate normal hypothesis to a combination of different well specified marginal models and a variety of possible dependence specifications. Copula has been widely used as an effective tool to predict multivariate distribution for assets in portfolio decision problems, see Patton (2004), Hong et al. (2007), Christoffersen et al. (2012) and Christoffersen and Langlois (2013), among others. Thus, using a copula-based mean-ES method is possibly a superior way to optimize portfolios.

In order to evaluate the performance of optimization, three measures will be considered. Sharpe ratio (SR), one of the most popular measures to quantify the balance between reward and risk, has been widely used as a standard tool to access investment since Sharpe (1966). One drawback of SR is that it chooses standard deviation to quantify risk, because standard deviation fails to differentiate good volatility and bad volatility. In addition, SR fails to consider the existence of arbitrage opportunities and it also may be misleading when the return is not normally distributed. To overcome these undesirable drawbacks, many other perfor-

[3] Although the literature of portfolio optimization typically uses the term “CVaR” instead of “Expected Shortfall (ES)”, to keep consistence with the expressions of the Bank for International Settlements (BIS) and Basel Committee on Banking Supervision (BCBS), we use the term “ES” in this paper.

mance measures have been developed, for instance, the gain loss ratio (GLR) proposed by Bernardo and Ledoit (2000). GLR is defined as the ratio of the mean excess return over the expectation of the negative tail. Cherny and Madan (2009) developed a more general framework termed acceptability indices to overcome the disadvantages of conventional measures. This is a consistent measure of comparing the performance of different portfolios even when the returns on the portfolios are not normally distributed.[4] Recently, this concept has been widely used in asset pricing (see Carr et al., 2011; Cherny and Orlov, 2011; Madan, 2010, etc). For the sake of robustness, all the performance measures above will be considered in this paper.

The main contributions of this paper are as follows:

First, understanding and quantifying the dependence of international financial markets, especially tail dependence, is one of the key issues in risk management and portfolio diversification for international investment. There are several studies in the literature on modelling the dependence structure between the exchange rates using copulas, however, to the best of our knowledge, there are few on applying dynamic copulas to study the co-movements between equity and currency returns. Our study pays particularly close attention to this matter, and uses two different methods to quantify this dependence. We find evidence that the dependence structures between equity and currency returns are not symmetric, which is different from the finding of Ning (2010). Using several tests, we show that the asymmetry of tail dependence is also statistically significant.

Second, verifying the existence of dynamic dependence is imperative, as a large number of studies have found the failure of traditional linear correlation and time-invariant dependence in portfolio hedging and asset allocation. We find striking evidence to demonstrate that the dependence structures between country equity index and corresponding currency change over time. Both markets move together more closely during currency crisis or financial crisis than during a normal period. We also find solid evidence to show that GAS-based time-varying copulas statistically outperform the corresponding time-invariant copulas in the modeling of equity-currency dependence. This finding implies that the time-varying dependence may offer economic benefits in real-time investment.

Third, the presence of multivariate asymmetry and time-variation of dependence motivate

[4] More details about the definition and properties of these measures can be found in Appendix I

us to consider a model that can accommodate all these documented properties. We propose a novel time-varying skewed t copula model allowing for multivariate non-normality and dynamic and asymmetric dependence. This model can be easily generalized from the bivariate case to the multivariate case. We will discuss more about the good properties of the TVAC model in Section 3.4.

Fourth, our findings of dynamic and asymmetric dependence between equity and currency have important implications for risk management in international financial markets. We construct long-short stock-currency portfolios from a US investor's perspective. Using various models of backtesting, we show that our TVAC model outperforms other popular models in general in VaR and ES forecasts. Our results show that the TVAC model substantially improves the accuracy of VaR and ES forecasts. This finding indicates the importance of modelling the dynamic and asymmetric dependence of international financial markets in risk management.

Lastly, our empirical findings and the proposed TVAC model also have practical implications for optimal asset allocation in international financial markets. The skewed t copula used in this paper can be generalized from the bivariate case to the high dimensional case, and this allows us to include a large amount of assets in the optimization problem.[5] We use a copula-based mean-ES framework, which combines the optimization method of Rockafellar and Uryasev (2000) with the TVAC model, to optimize our global equity portfolio. Our evaluation results indicate that significant economic value is earned when we choose this copula-based mean-ES approach instead of other traditional methods. In addition, we find that the optimization based on the TVAC model can provide the highest economic value.

The remainder of the paper is structured as follows. Section 3.2 clarifies the importance of the dependence structure of international financial markets using risk management and optimal asset allocation examples. Section 3.3 describes the data, as well as the empirical results on the asymmetric and dynamic dependence between the country stock index and the corresponding currency. Section 3.4 describes the modeling of the dynamic and asymmetric dependence. Section 3.5 and 3.6 study the economic importance of acknowledging dynamic and asymmetric dependence of international financial markets in risk management and the

[5] Although some Archimedean copulas, such as Gumbel, Clayton or their rotated versions, are also able to handle asymmetric dependence, they usually suffer from the issue of dimension restriction, see McNeil et al. (2005).

optimal asset allocation applications. Section 3.7 concludes.

3.2 Motivations

We study the nature of the dependence of international financial markets. We focus on equity portfolios and foreign exchange rates which are most actively traded in international financial markets. We are especially interested in how the dependence structure between the equity portfolio and the foreign exchange rate affect risk management and optimal asset allocation in international financial markets.

3.2.1 Risk Management

Suppose that a US investor invests \$1 in the UK stock market at time t. She firstly has to exchange her US dollars for GBP to buy stocks from the UK stock market. If the exchange rate (USD/GBP, *i.e.*, GBP per 1 USD) is X_t and the stock price is P_t, she could buy X_t/P_t shares. If the stock price is P_{t+1} at time $t+1$, the total value of the investment is $W_{t+1}^{UK} = (X_t/P_t)\,P_{t+1}$. We can represent this using a stock return $\left(R^E\right)$ as

$$W_{t+1}^{UK} = X_t\left(1+R^E\right). \tag{3.1}$$

Assuming that she sells her stocks and takes money back to US, she has to convert the pound denominated wealth into the dollar. If the exchange rate is X_{t+1} at time $t+1$, the dollar denominated wealth is

$$W_{t+1}^{US} = \frac{X_t}{X_{t+1}}\left(1+R^E\right). \tag{3.2}$$

Taking a natural logarithm to both sides produces

$$\ln\left(W_{t+1}^{US}\right) = \ln\left(\frac{X_t}{X_{t+1}}\right) + \ln\left(1+R^E\right), \tag{3.3}$$

and we replace $\ln\left(X_t/X_{t+1}\right)$ by

$$\ln\left(\frac{X_t}{X_{t+1}}\right) = \ln\left(\frac{1}{1+R^F}\right) = -\ln\left(1+R^F\right), \tag{3.4}$$

where R^F is the return on the exchange rate. Furthermore, the logarithm return of 3.3 can be represented using the return on the total wealth (R) as $\ln\left(W_{t+1}^{US}\right) = \ln(1+R)$. Since $\ln(1+z) \approx z$ for a small $z > 0$, the total return can be written by

$$R = R^E - R^F. \tag{3.5}$$

This investment can be interpreted as a portfolio taking a long position for the UK stock market and a short position for the exchange rate (USD/GBP, i.e. GBP per 1 USD).

Therefore, the dependence structure between equity and currency is of particular importand in risk management. International investors are keen to manage extreme event risks to avoid financial disaster. VaR or ES would be the most useful measures on extreme event risk:

$$\mathbb{P}\left[R \leq VaR_{t+1}(\alpha)\right] = 1 - \alpha, \tag{3.6}$$

where $\mathcal{F}_t$ represents the information available at t, and

$$ES_{t+1}(\alpha) = \mathbb{E}\left[R \mid R \leq VaR_{t+1}(\alpha)\right] \tag{3.7}$$

which is the expected value in the α worst cases over a given time horizon. Since both measures rely highly on tail dependence, the time-varying and asymmetric tail dependence of two assets is crucial for extreme event risk management (Christoffersen and Langlois, 2013). We study this extensively in Section 3.5.

3.2.2 Optimal Asset Allocation

Suppose that the US investor invests \$1 in a global equity portfolio containing N equity-portfolios from different countries at time t.[6] Thus the wealth from this investment should be a growth return $\$(1+R)$ at time $t+1$. We define returns on the equity and the exchange

[6] We use a stock index as a proxy for the equity portfolio.

rate by

$$R^E = \frac{P_{t+1} - P_t}{P_t}, \tag{3.8}$$

$$R^F = \frac{X_{t+1} - X_t}{X_t}, \tag{3.9}$$

where P denotes an equity price in a domestic currency and X denotes a domestic currency for US dollar. Then the wealth of the global equity portfolio (denominated by US dollar) is obtained by

$$W^{US} = 1 + R^{US}, \tag{3.10}$$

where $R^{US} = \sum_{i=1}^{N} w_i R_i^{US}$ and $R_i^{US} = \left(R_i^E - R_i^F\right) / \left(1 + R_i^F\right)$.

The US investor wants to construct an optimal equity portfolio using the N equity portfolios. That is, she wants to find the optimal weights on assets (w_i) in (9). To this end, she would consider various frameworks for optimal asset allocation. The mean-variance framework is a classical one. However, many studies criticize it for losing its efficiency when there is further dependence such as a time-varying or non-linear dependence structure. For this reason, we study the mean-ES portfolio optimization given by

$$\min_{w_{it} \in (0,1)} ES_{t+1}(\alpha) \tag{3.11}$$

subject to

$$\mathbb{E}\left[R_{t+1}^{US}\right] \geq \kappa, \ \sum_{i=1}^{N} w_{it} = 1,$$

where κ is the prespecified target return. The mean-ES efficient frontier can be obtained by optimization, given a series of target returns. The optimal portfolio weights with risk-free asset can be obtained by the following programming

$$\max_{w_{it} \in (0,1)} \frac{\mathbb{E}\left[R_{t+1}^{US} - R_f\right]}{ES_{t+1}(\alpha)} \tag{3.12}$$

subject to

$$\sum_{i=1}^{N} w_{it} = 1.$$

If the dependence structure is constant and linear for all pairs of assets under the normality assumption on asset returns, we cannot obtain any economic gains from this framework. Otherwise, this framework would perform better than the mean-variance framework.

The dependence structure of two assets is much more complicated in international financial markets than in domestic financial markets. For example, consider country i and j. Then we should consider three types of dependence, as described in Figure 3.1.

(1) Dependence between R_i^E $\left(R_j^E\right)$ and R_i^F $\left(R_j^F\right)$.

(2) Dependence between R_i^E $\left(R_i^F\right)$ and R_j^E $\left(R_j^F\right)$.

(3) Dependence between R_i^E $\left(R_i^F\right)$ and R_j^F $\left(R_j^E\right)$.

Therefore, it is important how well we model this complicated dependence structure of international financial markets in optimal asset allocation.

3.3 Equity Portfolios and Foreign Exchange Rates

We study daily currency spot rate expressed in a local currency per US dollar and the corresponding equity index measured in the local currency over the period from January 3, 2000 to December 31, 2014 (3,913 observations). We use two sets of countries for the sake of comparison. Our developed markets sample comprises the European Union, United Kingdom, Japan, Switzerland, Canada and Australia, and our emerging markets sample comprises Brazil, India, Russia, Turkey, South Korea and South Africa.[7] The stock market index from each country/region represents the equity portfolio of that country/region, and our country equity indices data are provided by Morgan Stanley Capital International (MSCI). The 3-month Treasury-bill rate is used as the risk-free rate. We also study weekly data for optimal asset allocation. All the data is obtained from Datastream.

[7] We use the emerging markets list provided by MSCI.

3.3.1 Returns

Descriptive statistics and distributional characteristics of the daily log returns are reported in Table 3.1. The non-zero values of skewness indicate that all the log return series of equities and currencies are either positively or negatively skewed. The values of kurtosis indicate the leptokurtosis and fat tails of return series. Both the skewness and the kurtosis clearly indicate that the return series are not normally distributed, and this departure from normality is also confirmed by the Jarque-Bera test (all the series are rejected by the JB test at 5% significant level). Ljung-Box Q-statistics of order 12 show the presence of autocorrelation in 17 out of 24 return series. Ljung-Box statistics of order 12 applied to squared returns are highly significant, implying significant heteroscedasticity. The significance of Lagrange Multiplier tests for ARCH effects in all return series also supports the usage of AR-GARCH-type models in our univariate modeling. Descriptive statistics of the weekly log returns are reported in Table 3.2.

The Pearson's linear correlation and Spearman's rank correlation between the country equity index and the corresponding currency are also reported at the bottom of the tabel for each country. All the correlations are negative except for Japan and Switzerland. The negative correlation between equity and currency is not surprising. Suppose that the equity index of UK is rising (probably outperforming the US market), this would lead an US investor to invest her money in the UK market. Thus she converts her US dollars to British Sterling and buys equity in the UK. The sale of US dollar and the buying of Sterling causes the appreciation of Sterling (i.e. the British Pound per US dollar goes down). Since the exchange rates are defined as local currency per USD, strong stock market performance in a country is accompanied by decrease of exchange rate. Meanwhile, the positive correlations of Japan and Switzerland are probably because both the Japanese Yen and Swiss Franc have been historically used as the funding currencies in the carry trade, see Galati et al. (2007) and Gyntelberg and Remolona (2007). Their safe haven properties are empirically supported by Ranaldo and Söderlind (2010). Figure 3.2 displays scatter plots of the equity indices and currencies for each country.

[INSERT TABLE 3.1, 3.2 AND FIGURE 3.2 ABOUT HERE]

3.3.2 Return Dynamics

To deal with some well-established distributional properties of univariate financial asset returns, we consider using an AR(1) model to compensate for autocorrelation

$$R_{i,t} = \phi_0 + \phi_1 R_{i,t-1} + \varepsilon_{i,t},\ i = 1, ..., n,\ \text{where } \varepsilon_{i,t} = \sigma_{i,t} z_{i,t} \tag{3.13}$$

and the GJR-GARCH(1,1,1) model of Glosten, et al. (1993) to capture volatility persistence, heteroskedasticity and leverage effect

$$\sigma_{i,t}^2 = \omega + \alpha \varepsilon_{i,t-1}^2 + \beta \sigma_{i,t-1}^2 + \gamma \varepsilon_{i,t-1}^2 I \tag{3.14}$$

where $R_{i,t}$ is the return of asset i at time t. The last term of Equation 4.2 incorporates asymmetry (leverage) into the variance by a Boolean indicator I that takes the value 1 if the prior model residual $\varepsilon_{i,t-1}$ is negative and 0 otherwise. The standardized residual is given by

$$z_{i,t} = \frac{(R_{i,t} - \mu_{i,t})}{\sigma_{i,t}} = \frac{(R_{i,t} - \phi_0 - \phi_1 R_{i,t-1})}{\sigma_{i,t}} \tag{3.15}$$

For parametric model, we assume that the standardized residuals $z_{i,t}$ follow the skewed t distribution of Fernández and Steel (1998):

$$z_{i,t} \sim F_{skew-t,i}(\eta_i, \lambda_i),\ u_{i,t} = F_{skew-t,i}(z_{i,t}; \eta_i, \lambda_i),\ \eta_i \in (2, \infty],\ \lambda_i \in \mathbb{R}_+ \tag{3.16}$$

where $F_{skew-t,i}$ is the cumulation distribution function of skewed t distribution, $u_{i,t}$ is the probability integral transformation. η_i is the degrees of freedom, which determines the thickness of the tails, and λ_i is the skewness parameter, which determines the the degree of asymmetry. The distribution is positively skewed if $\lambda_i > 1$ and it is negatively skewed if $\lambda_i < 1$. We assume any univariate asymmetry can be captured by this parametric marginal model. Appendix C provides the details of this univariate skewed t density (PDF). For the nonparametric model, we use the empirical distribution function (EDF) F_i:

$$\widehat{F}_i(z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1\{\hat{z}_{i,t} \leq z\},\ \hat{u}_{i,t} = \widehat{F}_i(\hat{z}_{i,t}) \tag{3.17}$$

The maximum likelihood estimation is used to estimate parameters in Equation 4.2 and 3.16. Using these models, the probability integral transformation for each asset can be estimated parametrically or nonparametrically for the copula estimation.

Table 3.3 and 3.4 report the results from the estimation of the AR-GJR-GARCH model with skewed t innovations for equity and currency returns on each market for the 2000-2014 data set. The standardized residuals in this step are used for the dependence structure modeling in Section 3.3.3.

All the estimations appear to be standard. All the leverage parameters for equity indices are significantly positive indicating higher volatility when the market goes down. Interestingly, most of the leverage parameters for currencies are significantly negative except for Japan and Switzerland. This exception can also be explained by the funding currency and frequent government intervention in these countries.

In order to examine the fitness of this univariate model, we use the Ljung-Box test again on the model residuals and find that the autocorrelation of log returns can be picked up well by the AR-GJR-GARCH model. Therefore, we can confirm that the AR-GJR-GARCH type models for the conditional mean and variance appear to fit the data well. Table 3.3 and 3.4 also report the results of goodness-of-fit tests of the skewed t distribution. We consider both Kolmogorov-Smirnov (KS) and Cramer-von Mises (CvM) tests following Patton (2012a). Impressively, all the p-values are greater than 0.05. Thus, we can verify that the skewed t distribution of Fernández and Steel (1998) is appropriate for modeling the conditional distribution of the standardized residuals.

[INSERT TABLE 3.3 AND 3.4 ABOUT HERE]

3.3.3 Asymmetric Dependence

In this section, we test whether the dependence between the equity portfolio and the corresponding foreign exchange rate is asymmetric. We consider both linear and copula-based approach to test the asymmetry.

First, we use the threshold correlation[8] in Ang and Chen (2002) and a model free test

[8]In our application, we apply a modified version of threshold correlation to measure the correlation be-

in Hong et al. (2007) to see whether the dependence is symmetric from a linear perspective. Surprisingly, all the exceedence correlations in developed markets are not significantly asymmetric. However, for the emerging markets, the linear asymmetries are significant in 4 markets (see Table 3.5). The rank correlations imply negative dependence and this can be easily accommodated by the Gaussian or Student's t copula (see Patton, 2012b).

Second, differently from the literature, we apply lower-upper and upper-lower tail dependence as we are concerned with the situation when the invested equity portfolio goes down and the exchange rate goes up (because we use the data measured by local currency per 1 USD, exchange rate going up means currency depreciation). The decline of equity portfolio and depreciation of corresponding currency will make a US investor suffer a dual loss if she wants to sell her equity and convert local currency back to USD. The test for copula-based tail dependence (see McNeil et al., 2005) provides clear evidence of asymmetry. This inconsistency may imply linear symmetric but nonlinear asymmetric dependence. Except for Switzerland, all the equity-currency pairs have stronger lower tail dependence than upper tail dependence, which is consistent with the empirical results of other asset classes, see for instance, Patton (2004), Patton (2006b) and Okimoto (2008). The abnormal pattern of the Swiss market is possibly caused by interventions from the Swiss National Bank. Appendix A and B describe a threshold correlation and a tail dependence in detail.

[INSERT TABLE 3.5 ABOUT HERE]

3.3.4 Time-varying Dependence

To investigate the time-varying dependence between the equity portfolio and the foreign exchange rate, we consider three tests to verify the presence of structural breaks in rank correlation. Table 3.6 presents the results of all the dependence break tests.

[INSERT TABLE 3.6 ABOUT HERE]

First, we investigate whether there is a break in the rank correlation at some specified point in the sample period. Following the naïve test used in Patton (2012a), we simply assume three points in the sample, at $t^*/T \in \{0.15, 0.50, 0.85\}$ which corresponds to the dates

tween equity and currency returns when equity market goes down but the exchange rate goes up (currency depreciation). The analytical expression of the modified threshold correlation is presented in Appendix A.

01-Apr-2002, 02-Jun-2007 and 28-Sep-2012. The results show that for each country there is at least one significant break point, indicating evidence against a constant rank correlation. In addition, we find it informative to further investigate whether the rank correlation between equity and currency statistically changed after the financial crisis broke out. We assume the first break point is September 15, 2008, which corresponds to the date of the collapse of Lehman Brothers. This break point is significant at 5% for 8 pairs (see the result in the column "US Crisis"). The second assumed break point is January 01, 2010, which corresponds to the break out of the European debt crisis.[9] Not surprisingly, this break is significant for the equity-currency pair of the European Union, and it is also significant for several other markets, such as that of Japan, Russia, India, *etc.* (see the result in the column "EU Crisis").

Second, we consider another test for the time-varying dependence used in Patton (2012a). It is based on the Engle (1982)'s ARCH LM test for time-varying volatility. Similarly, the result reported in column AR(1)-(3) also provides solid evidence against a constant rank correlation.

Third, the right column of Table 3.6 reports the p-values computed based on a generalized break test without an *a priori* point proposed by Andrews and Ploberger (1994). The results indicate that for all the equity-currency pairs, there is at least one dependence break point.

In order to demonstrate the dynamics of dependence, we present the variation of skewed t copula correlation estimated by the GAS model in Figure 3.3.

Overall, all the results above suggest that the dependence structures between country equity indices and corresponding currencies are regime dependent and time-varying.

3.4 Modeling Dependence

As shown in the previous section, the findings of time-varying and asymmetric dependence motivate us to consider using a TVAC in risk management and portfolio optimization. In this section, we introduce the modeling of time-varying and asymmetric dependence by copula. Furthermore, we investigate how much a TVAC improves the ability to explain data by comparing it to constant and symmetric copulas.

[9]The European debt crisis has hit several EU countries since the end of 2009. Thus, we arbitrarily assume that January 01, 2010 is a break point.

3.4.1 Copulas

The copula is the function that connects a multivariate distribution to its one-dimensional margins (see Sklar, 1959). If we let $\mathbf{z} = (z_1, ..., z_n)'$ denote an n-dimensional random vector and let $\mathbf{F}$ denote a multivariate distribution with marginal distributions $F_1, ...F_n$ given by $u_i = F_i(z_i) = \mathbb{P}(Z_i \leq z_i)$ for $i = 1, ..., n$, then there exists a copula $\mathbf{C} : [0,1]^n \rightarrow [0,1]$ such that,

$$\mathbf{F}(z_1, ..., z_n) = \mathbf{C}(F_1(z_1), ...F_n(z_n)), \forall \mathbf{z} \in \mathbb{R}^n \tag{3.18}$$

If the $F_1, ...F_n$ are continuous, this copula function $\mathbf{C}$ is unique. If F_i^{-1} denotes the inverse distribution function of F_i, where the $z_i = F_i^{-1}(u_i)$ for $0 \leq u_i \leq 1$ and $i = 1, ..., n$, then

$$\mathbf{C}(u_1, ...u_n) = \mathbf{F}\left(F_1^{-1}(u_1), ...F_n^{-1}(u_n)\right) \tag{3.19}$$

We define $u_i \equiv F_i(z_i)$ as the probability integral transformation variables, when F_i is continuous, then u_i is uniformly distributed.

$$u_i = F_i(z_i) \sim \text{Unif}(0,1), \; i = 1, 2, ..., n \tag{3.20}$$

We set $\mathbf{Z} = \left(F_1^{-1}(u_1), ..., F_n^{-1}(u_n)\right)'$, then we obtain

$$\begin{aligned} \mathbb{P}(Z_1 \leq z_1, ..., Z_n \leq z_n) &= \mathbb{P}\left(F_1^{-1}(u_1) \leq z_1, ..., F_n^{-1}(u_n) \leq z_n\right) \\ &= \mathbb{P}(u_1 \leq F_1(z_1), ..., u_n \leq F_n(z_n)) \\ &= \mathbf{C}(F_1(z_1), ..., F_n(z_n)) \end{aligned} \tag{3.21}$$

Patton (2006b) states that the joint distribution $\mathbf{F}$ can be decomposed into its n univariate margins $F_1, ..., F_n$ and an n-dimensional copula $\mathbf{C}$. If their densities $\mathbf{f}$ and $\mathbf{c}$ exist, then we obtain the representation of joint probability distribution function (PDF) implied by joint CDF in Equation (3.18):

$$\mathbf{f}(z_1, ..., z_n) = \mathbf{c}(F_1(z_1), ..., F_n(z_n)) \times \prod_{i=1}^{N} f_i(z_i), \tag{3.22}$$

where $\mathbf{c}(u_1,...,u_n) = \frac{\partial^n \mathbf{C}(u_1,...,u_n)}{\partial u_1 \cdot ... \cdot \partial u_n}$.

3.4.2 Asymmetric Copulas

Based on the multivariate skewed t distribution of Bauwens and Laurent (2005), Sklar's theorem (see Equation 3.18 and 3.19), the probability density function of the skewed t copula is given by

$$\mathbf{C}_{skt}(u_{1,t},...u_{n,t}) = \mathbf{F}_{\lambda,\eta}\left(F^{-1}_{\lambda_1,\eta}(u_1),...F^{-1}_{\lambda_n,\eta}(u_n) \mid 0,\Sigma_t\right) \tag{3.23}$$

where $\mathbf{C}_{skt}$ denotes the skewed t copula, $\mathbf{F}_{\lambda,\eta}$ denotes the cumulative distribution function (CDF) of the multivariate skewed t distribution with zero mean vector, correlation matrix Σ, the multivariate skewness parameter λ and the degree of freedom η. $F^{-1}_{\lambda_i,\eta}$ denotes the inverse CDF of univariate skewed t distribution with asymmetric parameter λ_i and degree of freedom η, which is identical with the degree of freedom of multivariate distribution.

From Patton (2006b), we know that a joint distribution can be decomposed into its marginal distributions and corresponding copula function. Therefore, we can decompose the joint PDF of skewed t distribution

$$\mathbf{c}_{skt}(u_{1,t},...,u_{n,t}) = \frac{\mathbf{f}_{\lambda,\eta}\left(F^{-1}_{\lambda_1,\eta}(u_{1,t}),...F^{-1}_{\lambda_n,\eta}(u_{n,t}) | 0,\Sigma_t\right)}{\prod_{i=1}^{N} f_{\lambda_i,\eta}\left(F^{-1}_{\lambda_i,\eta}(u_i)\right)} \tag{3.24}$$

where $\mathbf{f}_{\lambda,\eta}$ and f_{λ_i,υ_i} denote the multivariate and univariate skewed t density functions, respectively. Next, we define $\mathbf{z}_{i,t} = (z_{1,t},...,z_{n,t})$, $z_{i,t} = F^{-1}_{\lambda_i,\eta}(u_{i,t})$, $i = 1,...,n$ and $\mathbf{z}^*_t = \Sigma_t^{-\frac{1}{2}}(\mathbf{z}_t - \mu_t)$. Then the probability density function of the time-varying skewed t copula defined from the multivariate skewed t distribution of Bauwens and Laurent (2005) is given by

$$\mathbf{c}_t(u_{1,t},...,u_{n,t}) = \frac{\Sigma_t^{-\frac{1}{2}}\Gamma\left(\frac{\eta+n}{2}\right)\left[\Gamma\left(\frac{\eta}{2}\right)^{n-1}\right]\left(1+\frac{\mathbf{a}'_t\mathbf{a}_t}{\eta-2}\right)^{-\frac{\eta+n}{2}}}{\left[\Gamma\left(\frac{\eta+1}{2}\right)\right]^n \prod_{i=1}^{n}\left[1+\frac{\lambda_i^{-2I^C_{i,t}}\left(m_i^C+s_i^C x_{it}\right)^2}{\eta-2}\right]^{-\frac{\eta+1}{2}}} \tag{3.25}$$

where Σ_t denotes the covariance matrix and η is the degree of freedom. The vector $\mathbf{a} =$

$(a_1, ..., a_n)$ contains elements defined as $a_i = \lambda_i^{-I_{i,t}^{C*}} \left(m_i^C + s_i^C x_{i,t}^* \right)$,

$$I_{i,t}^{C*} = \begin{cases} 1 & \text{if } z_{i,t}^* \geq -\frac{m_i^C}{s_i^C} \\ -1 & \text{if } z_{i,t}^* < -\frac{m_i^C}{s_i^C} \end{cases}, \; I_{i,t}^{C} = \begin{cases} 1 & \text{if } z_{i,t} \geq -\frac{m_i^C}{s_i^C} \\ -1 & \text{if } z_{i,t} < -\frac{m_i^C}{s_i^C} \end{cases}, \tag{3.26}$$

$$m_i^C = \frac{\Gamma\left(\frac{\eta-1}{2}\right)\sqrt{\eta-2}}{\Gamma\left(\frac{\eta}{2}\right)\sqrt{\pi}} \left(\lambda_i - \frac{1}{\lambda_i}\right), \; \left(s_i^C\right)^2 = \lambda_i^2 + \frac{1}{\lambda_i^2} - 1 - \left(m_i^C\right)^2. \tag{3.27}$$

The PDF of this multivariate skewed t distribution can be found in Appendix D.

3.4.3 Time-varying Copulas

To deal with the time-varying characteristics of dependence structure, we implement the generalized autoregressive score (GAS) model of Creal et al. (2013). For the sake of simplicity, we let the correlation parameter δ_t of skewed t copula vary over time according to the GAS dynamics, holding other parameters constant. A strictly increasing transformation function $\delta_t = (1 - e^{-g_t}) / (1 + e^{-g_t})$ is used to ensure that $\delta_t \in (-1, 1)$, and the transformed correlation parameter is denoted by

$$g_t = h(\delta_t) \Leftrightarrow \delta_t = h^{-1}(g_t) \tag{3.28}$$

Following Creal et al. (2013) and Patton (2012a), we define the evolution of the transformed copula parameter g_{t+1} as a function of a constant ϖ, the lagged copula parameter g_t and the standardized score of the copula log-likelihood $Q_t^{-1/2} s_t$

$$g_{t+1} = \varpi + \eta Q_t^{-1/2} s_t + \varphi g_t \tag{3.29}$$

where

$$\mathbf{s}_t \equiv \frac{\partial \log c(u_{1,t}, ..., u_{n,t}; \delta_t)}{\partial \delta_t} \text{ and } Q_t \equiv \mathbb{E}_{t-1}\left[\mathbf{s}_t \mathbf{s}_t'\right].$$

The GAS model is an observation driven model and therefore its parameters can be easily estimated by the maximum likelihood estimation

$$\widehat{\delta_t} = \underset{\delta_t}{\operatorname{argmax}} \sum_{t=1}^{n} \log c\left(u_{1,t}, \ldots, u_{n,t}; \delta_t\right). \tag{3.30}$$

When $n > 2$, the estimation of the GAS dynamics for the covariance matrix of copula model is not straightforward. In our study, we follow the method of Lucas et al. (2013) to obtain the covariance matrix of the high-dimentional case.

3.4.4 Parametric and Semiparametric Estimation

The inference procedures for parametric and nonparametric models are different (see Patton, 2012a) and we provide some details about these two methods separately in Appendix E and F. The maximum likelihood estimation for parametric copula-based models is sometimes computationally difficult, thus we consider a two-stage maximum likelihood estimation, which estimates univariate parameters from maximizing univariate likelihoods, and then estimates dependence parameters from a multivariate likelihood, see Joe and Xu (1996), Joe (1997) and Joe (2005). Appendix E provides the details needed to implement this method. Intuitively, the two-stage maximum likelihood should be asymptotically less efficient than one-stage MLE, simulation results in Joe (2005) and Patton (2006b) suggest that the two-stage MLE has relatively good efficiency in general.

A semiparametric copula model is characterized by a parametric copula model of dependence and nonparametric models of marginal distributions. In the first step, the marginal data is transformed by the empirical distribution function, and in the second step the copula dependence parameter is estimated by maximizing the estimated log-likelihood function holding the marginal distributions fixed from the first step, see Chen and Fan (2006a) and Chen and Fan (2006b).

We estimate copula models by both parametric and semiparametric approaches. Overall, the semiparametric approach provides slightly better performance than the parametric approach. Hence, we provide all the results estimated by the semiparametric approach in our paper. The results estimated by the parametric approach are available in the internet appendix.

3.4.5 Improvements by TVAC

We investigate how accurately the TVAC model explains the dependence between the equity portfolio and the foreign exchange rate. We evaluate the log likelihoods (hereafter LL) of alternative copula models for 12 equity-currency pairs with daily and weekly data. Table 3.7 reports the LL values for four copula models: constant t copula, constant skewed t copula, time-varying t copula and time-varying skewed t copula. Note that a marginal distribution is modeled by the AR-GJR-GARCH model introduced in Section 3.3 and we use the semi-parametric approach to estimate the copula models.

[INSERT TABLE 3.7 ABOUT HERE]

For the constant copula, when we switch the symmetric copula (t copula) into the asymmetric one (skewed t copula), the LL is improved by 15 on average. For the time-varying copula, the LL is improved by 25 on average. These improvements are also observed for all individual equity-currency pairs. The results show that the asymmetric copula can explain the data better than the symmetric copula.

Next, we investigate how much the time-varying copula improves the LL. For the symmetric copula, when we switch the constant copula into the time-varying copula, the LL is improved by 83 on average. For the asymmetric copula, the LL is improved by 93 on average. These improvements are also observed for all individual equity-currency pairs. The results show that the time-varying copula can explain the data much better than the constant copula.

Interestingly, the improvements by the time-varying copula are much better than those by the asymmetric copula. When we switch the constant copula into the time-varying copula, the LL improves by 98 on average. This improvement is about five times that obtained by switching from the symmetric copula into the asymmetric copula.

Consequently, the results demonstrate that the TVAC is able to accurately model the time-varying and asymmetric dependence observed from the equity-currency data in Section 3. Hence, the use of the TVAC could improve the performance of risk management and the optimal asset allocation in international financial markets.

3.5 Robust Risk Management

We study the implications of the TVAC model by evaluating the performance of risk management in international financial markets. We consider the out-of-sample forecast of VaR and ES of the equity-currency portfolio. To this end, we use daily equity-currency data and construct a long-short portfolio described in Equation (3.5). We use Monte Carlo simulation to generate N simulated $\widetilde{R_t^E}$ and $\widetilde{R_t^F}$ from the estimated model, then form simulated portfolio returns $\widetilde{R_t} = \widetilde{R_t}^E - \widetilde{R_t}^F$. We use an empirical distribution of simulated portfolio returns to compute VaR and ES given a nominal probability α. More details about our forecasting algorithm can be found in Appendix G.

To forecast the one-step-ahead VaR and ES, we use a rolling window instead of the full sample and the rolling window size is set at 250 (one trading year) for all the return series. All the model parameters are recursively updated throughout the out-of-sample period and the correlation coefficients of copulas are predicted by the GAS model. In order to evaluate the coverage ability and the statistical accuracy of VaR forecasts, we implement three widely used backtestings including the empirical coverage probability (hereafter ECP), the conditional coverage test (hereafter CC test; Christoffersen, 1998), and the dynamic quantile test (hereafter DQ test; Engle and Manganelli 2004). In addition, we employ a mean absolute error (MAE) to evaluate the predictive power on ES.

We evaluate four copula models: constant t copula, constant skewed t copula, time-varying t copula and time-varying skewed t copula. The copula models are estimated by the semiparametric approach. For the purpose of comparison, widely used multivariate GARCH models are also considered as the benchmarks. Analogous to the copula models, we use Monte Carlos simulation to compute VaR and ES.

3.5.1 Coverage Ability

Table 3.8 presents the ECPs of the VaR forecast for both 95% and 99% nominal probabilities. The ECP should be close to α (small bias) and this small bias should be maintained for all portfolios (small variation). For this reason, we evaluate the root mean square error (RMSE) of ECPs. First, the RMSEs of copula models are much smaller than those of multivariate

GARCH models. Second, when we switch the symmetric copula into the asymmetric one, RMSEs decrease. Third, when we switch the constant copula into the time-varying one, RMSEs also decrease. Note that the TVAC has the smallest RMSE.

[INSERT TABLE 3.8 ABOUT HERE]

3.5.2 Statistical Accuracy

We evaluate the statistical accuracy of the VaR forecast by counting the number of rejections at the 5% significance level for both CC and DQ tests. A smaller number of rejections indicates a higher accuracy of the VaR forecast. Table 3.9 reports the CC test results. First, the copula models are less frequently rejected than the multivariate GARCH models. When the confidence interval is 99%, the copula models are rarely rejected while the multivariate GARCH models are rejected for all equity-currency pairs. Second, when we switch the symmetric copula into the asymmetric one, the rejection numbers decrease. Third, when we switch the constant copula into the time-varying one, the rejection numbers also decrease. Note that the rejection numbers of the TVAC are smallest at both nominal probabilities. Next, Table 3.10 reports the DQ test results. The results are qualitatively consistent with the CC test results despite the slight increase in the number of rejections.

[INSERT TABLE 3.9 and 3.10 ABOUT HERE]

3.5.3 Predictive Loss

We evaluate the predictive loss of the ES forecast. Table 3.11 reports the MAE computed by

$$MAE = \frac{1}{N} \sum_{s=1}^{N} \left| R_s - \widehat{ES}_s(\alpha) \right| I_s, \tag{3.31}$$

where $I_s = 1\left\{ R_s < \widehat{VaR}_s(\alpha) \right\}$. A smaller value indicates a more accurate forecast. First, the copula models have smaller MAEs than the multivariate GARCH models at the 99% confidence interval while only time-varying copula models have smaller MAEs at the 95% confidence interval. Second, when we switch the symmetric copula into the asymmetric one, the MAEs are decreased. Third, when we switch the constant copula into the time-varying one, the MAEs are also decreased.

[INSERT TABLE 3.11 ABOUT HERE]

To sum up, we perform a broad range of backtestings including coverage ability, statistical accuracy and predictive loss. When we switch the symmetric copula into the asymmetric one, the backtesting results are improved. When we switch the constant copula into the time-varying one, the backtesting results are also improved. Therefore, the use of the TVAC makes the risk management in the international financial market more robust.

3.6 Optimal Asset Allocation

In this section, we construct and monitor global equity portfolios to explore the economic importance of modeling the time-varying and asymmetric dependence of international financial markets. The naïve buy and hold strategy, the mean-variance framework (Markowitz, 1952) and the mean-absolute deviation (MAD) framework (Konno and Yamazaki, 1991) are considered to be classical benchmarks. We allow further dependence beyond the classical benchmarks and employ the copula-based optimization. First, we estimate the parameters of the skewed t copula model of global equity portfolio and then use the Monte Carlo simulation to generate scenarios for the computation of portfolio ES. Second, we use minimal ES as the objective function to optimize a portfolio to get the optimal weights for each asset. We repeat this process to form dynamically rebalanced portfolios with specified return and minimal ES described in Section 3.2. Weekly equity-currency data are used to construct optimal portfolios and monitor weekly portfolio returns. For the sake of comparison, we also consider several traditional optimization constrains. More details about our optimization algorithm can be found in Appendix H.

In order to evaluate the performance of optimal asset allocation, we consider two classical performance measures: Sharpe Ratio (SR) and Gain-Loss Ratio (GLR). Because of the limitations of these measures mentioned in Section 3.1, we also use the new measures termed indexes of acceptability recently proposed by Cherny and Madan (2009). The bigger value indicates better performance. Appendix I describes these measures in detail.

Table 3.12 reports the performance measures. First, the copula-based optimal portfolios dominate the classical benchmarks for all measures. This demonstrates that the dependence structure of the international financial markets is more complicated than that assumed by the classical frameworks. Thus the classical frameworks generate significant economic disutility by ignoring the further dependence structures in the optimal asset allocation.

Second, the bottom panel of 3.7 shows that the log-likelihood of the copula model can be significantly improved by taking into account the time-varying correlation and the result of likelihood ratio test also confirms that a significant improvement is obtained if we consider the asymmetric depdendence. For the 95% ES, when we switch the symmetric copula (Gaussian copula) into the asymmetric one (skewed t copula), performance is improved for all measures. When we switch the constant copula into the time-varying copula,[10] performance is also improved for all measures. Note that the TVAC shows the best performance for all measures.

Third, for the 99% ES, when we switch the constant copula into the time-varying copula, performance is improved for all measures. However, when we switch the symmetric copula into the asymmetric copula, performance is improved in the time-varying setting only. Note that the TVAC also shows the best performance for all measures.

[INSERT TABLE 3.12 ABOUT HERE]

Taken together, the evaluation results clearly indicate that the time-varying and asymmetric dependence structure of international financial markets is crucial for optimal asset allocation. The use of the TVAC model can properly balance return and risk by capturing the highly complicated dependence structure. When we switch the constant or symmetric models into the time-varying and asymmetic model, we can get more economic utility not obtained by the constant or symmetric framework.

[10]In our case, we estimate the GAS dynamics for the covariance matrix of high-dimensional case (12×12) following the method described in Lucas et al. (2013).

3.7 Conclusion

In this paper, we provide a comprehensive empirical study on the dependence structure of international financial markets. We find evidence of asymmetric dependence between the equity portfolio and the corresponding foreign exchange rate. Their lower tail dependence is significantly greater than the upper tail dependence in 6 out of 12 countries, indicating that the equity portfolio and the foreign exchange rate tend to move together more closely during a crash. In addition, we also find solid evidence against the constant dependence between the equity portfolio and the foreign exchange rate. This result implies that dynamic dependence may offer large economic benefits in real time investment.

A methodological contribution is offered in this paper. To accommodate some well-documented features and our empirical findings of the equity-currency relationship, we propose a novel time-varying asymmetric copula (TVAC) model that allows for non-linearity, asymmetry and time variation of the dependence, and deviations from multivariate normality. The time-varying mean and volatility are modeled by the AR-GJR-GARCH model and the univariate asymmetry and fat-tails are fitted by the univariate skewed *t* distribution in Fernández and Steel (1998). The multivariate asymmetry is captured by the skewed *t* copula derived from the mutlivariate skewed *t* distribution in Bauwens and Laurent (2005) and the time-varying dependence is captured by the GAS dynamics proposed by Creal et al. (2011). The estimation results indicate that our TVAC model can provide statistically better fitness than other constant or symmetric models.

Our findings about time-varying and asymmetric dependence have important implications for risk measurement. To show the usefulness of the TVAC model in market risk forecast, we apply it to forecast portfolio Value-at-Risk and Expected Shortfall. The backtesting results shows that the TVAC model yields better out-of-sample forecast performance of VaR and ES than other widely used multivariate models in general, at both 95% and 99% confidence intervals. Thus, our proposed TVAC model can be an ideal choice for financial institutions and regulatory authorities to make risk management in international financial markets more robust.

We also access the economic importance of acknowledging multivariate asymmetry and dynamic dependence from an investment perspective by comparing the TVAC-based optimization with several classical frameworks, such as mean-variance and mean-MAD optimization. Using different performance measures, we show that the TVAC-based mean-ES method can provide higher risk-adjusted reward than traditional optimization methods in general. This finding shows that the presence of statistically significant asymmetry and time-varying dependence are also economically important for the optimal asset allocation in international financial markets.

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Appendix

3.A Threshold Correlation

The threshold correlation (TC) is the linear correlation measuring a selected subset of data. Following Ang and Chen (2002), The threshold correlation with respect to the quantiles at probability level q of empirical distribution of asset return R^E and R^F is defined by

$$\rho^{--} = Corr\left(R_t^E, R_t^F | R_t^E \leq R^E(q) \text{ and } R_t^F \leq R^F(q)\right) \text{ if } q \leq 0.5 \tag{3.32}$$

$$\rho^{++} = Corr\left(R_t^E, R_t^F | R_t^E > R^E(q) \text{ and } R_t^F > R^F(q)\right) \text{ if } q > 0.5 \tag{3.33}$$

In our study, we want to measure the correlation between equity and currency returns when equity market goes down but the exchange rate goes up (currency depreciation). We rely on a modified expression of TC to measure the correlation between equity returns below their qth quantiles and exchange rate returns above their $(1-q)$th quantiles, or vise versa

$$\rho^{-+} = Corr\left(R_t^E, R_t^F | R_t^E \leq R^E(q) \text{ and } R_t^F \geq R^F(1-q)\right) \text{ if } q \leq 0.5 \tag{3.34}$$

$$\rho^{+-} = Corr\left(R_t^E, R_t^F | R_t^E > R^E(q) \text{ and } R_t^F < R^F(1-q)\right) \text{ if } q > 0.5 \tag{3.35}$$

3.B Tail Dependence

The tail dependence coefficient (TDC) is an important concept and useful measure to quantify the degree of dependence in multivariate analysis (see McNeil et al., 2005, among others). The upper tail dependence (UTD) is defined to measure the upper-upper tail case and the lower tail dependence (LTD) is used to measure the lower-lower tail case. Both of them have been widely applied in empirical finance literature (see Christoffersen et al., 2012; Elkamhi and Stefanova, 2014; Patton, 2009; Poon et al., 2004, among many others).

Let $z_i \sim F_i, i = 1,2$ be continuously distributed random variables. Then the coefficients of lower-lower, upper-upper, lower-upper and upper-lower tail dependence of z_1 and z_2 are

$$\lambda^{LL} = \lim_{q \to 0^+} \frac{\mathbb{P}\left\{z_2 \le F_2^{-1}(q), z_1 \le F_1^{-1}(q)\right\}}{\mathbb{P}\left\{z_1 \le F_1^{-1}(q)\right\}} = \lim_{q \to 0^+} \frac{C(q,q)}{q} \tag{3.36}$$

$$\lambda^{UU} = \lim_{q \to 1^-} \frac{\mathbb{P}\left\{z_2 > F_2^{-1}(q), z_1 > F_1^{-1}(q)\right\}}{\mathbb{P}\left\{z_l > F_l^{-1}(q)\right\}} = \lim_{q \to 1^-} \frac{1-2q+C(q,q)}{1-q} \tag{3.37}$$

$$\lambda^{LU} = \lim_{q \to 1^-} \frac{\mathbb{P}\left\{z_2 > F_2^{-1}(q), z_1 < F_1^{-1}(q)\right\}}{\mathbb{P}\left\{z_l < F_l^{-1}(q)\right\}} = \lim_{q \to 1^-} \frac{1-q-C(1-q,q)}{1-q} \tag{3.38}$$

$$\lambda^{UL} = \lim_{q \to 1^-} \frac{\mathbb{P}\left\{z_2 < F_2^{-1}(q), z_1 > F_1^{-1}(q)\right\}}{\mathbb{P}\left\{z_l < F_l^{-1}(q)\right\}} = \lim_{q \to 1^-} \frac{1-q-C(q,1-q)}{1-q} \tag{3.39}$$

If the copula C has an analytic solution, the coefficients can be easily calculated. The copula C has lower-lower tail dependence if $\lambda^{LL} \in (0,1]$ and no lower-lower tail dependence if $\lambda^{LL} = 0$. A similar conclusion holds for the other tail dependence coefficients. In our application, the Student's t copula is applied to compute tail dependence.

3.C The Univariate Skewed *t* Distribution

We assume that the standardized residuals of marginal distribution follow the univariate skewed Student's t distribution of Fernández and Steel (1998). They generate a skewed t distribution by introducing skewness into symmetric t distribution, and the probability density fucntion $f_{\gamma,\upsilon}$ of this distribution is given by:

$$f_{\lambda_i,\upsilon_i}(z_{i,t}) = \frac{2\lambda_i}{1+\lambda_i^2}\left[f_{\nu_i}\left(\frac{z_{i,t}}{\lambda_i}\right) I(z_{i,t} \ge 0) + f_{\nu_i}(z_{i,t}\lambda_i) I(z_{i,t} \le 0)\right], \ \lambda_i \in (0,\infty) \tag{3.40}$$

where I is the indicator variable equal to 1 when the condition is true and 0 otherwise, λ_i is the skewness parameter and υ_i is the degree of freedom, which controls the thickness of the tails. And f_{ν_i} is chosen to be the Student's t distribution with υ_i df, $\upsilon_i \in \mathbb{R}^+$, which is unimodal and symmetric around 0.

$$f_{\upsilon_i}\left(x_{i,t} \mid \mu_i, \sigma_i^2\right) = \frac{\Gamma\left(\frac{\nu_i+1}{2}\right)}{\Gamma\left(\frac{\nu_i}{2}\right)\sqrt{\left(\nu_i-2\right)\pi\sigma_i^2}}\left[1+\frac{\left(x_{i,t}-\mu_{i,t}\right)^2}{\left(\nu_i-2\right)\sigma_i^2}\right]^{-\frac{\nu_i+1}{2}}, \tag{3.41}$$

where $\mu_{i,t}$ is the conditional mean. Then the skewed t distribution is give by the PDF

$$f_{\lambda_i,\upsilon_i}\left(x_{i,t} \mid \mu_i, \sigma_i^2\right) = \frac{2\lambda_i s_i \Gamma\left(\frac{\nu_i+1}{2}\right)}{\left(1+\lambda_i^2\right)\Gamma\left(\frac{\nu_i}{2}\right)\sqrt{\left(\nu_i-2\right)\pi\sigma_i^2}}\left\{1+\frac{\lambda_i^{-2I}\left[m_i\sigma_i+s_i\left(x_{i,t}-\mu_{i,t}\right)\right]^2}{\left(\nu_i-2\right)\sigma_i^2}\right\}^{-\frac{\nu_i+1}{2}} \tag{3.42}$$

where

$$I = \begin{cases} 1 & \text{if } x_{i,t} \geq \mu_{i,t} - \frac{m_i\sigma_i}{s_i} \\ -1 & \text{if } x_{i,t} < \mu_{i,t} - \frac{m_i\sigma_i}{s_i} \end{cases}, \tag{3.43}$$

and

$$m_i = \frac{\Gamma\left(\frac{\nu_i-1}{2}\right)\sqrt{\nu_i-2}}{\Gamma\left(\frac{\nu_i}{2}\right)\sqrt{\pi}}\left(\lambda_i-\frac{1}{\lambda_i}\right),\ s_i^2 = \lambda_i^2+\frac{1}{\lambda_i^2}-1-m_i^2. \tag{3.44}$$

3.D The Multivariate Skewed *t* Distribution

Bauwens and Laurent (2005) propose a flexible approach to introduce skewness into multivariate Student's t distribution and extend this univariate skewed t distribution to a multivariate case. The PDF of their multivariate skewed t distribution is given by

$$\mathbf{f}_{\lambda,\eta}\left(\mathbf{x} \mid \mu,\Sigma\right) = \left(\frac{2}{\sqrt{\pi}}\right)^n\left(\prod_{i=1}^{n}\frac{\lambda_i s_i}{1+\lambda_i^2}\right)\frac{\Gamma\left(\frac{\eta+n}{2}\right)|\Sigma|^{-\frac{1}{2}}}{\Gamma\left(\frac{\eta}{2}\right)\left(\eta-2\right)^{\frac{n}{2}}}\left(1+\frac{\mathbf{a}'\mathbf{a}}{\eta-2}\right)^{-\left(\frac{\eta+n}{2}\right)} \tag{3.45}$$

where $\lambda = (\lambda_1, \ldots, \lambda_n)$ denotes the vector of asymmetric parameters, Σ denotes the covariance matrix and η is the degree of freedom. The vector $\mathbf{a} = (a_1, \ldots, a_n)$ contains elements defined as $a_i = \lambda_i^{-I_i}(m_i + s_i x_i^*)$, where $\mathbf{x} = (x_1^*, \ldots, x_n^*) = \Sigma^{-\frac{1}{2}}(\mathbf{x}-\mu)$ and

$$I = \begin{cases} 1 & \text{if } x_{i,t} \geq \mu_{i,t} - \frac{m_i\sqrt{\sigma_{ii}}}{s_i} \\ -1 & \text{if } x_{i,t} < \mu_{i,t} - \frac{m_i\sqrt{\sigma_{ii}}}{s_i} \end{cases}, \tag{3.46}$$

where

$$m_i = \frac{\Gamma\left(\frac{\eta-1}{2}\right)\sqrt{\eta-2}}{\Gamma\left(\frac{\eta}{2}\right)\sqrt{\pi}}\left(\lambda_i - \frac{1}{\lambda_i}\right),\ s_i^2 = \lambda_i^2 + \frac{1}{\lambda_i^2} - 1 - m_i^2. \tag{3.47}$$

3.E Estimation of Parametric Copula Model

The likelihood of a fully parametric copula model for conditional distribution of $\mathbf{r}_t$ takes the form:

$$\begin{aligned} L(\phi) &= \prod_{t=1}^{T} \mathbf{f}(\mathbf{z}_t|\mathcal{F}_{t-1};\phi) &(3.48)\\ &= \prod_{t=1}^{T}\left[c_t(u_{1,t},\ldots,u_{n,t}|\mathcal{F}_{t-1};\phi_C)\prod_{i=1}^{n} f_{i,t}(z_{i,t}|\mathcal{F}_{t-1};\phi_i)\right] \end{aligned}$$

with log-likelihood

$$\begin{aligned} \sum_{t=1}^{T}\log\mathbf{f}(\mathbf{z}_t|\mathcal{F}_{t-1};\phi) &= \sum_{t=1}^{T}\sum_{i=1}^{d}\log f_{i,t}(z_{i,t}|\mathcal{F}_{t-1};\phi_i) &(3.49)\\ &+ \sum_{t=1}^{T}\log c_t(F_{1,t}(z_{1,t}|\mathcal{F}_{t-1};\phi_1),\ldots,F_{n,t}(z_{n,t}|\mathcal{F}_{t-1};\phi_n)|\mathcal{F}_{t-1};\phi_C) \end{aligned}$$

where ϕ denotes the parameters for the full model parameters, ϕ_i denotes the parameters for the ith marginals, ϕ_C denotes the parameters of copula model and $\mathcal{F}_{t-1}$ denotes the information set at time $t-1$. Following the the two-stage maximum likelihood estimation (also known as the Inference method for margins) of Joe and Xu (1996), we first estimate the parameters of marginal models using maximum likelihood:

$$\hat{\phi}_{\mathbf{i}} = \underset{\phi_i}{\operatorname{argmax}}\sum_{t=1}^{T}\log f_{i,t}(z_{i,t} \mid \mathcal{F}_{t-1};\phi_i),\ i=1,\ldots,n \tag{3.50}$$

and then using the estimations in the first stage, we calculate $F_{i,t}$ and estimate the copula parameters via maximum likelihood:

$$\hat{\phi}_{\mathbf{C}} = \underset{\phi_C}{\operatorname{argmax}}\sum_{t=1}^{T}\log\mathbf{c}_t(F_{1,t}(z_{1,t} \mid \mathcal{F}_{t-1};\phi_1),\ldots,F_{n,t}(z_{n,t} \mid \mathcal{F}_{t-1};\phi_n) \mid \mathcal{F}_{t-1};\phi_C) \tag{3.51}$$

3.F Estimation of Semiparametric Copula Model

In the semiparametric estimation (also known as Canonical Maximum Likelihood Estimation), the univariate margins are estimated nonparametrically using the empirical distribution function (EDF) and the copula model is still parametrically estimated via maximum likelihood.

$$\hat{F}_i(z) \equiv \frac{1}{T+1}\sum_{t=1}^{T} 1\left\{\hat{z}_{i,t} \leq z\right\} \tag{3.52}$$

$$\hat{u}_{i,t} \equiv \hat{F}_i(z) \sim \text{ Unif } (0,1), \; i = 1,2,...,n \tag{3.53}$$

$$\hat{\phi}_{\mathbf{C}} = \arg\max_{\phi_{\mathbf{C}}} \sum_{t=1}^{T} \log \mathbf{c}_t \left(\hat{u}_{1,t},...,\hat{u}_{n,t} \mid \mathcal{F}_{t-1}; \phi_{\mathbf{C}}\right) \tag{3.54}$$

where $\mathrm{z}_{i,t}$ is the standardized residuals of marginal models and $\hat{F}_i$ is different from the standard empirical CDF in the scalar $\frac{1}{n+1}$, which is to ensure that the transformed data cannot be on the boundary of the unit interval $[0,1]$.

3.G Algorithm for TVAC based VaR and ES Forecasting

Step 1: It is necessary to determine the in sample and out-of-sample period for our forecasting.

Step 2: We forecast conditional mean, conditional volatility and residuals from the prespecified time series model on sliding window and do one step ahead forecasting.

Step 3: To get the estimation for the forecasted marginal distributions, we do parametric (univariate skewed *t*) and nonparametric (EDF) estimation on sliding window.

Step 4: We estimate parameters for full parametric and semiparetric copulas using in-sample data.

Step 5: Using the estimated parameters in Step 4 as initial values, we estimate time-varying dependence parameters for asymmetric (skew *t*) copulas based on GAS framework, see Equation 3.29.

Step 6: With the estimated time-varying copula parameters in hand, we can apply Monte Carlo simulation to generate *N* samples of innovations and portfolio returns.

Step 7: Based on the empirical $\alpha-$quantile of forecasted portfolio return, it is easy to forecast corresponding VaR.

Step 8: Given the N simulated portfolio returns, we can also calculate $\alpha-$quantile expected shortfall using Equation 3.7.

Step 9: Using realized portfolio returns to backtest VaR and ES forecasts.

3.H Algorithm for TVAC Copula based Portfolio Optimization

Step 1: Before copula modelling, we need to characterize individually the distribution of returns of each asset. Specifically, we use AR and GJR-GARCH to esimate conditional mean and conditional volatility, repectively, and use parametric or nonparametric method to get the probability integral transforms of the standardized residuals.

Step 2: Using the probability integral transforms we estimated from the last step, we estimate one-step-ahead skewed t copula parameters using rolling window[11].

Step 3: Given the copula parameters, the jointly-dependent uniform variate can be simulated by the skewed t copula random number generator.

Step 4: To generate the simulated daily returns, we transfer the uniform variate derived from the skewed t copula random number generator via the inverse CDF of each asset.

Step 5: The generated multivariate simulations from the skewed t copula model can be used to compute the single-period ES of a sample portfolio.

Step 6: Following Rockafellar and Uryasev (2000, 2002), we could find an optimal portfolio (weights) that gives us a minimum ES for a certain level of return at time t and repeat Step 1 to Step 6 using rolling window for weekly rebalance.

[11] A detailed method for the estimation of GAS dynamics for the covariance matrix of high-dimensional GH skewed t copulas can be found in Lucas et al. (2013).

3.I Sharpe Ratio, Gain-Loss Ratio and Index of Acceptability

The Sharpe Ratio of the portfolio return is defined by the ratio of the expected excess return over the standard deviation:

$$SR(R) = \begin{cases} \frac{E(R)-R_f}{\sigma(R)} & \text{if } E(R) - R_f > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.55}$$

where $E(R) - R_f$, R_f and $\sigma(R)$ denote the excess return, the risk free rate and the standard deviation of R, respectively. This measure satisfy the quasi-concavity, scale invariance, law invariance, expectation consistency and Fatou property. However, it is not monotonic, see a detailed proof in Cherny and Madan (2009).

The Gain-Loss Ratio proposed by Bernardo and Ledoit (2000) is defined as the ratio of the expected excess return over the expectation of the negative tail:

$$GLR(R) = \begin{cases} \frac{E(R)-R_f}{E(R^-)} & \text{if } E(R) - R_f > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.56}$$

where $R^- = \max\{-T, 0\}$. This measure satisfies all the desirable properties discussed before. Nevertheless, one major drawback of GLR is that the small losses and large losses are equally weighted in this measure.

The indices of acceptability are consistent measures which quantify how much a return on a portfolio is acceptable to rational investors. These measures satisfy a set of axioms including monotonicity, scale invariance, quasi-concavity and Fatou property. Bielecki et al. (2014) point out that these attractive properties also have corresponding finance interpretations. For instance, monotonicity implies that if a random variable X is acceptable at level α and it is dominated by another random variable Y, then Y is acceptable at level α as well. Quasi-concavity means that a combined portfolio always performs at higher acceptable level than its components. We consider the indices of acceptability based on the weighted

Value-at-Risk (WVaR), which is defined as the mixture of weighted ES at risk levels μ:

$$\mathrm{WVaR}_{\mu}(X) = -\int_{-\infty}^{\infty} xd\left[\Psi_{\mu}(F_X(x))\right], \tag{3.57}$$

where F_X is the distribution function of X and $\Psi_{\mu}(F_X(x))$ is a distortion function parameterized by μ. The numerical evaluation of WVaR is straightforward. If the $x_1, \ldots, x_N$ are historical portfolio returns and X has its empirical distribution, then:

$$\mathrm{WVaR}_{\mu}(X) = -\sum_{n=1}^{N} x_n\left[\Psi_{\mu}\left(\frac{n}{N}\right) - \Psi_{\mu}\left(\frac{n-1}{N}\right)\right], \tag{3.58}$$

where $x_{(1)}, \ldots, x_{(N)}$ are the portfolio returns $x_1, \ldots, x_N$ in the increasing order. Using the function above, we can find the largest stress level by distorting the distribution function of X, such that $\mathrm{E}(x) > 0$ under the corresponding distortion. Thus, the higher level of acceptability implies the better performance of portfolio. Following Cherny and Madan (2009), we consider four acceptability indices, including AIMIN, AIMAX, AIMAXMIN and AIMINMAX, under four concave distortion functions with different properties.

Figure 3.1 Dependence of International Financial Markets

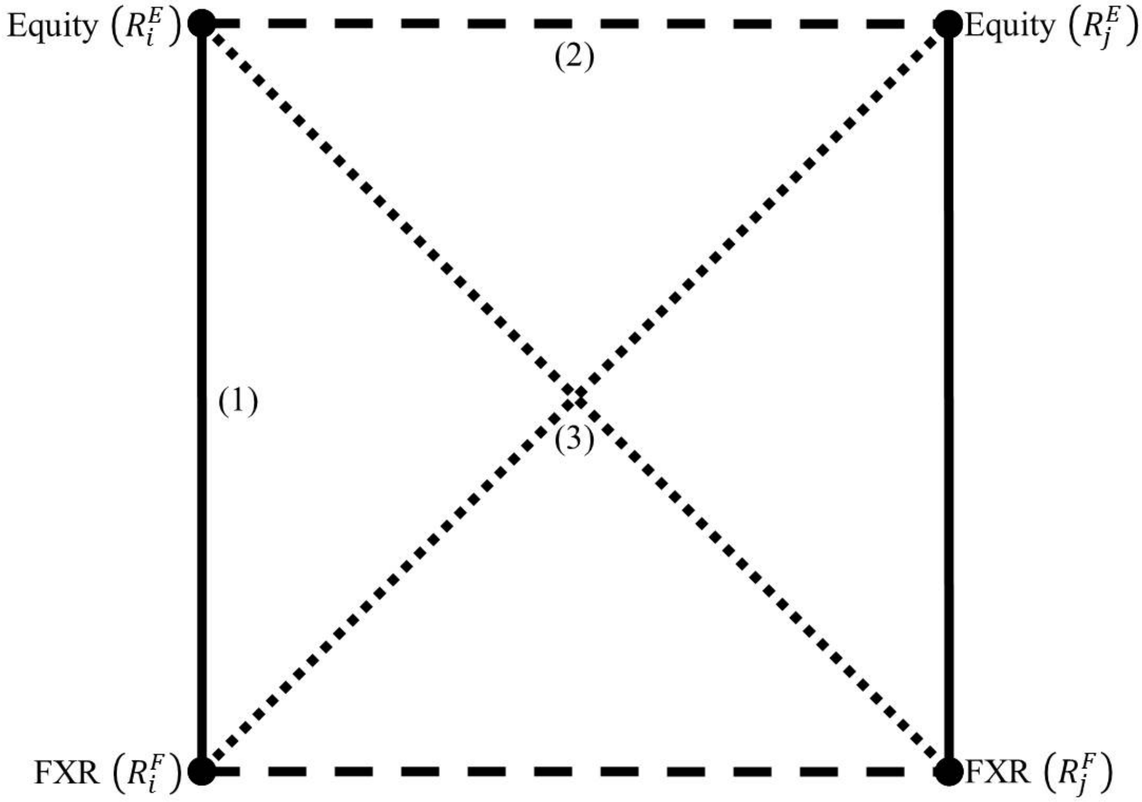
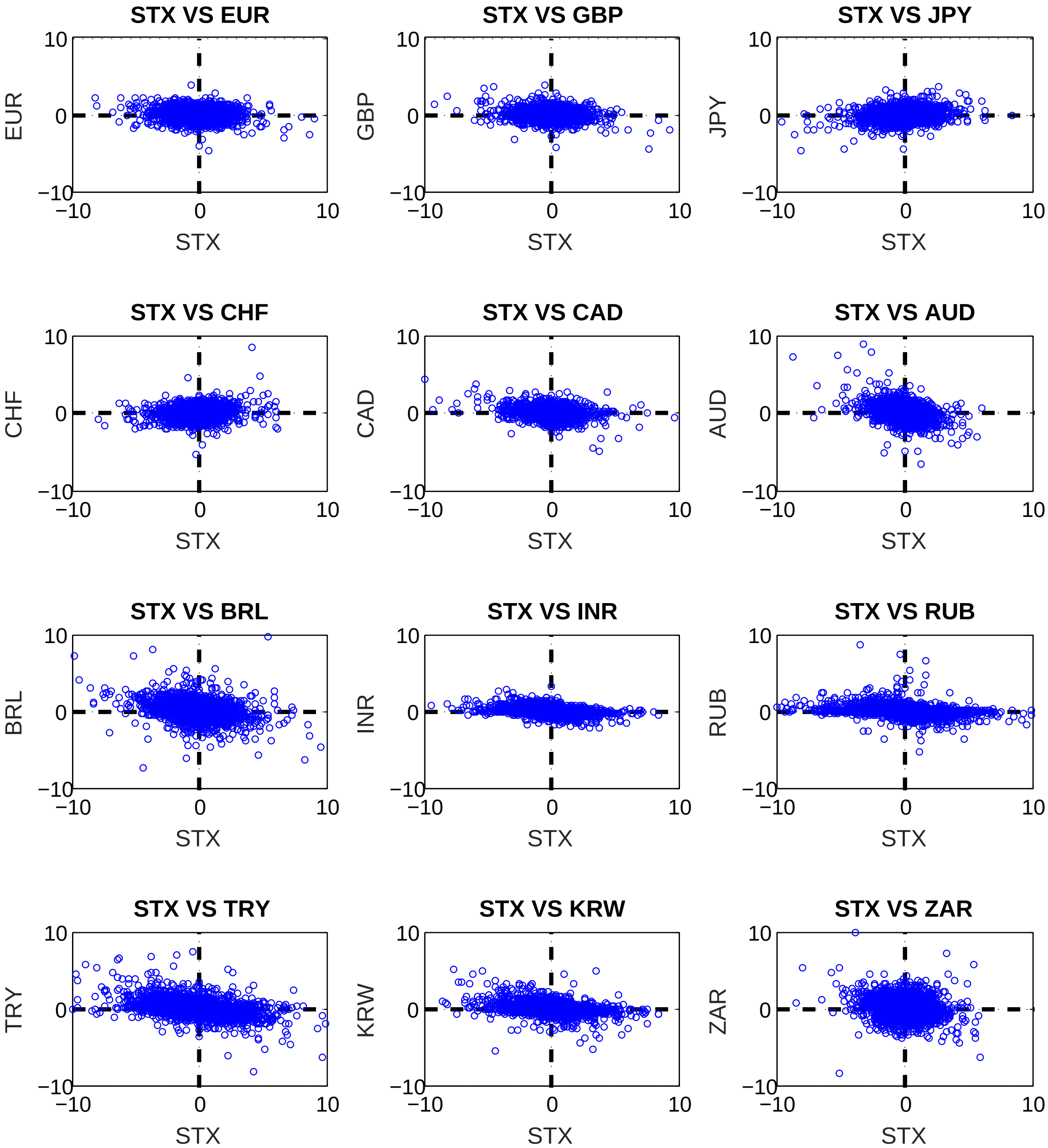
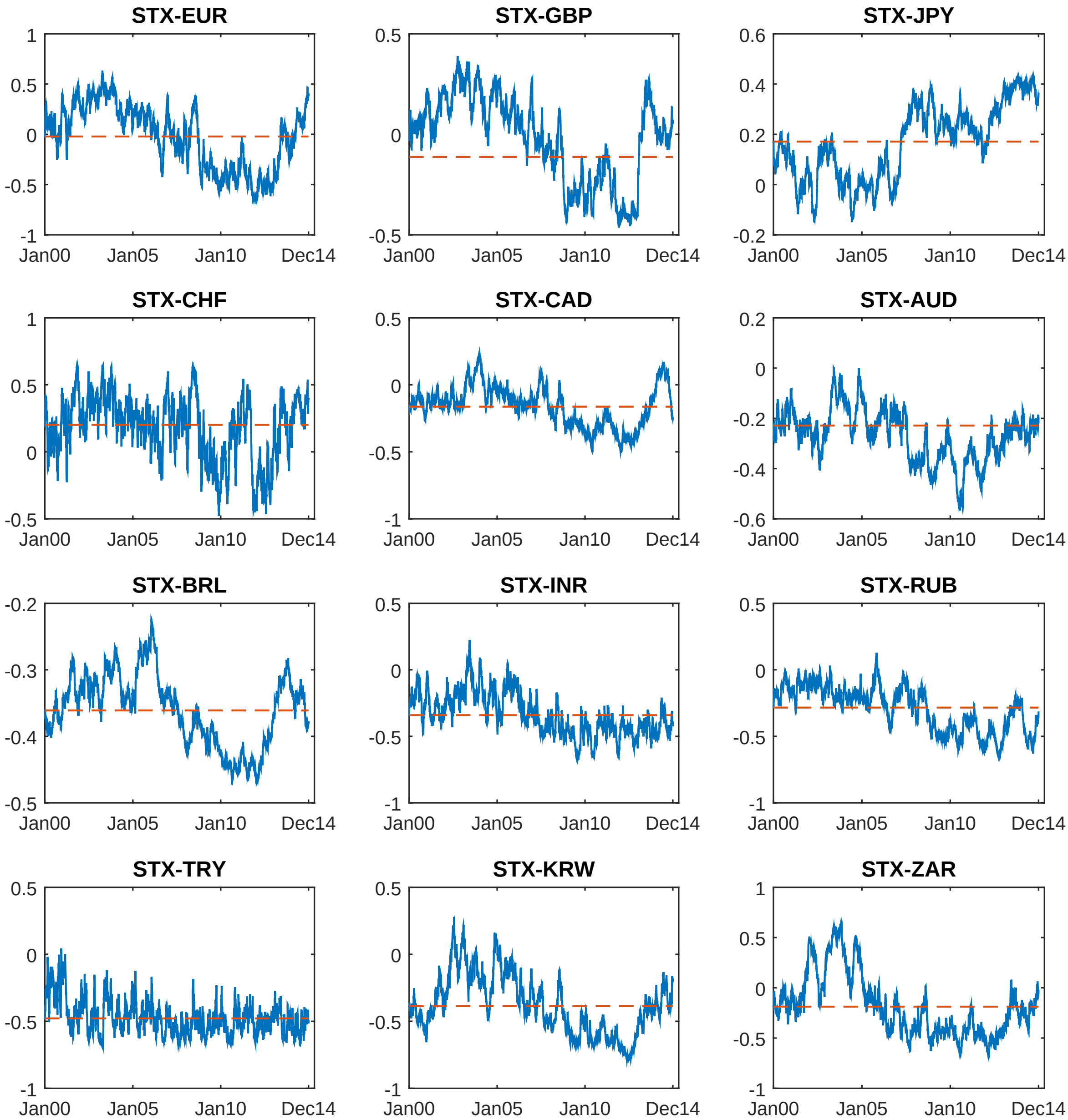


Figure 3.2 The Scatter Plots for Country Stock Index and Corresponding Currency



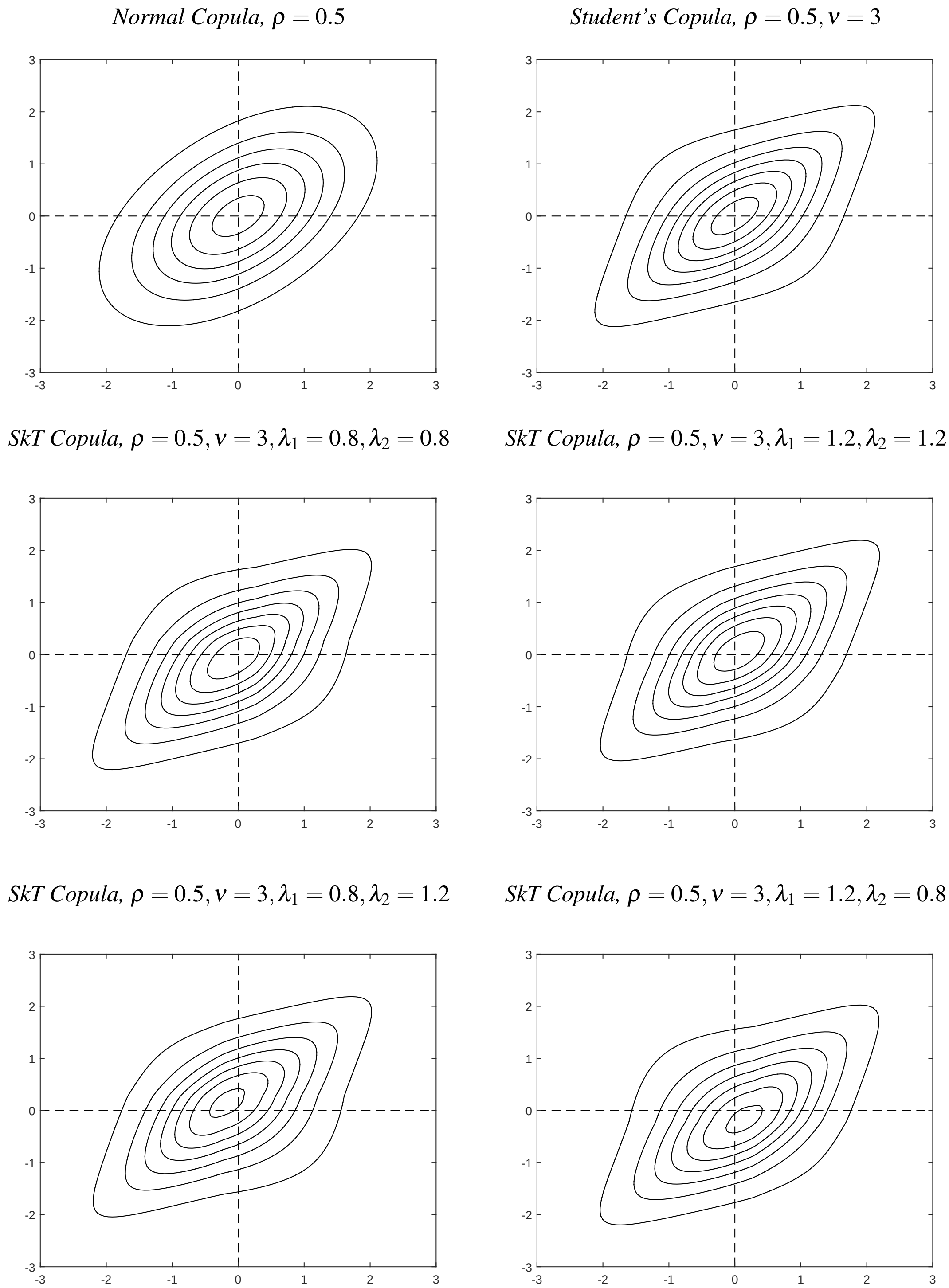
Note: This figure shows the scatter plots for different country equity indices (MSCI) and currencies pairs for the period from January 3, 2000 to December 31, 2014.

Figure 3.3 Dynamic Evolution of Dependence



Note: This figure shows the time-varying copula correlations between country equity indices (MSCI) and currencies implied by the GAS model (solid line) and constant copula correlation (dashed line) for the period from January 3, 2000 to December 31, 2014.

Figure 3.4 Contour Probability Plots for Copulas



Note: This figure shows contour probability plots for the normal, Student's t, and skewed t copulas derived from the multivariate skewed t distribution of Bauwens and Laurent (2005). The probability levels for each contour are held fixed across six panels. The marginal distributions are assumed to be normally distributed. ρ denotes the correlation coefficient, ν denotes the degree of freedom, and λ denotes the asymmetric parameters of copulas.

Table 3.1 Descriptive Statistics of Daily Log Return

Country	EU		UK		JAPAN		SWITZERLAND		CANADA		AUSTRALIA	
Asset	STX	EUR	STX	GBP	STX	JPY	STX	CHF	STX	CAD	STX	AUD
Mean	-0.004	-0.005	-0.001	0.001	-0.004	0.004	0.005	-0.012	0.014	-0.006	0.015	-0.006
Std.	1.267	0.622	1.208	0.576	1.394	0.642	1.168	0.677	1.214	0.583	1.033	0.835
Skewness	-0.123	-0.145	-0.168	0.054	-0.327	-0.252	-0.071	0.358	-0.595	-0.076	-0.396	0.867
Kurtosis	8.342	5.658	9.515	7.422	9.393	6.885	9.641	12.162	12.380	8.777	8.854	15.710
Min	-8.188	-4.617	-9.158	-4.474	-10.435	-4.610	-7.871	-5.451	-10.433	-5.046	-8.679	-6.701
Max	9.021	3.844	9.265	3.919	13.062	3.710	10.506	8.475	9.723	4.338	6.101	8.828
JB test	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LB Q(12)	0.000*	0.272	0.000*	0.001*	0.054	0.554	0.000*	0.199	0.000*	0.000*	0.070	0.000*
LB Q^2(12)	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LM	0.000*	0.014*	0.000*	0.000*	0.000*	0.000*	0.000*	0.003*	0.000*	0.000*	0.000*	0.000*
Linear	-0.097		-0.118		0.192		0.174		-0.229		-0.334	
Rho	-0.041		-0.020		0.130		0.155		-0.150		-0.230	
Country	BRAZIL		INDIA		RUSSIA		TURKEY		SOUTH KOREA		SOUTH AFRICA	
Asset	STX	BRL	STX	INR	STX	RUB	STX	TRY	STX	KRW	STX	ZAR
Mean	0.028	0.010	0.040	0.010	0.028	0.020	0.041	0.037	0.022	-0.001	0.042	0.016
Std.	1.672	0.997	1.589	0.395	2.353	0.664	2.291	1.176	1.719	0.677	1.254	1.047
Skewness	-0.132	0.111	-0.233	0.268	-0.482	0.790	0.051	8.765	-0.341	-0.702	-0.156	0.316
Kurtosis	8.921	16.843	10.487	10.627	16.349	152.776	9.925	292.104	8.336	62.744	6.119	8.573
Min	-14.068	-11.778	-12.050	-3.064	-25.279	-15.523	-19.715	-16.252	-13.097	-13.265	-8.448	-8.523
Max	13.441	9.677	16.423	3.251	23.950	14.268	17.816	37.462	11.722	10.351	5.962	9.808
JB test	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LB Q	0.007*	0.000*	0.000*	0.000*	0.000*	0.000*	0.002*	0.000*	0.268	0.000*	0.000*	0.108
LB Q^2(12)	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LM	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
Linear	-0.328		-0.361		-0.232		-0.232		-0.383		-0.184	
Rho	-0.308		-0.326		-0.279		-0.433		-0.359		-0.162	

Notes: This table reports descriptive statistics for daily returns on country equity index (MSCI) and currencies of 6 developed markets, including the European Union, United Kingdom, Japan, Switzerland, Australia and New Zealand, and 6 emerging markets, inluding Brazil, India, Russia, Turkey, South Korea and South Africa, over the period from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 observations for each market. STX denotes the country equity index provided by MSCI corresponding to each currency. Q(12) and Q^2(12) are the Ljung-Box statistics for serial correlation of order 12 in returns and squared returns. LM denotes the Lagrange Multiplier test for autoregressive conditional heteroscedasticity. We use * to indicate the rejection of the null hypothesis at the 5% significance level.

Table 3.2 Descriptive Statistics of Weekly Log Return

Country	EU		UK		JAPAN		SWITZERLAND		CANADA		AUSTRALIA	
Asset	STX	EUR	STX	GBP	STX	JPY	STX	CHF	STX	CAD	STX	AUD
Mean	-0.017	-0.025	-0.002	0.004	-0.018	0.021	0.026	-0.062	0.071	-0.029	0.074	-0.027
Std.	2.729	1.387	2.519	1.310	2.867	1.421	2.625	1.508	2.620	1.267	2.191	1.870
Skewness	-1.102	0.230	-1.110	0.798	-0.828	-0.476	-1.079	0.566	-0.807	0.896	-0.888	1.670
Kurtosis	12.201	3.908	14.773	8.161	7.695	4.989	17.160	8.196	9.685	9.015	8.592	15.886
Min	-23.996	-5.329	-23.430	-5.327	-22.318	-8.466	-24.788	-6.051	-16.968	-4.568	-16.490	-7.040
Max	11.559	5.624	12.549	8.664	9.642	5.421	15.521	12.096	14.130	9.090	9.733	17.805
JB test	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LB Q(12)	0.001*	0.226	0.000*	0.001*	0.811	0.049*	0.000*	0.078	0.001*	0.009	0.229	0.005*
LB $Q^2(12)$	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LM	0.002*	0.046*	0.014*	0.000*	0.000*	0.000*	0.000*	0.016*	0.000*	0.000*	0.000*	0.000*
Linear	-0.097		-0.130		0.337		0.187		-0.440		-0.463	
Rho	-0.005		-0.012		0.274		0.192		-0.325		-0.299	
Country	BRAZIL		INDIA		RUSSIA		TURKEY		SOUTH KOREA		SOUTH AFRICA	
Asset	STX	BRL	STX	INR	STX	RUB	STX	TRY	STX	KRW	STX	ZAR
Mean	0.142	0.052	0.200	0.049	0.143	0.087	0.204	0.186	0.113	-0.004	0.209	0.081
Std.	3.638	2.186	3.508	0.876	5.279	1.360	5.040	3.053	3.786	1.361	2.772	2.289
Skewness	-0.508	0.875	-0.591	0.268	-0.046	1.703	-0.099	7.264	-0.278	-0.029	-0.066	0.501
Kurtosis	6.737	8.322	5.986	7.092	12.623	17.554	7.090	152.853	6.792	11.523	5.818	5.524
Min	-21.357	-9.141	-19.000	-4.706	-27.797	-8.488	-29.275	-26.570	-21.403	-9.816	-10.804	-11.337
Max	16.891	12.579	13.660	4.489	44.916	11.067	26.146	56.003	18.819	6.988	16.264	10.897
JB test	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LB Q	0.026*	0.000*	0.001*	0.001*	0.506	0.000*	0.012*	0.000*	0.121	0.000*	0.036*	0.837
LB $Q^2(12)$	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LM	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
Linear	-0.587		-0.454		-0.302		-0.464		-0.495		-0.217	
Rho	-0.529		-0.449		-0.359		-0.523		-0.443		-0.162	

Notes: This table reports descriptive statistics for weekly returns on country equity index (MSCI) and currencies of 6 developed markets, including the European Union, United Kingdom, Japan, Switzerland, Australia and New Zealand, and 6 emerging markets, including Brazil, India, Russia, Turkey, South Korea and South Africa, over the period from January 7, 2000 to December 26, 2014, which correspond to a sample of 3,913 observations for each market. STX denotes the country equity index provided by MSCI corresponding to each currency. Q(12) and $Q^2(12)$ are the Ljung-Box statistics for serial correlation of order 12 in returns and squared returns. LM denotes the Lagrange Multiplier test for autoregressive conditional heteroscedasticity. We use * to indicate the rejection of the null hypothesis at the 5% significance level.

Table 3.3 Estimation for Univariate Distribution (Developed Markets)

Country	EU		UK		JAPAN		SWITZERLAND		CANADA		AUSTRALIA	
Asset	STX	EUR	STX	GBP	STX	JPY	STX	CHF	STX	CAD	STX	AUD
AR												
θ_0	0.023	-0.008	-0.003	-0.005	0.020	0.013	0.027	-0.009	0.048	-0.002	0.038	-0.018
	(0.014)	(0.008)	(0.011)	(0.008)	(0.018)	(0.009)	(0.013)	(0.009)	(0.013)	(0.007)	(0.012)	(0.010)
θ_1	-0.022	-0.001	-0.032	0.009	0.036	-0.034	0.005	-0.026	-0.007	-0.007	-0.032	0.012
	(0.018)	(0.017)	(0.016)	(0.016)	(0.017)	(0.016)	(0.017)	(0.016)	(0.017)	(0.017)	(0.017)	(0.017)
GARCH												
ω	0.016	0.001	0.014	0.001	0.045	0.004	0.020	0.002	0.010	0.001	0.010	0.004
	(0.003)	(0.000)	(0.002)	(0.001)	(0.008)	(0.001)	(0.003)	(0.001)	(0.002)	(0.000)	(0.002)	(0.001)
α	(0.000)	(0.035)	(0.000)	(0.050)	(0.023)	(0.032)	(0.000)	(0.024)	(0.016)	(0.054)	(0.003)	(0.064)
	(0.011)	(0.006)	(0.010)	(0.007)	(0.010)	(0.007)	(0.009)	(0.006)	(0.011)	(0.008)	(0.008)	(0.008)
δ	0.165	-0.011	0.282	-0.025	0.116	0.009	0.173	0.013	0.098	-0.014	0.120	-0.039
	(0.016)	(0.005)	(0.024)	(0.008)	(0.016)	(0.009)	(0.017)	(0.008)	(0.015)	(0.010)	(0.013)	(0.010)
β	0.903	0.969	0.859	0.958	0.892	0.955	0.893	0.965	0.921	0.950	0.922	0.947
	(0.009)	(0.007)	(0.010)	(0.006)	(0.011)	(0.007)	(0.010)	(0.005)	(0.008)	(0.006)	(0.008)	(0.007)
SkT												
η	12.523	8.987	11.316	11.879	9.179	5.804	8.653	7.694	9.505	12.112	11.938	8.391
λ	0.881	0.997	0.889	1.038	0.945	0.972	0.903	0.953	0.862	1.010	0.896	1.101
KS	0.271	0.223	0.947	0.384	0.376	0.114	0.221	0.186	0.192	0.701	0.248	0.327
CvM	0.126	0.187	0.941	0.209	0.453	0.229	0.265	0.142	0.172	0.660	0.273	0.268

Notes: This table summarizes parameter estimates from AR and GJR-GARCH models for conditional mean and conditional variance of country equity index returns and currency returns for developed markets. We estimate all parameters using the sample from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 observations for each series. The values in parenthesis represent the standard errors of the parameters.

Table 3.4 Estimation for Univariate Distribution (Emerging Markets)

Country	BRAZIL		INDIA		RUSSIA		TURKEY		SOUTH KOREA		SOUTH AFRICA	
Asset	STX	BRL	STX	INR	STX	RUB	STX	TRY	STX	KRW	STX	ZAR
AR												
θ_0	0.036	-0.010	0.070	-0.003	0.080	0.003	0.080	0.002	0.044	-0.020	0.041	0.011
	(0.021)	(0.010)	(0.017)	(0.002)	(0.023)	(0.002)	(0.027)	(0.009)	(0.019)	(0.006)	(0.016)	(0.013)
θ_1	0.024	0.035	0.081	-0.013	0.038	0.026	0.006	0.038	0.005	-0.025	0.044	0.022
	(0.017)	(0.017)	(0.017)	(0.015)	(0.016)	(0.014)	(0.016)	(0.017)	(0.017)	(0.016)	(0.017)	(0.017)
GARCH												
ω	0.055	0.013	0.050	0.000	0.059	0.000	0.062	0.011	0.015	0.003	0.027	0.011
	(0.010)	(0.003)	(0.008)	(0.000)	(0.012)	(0.000)	(0.014)	(0.002)	(0.004)	(0.001)	(0.006)	(0.003)
α	0.014	0.194	0.041	0.159	0.078	0.073	0.047	0.178	0.015	0.134	0.019	0.088
	(0.009)	(0.018)	(0.011)	(0.015)	(0.015)	(0.007)	(0.010)	(0.018)	(0.007)	(0.015)	(0.008)	(0.010)
δ	0.096	-0.108	0.139	-0.036	0.072	-0.004	0.059	-0.067	0.077	-0.052	0.113	-0.039
	(0.014)	(0.020)	(0.019)	(0.016)	(0.019)	(0.009)	(0.013)	(0.022)	(0.011)	(0.017)	(0.014)	(0.012)
β	0.914	0.850	0.867	0.859	0.880	0.929	0.912	0.852	0.941	0.892	0.906	0.923
	(0.010)	(0.012)	(0.012)	(0.008)	(0.011)	(0.005)	(0.009)	(0.012)	(0.006)	(0.010)	(0.010)	(0.008)
SkT												
η	8.986	7.685	7.314	4.222	5.419	4.431	6.229	5.429	6.406	4.793	11.284	8.472
λ	0.943	1.051	0.945	1.036	0.954	1.037	0.997	1.076	0.944	1.061	0.930	1.068
KS	0.178	0.414	0.345	0.843	0.185	0.947	0.215	0.165	0.213	0.924	0.326	0.459
CvM	0.258	0.547	0.336	0.901	0.147	0.941	0.264	0.387	0.267	0.965	0.342	0.562

Notes: This table summarizes parameter estimates from AR and GJR-GARCH models for conditional mean and conditional variance of country equity index returns and currency returns for emerging markets. We estimate all parameters using the sample from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 observations for each series. The values in parenthesis represent the standard errors of the parameters.

Table 3.5 Tests for Asymmetric Dependence between Equity and Currency

Test	Threshold correlaton		Tail dependence			
	HTZ	p-value	LTD	UTD	Diff	p-value
Panel A: Developed Markets						
STX-EUR	40.537	0.239	0.037	0.030	0.008	0.772
STX-GBP	31.515	0.637	0.002	0.000	0.002	0.729
STX-JPY	24.508	0.907	0.032	0.017	0.015	0.041*
STX-CHF	30.619	0.680	0.037	0.047	-0.010	0.633
STX-CAD	42.456	0.181	0.035	0.005	0.030	0.024*
STX-AUD	36.708	0.390	0.058	0.047	0.011	0.538
Panel B: Emerging Markets						
STX-BRL	38.065	0.332	0.068	0.030	0.038	0.022*
STX-INR	130.595	0.000*	0.104	0.089	0.015	0.575
STX-RUB	60.366	0.005*	0.076	0.049	0.027	0.044*
STX-TRY	50.731	0.042*	0.184	0.147	0.037	0.203
STX-KRW	31.339	0.646	0.198	0.067	0.131	0.000*
STX-ZAR	49.808	0.049*	0.061	0.015	0.046	0.016*

Notes: This table presents the statistics and p-values from two asymmetric tests. "HTZ" denotes the statistic from a model-free symmetry test proposed in (Hong et al., 2007) to examine whether the exceedance correlations between currency and corresponding country stock index returns are asymmetric at all. "LUTD", "ULTD" and "Diff" denote the coefficients of lower-upper tail dependence and upper-lower tail dependence estimated by Student's t copula, and the difference between them for all the portfolios pairs. The estimations are calculated by semiparametric approach in Patton (2012a). The p-values from the tests that the low tail and upper tail dependence coefficients are computed with 500 bootstrap replications. We use * to indicate significance at the 5%.

Table 3.6 Tests for Time-varying Dependence between Equity and Currency

Portfolio	0.15	0.5	0.85	Anywhere	AR(1)	AR(5)	AR(10)	US crisis	EU crisis	Quandt-Andrews
Panel A: Developed Markets										
STX-EUR	0.013	0.000	0.184	0.000	0.930	0.151	0.003	0.000	0.003	0.000
STX-GBP	0.078	0.000	0.075	0.000	0.165	0.064	0.005	0.002	0.063	0.000
STX-JPY	0.136	0.014	0.000	0.000	0.000	0.006	0.017	0.001	0.003	0.000
STX-CHF	0.570	0.011	0.088	0.020	0.805	0.587	0.155	0.001	0.249	0.000
STX-CAD	0.304	0.091	0.001	0.000	0.000	0.000	0.293	0.057	0.398	0.000
STX-AUD	0.022	0.002	0.000	0.010	0.004	0.958	0.200	0.096	0.748	0.000
Panel B: Emerging Markets										
STX-BRL	0.056	0.070	0.112	0.043	0.000	0.000	0.065	0.327	0.955	0.000
STX-INR	0.000	0.000	0.507	0.000	0.742	0.007	0.000	0.000	0.001	0.000
STX-RUB	0.000	0.000	0.890	0.000	0.403	0.684	0.005	0.000	0.001	0.000
STX-TRY	0.000	0.043	0.093	0.000	0.029	0.474	0.015	0.884	0.577	0.000
STX-KRW	0.077	0.000	0.260	0.000	0.903	0.365	0.031	0.000	0.000	0.000
STX-ZAR	0.001	0.000	0.323	0.000	0.070	0.193	0.023	0.000	0.021	0.000

Notes: This table reports the p-value from tests for time-varying dependence between currency and corresponding country stock index returns. Without a priori dates to consider for the timing of a break, we use naïve tests for breaks at three chosen points in sample period, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 01-Apr-2002, 02-Jun-2007, 28-Sep-2012. The "Any" column reports the results of test for dependence break of unknown timing proposed by Andrews (1993). The F-statistic in column "Quandt-Andrews" is based on a generalized break test without priori point in (Andrews and Ploberger, 1994). To detect whether the dependence structures between currency and equity significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points and the "Crisis" panel reports the results for this test. The "AR" panel presents the results from the ARCH LM test for time-varying volatility proposed by Engle (1982). Under the null hypothesis of a constant conditional copula, we test autocorrelation in a measure of dependence (see Patton, 2012a).

Table 3.7 Likelihood Ratio Test for Copula Models

Portfolio	STX-EUR	STX-GBP	STX-JPY	STX-CHF	STX-CAD	STX-AUD
T	64.34	23.02	56.91	81.90	44.40	100.98
SkT	67.15	33.42	67.77	100.94	54.66	118.45
TV T	261.26	102.21	101.36	185.27	93.16	129.00
TV SkT	279.50	127.90	109.58	202.47	114.30	195.85
LR test	0.000	0.000	0.000	0.000	0.000	0.000
Portfolio	STX-BRL	STX-INR	STX-RUB	STX-TRY	STX-KRW	STX-ZAR
T	214.48	247.82	169.32	477.02	338.84	107.45
SkT	263.55	266.09	174.89	487.40	350.44	120.25
TV T	221.23	304.10	235.16	533.05	465.33	289.39
TV SkT	279.80	322.61	246.70	568.59	475.13	301.16
LR test	0.000	0.000	0.000	0.000	0.000	0.000
Portfolio	STX_ALL					
T	3447.78					
SkT	3719.98					
TV T	8658.26					
TV SkT	9713.82					
LR test	0.000					

Notes: This table reports the log likelihood values for two constant copula models and two time-varying copula models, and the p-values for likelihood ratio test. "T", "SkT", "TV T" and "TV SkT" denote the "constant t copula", "constant skewed t copula", "time-varying t copula" and "time-varying skewed t copula" respectively. All copula models are semiparametrically estimated. "LR test" reports the p-values of likelihood ratio test of model specification with 2 degrees of freedom. We use the this test to assess whether our data provide enough evidence to favor the unrestricted model (TV SkT) over the restricted model (TV T). "STX_ALL" denotes the portfolio formed by all the equity indices from 12 markets in the application of asset allocation.

Table 3.8 Empirical Coverage Probability

Portfolio	T	SkT	TV T	TV SkT	BEKK	CCC	DCC
Panel A: 95% Value-at-Risk							
STX-EUR	5.16%	5.05%	5.00%	5.05%	5.62%	5.43%	5.49%
STX-GBP	4.86%	4.89%	4.83%	4.97%	5.68%	5.46%	5.35%
STX-JPY	4.75%	4.83%	4.91%	5.02%	5.46%	5.16%	5.00%
STX-CHF	5.13%	5.08%	4.97%	4.94%	5.24%	5.43%	5.35%
STX-CAD	5.11%	5.05%	5.00%	4.97%	5.76%	5.54%	5.51%
STX-AUD	5.21%	4.70%	5.08%	4.94%	5.43%	5.21%	4.97%
STX-BRL	5.21%	4.83%	5.27%	5.13%	5.24%	5.27%	5.16%
STX-INR	5.16%	4.64%	5.21%	5.21%	5.43%	5.43%	5.21%
STX-RUB	5.35%	4.94%	5.19%	5.00%	4.75%	5.08%	4.80%
STX-TRY	5.62%	5.60%	5.27%	5.05%	2.87%	4.10%	3.77%
STX-KRW	5.51%	5.08%	5.27%	5.00%	6.03%	5.87%	5.90%
STX-ZAR	5.19%	5.30%	4.70%	4.97%	5.27%	5.68%	5.32%
RMSE	0.31%	0.26%	0.20%	0.08%$\star$	0.84%	0.54%	0.54%
Panel B: 99% Value-at-Risk							
STX-EUR	0.98%	1.09%	0.96%	0.98%	1.64%	1.69%	1.58%
STX-GBP	1.15%	1.09%	1.04%	0.96%	1.88%	1.94%	1.64%
STX-JPY	0.76%	0.87%	0.85%	0.98%	1.37%	1.26%	1.28%
STX-CHF	0.98%	0.96%	1.06%	1.01%	1.61%	1.53%	1.45%
STX-CAD	1.09%	0.96%	1.04%	1.06%	1.69%	1.67%	1.56%
STX-AUD	1.26%	1.23%	1.34%	1.20%	1.99%	1.94%	1.94%
STX-BRL	1.15%	1.06%	1.15%	1.15%	1.97%	1.88%	1.80%
STX-INR	1.09%	1.12%	1.12%	0.98%	1.80%	1.83%	1.61%
STX-RUB	1.06%	0.93%	1.09%	1.01%	1.80%	1.91%	1.88%
STX-TRY	1.31%	1.31%	1.17%	1.28%	0.90%	1.50%	1.34%
STX-KRW	0.85%	1.04%	0.98%	1.01%	1.69%	1.72%	1.86%
STX-ZAR	1.06%	1.04%	0.98%	1.06%	1.53%	1.86%	1.53%
RMSE	0.17%	0.14%	0.14%	0.12%$\star$	0.72%	0.76%	0.66%

Notes: This table reports ECP for each equity-currency portfolio and VaR model. *Bias* summarizes the average deviation of 12 portfolios from the nominal coverage probability, 5% and 1%, for each VaR model, and *RMSE* (Root Mean Square Error) summarizes the fluctuation of the deviation across 12 portfolios for each VaR model, For the equity minus currency portfolios, we estimate the VaR and ES models using 250 business days over the period January 3, 2000 - December 15, 2000, and compute the one-day-ahead forecasts of the 95 and 99 percent VaR for December 18, 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for December 31, 2014. This generates 3,663 out-of-sample daily forecasts. * indicates that the VaR model is rejected at the 5% significance level. "T", "SkT", "TV T" and "TV SkT" denote the "constant *t* copula", "constant skewed *t* copula", "time-varying *t* copula" and "time-varying skewed *t* copula" respectively. All copula models are semiparametrically estimated. "BEKK", "CCC" and "DCC" denote the multivariate GARCH models. $\star$ indicates the smallest RMSE among 7 different models.

Table 3.9 Conditional Coverage Test

Portfolio	T	SkT	TV T	TV SkT	BEKK	CCC	DCC
Panel A: 95% Value-at-Risk							
STX-EUR	11.39*	8.78*	9.49*	3.28	24.59*	7.20*	1.79
STX-GBP	5.79	2.11	1.59	1.08	8.74*	2.50	1.07
STX-JPY	1.24	2.11	1.19	0.66	19.18*	17.36*	12.90*
STX-CHF	3.00	1.39	0.45	2.62	0.84	1.56	0.96
STX-CAD	15.77*	18.23*	17.04*	11.29*	18.15*	13.10*	8.53*
STX-AUD	14.74*	7.54*	10.26*	6.53*	10.14*	6.32*	7.93
STX-BRL	6.32*	3.48	4.14	0.73	1.99	2.82	0.74
STX-INR	3.99	1.80	2.85	0.79	5.96	4.85	0.79
STX-RUB	8.86*	5.21	6.53*	5.02	2.13	0.76	0.76
STX-TRY	10.00*	8.61*	10.69*	3.28	43.64*	11.54*	18.14*
STX-KRW	3.35	3.17	0.90	2.49	8.83*	7.12*	6.03*
STX-ZAR	5.10	4.85	4.72	0.15	4.89	6.61*	2.08
Average	6	4	5	2⋆	7	7	5
Panel B: 99% Value-at-Risk							
STX-EUR	0.73	1.19	0.75	0.73	12.64*	16.84*	10.72*
STX-GBP	1.74	1.19	0.85	0.75	23.29*	28.39*	14.64*
STX-JPY	3.79	1.76	2.16	0.82	18.58*	13.48*	12.64*
STX-CHF	0.82	0.96	0.76	0.74	11.66*	8.94*	8.06*
STX-CAD	3.39	4.06	0.72	3.40	15.42*	14.43*	15.97*
STX-AUD	7.55*	4.15	5.67	2.48	31.51*	27.08*	25.84*
STX-BRL	1.74	0.99	1.74	1.74	29.83*	23.03*	21.66*
STX-INR	0.85	1.00	1.00	0.82	21.27*	20.87*	12.57*
STX-RUB	0.99	1.16	0.85	0.74	21.27*	24.55*	24.55*
STX-TRY	11.80*	11.80*	7.07*	4.81	4.77	9.32*	23.62*
STX-KRW	1.45	0.85	0.73	0.76	16.84*	15.79*	21.74*
STX-ZAR	3.40	3.48	0.82	0.76	10.07*	28.40*	10.07*
Average	2	1	1	0⋆	11	12	12

Notes: This table presents the CC results. The CC test uses the LR statistic and it follows the Chi-squared distribution with two degrees-of-freedom under the null hypothesis. For the equity-currency (long equity and short currency) portfolios, we estimate the VaR and ES models using 250 business days over the period January 3, 2000 - December 15, 2000, and compute the one-day-ahead forecasts of the 95 and 99 percent VaR for December 18, 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for December 31, 2014. This generates 3,663 out-of-sample daily forecasts. * indicates that the VaR model is rejected at the 5% significance level. "T", "SkT", "TV T" and "TV SkT" denote the "constant *t* copula", "constant skewed *t* copula", "time-varying *t* copula" and "time-varying skewed *t* copula" respectively. All copula models are semiparametrically estimated. "BEKK", "CCC" and "DCC" denote the multivariate GARCH models. "Rejection" counts the number of rejection from the 12 equity-currency portfolios. ⋆ indicates the smallest rejection among 7 different models.

Table 3.10 Dynamic Quantile Test

Portfolio	T	SkT	TV T	TV SkT	BEKK	CCC	DCC
Panel A: 95% Value-at-Risk							
STX-EUR	26.08*	21.62*	24.93*	16.62*	48.85*	17.55*	4.70
STX-GBP	22.78*	19.63*	6.61	5.31	22.72*	12.43*	3.06
STX-JPY	15.63*	12.51*	8.89	10.41	42.25*	32.11*	24.32*
STX-CHF	21.71*	13.06*	11.11*	9.39	10.99	10.53	2.71
STX-CAD	28.99*	40.13*	37.02*	34.58*	33.44*	25.31*	14.57*
STX-AUD	32.99*	15.10*	24.93*	19.10*	23.39*	17.39*	13.88*
STX-BRL	26.90*	15.06*	25.00*	10.44	19.12*	19.49*	5.32
STX-INR	5.27	2.03	4.03	1.56	7.47	6.79	2.48
STX-RUB	23.92*	10.77	12.66*	10.72	7.89	8.40	8.40
STX-TRY	28.46*	29.61*	30.23*	32.31*	43.35*	24.93*	19.07*
STX-KRW	6.53	5.55	5.11	8.12	13.56*	11.45*	11.12*
STX-ZAR	25.23*	21.32*	10.14	11.02	13.11*	19.01*	3.37
Rejection	10	9	7	5$\star$	9	9	5$\star$
Panel B: 99% Value-at-Risk							
STX-EUR	10.59*	8.83	3.19	9.65	45.94*	42.53*	21.73*
STX-GBP	8.23	8.39	9.42	9.06	60.60*	48.73*	35.33*
STX-JPY	4.78	4.54	13.39*	3.22	81.09*	46.38*	48.78*
STX-CHF	22.26*	11.31*	19.53*	21.11*	21.18*	18.88*	12.56*
STX-CAD	13.75*	11.25*	15.04*	8.94	28.19*	22.65*	32.93*
STX-AUD	68.57*	27.39*	58.33*	64.98*	94.54*	129.63*	78.55*
STX-BRL	8.16	8.16	8.16	8.16	43.93*	34.19*	31.56*
STX-INR	24.14*	27.80*	13.35*	22.68*	55.83*	43.48*	29.87*
STX-RUB	8.21	3.74	8.64	9.43	39.60*	47.72*	47.72*
STX-TRY	33.67*	33.67*	30.98*	8.15	25.58*	34.52*	23.62*
STX-KRW	13.11*	9.55	9.63	10.03	35.83*	47.17*	33.58*
STX-ZAR	19.31*	20.11*	10.75	8.98	23.15*	62.36*	21.19*
Rejection	8	6	6	4$\star$	12	12	12

Notes: This table presents the DQ test results. The DQ test uses the Wald statistic and it follows the Chi-squared distribution with 6 degrees-of-freedom under the null hypothesis. For the equity-currency (long equity and short currency) portfolios, we estimate the VaR and ES models using 250 business days over the period January 3, 2000 - December 15, 2000, and compute the one-day-ahead forecast of VaR for December 18, 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for December 31, 2014. This generates 3,663 out-of-sample daily forecasts. * indicates that the VaR model is rejected at the 5% significance level. "T", "SkT", "TV T" and "TV SkT" denote the "constant *t* copula", "constant skewed *t* copula", "time-varying *t* copula" and "time-varying skewed *t* copula" respectively. All copula models are semiparametrically estimated. "BEKK", "CCC" and "DCC" denote the multivariate GARCH models. "Rejection" counts the number of rejection from the 12 equity-currency portfolios. $\star$ indicates the smallest rejection among 7 different models.

Table 3.11 Evaluation of Expected Shortfall: Mean Absolute Error

Portfolio	T	SkT	TV T	TV SkT	BEKK	CCC	DCC
Panel A: 95% Expected Shortfall							
STX-EUR	0.0331	0.0291	0.0223	0.0222⋆	0.0289	0.0307	0.0254
STX-GBP	0.0288	0.0294	0.0236	0.0232⋆	0.0304	0.0298	0.0264
STX-JPY	0.0290	0.0278	0.0238	0.0233⋆	0.0284	0.0253	0.0249
STX-CHF	0.0262	0.0241	0.0217	0.0211⋆	0.0239	0.0239	0.0221
STX-CAD	0.0300	0.0275	0.0273⋆	0.0274	0.0310	0.0303	0.0290
STX-AUD	0.0353	0.0326	0.0326	0.0319⋆	0.0374	0.0363	0.0331
STX-BRL	0.0482	0.0426	0.0432	0.0427	0.0471	0.0466	0.0451
STX-INR	0.0413	0.0343⋆	0.0378	0.0367	0.0392	0.0397	0.0356
STX-RUB	0.0640	0.0530⋆	0.0552	0.0550	0.0556	0.0617	0.0583
STX-TRY	0.0743	0.0751	0.0567	0.0556⋆	0.0475	0.0631	0.0559
STX-KRW	0.0466	0.0421	0.0391	0.0371⋆	0.0432	0.0429	0.0425
STX-ZAR	0.0369	0.0366	0.0306	0.0303⋆	0.0335	0.0377	0.0314
Average	0.0411	0.0378	0.0345	0.0339⋆	0.0372	0.0390	0.0358
Panel B: 99% Expected Shortfall							
STX-EUR	0.0061	0.0056	0.0031	0.0029⋆	0.0081	0.0096	0.0074
STX-GBP	0.0057	0.0057	0.0042	0.0033⋆	0.0095	0.0092	0.0080
STX-JPY	0.0088	0.0065	0.0051	0.0043⋆	0.0094	0.0086	0.0074
STX-CHF	0.0062	0.0047	0.0038	0.0030⋆	0.0076	0.0073	0.0063
STX-CAD	0.0077	0.0076	0.0065	0.0054	0.0097	0.0099	0.0085
STX-AUD	0.0103	0.0083	0.0067	0.0049⋆	0.0133	0.0133	0.0106
STX-BRL	0.0097	0.0101	0.0093	0.0084⋆	0.0146	0.0150	0.0133
STX-INR	0.0095	0.0069	0.0056	0.0036⋆	0.0135	0.0143	0.0129
STX-RUB	0.0181	0.0112⋆	0.0129	0.0126	0.0228	0.0253	0.0229
STX-TRY	0.0192	0.0197	0.0099	0.0089⋆	0.0203	0.0276	0.0221
STX-KRW	0.0075⋆	0.0118	0.0097	0.0094	0.0154	0.0159	0.0160
STX-ZAR	0.0070	0.0065	0.0062	0.0053	0.0094	0.0109	0.0089
Average	0.0096	0.0087	0.0069	0.0060⋆	0.0128	0.0139	0.0120

Notes: This table presents the mean absolute error (MAE) to evaluate the performance of ES predication. For the equity-currency (long equity and short currency) portfolios, we estimate the VaR and ES models using 250 business days over the period 26 Oct. 2005 - 10 Oct. 2006, and compute the one-day-ahead forecast of VaR for 11 Oct. 2006. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 4 Jan. 2012. This generates 1,677 out-of-sample daily forecasts. "T", "SkT", "TV T" and "TV SkT" denote the "constant *t* copula", "constant skewed *t* copula", "time-varying *t* copula" and "time-varying skewed *t* copula" respectively. All copula models are semiparametrically estimated. "BEKK", "CCC" and "DCC" denote the multivariate GARCH models. "Average" evaluates the average MAE of the 12 equity-currency portfolios. ⋆ indicates the smallest MAE among 7 different models.

Table 3.12 Evaluation of Optimal Asset Allocation

Portfolio	Sharp Ratio	GL Ratio	AIMIN	AIMAX	AIMAXMIN	AIMINMAX
Panel A: Traditional Frameworks						
Naïve	0.2492	0.1078	0.0422	0.0380	0.0441	0.0197
MEAN-VAR	0.4450	0.2001	0.0764	0.0686	0.0828	0.0353
MEAN-MAD	0.4696	0.2092	0.0802	0.0723	0.0872	0.0371
Panel B: Copula Optimization (95% Expected Shortfall)						
G	0.7565	0.3702	0.1378	0.1188	0.1598	0.0612
SkT	0.7665	0.3743	0.1404	0.1192	0.1634	0.0619
TV G	0.7555	0.3732	0.1399	0.1190	0.1627	0.0617
TV SkT	0.7738$\star$	0.3790$\star$	0.1415$\star$	0.1207$\star$	0.1648$\star$	0.0625$\star$
Panel C: Copula Optimization (99% Expected Shortfall)						
G	0.7638	0.3770	0.1393	0.1208	0.1619	0.0621
SkT	0.7500	0.3708	0.1367	0.1187	0.1584	0.0610
TV G	0.7666	0.3776	0.1396	0.1211	0.1623	0.0622
TV SkT	0.7771$\star$	0.3867$\star$	0.1430$\star$	0.1227$\star$	0.1669$\star$	0.0633$\star$

Notes: This table presents values of 6 performance measures for different optimization strategies. The first column reports the Sharpe Ratio and the second column reports the Gain-Loss Ratio. The other columns report four acceptability indices including AIMIN, AIMAX, AIMAXMIN and AIMINMAX, see Cherny and Madan (2009). "Naïve" denotes the equally weighted buy and hold strategy. "Mean-variance" denotes the optimal portfolio constructed by the mean-variance framework. "Mean-MAD" denotes the optimal portfolio constructed by the mean-MAD framework proposed by Konno and Yamazaki (1991). All the copla based optimal portfolios are constructed by mean-ES framework (The portfolio ES is estimated by the copula model at both 95% and 99% confidence levels). "G", "SkT", "TV G" and "TV SkT" denote the "constant Gaussian copula", "constant skewed *t* copula", "time-varying Gaussian copula" and "time-varying skewed *t* copula" respectively. All copula models are semiparametrically estimated. $\star$ indicates the maximum value for each performance measure.

Chapter 4

Correlated Defaults of UK Banks: Dynamics and Asymmetries

We document asymmetric and time-varying features of dependence between the credit risk of UK top tier banks using a new CDS dataset. The market-implied probability of default for individual banks is derived from observed market quotes of CDS. The default dependence between banks is modeled by a novel dynamic asymmetric copula framework. We show that all the empirical features of CDS spreads, such as heavy-tailedness, skewness, time-varying volatility, multivariate asymmetries and dynamic dependence, can be captured well by our model. Given the marginal default probability and estimated copula model, we compute the joint and conditional probability of default of UK banks by applying a fast simulation algorithm. Comparing our model with traditional copula models, we find that the traditional models may underestimate the joint credit risk most of the time, especially during the recent crisis. Furthermore, we perform an extensive regression analysis and find solid evidence that time-varying asymmetries between CDS spreads of UK banks contain useful information to explain and predict their joint and conditional default probabilities.

4.1 Introduction

The recent global financial crisis and EU sovereign debt crisis have caused great concern about the credit risk of large financial institutions and sovereign entities. The central banks and financial authorities have paid much more attention to the supervision of credit risk in large financial institutions since then. Understanding the joint credit risk of financial institutions is of particular important because their failures and losses can impose serious externalities on the rest of the economy (Acharya et al., 2010). Acharya et al. (2014) also document that the bailouts of large banks in Eurozone triggered a significant increase of sovereign credit risk from 2007 to 2011.

Recent empirical literature shows that modeling default dependence and joint default probability plays an important role in banking supervision (see Erlenmaier and Gersbach, 2014; Pianeti et al., 2012). This is because joint default probability can be viewed as an efficient systemic risk measure, as the systemic default risk arises from simultaneous default of multiple large banks. Some giant banks are "too big to fail" and the default of one bank can probably trigger a series of defaults of other banks and financial companies; for instance, the collapse of the Lehman Brothers in September 2008 triggered turmoil in financial markets and exacerbated the global financial crisis of 2007-2009. Das et al. (2006) show that the defaults of individual firms may cluster when their default risk is driven by some common factors.

From the perspective of practitioners, modeling the joint default probability of banks is also of great interest for risk management and derivative pricing. For instance, a protection contract (e.g. Credit Default Swap (CDS)) written by one bank (CDS seller) to insure against the default of another bank (debtor) is exposed to the risk that both banks default. In other words, CDS buyer also takes the counterparty risk that CDS seller will fail to fulfill their obligations because of the OTC nature of the CDS market.

There are two important aspects of modeling the joint default probability of two or several reference entities:

First, it is essential to obtain reliable probability of default (PD) and capture the default dynamics of a single reference entity. A number of statistical and econometric models have been proposed to obtain the term structure of default rates and they can be classified into three methods: (i) Historical default rate based on the internal rating systems from rating agencies (e.g. Moody's publishes historical default information regularly); (ii) Structural credit pricing models based on the option theoretical approach of Merton (1974); (iii) Reduced-form models. In our study, we consider using one reduced-form model based on the bootstrapping method proposed in Hull and White (2000a) and O'Kane and Turnbull (2003) to calculate the risk neutral default probability for each bank using CDS market quotes.[1] The reason we choose this approach is as follows: Firstly, the rating information provided by agencies is not able to change as fast as the market movement. Whereas, the market information used in the approach of Hull and White (2000a) can reflect well the market agreed anticipation of evolution of future credit quality; Secondly, although credit rating agencies such as Moody's regularly publish short-term and long-term credit ratings (PDs) for firms, this rating information normally lacks granularity. Different from the rating information provided by agencies, CDS market quotes normally have different maturities (6 month, 1-year, 2-year, 3-year, 4-year. 5-year, 7-year and 10-year) and thus could imply the full term structure of default probability; Thirdly, the bootstrapping procedure is a standard method for marking CDS positions to market and has been widely used by the overwhelming majority of credit derivative trading desks in financial practice, see Li (2000) and O'Kane and Turnbull (2003). Recently, this procedure has also has been applied in empirical financial studies, see Huang et al (2009), Creal et al. (2014b) and Lucas et al. (2014), etc.

Second, an empirically reliable model of correlated defaults between the reference entities plays a central role in credit risk modeling and pricing. Various approaches have been proposed to model correlated defaults and these models can be roughly classified into four categories: (i) CreditMetrics; (ii) Intensity-based models; (iii) Barrier-based firm's value models; (iv) Copula-based correlation models. However, these popular methods have their own limitations. For instance, the calibration and implementation of barrier-based models are difficult and the intensity-based approach normally has computational complexities because of a large number of parameters (see Schönbucher, 2003).

[1] CDS is essentially a protection contract to insure against the default of a reference entity. The CDS spread can be viewed as a more direct measure of credit risk compared to bond or loan spreads. This is because the bond or loan spread is also driven by other factors, such as interest rate movements and firm-specific equity volatility, see Campbell and Taksler (2003).

A Copula function has several attractive mathematical properties in default correlation modeling. First, it allows more flexibility and heterogeneity in the marginal distribution modeling. It is straightforward and convenient to link random variables with different marginal distributions with one copula function. Second, there are various versions of copula function and that allows us to fit different default dependence between the reference entities. Earlier studies on default dependence modeling normally rely on the Gaussian copula assumption (see Andersen and Sidenius, 2004; Das et al., 2007; Glasserman and Li, 2005; Giesecke, 2004; Hull and White, 2004; Li, 2000, among many others), however empirical finance literature provides strong evidence against the assumption of Gaussian dependence.[2] One drawback of the Gaussian copula is the lack of tail dependence. This means that tail events in the Gaussian copula are asymptotically independent of each other. Thus, the Gaussian copula-based models lack the ability to model the dependence between extreme values and therefore substantially underestimate the risk under extreme circumstance of financial market, such as the recently financial crisis caused by the collapse of Lehman Brothers in 2008. Substantial evidence has been found to show that default dependence between reference entities is non-Gaussian and time-varying, see for instance, Christoffersen et al. (2013). Meneguzzo and Vecchiato (2004) use the Student's t copula to price the CDO and basket CDS and find it could provide better fit than the Gaussian copula because of its flexibility in capturing the tail dependence. Thus the t copula is able to generate simultaneous extreme events with higher probabilities than the Gaussian copula. This property is of particular importance in credit risk modeling, as it leads to higher probabilities of joint defaults. However, the t copula is limited by its nature of symmetry and thus it is not able to well capture any asymmetries between assets.

Another important feature of default correlation is the time variation. Some conventional models use historical data to estimate the default correlation (e.g., Gupton et al., 1997; Lucas, 1995), however historical data cannot reflect well the current perception of the market. First, the default correlation changes over time as the credit quality of firms is dynamic. A

[2]Before the 2007-2008 global financial crisis, the Gaussian copula was the most popular copula model in derivatives pricing, especially the valuation of collateralized debt obligations (CDOs), because of its computational simplicity. However, many financial media commentators believed that the abuse of the Gaussian copula was one of the major reasons contributing to this crisis, see for instance, "*Recipe for Disaster: The Formula That Killed Wall Street*" (Wired Magazine, 2009), "*Wall Street Wizards Forgot a Few Variables*" (New York Times, 2009), and "*The Formula That Felled Wall Street*" (The Financial Times, 2009).

substantial improvement of credit quality may cause a significant drop of default correlation, whereas a deterioration of credit quality may lead to a considerable increase of default correlation (Zhou, 2001). Second, the default correlation also varies with systemic risk factors, such as the state of the economy in the business cycle and the conditions of the financial market (Crouhy et al., 2000).

In this paper, we apply a novel multivariate econometric framework to model the default correlation and assess the joint default probability of UK banks following Lucas et al. (2014). Specifically, we use a dynamic asymmetric copula model which combines the generalized hyperbolic skewed t (hereafter GHST) copula with the generalized autoregressive score (hereafter GAS) model. Our model is able to capture all the empirical features of univariate and multivariate financial time series, such as heavy-tailedness, skewness, time-varying volatility, multivariate asymmetries and dynamic dependence. This dynamic framework is closely related to two strands of literature on copula modeling. One strand of literature focuses on modeling multivariate asymmetries using the GHST copula, see for example Demarta and McNeil (2005), Smith et al (2012), Christoffersen et al. (2012) and Christoffersen and Langlois (2013), among others. Another strand of literature which uses the GAS model to capture the dependence dynamics, is pioneered by Creal et al. (2013). It has several attractive econometric properties and therefore has become increasingly popular in empirical finance studies in recent years, see for instance, Creal et al. (2014a), Janus et al (2014), Lucas et al. (2014) and Salvatierra and Patton (2015). There are two clear advantages of this dynamic asymmetric copula framework. First, it allows for non-zero tail dependence and multivariate asymmetries. Second, the time-varying nature of default correlation can be captured well by the GAS process.

Given the joint probability of default, we can further estimate the conditional default probabilities under "what if" circumstances. Inspired by Lucas et al. (2014), we further investigate the default risk of one bank given a credit event occurring in another bank. From the perspective of financial institutions and authorities, a robust stress testing and monitoring framework are obviously useful for quantifying the interaction and contagions of corporate credit risks during the crisis.

In addition, the ongoing debate on the source of banking credit risk also motivates us to investigate if the tail dependence have explanatory and predictive power to joint and conditional default risk for the top-tier banks in UK markets. Although the determinants of credit spreads have been extensively studied by both theoretical and empirical finance literature,[3] research on determinants of joint and conditional credit risk is very limited. This is important because understanding determinants of systemic credit risks of banks can not only help us explain the time-variation of default risk, but also improve the predictive accuracy of joint and conditional probabilities of default in the future.

We make five empirical contributions to the literature:

First, differently from existing literature on the joint credit risk of UK banks, such as Li and Zinna (2014), we document two important features of CDS spreads for UK top tier banks: multivariate asymmetry and dynamic dependence. Using threshold correlation and a model free test proposed by Hong et al. (2007), we find there is no significant linear asymmetries between CDS spreads of UK banks. However, significant asymmetries are found by performing a test based on the tail dependence in Patton (2012). In addition, we also apply several widely used structure break tests and identify the presence of dynamic dependence of CDS spreads. These documented features provide us with strong motivation to consider a econometric model which is able to accommodate them.

Second, we apply a novel dynamic asymmetric (i.e. GHST) copula framework to capture the variation of credit dependence between banks. Differently from the copula literature on CDS market, such as Christoffersen et al. (2013) and Lucas et al. (2014), we consider not only a full parametric method, but also a semiparametric dynamic copula framework that relies on fewer amounts of distributional assumptions, see Chen and Fan (2006a) and Chen and Fan (2006b). Surprisingly, we find that semiparametric copula models slightly underperform compared with a full parametric copula. We attribute this result to the better fitness provided by the univariate skewed t distribution in full parametric modeling. In general, we find the dynamic asymmetric copula outperforms the dynamic model based on the Gaussian or Student's t copula, as our framework is able to capture the multivariate asymmetry and dependence dynamics simultaneously. In addition, from the copula implied default correlation, we find that correlation between banks dramatically increases during times of stress and gradually decreases after 2013.

[3]Important contributions include Merton (1974), Collin-Dufresne et al. (2001), Campbell and Taksler (2003), Ericsson et al. (2009) and Christoffersen et al. (2013), among many others.

Third, we perform a copula-based simulation algorithm to estimate the joint default probability of UK top tier banks. Our empirical results show that the joint probability of default estimated by dynamic asymmetric copula is higher than the probability estimated by dynamic Gaussian or Student's t copula in most of the time during our sample period. This indicates that the Gaussian or Student's t copula-based models may underestimate the potential risk as neither of them can accommodate the multivariate asymmetries between credit risk of banks. Using marginal and joint probability, we also investigate the conditional probability of default under a hypothetical adverse market scenario.

Fourth, we find that the joint credit risk of UK banks implied by the copula model has dramatic variation during 2007-2015. The joint probability of default remarkably increases during the global financial crisis, Eurozone debt crisis and after the downgrade of US sovereign debt. In addition, our result also implies that the monetary policy implemented by the Bank of England and European Central Bank also significantly affect the joint credit risk of UK banks.

Fifth, we perform an extensive regression analysis to investigate two questions: (1) Whether the tail dependence of CDS spreads between banks implied by dynamic copulas are related to the their joint and conditional probabilities of default; (2) Whether the tail dependence can provide useful information to predict future joint and conditional probabilities of default. We find that the tail dependence contains useful information which not only explains the contemporaneous joint and conditional default probability but also predicts the future risk of joint default in the bank industry.

The remainder of the paper is organized as follows. In Section 4.2, we introduce the model for marginal default probability and the dynamic asymmetric copula model. Section 4.3 presents the main empirical results on asymmetric and dynamic dependence, marginal default probability, joint default probability estimates and conditional default probability. Section 4.4 contains a regression analysis to investigate if the estimated dependence measures (i.e. the lower and upper tail dependence) are related to cross-sectional joint probabilities of default as well as conditional probabilities of default between banks. We further investigate if the time-varying tail dependence have predictive power of future joint and conditional default probability. Section 4.5 concludes.

4.2 Model Specification

4.2.1 Dynamic Model of CDS Spread

First, we define an observed d-dimensional time series vector $y_{i,t} \in \mathbb{R}^d$, $t = 1,...,T$, as the log-difference of weekly CDS spread. In order to obtain the standardized residuals for consistent modeling of dependence dynamics, we consider an ARMA(1,1) model to capture the variation of conditional mean of the log-differenced CDS spreads,[4]

$$y_{i,t} = c_i + \varphi_i y_{i,t-1} + \theta_i \varepsilon_{i,t-1} + \varepsilon_{i,t} \tag{4.1}$$

where $\varepsilon_{i,t}$ is assumed to be independent of $y_{i,t}$ and thus the conditional mean is $\mu_{i,t} = c_i + \varphi_i y_{i,t-1} + \theta_i \varepsilon_{i,t-1}$. Second, the conditional variance is fitted by the GJR-GARCH(1,1,1) model, which is able to capture the asymmetric volatility clustering (see Glosten et al., 1993).

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \delta \varepsilon_{i,t-1}^2 I_{i,t-1} \tag{4.2}$$

where ω_i, α_i and β_i are constrained to be positive and the indicator function $I_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$ and $I_{i,t-1} = 0$ if $\varepsilon_{i,t-1} > 0$. The standardized residual is given by $z_{i,t} = (y_{i,t} - \mu_{i,t})/\sigma_{i,t}$. For the parametric modeling, we assume that $z_{i,t}$ follows the univariate skewed t distribution F_{skew-t} from Hansen (1994) to accommodate its skewed and heavy-tailed features. For the semiparametric modeling, we use the empirical distribution function $\hat{F}_i$ to transfer $z_{i,t}$ to the uniformly distributed probabilities.[5]

Then we further investigate asymmetric and time-varying default dependence across different banks in US and EU markets. The linear asymmetry can be tested using a Wald test proposed in Hong et al. (2007) and the tail dependence asymmetry can be tested by the bootstrap method in Patton (2012). The time-varying nature of default dependence between banks can be examined by the structural break tests used in Patton (2012). Verifying these

[4]We first consider all the possible models nested within the ARMA(2,2) and choose the optimal order according to the Bayesian Information Criterion (BIC). It turns out that for most banks, ARMA(1,1) gains the smallest BIC.

[5]More details of semiparametric copula model can be found in Chen and Fan (2006a) and Chen and Fan (2006b)

properties of CDS spreads in the banking sector will provide strong motivation and support for using dynamic asymmetric copula models.

4.2.2 Calibrating the Marginal Default Probability Curve

It is essential to estimate the firm-specific default probability before computing the joint default probability of selected banks. The firm-specific default probabilities can be estimated from the observed spreads of corporate CDS using conventional way. First, we define the default probability function $F_i(t)$ of bank i at time t by

$$F_i(t) = \mathbb{P}(\tau \le t) = 1 - \mathbb{P}(\tau > t) = 1 - Q(t), t \ge 0 \tag{4.3}$$

where the τ denotes the time to default (survival time). $Q_i(t)$ is the survival function, which is defined in terms of a piecewise hazard rate $\lambda(t)$

$$Q_i(t) = \exp\left(-\int_t^{t_N} \lambda(s)\, ds\right) \tag{4.4}$$

More discussions and proofs of hazard rate function can be found in Appendix 4.A. Following the "bootstrapping" method[6] described in Hull and White (2000a), O'Kane and Turnbull (2003) and O'Kane (2008), the risk neutral default probabilities can be calculated by inverting a CDS pricing formula.[7] Defining the time to default $\tau = T - t_V$, we have

$$Q_i(t,T) = \begin{cases} \exp(-\lambda_{0,1}\tau) & \text{if } 0 < \tau < 1 \\ \exp(-\lambda_{0,1}, -\lambda_{1,3}(\tau - 1)) & \text{if } 1 < \tau < 3 \\ \exp(-\lambda_{0,1}, -2\lambda_{1,3} - \lambda_{3,5}(\tau - 3)) & \text{if } 3 < \tau < 5 \\ \exp(-\lambda_{0,1}, -2\lambda_{1,3} - 2\lambda_{3,5} - \lambda_{5,7}(\tau - 5)) & \text{if } 5 < \tau < 7 \\ \exp(-\lambda_{0,1}, -2\lambda_{1,3} - 2\lambda_{3,5} - 2\lambda_{5,7} - \lambda_{7,10}(\tau - 7)) & \text{if } \tau > 7 \end{cases}$$

[6] Here, the bootstrap approach is different from the concept of the bootstrap used in statistics. It is an iterative process to construct the default probability curve using CDS market quotes. This method has been widely used in financial practice because of its computational simplicity and stability.

[7] More details of premium leg, protection leg and breakeven quotes of CDS can be found in Appendix 4.B and 4.C.

where $Q_i(t,T)$ denotes the arbitrage-free survival probability of the reference entity to time T conditional on surviving to time t_V, λ_{t_0,t_1} denotes the hazard rate from time t_0 to t_1. Given the market quotes of CDS spreads $S_1,\ldots,S_N$ at market dates $t_1,\ldots,t_N$, we can calibrate the hazard rate $\lambda(t)$ and calculate the default probability $p_{i,t} = F_i(t)$. We provide a detailed bootstrapping algorithm in Appendix 4.D.

4.2.3 Generalized Hyperbolic Skewed *t* Copula Model

Copula has been widely applied in default dependence modeling in the financial industry since Li (2000). The existence of multivariate asymmetry and time-variation of default dependence between CDS spreads necessitates the usage of a new class of dynamic asymmetric copula models. Following the study of Christoffersen et al. (2012), Christoffersen and Langlois (2013) and Lucas et al. (2014), we apply an asymmetric copula framework implied by the generalized hyperbolic (GH) skewed t distribution discussed in Demarta and McNeil (2005). This distribution belongs to the class of multivariate normal variance mixtures and has the stochastic representation

$$\mathbf{X} = \mu + \gamma W + \sqrt{W}\mathbf{Z} \tag{4.5}$$

for a d-dimensional parameter vector γ. Further, W is a scalar valued random variable following an inverse gamma distribution $W \sim IG(\nu/2, \nu/2)$ and $\mathbf{Z}$ is a d-dimensional random vector following a normal distribution $\mathbf{Z} \sim N(\mathbf{0}, \Sigma)$ and is independent of W (see Demarta and McNeil, 2005).

The density function of multivariate GH skewed t distribution is given by

$$\mathbf{f}_{skt}(\mathbf{z};\gamma,\nu,\Sigma) = \frac{2^{\frac{2-(\nu+d)}{2}} K_{\frac{\nu+d}{2}}\left(\sqrt{(\nu + \mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^*)\,\gamma'\Sigma^{-1}\gamma}\right) e^{\mathbf{z}^{*\prime}\Sigma^{-1}\gamma}}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{\frac{d}{2}}|\Sigma|^{\frac{1}{2}}\left(\nu + \mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^*\right)^{\frac{-\nu+d}{2}}\left(1 + \frac{1}{\nu}\mathbf{z}^{*\prime}\Sigma^{-1}\mathbf{z}^*\right)^{\frac{-\nu+d}{2}}} \tag{4.6}$$

where K_λ, ν and γ denote the modified Bessel function of the third kind, the degree of freedom and skewed parameter vector, respectively. The density of multivariate GH skewed t converges to the conventional symmetric t density when γ tends to $\mathbf{0}$. For the parametric case, we define the shocks $z^*_{i,t} = F^{-1}_{skt,i}(u_{i,t}) = F^{-1}_{skt,i}\left(F_{skew-t,i}(z_{i,t})\right)$ where $F^{-1}_{skt,i}(u_{i,t})$ denotes the inverse cumulative distribution function of the univariate GH skewed t distribution and it is not known in closed form but can be approximated well via simulation. $F_{skew-t,i}$ denotes the cumulative distribution function of skewed t distribution in Hansen (1994). Note that we use $z_{i,t}^*$ not the standardized return $z_{i,t}$. For the nonparametric case, we use the EDF to obtain the estimate of $u_{i,t}$. A more detailed discussion can be found in Christoffersen et al. (2012).

The probability density function of the GHST copula defined from above multivariate GH skewed t density of Eq. 4.6 is given by

$$c_{skt}(\mathbf{z};\gamma,\nu,\Sigma) = \frac{2^{\frac{(\nu-2)(d-1)}{2}} K_{\frac{\nu+d}{2}}\left(\sqrt{\left(\nu+\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^*\right)\gamma'\Sigma_t^{-1}\gamma}\right) e^{\mathbf{z}^{*\prime}\Sigma_t^{-1}\gamma}}{\Gamma\left(\frac{\nu}{2}\right)|\Sigma|^{\frac{1}{2}}\left(\nu+\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^*\right)^{\frac{-\nu+d}{2}}\left(1+\frac{1}{\nu}\mathbf{z}^{*\prime}\Sigma_t^{-1}\mathbf{z}^*\right)^{\frac{-\nu+d}{2}}} \times \prod_{i=1}^{d}\frac{\left(\sqrt{\left(\nu+(z_i^*)^2\right)\gamma_i^2}\right)^{-\frac{\nu+1}{2}}\left(1+\frac{1}{\nu}(z_i^*)^2\right)^{\frac{\nu+1}{2}}}{K_{\frac{\nu+1}{2}}\left(\sqrt{\left(\nu+(z_i^*)^2\right)\gamma_i^2}\right)e^{z_i^*\gamma_i}} \tag{4.7}$$

where Σ_t is the time-varying covariance matrix. Specifically, $\Sigma_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$, where $\mathbf{D}_t$ is an identity matrix in copula modeling and $\mathbf{R}_t$ is the time-varying correlation matrix. So we only need to model the correlation matrix $\mathbf{R}_t$ in this case. Figure 4.1 illustrates the differences between the Gaussian copula, t copula and GHST copula in dependence modeling.

[INSERT FIGURE 4.1 ABOUT HERE]

4.2.4 GAS Dynamics

Next, we investigate the evolution of dependence structure of credit risk across UK top tier banks during our sample period. The dynamics of copula correlation matrix $\mathbf{R}_t$ are driven by the Generalized Autoregressive Score (GAS) model of Creal et al. (2013) and Lucas et al. (2014). We assume that the correlation parameter δ_t is dynamic and is updated as function of its own lagged value and the standardized score of the log-likelihood.[8] To make sure that it always lies in a pre-determined range e.g. $\delta_t \in (-1,,1)$, the GAS model utilizes a strictly increasing transformation. Following Patton (2012) and Lucas et al. (2014), the transformed

[8]In the baivariate case, the copula correlation is a scalar and it can be obtained from the correlation matrix $\mathbf{R}_t = \begin{bmatrix} 1 & \delta_t \\ \delta_t & 1 \end{bmatrix}$ estimated in Section 4.2.3. When the dimension is three or above, we assume that $\Sigma_t = \Sigma_t(\delta_t) = \mathbf{L}(\delta_t)\mathbf{L}(\delta_t)' = \mathbf{D}_t(\delta_t)\mathbf{R}_t(\delta_t)\mathbf{D}_t(\delta_t)$, see more detailed discussion in Lucas et al. (2014).

correlation matrix is denoted by g_t:

$$g_t = h(\delta_t) \Leftrightarrow \delta_t = h^{-1}(g_t) \tag{4.8}$$

where $\delta_t = (1 - e^{-g_t}) / (1 + e^{-g_t})$. Furthermore, the updated transformed parameter g_{t+1} is a function of a constant $\bar{\omega}$, the lagged transformed parameter g_t, and the standardized score of the copula log-likelihood $s_t = Q_t^{-1/2} \nabla_t$:

$$g_{t+1} = \bar{\omega} + \eta Q_t^{-1/2} \nabla_t + \varphi g_t \tag{4.9}$$

where

$$\nabla_t \equiv \frac{\partial \log c\left(u_{1,t}, ..., u_{d,t}; \delta_t\right)}{\partial \delta_t} \text{ and } Q_t \equiv \mathbb{E}_{t-1}\left[\nabla_t \nabla_t'\right].$$

where ∇_t is the score of the conditional copula density with respect to δ_t and Q_t is the scaling matrix for the score ∇_t. Since the GAS model is an observation driven model, we estimate the parameters using the maximum likelihood estimation

$$\widehat{\delta_t} = \underset{\delta_t}{\operatorname{argmax}} \sum_{t=1}^{n} \log c\left(u_{1,t}, ..., u_{d,t}; \delta_t\right) \tag{4.10}$$

The dynamic copulas are parametrically estimated using maximum likelihood estimation. When the marginal distributions are estimated using the skewed t distribution, the resulting joint distribution is fully parametric. When the marginal distribution is estimated by the empirical distribution function, then the resulting joint distribution is semiparametric.

4.2.5 Modeling Joint and Conditional Probability of Default

From the last section, we know the distribution function (marginal default probability) $p_{i,t}$ of bank i at time t; then the joint probability of default $p_{i,j,t}$ for banks i and j at time t is given by

$$p_{i,j,t} = \mathbb{P}\left(z_{i,t} > F_{i,t}^{-1}(1 - p_{i,t}), z_{j,t} > F_{j,t}^{-1}\left(1 - p_{j,t}\right)\right) \tag{4.11}$$

where $z_{i,t}$ denotes the filtered CDS spread changes of bank i at time t and $F_{i,t}^{-1}(\cdot)$ denotes the inverse univariate GHST distribution. The joint probability of exceedance can be computed using the dependence estimated by GHST copula. The conditional probability of a default for bank i given a default of bank j is defined by

$$p_{i|j,t} = \mathbb{P}\left(z_{i,t} > F_{i,t}^{-1}(1-p_{i,t}), z_{j,t} > F_{j,t}^{-1}\left(1-p_{j,t}\right) \mid z_{j,t} > F_{j,t}^{-1}\left(1-p_{j,t}\right)\right) = \frac{p_{i,j,t}}{p_{j,t}} \quad (4.12)$$

4.3 Empirical Analysis

4.3.1 Data

We use a novel dataset of weekly corporate credit default swap (CDS) spreads with different maturities (6-month, 1-year, 2-year, 3-year, 4-year and 5-year) of top-tier UK banks including Barclays, HSBC Holdings (hereafter HSBC), Lloyds Banking Group (hereafter Lloyds), Royal Bank of Scotland Group (hereafter RBS) and Standard Chartered (hereafter Standard). All the CDS contracts are denominated in Euros. The London Interbank Offered Rate (henceforth Libor) data with different maturities are also collected to calibrate the marginal default probability curve. Our data cover the period from September 7, 2007 to April 17, 2015.[9] We use weekly data to avoid non-synchronicity and other problems with daily data. All the CDS market quotations are collected from Bloomberg. For the dependence analysis, we mainly focus on 5-year CDS contracts on all banks as these contracts are the most liquid and take up the largest percentage of the entire CDS market.

Table 4.1 reports descriptive statistics and time series test results for log-differences of weekly 5-year CDS spreads across five top tier UK banks in FTSE 100 index from September 7, 2007 to April 17, 2015. The basic statistics in Panel A describe the main features of CDS spread, such as univariate asymmetry, heavy-tailness and leptokurtosis. The non-zero skewness and large value of kurtosis clearly indicate the non-Gaussian features of CDS spreads. In particular, we find that the Standard Chartered obtains the largest skewness (0.793) as well as the largest kurtosis (10.041). Panel B reports that the results of the Jarque-Bera test for normality, the Ljung-Box Q-test for autocorrelation, as well as the Engle's Lagrange Multiplier test for the ARCH effect. The basic statistics and the p-values of JB test show solid evidence against the assumption of normality. Also the results for the Ljung-Box Q-test and Engle's Lagrange Multiplier test indicate the necessity for modeling conditional mean and

[9]The CDS data of Standard Chartered is only available since June 27, 2008.

conditional variance before modeling the dependence structure between the CDS spread of UK banks. Table 4.2 reports the Pearson's linear correlation coefficients as well as the Spearman's rank correlation coefficients between UK banks and it indicates that the credit risk of banks are highly correlated with each other. It is worth noting that the correlations of Standard Chartered with other banks are clearly lower than the correlations between other four banks. This is possibly because the Standard Chartered does not have retail banking business in the UK, and about 90% of its profits come from the Asian, African and the Middle Eastern markets according to its annual report in 2013. Althrough HSBC and Barclays are also multinational banking and financial services companies, the UK market is still targeted as their "home market".

[INSERT TABLE 4.1 AND 4.2 ABOUT HERE]

Figure 4.2 shows the levels and conditional volatility of average CDS spread of five UK top tier banks. It indicates some important patterns regarding the CDS spread in our sample period. Panel A illustrates the trend of the average CDS spread across five top tier banks. The arrows in each figure indicate several major events in CDS market from 2007 to 2015. We can see that the occurrence of major credit events is always accompanied with the CDS spread skyrocketing. For instance, after the S&P downgrades the US sovereign debt, the average CDS spread goes up to 285 in November 2011. Panel B plots the timer series of the conditional volatility for the average CDS spread changes estimated by the GJR-GARCH of Glosten et al. (1993). First, this shows that the CDS spread is extraordinarily volatile during the financial crisis in 2008-2009. Second, it also indicates that the turbulence of CDS spreads of UK banks is closely related to the credit events in global financial market. Another worth noting fact is that conditional volatility has been stabilized since the end of the global and EU financial crisis. It is significantly smaller than volatility during crisis even when the average CDS spread shows skyrocketing after the S&P downgraded the US government debt in August 2011. It would indicate that the CDS spread was widely fluctuated during global financial crisis due to high uncertainty while it kept maintaining high CDS spread without big fluctuation since 2011.

[INSERT FIGURE 4.2 ABOUT HERE]

Table 4.3 presents the parameter estimations and results of goodness-of-fit test for univariate models, i.e. ARMA for conditional mean, GJR-GARCH for conditional volatility and skewed t distribution for standardized residuals. First, we model the conditional mean dynamics using the ARMA model up to order (2,2) and use Bayesian Information Criterion (BIC) to select the optimal order. It turns out that the ARMA(1,1) is the best candidate for all the cases except the Standard Chartered. Second, the time-varying volatility is captured by the GARCH-type model. We experiment with ARCH, GARCH and GJR-GARCH models up to order (2,2) and choose the best candidate according to the BIC. The values of BIC indicate that GJR-GARCH provides the best performance. All the leverage parameters of the GJR-GARCH model are significantly negative indicating asymmetric volatility clustering, i.e. large positive changes of CDS spread are more likely to be clustered than negative changes. This finding is consistent with the phenomenon that the CDS spread normally has sharp and continuous increase near or after the occurrence of major credit events. The bottom of Table 4.3 reports p-values from the Kolmogorov-Smirnov and Cramer-von Mises goodness-of-fit tests for the conditional marginal distributions modeling. The p-values are obtained using the simulation approach in Patton (2012). All the p-values are clearly greater than 0.05, so we fail to reject the null hypothesis that the filtered returns are well-specified by the skew t distribution of Hansen (1994).

[INSERT TABLE 4.3 ABOUT HERE]

4.3.2 Calibrating CDS-implied Marginal Default Probability

In this section, we calibrate the model using the market quotes of CDS with different maturities (6-month, 1-year, 2-year, 3-year, 4-year and 5-year) at each time t, and bootstrap the default probability term structure following the procedure proposed in Hull and White (2000a) and O'Kane and Turnbull (2003). This mark-to-market probability of default of individual firm is derived from the observed spread of CDS contract by inverting a CDS formula. Specifically, we use the Libor rate with different maturities as discount factors and assume that the recovery rate is 40% (see O'Kane and Turnbull, 2003). Following recent finance literature, such as Huang et al (2009), Black et al. (2013), Creal et al. (2014b) and Lucas et al. (2014), we consider a CDS pricing formula with no counterparty default risk. Given the assumptions above, the default intensity can be obtained using a bootstrap algorithm, see Appendix 4.D. Given the default intensity, we can also compute the probability of default for different maturities as this is just a function of the default intensity. Note that the probability we obtain is risk neutral as the bootstrap method assumes that the present value premium leg should be exactly equal to the present value of the protection leg, see a detailed discussion in Appendix 4.C.

[INSERT FIGURE 4.3 ABOUT HERE]

Figure 4.3 illustrates the risk neutral default probabilities for individual banks inferred directly from the market quotes of CDS spread. Panel A plots the bank-specific marginal probabilities of default over a one year horizon and Panel B plots the bank-specific marginal probabilities of default over a five year horizon. The market-implied default probabilities vary over time. They significantly rise after the bankruptcy of Lehman Brothers and the downgrade of US sovereign debt. After May 2012, the probabilities of default for all the banks dramatically decline and stay at a low level in the last two years.

4.3.3 Asymmetric Dependence between Credit Risk of Banks

The asymmetric dependence assets in equity, currency and energy markets have been extensively studied in empirical finance literature. In this section, we verify the existence of asymmetries in credit market. Specifically, we investigate whether the dependence between the CDS spreads of banks is asymmetric. Two methods are considered to test for the presence of asymmetric dependence: a model-free test for asymmetric correlations proposed by Hong et al. (2007) and a tail dependence-based asymmetric test described in Patton (2012).

[INSERT TABLE 4.4 ABOUT HERE]

Table 4.4 reports the results of two tests on bivariate asymmetry. Given the number of banks $n = 5$, there are $n(n-1)/2 = 10$ different pairwise combinations of banks. Panel A reports the statistics and corresponding p-values of the model-free test of Hong et al. (2007) on threshold correlation. We find that there is no statistically significant asymmetry on the threshold correlations. We compute the threshold correlations using the standardized residuals of CDS spreads. Panel B presents the estimates of the lower and upper tail dependence coefficients for the standardized residuals of CDS spreads based on the full parametric copula model. It also reports bootstrap p-values from tests on the null hypothesis that the dependence structure is symmetric (i.e. the upper and lower tail dependence coefficients are equal). Differently from the test based on the linear correlation, half of the pairs are rejected at 5% significance level showing evidence of significant difference between the upper and lower tail dependence coefficients. Interestingly, different from the asymmetries of other assets which exhibit greater correlation during market downturns than market upturns, CDS spreads have higher upper tail dependence than lower tail dependence. This is because of the nature of CDS as a credit derivative contract to insure the protection buyer against any uncertain reference loan defaulting. The spread of CDS is the cost that the protection buyer needs to pay to the protection seller in order to obtain a payoff if the loan defaults. The higher upper tail dependence of CDS market is due to the asymmetric reaction of CDS spreads to negative and positive news. The spreads normally incorporate negative news much faster than positive news, see for instance Lehnert and Neske (2006). Thus, when the credit market deteriorated sharply during the crisis, the CDS spreads (insurance costs) of firms tend to increase together more rapidly. Panel C presents the estimates of the lower and upper tail dependence coefficients based on the semiparametric copula model and the results confirm the presence of asymmetric dependence between CDS spreads of UK banks.

4.3.4 Time-varying Dependence between Credit Risk of Banks

From Figure 4.2 in Section 4.2.1, we find that the CDS spreads and volatility are time-varying. This leads us to consider a reasonable conjecture that the dependence structure between CDS spreads also may vary through time. In order to examine the presence of time-varying dependence, we consider three tests widely used in literature: (i) A simple test that examines a structure break in rank correlation at some specified point in the sample period, see Patton (2012); (ii) A test for unknown break points in rank correlation, see Andrews (1993); (iii) A generalized break test without an *a priori* point, see Andrews and Ploberger (1994).

[INSERT TABLE 4.5 ABOUT HERE]

We implement these tests for time-varying dependence between the standardized residuals of 5-year weekly CDS spreads and summary results in Table 4.5. First, without *a priori* knowledge of breaking points, we consider using naïve tests for breaks at three chosen points in sample period, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 24-Oct-2008, 24-Jun-2011, 21-Feb-2014. Second, the "Any" column reports the results of the test for dependence break of unknown timing proposed by Andrews (1993). The *p*-values in column "QA" are based on a generalized break test without an *a priori* point in (Andrews and Ploberger, 1994). In order to detect whether the dependence structures between CDS spreads of different banks significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points in rank correlation and the "US" and "EU" panels report the results for this test. Overall, the results indicate that for all the bank pairs, except for Lloyds and Standard Chartered, the null hypothesis (that there is no break point in rank correlation in the sample period) is significantly rejected by at least one test at 5%. This finding motivates us to choose the dynamic copula model instead of constant copula model in dependence modeling.

4.3.5 Joint Credit Risk of UK Banking System

The presence of multivariate asymmetry and time-varying dependence between the credit risk (CDS spreads) of UK top tier banks provides us with strong motivation to apply the GAS-based GHST copula model proposed by Lucas et al. (2014). It is able to capture all the stylized facts of CDS spread in univariate time series, the asymmetry as well as the time-varying dependence in multivariate CDS spread series. In this section, we consider this framework to estimate the dependence structure between banks and further use the estimated dependence to simulate the joint probability of default (JPD) of UK top tier banks.

[INSERT TABLE 4.6 AND 4.7 ABOUT HERE]

First, we estimate the time-varying correlation coefficients of 10 pairs of banks using the GAS-based GHST copula. For the sake of comparison, the GAS-based Gaussian copula and

t copula are also considered. Table 4.6 and Table 4.7 report the estimates for parametric and semiparametric dynamic copula models, respectively. Comparing these two tables, we find that their estimates are very close and the parametric copula models are able to provide relatively higher log-likelihood in general. This is probably due to the better fitness in univariate modeling of the skewed t distribution in Hansen (1994).

[INSERT FIGURE 4.4 ABOUT HERE]

Figure 4.4 shows the dynamic evolution of average correlation, which is averaged across time-varying correlations of 10 bivariate pairs. We plot the time-varying average correlation implied by the GAS-Gaussian copula, GAS-Student's t copula and GAS-GHST copula. For each model specification, we use the same standardized residuals obtained in Section 4.3.1. The figure shows that the average correlation significantly increased during the crisis. It goes up to over 0.9 during the global financial crisis in 2008 and has a remarkable decrease after 2013. Notice that the sudden decreases of correlations on June 27, 2008 are caused by the fact that we include the data of the Standard Chartered, which has much lower average correlation with other banks. We find that the correlation implied by the GHST copula is greater than the correlations implied by the Gaussian and Student's t copulas. This result is in accordance with the properties and connection of these copula models. Specifically, the Gaussian copula is nested in the Student's t copula, as the Student's t copula asymptotically approaches the Gaussian copula when the degree of freedom increases. Unlike the Gaussian copula, the Student's t copula is able to take into account the tail dependence, which is of particular importance in risk modeling. Further, the Student's t copula is also nested in the GHST copula. The GHST copula is equivalent to the t copula when its skewness parameter is equal to zero. The skewness parameter enables the GHST copula to capture the multivariate asymmetry in dependence modeling. Thus, the GHST copula is a more general framework which is able to capture the tail dependence as well as the asymmetric dependence.

[INSERT FIGURE 4.5 ABOUT HERE]

Given the marginal probability of default for each bank, the estimated time-varying correlation matrix and the copula parameters, we can simulate the joint probability that two or more credit events occur in five UK top tier banks during the sample period. Figure 4.5 shows the market-implied joint probability of default (i.e. two or more credit events occurring) among five UK banks over a five year horizon. The probabilities are estimated based on three different multivariate models: GAS-Gaussian copula, GAS-Student's t copula and GAS-GHST copula. The arrows indicate time points of several major events in the global financial market. First, joint probability of default sharply rises during the crisis or after the major credit events take place. The highest default probability arises after S&P downgrades the US sovereign debt. Second, the joint probability is also affected by the monetary policy implemented by the Bank of England and European Central Bank, and gradually decreases after the cut of interest rate. Third, compared with the GHST copula, the Gaussian and Student's t copula tend to underestimate the joint probability of default most of the time.

4.3.6 Conditional Risk of UK Banks

Given the joint probability of default and the marginal default probability of individual banks, we can further investigate the conditional probability of default under a hypothetical adverse market scenario. Recently, the Bank of England published a new document to list all the key elements of stress testing for UK banks. Counterparty default is considered as a key risk of many traded risk scenarios in this document, because a large amount of risk exposures to individual counterparties is contained in the banks' trading books (Bank of England, 2015). In our case, we estimate the conditional probability of default for top tier banks in UK assuming that a default event occurs in one of them. We consider a hypothetical scenario that a credit event happens in RBS and estimate the default probabilities of other banks conditional on this adverse market scenario. The reason that we choose RBS instead of other banks is because RBS has the highest average market-implied default probability among five selected banks (0.1287).

[INSERT FIGURE 4.6 ABOUT HERE]

Figure 4.6 shows conditional probability of default for four UK top tier banks assuming a credit event of RBS. The conditional probabilities are estimated by the Gaussian copula, Student's t copula and GHST copula models. First, we find that the GHST copula obtains highest conditional probability estimates and the Gaussian copula model has the lowest estimates. The conditional probabilities estimated by the Gaussian copula are about 17% (or 24%) lower those estimated by the Student's t copula (or the GHST copula). This is because the Gaussian copula is unable to model tail dependence and neither the Student's t copula nor the Gaussian copula are able to take into account the asymmetries of dependence. Second, the probability estimates sharply increase during the financial crisis. In addition, the credit risk (default probability) also remarkably increases after S&P downgrades the US sovereign debt.

In this section, first we provide a brief analysis based on the summary statistics of CDS spreads of five top-tier banks listed in FTSE 100 and we find that occurrence of major credit events is always accompanied with the significant rise of CDS spread. Also, we find that the volatility of CDS spreads is extremely fluctuate during the global financial crisis but the volatility tends to be stabilized in recent years. Second, we calibrate the CDS-implied marginal default probability for each bank using its CDS market quotes. Third, we document two important features of dependence structures between CDS spreads: asymmetry and time-variation. Given the presence of multivariate asymmetries and time-varying dependence, we apply a dynamic asymmetric (GHST) copula model to compute the joint and conditional probability of default of UK banks. Our results show that the default probabilities estimated by the GHST copula are higher than the probabilities estimated by Gaussian or t copula implying that the Gaussian or t copula-based mode may underestimate the default probabilities due to their symmetric natures.

4.4 Further Analysis of Asymmetry

In this section, we further study how the asymmetric dependence structures (the upper and lower tail dependence coefficients) play in measuring and predicting joint (or conditional) default probability using insightful regression analysis. This analysis would further support the empirical findings in the previous section.

4.4.1 Asymmetry and Joint Default Probability

Let the joint default probability of bank i and j be $p_{i,j,t}$ and their tail dependence be $\lambda_{i,j,t}$ at time t. We define the joint default probability by Equation (4.11) and we compute average joint probability of default $\bar{p}_{i,j,t}$ implied by six GAS-based copula models, including parametric and semiparametric Gaussian, Student's t and GHST copulas. We test the impact of the asymmetric dependence on the joint default probability. We measure the asymmetric dependence using lower and upper tail dependence implied by both parametric and semiparametric dynamic asymmetric copula models. Following McNeil et al. (2005) the lower tail dependence (LTD) is defined by

$$\lambda_{i,j,t}^{LL} = \lim_{q \to 0+} \frac{C_t(q,q)}{q}, \tag{4.13}$$

and the upper tail dependence (UTD) is defined by

$$\lambda_{i,j,t}^{UU} = \lim_{q \to 1-} \frac{1-2q+C_t(q,q)}{1-q}. \tag{4.14}$$

We estimate the bivariate dynamic GHST copula model across all possible pairs of banks. For each pair, we compute average tail dependence $\bar{\lambda}_{i,j,t}^{LL}$ and $\bar{\lambda}_{i,j,t}^{UU}$ by taking average of parametric and semiparametric tail dependence coefficients. Then, we regress $\bar{p}_{i,j,t}$ on $\bar{\lambda}_{i,j,t}^{LL}$ and $\bar{\lambda}_{i,j,t}^{UU}$, and test the impact of LTD and UTD on the joint default probability. Hence, the regression equation is

$$\bar{p}_{i,j,t} = \alpha + \beta_{i,j}^{LL} \bar{\lambda}_{i,j,t}^{LL} + \beta_{i,j}^{UU} \bar{\lambda}_{i,j,t}^{UU} + \varepsilon_{i,j,t}. \tag{4.15}$$

Panel A of Table 4.9 presents the regression results of Equation (4.15).

We consider three possible estimators with panel data: (i) pooled OLS (POLS); (ii) fixed effects (FE); (iii) random effects (RE). First, we test the existence of fixed effects by comparing POLS and FE. We perform the F-test under the null of no fixed effects and reject the F-test statistic. Hence, we should consider the fixed effects in the regression to get consistent and efficient results. Second, we test if regressors are correlated with the fixed effects using Hausman approach. The Hausman test statistic is also rejected. Thus FE is consistent while RE is inconsistent. We interpret the estimation results based on FE.

We find that $\hat{\beta}^{LL}$ is insignificant while $\hat{\beta}^{UU}$ is significantly positive, and $\hat{\beta}^{UU} > \hat{\beta}^{LL}$. This implies that a higher upper tail dependence from credit deterioration is closely associated with a higher joint default risk. We also find that both tail dependences can explain the variation of the conditional default probability by 12.1%. The regression results show that the upper tail dependence is more informative than the lower tail dependence. The upper tail plays an important role for measuring the joint default probability.

Next, we test the predictability of the tail dependence. To this end, we included a lagged tail dependence coefficients as regressors in the regression equation:

$$\bar{p}_{i,j,t} = \alpha + \beta^{LL}\bar{\lambda}^{LL}_{i,j,t-k} + \beta^{UU}\bar{\lambda}^{UU}_{i,j,t-k} + \varepsilon_{t,j,t}. \tag{4.16}$$

We estimate (4.16) by FE for $k = 1, \ldots, 5$ and report results in Panel B.

Only $\hat{\beta}^{UU}$ is significant and positive for all lags and its magnitude is slowly decreased as the lag increases. On the other hand, $\hat{\beta}^{LL}$ is insignificant for all lags. R^2 is also slowly decreased as the lag increases. It ranges from 0.119 (first lag) to 0.116 (fifth lag). Hence, both the current and lagged upper tail dependences contains a significant and strong signal for the future joint default risk.

[INSERT TABLE 4.9 ABOUT HERE]

In sum, the upper tail dependence contains useful information which not only explains the current joint default probability but also predicts the future risk of joint default of banks. Thus modeling of asymmetric tail dependence between banks can improve the accuracy of measuring and forecasting the joint default risk of banks.

4.4.2 Asymmetry and Conditional Default Probability

We apply the regression analysis to the conditional default probability. We regress $\bar{p}_{i|j,t}$ on $\bar{\lambda}^{LL}_{i,j,t}$ and $\bar{\lambda}^{UU}_{i,j,t}$ and test the impact of lower and upper tail dependences between bank i and j on the conditional default probability. The regression equation is thus given by

$$\bar{p}_{i|j,t} = \alpha + \beta^{LL}_{i,j}\bar{\lambda}^{LL}_{i,j,t} + \beta^{UU}_{i,j}\bar{\lambda}^{UU}_{i,j,t} + \varepsilon_{i,j,t}. \tag{4.17}$$

Panel A of Table 4.10 presents the regression results of Equation (4.17).

We perform the F-test under the null of no fixed effects and we reject the F-test statistic. The Hausman test statistic is also rejected. FE is thus consistent while RE is inconsistent. We interpret the estimation results based on FE.

Both $\hat{\beta}^{LL}$ and $\hat{\beta}^{UU}$ are significant and positive, implying that higher dependence under extreme circumstances is closely associated with higher conditional default risk. We apply the equality test to $H_0 : \beta^{LL} = \beta^{UU}$ to test the asymmetric effect. Although quantitatively $\hat{\beta}^{LL} < \hat{\beta}^{UU}$, the null of equality is statistically not rejected. We find that both tail dependences can explain the variation of the conditional default probability by 17.4%. The estimation results therefore show that the upper tail dependence is quantitatively more informative than the lower tail dependence while it is not statistically validated. Thus we conclude that the tail dependence plays an important role for measuring the conditional default probability.

Next, we test the predictability of the tail dependence. To this end, we included a lagged tail dependence coefficients as regressors in the regression equation:

$$\bar{p}_{i|j,t} = \alpha + \beta^{LL}\bar{\lambda}^{LL}_{i,j,t-k} + \beta^{UU}\bar{\lambda}^{UU}_{i,j,t-k} + \varepsilon_{t,j,t}. \tag{4.18}$$

We estimate (4.18) by FE for $k = 1, \ldots, 5$ and report results in Panel B.

Both $\hat{\beta}^{LL}$ and $\hat{\beta}^{UU}$ are significant and positive for all lags and its magnitude is slowly decreased as the lag increases. This implies that higher dependence under extreme circumstances leads to higher conditional default risk. Quantitatively, $\hat{\beta}^{LL} < \hat{\beta}^{UU}$ for all lags but the inequality is not statistically validated. R^2 is also slowly decreased as the lag increases. It ranges from 0.170 (first lag) to 0.156 (fifth lag). These results indicate that both tail dependence coefficients contain a significant and strong signal for the future conditional default risk.

[INSERT TABLE 4.10 ABOUT HERE]

In sum, both tail dependence coefficients contain useful information which not only explains the current conditional default probability but also predicts the future risk of conditional default of banks. Although the upper tail dependence is quantitatively more informative than the lower tail dependence, it is not statistically validated. Thus the modeling of asymmetric tail dependence between banks can improve the accuracy of measuring and forecasting the conditional default risk of banks.

Consequently, we find that asymmetry contains useful information for measuing and forecasting joint (or conditional) default risk of banking system, from the regression analysis. This has two important economic implications. First, from the perspective of the credit risk modeller, asymmetric dependence should be considered in the credit risk model to avoid the past nightmare in 2007. Ignoring both multivariate asymmetries must underestimate a joint default risk of banking system, from the risk manager's (regulator's) perspective, during a crisis. Second, from the perspetive of risk manager, tail dependence coefficients should be good indicators to predict a future joint default risk of banking system. They can monitor the banking system, by keep watching if a high tail dependence is frequently observed to give an early warning to both banks and financial authorities.

4.5 Conclusion

We characterize asymmetric and time-varying dependence between CDS spreads using a new dataset on top tier UK banks. We find substantial evidence that the upper tail dependence of CDS spreads is significantly higher than lower tail dependence. Also, the results from structural break tests are strongly against the constant dependence structure over the sample period. Our findings highlight the importance of using a multivariate econometric framework to capture these documented features simultaneously. We calibrate a marginal model using market quotes of CDS to obtain the market-implied risk neutral default probability for each bank using a bootstrap algorithm and apply a novel dynamic asymmetric copula approach to model the default dependence between banks. We find that the default dependence between banks dramatically increases during times of stress and gradually decreases after 2013. Using marginal default probability and estimated copula model, we perform a simulation algorithm to obtain the joint and conditional probability of default of all the selected banks. Our empirical results show that the joint probability of default estimated by the dynamic asymmetric copula is higher than the probability estimated by the dynamic Gaussian or Student's t copula in most of the time during our sample period indicating that the Gaussian or Student's t copula-based models may underestimate the potential risk as neither of them can accommodate the multivariate asymmetries between credit risk of banks. Furthermore, we perform a panel regression analysis and find clear evidence that the time-varying tail dependence coefficients are closely related to the joint and conditional default probabilities of banks. Also, we empirically show that the time-varying tail dependence coefficients are very informative to predict future joint and conditional default risks. Overall, our empirical findings have important implications for credit risk modeling and derivative pricing in financial practice and a possible extension for further studies is to apply our framework to the firms in other sectors or markets.

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Appendix

4.A Hazard Rate Function

The hazard rate function $\lambda(t)$ is the conditional instantaneous default probability of reference entity, given that it survived until time t.

$$\mathbb{P}(t < \tau \leq t + \triangle t \mid \tau > t) = \frac{F(t + \triangle t) - F(t)}{1 - F(t)} \approx \frac{f(t)\triangle t}{1 - F(t)} \tag{4.19}$$

The association of the hazard rate function $\lambda(t)$ at time t with the default probability $F(t)$ and survival probability $S(t)$ is as follows

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = -\frac{Q^{'}(t)}{Q(t)} \tag{4.20}$$

The survival function $Q(t)$ can be defined in terms of the hazard rate function $\lambda(t)$

$$Q(t) = \exp\left(-\int_t^{t_N} \lambda(s)\, ds\right)$$

Proof:

$$Q^{'}(t) = \frac{d(Q(t))}{dt} = \frac{d(1 - F(t))}{dt} = -f(t)$$

$$\lambda(t) = -\frac{d(Q(t))}{dt}\frac{1}{Q(t)} = \frac{f(t)}{Q(t)} = -\frac{d\log(Q(t))}{d(Q(t))} \cdot \frac{d(Q(t))}{dt} = -\frac{d\log(Q(t))}{dt}$$

Taking integral on both sides

$$-\log(Q(t)) = \int_t^{t_N} \lambda(s)\, ds$$

and taking exponentials of both sides, we get

$$Q(t) = \exp\left(-\int_{t}^{t_N} \lambda(s)\, ds\right)$$

4.B Valuing the Premium Leg and Protection Leg

The premium leg is a stream of the scheduled fee payments of CDS made to maturity if the reference entity survivies or to the time of the first credit event occurs. The present value of the premium leg of an existing CDS contract is given by

$$\mathrm{PV}_{\mathrm{premium}}(t, t_N) = S_0 \cdot \mathrm{RPV01}(t, t_N) \tag{4.21}$$

$$\begin{aligned} \mathrm{RPV01}(t, t_N) &= \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B)\, Z(t, t_n)\, Q(t, t_n) \\ &+ \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, s)\, Z(t, s)\, (-dQ(t, s)),\ n = 1, \ldots, N \end{aligned} \tag{4.22}$$

where t, t_n, t_N denotes the effective date, the contractual payment dates, and the maturity date of the CDS contract, respectively. $S(t_0, t_N)$ represents the fixed contractual spread of CDS with maturity date t_N at time t_0, $\mathrm{RPV01}(t, t_N)$ represents the present value at time t of 1bp paid on the premium leg until default or maturity, whichever is sooner. $\Delta(t_{n-1}, t_n, B)$ represents the day count fraction between premium date t_{n-1} and t_n in the selected day count convention B, $Z(t, t_n)$ is the Libor discount factor from the valuation date t to premium payment date t_n and $Q(t, t_n)$ is the arbitrage-free survival probability of the reference entity from t to t_n. O'Kane (2008) show that in practice, the integral part can be approximated by

$$\int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, s)\, Z(t, s)\, (-dQ(t, s)) \backsimeq \frac{1}{2} \Delta(t_{n-1}, t_n)\, Z(t, t_n)\, (Q(t, t_{n-1}) - Q(t, t_n)) \tag{4.23}$$

Thus, it can be simplified as

$$\mathrm{RPV01}(t, t_N) = \frac{1}{2} \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B)\, Z(t, t_n)\, (Q(t, t_{n-1}) + Q(t, t_n))$$

The protection leg is the compensation that the protection seller pays to the buyer for the loss associated to a given reference entity at the time of default. It is a contingent payment of

$(100\% - R)$ on the par value of the protection when the credit event occurs. R is the expected recovery rate of the cheapest-to-deliver (CTD) obligation into the protection at the time of the credit event. So the expected present value of the protection payment is given by

$$\mathrm{PV}_{\text{protection}}(t, t_N) = (1 - R) \int_t^{t_N} Z(t, s)\left(-dQ(t, s)\right) \tag{4.24}$$

The computation of the integral part is normally tedious. Nevertheless, following O'Kane and Turnbull (2003) and O'Kane (2008), we could assume that the credit event only happens on a finite number M of several specific discrete points per year without much loss of accuracy. We can discrete the time between t and t_N into K equal intervals, where $K = \mathrm{int}(M \times (T - t) + 0.5)$. Defining $\varepsilon = (T - t)/K$, we can calculate the approximation of expected present value of the protection payment as

$$\mathrm{PV}_{\text{protection}} = (1 - R) \sum_{k=1}^{K} Z(t, k\varepsilon)\left(Q(t, (k-1)\varepsilon) - Q(t, k\varepsilon)\right) \tag{4.25}$$

Clearly, more accurate results can be obtained by increasing discrete points M.

4.C Relationship between Market Quotes and Survival Probability

In order to compute the survival probabilities from the market quote of CDS spread, it is important to understand their relationship. For a fair market trade, the present value premium leg should be exactly equal to the present value of the protection leg

$$\mathrm{PV}_{\text{premium}} = \mathrm{PV}_{\text{protection}}$$

New quotes for CDS contracts at time t_0 can be obtained by substituting and rearranging Equation 4.21 and 4.25

$$S(t_0, t_N) = \frac{(1-R)}{2} \frac{\sum_{k=1}^{K}\left(Z(t_0, t_{k-1}) + Z(t_0, t_k)\right)\left(Q(t_0, t_{k-1}) + Q(t_0, t_k)\right)}{\mathrm{RPV01}(t_0, t_N)} \tag{4.26}$$

where the RPV01 is given by

$$\text{RPV01}\left(t_0,t_N\right)=\frac{1}{2}\sum_{n=1}^{N}\Delta\left(t_{n-1},t_n,B\right)Z\left(t_0,t_n\right)\left(Q\left(t_0,t_{n-1}\right)+Q\left(t_0,t_n\right)\right)$$

4.D Bootstrapping a Survival Probability Curve

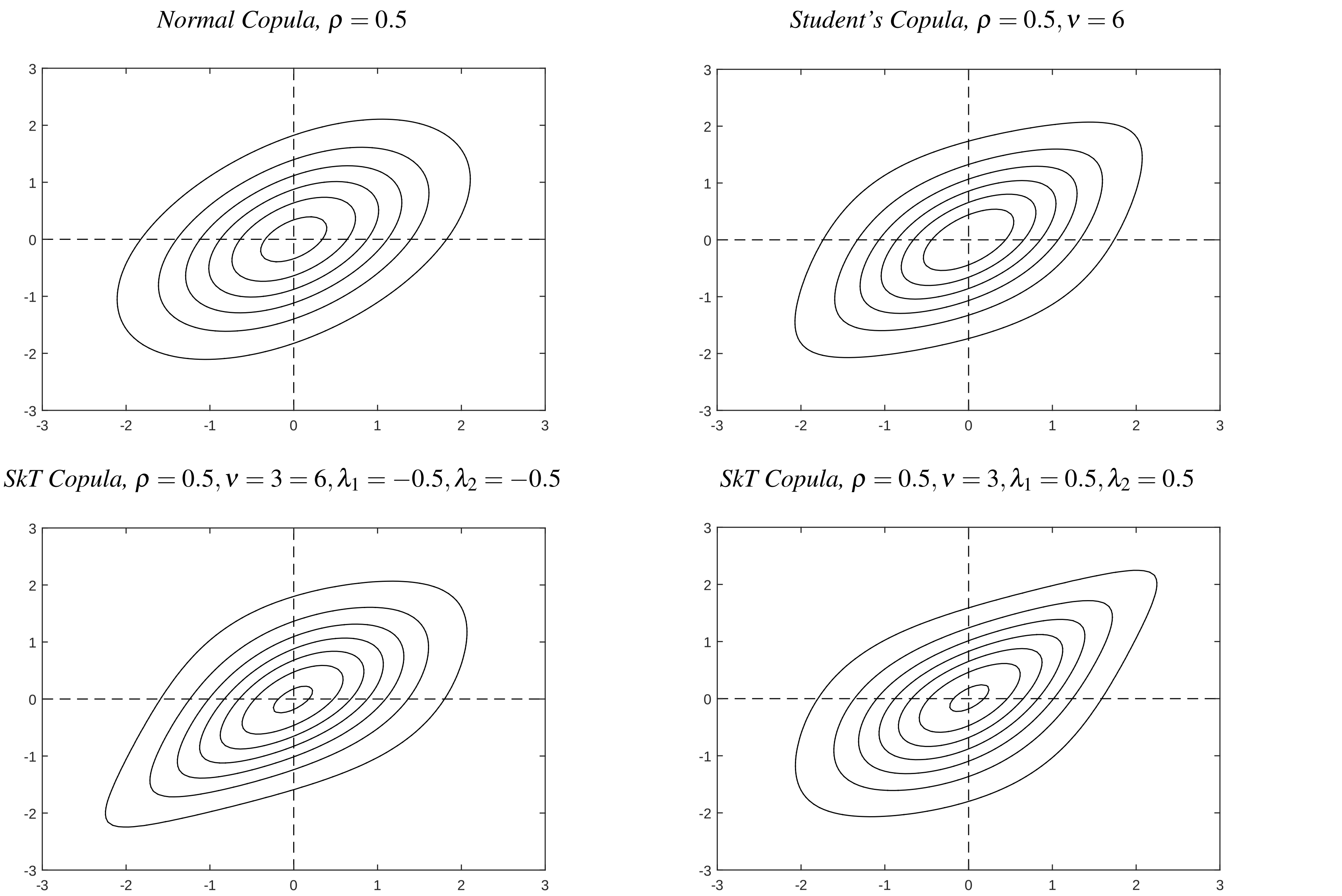
The bootstrap is a fast and stable curve construction approach, which has been widely used in financial practice as a standard method for constructing CDS survival curves. The bootstrap algorithm works by starting with shortest maturity contract and works out to the CDS contract with the longest maturity. At each step it uses the spread of the next CDS contract to solve for the next maturity survival probability and to extend the survival curve (see Hull and White, 2000a; O'Kane and Turnbull, 2003; O'Kane, 2008; Schönbucher, 2003, etc.). The default probability can be easily obtained by calculating the complement of survival probability.

First, we define the market quotes of CDS as a set of maturity dates $T_1,T_2,...,T_M$ and corresponding CDS spread $S_1,S_2,...,S_M$. All the CDS quotes are sorted in order of increasing maturity. Second, we need to extrapolate the survival curve below the shortest maturity CDS by assuming that the forward default rate is flat at a level of 0, and we also extrapolate the survival curve beyond the longest maturity T_M by assuming that the forward default rate is flat at its latest interpolated value.

The bootstrap algorithm to calculate the survival probability from CDS market quotes is as follows:

1. We initialize the first point of survival curve by defining $Q\left(T_0=0\right)=1$ and $m=1$.
2. The survival probability $Q\left(T_m\right)$ can be calculated by solving Equation (4.26). Note that the no-arbitrage bound on $Q\left(T_m\right)$ is $0<Q\left(T_m\right)\leq Q\left(T_{m-1}\right)$.
3. Given the value of $Q\left(T_m\right)$ which reprices the CDS with maturity T_m, we can extend the survival curve to time T_m.
4. Set $m=m+1$ and go back and repeat step 2 - 4 iteratively until $m\leq M$.
5. Given $M+1$ points values of survival probability $1,Q\left(T_1\right),Q\left(T_2\right),...,Q\left(T_M\right)$ at time $0,T_1,T_2,...,T_M$.

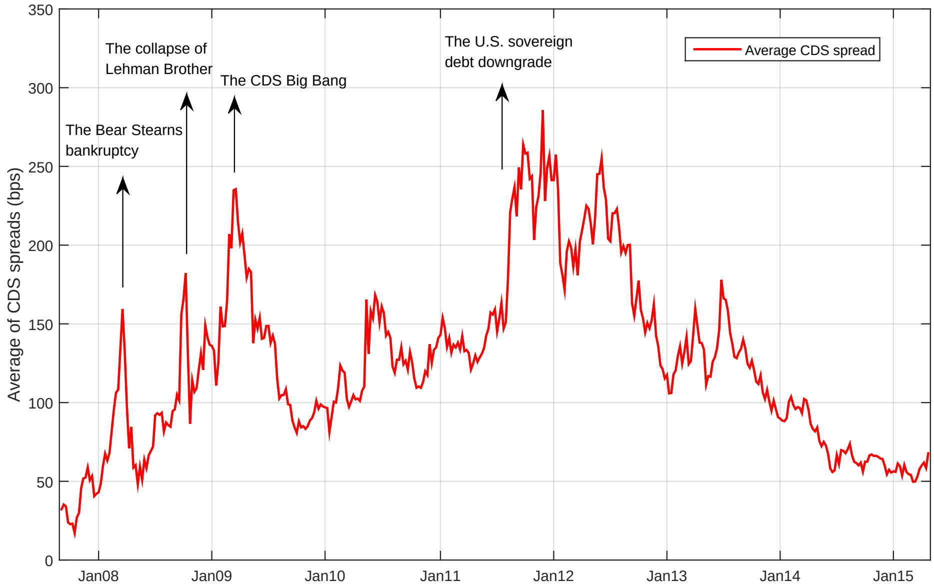
Figure 4.1 Contour Probability Plots for Copulas



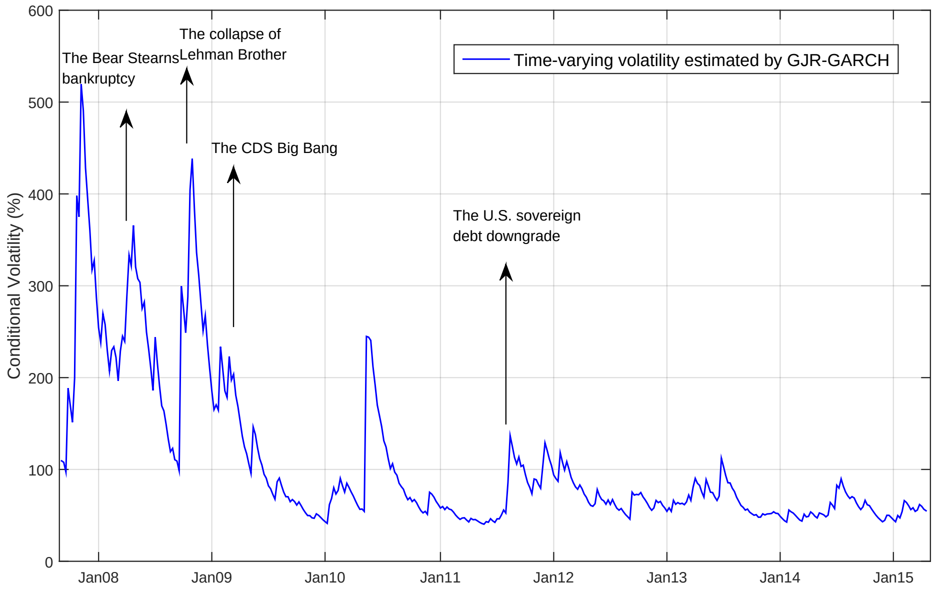
Note: This figure shows contour probability plots for the normal, Student's t, and GHST copulas. The probability levels for each contour are held fixed across four panels. The marginal distributions are assumed to be normally distributed. ρ denotes the correlation coefficient, ν denotes the degree of freedom, and λ denotes the asymmetric parameters of copulas.

Figure 4.2 Dynamics of CDS Spread from 2007 to 2015

A. Average CDS Spread



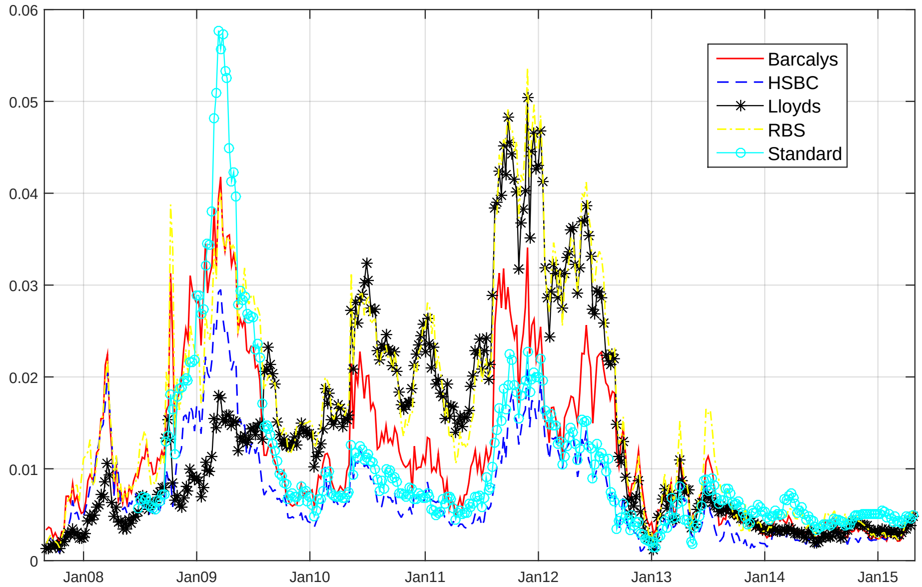
B. Conditional Volatility of Average CDS spread



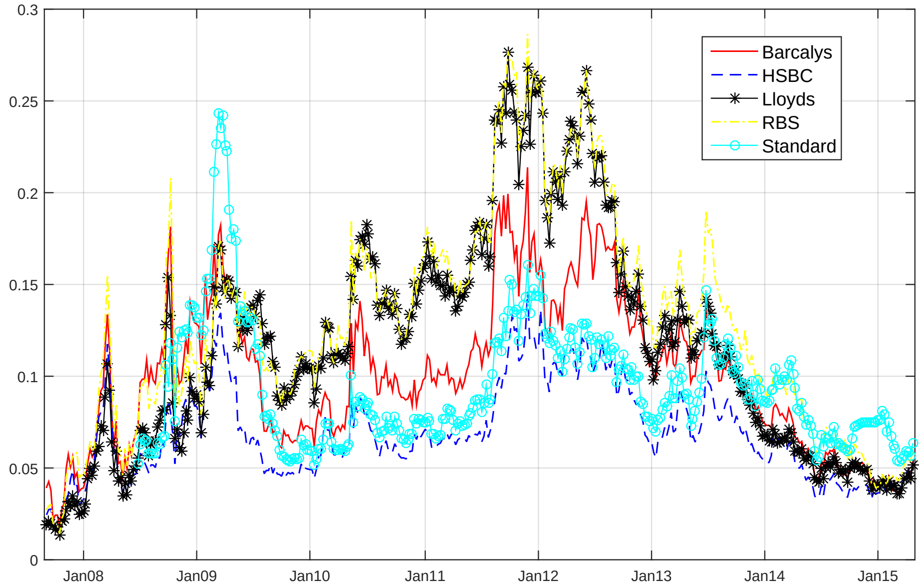
Notes: This figure shows the levels and conditional volatility of average 5-year CDS spread of five UK top tier banks from September 7, 2007 to April 17, 2015. The conditional volatility of average CDS spread changes is estimated by the GJR-GARCH(1,1,1) of Glosten et al. (1993). The arrows in each figure indicate several major events in CDS market during the sample period.

Figure 4.3 CDS-implied Marginal Risk Neutral Default Probabilities

A. Probability of Default over One Year Horizon

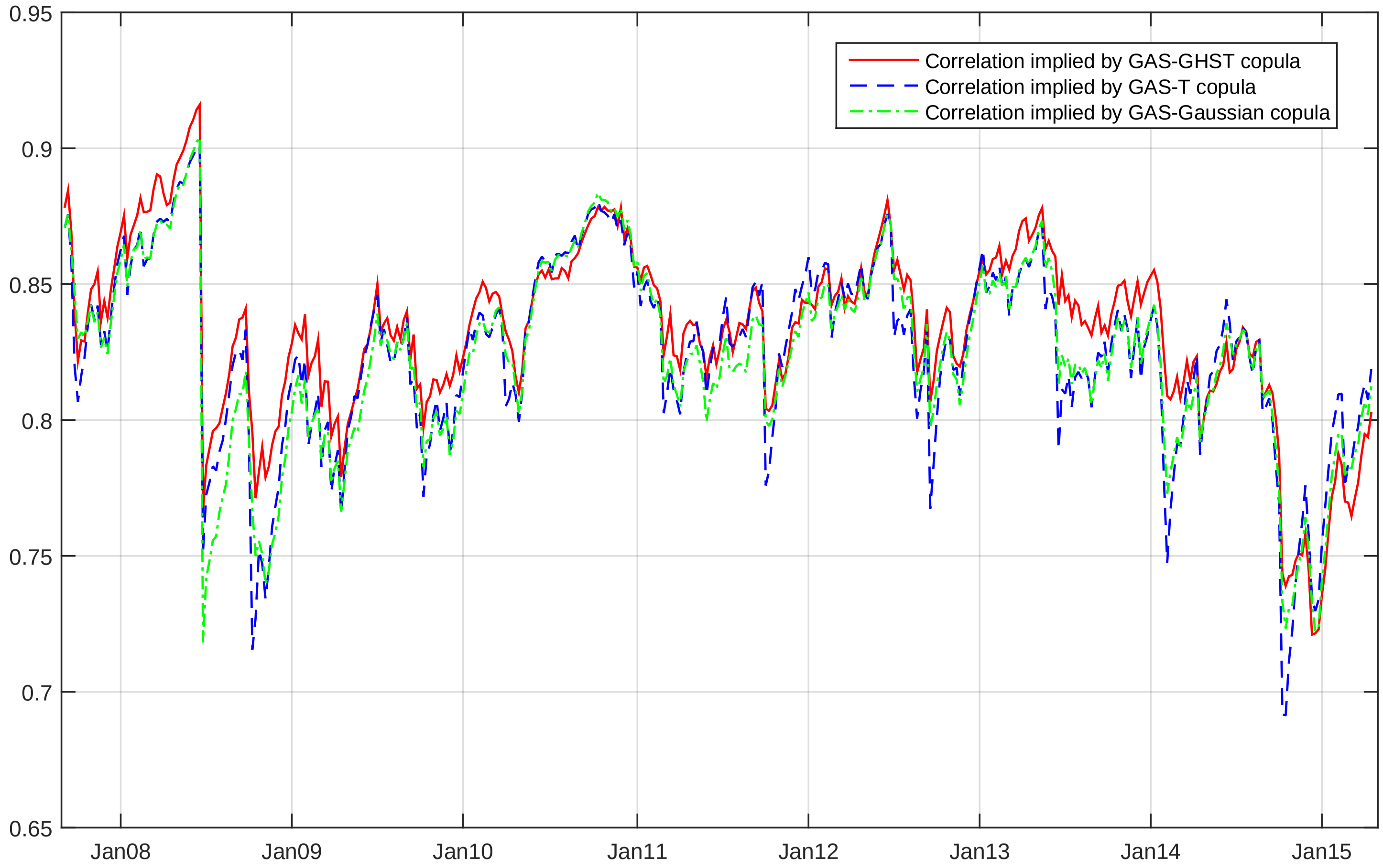


B. Probability of Default over Five Year Horizon



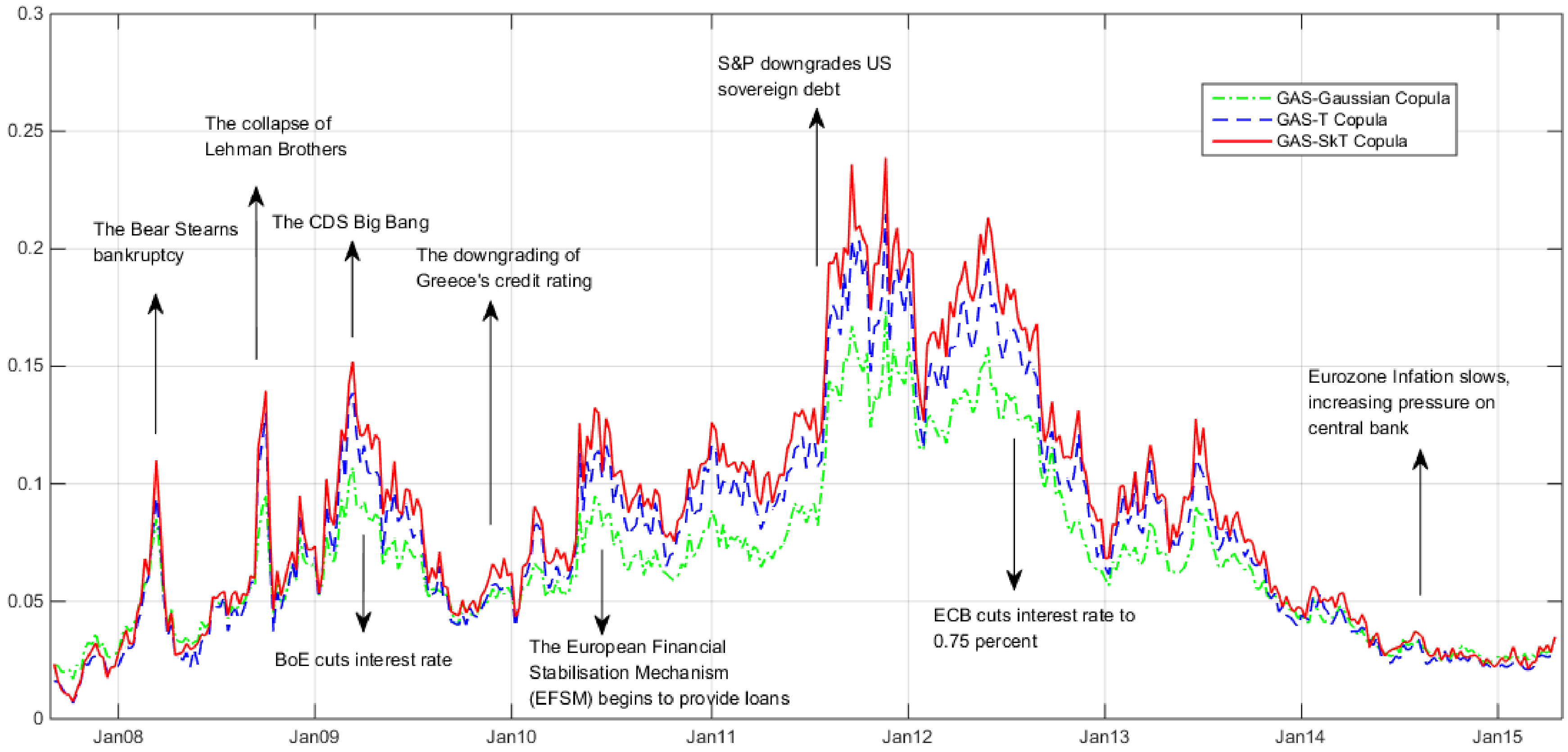
Notes: This figure plots risk neutral marginal probabilities of default for five top tier banks in UK. These probabilities are directly inferred from weekly CDS prices with different maturities using bootstrap algorithm described in Appendix 4.D. The sample period is from September 7, 2007 to April 17, 2015.

Figure 4.4 Average Correlation Implied by GAS Over Time



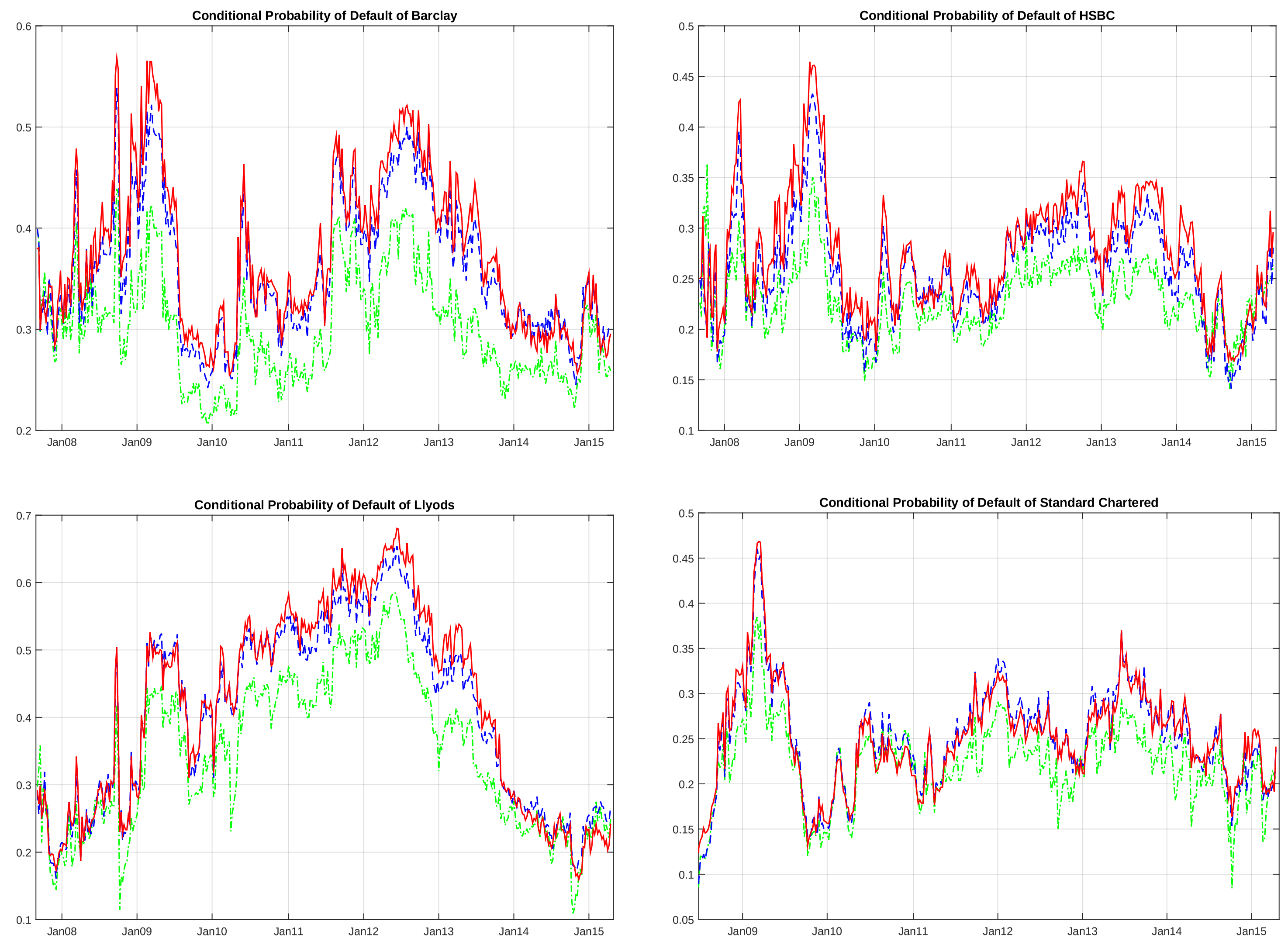
Notes: This figures shows the estimated average correlation implied by three GAS-based copula models from August 15, 2008 to April 17, 2015. The copula correlations are obtained by taking average of estimated correlation series between 10 pairs of banks. The correlation coefficients are estimated by both parametric copula and semiparametric copula. The solid line represents the time-varying correlation estimated by the GAS-GHST copula. The dashed line and dash-dot line represent the time-varying correlation estimated by the GAS-Student's t copula and GAS-Gaussian copula. The sudden decreases of correlations on June 27, 2008 are caused by the fact that we include the data of the Standard Chartered, which has much lower average correlation with other banks.

Figure 4.5 The Joint Credit Risk of UK Top-tier Banks from 2007 to 2015



Notes: This figure plots the estimated time-varying probabilities of two or more credit events over the five year horizon from September 7, 2007 to April 17, 2015. The probabilities are estimated based on three different multivariate models: GAS-Gaussian copula, GAS-Student's *t* copula and GAS-GHST copula. The solid line represents the joint probability of default estimated by the GHST copula. The dashed line and dash-dot line represent the joint probability of default estimated by the Student's *t* copula and Gaussian copula. The arrows indicate time points of several major events in global financial market.

Figure 4.6 Conditional Probabilities of Default Given the Default of RBS



Note: This figure plots the conditional probability of a credit event for four top tier UK banks given a credit event of RBS. The solid line represents the conditional probability of default estimated by the GHST copula. The dashed line and dash-dot line represent the conditional probability of default estimated by the Student's *t* copula and Gaussian copula.

Table 4.1 Descriptive Statistics and Time Series Tests on 5-year CDS Spreads

	Barclays	HSBC	Lloyds	RBS	Standard
			A. Descriptive Statistics		
Mean	0.079	0.200	0.256	0.211	0.054
Median	-0.243	-0.012	0.141	0.432	0.000
Std.	11.857	10.466	10.872	11.968	8.707
Skewness	-0.176	0.122	0.287	-0.055	0.793
Kurtosis	6.443	5.305	6.112	8.783	10.041
Max	49.320	48.432	51.173	57.738	48.906
Min	-55.131	-36.795	-44.802	-68.245	-38.349
			B. Time Series Tests		
JB test	0.000*	0.000*	0.000*	0.000*	0.000*
LB Q(12)	0.101	0.047*	0.083	0.011*	0.077
LB Q(12)^2	0.000*	0.000*	0.000*	0.000*	0.000*
LM ARCH	0.000*	0.000*	0.001*	0.001*	0.000*

Notes: This table reports descriptive statistics and time series test results for log-differences of 5-year weekly CDS spreads across five top tier UK banks in FTSE 100 index from September 7, 2007 to April 17, 2015, which correspond to a sample of 398 observations for Barclays, HSBC, Lloyds and RBS and a sample of 356 Standard Chartered (Note: The CDS data of Standard Chartered is available from June 27, 2008.). Note that the means, standard deviations, minima, and maxima are reported in %. JB test denotes the Jarque–Bera test for normal distribution. LB test lag 5 and 10 denote the *p*-values of the Ljung-Box Q-test for autocorrelation at lags 5 and 10, respectively. In addition, we report the p-values of Engle's Lagrange Multiplier test for the ARCH effect on the residual series. We use * to indicate the rejection of the null hypothesis at the 5% significance level.

Table 4.2 Correlation Matrix of Weekly Log-differences of CDS spreads

A. Linear Correlation

	Barclays	HSBC	Lloyds	RBS	Standard
Barclays	1.000				
HSBC	0.854	1.000			
Lloyds	0.873	0.847	1.000		
RBS	0.897	0.855	0.862	1.000	
Standard	0.762	0.803	0.711	0.781	1.000

B. Rank Correlation

	Barclays	HSBC	Lloyds	RBS	Standard
Barclays	1.000				
HSBC	0.835	1.000			
Lloyds	0.879	0.837	1.000		
RBS	0.885	0.840	0.879	1.000	
Standard	0.746	0.785	0.710	0.727	1.000

Notes: This table reports the correlation matrix for log-differences of 5-year weekly CDS spreads across five top tier UK banks in FTSE 100 index from September 7, 2007 to April 17, 2015, which correspond to a sample of 398 observations for Barclays, HSBC, Lloyds and RBS and a sample of 356 Standard Chartered (The CDS data of Standard Chartered is available from June 27, 2008.). Panel A reports the Pearson's linear correlation coefficients and Panel B reports the Spearman's rank correlation coefficients.

Table 4.3 Summary of ARMA-GJR-GARCH Estimation on Weekly Log-Differences

	Barclays	HSBC	Lloyds	RBS	Standard
ARMA					
$\phi 1$	-0.671***	-0.848***	-0.794***	-0.697***	-0.094*
	(0.238)	(0.114)	(0.173)	(0.203)	(0.053)
$\phi 2$	0.580**	0.791***	0.757***	0.593***	_
	(0.261)	(0.132)	(0.187)	(0.228)	_
GARCH					
ω	3.168***	3.28***	2.663***	3.361***	7.089***
	(0.099)	(1.761)	(1.683)	(1.599)	(0.149)
α	0.096***	0.101**	0.078*	0.067*	0.106***
	(0.026)	(0.047)	(0.040)	(0.036)	(0.000)
δ	-0.096***	-0.080	-0.027	-0.044	-0.107***
	(0.031)	(0.054)	(0.059)	(0.049)	(0.007)
β	0.916***	0.894***	0.905***	0.915***	0.850***
	(0.022)	(0.041)	(0.036)	(0.033)	(0.003)
SkT					
υ	5.511***	7.385***	7.179***	5.269***	3.159***
η	0.017*	0.079***	0.001	-0.012*	0.016*
KS p-value	0.83	0.94	0.22	0.23	0.16
CvM p-value	0.58	0.91	0.15	0.41	0.25

Note: This table presents the estimated parameters with p-values from the ARMA model for the conditional mean and GJR-GARCH(1,1) models for the conditional variance of log-differences of 5-year weekly CDS spread. We estimate all parameters using the sample from September 7, 2007 to April 17, 2015, which correspond to a sample of 398 observations for Barclays, HSBC, Lloyds and RBS and a sample of 356 Standard Chartered (The CDS data of Standard Chartered is available from June 27, 2008.). The values in parenthesis represent the standard errors of the parameters. We also report the p-values of two goodness-of-fit tests for the skewed Student's t distribution. KS and CvM denote Kolmogorov-Smirnov test and Cramer-von Mises test, respectively.

Table 4.4 Tests of Bivariate Asymmetry

	A. Threshold Correlation		B. Full parametric tail dependence				C. Semiparametric tail dependence			
	HTZ	p-value	Lower	Upper	Diff	p-value	Lower	Upper	Diff	p-value
B-H	16.445	0.997	0.379	0.509	-0.130	0.315	0.279	0.390	-0.111	0.468
B-L	19.702	0.983	0.287	0.543	-0.255	0.015	0.428	0.661	-0.233	0.045
B-R	12.524	0.998	0.328	0.647	-0.319	0.002	0.274	0.598	-0.324	0.004
B-S	31.214	0.652	0.299	0.220	0.079	0.462	0.305	0.282	0.023	0.867
H-L	14.342	0.999	0.217	0.239	-0.022	0.869	0.339	0.341	-0.002	0.992
H-R	15.487	0.998	0.217	0.537	-0.320	0.002	0.257	0.606	-0.349	0.013
H-S	18.250	0.991	0.511	0.153	0.358	0.003	0.559	0.171	0.389	0.015
L-R	9.652	1.000	0.312	0.538	-0.226	0.037	0.242	0.698	-0.457	0.001
L-S	41.811	0.199	0.233	0.300	-0.066	0.673	0.200	0.453	-0.253	0.109
R-S	25.528	0.879	0.291	0.167	0.125	0.244	0.220	0.186	0.035	0.747
Average	20.495		0.308	0.385	-0.078		0.310	0.438	-0.128	

Notes: This table presents the statistics and p-values from two asymmetric tests. Given the number of banks $n = 5$, there are $n(n-1)/2 = 10$ different pairwise combinations of banks. “B”, “H”, “L”, “R” and “S” denote Barclays, HSBC, Lloyds, RBS and Standard Charted, respectively. “HTZ” denotes the statistic from a model-free symmetry test proposed in (Hong et al., 2007) to examine whether the exceedance correlations between the 5-year weekly CDS spreads of different banks are asymmetric at all. “LUTD”, “ULTD” and “Diff” denote the coefficients of lower-upper tail dependence and upper-lower tail dependence estimated by Student’s t copula, and the difference between them for all the portfolios pairs. The estimations are calculated by both parametric and semiparametric approach in Patton (2012). The p-values from the tests that the low tail and upper tail dependence coefficients are computed with 500 bootstrap replications.

Table 4.5 Structural Break Test for Time-varying Dependence Structures

	0.15	0.5	0.85	Any	US	EU	QA
B-H	0.053	0.310	0.369	0.110	0.035	0.360	0.020
B-L	0.040	0.106	0.296	0.080	0.019	0.282	0.152
B-R	0.021	0.087	0.190	0.040	0.014	0.250	0.030
B-S	0.837	0.767	0.357	0.500	0.951	0.247	0.048
H-L	0.026	0.154	0.485	0.020	0.015	0.226	0.595
H-R	0.024	0.227	0.407	0.070	0.012	0.225	0.005
H-S	0.542	0.403	0.571	0.720	0.806	0.161	0.048
L-R	0.043	0.062	0.320	0.090	0.013	0.241	0.010
L-S	0.721	0.965	0.540	0.460	0.945	0.319	0.521
R-S	0.993	0.840	0.280	0.240	0.883	0.490	0.014

Notes: This table reports the p-values from tests for time-varying dependence between 5-year weekly CDS spreads changes of different banks. "B", "H", "L", "R" and "S" denote Barclays, HSBC, Lloyds, RBS and Standard Charted, respectively. Without *a priori* knowledge of breaking points, we consider using naïve tests for breaks at three chosen points in sample period, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 24-Oct-2008, 24-Jun-2011, 21-Feb-2014. The "Any" column reports the results of test for dependence break of unknown timing proposed by Andrews (1993). The p-values in column "QA" is based on a generalized break test without priori point in (Andrews and Ploberger, 1994). In order to detect whether the dependence structures between CDS spreads changes of different banks significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points in rank correlation and the "US" and "EU" panels report the results for this test. We use * and ** to indicate significance at the 5% and 1%, respectively.

Table 4.6 Full Parametric Dynamic Copula Parameter Estimation

		B-H	B-L	B-R	B-S	H-L	H-R	H-S	L-R	L-S	R-S	Joint
GAS-Gaussian	ω	0.377	0.410	0.401	0.290	0.376	0.389	0.262	0.367	0.268	0.301	0.347
		0.013	0.019	0.012	0.011	0.018	0.006	0.007	0.004	0.004	0.004	0.010
	α	0.058	0.070	0.058	0.136	0.068	0.043	0.149	0.190	0.175	0.225	0.117
		0.020	0.003	0.009	0.004	0.018	0.016	0.003	0.004	0.010	0.006	0.009
	β	0.852	0.857	0.860	0.838	0.849	0.843	0.877	0.878	0.851	0.826	0.854
		0.008	0.008	0.009	0.005	0.003	0.013	0.003	0.009	0.013	0.015	0.009
	log L	259	309	317	149	249	250	181	338	143	140	1238
GAS-T	ω	0.382	0.403	0.417	0.274	0.371	0.398	0.279	0.376	0.268	0.317	0.348
		0.011	0.010	0.008	0.009	0.012	0.005	0.007	0.005	0.008	0.012	0.009
	α	0.075	0.083	0.098	0.174	0.091	0.108	0.190	0.196	0.200	0.242	0.145
		0.015	0.003	0.002	0.005	0.018	0.004	0.004	0.003	0.014	0.003	0.007
	β	0.850	0.860	0.856	0.854	0.852	0.837	0.866	0.881	0.854	0.833	0.856
		0.003	0.014	0.004	0.005	0.018	0.011	0.003	0.016	0.010	0.006	0.009
	η^{-1}	0.215	0.200	0.186	0.168	0.201	0.200	0.166	0.178	0.211	0.166	0.189
		0.056	0.024	0.066	0.032	0.064	0.058	0.043	0.072	0.020	0.020	0.045
	log L	268	318	324	155	256	257	190	350	155	154	1296
GAS-GHST	ω	0.390	0.402	0.406	0.269	0.364	0.382	0.285	0.380	0.269	0.330	0.348
		0.001	0.008	0.013	0.008	0.012	0.006	0.003	0.005	0.009	0.011	0.008
	α	0.047	0.072	0.038	0.026	0.060	0.142	0.199	0.192	0.155	0.168	0.109
		0.008	0.003	0.003	0.006	0.017	0.004	0.003	0.003	0.003	0.007	0.006
	β	0.866	0.858	0.840	0.854	0.852	0.890	0.908	0.871	0.825	0.890	0.867
		0.004	0.016	0.007	0.004	0.013	0.013	0.005	0.020	0.005	0.006	0.009
	η^{-1}	0.210	0.200	0.185	0.164	0.192	0.190	0.168	0.177	0.192	0.165	0.184
		0.005	0.029	0.068	0.039	0.056	0.056	0.046	0.065	0.013	0.021	0.040
	λ	0.122	0.216	0.120	-0.043	0.105	0.117	-0.179	0.104	0.138	-0.119	0.127
		0.012	0.012	0.014	0.026	0.022	0.024	0.008	0.011	0.007	0.070	0.021
	log L	298	343	341	174	279	281	210	361	180	163	1412

Notes: This table reports parameter estimates for three different parametric dynamic copula models: Gaussian copula, Student's *t* copula and GHST copula. The sample period is from September 7, 2007 to April 17, 2015. ω, α and β denote the parameters of GAS model, η^{-1}denotes the inverse of degree of freedom of *t* and GHST copula, λ denotes the skewness parameter of GHST copula and log L denotes the log-likelihood of estimated copula model. The "Joint" column reports the estimates of parameters for five-dimensional copula models. Notice that we estimate this high-dimensional copula following the method described in Lucas et al. (2014).

Table 4.7 Semiparametric Dynamic Copula Parameter Estimation

		B-H	B-L	B-R	B-S	H-L	H-R	H-S	L-R	L-S	R-S	JOINT
GAS-Gaussian	ω	0.369	0.410	0.400	0.290	0.379	0.389	0.261	0.360	0.269	0.301	0.344
		0.013	0.014	0.012	0.009	0.020	0.004	0.007	0.019	0.003	0.006	0.011
	α	0.064	0.069	0.050	0.132	0.061	0.045	0.150	0.200	0.168	0.220	0.115
		0.016	0.002	0.003	0.001	0.011	0.008	0.006	0.007	0.002	0.003	0.006
	β	0.858	0.859	0.862	0.850	0.850	0.844	0.880	0.881	0.851	0.843	0.858
		0.008	0.030	0.004	0.003	0.008	0.009	0.009	0.007	0.014	0.017	0.011
	log L	255	310	312	150	248	247	183	329	140	144	1209
GAS-T	ω	0.376	0.404	0.428	0.282	0.375	0.401	0.279	0.376	0.268	0.310	0.350
		0.011	0.007	0.010	0.007	0.014	0.004	0.004	0.018	0.002	0.009	0.009
	α	0.072	0.087	0.098	0.180	0.088	0.105	0.190	0.194	0.196	0.250	0.146
		0.010	0.013	0.004	0.005	0.009	0.014	0.004	0.003	0.003	0.003	0.007
	β	0.855	0.862	0.859	0.849	0.852	0.841	0.870	0.875	0.854	0.837	0.855
		0.003	0.004	0.006	0.005	0.006	0.018	0.008	0.018	0.004	0.012	0.008
	η^{-1}	0.218	0.209	0.186	0.161	0.201	0.200	0.166	0.178	0.215	0.161	0.190
		0.070	0.025	0.061	0.047	0.072	0.012	0.070	0.017	0.018	0.013	0.041
	log L	266	321	326	155	252	254	186	342	150	148	1251
GAS-GHST	ω	0.390	0.396	0.410	0.281	0.364	0.373	0.284	0.380	0.274	0.320	0.348
		0.008	0.007	0.009	0.069	0.012	0.007	0.003	0.004	0.009	0.015	0.014
	α	0.090	0.073	0.039	0.199	0.069	0.194	0.199	0.192	0.155	0.166	0.137
		0.008	0.009	0.023	0.045	0.010	0.011	0.020	0.007	0.009	0.001	0.014
	β	0.860	0.857	0.858	0.841	0.851	0.836	0.851	0.875	0.852	0.886	0.857
		0.004	0.006	0.007	0.004	0.006	0.022	0.011	0.017	0.005	0.009	0.009
	η^{-1}	0.210	0.209	0.185	0.165	0.199	0.190	0.163	0.177	0.206	0.165	0.187
		0.007	0.026	0.063	0.048	0.070	0.009	0.064	0.020	0.012	0.306	0.062
	λ	0.119	0.177	0.141	-0.050	0.115	0.132	-0.198	0.100	0.141	-0.136	0.125
		0.015	0.016	0.012	0.025	0.026	0.020	0.009	0.014	0.008	0.005	0.015
	log L	296	351	340	171	282	287	208	358	183	168	1390

Notes: This table reports parameter estimates for three different semiparametric dynamic copula models: Gaussian copula, Student's t copula and GHST copula. The sample period is from September 7, 2007 to April 17, 2015. ω, α and β denote the parameters of GAS model, η^{-1}denotes the inverse of degree of freedom of t and GHST copula, λ denotes the skewness parameter of GHST copula and log L denotes the log-likelihood of estimated copula model. The "Joint" column reports the estimates of parameters for five-dimensional copula models. Notice that we estimate this high-dimensional copula following the method described in Lucas et al. (2014).

Table 4.8 Log-Likelihood, AIC and BIC for Model Comparisons

Portfolios	B-H	B-L	B-R	B-S	H-L	H-R	H-S	L-R	L-S	R-S	JOINT
Panel A: Log-Likelihood											
Gaussian	259	309	317	149	249	250	181	338	143	140	1238
GAS-T	268	318	324	155	256	257	190	350	155	154	1296
GAS-GHST	298	343	341	174	279	281	210	361	180	163	1412
LR test	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel B: Akaike Information Criterion (AIC)											
Gaussian	-509	-610	-626	-290	-490	-493	-354	-668	-277	-272	-2449
GAS-T	-527	-627	-638	-300	-503	-505	-369	-690	-299	-298	-2565
GAS-GHST	-584	-674	-670	-335	-545	-550	-408	-710	-348	-315	-2793
Panel C: Bayesian Information Criterion (BIC)											
Gaussian	-493	-594	-610	-274	-475	-477	-338	-652	-261	-256	-2398
GAS-T	-507	-607	-618	-280	-483	-485	-349	-670	-279	-279	-2509
GAS-GHST	-560	-650	-646	-311	-521	-526	-384	-686	-324	-291	-2733

Notes: This table presents the results of model comparison via statistical criteria. Panel A reports the log-likelihood from three GAS based copula models: Gaussian, Student's t and GHST. The bottom row of Panel A shows the p-values of likelihood ratio test for Student's t and GHST copula models. Panel B and C report the values of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for different model specifications, respectively.

Table 4.9 Joint Default Probability and Asymmetry

A. Contemporaneous Relationship

	POLS	FE	RE
LTD	0.022***	0.020	0.054***
	(0.003)	(0.054)	(0.011)
UTD	0.049***	0.174*	0.118***
	(0.002)	(0.087)	(0.010)
Cons	0.013	-0.035	-0.023
	(0.001)	(0.028)	(0.004)
R^2	0.125	0.121	0.125
T-test (LTD)	[0.000]	[0.361]	[0.000]
T-test (UTD)	[0.000]	[0.038]	[0.000]
F-test	[0.000]		
Hausman		[0.000]	

B. Predictability

	Lag(1)	Lag(2)	Lag(3)	Lag(4)	Lag(5)
LTD(-1)	0.013				
	(0.053)				
UTD(-1)	0.177**				
	(0.088)				
LTD(-2)		0.010			
		(0.054)			
UTD(-2)		0.176*			
		(0.090)			
LTD(-3)			0.008		
			(0.053)		
UTD(-3)			0.174*		
			(0.090)		
LTD(-4)				0.009	
				(0.053)	
UTD(-4)				0.165*	
				(0.092)	
LTD(-5)					0.008
					(0.053)
UTD(-5)					0.160*
					(0.092)
Cons	-0.034	0.090	-0.031	-0.028	-0.026
	(0.028)	(-0.033)	(0.028)	(0.029)	(0.029)
R^2	0.119	0.118	0.118	0.117	0.116
T-test (LTD)	[0.403]	[0.427]	[0.442]	[0.433]	[0.437]
T-test (UTD)	[0.022]	[0.025]	[0.026]	[0.036]	[0.041]

Notes: This table reports the regression analysis of the impact of the tail dependence on the joint default probability. We estimate the bivariate dynamic GHST copula models across all possible pairs of banks. For each pair, we compute average tail dependence by taking average of parametric and semiparametric tail dependence coefficients. The average joint default probability is computed by taking average of joint default probabilities estimated from six time-varying copula models (parametric and semiparametric Gaussian, Student's t and GHST copulas). In Panel A, we regress the joint default probability on the lower tail dependence (LTD) and the upper tail dependences (UTD) in Equation (4.13). We consider three panel data estimators; pooled OLS (POLS), fixed effects (FE), and random effects (RE), and choose a consistent and efficient estimator. We test the existence of fixed effects by F-test and apply Hausman approach to test if regressors are correlated with the fixed effects. T-test (LTD) (T-test (UTD)) tests the null of $\beta^{LL} = 0$ against $\beta^{LL} > 0$ ($\beta^{UU} = 0$ against $\beta^{UU} > 0$). In Panel B, we regress the joint default probability on the lagged tail dependences in Equation (4.16). We estimate regression equations by one selected from Panel A. In both panels, $[\cdot]$ reports the p-value of the test and $(\cdot)$ reports the standard error of the estimate, respectively. We use *, ** and *** to indicate the significance levels at 10%, 5% and 1%.

Table 4.10 Conditional Default Probability and Asymmetry

A. Contemporaneous Relationship

	POLS	FE	RE
LTD	0.269***	0.506***	0.532***
	(0.011)	(0.152)	(0.143)
UTD	0.216***	0.639***	0.592***
	(0.008)	(0.159)	(0.138)
Cons	0.190***	-0.045	-0.032
	(0.004)	(0.062)	(0.061)
R^2	0.179	0.174	0.177
t-test (LTD)	[0.000]	[0.002]	[0.000]
t-test (UTD)	[0.000]	[0.001]	[0.000]
F-test	[0.000]		
Hausman		[0.000]	

B. Predictability

	Lag(1)	Lag(2)	Lag(3)	Lag(4)	Lag(5)
LTD(-1)	0.477***				
	(0.153)				
UTD(-1)	0.630***				
	(0.162)				
LTD(-2)		0.460***			
		(0.154)			
UTD(-2)		0.610***			
		(0.168)			
LTD(-3)			0.446***		
			(0.150)		
UTD(-3)			0.588***		
			(0.171)		
LTD(-4)				0.438***	
				(0.149)	
UTD(-4)				0.557***	
				(0.179)	
LTD(-5)					0.422***
					(0.143)
UTD(-5)					0.534***
					(0.177)
Cons	-0.032	-0.019	-0.006	0.009	0.022
	(0.062)	(0.063)	(0.064)	(0.065)	(0.065)
R^2	0.170	0.166	0.162	0.159	0.156
t-test (LTD)	[0.001]	[0.001]	[0.001]	[0.002]	[0.002]
t-test (UTD)	[0.000]	[0.000]	[0.000]	[0.001]	[0.001]

Notes: This table reports the regression analysis of the impact of the tail dependence on the conditional default probability. We estimate the bivariate dynamic GHST copula models across all possible pairs of banks. For each pair, we compute average tail dependence by taking average of parametric and semiparametric tail dependence coefficients. The average conditional default probability is computed by taking average of conditional default probabilities estimated from six time-varying copula models (parametric and semiparametric Gaussian, Student's t and GHST copulas). In Panel A, we regress the conditional default probability on the lower tail dependence (LTD) and the upper tail dependences (UTD) in Equation (4.17). We consider three panel data estimators; pooled OLS (POLS), fixed effects (FE), and random effects (RE), and choose a consistent and efficient estimator. We test the existence of fixed effects by F-test and apply Hausman approach to test if regressors are correlated with the fixed effects. T-test (LTD) (T-test (UTD)) tests the null of $\beta^{LL} = 0$ against $\beta^{LL} > 0$ ($\beta^{UU} = 0$ against $\beta^{UU} > 0$). In Panel B, we regress the conditional default probability on the lagged tail dependences in Equation (4.18). We estimate regression equations by one selected from Panel A. In both panels, $[\cdot]$ reports the p-value of the test and $(\cdot)$ reports the standard error of the estimate, respectively. We use *, ** and *** to indicate the significance levels at 10%, 5% and 1%.

Chapter 5

Conclusions and Further Research

Copula method provides a flexible way of modeling multivariate distributions, as it allows us to specify the marginal behavior of individual risk factors separately from their dependence structure. The concept of copula and its mathematical properties are extremely useful in quantitative risk modeling. There are several attractive advantages of copula-based models. First, copulas can help us understand the dependence between risk factors at a deeper level, because copula functions describe dependence on a quantile scale and therefore overcome several disadvantages of linear correlation (see McNeil et al., 2015; Embrechts et al., 2002 for a detailed discussion). More importantly, copula dependence is able to provide insightful information about joint extreme outcomes of risk factors (more on this below). Second, copulas can be utilized to build a bottom-up framework for multivariate modeling, as they enable us to combine different well-developed marginal models with various possible dependence structures. This provides us flexibility to experiment with different kinds of marginal specifications either parametrically or nonparametrically. Third, most of the copulas can be easily simulated and this convenience allows us to apply them to the Monte Carlo studies of risk in practice.

This thesis reviews the growing literature on copula-based methods in economics and finance, and investigates empirical applications of copula theory in three different areas: market risk, portfolio optimization and credit risk. Chapter 1 reviews recent advances in copulas and some empirical applications of copula-based models for economic and financial time series in literature to date. Chapter 2 investigates the dependence structure between equity portfolios from the US and UK, and demonstrates the statistical significance of dynamic asymmetric copula models in modeling and forecasting market risk. We show consistent and robust evidence that our dynamic asymmetric copula model provides the most accurate Value-at-Risk and Expected Shortfall forecasts, indicating the importance of incorporating the dynamic and asymmetric dependence structure in market risk modeling. Chapter 3 studies the dependence between equity return and currency return in international financial markets, and explores its economic importance in portfolio allocation. We find empirical evidence of time-varying and asymmetric dependence between the equity portfolio and the corresponding foreign exchange rate across the developed and emerging markets. To account for these empirical characteristics, we further introduce a new time-varying asymmetric copula (TVAC) model, which allows for non-linearity, asymmetry and time variation of the dependence. Our empirical applications indicate that the use of this TAVC model is generally preferred for making risk management more robust and asset allocation more optimal in international financial markets. Chapter 4 studies the credit risk of UK's top-tier banks. We document the dynamics of joint credit risk by relying on a copula approach using weekly CDS data for the UK banks. We find that the tail dependence between CDS spreads contains useful information not only for explaining current joint default probabilities but also predicting systemic credit events in the banking sector.

Recent econometric and finance literature shows that a theoretically ideal and empirically practical copula model should contain several properties. First, a truly ideal copula model should be able to accommodate both positive and negative dependence, because in the real financial world, the dependence structure (or correlation) between assets can be either positive or negative and sometimes varies from positive to negative, or *vice versa*. Second, this copula model should also allow for non-zero tail dependence, as the tail dependence is able to provide measures of the strength of dependence in the tails. For instance, the Student's *t* copula and its skewed versions are asymptotically dependent in both the upper and the lower tails. This property is particularly useful for describing extreme co-movements between risk factors in financial practice. Third, besides taking into account the possibility of non-zero tail dependence, an ideal copula model should also allow for both symmetric and asymmetric dependence. In other words, it allows the lower tail dependence to be greater than the upper one, or *vice versa*. The properties of non-zero tail dependence and asymmetry are important in a risk management context, as risk modelers and decision makers are more concerned about large simultaneous losses in risk factors. Fourth, a flexible copula should be able to handle not only the bivariate case but also the higher dimensional case. Elliptical copulas (i.e. the Gaussian and the Student's t copula) are quite flexible in high dimensional modeling, however a computational complexity still exists when we consider using skewed t copulas. In this thesis, we applied high-dimensional copulas in empirical studies of asset allocation and credit risk modeling, but we only used the bivariate copulas for market risk prediction. As we discussed in Chapter 1, although several models have been proposed in the literature to mitigate the curse of dimensionality in copula modeling (see Aas et al., 2009; Oh and Patton, 2015a), this topic is still an open issue. Fifth, an empirically practical copula model should be able to capture the variation of copula parameters over time. Several mechanisms have been applied to update the correlation matrix of copula models, see for instance, Patton (2006), Jondeau and Rockinger (2006), Christoffersen et al. (2012) and Creal et al. (2013). Empirical studies in this thesis also only focus on the time-varying correlations of copula models holding other parameters constant. However, other parameters of copulas, for instance the degrees of freedom and the skewness parameter of the skewed t copula, are also likely to be time-varying. Therefore, taking into account the dynamics of other parameters in copula modeling will be an active area of research. Taken together, these properties underline the attractiveness of the dynamic (or time-varying) asymmetric copula models in quantitative risk management and asset allocation.

The empirical findings in this thesis have very important implications for risk management and asset allocation. Several important challenges are still left for future research. First, we only focus on the low-dimensional cases in this thesis and extending our studies to high-dimensional cases would be interesting. Second, it would also be interesting to investigate which economic and financial variables drive the variations of correlation, asymmetry, and joint probability of default between banks. We could conduct this analysis using the methodology of Christoffersen et al. (2015). Third, our analysis on the impact of the monetary policy on the credit risk of banks in the UK is preliminary and it would be interesting to further investigate this topic using appropriate models. Finally, a possible extension is to investigate the usefulness of the dynamic asymmetric copula model in systemic risk measurement.

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