Essays on Fiscal Policy in Heterogeneous Agent Models

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Abstract

This thesis consists of three inter-related chapters designed to study the effects of fiscal policy on unemployment, the distribution of income, and social welfare in heterogeneous agent models incorporating unemployment. Each chapter employs a different setup for unemployment in a general equilibrium framework. These include models of equilibrium unemployment, right-to-manage union bargaining, and search and matching.

Chapter 1 develops a model with equilibrium unemployment to study the effects of optimal taxation under commitment. Two models are explored: a model with zero economic profits and a model with non-zero economic profits due to the presence of productive public investment. We find that the optimal policy in these two models results in a different labour wedge which defines the gap between the marginal rate of substitution between labour and consumption and the marginal product of labour. In particular, the labour wedge can only be completely eliminated when the profits are absent from the model. It is further demonstrated that there exists a trade-off between efficiency and equity for the government in the model with non-zero economic profits.

Chapter 2 examines the importance of imperfect competition in labour and product markets in determining the welfare effects of tax reforms assuming agent heterogeneity in capital holdings. The analysis shows that each of these market distortions, independently, results in welfare losses for at least one segment of the population after a capital tax cut and a concurrent labour tax increase. However, with both present in the model, the tax reform is Pareto improving in a realistic calibration to the UK economy.

Chapter 3 extends a Mortensen-Pissarides search-and-matching framework with household heterogeneity to investigate the importance of search frictions in determining the welfare and distributional effects of tax reforms which reallocate the tax burden from capital to labour income. The optimal tax policy under commitment is also analysed. We find that the tax reforms are Pareto improving in the long run, despite welfare losses for at least one segment of the population in the transition period. Finally, the long-run Ramsey policy implies a negative capital tax which is associated with a rise in the labour tax and a fall in the unemployment benefit.

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Declaration

I declare that this thesis is the result of my own work and has not been previously submitted for any other degree at the University of Glasgow or any other institution.

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Introduction

This thesis is generally concerned with the relationship between fiscal policy and unemployment in general equilibrium models populated by agents who differ in their endowments. In each of the three chapters, a different setup for unemployment is used. The purpose of this thesis is threefold. It first aims to analyse optimal fiscal policy under commitment by assuming that the government can choose different policy instruments under different assumptions about unemployment. Second, it examines the importance of the combination of labour and product market failures in determining the welfare effects of tax reforms. Finally, it investigates the relevance of job search frictions in determining the long- and short-run welfare effects of tax reforms using a search-and-matching framework. All the objectives contribute to the growing literature on fiscal policy study in macroeconomics.

Chapter 1 studies the optimal design of steady-state taxation under commitment in heterogeneous agent models with equilibrium unemployment. It particularly examines the effects of optimal taxation on unemployment, the distribution of income, and welfare of agents. It further examines the importance of the presence of economic profits in determining the optimal taxation. In addition, this chapter evaluates the optimal labour wedge which captures the gap between the marginal rate of substitution between labour and consumption, and the marginal product of labour. The labour wedge is considered as an indicator of the labour market distortions.[1]

In Chapter 1 households are divided into capitalists and workers and the agent heterogeneity lies in the working and saving propensities of households. Following Judd (1985), Lansing (1999) and Ardagna (2007), we assume that only capitalists can participate in the asset market and only workers can work. We then employ a simple and tractable setup for unemployment. Following Pissarides (1998), Ardagna (2001) and Ljungqvist and Sargent (2006), unemployment arises in the competitive labour markets as the outcome of optimal choices made by workers. In particular, time off work (leisure) is treated as unemployment. Unemployment can generate both leisure and unemployment benefits for the workers. The model is calibrated in a way that unemployment benefits are always below the net wage rate. This implies that leisure is always costly to the workers. Unemployment benefits in the economy can mimic the role that they have in non-competitive labour markets, and in effect they are equivalent

[1] See Chari *et al.* (2002 and 2007a) and Shimer (2009). But they use this concept to study the business cycle accounting.

to a labour income tax. However, by separating the unemployment benefits from the labour tax, it makes the model capable of evaluating their different effects on workers and government's budget. In the modified model, profits are present in the economy when the productive government investment is introduced (see e.g. Lansing (1998) and Malley *et al.* (2009)).

Chapter 1 contributes to the optimal policy literature by showing the importance of economic structure in determining optimal taxation, and focusing on the effects of optimal taxation on unemployment and its distributional effects on different agents. In this chapter, we show that the standard Chamley-Judd result of long-run optimal zero capital tax survives in the model with zero economic profits. The optimal policy completely eliminates the labour wedge. These results hold even if the government cares only about the workers. This implies that there is no conflict of interests between agents. The study offers new results in the model with non-zero economic profits. The optimal taxation implies a negative capital tax. The negative capital tax can help to correct the under-investment of capitalists which is consistent with Guo and Lansing (1999) and Judd (1997 and 2002). The labour wedge cannot be completely eliminated. The optimal allocation of resources varies when the government changes the weight placed on the welfare of agent in its objective function.

Chapter 2 investigates the effects of tax reforms of cutting capital tax in models with labour and product market distortions. We study the interactions between labour and product market failures in determining the welfare and inequality effects of factor income taxation. The studies of tax reforms have recently received a great deal of attention from both academics and policymakers. However, the potential distributional effects of tax reforms have not been examined under imperfect competition in both labour and product markets.

In Chapter 2 capitalists and workers are distinguished by differences in their capital holdings which is motivated by imperfections in the asset market that require agents to pay different participation premia (see e.g. Aghion and Howitt (2009)). On the other hand, households are identical in the labour market since labour unions guarantee that they have equal employment and wages (see e.g. Pissarides (1998), Maffezzoli (2001) and Ardagna (2007)). In this chapter the labour market distortion is introduced in the unionised labour market. Following Nickell and Andrews (1983), Farber (1986), Pissarides (1998) and Kass and von Thadden (2004), a right-to-manage union bargaining setup is employed where unions and firms bargain over the wage rate to maximise a weighted average of labour income and profits. Following Dixit and Stiglitz (1977), Benhabib and Farmer (1994) and Guo and Lansing (1999), the product market failure lies in the intermediate goods production as the intermediate goods producers have monopoly power and can earn strictly positive profits in equilibrium.

The over-arching finding in Chapter 2 is that the presence of labour and product market distortions is critical in determining not only the size but also the direction of welfare effects after the tax reforms, in particular, whether a capital tax cut can be Pareto improving. We find that each of these two market distortions can result in welfare losses for at least one segment of the population after the tax reforms. However, when they are combined in a realistic calibration to the UK economy, a capital tax cut will be Pareto improving. Our analysis makes clear that although each market distortion independently has similar welfare effects, they two together can lead to completely reversed welfare implications. Therefore, our results suggest that the omission of relevant market and policy failures may lead to biased results that cannot always be predicted *ex ante*.

Chapter 3 evaluates both the long- and short-run effects of re-allocating the tax burden from capital to labour income using a Mortensen-Pissarides search-and-matching framework. We examine the importance of search frictions in determining the welfare effects of such tax reforms. To the best of our knowledge, this question has not yet been answered in the search-and-matching literature. The steady-state optimal tax policy is also studied. We compare our results with those in Domeij (2005) by making several distinct model assumptions. Our analysis shows that incorporating the search frictions into the model cannot only change the size but also the sign of optimal capital tax.

In Chapter 3 the setup for agent heterogeneity employed is as in Chapter 1. Specifically, the capitalists by assumption do not work and workers do not save. Following Shi and Wen (1999), Pissarides (2001) and Domeij (2005), the wage paid in any given job is determined through a Nash bargain between a pairing of matched worker and firm. They bargain over the wage rate to maximise a weighted average of worker's and firm's surpluses. Once they reach an agreement, the worker supplies one unit of labour endowment to the firm in the following period. If the worker rejects the offer, he is not entitled to the unemployment benefits. In this sense, the employment is pre-determined at any given time. As a model extension, a different specification of unemployment benefits is studied. Unemployment benefits by assumption depend on past wages due to some institutional features in the labour markets, following Blanchard and Katz (1999) and Chéron and Langot (2010). As a result, the unemployment benefit is proportional to past wages by a constant in the transition period. However, at the steady-state, it is constant. Given the relevance of unemployment benefits in determining the wage rate in the bargaining, this new specification can offer new results of the effects of tax reforms and allow the study of the mechanisms driving the results.

Chapter 3 contributes to the search-and-matching literature on the study of the tax reforms and optimal tax policy by examining the relevance of search frictions. The results suggest that the tax reform will be Pareto improving in the long run although it hurts the workers and worsens the aggregate welfare during the transition. Generally, there are higher welfare gains for all agents by reducing workers' bargaining power or under the new specification of unemployment benefits. Finally, the optimal tax policy implies a negative capital tax in the long run, while the labour tax increases and unemployment benefits decrease.

Chapter 1: Optimal taxation in heterogeneous agent models with equilibrium unemployment

Abstract: This chapter examines the effects of optimal taxation on unemployment, the distribution of income and welfare of agents in dynamic general equilibrium models with capitalists and workers. Equilibrium unemployment is generated in a competitive labour market as the outcome of optimal choices made by workers. Two models are studied: a model without economic profits and a model with economic profits due to the presence of productive public investment. First, we find that, in the absence of economic profits, the optimal taxation implemented by the government relative to the historical taxation indicates that: (i) the long-run optimal capital tax is zero; (ii) it is optimal to tax the leisure which is equivalent to a subsidy to the labour supply of workers; (iii) the labour wedge is completely eliminated and there are welfare gains for all agents; and (iv) all the results hold when the government varies the weight attached to the welfare of agent in its objective function, so that there is no conflict of interests between agents. Second, in the presence of economic profits, we show that: (i) the government subsidizes the capital income in the long run; (ii) the labour wedge cannot be eliminated, as a result, the welfare of workers worsens despite welfare gains for the capitalists; and (iii) the optimal taxation has redistributional effects on different agents when the weight placed on the welfare of capitalists by the government exceeds a critical value. This leads to a trade-off between efficiency and equity.

1.1 Introduction

This chapter uses a heterogeneous agent dynamic general equilibrium model with unemployment to examine the effects of optimal taxation on unemployment, the distribution of income and welfare of agents in the economy.

Since the 1980s there has been an extensive literature studying optimal taxation in macroeconomics. For example, Chamley (1986) studies the optimal taxation using a representative agent model. He shows that the government should use a zero tax rate on capital income in the long run. Chari *et al.* (1994) and Chari and Kehoe (1999) also conclude that a permanent positive tax rate on capital income is not efficient in a Ramsey-type setup of the government. This family of models, however, is silent on the research question of inequality which has resulted from the conflict of interests between different agents. In this sense, the distributional effect of optimal taxation has been neglected in these papers. In this context, heterogeneous agent models obviously become a good candidate to study the distributional effect of optimal taxation.

Within the heterogeneous agent framework, the seminal research of Judd (1985) makes a distinction between "capitalists" and "workers" in order to investigate the redistributive potential of capital taxation in the economy. He suggests that the optimal tax policy under commitment is to not tax capital income in the long run and to raise all the required tax revenues by taxing labour income. This result even holds when the government cares only about the workers. This implies that there is no conflict of interests between agents in the economy.

All these studies point out that the government should not tax capital income in the long run. However, the robustness of this result has recently been challenged. Whether capital income should be taxed or not in the long run still remains an open question in the literature. Guo and Lansing (1999) introduce imperfect competition and profits in the product market and they show that the optimal zero capital tax rate might not be obtained in the long run assuming that the government has access to a commitment technology. The introduction of profits via firms' monopoly power creates distortions in the capital market that break down the normative long-run result of optimal zero tax rate on capital income. Koskela and von Thadden (2008) model the non-Walrasian labour market with Nash bargaining between firms and labour unions. They suggest that both capital and labour taxes should be used in the long run. Also the result of non-zero optimal capital income tax can also be obtained in the models without commitment technology for the government (see e.g. Krusell (2002) and Angelopoulos *et al.* (2011a)), or assuming that households are endowed with different skills in the labour markets (see e.g. Conesa *et al.* (2009)). All these studies have shown the importance of economic structure in determining optimal taxation of the government.

This chapter contributes to the optimal taxation literature by examining the determination and effects of optimal taxation under different market structures. We stay as close as possible to Judd (1985), but two new concepts are introduced into the model: equilibrium unemployment and profits in the product market. In addition, this study sheds some light on the determination of optimal labour wedge which captures the gap between the marginal rate of substitution between labour and consumption, and the marginal product of labour. This concept has never been studied in the optimal policy literature. In the past, it was only used to study the business cycle accounting (see e.g.

Chari *et al.* (2002 and 2007a) and Shimer (2009)). In the competitive labour market, unemployment is generated, following for example, Pissarides (1998), Ardagna (2001) and Ljungqvist and Sargent (2006), as the outcome of optimal choices made by workers. In this chapter their models are extended to allow for agent heterogeneity by assuming different economic roles of agents in the economy. Following Judd (1985), Lansing (1999) and Ardagna (2007), we assume that capitalists do not work and workers do not save. In this setup, the government taxes labour income and interest income from capital and profits to finance its public spending. The unemployment benefits in this model mimic the role that they have in a non-competitive labour market.[2] An increase in unemployment benefits tends to decrease the labour supply of workers and then put some pressure on the equilibrium wage rate. Alternatively, unemployment benefits can be considered as another tax rate on the labour income. By separating unemployment benefits from the explicit tax rate on labour income, it is possible to investigate the different effects of these two policy instruments on workers and government's budget.

Two different heterogeneous agent models are studied in this chapter. In the first model which is referred to as the benchmark model, firms earn zero economic profits in the product market. The second model extends the benchmark model to allow for productive public investment in the production. This model is referred to as the modified model. Following Lansing (1998) and Malley *et al.* (2009), we assume that the government can provide individual firms with public capital without asking for rents. In the modified model, the production is constant-returns-to-scale (CRTS) in three productive inputs: private capital, labour and public capital. The equilibrium profits are equal to the difference between the value of output and the production costs of inputs employed in the private sectors. This setup allows us to examine the relevance of profits in determining the optimal taxation of Benthamite (non-partisan) government. The case of partisan government is also examined, in the sense that, the government is biased towards one agent and places higher weight to the welfare of the agent in its objective function.

The model with exogenous policy instruments is calibrated so that its steady state can reflect the main empirical characteristics of the current UK economy, with particular focus on its long-run unemployment rate. The UK is chosen for the quantitative analysis since the high and rising unemployment rate has been a feature of the UK

[2] Unemployment can also be generated in the unionised labour market (see e.g. Maffezzoli (2001) and Ardagna (2007)), or in the model with search frictions (see e.g. Mortensen and Pissarides (1994) and Pissarides (2001)).

economy compared to other European countries.

The main findings can be summarized as follows. First, in the benchmark model with zero profits, we find that the optimal tax rate on capital income is zero in the long run. The government chooses to tax the leisure of workers in the long run. This is equivalent to a subsidy to the labour supply of workers. Meanwhile, the government slightly increases the tax rate on labour income. We also find that the distortions in the labour market caused by the distortionary labour tax can be completely eliminated as a consequence of the equal amount of government subsidies to the workers in the form of taxation on the leisure. In other words, the positive effect of negative replacement rate and the negative effect of increase in labour tax on workers net out in the long run. Therefore, the gap between the marginal rate of substitution between labour and consumption, and the marginal product of labour totally disappears in the long run. As a result, the labour supply of workers increases which is beneficial to the workers. The income, consumption and welfare of workers improves. In addition, as in Judd (1985), the results show that the optimal taxation and allocation under commitment are independent of the weight to the welfare of agent in the Ramsey setup of the government. This implies that there is no conflict of interests between agents in the long run.

Second, the result of long-run optimal zero capital tax cannot be obtained in the model with non-zero economic profits due to the presence of productive public investment. The optimal tax rate on capital income is negative which means the government chooses to subsidize the capital income in the long run. There are two opposing effects in determining the direction of optimal capital taxation: the under-investment effect and the profit effect (see e.g. Guo and Lansing (1999)). In our model, on one hand, the crowding-out effect of public investment is equivalent to the under-investment effect which motivates a Benthamite government to use a subsidy to the capital income in order to reduce the distortions in the capital market. On the other hand, the presence of profits motivates the government to use a positive tax rate on capital income as taxing profits is not distortionary. In our case, we show that the under-investment effect outweighs the profit effect. As a result, the government subsidizes the capital income in the long run. The optimal capital tax directly increases the investment of capitalists and therefore the income, consumption and welfare of capitalists increase. As in the benchmark model, the government subsidizes the labour supply of workers while the tax rate on labour income slightly increases. These two policy instruments have opposing effects on the labour supply of workers. We find that the positive effect of labour subsidy dominates so that the labour supply of workers is higher than it would be in the model with given policy. In the presence of profits, there is a gap between the marginal rate of substitution between labour and consumption, and the marginal product of labour, so that the tax distortion in the labour market cannot be completely eliminated in the long run. The distortion causes welfare losses for workers. In contrast to the benchmark model, the weight to the welfare of agent matters for the optimal taxation in the modified model. The effects are found to be monotonic. This implies that the optimal taxation generates conflict of interests between agents and it has redistributional effects in the long run. As the weight to the welfare of capitalists increases, the capital taxation decreases and it turns into a subsidy after a critical value. The tax rate on labour income increases in order to make up for the losses in government's tax revenues. In this case, a trade-off between efficiency and equity needs to be taken into account by the government in the Ramsey setup of government.

The rest of this chapter is organized as follows: Section 1.2 sets out the benchmark model structure. Section 1.3 discusses the calibration of the model and gives the steady-state and dynamic solution of the model. Section 1.4 analyses the impulse responses of variables to model's exogenous shocks. Section 1.5 studies optimal policy under commitment. Section 1.6 presents one extension to the benchmark model and provides an analysis of steady-state optimal policy. Section 1.7 constructs welfare analysis for two models and Section 1.8 offers a summary and conclusion.

1.2. Equilibrium unemployment in a model without profits

1.2.1 The model

The main features of the economy are summarized as follows. Infinitely lived households, firms and a government populate the economy. There is a large but fixed number of households which can be divided into two types in terms of their different roles in the economy: capitalists and workers. Following Judd (1985), Lansing (1999) and Ardagna (2007), capitalists by assumption do not work and workers do not save. Capitalists can participate in the capital market and they are owners of the firms. Their income includes interest income from private capital and dividends of firms. Employed workers supply labour to the firms and obtain wage incomes. If workers are unemployed, they can receive unemployment benefits from the government. Workers consume all their disposable income in each period. Following Pissarides (1998), Ardagna (2001) and Ljungqvist and Sargent (2006), equilibrium unemployment is generated in the competitive labour market as the outcome of optimal choices made by workers. Firms are perfectly competitive and they produce a single product in the goods sector with a constant-returns-to-scale technology. Finally, the government purchases goods and services from the private sector which could enhance the utility of households. It also provides unemployment benefits to unemployed workers. The government finances all its spending requirements by taxing labour income and interest income from capital and profits.

1.2.2 Population composition

The whole population size of the households is given by N. The population sizes of capitalists and workers are assumed to be: N^k and N^w.[3] The population shares of capitalists and workers are assumed to be: $N^k/N \equiv n^k$, and $N^w/N \equiv n^w = 1 - n^k$. The population composition is taken as given and fixed over time. The firms are indexed by the superscript f. Each capitalist owns one single firm. This implies that the number of firms is equal to the number of capitalists, i.e. $N^f = N^k$.

1.2.3 Capitalists

The utility function of households is of the constant elasticity of substitution (CES) variety which is defined over a composite good and leisure as follows:

$$U_t^i = \left[\mu \left(C_t^i + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1 - H_t^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where C_t^i is household i's private consumption; $\overline{G}_t^c$ represents per capita government consumption, i.e. $\overline{G}_t^c = G_t^c/N$, where G_t^c denotes aggregate government consumption; H_t^i is the labour supply. We fix $H_t^k = 0$ for the capitalists in their utility function as they are assumed to not work in the economy. The parameter $\sigma > 0$ measures the elasticity of substitution between consumption and leisure; and $0 < \mu < 1$ is the weight given to consumption relative to leisure in the utility.

The utility function differs from the conventional neoclassical utility function by including the term of per capita government consumption, $\overline{G}_t^c$. The private consumption and government consumption are assumed to be substitutes in the utility function. The

[3] Variables for capitalists are indexed by the superscript k and variables for workers are indexed by the superscript w in what follows.

degree of substitutability is determined by the constant parameter $0 < \omega \leq 1$. In this way, the government could affect households via the utility effect of $\overline{G}_t^c$. Barro (1981 and 1989) suggests that government consumption expenditure on goods and services can provide direct utility for the households. This argument is supported by some empirical studies in the literature. Kormendi (1983) and Aschauer (1985) test the parameter which defines the relationship between private and government consumption for the US economy, and Ahmed (1986) for the UK economy. They all support the substitutability relationship. This specification of substitutability between private and government consumption is widely used in the RBC literature.[4]

The objective function of the representative capitalist is to maximise his present discounted value of lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t U_t^k (C_t^k, \overline{G}_t^c) \tag{2}$$

where $0 < \beta < 1$ is the constant discount factor and U_t^k is given by:

$$U_t^k = \left[\mu \left(C_t^k + \omega \overline{G}_t^c \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1-0)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

The budget constraint of each capitalist at time t is given by:

$$C_t^k + I_t^k = r_t K_t^k - \tau_t^k (r_t - \delta^p) K_t^k + (1 - \tau_t^k) \pi_t^k \tag{4}$$

where K_t^k is the private capital stock at the beginning of time t; I_t^k is the investment; r_t is the gross return to capital; π_t^k denotes profits; $0 < \delta^p < 1$ is the constant depreciation rate of capital stock; and $0 \leq \tau_t^k < 1$ is the tax rate on capital income and profits.[5]

The evolution equation of capital stock is:

$$K_{t+1}^k = (1 - \delta^p) K_t^k + I_t^k. \tag{5}$$

The capitalist chooses $\left\{ C_t^k, K_{t+1}^k \right\}_{t=0}^{\infty}$ to maximize (2) subject to the constraints (3), (4) and (5) by taking market prices $\{r_t\}_{t=0}^{\infty}$, policy variables $\left\{ \tau_t^k, \overline{G}_t^c \right\}_{t=0}^{\infty}$ and an initial

[4] See Aschauer and Greenwood (1985), Ambler and Paquet (1996) and Finn (1998).

[5] Following Guo and Lansing (1999), we assume that the government cannot distinguish between returns to capital stock and profits received from firms, so that they are taxed at the same rate. In equilibrium, the firms earn zero economic profits, and hence the capitalists receive zero profits from firms, i.e. $\pi_t^k = 0$. Hereafter, the last term involving π_t^k will be dropped from capitalist's budget constraint in the benchmark model. In addition, the capital taxes are assumed to be net of depreciation (see e.g. Lansing (1998)).

condition for the capital stock, K_0^k, as given.

The optimization problem of the capitalist can be expressed mathematically as follows:

$$\max_{\left\{C_t^k, K_{t+1}^k\right\}_{t=0}^{\infty}} \left\{\sum_{t=0}^{\infty} \beta^t \left[\mu \left(C_t^k + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1-0)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right\}$$

$$s.t. \qquad C_t^k + K_{t+1}^k - (1-\delta^p) K_t^k = r_t K_t^k - \tau_t^k (r_t - \delta^p) K_t^k.$$

The Lagrangian function of the capitalist is then written as:

$$\begin{aligned} L^k = & \sum_{t=0}^{\infty} \beta^t \left\{\left[\mu \left(C_t^k + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1-0)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \right. \\ & \left. + \xi_t \left[r_t K_t^k - \tau_t^k (r_t - \delta^p) K_t^k - C_t^k - K_{t+1}^k + (1-\delta^p) K_t^k\right]\right\} \end{aligned}$$

where ξ_t is the Lagrangian multiplier on the capitalist's budget constraint.

The first-order condition (FOC) for C_t^k is:

$$\left[\mu \left(C_t^k + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\right]^{\frac{1}{\sigma-1}} \cdot \mu \left(C_t^k + \omega \overline{G}_t^c\right)^{-\frac{1}{\sigma}} = \xi_t. \tag{6}$$

The FOC for K_{t+1}^k is:

$$\beta \xi_{t+1} \left[1 + (1-\tau_{t+1}^k)(r_{t+1} - \delta^p)\right] = \xi_t. \tag{7}$$

Consolidating these two FOCs yields the following optimality condition of the capitalist:

$$\begin{aligned} & \left(C_t^k + \omega \overline{G}_t^c\right)^{-\frac{1}{\sigma}} \left[\mu \left(C_t^k + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\right]^{\frac{1}{\sigma-1}} \\ = & \ \beta \left(C_{t+1}^k + \omega \overline{G}_{t+1}^c\right)^{-\frac{1}{\sigma}} \left[\mu \left(C_{t+1}^k + \omega \overline{G}_{t+1}^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\right]^{\frac{1}{\sigma-1}} \cdot \\ & \cdot \left[1 + (1-\tau_{t+1}^k)(r_{t+1} - \delta^p)\right]\} . \end{aligned} \tag{8}$$

This is the consumption Euler equation of the capitalist which describes the optimal intertemporal choice made by the capitalist in equilibrium. It implies that the marginal utility of foregone consumption at time t should be equal to the expected marginal benefit of discounted $t+1$ returns from investing one more unit at time t in equilibrium.

1.2.4 Workers

The workers are assumed to be identical in the labour market. Hence, the labour supply of workers to firms is homogenous. They work and consume all their disposable income in each period. Unemployment is generated in the competitive labour market as the outcome of optimal choices made by the workers. Time off work is then treated as unemployment in the model. If unemployed, workers receive unemployment benefits from the government. The time constraint of workers is crucial in the workers' setup. It is described as follows. At time t, the workers are endowed with the fixed amount of time. The time spent on physiological needs is treated as the exogenous leisure of workers. Apart from this, the workers are expected to work for the firms and obtain wage income from working. This portion of time is taken as potential labour supply of workers which is normalised to unity. In the competitive labour market, both firms and workers are assumed to be price takers. The wage rate is determined when the aggregate labour supply is equal to the aggregate labour demand. The equilibrium labour supply generated by the model is less than the potential labour supply of workers. The difference between these two is then treated as unemployment. In other words, time off work is considered as unemployment in this model. Unemployment by assumption could generate both leisure and unemployment benefits for the workers. The structural parameters is calibrated so that per capita unemployment benefits are always below the net return to labour. In other words, leisure is costly to workers as working can generate higher labour income. The workers do not save so that they do not have to make intertemporal choices. The optimization problem for the workers is thus static.

At time t, the objective function of the representative worker is given by:

$$\max U_t^w(C_t^w, 1 - H_t^w, \overline{G}_t^c) \tag{9}$$

and the utility function is:

$$U_t^w = \left[\mu \left(C_t^w + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1 - H_t^w\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{10}$$

The time constraint of the worker is given by:

$$L_t^w = 1 - H_t^w \tag{11}$$

where L_t^w denotes the leisure of the worker.

The worker has the following within-period budget constraint:

$$C_t^w = (1 - \tau_t^w)\, w_t H_t^w + \overline{G}_t^u (1 - H_t^w) \tag{12}$$

where w_t is the wage rate; $0 \leq \tau_t^w < 1$ is the tax rate on labour income; and $\overline{G}_t^u$ is per capita unemployment benefits which are assumed to be proportional to the wage rate, i.e. $\overline{G}_t^u = \overline{r}_t w_t$, where $\overline{r}_t$ is the replacement rate measuring the imputed value of leisure. As discussed above, unemployment benefits are less than the net wage rate, i.e. $\overline{r}_t w_t < (1 - \tau_t^w)\, w_t$, so that unemployment is costly to the worker although it yields leisure.

The value of the free parameter in the utility function, μ, is calibrated, such that the model's steady-state unemployment is in line with the data average between 1970 and 2009.[6]

At time t, the worker takes the market price, w_t, per capita government consumption and unemployment benefits, $\overline{G}_t^c$ and $\overline{G}_t^u$, and the tax rate on labour income, τ_t^w, as given, and chooses C_t^w and H_t^w to maximize (9) subject to the constraints (10), (11) and (12).

The optimization problem for the worker is shown as follows:

$$\max_{C_t^w, H_t^w} \left\{ \left[\mu \left(C_t^w + \omega \overline{G}_t^c \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1 - H_t^w)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

$$s.t. \quad C_t^w = (1 - \tau_t^w)\, w_t H_t^w + G_t^u (1 - H_t^w).$$

The Lagrangian function of the worker is written as:

$$\begin{aligned} L^w = \quad & \left[\mu \left(C_t^w + \omega \overline{G}_t^c \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1 - H_t^w)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \\ & + \phi_t \left[(1 - \tau_t^w)\, w_t H_t^w + \overline{G}_t^u (1 - H_t^w) - C_t^w \right] \end{aligned} \tag{13}$$

where ϕ_t is the Lagrangian multiplier on the worker's budget constraint.

The FOC for C_t^w is:

$$\left[\mu \left(C_t^w + \omega \overline{G}_t^c \right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)(1 - H_t^w)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \mu \left(C_t^w + \omega \overline{G}_t^c \right)^{-\frac{1}{\sigma}} = \phi_t. \tag{14}$$

[6] In the UK economy, the date average of unemployment rate was 7% between 1970 and 2009.

The FOC for H_t^w is:

$$\left[\mu\left(C_t^w+\omega\overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\left(1-H_t^w\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}(\mu-1)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+$$

$$+\phi_t\left(1-\tau_t^w\right)w_t-\phi_t\overline{G}_t^u=0. \tag{15}$$

These two FOCs are next combined into one equation as follows:

$$\begin{aligned}&(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu G_t^u\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\\=\;&\mu w_t\left(1-\tau_t^w\right)\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\end{aligned} \tag{16}$$

which can be re-written as:

$$\begin{aligned}&(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu\overline{r}_t w_t\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\\=\;&\mu w_t\left(1-\tau_t^w\right)\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\end{aligned} \tag{17}$$

by replacing $\overline{G}_t^u$ with $\overline{r}_t w_t$.

The expression above is re-arranged to obtain the following condition:

$$1-\tau_t^w-\overline{r}_t=\frac{\frac{(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}}{\mu\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}}}{w_t} \tag{18}$$

where $\frac{(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}}{\mu\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}}=MRS_{H_t^w,C_t^w}$ is the marginal rate of substitution between labour and consumption. Therefore, the r.h.s of the equation reflects the gap between the marginal rate of substitution between labour and consumption, and the marginal product of labour.[7] Chari *et al.* (2002 and 2007a) and Shimer (2009) define it as the labour wedge. The labour wedge is interpreted as an indicator of the labour market distortions.

1.2.5 Firms

A representative firm produces its individual output using a technology that exhibits constant-returns-to-scale in capital and labour. The production function of the representative firm is given by:

$$Y_t^f=A_t\left(K_t^f\right)^{\alpha_1}\left(H_t^f\right)^{\alpha_2} \tag{19}$$

[7]In equilibrium, the firms hire the workers until the wage rate is equal to the marginal product of labour. This is shown in the profit-maximizing problem of the firm as follows.

where Y_t^f represents the firm's output; K_t^f is the capital stock employed by the firm in the production; H_t^f is the labour input; and $0 < \alpha_1, \alpha_2 < 1$ denote the capital's and labour's shares of output. The CRTS property implies: $\alpha_1 + \alpha_2 = 1$.

The aggregate output denoted by, Y_t, measuring the gross product of the economy, is the sum of individual firm's output:

$$Y_t = N^f Y_t^f. \tag{20}$$

The law of motion for the total factor productivity (TFP), A_t, is an exponential $AR(1)$ process:

$$A_{t+1} = A_0^{(1-\rho^a)} A_t^{\rho^a} e^{\varepsilon_{t+1}^a} \tag{21}$$

or

$$\ln(A_{t+1}) = (1-\rho^a)\ln(A_0) + \rho^a \ln(A_t) + \varepsilon_{t+1}^a \tag{22}$$

where $A_0 > 0$ is a parameter which gives the steady-state value of TFP; $0 < \rho^a < 1$ is the first-order auto-regressive parameter; and $\varepsilon_t^a \sim iid\,(0, \sigma_a^2)$ is the technology innovation which follows a normal distribution with the standard deviation being $\sigma_a > 0$.

The profits earned by the firm at time t are given by:[8]

$$\pi_t^f = Y_t^f - r_t K_t^f - w_t H_t^f. \tag{23}$$

At time t, the firm chooses the quantities of capital and labour in order to maximise profits taking the market prices of them as given.

The optimization problem for the firm can be summarized in the following:

$$\max_{K_t^f, N_t^f} \left\{ Y_t^f - r_t K_t^f - w_t H_t^f \right\}$$

$$s.t. \quad Y_t^f = A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2}.$$

The Lagrangian function of the firm is written as:

$$L^f = A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2} - r_t K_t^f - w_t H_t^f. \tag{24}$$

[8]The price of goods is fixed to be 1, so that all the variables in the model are written in real terms.

The FOC for K_t^f is:

$$\alpha_1 A_t \left(K_t^f\right)^{\alpha_1 - 1} \left(H_t^f\right)^{\alpha_2} - r_t = 0 \tag{25}$$

which can be re-written as:

$$\begin{aligned} r_t &= \alpha_1 A_t \left(K_t^f\right)^{\alpha_1 - 1} \left(H_t^f\right)^{\alpha_2} \\ &= \frac{\alpha_1 Y_t^f}{K_t^f}. \end{aligned} \tag{26}$$

The FOC for H_t^f is:

$$\alpha_2 A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2 - 1} - w_t = 0$$

which can be re-written as:

$$\begin{aligned} w_t &= \alpha_2 A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2 - 1} \\ &= \frac{\alpha_2 Y_t^f}{H_t^f}. \end{aligned} \tag{27}$$

These two optimality conditions of the firm imply that factor rentals are equal to their marginal products in equilibrium.

The firm's profits in equilibrium are:

$$\begin{aligned} \pi_t^f &= Y_t^f - r_t K_t^f - w_t H_t^f \\ &= Y_t^f - \frac{\alpha_1 Y_t^f}{K_t^f} K_t^f - \frac{\alpha_2 Y_t^f}{H_t^f} H_t^f \\ &= 0. \end{aligned} \tag{28}$$

This implies that the firm earns zero profits in equilibrium.

1.2.6 Government

In the absence of government debt, the government has a balanced budget in each period. The aggregate budget constraint of the government is:

$$G_t^c + N^w \overline{r}_t w_t \left(1 - H_t^w\right) = N^k \tau_t^k \left(r_t - \delta^p\right) K_t^k + N^w \tau_t^w w_t H_t^w. \tag{29}$$

Government expenditures include unemployment benefits and government consumption which is utility-enhancing. They are financed by the tax revenues from capitalists and workers.

Both sides of the constraint (29) are divided by the total population, N, and we make use of the population relationships, $N^k/N = n^k$, $N^w/N = 1 - n^k$ and $N^f = N^k$, to get the per capita government budget constraint as follows:

$$\begin{aligned} & \overline{G}_t^c + \left(1 - n^k\right) \overline{r}_t w_t \left(1 - H_t^w\right) \\ = \ & n^k \tau_t^k \left(r_t - \delta^p\right) K_t^k + \left(1 - n^k\right) \tau_t^w w_t H_t^w . \end{aligned} \tag{30}$$

The policy instruments of government include τ_t^k, τ_t^w, $\overline{r}_t$ and $\overline{G}_t^c$. In the exogenous policy case, τ_t^k, τ_t^w and $\overline{r}_t$ are all assumed to follow stochastic exponential $AR(1)$ processes given by:

$$\ln(\tau_{t+1}^k) = \left(1 - \rho^k\right) \ln\left(\tau_0^k\right) + \rho^k \ln\left(\tau_t^k\right) + \varepsilon_{t+1}^k \tag{A1}$$

$$\ln(\tau_{t+1}^w) = \left(1 - \rho^w\right) \ln\left(\tau_0^w\right) + \rho^w \ln\left(\tau_t^w\right) + \varepsilon_{t+1}^w \tag{A2}$$

and

$$\ln(\overline{r}_{t+1}) = \left(1 - \rho^r\right) \ln\left(\overline{r}_0\right) + \rho^r \ln\left(\overline{r}_t\right) + \varepsilon_{t+1}^r \tag{A3}$$

where τ_0^k, τ_0^w and $\overline{r}_0$ are constant which give the steady-state values of these policy variables; $0 < \rho^k, \rho^w, \rho^r < 1$ are first-order autoregressive parameters; and ε_t^k, ε_t^w and ε_t^r are random shocks to policy instruments that are all characterised by the normal distribution, i.e. $N\left(0, \sigma_{k,w,r}^2\right)$. In Section 1.4, the dynamic responses of key variables to each of these random shocks plus TFP shock are examined.

The per capita government consumption, $\overline{G}_t^c$, is allowed to be residually determined ensuring that the government budget constraint is balanced at any given period of time. The change in any other exogenous policy variable will be met by the change in $\overline{G}_t^c$.

1.2.7 Market clearing conditions and resource constraint

In the capital market, the aggregate demand of capital is equal to the aggregate supply of capital. This implies:

$$N^k K_t^k = N^f K_t^f . \tag{31}$$

It has been assumed that $N^k = N^f$, so that the above condition implies $K_t^k = K_t^f$.[9]

[9]In what follows, the superscript p is used to denote the private capital stock in the model, i.e. $K_t^k = K_t^f \equiv K_t^p$ in order to distinguish from the public capital stock to be introduced in Section 1.6.

The per capita market clearing condition for the labour is:

$$H_t^w = \frac{n^k}{(1-n^k)} H_t^f. \tag{32}$$

Finally, in the goods market, the economy's aggregate resource constraint is given by:

$$Y_t = N^k C_t^k + N^w C_t^w + I_t^p + G_t^c \tag{33}$$

which can be re-written in per capita terms:

$$n^k Y_t^f = n^k C_t^k + \left(1-n^k\right) C_t^w + n^k I_t^p + \overline{G}_t^c. \tag{34}$$

1.2.8 Decentralized competitive equilibrium (given policy)

We now summarise the decentralized competitive equilibrium (DCE) conditions in the benchmark model. Given the paths of policy instruments $\left\{\tau_t^k, \tau_t^w, \overline{r}_t\right\}_{t=0}^{\infty}$, prices $\{r_t, w_t\}_{t=0}^{\infty}$, the TFP $\{A_t\}_{t=0}^{\infty}$ and the initial condition for K_0^p, a decentralized competitive equilibrium is defined to be an allocation $\left\{C_t^k, K_{t+1}^p, C_t^w, H_t^w, Y_t^f\right\}_{t=0}^{\infty}$ and one residually determined policy instrument $\left\{\overline{G}_t^c\right\}_{t=0}^{\infty}$, such that (i) capitalists, workers and firms undertake their respective optimization problems; (ii) all budget constraints satisfied; and (iii) all markets clear.

Thus, the DCE consists of the capitalist's and worker's optimality conditions, i..e. OC^k and OC^w; the firm's first-order conditions for K_t^f and L_t^f; the budget constraints of capitalist, worker and government, i.e. BC^k, BC^w and BC^g; the production function, i.e. PF; and the per capita market clearing conditions in capital and labour markets, i.e. MC_K and MC.[10]

1.3 Calibration and model solution

1.3.1 Calibration and steady-state solution

The structural parameters of the model are calibrated using the annual data of the UK economy over the period 1970-2009. All the data is obtained from International Monetary Fund (IMF), United Nation Statistics Division (UNSD), the Office for National Statistics (ONS), OECD International Sectorial Data Base (ISDB) and OECD Eco-

[10] The full DCE conditions are provided in the Appendix 1.A.1. Relying on the Walras's law, if the budget constraints of capitalists and workers and the government budget constraint are satisfied, then the resource constraint is redundant and it can be dropped from the equilibrium.

nomic Outlook. The IMF data is from the World Economic Outlook (WEO) database. The UNSD databases include: (i) World Bank (WB) database; (ii) National Accounts Statistics (NAS) database; and (iii) International Financial Statistics (IFS) database. The ONS data is from Labour Force Survey (LFS) database. The OECD data is from OECD tax database.

The structural parameters of the model are assigned values so that the model's steady-state solution can reflect the main empirical characteristics of the current UK economy with particular focus on its unemployment rate. The calibrated values for the structural parameters are reported in Table 1.1 as follows.

Table 1.1: Calibration in the benchmark model

Parameter	Definition	Value
$0 < \beta < 1$	rate of time preference	0.970
$0 < \alpha_1 < 1$	capital's share of output	0.400
$0 < \alpha_2 < 1$	labour's share of output	0.600
$0 < \omega \leq 1$	degree of substitutability	0.400
$0 < \mu < 1$	weight of consumption	0.841
$0 < \delta^p < 1$	depreciation rate on capital	0.100
$0 < n^k < 1$	population share of capitalists	0.115
$\sigma > 0$	elasticity of substitution	2.000
$A_0 > 0$	TFP process	1.000
$0 \leq \tau_0^k < 1$	tax rate on capital income	0.344
$0 \leq \tau_0^w < 1$	tax rate on labour income	0.188
$0 < \overline{r}_0 < 1$	replacement rate	0.204
$0 < \rho^a < 1$	persistent parameter of A_t	0.914
$0 < \rho^k < 1$	persistent parameter of τ_t^k	0.971
$0 < \rho^w < 1$	persistent parameter of τ_t^w	0.971
$0 < \rho^r < 1$	persistent parameter of $\overline{r}_t$	0.972
$\sigma^a > 0$	s.d. of innovation ε_t^a	0.025
$\sigma^k > 0$	s.d. of innovation ε_t^k	0.015
$\sigma^w > 0$	s.d. of innovation ε_t^w	0.029
$\sigma^r > 0$	s.d. of innovation ε_t^r	0.013

The labour's share of output, $\alpha_2 = 0.6$, is obtained directly from the ISDB dataset. The capital's share of output is therefore: $\alpha_1 = 1 - \alpha_2 = 0.4$. The annual depreciation rate of capital stock is 10%, which is consistent with 2.5% quarterly depreciation rate of capital stock. The degree of substitutability across private and public consumption, ω, is set to 0.4. This is in line with Ahmed (1986, see Tables 1 and 2) who estimated this parameter for the UK economy. The elasticity of substitution between consumption and leisure, σ, is set to 2 which is common in the DGE literature. The steady-state TFP is normalised to 1. The normal distribution parameters are estimated by the TFP process. The steady-state values of exogenous policy instruments, $\left\{\tau_0^k, \tau_0^w, \overline{r}_0\right\}$, are set to their respective data averages and the parameters in $AR(1)$ equations, $(A1)-(A3)$, are estimated from the data series. All the tax data is obtained from OECD tax database.[11]

There are two common methods in the literature to calibrate the annual rate of time preference, β. It can be calibrated so that, $1/\beta - 1$, corresponds to the annual *ex-post* real interest rate. Alternatively, the consumption Euler equation of the capitalist can be used to calibrate the value for β. In this model, the second method for the calibration of β is applied in order to have the model's steady-state ratio of $\frac{K^p}{Y^f}$ to be in line with its data average.[12] The steady-state version of the consumption Euler equation is now re-arranged for β.[13] It yields:

$$\beta = \frac{1}{\left[1 + \left(1-\tau^k\right)\left(\alpha_1 Y^f/K^p - \delta^p\right)\right]}. \tag{35}$$

Using the values for δ^p and α_1 and the data average of $\frac{K^p}{Y^f}$, the calibrated value for β is 0.97. This is very close to 0.972 which is calibrated applying the first method.

The capitalists do not work in the model economy, but they can save in the form of private capital stock, own firms and receive dividends of firms. Following Ardagna (2007), the self-employed are treated as capitalists in the economy in order to calibrate the population share of capitalists, n^k. The data of self-employment only became available from 1992 for the UK economy in the LFS database. The data average is 0.115, so that $n^k = 0.115$. Finally, the value for μ is calibrated in order to get the steady-state unemployment rate of 7% which coincides with the data average between 1970 and 2009.

In the long run, the economy converges to a steady state when all the variables

[11] The average marginal tax rates on capital and labour income in the data are used for τ^k and τ^w. The replacement rate, $\overline{r}_0$, is a net rate after the deduction of taxes. The value for $\overline{r}_0$ is similar to Ardagna (2007).

[12] Data of aggregate capital stock is generated using perpetual inventory method.

[13] In what follows variables without time subscripts denote their steady-state values.

remain constant. The steady-state solution of the benchmark model for the above parameterization is shown in Table 1.2 below.[14]

Table 1.2: Steady-state solution of the benchmark model

Variable	Description	Value
C^k	consumption of capitalist	1.172
K^p	private capital stock	37.895
I^p	private investment	3.790
C^w	consumption of worker	0.899
H^w	labour supply of worker	0.930
Y^f	output of firm	13.940
r	return to private capital	0.147
w	real wage rate	1.169
$\overline{G}^c$	per capita government consumption	0.237
U^k	utility of capitalist	1.222
U^w	utility of worker	0.776
U	average utility	0.827

Table 1.3 below shows the steady-state ratios of aggregate capital stock, investment and consumption to output, and the steady-state employment generated by the model. The same table also gives their data averages.

Table 1.3: Data averages and benchmark model's steady-state values

Variable	Data average	Model
K^p/Y	2.720	2.720
I^p/Y	0.201	0.272
C/Y	0.597	0.581
G^c/Y	0.202	0.148
H^w	0.930	0.930

As can be seen from Table 1.3, the model's long-run solution matches most of the data averages well.

[14]In Table 1.2, U is defined in a Benthamite fashion as the aggregate welfare of capitalist and worker at steady-state, i.e. $U = \alpha^k U^k + \left(1 - \alpha^k\right) U^w$.

1.3.2 Schur decomposition

To solve the model, the method of generalised Schur decomposition is applied.[15] The non-linear DCE conditions and expectations of the exogenous $AR(1)$ processes can be expressed in the form of:[16]

$$E_t f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{x}_t\right) = 0 \tag{36}$$

where E_t stands for the expectation operator which is conditional on the information available at time t; $\mathbf{x}_t$ is a vector of the predetermined (state) variables of size $n_x \times 1$; and $\mathbf{y}_t$ is a vector of the non-predetermined (control) variables of size $n_y \times 1$. In the vector of $\mathbf{x}_t$, the endogenous state variables are written firstly. In our model, we have $\mathbf{x}_t = [K_t^p, A_t, \tau_t^k, \tau_t^w, \overline{r}_t]'$ and $\mathbf{y}_t = [C_t^k, C_t^w, H_t^w, Y_t^f, r_t, w_t, \overline{G}_t^c]'$.

All the non-linear equilibrium conditions in (36) are log-linearised round their steady state values of the variables.[17] The log-linearised system can be expressed in the following matrix form:

$$\mathbf{A} E_t \begin{bmatrix} \widehat{\mathbf{x}}_{t+1} \\ \widehat{\mathbf{y}}_{t+1} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \widehat{\mathbf{x}}_t \\ \widehat{\mathbf{y}}_t \end{bmatrix} \tag{37}$$

where the hatted variable denotes the natural log difference of the variable, e.g. $\widehat{K}_t^p = \ln K_t^p - \ln K^p$. It approximates the percentage deviation of variable from its steady-state value. $\mathbf{A}$ and $\mathbf{B}$ are two $\mathbf{12} \times \mathbf{12}$ coefficient matrices which specify the log-linearised equations.

The above system can be solved using generalised Schur or QZ decomposition algorithm. In the solution, it is found that all the eigenvalues are real. There is one eigenvalue with absolute value less than one. Therefore, this model exhibits saddle path stability as there is one endogenous state variable in it, i.e. K_t^p. This implies a unique convergent solution to the model. The model's solution can be represented in the form of first-order linear difference equations:

$$E_t \widehat{\mathbf{x}}_{t+1} = \mathbf{E} \widehat{\mathbf{x}}_t \tag{38}$$

[15] For more details, see Klein (2000).

[16] The variables K_t^f and L_t^f have been substituted out using the per capita market clearing conditions for private capital and labour, i.e. MC_K and MC_L.

[17] See Appendix 1.A.2 for the mathematical details of log-linearization.

and

$$\widehat{\mathbf{y}}_t = \mathbf{F}\widehat{\mathbf{x}}_t \tag{39}$$

where $\mathbf{E}$ and $\mathbf{F}$ are $\mathbf{5} \times \mathbf{5}$ and $\mathbf{7} \times \mathbf{5}$ coefficient matrices specifying the laws of motion for the variables in vectors $\mathbf{x}_t$ and $\mathbf{y}_t$.[18]

1.4 Impacts of TFP and fiscal policy shocks

Using the dynamic solution of the benchmark model given in (38) and (39), we now study the dynamic behavior of the model economy in response to a temporary unitary innovation to TFP and the exogenous policy instruments: capital income tax, labour income tax and replacement rate.[19] The exogenous and temporary policy changes can affect both the supply- and demand-side of the economy during the transition to the initial steady-state, but not in the long run. The household heterogeneity has been assumed by isolating the different economic roles in the economy. In effect, policy changes would impact on their behaviors differently so that these fiscal policy shocks will have different effects on the welfare of capitalists and workers.

The effects of one-period, unanticipated 1% increase in each of the exogenous variables are studied in the following sub-sections. We keep all the other exogenous variables fixed at their steady-states. In each experiment, the post-shock economy is simulated for 100 years and the data are presented as percent deviations from variables' respective steady-states. The impulse responses of key variables are illustrated in Figures 1.1-1.4, respectively. In Tables 1.4-1.7, the present discounted values of percent deviations from the steady-states of variables are reported in years after the shock which are calculated using the following formula:

$$D_j^X = \frac{\sum_{t=1}^{j} \beta^{t-1} X_t}{\sum_{t=1}^{j} \beta^{t-1} X} - 1 \tag{40}$$

where j denotes the number of years after the shock; X is the steady-state value of variable; X_t is the value of variable for year t; and D_j^X is the percent deviation after j years. All the $t > 1$ values are discounted by the parameter, $0 < \beta < 1$.

[18] In order to save space, we have not presented the policy rules in (38) and (39). But they are available upon request.

[19] See Appendix 1.A.3 for the mathematical details of the simulation.

1.4.1 Shock to TFP

Figure 1.1 illustrates the effects of the one-period, unanticipated 1% increase in TFP on the key variables and Table 1.4 reports the numeric results of percent deviations of variables after j years in the post-shock economy.

[Figure 1.1 about here]

Table 1.4: Percent deviation from steady-state after A_t shock

	Years after A_t shock							
	1	3	5	10	20	50	80	100
C_t^k	-3.341	-2.288	-1.512	-0.363	0.370	0.433	0.377	0.361
C_t^w	1.027	1.019	0.997	0.916	0.746	0.495	0.426	0.408
H_t^w	0.064	0.065	0.064	0.059	0.049	0.033	0.028	0.027
L_t^w	-0.853	-0.857	-0.846	-0.787	-0.648	-0.432	-0.372	-0.356
Y_t^f	1.044	1.036	1.015	0.932	0.759	0.504	0.433	0.415
K_t^p	0.000	0.185	0.311	0.468	0.498	0.364	0.314	0.301
$\overline{G}_t^c$	1.769	1.638	1.520	1.281	0.969	0.623	0.535	0.513
U_t^k	-2.537	-1.710	-1.102	-0.207	0.354	0.383	0.333	0.319
U_t^w	1.004	0.985	0.956	0.867	0.699	0.463	0.398	0.381
U_t	0.402	0.527	0.607	0.685	0.641	0.449	0.387	0.370

The innovation to TFP impacts on the model economy via two effects: wealth effect and substitution effect. They are accompanied by the changes in input prices. This positive TFP shock generates one immediate positive effect on firms' output which can be seen from the production function. The output, Y_t^f, goes up immediately following the shock (i.e. 1.044%). This is a positive wealth effect generated by the innovation in TFP. As a result, more goods are available for consumption and to be invested at the aggregate level. It leads to an immediate increase in the government consumption by holding all the other policy instruments unchanged in its budget constraint. Government consumption increases by 1.769% relative to its initial steady-state. Meanwhile, the return to capital and to labour increases immediately in response to the shock as can be seen in Figure 1.1. This is because the innovation in TFP increases the marginal product of capital and labour. The capitalists spend the disposable income on consumption or investment. The higher return to capital increases the attractiveness of investment. The capitalists increase investment and this could bring them higher income to finance the future spending. In turn, the accumulation of capital stock increases in response to the TFP shock. The transition path of capital stock is converted U-shaped. Figure 1.1 shows that the consumption of capitalists decreases in the first few years before it rises above its initial steady-state. There are two effects resulting in the initial decrease in the consumption of capitalists. On one hand, there exists substitutability between private consumption and government consumption. On the other hand, the increase in private investment crowds out the private consumption.

The workers are assumed to not save in the economy. If the workers are employed, they obtain wage incomes from the firms. The return to work increases immediately in the post-shock economy. This tends to increase the attractiveness of working. There exist two opposing effects in determining the labour supply of workers as a result of the higher return to labour, in other words, the higher wage rate. On one hand, the higher wage rate implies an increase in the opportunity cost of leisure. This increases the benefits of working and it induces workers to substitute labour for leisure. This is called a positive substitution effect (SE) resulting from a higher wage rate. On the other hand, the higher wage rate increases the wage income of workers. As a result, they value leisure more relative to working. This is called a negative income effect (IE). The dynamic path of the labour supply illustrates that the substitution effect outweighs the income effect in our model. This result is consistent with the calibrated value for the elasticity of substitution in the CES utility function. The parameter $\sigma > 1$ implies that the substitution effect is bigger than the income effect. As a result, the labour supply increases immediately after the shock (i.e. 0.064%) which is beneficial for the workers. The higher labour supply increases the income of workers as working can generate higher income for workers. As a result, the consumption of workers increases immediately in response to the innovation in TFP (i.e. 1.027%).

In the long run, the return to capital and labour recovers to their respective initial steady-state. Subsequently, all the other variables gradually return to their steady-states as the positive TFP shock dies out. In the process of adjustment, all the variables display some inherent persistence. On the whole, as can be seen in Figure 1.1, the TFP shock has a positive impact on the aggregate economy.

1.4.2 Shock to capital income tax

In the following three sub-sections, the effects of shocks to fiscal policy instruments on the economy are analysed. Figure 1.2 provides the dynamic paths of variables to the one-period, unanticipated 1% increase in the tax rate on capital income, τ_t^k, and Table 1.5 shows the present values of percent deviations in different j years after this positive shock.

[Figure 1.2 about here]

Table 1.5: Percent deviation from steady-state after τ_t^k shock

	Years after τ_t^k shock							
	1	3	5	10	20	50	80	100
C_t^k	0.782	0.598	0.456	0.224	0.025	-0.075	-0.077	-0.076
C_t^w	-0.001	-0.016	-0.027	-0.045	-0.056	-0.054	-0.049	-0.047
H_t^w	-0.002	-0.003	-0.004	-0.005	-0.006	-0.005	-0.004	-0.003
L_t^w	0.026	0.040	0.050	0.066	0.075	0.068	0.062	0.060
Y_t^f	-0.001	-0.017	-0.028	-0.046	-0.058	-0.055	-0.050	-0.048
K_t^p	0.000	-0.037	-0.065	-0.108	-0.136	-0.130	-0.119	-0.115
$\overline{G}_t^c$	0.295	0.284	0.274	0.253	0.223	0.175	0.157	0.151
U_t^k	0.638	0.492	0.379	0.194	0.034	-0.048	-0.051	-0.050
U_t^w	0.027	0.014	0.004	-0.012	-0.029	-0.027	-0.025	-0.024
U_t	0.131	0.095	0.068	0.023	-0.015	-0.031	-0.030	-0.029

The isolating positive shock to capital income tax immediately generates a positive wealth effect on the government consumption since more tax revenues can be provided to finance government spending. The increase in τ_t^k results in a concurrent increase in $\overline{G}_t^c$ by holding all the other policy instruments constant in the budget constraint of government (i.e. 0.295%). The dynamic responses of capitalists are then analysed. This positive shock causes a negative wealth effect on the investment of capitalists. A higher tax rate on capital income implies lower net return to capital. In this case, the investment of capitalists decreases dramatically immediately in response to the capital tax increase. This negative wealth effect is transformed into a decrease in the capital stock. On one hand, the decrease in K_t^p results in a fall in the output, Y_t^f (i.e. -0.001%). On the other hand, the lower level of capital stock implies a higher return to capital which stimulates the investment of capitalists. The private investment, I_t^p, gradually rises and in the end recovers to its initial steady-state. The consumption of capitalists increases in the first few years although it falls short of the initial steady-state afterwards. The increase in the consumption of capitalists in the first few years is resulted from the dramatic decrease in investment.

In the labour market, the marginal product of labour decreases when the output falls. This implies a decrease in the return to labour. As a result, the labour supply of workers decreases. The decrease in labour supply reinforces the fall in the output. Table 1.5 shows that the output, Y_t^f, continuously falls in the first twenty years (i.e. from -0.001% to -0.058%). The labour supply gradually recovers to the steady-state, the firms increase the production and output finally returns to its initial steady-state.

1.4.3 Shock to labour income tax

The time paths of variables generated by the one-period, positive shock to the labour income tax are shown in Figure 1.3. Table 1.6 shows the results calculated using the formula (40) for different variables. In general, the temporary, unanticipated increase in labour income tax has a negative impact on the aggregate economy.

[Figure 1.3 about here]

Table 1.6: Percent deviation from steady-state after τ_t^w shock

	Years after τ_t^w shock							
	1	3	5	10	20	50	80	100
C_t^k	0.084	0.063	0.048	0.022	0.001	-0.010	-0.011	-0.010
C_t^w	-0.238	-0.233	-0.228	-0.216	-0.194	-0.155	-0.139	-0.134
H_t^w	-0.033	-0.032	-0.031	-0.030	-0.027	-0.021	-0.019	-0.018
L_t^w	0.444	0.433	0.422	0.398	0357	0.284	0.254	0.245
Y_t^f	-0.020	-0.021	-0.022	-0.023	-0.022	-0.019	-0.017	-0.016
K_t^p	0.000	-0.004	-0.007	-0.012	-0.015	-0.014	-0.013	-0.012
$\overline{G}_t^c$	0.704	0.684	0.665	0.622	0.554	0.440	0.393	0.379
U_t^k	0.111	0.094	0.080	0.057	0.036	0.020	0.017	0.016
U_t^w	-0.120	-0.118	-0.116	-0.111	-0.100	-0.080	-0.072	-0.069
U_t	-0.081	-0.082	-0.083	-0.082	-0.077	-0.063	-0.057	-0.055

The increase in tax rate on labour income brings about more tax revenues which could directly increase government consumption for the first year (i.e. 0.704%). Meanwhile, higher labour income tax has a negative revenue effect on workers. It reduces the disposable income of workers. The consumption of workers reduces by 0.238% for the first year relative to its initial steady-state. The increase in τ_t^w decreases the net return to work. This has a negative effect on the labour supply of workers. The wage rate is determined in the competitive labour market. The gross return to work becomes higher as the supply of labour decreases. As a result, the workers increase labour supply gradually. In the long run they will offer the initial level of labour supply.

The output, Y_t^f, goes down for the first year (i.e. -0.020%). As a consequence, fewer goods are available for consumption and investment in the post-shock economy such that the investment of capitalists reduces. This negative effect is transformed into lower capital stock, which reinforces the fall in firms' output. The consumption of capitalists increases for the first few years. This is because the fall in private investment outweighs the decrease in output. In turn, more income is used for consumption. However as the capitalists increase investment over time, the consumption falls short of its initial steady-state. The higher capital stock due to increase in investment leads to an increase in firms' output.

1.4.4 Shock to replacement rate

Finally, the responses of variables to the one-period, unanticipated 1% increase in replacement rate, $\overline{r}_t$ are investigated. This is a positive shock to government spending. The impact of this shock is illustrated by Figure 1.4 and Table 1.7 reports the percentage deviations of key variables in the post-shock economy.

[Figure 1.4 about here]

Table 1.7: Percent deviation from steady-state after $\overline{r}_t$ shock

	Years after $\overline{r}_t$ shock							
	1	3	5	10	20	50	80	100
C_t^k	0.133	0.102	0.078	0.039	0.005	-0.012	-0.013	-0.012
C_t^w	0.003	2E-04	-0.002	-0.004	-0.007	-0.008	-0.007	-0.006
H_t^w	-0.047	-0.046	-0.045	-0.042	-0.038	-0.030	-0.027	-0.026
L_t^w	0.624	0.609	0.595	0.562	0.506	0.405	0.363	0.350
Y_t^f	-0.028	-0.030	-0.031	-0.033	-0.032	-0.027	-0.025	-0.024
K_t^p	0.000	-0.006	-0.011	-0.018	-0.023	-0.022	-0.020	-0.019
$\overline{G}_t^c$	-0.150	-0.146	-0.143	-0.135	-0.121	-0.097	-0.087	-0.084
U_t^k	0.096	0.071	0.053	0.022	-0.003	-0.016	-0.015	-0.014
U_t^w	0.018	0.016	0.014	0.010	0.007	0.005	0.004	0.003
U_t	0.032	0.025	0.020	0.012	0.005	0.001	4E-04	3E-04

In this model, there are two types of government expenditures: government consumption on goods and services and unemployment benefits. The positive shock to replacement rate implies that the government increases unemployment benefits to unemployed workers. It causes a negative crowding-out effect on the government consumption so that it decreases in the post-shock economy (i.e. -0.150%). The unemployment benefits are equivalent to a subsidy to labour. There are two opposite effects in determining the consumption of workers. On one hand, the positive shock to replacement rate generates a positive wealth effect on the consumption of workers as higher replacement rate increases the income of workers. On the other hand, higher replacement rate increases the attractiveness of leisure. This discourages the labour supply of workers. As a result, the labour supply of workers decreases dramatically for the first year (i.e. -0.047%). The consumption of workers decreases as the leisure is costly to workers. As can be seen in Figure 1.4, the negative effect dominates and the consumption of workers decreases in the post-shock economy.

The firms' output, Y_t^f, goes down for the first year (i.e. -0.028%). This has a positive effect on the wage rate. The wage rate rises and higher return to work increases the labour supply of workers. The marginal product of capital decreases in the post-shock economy. In the capital market, the return to capital falls and it leads to a decrease in the private investment of capitalists. This is transformed into lower capital stock. In turn, the consumption of capitalists increases. In addition, the decrease in government consumption generates a further increase in the consumption of capitalists via the substitution effect.

All the variables will return to their respective steady-states in the long run. In response to this positive shock to replacement rate, most variables display large persistence, with the exception of two input prices, i.e. r_t and w_t.

1.5 Optimal policy with commitment

1.5.1 Ramsey problem

In the commitment framework, the government takes into account that the households and firms will behave in their own best interest by taking all the fiscal policy variables as given. Each applicable fiscal policy implies a feasible equilibrium allocation that fully reflects the optimal behavioral responses of resources. Given a welfare criterion, the optimization problem for the government is to pick the best fiscal policy which can produce an equilibrium allocation giving the highest aggregate welfare. To avoid the general time inconsistency problem in policy making, the government is assumed to commit itself once-and-for-all to one fiscal policy which is announced at initial period and never re-optimizes.[20] This problem is usually referred to as the Ramsey problem of government under commitment.

The government now optimally chooses some of its policy instruments. Meanwhile, it also chooses the allocation of private agents. This is called the dual approach to the Ramsey problem.[21] The objective of government is to maximize the present discounted value of a weighted average of capitalists' and workers' welfare:

$$\sum_{t=0}^{\infty} \beta^t \left[\gamma U_t^k + (1-\gamma) U_t^w\right] \tag{41}$$

where the government is assumed to have the same discount rate as households; and $0 < \gamma, (1-\gamma) < 1$ are the weights attached to the welfare of capitalists and workers by the government.

The optimal policy approach emphasizes the constraints under which the government must operate. These constraints include the requirement to raise enough tax revenues and the behavioral responses of households and firms. These are summarized in the DCE conditions. In order to simplify the optimization problem of the government - it is necessary to reduce the number of choice variables for the government, we substitute out, r_t, w_t, K_t^f, L_t^f and Y_t^f, by making use of some DCE conditions. The per capita government consumption, $\overline{G}_t^c$, is assumed to be constant in the Ramsey problem, i.e. $\overline{G}_t^c \equiv \overline{G}^c$ for all periods.[22] To summarize, in the dual approach of the Ramsey problem, the choice variables for the government are four allocation variables, $\left\{C_t^k, H_t^w, C_t^w, K_{t+1}^p\right\}_{t=0}^{\infty}$ and three policy variables $\left\{\tau_t^k, \tau_t^w, \overline{r}_t\right\}_{t=0}^{\infty}$. The initial condition for K_0^p is taken as given. The optimization problem can thus be summarized as follows:

$$\max_{\left\{C_t^k, H_t^w, C_t^w, K_{t+1}^p, \tau_t^k, \tau_t^w, \overline{r}_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma U\left(C_t^k\right) + (1-\gamma) U\left(C_t^w\right)\right] \tag{42}$$

[20] The time inconsistency refers to that when the government revises its policy annouced initially if it has a chance to do so.

[21] In contrast to the dual approach, the government only chooses the allocation of private agents and all the policy variables are substituted out using DCE conditions in the primal approach.

[22] Its value is set to the steady-state value as in Table 1.2.

subject to the DCE conditions of

$$\begin{aligned}
& \left(C_t^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_t^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}} \\
= \; & \beta E_t\left\{\left(C_{t+1}^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_{t+1}^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}}\cdot\right. \\
& \left.\cdot\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}-\delta^p\right)\right]\right\}
\end{aligned} \tag{D1}$$

$$\begin{aligned}
& (1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu\overline{r}_t w_t\left(C_t^w+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}} \\
= \; & \mu w_t\left(1-\tau_t^w\right)\left(C_t^w+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}
\end{aligned} \tag{D2}$$

$$C_t^k+K_{t+1}^p-\left(1-\delta^p\right)K_t^p=r_tK_t^p-\tau_t^k\left(r_t-\delta^p\right)K_t^p \tag{D3}$$

$$C_t^w=\left(1-\tau_t^w\right)w_tH_t^w+\overline{r}_tw_t\left(1-H_t^w\right) \tag{D4}$$

$$\overline{G}^c+\left(1-n^k\right)\overline{r}_tw_t\left(1-H_t^w\right)=n^k\tau_t^k\left(r_t-\delta^p\right)K_t^p+\left(1-n^k\right)\tau_t^ww_tH_t^w. \tag{D5}$$

The Lagrangian function of the government can be written as:

$$\begin{aligned}
L^g = \; & E_0\sum_{t=0}^{\infty}\beta^t\left\{\gamma\left[\mu\left(C_t^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)(1-0)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}+\right. \\
& +(1-\gamma)\left[\mu\left(C_t^w+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\left(1-H_t^w\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}+ \\
& +\lambda_t^1\left[\beta\left(C_{t+1}^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_{t+1}^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}}\cdot\right. \\
& \cdot\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}-\delta^p\right)\right]- \\
& \left.-\left(C_t^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_t^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}}\right]+ \\
& +\lambda_t^2\left[\mu w_t\left(1-\tau_t^w\right)\left(C_t^w+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\right. \\
& \left.-(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}-\mu\overline{r}_tw_t\left(C_t^w+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\right]+ \\
& +\lambda_t^3\left[r_tK_t^p-\tau_t^k\left(r_t-\delta^p\right)K_t^p-C_t^k-K_{t+1}^p+\left(1-\delta^p\right)K_t^p\right]+ \\
& +\lambda_t^4\left[\left(1-\tau_t^w\right)w_tH_t^w+\overline{r}_tw_t\left(1-H_t^w\right)-C_t^w\right]+ \\
& +\lambda_t^5\left[n^k\tau_t^k\left(r_t-\delta^p\right)K_t^p+\left(1-n^k\right)\tau_t^ww_tH_t^w-\right. \\
& \left.\left.-\overline{G}^c-\left(1-n^k\right)\overline{r}_tw_t\left(1-H_t^w\right)\right]\right\}
\end{aligned} \tag{43}$$

where λ_t^i, $i=1,2,\cdots,5$, represents the multiplier associated with each constraint in $(D1)-(D5)$. The constraints in the Lagrangian function have been rearranged so that all the multipliers are non-negative at the steady-state.

Some FOCs of the government at time 0 are different from the same rules governing behavior from time 1 on. Specifically, these include the FOCs of C_t^k, H_t^w and τ_t^k and these variables appear in the forward-looking inter-temporal optimality condition $(D1)$. To avoid this problem, it is necessary to consider the Ramsey problem in the economy starting from time 1 and assume that time 0 optimality conditions of the government do not alter the results in equilibrium.

In addition, the FOCs of the government should also include the constraints to the Ramsey problem, i.e. $(D1) - (D5)$.[23]

1.5.2 Benthamite (non-partisan) optimal taxation

The first case to be studied is that of a Benthamite government. This implies that the weights attached to the welfare of capitalists and workers in the objective function of government are equal to their respective population shares, i.e. $\gamma = n^k$ and $1 - \gamma = 1 - n^k$. Using the parameters in Table 1.1, we can get the steady-state solution of optimal policy which is shown in Column (1) of Table 1.8. It is compared to the steady-state solution with exogenous policy as reported in Column (2) of Table 1.8.

Table 1.8 incorporates the following findings. First, in the absence of profits, the celebrated result of Judd (1985) and Chamley (1986) is verified: the optimal capital tax is zero in the long run. This implies that capitalists are exempted from paying taxes in the long run. All the government expenditures should be financed by the taxes on workers.[24] This result is silent about the transition to the steady-state. If τ^k is positive, it reduces the return from today's savings and therefore makes the consumption of next period more expensive relative to current period. In the model with infinitely-lived households, the long-run positive tax rate on capital income implies that the implicit tax rate on consumption of future periods increases without bound. However, the relevant elasticity of demand for consumption in all periods is constant. Therefore taxing consumption at different rates violates the general public finance principle stating that tax rates should be inversely proportional to the demand elasticities of consumption. The assumption of constant demand elasticity of consumption implies that the capital income tax rate should be zero in the long run. As a result, zero capital income tax stimulates the investment of capitalists (i.e. from 3.79 to 4.833), and this

[23] We did not show the FOCs of government in the Ramsey problem to preserve space. But they are available upon request.

[24] In the model with non-zero economic profits, this result does not hold any more. The two opposing effects on the sign of optimal tax rate on capital income will be demonstrated later.

is transformed into higher capital stock (i.e. from 37.895 to 48.332).

Table 1.8: Steady-state of Benthamite optimal taxation in the model without profits

Variable	Ramsey (1)	Exogenous (2)
C^k	1.495	1.172
K^p	48.332	37.895
I^p	4.833	3.790
C^w	0.966	0.899
H^w	0.976	0.930
Y^f	15.820	13.940
K^p/Y	3.055	2.720
I^p/Y	0.306	0.272
C/Y	0.564	0.580
G^c/Y	0.130	0.148
r	0.131	0.147
w	1.263	1.168
τ^k	0	0.344
τ^w	0.212	0.188
$\overline{r}$	-0.212	0.204
U^k	1.487	1.222
U^w	0.794	0.776
U	0.873	0.827

Second, the optimal replacement rate turns out to be negative in the long run. It predicts that the government taxes unemployed workers rather than offer unemployment benefits. In this model, the difference between the level of potential labour supply and the level of labour supply chosen by the workers is treated as unemployment. A negative replacement rate implies that the government taxes those workers who do not provide the potential level of labour. In this sense, leisure generates income losses for workers. Alternatively, we can understand the negative replacement rate as a subsidy to the labour supply of workers.[25] Therefore, the negative, $\overline{r}$, leads to an increase in the

[25] The budget constraint of workers (12) at the steady state can be rewritten as: $C^w = (1-\tau^w-\overline{r})\,wH^w + \overline{G}^u$. A negative replacement rate therefore implies that the government subsidizes the labour supply of workers. The last term, $\overline{G}^u = \overline{r}w < 0$, can be considered as a lump-sum tax paid by the workers at the steady-state which does not generate any distortion in the economy.

labour supply of workers. The optimal tax rate on labour income, τ^w, slightly increases relative to the exogenous policy case (i.e. from 0.188 to 0.212). This, in contrast, has a negative effect on the labour supply. Overall, the labour supply increases resulting from the dominant positive effect of negative replacement rate (i.e. from 0.930 to 0.976).

Third, the labour wedge defined by $1 - \tau^w - \overline{r}$ is equal to one at the steady state. This implies that the marginal product of labour is equal to the marginal rate of substitution between labour and consumption. In the words, in absence of profits, the labour wedge can be completely eliminated by the government in the long run. This happens because, in Ramsey, the government optimally chooses two tax rates on workers, i.e. τ_t^w and $\overline{r}_t$. At the steady state, the wedge between the marginal product of labour and the marginal rate of substitution between labour and consumption created by τ^w is exactly canceled out by the negative $\overline{r}$. As a result, there is no distortion in the labour market. This result also explains the large increase in labour supply which is consistent with the finding in Prescott (2002 and 2004).

Finally, the output, Y^f, increases substantially at the steady-state (i.e. from 13.940 to 15.820) as two production inputs, capital and labour, both increase in the production. This generates positive welfare effects on private consumption and investment as can be seen in aggregate resource constraint. The consumption of capitalists and workers increases more than it would be in the exogenous policy case. The welfare of all agents improves and the Ramsey solution is Pareto improving in the long run.

1.5.3 Non-Benthamite (partisan) optimal taxation

The next case to be investigated is that of a partisan government. In other words, the weights attached to the welfare of each agent in Ramsey problem are not equal to the population share of each agent so that the government is biased towards one party. Table 1.9 reports the steady-state solutions of the optimal policy under different values of γ.[26] The case of Benthamite government is in bold.

As in Judd (1985), we find that the optimal taxation and allocation under commitment are independent of weights attached to the welfare of agents. This implies that, for all agents, the zero capital tax and elimination of labour wedge are the best option to adopt in the Ramsey set-up of government. This holds even if the government cares only about the workers, so that there is no conflict of interests between agents. In the next section, the relevance of economic profits in determining this result will be

[26] The range of γ corresponds to that in the modified model (see Table 1.11).

investigated, in other words, we examine whether the commonality of interests still holds in a model with strictly positive profits.

Table 1.9: Steady-state of Partisan optimal taxation in the model without profits

	$\gamma = 0$	$\gamma = 0.105$	$\gamma = 0.110$	$\boldsymbol{\gamma = 0.115}$	$\gamma = 0.120$	$\gamma = 0.125$
	(1)	(2)	(3)	**(4)**	(5)	(6)
C^k	1.495	1.495	1.495	**1.495**	1.495	1.495
C^w	0.966	0.966	0.966	**0.966**	0.966	0.966
H^w	0.976	0.976	0.976	**0.976**	0.976	0.976
τ^k	0.000	0.000	0.000	**0.000**	0.000	0.000
τ^w	0.212	0.212	0.212	**0.212**	0.212	0.212
$\overline{r}$	-0.212	-0.212	-0.212	**-0.212**	-0.212	-0.212
U^k	1.487	1.487	1.487	**1.487**	1.487	1.487
U^w	0.794	0.794	0.794	**0.794**	0.794	0.794
U	0.873	0.873	0.873	**0.873**	0.873	0.873

1.6 Equilibrium unemployment in a model with profits

In this section, one extension to the benchmark model is made by introducing equilibrium profits into the model. Specifically the profits appear in the economy when the public investment appears in the production. Next to be studied is the optimal taxation and its effects on unemployment, the distribution of income and welfare of agents. We intend to investigate the implications of this modification for the results discussed in the benchmark model above.

1.6.1 Model extension

It is now assumed that the government can invest in the production of goods. The government provides individual firms with public capital without asking for rents. Following Lansing (1998) and Malley *et al.* (2009), the firm produces homogeneous goods with a CRTS technology in labour, private capital and public capital.[27] The production function of the representative firm is given by:

$$Y_t^f = A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2} \left(\overline{K}_t^g\right)^{\alpha_3} \tag{44}$$

[27]See Aschauer (1989), Munell (1990) and Ai and Cassou (1995). These empirical studies support for the specification of CRTS in these three inputs.

where $\overline{K}_t^g$ denotes the per capita public capital which is exogenously provided by the government; and $0 < \alpha_3 < 1$ measures the public capital's share of output. The CRTS technology implies $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

The specification of CRTS in all three inputs implies that the profits of individual firm are non-zero in equilibrium. Profit-maximization of the firm yields:

$$\begin{aligned} \pi_t^f &= Y_t^f - r_t K_t^p - w_t H_t^f \\ &= Y_t^f - \frac{\alpha_1 Y_t^f}{K_t^p} K_t^p - \frac{\alpha_2 Y_t^f}{H_t^f} H_t^f \\ &= (1 - \alpha_1 - \alpha_2) Y_t^f \end{aligned} \tag{45}$$

where the optimality conditions of the firm are the same as in the previous model, i.e. $r_t = \frac{\alpha_1 Y_t^f}{K_t^p}$ and $w_t = \frac{\alpha_2 Y_t^f}{H_t^f}$. In equilibrium, the firm earns strictly positive economic profits which are equal to the difference between the value of output and the production costs of inputs employed from the capitalists and workers. The profits are equally distributed to the capitalists. The per capita market clearing condition for dividends is given by:

$$\pi_t^k = \pi_t^f \equiv \pi_t^p. \tag{46}$$

This extension does not alter the optimality conditions of the worker which are described by (17) and (12). The consumption Euler equation of the capitalist still holds but the term involving, π_t^p, should be recovered in his budget constraint. The budget constraint of the capitalist is rewritten as:

$$C_t^k + K_{t+1}^p - (1 - \delta^p) K_t^p = r_t K_t^p - \tau_t^k (r_t - \delta^p) K_t^p + (1 - \tau_t^k) \pi_t^p. \tag{47}$$

Finally, the per capita government budget constraint should be rewritten:

$$\begin{aligned} & \overline{G}_t^c + \left(1 - n^k\right) \overline{r}_t w_t (1 - H_t^w) + n^k \overline{I}_t^g \\ = \; & n^k \left[\tau_t^k (r_t - \delta^p) K_t^p + \tau_t^k \pi_t^p\right] + \left(1 - n^k\right) \tau_t^w w_t H_t^w \end{aligned} \tag{48}$$

where $\overline{I}_t^g = \frac{I_t^g}{N^f}$ is the per capita public investment where I_t^g is the aggregate public investment

The aggregate public capital stock, K_t^g, evolves according to:[28]

$$K_{t+1}^g = (1-\delta^g)K_t^g + I_t^g \tag{49}$$

where δ^g is the constant depreciation rate of public capital stock. The public capital and private capital are assumed to depreciate at the same rate, so that $\delta^g = 0.1$.[29]

1.6.2 Benthamite (non-partisan) optimal taxation

Columns (1) and (2) in Table 1.10 report the stead-states of the modified model with optimal and exogenous fiscal policy, respectively.[30] The case of a Benthamite government is first studied. We intend to examine the relevance of non-zero profits in determining the long-run optimal taxation and therefore the steady-state allocation of resources.

First, the result of long-run optimal zero capital tax rate cannot be obtained in the modified model. This result is consistent with what has been found by Lansing (1998). He argues that the existence of profits, together with the assumption that the government cannot distinguish between profits and other asset incomes can result in the non-zero optimal capital income tax in the long run. The steady-state optimal tax rate on capital income is negative.[31] This implies that it is optimal for the government to subsidize the interest income from capital and profits in the long run and it is accomplished by increasing the labour income tax. Guo and Lansing (1999) show that in an imperfectly competitive economy, the sign of the steady-state optimal capital income tax is ambiguous and find that this ambiguity mainly results from two opposite effects: under-investment effect and profit effect. The under-investment effect arises when the private agent under-invests relative to the socially optimal level as the interest rate that determines the equilibrium investment is smaller than the social marginal

[28] The aggregate public capital stock is the sum of public capital stock that each firm receives from the government, i.e. $K_t^g = N^f \overline{K}_t^g$.

[29] Because in what follows, the focus will only be on the steady-state analysis of the model, the ratio of aggregate public investment to aggregate output, g^i, is set to the data average. In the Ramsey setup, the government optimally chooses K_{t+1}^g, and I_t^g is substituted out using the public capital evolution equation, (49).

[30] The results reported in Column (2) are obtained using the parameters in Table 1.1, except that, μ, is re-calibrated so that the steady-state value of $H^w = 0.930$ can be achieved at the steady-state of the modified model.

[31] Judd (1997) shows that the tax rate on capital income is ambiguous if the government does not distinguish between taxing returns on new investment and taxing economic profits. His paper, however, mainly studies on the sub-optimality of a capital income tax. Judd (1999) argues that a tax on capital cannot be optimal as its distortions accumulate over time, a pattern that is inconsistent with the commodity tax principle. Later, Judd (2002) proposes an optimal capital income subsidy referring to the repealed Investment Tax Credit scheme in the US economy.

product of capital. Therefore, a negative tax rate on capital income helps to correct the existence of under-investment in the capital market. The profit effect, in contrast, motivates the use of a positive tax rate on capital income, because taxing profits does not affect private agent's decisions at the margin such that it does not distort incentives of investment. In this case, the government has an incentive to fully confiscate the profits. This motivates a positive tax rate on capital income. In the model with the presence of public investment, the crowding-out of the public investment is equivalent to the under-investment effect and it dominates the profit effect. As a result, the steady-state optimal tax rate on capital income turns out to be negative. The negative capital tax increases the private investment of capitalists (i.e. from 3.089 to 4.047). This is transformed into higher private capital (i.e. from 30.892 to 40.472). In turn, the output, Y^f, goes up at the steady-state (i.e. from 12.122 to 13.760).

Second, the government increases the tax rate on labour income. As in the benchmark model, the long-run optimal replacement rate is negative. In other words, the government provides a subsidy to the labour supply of workers. The optimal labour income tax and replacement rate generate two opposite effects on the labour supply of workers in the long run. On one hand, higher labour tax implies lower return to work. This tends to reduce the labour supply of workers. On the other hand, the negative replacement rate working as a subsidy to work tends to increase the labour supply of workers. On the whole, the replacement rate effect dominates and labour supply goes up relative to the exogenous policy case (i.e. from 0.930 to 0.976).

Finally, the labour wedge, $1-\tau^w-\overline{r}$, is no longer equal to one at the steady-state in the modified model. This implies that there exists a discrepancy between the marginal rate of substitution between labour and consumption and the marginal product of labour. The labour market distortion exists in the modified model. The Benthamite optimal taxation, with the presence of profits, generates conflict of interests between agents. It leads to distributional effects in the long run. This is because the under-investment distortion is so large in the capital market that it increases the incentive for the government to impose a subsidy to capital income. In turn, this reduces the incentive for the government to eliminate the distortion in the labour market. Because of labour market distortion, the welfare of workers decreases in the Ramsey setup (i.e. from 0.680 to 0.676). The welfare of capitalists increases (i.e. from 1.191 to 1.592) as the subsidy to capital income together with the profits increases the income and consumption of capitalists. The optimal policy increases the aggregate welfare (from

0.739 to 0.782) despite welfare losses for the workers in the long run.

Table 1.10: Steady-state of Benthamite optimal taxation in the model with profits

	Ramsey	Exogenous
Variable	(1)	(2)
C^k	1.639	1.154
K^p	40.472	30.892
I^p	4.047	3.089
C^w	0.813	0.782
H^w	0.976	0.930
Y^f	13.760	12.122
K^p/Y	2.941	2.549
I^p/Y	0.294	0.255
K^g/Y	0.189	0.250
I^g/Y	0.019	0.025
C/Y	0.574	0.592
G^c/Y	0.113	0.129
r	0.128	0.147
w	1.099	1.016
τ^k	-0.125	0.344
τ^w	0.237	0.188
$\overline{r}$	-0.226	0.204
U^k	1.592	1.191
U^w	0.676	0.680
U	0.782	0.739

1.6.3 Non-Benthamite (partisan) optimal taxation

The steady-state of Ramsey problem is next studied when the government becomes partisan. We solve the model and evaluate the model's steady-state for different weight attached to the welfare of capitalists in government's objective function.[32] Figure 1.5

[32] The steady-state is calculated when γ lies in [0, 0.125]. Beyond 0.125, the incentive for the government to subsidise capital income in order to eliminate capital distortion is so high that an interior solution cannot be obtained. However, our evaluation captures all three cases of the government: Benthamite, "capitalist bias", and "worker bias". Moreover, the changes of variables at steady-state are monotonic with the magnitude of γ.

below plots the steady-state values for the policy instruments, equilibrium allocations and welfare of different agents against the weight attached to the welfare of capitalists, γ.

[Figure 1.5 about here]

We also produce Table 1.11 to compare with the steady-state values in the model with zero profits as in Table 1.9.

Table 1.11: Steady-state of Partisan optimal taxation in the model with profits

	$\gamma = 0$	$\gamma = 0.105$	$\gamma = 0.110$	$\boldsymbol{\gamma = 0.115}$	$\gamma = 0.120$	$\gamma = 0.125$
	(1)	(2)	(3)	**(4)**	(5)	(6)
C^k	1.200	1.537	1.581	**1.639**	1.723	1.929
C^w	0.827	0.819	0.817	**0.813**	0.807	0.789
H^w	0.974	0.976	0.976	**0.977**	0.977	0.978
τ^k	0.342	2E-04	-0.053	**-0.125**	-0.236	-0.539
τ^w	0.152	0.218	0.227	**0.237**	0.252	0.286
$\overline{r}$	-0.189	-0.218	-0.222	**-0.226**	-0.231	-0.248
U^k	1.229	1.508	1.544	**1.592**	1.660	1.828
U^w	0.689	0.681	0.679	**0.676**	0.671	0.658
U	0.751	0.776	0.778	**0.782**	0.785	0.792

In Table 1.11, the case of Benthamite government is in bold. Apparently, in contrast to the benchmark model, the value of γ matters for the steady-state solution in the modified model. In addition, all the changes of variables are monotonic with the magnitude of γ. The magnitude of change in capital income tax is very large. The case when the government cares only about the workers is first examined. As can be seen in Column (1), the capital income rate is positive and well below the date average of 34.4%. The incentive for the government to tax labour income is reduced. The labour income tax is below the data average of 18.8%. In this case, the replacement rate is negative which implies the government subsidizes the labour supply in the long run. When $\gamma = 0$, the welfare of workers improves at the steady-state of Ramsey relative to the exogenous policy case. (i.e. from 0.68 to 0.689). As weight for the welfare of capitalists increases, the capital income tax falls very quickly, as can be seen in Figure 1.5. The steady-state optimal capital tax turns into a subsidy when γ reaches 0.110, i.e. $\tau^k = -0.053$. In turn, the labour income tax increases to make up for the tax revenue losses from capital. This optimal policy hurts the workers and the welfare of workers decreases relative to the exogenous policy case (i.e. from 0.680 to 0.679). This implies that the government redistributes the welfare towards capitalists if γ exceeds 0.110. The replacement rate decreases as γ increases. This implies the government increases the subsidy to labour as γ increases. This policy increases the incentive for the workers to provide labour to firms. As a result, employment goes up (i.e. from 0.974 to 0.978).

As can be seen in Figure 1.5, as the weight to the capitalists increases, the steady-state welfare of capitalists increases. It is because the capitalists directly benefit from the substantial reductions in capital tax. In contrast, the increase in γ worsens the welfare of workers. However, the workers are slightly hurt by the labour tax increases since the subsidy to labour increases in the meanwhile. The output, Y^f, goes up as a result of increases in three inputs, K^p, H^w and $\overline{K}^g$. Moreover, the aggregate welfare improves as γ increases. This implies that the efficiency of the whole economy has improved as the government becomes biased towards the capitalists.

The above discussion suggests that, in the model with strictly positive profits, when the government cares more about the capitalists, it helps to reduce the inefficiently high capital tax and eventually it turns into a subsidy after a critical level of the weight attached to the capitalists placed by the government. The welfare of capitalists substantially improves as the capital distortion reduces. Meanwhile, the optimal policy hurts the workers as the government has to raise the revenue to the required level by increasing labour income tax. As a result, the welfare of workers worsens. This implies a conflict of interests between the agents and hence a trade-off between efficiency and equity. This result is consistent with Angelopoulos *et al.* (2011a).

1.7 Welfare analysis

This section examines the welfare effects of optimal taxation at the steady-state. In particular, the steady-state welfare costs or benefits for all agents are computed when the government chooses the optimal policy relative to the exogenous policy. This has become one popular way to evaluate fiscal policies in recent literature (see e.g. Baier and Glomm (2001) and Ardagna (2007)). Following Lucas (1990), Cooley and Hansen (1992) and Ohanian (1997), the additional level of consumption, ζ^i to give to the agent is calculated so that he is equally well off in two cases of exogenous policy and optimal policy. Mathematically, ζ^i satisfies the following equation:

$$U_R^i = \overline{U}_E^i = \left[\mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{50}$$

The welfare losses and gains for the capitalists and the workers are denoted by ζ^k and ζ^w, respectively, together with the welfare losses and gains at the aggregate level, ζ.[33] The subscript E denotes the exogenous policy while the subscript R denotes the Ramsey policy $\overline{U}_E^i$ is the contingent utility of agent i in the model with exogenous policy in which he would increase ζ^i fraction of the consumption such that he can enjoy the same utility as in the model with optimal policy.

A positive ζ^i implies that the agent is better off in the optimal policy case while a negative ζ^i implies that the agent is better off in the exogenous policy case. The agent will be indifferent about two policies if ζ^i is zero.

1.7.1 Model without profits

The values for ξ^i for agents in the benchmark model are first computed by varying the values for γ. Table 1.12 in what follows reports the different values of ξ^i under different values of γ. We can refer to different columns in this table for welfare losses or gains for different agents.

Table 1.12: Welfare losses/gains in the model without profits

γ	Capitalist $\left(\zeta^k\right)$ (1)	Worker (ζ^w) (2)	Aggregate (ζ) (3)
0.000	0.276	0.028	0.052
0.105	0.276	0.028	0.052
0.110	0.276	0.028	0.052
0.115	0.276	0.028	0.052
0.120	0.276	0.028	0.052
0.125	0.276	0.028	0.052

As can be seen in Table 1.12, the optimal taxation in Ramsey can improve the welfare of all agents in all cases of different γ. There does not exist a conflict of interests between agents. In the benchmark model, the long-run optimal zero capital taxation holds no matter the weight attached to the welfare capitalists, i.e. $\zeta^k = 0.276 > 0$.

[33] The derivation of the formula for ζ^i is provided in the Appendix 1.A.4.

This is because the long-run optimal zero capital tax increases the private investment and therefore capital stock. The income and consumption of capitalists increases in the long run. Recalling the utility function of capitalists, the welfare of capitalists depends on the private consumption and per capita government consumption because the capitalists do not work in the economy. The increase in private consumption increases the welfare of capitalists.

The welfare of workers improves as well. It has been demonstrated, in the steady-state analysis above, that the long-run optimal negative replacement rate is equivalent to a subsidy to work. This leads to a rise in the labour supply of workers. On one hand, the income, consumption welfare of workers increases as a result of higher labour supply as working can generate higher income for the workers. On the other hand, the welfare of workers decreases because the utility of workers negatively depends on the labour supply. As can be seen in Table 1.12, the positive effect dominates and the workers are better off in the setup of Ramsey, i.e. $\zeta^w = 0.028 > 0$.

Both capitalists and workers are better off at the steady-state of Ramsey setup no matter whether the government is Benthamite or partisan. The optimal policy is Pareto improving in the long run, but the welfare gains for the capitalists relative to the workers are bigger. This implies that the optimal taxation increases the welfare inequality.

1.7.2 Model with profits

The welfare gains or losses for agents in the model with profits are next analysed. Table 1.13 shows different values for ζ^i for different values of γ. The results are compared with those in Table 1.12 in order to investigate the distributional effects of optimal taxation.

Apparently, the commonality of interests no longer holds in the modified model with strictly positive profits. The presence of profits creates the conflict of interests between agents. As can be seen in Table 1.11 above, when the government cares more about the capitalists, it substantially decreases the capital income tax in order to reduce the distortion in the capital market. The capital income tax turns into a subsidy when γ exceeds 0.110. The capital tax cut is associated with a higher labour income tax. Thus, the welfare of workers goes down as γ increases. When the weight attached to the welfare of capitalists increases above the critical value of 0.110, the steady-state welfare of workers decreases in the optimal policy case compared to the exogenous policy case, i.e. $\zeta^w = -0.003 < 0$.

Table 1.13: Welfare losses/gains in the model with profits

	Capitalist (ζ^k)	Worker (ζ^w)	Aggregate (ζ)
γ	(1)	(2)	(3)
0.000	0.040	0.014	0.017
0.105	0.332	0.001	0.036
0.110	0.370	-0.003	0.039
0.115	0.420	-0.008	0.043
0.120	0.492	-0.016	0.050
0.125	0.671	-0.038	0.061

The above findings show that, with the presence of profits, the government redistributes welfare towards capitalists, when γ reaches a critical level. In the modified model with non-zero profits, the government values distortion in the capital market more than labour market distortion. This incentive leads to a decrease in the optimal capital income tax and therefore the long-run welfare gains for capitalists increase while the welfare gains for workers decrease as γ increases. Therefore, there is a conflict of interests between agents.

1.8 Summary and concluding remarks

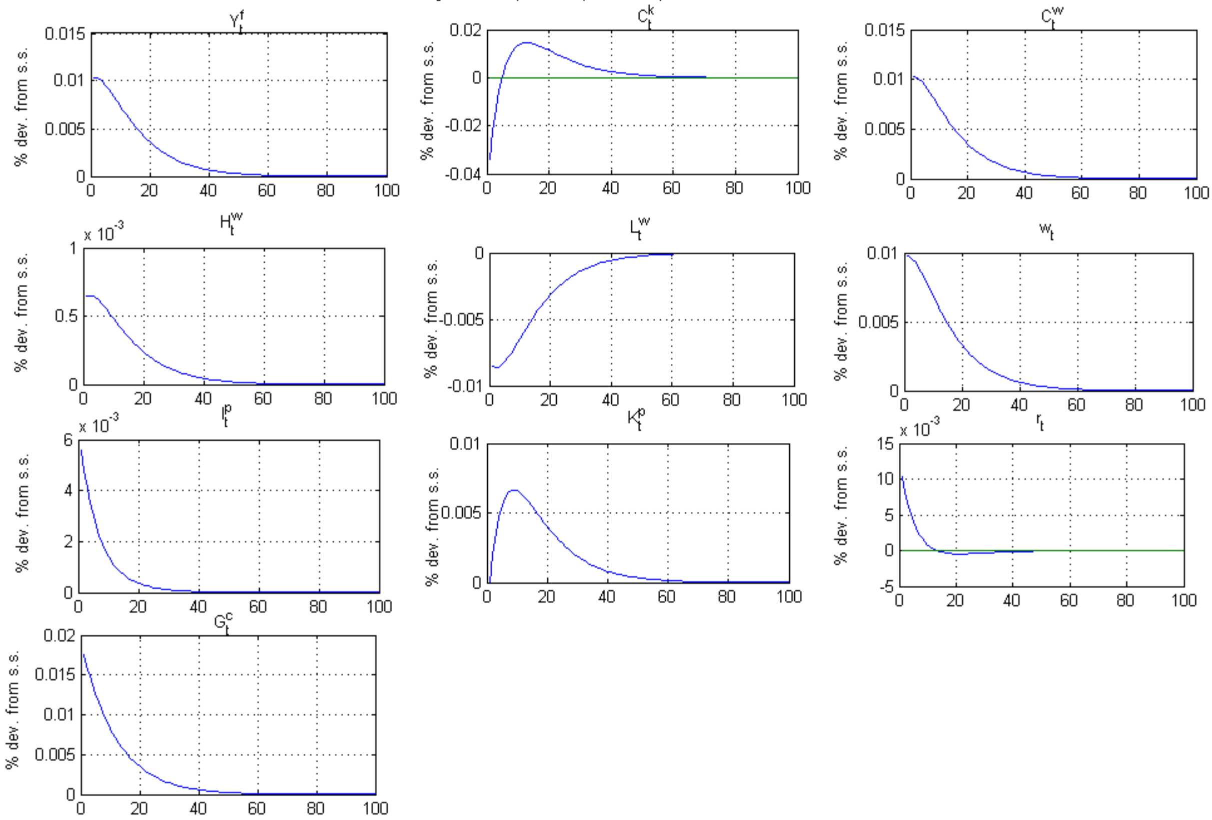
This chapter used two different heterogeneous agent models with equilibrium unemployment to study the effects of optimal taxation on unemployment, the distribution of income and welfare of agents. The agent heterogeneity lay in the working and saving propensities of households. The capitalists by assumption did not work and the workers did not save. In the first model the firms earned zero economic profits in equilibrium while in the second model the equilibrium profits were non-zero due to the presence of productive public investment. In both models, equilibrium unemployment was generated in the competitive labour market as the outcome of optimal choices made by workers. The main findings can be summarized as follows.

First, in the model with zero economic profits, we show that the optimal tax rate on capital income should be zero in the long run which is consistent with Judd (1985) and Chamley (1986). It is optimal for the government to tax the leisure of workers in the long run. This is equivalent to a subsidy to the labour supply of workers. Meanwhile, the government slightly increases the tax rate on labour income. The distortions in the labour market caused by the distortionary labour tax can be completely eliminated as a consequence of the equal amount of government subsidies to the workers in the form of taxation on the leisure. As a result, the labour supply of workers increases which is beneficial to the workers. The income, consumption and welfare of workers improves in the long run. In addition, as in Judd (1985), The weight to the welfare of agent in the Ramsey setup of the government does not matter for the long-run optimal policy. This implies that there is no conflict of interests between agents in the long run in the benchmark model.

Second, the result of long-run optimal zero capital tax cannot be obtained in the modified model. The optimal tax rate on capital income is found to be negative in the long run which means the government chooses to subsidize the capital income in the long run. There are two opposing effects in determining the direction of optimal capital taxation: the under-investment effect and the profit effect (see e.g. Guo and Lansing (1999)). In our model, on one hand, the crowding-out effect of public investment is equivalent to the under-investment effect which motivates a Benthamite government to use a subsidy to the capital income to help reduce the distortions in the capital market. On the other hand, the presence of profits motivates the government to use a positive tax rate on capital income as taxing profits is not distortionary. In our case, we show that the under-investment effect outweighs the profit effect. As a result, the government subsidizes the capital income in the long run. The negative capital income tax directly increases the investment of capitalists and therefore the income, consumption and welfare of capitalists increase. As in the benchmark model, the government subsidizes the labour supply of workers while the tax rate on labour income slightly increases. These two policy instruments have opposing effects on the labour supply of workers. We find that the positive effect of labour subsidy dominates so that the labour supply of workers is higher than it would be in the model with given policy. In the presence of profits, the tax distortion in the labour market cannot be completely eliminated in the long run. The distortion causes welfare losses for workers. Finally, in contrast to the benchmark model, the weight to the welfare of agent matters for the optimal taxation under commitment in the modified model. The effects are found to be monotonic. This implies that the optimal taxation generates conflict of interests between agents and it has redistributional effects in the long run. As the weight to the welfare of capitalists increases, the capital taxation decreases and it turns into a subsidy after a critical value. The tax rate on labour income increases in order to make up for the losses in government's tax revenues. In this case, a trade-off between efficiency and equity needs to be taken into account in the Ramsey setup of government.

1.9 Figures

Figure 1.1 Impulse responses to positive TFP shock



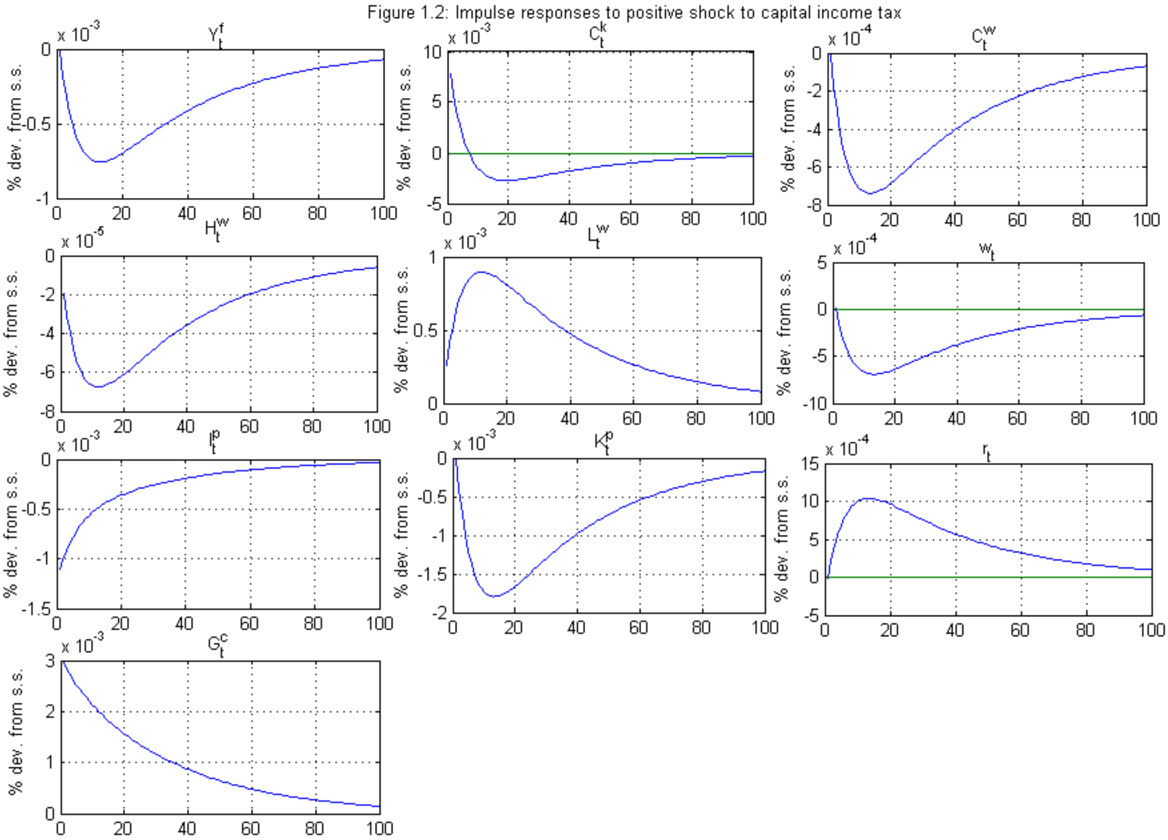


Figure 1.2: Impulse responses to positive shock to capital income tax

Figure 1.3: Impulse responses to positive shock to labour income tax

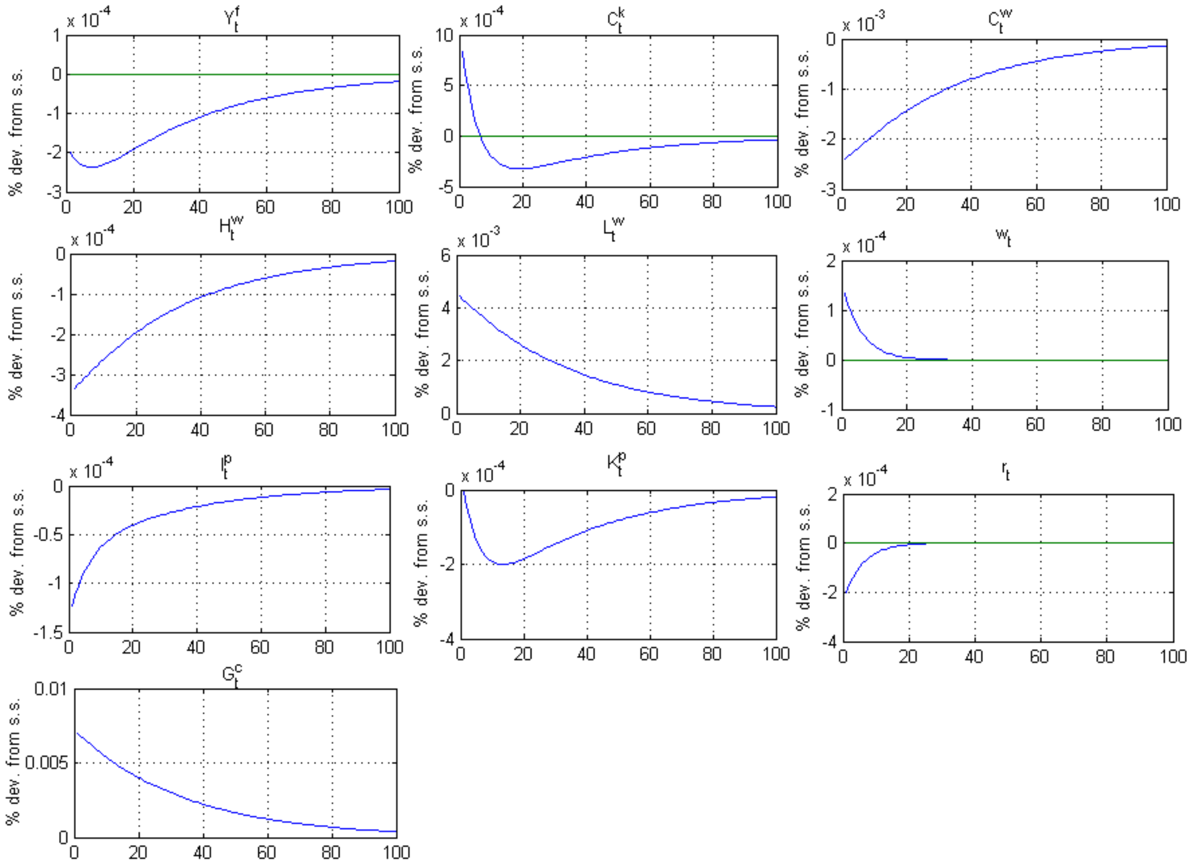


Figure 1.4: Impulse responses to positive shock to replacement rate

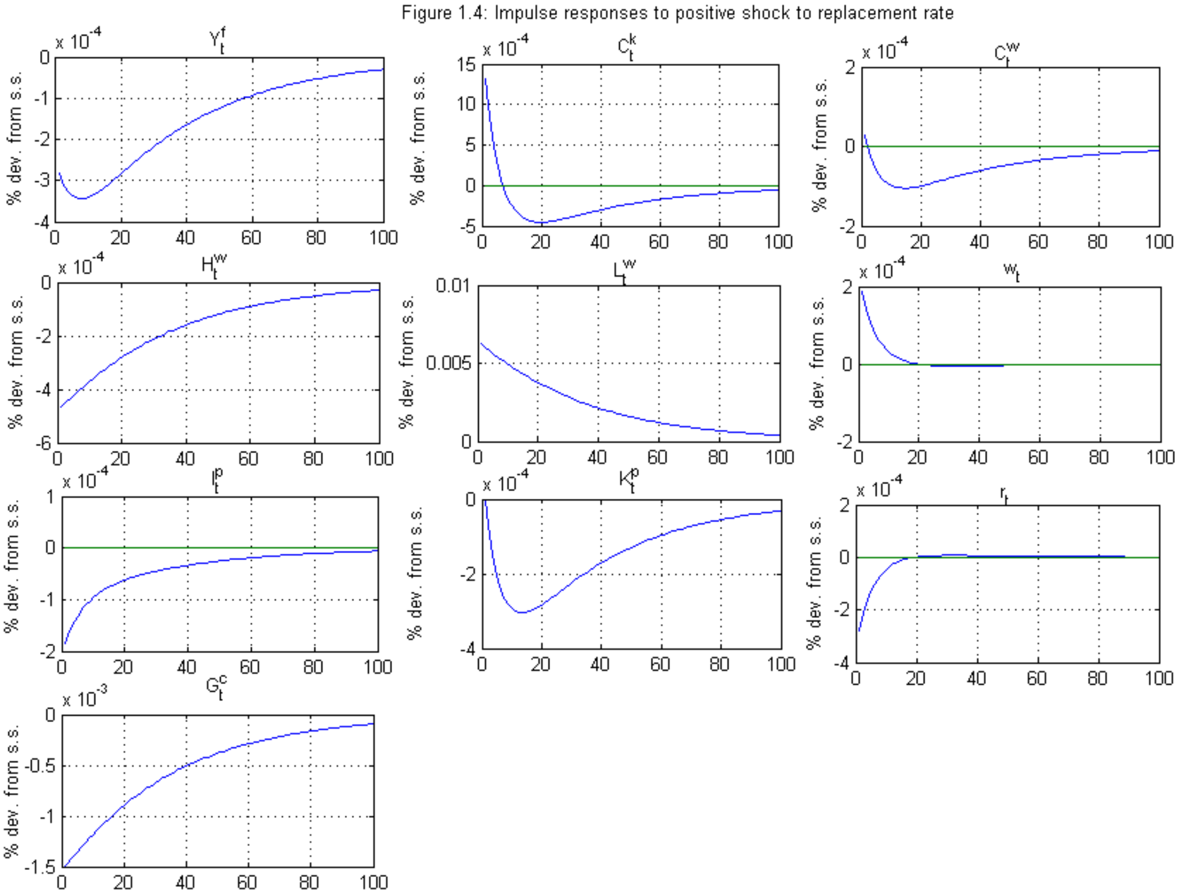
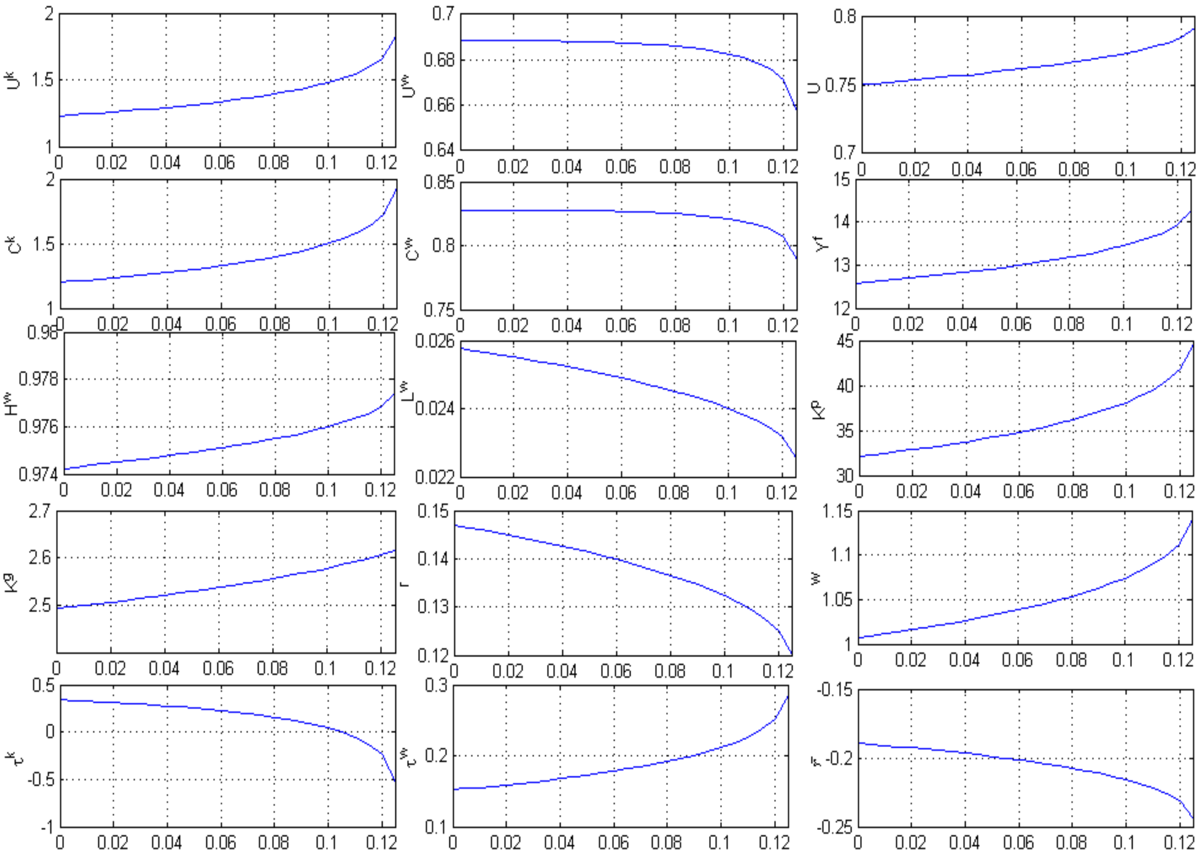


Figure 1.5: Non-Benthamite (partisan) preferences in the model with profits



1.A Appendix

1.A.1 DCE conditions

The DCE consists of the following conditions:

$$
\begin{aligned}
OC^k \quad &: \quad \left(C_t^k + \omega \overline{G}_t^c\right)^{-\frac{1}{\sigma}} \left[\mu \left(C_t^k + \omega \overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\right]^{\frac{1}{\sigma-1}} \\
&= \quad \beta \left(C_{t+1}^k + \omega \overline{G}_{t+1}^c\right)^{-\frac{1}{\sigma}} \left[\mu \left(C_{t+1}^k + \omega \overline{G}_{t+1}^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\right]^{\frac{1}{\sigma-1}} \cdot \\
&\quad \cdot \left[1 + \left(1 - \tau_{t+1}^k\right)\left(r_{t+1} - \delta^p\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
OC^w \quad &: \quad (1-\mu)\left(1 - H_t^w\right)^{-\frac{1}{\sigma}} + \mu \overline{r}_t w_t \left(C_t^w + \omega \overline{G}_t^c\right)^{-\frac{1}{\sigma}} \\
&= \quad \mu w_t \left(1 - \tau_t^w\right)\left(C_t^w + \omega \overline{G}_t^c\right)^{-\frac{1}{\sigma}}
\end{aligned}
$$

$$K_t^f : r_t = \frac{\alpha_1 Y_t^f}{K_t^f}$$

$$L_t^f : w_t = \frac{\alpha_2 Y_t^f}{H_t^f}$$

$$BC^k : C_t^k + K_{t+1}^p - \left(1 - \delta^p\right) K_t^p = r_t K_t^p - \tau_t^k \left(r_t - \delta^p\right) K_t^p$$

$$BC^w : C_t^w = \left(1 - \tau_t^w\right) w_t H_t^w + \overline{r}_t w_t \left(1 - H_t^w\right)$$

$$
\begin{aligned}
BC^g \quad &: \quad \overline{G}_t^c + \left(1 - n^k\right) \overline{r}_t w_t \left(1 - H_t^w\right) \\
&= \quad n^k \tau_t^k \left(r_t - \delta^p\right) K_t^p + \left(1 - n^k\right) \tau_t^w w_t H_t^w
\end{aligned}
$$

$$PF : Y_t^f = A_t \left(K_t^f\right)^{\alpha_1} \left(H_t^f\right)^{\alpha_2}$$

$$MC_K : K_t^k = K_t^f \equiv K_t^p$$

$$MC_L : H_t^w = \frac{n^k}{\left(1 - n^k\right)} H_t^f$$

1.A.2 Log-linearization

We first take natural logs of equations. We then differentiate the resulting logged equations at the steady-state with respect to time. The log-linearization of the non-linear DCE conditions and $AR(1)$ processes is shown as follows:

$$\begin{aligned}
& \left(C_t^k+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_t^k+\omega\overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}} \\
= & \ \beta\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)^{-\frac{1}{\sigma}}\left[\mu\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]^{\frac{1}{\sigma-1}} \cdot \\
& \cdot\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}-\delta^p\right)\right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\sigma}\ln\left(C_t^k+\omega\overline{G}_t^c\right)+\frac{1}{\sigma-1}\ln\left[\mu\left(C_t^k+\omega\overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right] \\
= & \ \ln\beta-\frac{1}{\sigma}\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)+\frac{1}{\sigma-1}\ln\left[\mu\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]+ \\
& +\ln\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}-\delta^p\right)\right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\sigma}\frac{d\ln\left(C_t^k+\omega\overline{G}_t^c\right)}{dt}+\frac{1}{\sigma-1}\frac{d\ln\left[\mu\left(C_t^k+\omega\overline{G}_t^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]}{dt} \\
= & \ \frac{d\ln\beta}{dt}-\frac{1}{\sigma}\frac{d\ln\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)}{dt}+\frac{1}{\sigma-1}\frac{d\ln\left[\mu\left(C_{t+1}^k+\omega\overline{G}_{t+1}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]}{dt}+ \\
& +\frac{d\ln\left[1+\left(1-\tau_{t+1}^k\right)\left(r_{t+1}-\delta^p\right)\right]}{dt}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\sigma}\frac{1}{\left(C^k+\omega\overline{G}^c\right)dt}\left[d\left(C_t^k\right)+\omega d\left(\overline{G}_t^c\right)\right]+ \\
& +\frac{1}{\sigma-1}\frac{\mu\frac{\sigma-1}{\sigma}\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[d\left(C_t^k\right)+\omega d\left(\overline{G}_t^c\right)\right]}{\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]dt} \\
= & \ -\frac{1}{\sigma}\frac{1}{\left(C^k+\omega\overline{G}^c\right)dt}\left[d\left(C_{t+1}^k\right)+\omega d\left(\overline{G}_{t+1}^c\right)\right]+ \\
& +\frac{1}{\sigma-1}\frac{\mu\frac{\sigma-1}{\sigma}\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left[d\left(C_{t+1}^k\right)+\omega d\left(\overline{G}_{t+1}^c\right)\right]}{\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]dt}+ \\
& +\frac{1}{\left[1+\left(1-\tau^k\right)\left(r-\delta^p\right)\right]dt}\left[(1-\tau^k)dr_{t+1}-\left(r-\delta^p\right)d\tau_{t+1}^k\right]
\end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{\sigma\left(C^k+\omega\overline{G}^c\right)}\left[C^k\frac{d\left(C_t^k\right)}{C^k dt}+\omega\overline{G}^c\frac{d\left(\overline{G}_t^c\right)}{\overline{G}^c dt}\right]+ \\
& +\frac{\mu\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}}{\sigma\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]}\left[C^k\frac{d\left(C_t^k\right)}{C^k dt}+\omega\overline{G}^c\frac{d\left(\overline{G}_t^c\right)}{\overline{G}^c dt}\right] \\
= & -\frac{1}{\sigma\left(C^k+\omega\overline{G}^c\right)}\left[C^k\frac{d\left(C_{t+1}^k\right)}{C^k dt}+\omega\overline{G}^c\frac{d\left(\overline{G}_{t+1}^c\right)}{\overline{G}^c dt}\right]+ \\
& +\frac{\mu\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}}{\sigma\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]}\left[C^k\frac{d\left(C_{t+1}^k\right)}{C^k dt}+\omega\overline{G}^c\frac{d\left(\overline{G}_{t+1}^c\right)}{\overline{G}^c dt}\right]+ \\
& +\frac{1}{\left[1+\left(1-\tau^k\right)\left(r-\delta^p\right)\right]}\left[(1-\tau^k)r\frac{d\left(r_{t+1}\right)}{r dt}-\left(r-\delta^p\right)\tau^k\frac{d\left(\tau_{t+1}^k\right)}{\tau^k dt}\right]
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{\left(C^k\widehat{C}_t^k+\omega\overline{G}^c\widehat{\overline{G}}_t^c\right)}{\sigma\left(C^k+\omega\overline{G}^c\right)}+\frac{\mu\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left(C^k\widehat{C}_t^k+\omega\overline{G}^c\widehat{\overline{G}}_t^c\right)}{\sigma\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]} \\
= & -\frac{\left(C^k E_t\widehat{C}_{t+1}^k+\omega\overline{G}^c E_t\widehat{\overline{G}}_{t+1}^c\right)}{\sigma\left(C^k+\omega\overline{G}^c\right)}+\frac{\mu\left(C^k+\omega\overline{G}^c\right)^{-\frac{1}{\sigma}}\left(C^k E_t\widehat{C}_{t+1}^k+\omega\overline{G}^c E_t\widehat{\overline{G}}_{t+1}^c\right)}{\sigma\left[\mu\left(C^k+\omega\overline{G}^c\right)^{\frac{\sigma-1}{\sigma}}+(1-\mu)\right]}+ \\
& +\frac{\left[(1-\tau^k)rE_t\widehat{r}_{t+1}-\left(r-\delta^p\right)\tau^k E_t\widehat{\tau}_{t+1}^k\right]}{\left[1+\left(1-\tau^k\right)\left(r-\delta^p\right)\right]} \qquad (51)
\end{aligned}
$$

$$
\begin{aligned}
& (1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu\overline{r}_t w_t\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}} \\
= & \ \mu w_t\left(1-\tau_t^w\right)\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}
\end{aligned}
$$

$$
\begin{aligned}
& \ln\left[(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu\overline{r}_t w_t\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\right] \\
= & \ \ln\mu+\ln w_t+\ln\left(1-\tau_t^w\right)-\frac{1}{\sigma}\ln\left(C_t^w+\omega\overline{G}_t^c\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d\ln\left[(1-\mu)\left(1-H_t^w\right)^{-\frac{1}{\sigma}}+\mu\overline{r}_t w_t\left(C_t^w+\omega\overline{G}_t^c\right)^{-\frac{1}{\sigma}}\right]}{dt} \\
= & \ \frac{d\ln\mu}{dt}+\frac{d\ln\left(w_t\right)}{dt}+\frac{d\ln\left(1-\tau_t^w\right)}{dt}-\frac{1}{\sigma}\frac{d\ln\left(C_t^w+\omega\overline{G}_t^c\right)}{dt}
\end{aligned}
$$

$$\frac{1}{\mu w\left(1-\tau^{w}\right)\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}dt}\left[\frac{1}{\sigma}\left(1-\mu\right)\left(1-H^{w}\right)^{\frac{-1-\sigma}{\sigma}}d\left(H_{t}^{w}\right)+\right.$$
$$+\mu w\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}d\overline{r}_{t}+\mu\overline{r}\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}d\left(w_{t}\right)-$$
$$\left.-\frac{1}{\sigma}\mu\overline{r}w\left(C^{w}+\omega\overline{G}^{c}\right)^{\frac{-1-\sigma}{\sigma}}\left(d\left(C_{t}^{w}\right)+\omega d\left(\overline{G}_{t}^{c}\right)\right)\right]$$
$$=\frac{d\left(w_{t}\right)}{wdt}-\frac{d\left(\tau_{t}^{w}\right)}{\left(1-\tau^{w}\right)dt}-\frac{1}{\sigma}\frac{d\left(C_{t}^{w}\right)+\omega d\left(\overline{G}_{t}^{c}\right)}{\left(C^{w}+\omega\overline{G}^{c}\right)dt}$$

-

$$\frac{1}{\mu w\left(1-\tau^{w}\right)\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}}\left[\frac{1}{\sigma}\left(1-\mu\right)\left(1-H^{w}\right)^{\frac{-1-\sigma}{\sigma}}H^{w}\frac{d\left(H_{t}^{w}\right)}{H^{w}dt}+\right.$$
$$+\mu w\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}\overline{r}\frac{d\left(\overline{r}_{t}\right)}{\overline{r}dt}+\mu\overline{r}\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}w\frac{d\left(w_{t}\right)}{wdt}-$$
$$\left.-\frac{1}{\sigma}\mu\overline{r}w\left(C^{w}+\omega\overline{G}^{c}\right)^{\frac{-1-\sigma}{\sigma}}\left(C^{w}\frac{d\left(C_{t}^{w}\right)}{C^{w}dt}+\omega\overline{G}^{c}\frac{d\left(\overline{G}_{t}^{c}\right)}{\overline{G}^{c}dt}\right)\right]$$
$$=\frac{dw_{t}}{wdt}-\frac{\tau^{w}}{\left(1-\tau^{w}\right)}\frac{d\left(\tau_{t}^{w}\right)}{\tau^{w}dt}-\frac{1}{\sigma\left(C^{w}+\omega\overline{G}^{c}\right)}\left[C^{w}\frac{d\left(C_{t}^{w}\right)}{C^{w}dt}+\omega\overline{G}^{c}\frac{d\left(\overline{G}_{t}^{c}\right)}{\overline{G}^{c}dt}\right]$$

$$\frac{1}{\mu w\left(1-\tau^{w}\right)\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}}\left[\frac{1}{\sigma}\left(1-\mu\right)\left(1-H^{w}\right)^{-\frac{1+\sigma}{\sigma}}H^{w}\widehat{H}_{t}^{w}+\right.$$
$$+\mu w\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}\overline{r}\widehat{\overline{r}}_{t}+\mu\overline{r}\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1}{\sigma}}w\widehat{w}_{t}-$$
$$\left.-\frac{1}{\sigma}\mu\overline{r}w\left(C^{w}+\omega\overline{G}^{c}\right)^{-\frac{1+\sigma}{\sigma}}\left(C^{w}\widehat{C}_{t}^{w}+\omega\overline{G}^{c}\widehat{\overline{G}}_{t}^{c}\right)\right]$$
$$=\widehat{w}_{t}-\frac{\tau^{w}}{\left(1-\tau^{w}\right)}\widehat{\tau}_{t}^{w}-\frac{1}{\sigma\left(C^{w}+\omega\overline{G}^{c}\right)}\left(C^{w}\widehat{C}_{t}^{w}+\omega\overline{G}^{c}\widehat{\overline{G}}_{t}^{c}\right)\tag{52}$$

$$r_{t}=\frac{\alpha_{1}Y_{t}^{f}}{K_{t}^{p}}$$
$$\ln r_{t}=\ln\alpha_{1}+\ln Y_{t}^{f}-\ln K_{t}^{p}$$
$$\frac{d\ln r_{t}}{dt}=\frac{d\ln\alpha_{1}}{dt}+\frac{d\ln Y_{t}^{f}}{dt}-\frac{d\ln K_{t}^{p}}{dt}$$
$$\frac{d\left(r_{t}\right)}{rdt}=\frac{d\left(Y_{t}^{f}\right)}{Y^{f}dt}-\frac{d\left(K_{t}^{p}\right)}{K^{p}dt}$$
$$\widehat{r}_{t}=\widehat{Y}_{t}^{f}-\widehat{K}_{t}^{p}\tag{53}$$

$$w_t = \frac{\alpha_2 Y_t^f}{\frac{1-n^k}{n^k} H_t^w}$$

$$\ln w_t = \ln \alpha_2 + \ln Y_t^f - \ln\left(1-n^k\right) + \ln n^k - \ln H_t^w$$

$$\frac{d\ln w_t}{dt} = \frac{d\ln \alpha_2}{dt} + \frac{d\ln Y_t^f}{dt} - \frac{d\ln\left(1-n^k\right)}{dt} + \frac{d\ln n^k}{dt} - \frac{d\ln H_t^w}{dt}$$

$$\frac{d\left(w_t\right)}{wdt} = \frac{d\left(Y_t^f\right)}{Y^f dt} - \frac{d\left(H_t^w\right)}{H^w dt}$$

$$\widehat{w}_t = \widehat{Y}_t^f - \widehat{H}_t^w \tag{54}$$

$$C_t^k + K_{t+1}^p - \left(1-\delta^p\right) K_t^p = r_t K_t^p - \tau_t^k \left(r_t - \delta^p\right) K_t^p$$

$$\ln\left[C_t^k + K_{t+1}^p - \left(1-\delta^p\right) K_t^p\right] = \ln\left[r_t K_t^p - \tau_t^k \left(r_t - \delta^p\right) K_t^p\right]$$

$$\frac{d\ln\left[C_t^k + K_{t+1}^p - \left(1-\delta^p\right) K_t^p\right]}{dt} = \frac{d\ln\left[r_t K_t^p - \tau_t^k \left(r_t - \delta^p\right) K_t^p\right]}{dt}$$

$$\begin{aligned} & \frac{1}{\left[C^k + K^p - \left(1-\delta^p\right) K^p\right] dt}\left[d\left(C_t^k\right) + d\left(K_{t+1}^p\right) - \left(1-\delta^p\right) d\left(K_t^p\right)\right] \\ = \quad & \frac{K^p d\left(r_t\right) + rd\left(K_t^p\right) - \left(r-\delta^p\right) K^p d\left(\tau_t^k\right) - \tau^k K^p d\left(r_t\right) - \tau^k\left(r-\delta^p\right) d\left(K_t^p\right)}{\left[rK^p - \tau^k\left(r-\delta^p\right) K^p\right] dt} \end{aligned}$$

$$\begin{aligned} & \frac{1}{C^k + \delta^p K^p}\left[C^k \frac{d\left(C_t^k\right)}{C^k dt} + K^p \frac{d\left(K_{t+1}^p\right)}{K^p dt} - \left(1-\delta^p\right) K^p \frac{d\left(K_t^p\right)}{K^p dt}\right] \\ = \quad & \frac{1}{\left[rK^p - \tau^k\left(r-\delta^p\right) K^p\right]}\left[K^p r \frac{d\left(r_t\right)}{rdt} + K^p r \frac{d\left(K_t^p\right)}{K^p dt} - \left(r-\delta^p\right) K^p \tau^k \frac{d\left(\tau_t^k\right)}{\tau^k dt}\right] - \\ & - \frac{1}{\left[rK^p - \tau^k\left(r-\delta^p\right) K^p\right]}\left[\tau^k K^p r \frac{d\left(r_t\right)}{rdt} + \tau^k\left(r-\delta^p\right) K^p \frac{d\left(K_t^p\right)}{K^p dt}\right] \end{aligned}$$

$$\begin{aligned} & \left[C^k \widehat{C}_t^k + K^p E_t \widehat{K}_{t+1}^p - \left(1-\delta^p\right) K^p \widehat{K}_t^p\right] \\ = \quad & \left[K^p r \widehat{r}_t + K^p r \widehat{K}_t^p - \left(r-\delta^p\right) K^p \tau^k \widehat{\tau}_t^k\right] - \left[\tau^k K^p r \widehat{r}_t + \tau^k\left(r-\delta^p\right) K^p \widehat{K}_t^p\right] \end{aligned} \tag{55}$$

$$C_t^w = (1-\tau_t^w) w_t H_t^w + \overline{r}_t w_t (1-H_t^w)$$

$$\ln (C_t^w) = \ln [(1-\tau_t^w) w_t H_t^w + \overline{r}_t w_t (1-H_t^w)]$$

$$\frac{d\ln (C_t^w)}{dt} = \frac{d\ln [(1-\tau_t^w) w_t H_t^w + \overline{r}_t w_t (1-H_t^w)]}{dt}$$

$$\begin{aligned}\frac{d(C_t^w)}{C^w dt} &= \frac{-wH^w d(\tau_t^w) + (1-\tau^w) H^w d(w_t) + (1-\tau^w) wd(H_t^w)}{C^w dt} + \\ &+ \frac{w(1-H^w) d(\overline{r}_t) + \overline{r}(1-H^w) d(w_t) - \overline{r} wd(H_t^w)}{C^w dt}\end{aligned}$$

$$\begin{aligned}\frac{d(C_t^w)}{C^w dt} &= \frac{1}{C^w}\left[(1-\tau^w) H^w w \frac{d(w_t)}{wdt} - wH^w \tau^w \frac{d(\tau_t^w)}{\tau^w dt}\right] + \\ &+ \frac{1}{C^w}\left[(1-\tau^w) wH^w \frac{d(H_t^w)}{H^w dt} + w(1-H^w)\overline{r}\frac{d(\overline{r}_t)}{\overline{r} dt}\right] + \\ &+ \frac{1}{C^w}\left[\overline{r}(1-H^w) w \frac{d(w_t)}{wdt} - \overline{r} wH^w \frac{d(H_t^w)}{H^w dt}\right]\end{aligned}$$

$$\begin{aligned}\widehat{C}_t^w &= \frac{1}{C^w}\left[(1-\tau^w) H^w w\widehat{w}_t - wH^w \tau^w \widehat{\tau}_t^w\right] + \\ &+ \frac{1}{C^w}\left[(1-\tau^w) wH^w \widehat{H}_t^w + w(1-H^w)\overline{r}\widehat{\overline{r}}_t\right] + \\ &+ \frac{1}{C^w}\left[\overline{r}(1-H^w) w\widehat{w}_t - \overline{r} wH^w \widehat{H}_t^w\right] \qquad (56)\end{aligned}$$

$$\begin{aligned}&\overline{G}_t^c + \left(1-n^k\right)\overline{r}_t w_t (1-H_t^w) \\ = \; & n^k \tau_t^k (r_t - \delta^p) K_t^p + \left(1-n^k\right)\tau_t^w w_t H_t^w\end{aligned}$$

$$\begin{aligned}&\ln\left[\overline{G}_t^c + \left(1-n^k\right)\overline{r}_t w_t (1-H_t^w)\right] \\ = \; & \ln\left[n^k \tau_t^k (r_t - \delta^p) K_t^p + \left(1-n^k\right)\tau_t^w w_t H_t^w\right]\end{aligned}$$

$$\begin{aligned}&\frac{d\ln\left[\overline{G}_t^c + \left(1-n^k\right)\overline{r}_t w_t (1-H_t^w)\right]}{dt} \\ = \; & \frac{d\ln\left[n^k \tau_t^k (r_t - \delta^p) K_t^p + \left(1-n^k\right)\tau_t^w w_t H_t^w\right]}{dt}\end{aligned}$$

$$
\begin{aligned}
& \frac{d\left(\overline{G}_t^c\right)+\left(1-n^k\right)w\left(1-H^w\right)d\left(\overline{r}_t\right)+\left(1-n^k\right)\overline{r}\left(1-H^w\right)d\left(w_t\right)}{\left[\overline{G}^c+\left(1-n^k\right)\overline{r}w\left(1-H^w\right)\right]dt}- \\
& -\frac{-\left(1-n^k\right)\overline{r}wd\left(H_t^w\right)}{\left[\overline{G}^c+\left(1-n^k\right)\overline{r}w\left(1-H^w\right)\right]dt} \\
= \; & \frac{n^k\left(r-\delta^p\right)K^pd\left(\tau_t^k\right)+n^k\tau^kK^pd\left(r_t\right)+n^k\tau^k\left(r-\delta^p\right)d\left(K_t^p\right)}{\left[n^k\tau^k\left(r-\delta^p\right)K^p+\left(1-n^k\right)\tau^wwH^w\right]dt}+ \\
& +\frac{\left(1-n^k\right)wH^wd\left(\tau_t^w\right)+\left(1-n^k\right)\tau^wH^wd\left(w_t\right)+\left(1-n^k\right)\tau^wwd\left(H_t^w\right)}{\left[n^k\tau^k\left(r-\delta^p\right)K^p+\left(1-n^k\right)\tau^wwH^w\right]dt}
\end{aligned}
$$

$$
\begin{aligned}
& \overline{G}^c\frac{d\left(\overline{G}_t^c\right)}{\overline{G}^cdt}+\left(1-n^k\right)w\left(1-H^w\right)\overline{r}\frac{d\left(\overline{r}_t\right)}{\overline{r}dt}+ \\
& +\left(1-n^k\right)\overline{r}\left(1-H^w\right)w\frac{d\left(w_t\right)}{wdt}-\left(1-n^k\right)\overline{r}wH^w\frac{d\left(H_t^w\right)}{H^wdt} \\
= \; & n^k\left(r-\delta^p\right)K^p\tau^k\frac{d\left(\tau_t^k\right)}{\tau^kdt}+n^k\tau^kK^pr\frac{d\left(r_t\right)}{rdt}+ \\
& +n^k\tau^k\left(r-\delta^p\right)K^p\frac{d\left(K_t^p\right)}{K^pdt}+\left(1-n^k\right)wH^w\tau^w\frac{d\left(\tau_t^w\right)}{\tau^wdt}+ \\
& +\left(1-n^k\right)\tau^wH^ww\frac{d\left(w_t\right)}{wdt}+\left(1-n^k\right)\tau^wwH^w\frac{d\left(H_t^w\right)}{H^wdt}
\end{aligned}
$$

$$
\begin{aligned}
& \overline{G}^c\widehat{\overline{G}}_t^c+\left(1-n^k\right)w\left(1-H^w\right)\overline{r}\widehat{\overline{r}}_t+ \\
& +\left(1-n^k\right)\overline{r}\left(1-H^w\right)w\widehat{w}_t-\left(1-n^k\right)\overline{r}wH^w\widehat{H}_t^w \\
= \; & n^k\left(r-\delta^p\right)K^p\tau^k\widehat{\tau}_t^k+n^k\tau^kK^pr\widehat{r}_t+ \\
& +n^k\tau^k\left(r-\delta^p\right)K^p\widehat{K}_t^p+\left(1-n^k\right)wH^w\tau^w\widehat{\tau}_t^w+ \\
& +\left(1-n^k\right)\tau^wH^ww\widehat{w}_t+\left(1-n^k\right)\tau^wwH^w\widehat{H}_t^w
\end{aligned} \tag{57}
$$

$$
Y_t^f=A_t\left(K_t^p\right)^{\alpha_1}\left[\frac{1-n^k}{n^k}H_t^w\right]^{\alpha_2}
$$

$$
\ln\left(Y_t^f\right)=\ln\left(A_t\right)+\alpha_1\ln\left(K_t^p\right)+\alpha_2\ln(1-n^k)-\alpha_2\ln(n^k)+\alpha_2\ln\left(H_t^w\right)
$$

$$
\begin{aligned}
\frac{d\ln\left(Y_t^f\right)}{dt} = \; & \frac{d\ln\left(A_t\right)}{dt}+\alpha_1\frac{d\ln\left(K_t^p\right)}{dt}+ \\
& +\alpha_2\frac{\ln(1-n^k)}{dt}-\alpha_2\frac{\ln(n^k)}{dt}+\alpha_2\frac{\ln\left(H_t^w\right)}{dt}
\end{aligned}
$$

$$
\frac{d\left(Y_t^f\right)}{Y^fdt}=\frac{d\left(A_t\right)}{Adt}+\alpha_1\frac{d\ln\left(K_t^p\right)}{K^pdt}+\alpha_2\frac{\ln\left(H_t^w\right)}{H^wdt}
$$

$$\widehat{Y}_t^f = \widehat{A}_t + \alpha_1 \widehat{K}_t^p + \alpha_2 \widehat{H}_t^w \tag{58}$$

$$\begin{aligned}
\ln(A_{t+1}) &= (1-\rho^a)\ln(A_0) + \rho^a \ln(A_t) + \varepsilon_{t+1}^a \\
E_t \ln(A_{t+1}) &= (1-\rho^a)\ln(A_0) + \rho^a \ln(A_t) \\
\frac{E_t d\ln(A_{t+1})}{dt} &= (1-\rho^a)\frac{d\ln(A_0)}{dt} + \rho^a \frac{\ln(A_t)}{dt} \\
\frac{E_t d(A_{t+1})}{Adt} &= \rho^a \frac{d(A_t)}{Adt} \\
E_t \widehat{A}_{t+1} &= \rho^a \widehat{A}_t
\end{aligned} \tag{59}$$

$$\begin{aligned}
\ln(\tau_{t+1}^k) &= \left(1-\rho^k\right)\ln\left(\tau_0^k\right) + \rho^k \ln\left(\tau_t^k\right) + \varepsilon_{t+1}^k \\
E_t \ln(\tau_{t+1}^k) &= \left(1-\rho^k\right)\ln\left(\tau_0^k\right) + \rho^k \ln\left(\tau_t^k\right) \\
\frac{E_t d\ln(\tau_{t+1}^k)}{dt} &= \left(1-\rho^k\right)\frac{d\ln\left(\tau_0^k\right)}{dt} + \rho^k \frac{\ln\left(\tau_t^k\right)}{dt} \\
\frac{E_t d(\tau_{t+1}^k)}{\tau^k dt} &= \rho^k \frac{d\left(\tau_t^k\right)}{\tau^k dt} \\
E_t \widehat{\tau}_{t+1}^k &= \rho^k \widehat{\tau}_t^k
\end{aligned} \tag{60}$$

$$\begin{aligned}
\ln(\tau_{t+1}^w) &= (1-\rho^w)\ln(\tau_0^w) + \rho^w \ln(\tau_t^w) + \varepsilon_{t+1}^w \\
E_t \ln(\tau_{t+1}^w) &= (1-\rho^w)\ln(\tau_0^w) + \rho^w \ln(\tau_t^w) \\
\frac{E_t d\ln(\tau_{t+1}^w)}{dt} &= (1-\rho^w)\frac{d\ln(\tau_0^w)}{dt} + \rho^w \frac{\ln(\tau_t^w)}{dt} \\
\frac{E_t d(\tau_{t+1}^w)}{\tau^w dt} &= \rho^w \frac{d(\tau_t^w)}{\tau^w dt} \\
E_t \widehat{\tau}_{t+1}^w &= \rho^w \widehat{\tau}_t^w
\end{aligned} \tag{61}$$

$$\begin{aligned}
\ln(\overline{r}_{t+1}) &= (1-\rho^r)\ln(\overline{r}_0) + \rho^r \ln(\overline{r}_t) + \varepsilon_{t+1}^r \\
E_t \ln(\overline{r}_{t+1}) &= (1-\rho^r)\ln(\overline{r}_0) + \rho^r \ln(\overline{r}_t)
\end{aligned}$$

$$
\begin{aligned}
\frac{E_t d\ln(\overline{r}_{t+1})}{dt} &= (1-\rho^r)\,\frac{d\ln\left(\overline{r}_0\right)}{dt} + \rho^r \frac{\ln\left(\overline{r}_t\right)}{dt} \\
\frac{E_t d(\overline{r}_{t+1})}{\overline{r}dt} &= \rho^r \frac{d\left(\overline{r}_t\right)}{\overline{r}dt} \\
E_t \widehat{\overline{r}}_{t+1} &= \rho^r \widehat{\overline{r}}_t \qquad (62)
\end{aligned}
$$

1.A.3 Simulation of the model

The random shocks in the model economy are $\left\{\varepsilon_t^a, \varepsilon_t^k, \varepsilon_t^w, \varepsilon_t^r\right\}_{t=0}^T$ which are all normally distributed with the sample size of $T = 100$. The evolution equations of the predetermined and non-predetermined variables are given by the following equations, respectively:

$$E_t\widehat{\mathbf{x}}_{t+1} = \mathbf{E}\widehat{\mathbf{x}}_t \tag{63}$$

and

$$\widehat{\mathbf{y}}_t = \mathbf{F}\widehat{\mathbf{x}}_t \tag{64}$$

where $\widehat{\mathbf{x}}_t = [\widehat{K}_t^p, \widehat{A}_t, \widehat{\tau}_t^k, \widehat{\tau}_t^w, \widehat{r}_t]'$; $\mathbf{y}_t = [\widehat{C}_t^k, \widehat{C}_t^w, \widehat{H}_t^w, \widehat{Y}_t^f, \widehat{r}_t, \widehat{w}_t, \widehat{\overline{G}}_t^c]'$; and $\mathbf{E}$ and $\mathbf{F}$ are $\mathbf{5} \times \mathbf{5}$ and $\mathbf{7} \times \mathbf{5}$ coefficient matrices specifying the laws of motion for the variables in vectors $\mathbf{x}_t$ and $\mathbf{y}_t$.

Then, the simulation for these variables in conducted using the following first-order linear difference equations:

$$\widehat{\mathbf{x}}_{t+1} = \mathbf{E}\widehat{\mathbf{x}}_t + \mathbf{G}\boldsymbol{\varepsilon}_{t+1} \tag{65}$$

and

$$\widehat{\mathbf{y}}_t = \mathbf{F}\widehat{\mathbf{x}}_t + \mathbf{H}\boldsymbol{\varepsilon}_t \tag{66}$$

where the vector $\boldsymbol{\varepsilon}_t$ contains all the random shocks $\left[\varepsilon_t^a, \varepsilon_t^k, \varepsilon_t^w, \varepsilon_t^r\right]'$ and $\mathbf{H}$ is a $\mathbf{7} \times \mathbf{4}$ zero matrix. The coefficient matrix $\mathbf{G}$ is given by:

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{67}$$

Based on the above system, the function *dimpulse* in the Matlab can be used to simulate the impulse responses of the variables to the random shock.

1.A.4 Derivation of ξ^i

ξ^i satisfies the follow equation implying that the agent i is as well off in the exogenous policy model as in the Ramsey model.

$$U_R^i = \overline{U}_E^i = \left[\mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

We can solve for ξ^i in the equation above by taking the following algebra:

$$\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} = \left[\mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}\cdot\frac{\sigma-1}{\sigma}}$$

$$\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} = \mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}} + (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}$$

$$\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}} = \mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}}$$

$$\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = \left[\mu\left(C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = \mu^{\frac{\sigma}{\sigma-1}}\left[C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c\right]$$

$$\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\mu^{\frac{\sigma}{1-\sigma}} = C_E^i\left(1+\zeta^i\right)+\omega\overline{G}_E^c$$

$$\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\mu^{\frac{\sigma}{1-\sigma}} - \omega\overline{G}_E^c = C_E^i\left(1+\zeta^i\right)$$

$$\frac{\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\mu^{\frac{\sigma}{1-\sigma}} - \omega\overline{G}_E^c}{C_E^i} = 1+\zeta^i.$$

$$\Rightarrow \zeta^i = \frac{\left[\left(U_R^i\right)^{\frac{\sigma-1}{\sigma}} - (1-\mu)\left(1-H_E^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\mu^{\frac{\sigma}{1-\sigma}} - \omega\overline{G}_E^c}{C_E^i} - 1 \tag{68}$$

Chapter 2: The welfare effects of tax reforms under market distortions

Abstract: This chapter examines the importance of imperfect competition in product and labour markets in determining the welfare effects of tax reforms assuming agent heterogeneity in capital holdings. The analysis shows that each of these market failures, independently, results in welfare losses for at least one segment of the population, after a capital tax cut and a concurrent labour tax increase. However, when these two market failures are combined in a realistic calibration to the UK economy, they imply that a capital tax cut will be Pareto improving. Consistent with the theory of second-best, these two distortions in this context work to correct the negative distributional effects of a capital tax cut that each one, on its own, creates.

2.1 Introduction

The seminal research of Judd (1985) and Chamley (1986) established that the optimal tax policy under commitment is to set a zero tax rate on capital in the long run, while, concurrently, increasing labour taxes to the level required to finance public spending.[34] A striking implication of this result is that despite the higher tax burden for those agents not holding capital, optimal tax policy is Pareto improving in the long run. The intuition is that the gains from increased labour productivity, induced by higher capital accumulation, compensate for the costs of higher labour taxes and hence labour income is increased.[35]

Another strand in the literature has focused on the distributional effects of tax reforms in models where the optimal long-run zero capital tax may or may not obtain (see e.g. Garcia-Milà *et al.* (2010) for a review of this literature). For instance, Domeij and Heathcote (2004) show that there can be distributional effects from cutting capital taxes in the presence of uninsured idiosyncratic productivity shocks. Garcia-Milà *et al.* (2010) find that there will be lifetime welfare losses for labour from a tax reform

[34] A significant body of literature has examined the conditions under which an optimal non-zero capital tax might be obtained in the long run. For example Guo and Lansing (1999) and Domeij (2005) introduce product and labour market power respectively into representative agent models assuming governments have access to a commitment technology. Also see Klein *et al.* (2008) who, in contrast, use a representative agent framework without market frictions assuming time-consistent Markov policies.

[35] Note that a non-zero optimal capital tax can be obtained in models with heterogeneous agents, under skill differences (see e.g. Conesa *et al.* (2009)) or lack of commitment mechanisms on the part of the government (see e.g. Krusell (2002) and Angelopoulos *et al.* (2011a)).

that implements the zero capital tax. This is because the benefits to labour, through the high capital and productivity channel, occur in the long run, whereas the costs, in the form of higher labour taxes, have an immediate effect. Also Angelopoulos *et al.* (2011b) show that if capital complements skilled labour more than unskilled, capital tax cuts can be skill-biased and thus hurt unskilled labour.

With a view to contributing to the tax policy literature focussed on distributional issues, the importance of unionised labour markets and monopolistically competitive product markets in determining the welfare effects of tax reforms that re-allocate the tax burden from capital to labour is investigated. In this setup the government taxes capital income, including interest on savings and profits, and labour income by using two different tax rates. In the unionised labour market, the wage rate is determined, following Nickell and Andrews (1983), Farber (1986), Pissarides (1998) and Kass and von Thadden (2004), as the outcome of a Nash-bargain between unions and firms. Also, given the importance of the unemployment benefit as the outside option in the Nash-bargaining process, we include it as a component of government spending along with non-employment related public transfers. In the monopolistically competitive product market, following Dixit and Stiglitz (1977), Benhabib and Farmer (1994) and Guo and Lansing (1999), intermediate goods producers earn non-zero economic profits. To highlight the importance of union bargaining and firm power in the product market relative to the competitive model of Judd (1985) we assume a standard neoclassical production technology without skill heterogeneity. Therefore, agents are distinguished in this setup by differences in their capital holdings, which can be motivated by imperfections in the asset markets that require agents to pay different participation premia, see e.g. Aghion and Howitt (2009). On the other hand, households are identical in the labour market since unions guarantee that their members have equal employment and wages (see e.g. Pissarides (1998), Maffezzoli (2001) and Ardagna (2007)). Following, for example, Judd (1985), Lansing (1999), Krusell (2002) and Blanchard and Giavazzi (2003), we allow for two types of households that are termed as capitalists and workers and assume that workers do not participate in the asset market. The capitalists, on the other hand, own the firms and invest in capital in the economy.

The setup regarding heterogeneity in firm ownership is similar to Blanchard and Giavazzi (2003), who also consider a model with an entrepreneur and a worker assuming unionised labour markets and monopolistic product markets to examine the effects of de-regulation in the product and labour markets. We build on this framework to focus instead on the interactions between product and labour market failures in determining the welfare and inequality effects of factor income taxation. This requires that we extend Blanchard and Giavazzi (2003) by introducing tax policy and productive capital, which the entrepreneurs-capitalists own.

To understand the quantitative implications of distortions in the labour and product markets when assessing the welfare effects of tax reforms, the model is calibrated so that its steady-state reproduces the main features of the current UK economy and, in particular, its tax structure and long-run unemployment rate. The UK is used to illustrate the quantitative analysis since unions play an important role in wage-bargaining at the firm-level compared to other EU economies[36] and because its tax structure stands in stark contrast with other European countries, by having a very high capital to labour income tax ratio.[37]

Since the effects of tax reforms that reduce the capital tax are monotonic in the setup, implementing a reform that is consistent with the "zero capital tax" prescription from the optimal taxation literature is focused on.[38] Tax reforms have recently received a great deal of attention by both academics and policymakers (see e.g. the discussion in Garcia-Milà *et al.* (2010) for OECD countries and the Mirrlees Review, Mirrlees *et al.* (2010, 2011) for the UK). However, to the best of our knowledge, the potential distributional effects of such reforms have not been examined under imperfect competition in both product and labour markets. Given the relevance of these market failures, which the fiscal authorities must largely take as given institutional features when designing tax reforms, our analysis aims to inform current policy discussions in the UK and other advanced economies.

The last section of this chapter solves for the laws of motion of variables and studies the dynamic behavior of the economy in response to a temporary and unanticipated 1% cut in the tax rate on capital income. The results of post-policy change are compared with the initial steady-state. The results emphasize the importance of labour and

[36] For example, see the OECD Employment Outlook 2004 which distinguishes levels of bargaining in terms of where labour contracts are negotiated for the period 1970-2000. The data show that wage bargaining in the UK mainly occurs at the firm-level since 1980. The evidence also suggests that there is little or no coordination by upper-level associations. In contrast, in many other European countries, e.g. Belgium, Germany and Spain, bargaining takes place at the industry-level.

[37] According to the ECFIN tax rates (see Martinez-Mongay (2000)) the UK implicit tax rate on labour is 26.5% compared to 37.5% in the EU-11. In contrast, the implicit tax rate on capital of 47% in the UK is well above the rate of 30% in the EU-11, and indeed is one of the highest in the EU. More details on tax and other data used for the calibration are provided later in Section 2.3.

[38] See, however, Atkinson and Stiglitz (1980) for a discussion of Ramsey taxation in models with distortions and Hagedorn (2010) on how labour market distortions can result in non-convexities in the Ramsey problem.

product market distortions in determining the dynamic effects of tax policy changes on unemployment, the distribution of income and welfare. Moreover, our analysis highlights the similarities and differences in the transmission mechanisms of tax policy changes across four alternative models.

The over-arching finding is that the presence of labour and product market distortions is critical in determining not only the size but also the direction of welfare effects after the capital tax cut and, in particular, whether a capital tax cut can be Pareto improving. Previous research suggests that, on their own, each of these failures should result in welfare losses for at least a segment of the population, after a capital tax cut and a concurrent labour tax increase. For instance, Ardagna (2007) employs a model with monopoly union power and documents negative welfare effects for workers after increases in labour taxes, while Guo and Lansing (1999) show that the optimal capital tax in a model with monopolistic competition in the product market can be non-zero. While these implications are confirmed in the model, it has been found that when both failures characterise the economy, one distortion effectively corrects the other in a way that a capital tax cut can be Pareto improving.

The specific results can be summarised as follows. First, in the model with only the labour market distortion, a tax reform that implements a zero capital tax will imply welfare losses for the workers in the long run, whereas capitalist's and aggregate welfare increase. As in the model with perfectly competitive labour and product markets, when the capital tax is set to zero, the labour tax will have to increase to make up for the loss in tax revenue. However, given the non-competitive labour market, this increase in labour taxes will lead to a rise in unemployment, because it lowers the returns to work and thus makes the outside option, in the form of unemployment benefits, more attractive to the union. The unemployment channel is the critical link that modifies the results from the benchmark model with competitive labour markets.[39] In particular, although labour productivity and the wage rate increase in the long run, thanks to the higher capital accumulation, the workers cannot capture the full benefit of this as unemployment has also increased and the return to unemployment, i.e. the unemployment benefit, is less than the wage rate.

Second, under competitive labour markets but non-competitive product markets,

[39] The quantitative importance of the unemployment channel is confirmed by contrasting the above results to those obtained by performing the same experiment in the model with perfectly competitive labour markets, also calibrated to the same tax structure for the UK economy. In this case there are welfare gains for all agents.

there are welfare gains for the capitalists, losses for the workers, and also losses at the aggregate level, by a reform that implements a zero tax on capital income. This happens because, in this case, the government foregoes revenue from a non-distortionary tax base comprised of "pure profits" so that the required increase in the labour tax is larger and thus the after-tax wage decreases after the tax reform.

Third, monopolistic competition in the product market under unionised labour markets, introduces a market failure that works to correct the negative implications of imperfect labour markets. In particular, monopolistic competition tends to reduce the positive revenue effects of the increase in output after a capital tax cut and thus reduces the benefits to the firm for a successful outcome of wage bargaining. In turn, this implies that the relative attractiveness of the firm's outside option in bargaining and its power relative to the union increases. This tends to increase employment and the benefits to all agents after a tax cut.

The rest of this chapter is organised as follows. Section 2.2 describes the model structure and Section 2.3 discusses the calibration of the model to the current UK economy. The results of tax reforms are analysed in Section 2.4. Section 2.5 solves the model and studies the dynamic behavior of the economy in response to a temporary and unanticipated capital tax cut and finally Section 2.6 gives a summary of the model and concludes.

2.2 The model

A model economy that allows for imperfect competition in both labour and product markets is described in this section. The economy consists of infinitely lived households, firms, trade unions, and a government. Households are comprised of capitalists and workers. Capitalists can work and save in the form of capital, own firms and receive profits. Workers, in contrast, do not save and thus consume all their disposable income in each period. Both capitalists and workers can spend part of their time endowment either employed or unemployed and receive unemployment benefits from the government when not working. All households are represented by firm-level trade unions which determine work hours and bargain with firms over the wage rate with the aim of maximising the average labour income of their members. Firms include final and intermediate goods producers. Final goods producers are competitive, but intermediate goods producers have monopoly power in the product market and seek to maximise profits employing workers from the unionised labour market and capital from the perfectly competitive asset market. Finally, exogenous public policy consists of the government taxing interest income from capital, profits and labour income to finance unemployment benefits and other non-employment related public transfers.

The timing of events in this setup is as follows. Given fiscal policy, unions and intermediate goods producers bargain over the wage rate, subject to the demand functions for labour and intermediate goods, by taking capital accumulation as given. Next, each intermediate goods producer, taking factor prices, final output and government policy as given, chooses factor quantities to maximise profits, subject to the demand function by the final goods producer for its output. Finally, final goods producers generate output and households make their savings decisions, taking all prices and policy variables as given.

2.2.1 Population composition

Total population, N, is exogenous and constant over time with capitalists and workers respectively being denoted as N^k and N^w. The population share of capitalists is defined as: $N^k/N \equiv n^k$, and the workers share as $N^w/N \equiv n^w = 1-n^k$. Finally each capitalist is assumed to own one intermediate goods-producing firm, hence the number of firms, $N^f = N^k$.

2.2.2 Households

As pointed out above, the households which populate the model have unequal access to the financial markets. This is motivated by imperfections in the asset markets that require agents to pay transactions costs to participate. These participation premia differ between the agents due to, for instance, past experience, socioeconomic background, networks, or firm ownership that gives an insider advantage in financial transactions (see e.g. Galor and Zeira (1993), Benabou (1996) and Aghion and Howitt (2009) for further discussion and micro-foundations for capital market imperfections and income inequality). On the other hand, households are identical in the labour market since unions guarantee that their members have equal employment and wages (see e.g. Pissarides (1998)).[40]

Following, for example, Judd (1985), Lansing (1999), Krusell (2002) and Blanchard and Giavazzi (2003) there are two types of households, termed as capitalists and work-

[40] For a model with price taking firms and uninsured idiosyncratic employment risk due to incomplete financial markets and search frictions, see Krusell *et al.* (2010).

ers, that face, respectively, the minimum (zero) and maximum (infinity) participation costs in the asset markets. Each household $i = k, w$, maximises the discounted sum of his lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^i\right)^{1-\sigma}}{1-\sigma} \tag{69}$$

where C_t^i is household i's private consumption; $0 < \beta < 1$ is the constant discount rate; and $\sigma > 1$ is the coefficient of relative risk aversion.

The budget constraint of each capitalist at t is given by:

$$\begin{aligned} C_t^k + I_t^k &= r_t K_t^k - \tau_t^k \left(r_t - \delta\right) K_t^k + (1-\tau_t^k)\pi_t^k + \\ &\quad + \left(1-\tau_t^w\right) w_t e_t + \overline{G}_t^u \left(1-e_t\right) + \overline{G}_t^t \end{aligned} \tag{70}$$

where I_t^k is the investment; K_t^k is the capital stock held at the beginning of time t; r_t is the gross return to capital; π_t^k is profits; $e_t = 1 - u_t$ is the per capita employment rate with u_t denoting the per capita unemployment rate; w_t is the gross wage rate; $\overline{G}_t^u$ is per capita unemployment benefits; $\overline{G}_t^t$ is per capita government transfers; $0 \leq \tau_t^w < 1$ is the tax rate on labour income; $0 \leq \tau_t^k < 1$ is the tax rate on capital income; and $0 < \delta < 1$ is the constant depreciation rate of capital stock.[41]

The capital stock evolves according to:

$$K_{t+1}^k = \left(1-\delta\right) K_t^k + I_t^k. \tag{71}$$

Each worker's within period budget constraint is given by:

$$C_t^w = \left(1-\tau_t^w\right) w_t e_t + \overline{G}_t^u \left(1-e_t\right) + \overline{G}_t^t. \tag{72}$$

Each household is randomly allocated to a union which bargains with a firm to determine employment, e_t, and the wage rate, w_t. Given that we will work with a symmetric equilibrium, employment and the wage rate will be the same for all households, so that the allocation of households to unions does not matter. In other words, the hetererogeneity in the labour market is not examined in the model. Instead, all heterogeneity in our model is driven by differences in asset ownership.[42]

[41] Note that capital taxes are assumed to be net of depreciation as in e.g. Lansing (1998). Also, as in Guo and Lansing (1999), the fiscal authority cannot impose a separate tax rate on profits and on interest income from savings, since it is difficult, in practice, to distinguish these two sources of capital income. If a separate profit tax was available, welfare could be improved by using this tax instrument, relative to the others, given that taxing profits does not distort incentives.

[42] To simplify notation, household subscripts are not used for e_t and w_t, since these quantities will

Therefore, the capitalist's problem is to choose $\left\{C_t^k, K_{t+1}^k\right\}_{t=0}^{\infty}$ to maximize (69) subject to (70) and (71) taking market prices $\{r_t, w_t\}_{t=0}^{\infty}$, the employment rate $\{e_t\}_{t=0}^{\infty}$, policy variables $\left\{\tau_t^k, \tau_t^w, \overline{G}_t^u, \overline{G}_t^t\right\}_{t=0}^{\infty}$, and K_0^k as given. The work hours for capitalists and the wage rate are determined by the bargain between the union and firm.

The optimization problem of the capitalist can be expressed mathematically as follows:

$$\max_{\left\{C_t^k, K_{t+1}^k\right\}_{t=0}^{\infty}} \left\{\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma}\right\}$$

$$\begin{aligned} s.t. \qquad C_t^k + K_{t+1}^k - (1-\delta) K_t^k &= r_t K_t^k - \tau_t^k (r_t - \delta) K_t^k + (1-\tau_t^k)\pi_t^k + \\ &\quad + (1-\tau_t^w) w_t e_t + \overline{G}_t^u (1-e_t) + \overline{G}_t^t. \end{aligned}$$

The Lagrangian function of the capitalist is then written as:

$$\begin{aligned} L^k = \quad & \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma} + \lambda_t^k \left[r_t K_t^k - \tau_t^k (r_t - \delta) K_t^k + (1-\tau_t^k)\pi_t^k + \right.\right. \\ & \left.\left. + (1-\tau_t^w) w_t e_t + \overline{G}_t^u (1-e_t) + \overline{G}_t^t - C_t^k - K_{t+1}^k + (1-\delta) K_t^k \right\} \end{aligned}$$

where λ_t^k is the Lagrangian multiplier associated with capitalist's budget constraint.

The first-order condition (FOC) for C_t^k is:

$$(1-\sigma) \frac{\left(C_t^k\right)^{-\sigma}}{1-\sigma} - \lambda_t^k = 0$$

$$\lambda_t^k = \left(C_t^k\right)^{-\sigma}. \tag{73}$$

The FOC for K_{t+1}^k is:

$$-\lambda_t^k + \beta \lambda_{t+1}^k \left[r_{t+1} - \delta + \tau_{t+1}^k (\delta - r_{t+1}) + 1\right]$$

$$\lambda_t^k = \beta \lambda_{t+1}^k \left[r_{t+1} - \delta + \tau_{t+1}^k (\delta - r_{t+1}) + 1\right]. \tag{74}$$

These two optimality conditions from capitalist's utility maximization problem describe the optimal intertemporal choice made by the capitalist in equilibrium.

Since the worker does not save and given that his work hours also depend on the outcome of the Nash bargain, optimal consumption simply follows residually from his

be the same for capitalists and workers in equilibrium. Thus, e_t and w_t, like $\overline{G}_t^u$ and $\overline{G}_t^t$, denote average, or per capita outcomes.

budget constraint in (72).

2.2.3 Firms

There are two production sectors: intermediate goods sector and final goods sector. Following for example, Guo and Lansing (1999), the model allows for monopolistic power in the intermediate goods market. A unique final good, Y_t, is produced according to the following constant returns to scale technology:

$$Y_t = \left[\sum_{f=1}^{N^f} \lambda^f \left(Y_t^f \right)^{\theta} \right]^{1/\theta} \tag{75}$$

where $\sum_{f=1}^{N^f} \lambda^f = 1$ are weights attached to intermediate goods producers; and $0 < \theta \leq 1$ implies the degree of monopoly power of intermediate goods producers. Final goods producers behave competitively and choose intermediate inputs, Y_t^f , to maximize profits, Π_t, taking the price of these inputs, P_t^f, as given:

$$\Pi_t = Y_t - \sum_{f=1}^{N^f} \lambda^f P_t^f Y_t^f. \tag{76}$$

The profits function can be rewritten by substituting out Y_t using (75):

$$\Pi_t = \left[\sum_{f=1}^{N^f} \lambda^f \left(Y_t^f \right)^{\theta} \right]^{1/\theta} - \sum_{f=1}^{N^f} \lambda^f P_t^f Y_t^f \tag{77}$$

and take the first-order condition with respect to Y_t^f:

$$\frac{\partial \Pi_t}{\partial Y_t^f} = \frac{1}{\theta} \left[\sum_{f=1}^{N^f} \lambda^f \left(Y_t^f \right)^{\theta} \right]^{1/\theta - 1} \lambda^f \theta \left(Y_t^f \right)^{\theta - 1} - \lambda^f P_t^f = 0. \tag{78}$$

The final goods function (75) is then rewritten as:

$$\sum_{j=1}^{N^f} \lambda^f \left(Y_t^f \right)^{\theta} = (Y_t)^{\theta} \tag{79}$$

and substitute it back into (78):

$$\left[(Y_t)^{\theta} \right]^{1/\theta - 1} \lambda^f \left(Y_t^f \right)^{\theta - 1} = \lambda^f P_t^f.$$

Solving for individual price, P_t^f, in above equation yields:

$$P_t^f = \left(\frac{Y_t}{Y_t^f}\right)^{1-\theta} \tag{80}$$

which is the demand function for intermediate input in the final goods production and can be rewritten as follows:

$$Y_t^f = Y_t \left(P_t^f\right)^{\frac{1}{\theta-1}}. \tag{81}$$

Final goods production is completely competitive and this implies zero profits of the firm:

$$\Pi_t = Y_t - \sum_{f=1}^{N^f} \lambda^f P_t^f Y_t^f = 0 \tag{82}$$

$$Y_t - \sum_{f=1}^{N^f} \lambda^f P_t^f Y_t \left(P_t^f\right)^{\frac{1}{\theta-1}} = 0$$

$$Y_t = Y_t \sum_{f=1}^{N^f} \lambda^f \left(P_t^f\right)^{\frac{1}{\theta-1}+\frac{\theta-1}{\theta-1}}$$

$$1 = \sum_{f=1}^{N^f} \lambda^f \left(P_t^f\right)^{\frac{\theta}{\theta-1}} \tag{83}$$

which could be thought of as zero-profit condition in the final goods sector.

In intermediate goods sector, each firm produces its individual output, Y_t^f, with a constant-returns-to-scale technology in two productive inputs: capital, K_t^f, and labour, L_t^f:

$$Y_t^f = A \left(K_t^f\right)^{\alpha} \left(L_t^f\right)^{1-\alpha} \tag{84}$$

where A is neutral technical progress and $0 < \alpha < 1$ denotes capital's share of output. The profits earned by the intermediate goods producer at time t are:

$$\pi_t^f = P_t^f Y_t^f - r_t K_t^f - w_t^f L_t^f. \tag{85}$$

Taking factor prices, r_t and w_t^f, and final output, Y_t, as given, the intermediate firm chooses K_t^f and L_t^f to maximize profits (85) subject to its production function (84), and the demand function for its output (80).

The first-order condition with respect to K_t^f:

$$\begin{aligned}\frac{\partial \pi_t^f}{\partial K_t^f} &= 0 = \frac{\partial \left[P_t^f Y_t^f\right]}{\partial K_t^j} - r_t \\ &= \frac{\partial \left[\left(\frac{Y_t}{Y_t^f}\right)^{1-\theta} Y_t^f\right]}{\partial K_t^f} - r_t\end{aligned}$$

$$\frac{\partial \left[(Y_t)^{1-\theta}\left(Y_t^f\right)^{\theta}\right]}{\partial K_t^f} = r_t.$$

After substituting out Y_t^f using (84), the FOC becomes:

$$(Y_t)^{1-\theta} \frac{\partial \left[\left(A\left(K_t^f\right)^{\alpha}\left(L_t^f\right)^{1-\alpha}\right)^{\theta}\right]}{\partial K_t^f} = r_t$$

$$(Y_t)^{1-\theta}\,\theta \left[A\left(K_t^f\right)^{\alpha}\left(L_t^f\right)^{1-\alpha}\right]^{\theta-1} \alpha A\left(K_t^f\right)^{\alpha-1}\left(L_t^f\right)^{1-\alpha} = r_t$$

$$(Y_t)^{1-\theta}\,\theta\left(Y_t^f\right)^{\theta-1}\frac{\alpha Y_t^f}{K_t^f} = r_t.$$

By making use of (80), this condition can be simplified to be:

$$\theta P_t^f \frac{\alpha Y_t^f}{K_t^f} = r_t \tag{86}$$

which is the demand function for physical capital services in the intermediate goods sector.

Analogously, the demand function for labour can be derived which turns out to be:

$$\theta P_t^f \frac{(1-\alpha)Y_t^f}{L_t^f} = w_t^f. \tag{87}$$

In equilibrium, the resulting profits in the intermediate goods sector are:

$$\pi_t^f = (1-\theta)\, P_t^f Y_t^f. \tag{88}$$

If we restrict our attention to a symmetric equilibrium, all the intermediate producers are identical so that the products they produce are identical. In this case, the same wage rate is paid to the households by all firms, i.e. $w_t^f = w_t$ for all f. These two implications of $P_t^f = 1$ and $Y_t^f = Y_t$ can be obtained in equilibrium conditions (83) and (80). Thus optimality conditions from the firm's profit-maximization problem, (86)-(88), can become:

$$\theta \frac{\alpha Y_t^f}{K_t^f} = r_t, \tag{89}$$

$$\theta \frac{(1-\alpha) Y_t^f}{L_t^f} = w_t \tag{90}$$

and

$$\pi_t^f = (1-\theta) Y_t^f. \tag{91}$$

When $\theta = 1$, intermediate goods are perfect substitutes in the production of the final good implying that intermediate goods producers have no power in the product market. In this case, prices are given for these producers and thus there is perfect competition. However, when $\theta < 1$, the demand function is downward sloping and they can exploit their monopoly power to maximise non-zero profits.

2.2.4 Unions

Following the literature cited in the introduction, the right-to-manage setup is employed where unions and firms (intermediate goods producer) bargain over the wage rate. For simplicity, each union is assumed to bargain with one firm to determine the wage rate (see e.g. Pissarides (1998)). Given that we will work with a symmetric equilibrium, this assumption is not important. Moreover, for tractability, and following for example, Domeij (2005) and Koskela and von Thadden (2008), two simplifying assumptions are made regarding this bargaining process.[43] First, unions are small enough so that they do not internalise the effects of the wage rate on capital accumulation and thus on future prices. Second, firms are also small enough so that they do not internalise the effects of the outcome of wage bargaining on capital accumulation.

The above assumptions imply that unions and firms take capital as given when bargaining over the wage rate. This form of myopia allows for a technical simplification in that it effectively reduces the wage-bargaining problem to a series of static problems, as in for example, Pissarides (1998), Maffezzoli (2001) and Ardagna (2007). The union and the intermediate goods producer bargain over the wage rate to maximise a weighted

[43] See Domeij (2005), Koskela and von Thadden (2008) and the references therein for the empirical relevance of these assumptions.

average of labour income and profits:

$$U_t^N = \left[(1-\tau_t^w)\, w_t n^k L_t^f + \overline{G}_t^u \left(1 - n^k L_t^f\right) - \overline{G}_t^u\right]^\phi \left[\pi_t^f + r_t K_t^f\right]^{1-\phi} \tag{92}$$

subject to the labour demand function given by the firm's first-order condition for labour, $\theta \frac{(1-\alpha) Y_t^f}{L_t^f} = w_t$, and the firm's product demand function, $P_t^f = \left(Y_t / Y_t^f\right)^{1-\theta}$, taking the capital stock, K_t^f, final output, Y_t, and the fiscal policy variables, $\left\{\tau_t^k, \tau_t^w, \overline{G}_t^u, \overline{G}_t^t\right\}$, as given.

In the above setup, $n^k L_t^f \equiv e_t$ is the average employment rate, so that $\left(1 - n^k L_t^f\right)$ is the unemployment rate and $0 \leq \phi \leq 1$ describes the relative bargaining power of the union with $\phi = 1$ representing the monopoly union case. Note that the union targets average labour income, $(1-\tau_t^w)\, w_t n^k L_t^f + +\overline{G}_t^u \left(1 - n^k L_t^f\right)$, while the firm targets average profits, π_t^f. The outside option for the union is the unemployment benefit, $\overline{G}_t^u$, while for the firm it is the sunk cost of capital, $-r_t K_t^f$, which is a consequence of the assumption that the representative firm takes the average capital accumulation as given. It is important to note that while the agents involved in Nash-bargaining over the wage rate do not internalise the effects of the wage rate on capital accumulation which is consistent with Domeij (2005) and Koskela and von Thadden (2008).

To get the optimality conditions of the union, the labour demand function is used as a constraint in the Lagrangian function of the union and we derive the first-order conditions with respect to w_t and e_t. The variable λ_t^u is the Lagrangian multiplier on the labour demand function. Alternatively, the variable, e_t, can be substituted out in the bargaining function (92) by using the labour demand function. The choice variable of union is therefore only w_t. In practice, these two methods are analytically equivalent. We have examined that both methods yield the same equilibrium conditions.[44]

2.2.5 Government and market clearing conditions

The per-capita government budget constraint equating public spending and revenues is given by:

$$\overline{G}_t^t + \overline{G}_t^u (1 - e_t) = n^k \tau_t^k (r_t - \delta) K_t^k + n^k \tau_t^k \pi_t^k + \tau_t^w w_t e_t. \tag{93}$$

To ensure that the government budget is balanced at each time t, the wage income

[44] The analytical solution to the bargaining problem is obtained using Matlab programs. In order to save space, we did not present the intermediate mathematical steps. But they are available upon request.

tax, τ_t^w, is residually determined. In what follows, the government spending instruments will be fixed to their steady-state values $\overline{G}^t$ and $\overline{G}^u$, respectively, so that any changes in the capital tax rate, τ_t^k, will be met by changes in τ_t^w, ensuring that the budget constraint of the government is satisfied.

2.2.6 Market-clearing conditions and recourse constraint

The capital market clears when the supply is equal to the demand for capital per capita:

$$K_t^k = K_t^f. \tag{94}$$

The market clearing condition for per capita dividends is:

$$\pi_t^k = \pi_t^f. \tag{95}$$

In the labour market the equality of per capita labour supply and demand is given by:

$$e_t = n^k L_t^f. \tag{96}$$

Finally, in the goods market, the economy's per capita resource constraint is:

$$n^k Y_t^f = n^k C_t^k + n^w C_t^w + n^k \left[K_{t+1}^k - (1-\delta) K_t^k\right]. \tag{97}$$

2.2.7 Decentralized equilibrium (given policy)

The decentralized equilibrium conditions are now summarised in real terms and as a symmetric equilibrium implying that $Y_t^f = Y_t$, $w_t^f = w_t$ and $P_t^f = 1$ for all f. Given the paths of prices $\{w_t, r_t\}_{t=0}^{\infty}$ the policy instruments $\left\{\tau_t^k, \overline{G}_t^u, \overline{G}_t^t\right\}_{t=0}^{\infty}$ and an initial condition for K_0^k, a decentralized equilibrium (DE) is defined to be an allocation $\left\{C_t^k, K_{t+1}^k, C_t^w, e_t\right\}_{t=0}^{\infty}$ and one residually determined policy instrument, τ_t^w, such that (i) households, firms and unions undertake their respective optimization problems outlined above; (ii) all budget constraints are satisfied; and (iii) all markets clear.

To summarize, the DE[45] consists of the capitalist's optimality conditions for C_t^k and K_{t+1}^k; the firm's first-order conditions for K_t^f and L_t^f; the budget constraints of workers and government, i.e. BC^w and BC^g; the aggregate resource constraint, RC; the market clearing conditions in the capital, dividend and labour markets, i.e. MC_K,

[45] The full DE conditions are provided in the Appendix 2.A.1. Note that relying on Walras's law, the budget constraint of the capitalist can be dropped from the DE.

MC_π and MC_L; the union's optimality condition for the wage rate, w_t, and constraint for the employment rate, e_t.

2.3 Calibration to the UK economy

2.3.1 Calibration

The structural parameters of the model with product and labour market distortions are next calibrated so that its steady-state solution reflects the main empirical characteristics of the UK economy, with particular emphasis on the tax rates and the unemployment rate. The structural parameters for the full model, including both labour and product market distortions, are reported in Table 2.1 with the implied steady-state solution in Column (1) of Table 2.2.

Table 2.1: Calibration (labour and product market distortions)

Parameter	Definition	Value
$0 < \beta < 1$	rate of time preference	0.970
$0 < \alpha < 1$	capital's share	0.350
$0 < \delta < 1$	depreciation rate on capital	0.100
$0 < n^k < 1$	population share of capitalists	0.300
$\sigma > 1$	relative risk aversion coefficient	2.000
$A > 0$	TFP level	1.000
$0 \leq \phi \leq 1$	union power	0.500
$0 < \theta \leq 1$	product market power	0.900
$\overline{G}^t > 0$	per capita public transfers	0.309
$\overline{G}^u > 0$	per capita unemployment benefit	0.475
$0 \leq \tau^k < 1$	tax rate on capital income	0.442

According to the Family Resources Survey in 2008-2009, about 30% of households have savings and investments above £10,000.[46] Assuming, for the households with savings below this threshold, that capital income does not constitute a significant portion of their budget, the share of capitalists, n^k, is set to 0.3. The productivity parameter, A, is normalised to 1. The common values from the literature are used for the: (i) intertemporal elasticity of substitution, $1/\sigma = 0.5$ or $\sigma = 2$; (ii) depreciation

[46] The survey is sponsored by the Department for Work and Pensions (see their Table 4.9 in Chapter 4: Savings and Investments of the 2008-2009 Annual Report for the information reported here).

rate, $\delta = 0.1$; and (iii) annual rate of time preference, $\beta = 0.97$, (see, e.g. Angelopoulos *et al.* (2011b) and references therein). Together with a standard value for the capital productivity parameter, $\alpha = 0.35$,[47] these parameters imply that in the steady-state the capital-to-output ratio is about 2.

The value for union power, $\phi = 0.5$, is chosen to be the middle of the range (i.e. 0.4 to 0.6) of values typically used in the literature (see e.g. Domeij (2005) for a discussion of the relevant studies and empirical evidence). It is shown later that the results which follow in the remainder of the study remain qualitatively robust to lower and higher values of ϕ encompassing this range.

The base calibration also allows for market power in the product market by setting $\theta = 0.9$, implying that profits, in equilibrium, amount to 10% of GDP. This value approximates the magnitude typically employed in New Keynesian models to capture the price mark-up over marginal costs (see, e.g. Leith and Malley (2005)). As with union power, it is shown below that the results which follow, generally, do not change qualitatively when different values of θ are considered.

Effective average tax rates for capital and labour income from 1970-2005 are constructed by following the approach in Conesa *et al.* (2007) using data from the National Accounts and the Public Sector, Taxation and Market Regulation databases (available from OECD.Stat database). The average capital tax rate over the time period is $\tau^k = 0.442$. This dataset also implies that the average tax rate on labour income is 0.27.[48] The spending instruments, $\overline{G}^t$ and $\overline{G}^u$, are thus calibrated so that the implied model solution for τ^w is 0.27 and the unemployment rate is 7%. The unemployment rate corresponds to the average from 1970 to 2010 from the UK Office for National Statistics.

The steady-state solution of four different models given the above parameterisation, as shown in Table 2.2 Column (1), implies the following shares of public spending in GDP: $\frac{G^t}{Y} = 0.227$ and $\frac{G^u}{Y} = 0.024$, which further implies that government spending in transfers is about 25% of GDP, consistent with UK data from the OECD.Stat database. In addition, it suggests a replacement ratio, $\frac{\overline{G}^u}{w}$, of about 50% in the long run. This rate is comparable with data for industrialised countries (see e.g. Nickell and Nunziata

[47] Note that the value for α is different from the one employed in Chapter 1. This is because the calibration in Chapter 1 is completely based on the data. However, in this chapter, this value is chosen to yield the steady-state capital-to-output ratio of 2.

[48] Following the approach in Conesa *et al.* (2007) gives similar qualitative tax rates for the UK to those in Martinez-Mongay (2000), and has the advantage that the data for a longer period can be employed.

(2001)) and values used in previous studies, ranging from 45% (Shi and Wen (1999)) to 60% (Pissarides (1998)).

2.3.2 Steady-state solution

Table 2.2 also reports the net returns to labour and capital, $\widetilde{w} = w(1-\tau^w)$ and $\widetilde{r} = (r-\delta)\left(1-\tau^k\right)$, respectively, which will be useful for the analysis which follows. Finally aggregate or social welfare, U, is defined in the Benthamite fashion as the average welfare of all agents in the economy.

Table 2.2: Pre-reform steady-states

	Full Model (1)	Comp. Model (2)	Union Model (3)	Profits Model (4)
$\frac{C}{Y}$	0.797	0.775	0.775	0.797
$\frac{I^k}{Y}$	0.203	0.225	0.225	0.203
$\frac{K^k}{Y}$	2.027	2.252	2.252	2.027
$\frac{G^t}{Y}$	0.227	0.231	0.203	0.252
$\frac{G^u}{Y}$	0.024	0.000	0.028	0.000
$\frac{\pi}{Y}$	0.100	0.000	0.000	0.100
$\frac{\overline{G}^u}{w}$	0.499	0.000	0.575	0.000
τ^w	0.270	0.270	0.270	0.270
$(1-e)$	0.070	0.000	0.070	0.000
w	0.951	1.006	1.006	0.856
$\widetilde{w}$	0.694	0.735	0.735	0.625
r	0.173	0.155	0.155	0.155
$\widetilde{r}$	0.041	0.031	0.031	0.031
C^k	1.461	1.451	1.350	1.571
C^w	0.924	1.092	1.015	0.993
U^k	-22.82	-22.97	-24.70	-21.22
U^w	-36.09	-30.53	-32.83	-33.57
U	-32.11	-28.26	-30.39	-29.86

To obtain benchmarks that will help contextualise the importance of the two distortions in the labour and product markets, in addition to the full model shown in

Column (1) of Table 2.2, the relevant special cases are presented. For example, Column (2) reports the steady-state results for the competitive model.[49] This model effectively belongs to the set of models discussed in Judd (1985) and assumes an inelastic labour supply implying that $e^k = e^w = e = 1$, or that unemployment is zero.[50] Column (3) covers the case when both final and intermediate product markets are completely competitive but labour markets are unionised.[51] Following the setup in Guo and Lansing (1999), albeit with inelastic labour supply, Column (4) shows the results for the model when intermediate product markets are monopolistic but labour markets are competitive.[52]

To understand the effects of introducing union power to the competitive model and to the profits model, compare respectively the results in Column (2) with (3) and (4) with (1). It is clear that the labour market distortion worsens relative outcomes for both agents through higher unemployment, lower labour income and lower consumption. Hence welfare for both agents and thus aggregate welfare is lower in the models incorporating unions.

Similarly the effects of allowing for market power in the competitive model and in the union model can be seen by comparing, respectively, the results in Column (2) with (4) and (3) with (1). In both cases it can be seen that the welfare of capitalist has increased though worker's and aggregate welfare has decreased. This finding is driven by higher relative consumption for the capitalist arising from non-zero economic profits resulting from the intermediate goods production.

2.4 Steady-state effects of tax reforms

We are now in a position to examine the distributional effects of tax reforms that reduce the tax burden on capital under market distortions in labour and product markets. In all cases, the effects of capital tax reductions are found to be monotonic and increase

[49]The results reported in Column (2) have been obtained using the parameters in Table 2.1 except that ϕ and $\overline{G}^u$ are not relevant since unemployment is zero, θ is equal to unity and $\overline{G}^t$ is re-calibrated so that the steady-state value of $\tau^w = 0.27$ can be achieved.

[50]The case of inelastic labour supply was also considered by Judd (1985) and is employed here so that exogenous leisure is treated consistently across the models we employ. However, note that the results reported below do not change qualitatively when we allow for endogenous leisure in the perfectly competitive model.

[51]To obtain the results reported in Column (3), the parameters from Table 2.1 are employed, except that θ is equal to unity and the values for $\overline{G}^t$ and $\overline{G}^u$ are re-calibrated, so that the implied model solutions for τ^w and $(1-e)$ are 0.27 and 0.07 respectively.

[52]The results reported in Column (4) have been obtained using the parameters in Table 2.1, except that, ϕ and are not relevant since unemployment is zero and G^t is re-calibrated so that the steady-state value of $\tau^w = 0.27$ can be achieved.

with the magnitude of the capital tax cut. Hence, we focus on a policy reform that imposes a zero capital tax given, as pointed out in the introduction, its prominence in the tax reform literature.

The effects of the tax reform are evaluated by comparing the post- with the pre-reform steady states since the distributional effects of capital tax cuts in the long run will be examined.[53] In the Tables 2.3-2.6 which follow, for ease of comparison, the relevant pre-reform results alongside the post-reform ones are repeated. To contextualise the importance of market imperfections, the results for the benchmark case of competitive markets are first discussed. We next analyse the role of unionised labour markets and imperfect product markets in isolation and then add product market power to the labour market distortion. Finally, as a robustness exercise, in Tables 2.7 and 2.8 respectively, the quantitative importance of firm and union power is examined.

2.4.1 The competitive model

The steady-state allocations together with welfare in the competitive model are shown in Table 2.3. The steady-state welfare gains and losses for capitalists and workers, ζ^k and ζ^k, respectively, together with the welfare gains at the aggregate level, ζ are also reported. These have been calculated as the consumption supplement required to make the agent as well off in both regimes.[54] As can be seen, in the competitive model, implementation of the tax reform will be Pareto improving in the long run, even if it increases inequality. In other words, there are welfare gains for both type of agents, although the gains for the capitalists compared to the workers are much higher (i.e. 8.3% versus 1.1%) and thus their relative welfare position improves. This is consistent with Judd's (1985) results that it is optimal for both types of agents to choose a zero capital tax.

The trade-off for the worker after implementing the zero capital tax can be seen by noting that, although the labour tax, τ^w, increases (i.e. from 0.27 to 0.324) to make up for the loss in the tax revenue, due to the elimination of the capital tax, the wage rate, w, rises as well (i.e. from 1.006 to 1.104). This implies that the net return to labour, $\widetilde{w}$, also increases (i.e. 0.735 to 0.747) and thus income, consumption and welfare rise. The

[53] Note that it is already known that including the transition period will affect the Pareto superiority of capital tax cuts, even in models where a reduction in the capital tax is Pareto improving in the long run (see e.g. Garcia-Milà *et al.* (2010) and Angelopoulos *et al.* (2011b)).

[54] In particular, they have been obtained using the formula $\left(\frac{W_A}{W_B}\right)^{\frac{1}{1-\sigma}} - 1$, where W_A and W_B is welfare post- and pre-reform, respectively.

reason the wage rate increases by more than the tax rate is that the elimination of the capital tax boosts investment and capital, which in turn increases labour productivity and this is translated into higher wages. Therefore, everyone benefits in the long run by a reform that implements the zero capital tax.

Table 2.3: Effects of tax reform in the competitive model

	post-reform (1)	pre-reform (2)
$\frac{C}{Y}$	0.733	0.775
$\frac{I^k}{Y}$	0.267	0.225
$\frac{K^k}{Y}$	2.673	2.252
$\frac{G^t}{Y}$	0.210	0.231
$\frac{G^u}{Y}$	0.000	0.000
$\frac{\pi}{Y}$	0.000	0.000
$\frac{\overline{G}^u}{w}$	0.000	0.000
τ^w	0.324	0.270
$(1-e)$	0.000	0.000
w	1.104	1.006
$\widetilde{w}$	0.747	0.735
r	0.131	0.155
$\widetilde{r}$	0.031	0.031
C^k	1.572	1.451
C^w	1.104	1.092
U^k	-21.21	-22.97
U^w	-30.20	-30.53
U	-27.50	-28.26
ζ^k	0.083	–
ζ^w	0.011	–
ζ	0.028	–

2.4.2 Unionised labour markets

The results for the model with distortions in the labour market are shown in Table 2.4. Under unionised labour markets, the welfare gains for all agents are generally lower, compared to those obtained in the competitive model (see Table 2.3), and, more importantly, there are now welfare losses for the workers in the long run. This occurs since imperfect competition in the labour market negatively affects the trade-off for the workers that arises with the implementation of the zero capital tax.

Table 2.4: Effects of tax reform in the union model

	post-reform	pre-reform
	(1)	(2)
$\frac{C}{Y}$	0.733	0.775
$\frac{I^k}{Y}$	0.267	0.225
$\frac{K^k}{Y}$	2.673	2.252
$\frac{G^t}{Y}$	0.187	0.203
$\frac{G^u}{Y}$	0.030	0.028
$\frac{\pi}{Y}$	0.000	0.000
$\frac{\overline{G}^u}{w}$	0.524	0.575
τ^w	0.334	0.270
$(1-e)$	0.082	0.070
w	1.104	1.006
$\widetilde{w}$	0.735	0.735
r	0.131	0.155
$\widetilde{r}$	0.031	0.031
C^k	1.443	1.350
C^w	1.014	1.015
U^k	-23.10	-24.70
U^w	-32.89	-32.83
U	-29.95	-30.39
ζ^k	0.069	–
ζ^w	-0.002	–
ζ	0.015	–

For example, Table 2.4 shows that the rise in the wage rate (i.e. from 1.006 to 1.104), due to higher productivity, just makes up for the rise in the labour income tax (i.e. from 0.27 to 0.334), so that the net wage for workers is the same before and after the zero capital tax reform (i.e. $\widetilde{w} = 0.735$).

Given the existence of unemployment, the labour income tax has to increase by more under imperfect competition to raise the necessary tax revenue after the loss in capital tax revenue. However, this rise in the labour tax has an additional side effect which ultimately hurts worker's labour income. In particular, a higher labour tax increases the unemployment rate, $(1-e)$, (i.e. from 7% to 8.2%), given that, *ceteris paribus* at the union-firm bargaining level, it decreases the returns to work and thus makes the outside option more attractive to the union. At the same time, again at the union-firm bargaining level, the rise in firms' capital stock increases output and this tends to increase the desire of the firms for a successful outcome of the wage bargain and thus further enhances the power of the unions.[55]

Since both effects tend to increase unions' power, unemployment increases. This in turn implies that the labour income of the worker falls since the wage rate is higher than the unemployment benefit. Therefore, the distortion in the labour market implies that workers cannot fully benefit from the positive effects created from capital accumulation and overall they are worse off by a tax reform that eliminates capital and increases labour taxes.

Our results for aggregate welfare are consistent with the long-run welfare gains to the representative agent in a unionised labour market generated by a tax reform that cuts capital taxes and increases labour taxes, see e.g. Daveri and Maffezzoli (2001). The quantitative magnitudes we find suggest that such a reform would generate enough gains to compensate the losers. Our findings regarding the distributional effects of the tax reform are also consistent with those in Ardagna (2007) who uses a model with monopoly union power following Maffezzoli (2001). Under perfectly competitive product markets, Ardagna (2007) includes a rich fiscal policy menu and government sector employment to examine exogenous changes in fiscal instruments accommodated by changes in government debt and finds that workers' utility decreases after increases in labour taxes.

2.4.3 Monopolistic product markets

The results for the model with distortions in the product markets are shown in Table 2.5. These suggest that there are welfare losses for the workers and at the aggregate level, but big welfare gains for the capitalists. This occurs since imperfect competition in the product market negatively affects the trade-off for the worker that arises with

[55]Note, given that there is perfect competition in the product market, prices are given for the firm and thus the increase in output implies a similar increase in revenue. The importance of this will become clearer when the case of monopolistic competition in product markets is considered below.

the implementation of the zero capital tax.

Table 2.5: Effects of tax reform in the profits model

	post-reform (1)	pre-reform (2)
$\frac{C}{Y}$	0.759	0.797
$\frac{I^k}{Y}$	0.241	0.203
$\frac{K^k}{Y}$	2.406	2.027
$\frac{G^t}{Y}$	0.230	0.252
$\frac{G^u}{Y}$	0.000	0.000
$\frac{\pi}{Y}$	0.100	0.100
$\frac{\overline{G}^u}{w}$	0.000	0.000
τ^w	0.393	0.270
$(1-e)$	0.000	0.000
w	0.939	0.856
$\widetilde{w}$	0.570	0.625
r	0.131	0.155
$\widetilde{r}$	0.031	0.031
C^k	1.871	1.571
C^w	0.939	0.993
U^k	-17.81	-21.22
U^w	-35.52	-33.57
U	-30.20	-29.86
ζ^k	0.192	–
ζ^w	-0.055	–
ζ	-0.011	–

More specifically, although the wage rate increases after the elimination of the capital tax (i.e. from 0.856 to 0.939), due to the usual channel of increased capital stock and labour productivity, the net wage decreases (i.e. from 0.625 to 0.57), as the rise in the labour tax (i.e. from 0.27 to 0.393) is higher than the increase in the wage rate. This implies that labour income is reduced and the workers are worse-off after the tax reform. This results is driven by the fact that the loss in tax revenue from capital is bigger under monopolistic profits, since profits represent an inelastic tax base. In turn, consistent with the findings in Guo and Lansing (1999), this implies that the tax rate on labour has to increase by more to make up for the loss in revenue, so that the net wage is reduced.

2.4.4 Labour and product market distortions

The post-reform findings in the model with distortions in both product and labour market are shown in Table 2.6. These reveal that the zero capital tax is Pareto superior to the current tax regime. This transpires since distortions in the product market work to offset the effects of imperfect competition in the labour market, resulting in a trade-off for the workers implying welfare gains in the long run after the capital tax cut.

Although again, after the tax reform, the net return to labour has remained the same as in the pre-reform economy, unemployment falls from 7% to 0.6% and, given that the wage rate is higher than the unemployment benefit, income, consumption and welfare for the workers increase as well. The two distortions create two opposing effects on unemployment after the tax reform. On one hand, the unionization of the labour market implies that, as previously, the rise in the labour tax (i.e. from 0.27 to 0.334) decreases the return to work and thus makes the outside option more attractive to the unions. However, the fall in the capital tax increases capital accumulation, which increases each individual firm's output, i.e. Y^f goes up. Given that final, per capita output, Y, is taken as given, this increase in individual firm's supply in the monopolistic market implies a fall in its individual price, P^f and tends to decrease the expected revenue of the representative firm. Monopolistic competition, therefore, tends to reduce the positive revenue effects of the increase in output and overall reduces the desire of the firms for a successful outcome of the wage bargain. In turn, this implies that the relative attractiveness of the firms' outside option in bargaining has increased, or else that the firms' power relative to the unions has increased. The result of these opposing effects is a rise in employment which is beneficial to the workers. Finally, note that the welfare gains to the capitalists are bigger relative to the worker, so that, although the elimination of the tax cut is Pareto superior, welfare inequality has increased.

In general, our findings for this case are consistent with results from policy analysis in second-best environments, which suggest that adding more frictions may lead to improved outcomes, given that one distortion might, effectively, correct another. Independently, each market failure implies that capital tax cuts will result in welfare losses for at least one segment of the population. Together, however, decreases in the capital tax under monopolistic competition in the product markets and unionization in labour markets are welfare improving for all agents.

Table 2.6: Effects of tax reform in the full model

	post-reform	pre-reform
	(1)	(2)
$\frac{C}{Y}$	0.759	0.797
$\frac{I^k}{Y}$	0.241	0.203
$\frac{K^k}{Y}$	2.406	2.027
$\frac{G^t}{Y}$	0.194	0.227
$\frac{G^u}{Y}$	0.002	0.024
$\frac{\pi}{Y}$	0.100	0.100
$\frac{\overline{G}^u}{w}$	0.455	0.499
τ^w	0.334	0.270
$(1-e)$	0.006	0.070
w	1.043	0.951
$\widetilde{w}$	0.694	0.694
r	0.146	0.173
$\widetilde{r}$	0.046	0.041
C^k	1.861	1.461
C^w	0.933	0.924
U^k	-17.92	-22.82
U^w	-35.72	-36.09
U	-30.38	-32.11
ζ^k	0.273	–
ζ^w	0.010	–
ζ	0.057	–

2.4.5 Changes in firm power

Using the full model, the importance of firm power in the product market on the welfare gains/losses of a tax reform that eliminates the capital tax is examined, given a unionised labour market (i.e. $\phi = 0.5$). The results for θ between 0.9 and 1 are summarised. For each case considered, we report the changes for consumption and the unemployment rate from a tax reform that eliminates the capital tax relative to the current tax policy as well as the compensating consumption supplement for each agent and the aggregate economy.[56]

Under unionised labour markets, the positive welfare effects for both types of agents are increased when the extent of monopolistic competition in the product market increases. In particular, when firms' power in the product market is sufficiently high (for our calibration, this occurs for $\theta < 0.98$), there are welfare gains to the workers to be observed after the capital tax cut. As can be seen, the effect of the tax reform on the labour market is positive in these cases, in the sense that employment is increased. This is because the monopolistic effect on firms' revenue is strong enough to outweigh, in the wage bargaining problem, the negative effects of increased labour taxes in the trade-off between the two market distortions discussed above.

Table 2.7: Firm power (θ) in the product market (changes relative to current policy)

	0.900	**0.950**	**0.980**	**0.990**	**1.000**
C^k	0.400	0.240	0.150	0.122	0.093
C^w	0.009	0.004	0.000	-0.001	-0.001
$(1-e)$	-0.064	-0.024	-0.002	0.005	0.012
ζ^k	0.273	0.170	0.109	0.090	0.069
ζ^w	0.010	0.004	0.000	-0.001	-0.002
ζ	0.057	0.038	0.024	0.020	0.015

2.4.6 Changes in union power

The importance of union power in the labour markets on the welfare gains of a tax reform the eliminates the capital tax, for given firms' power in the product market (i.e. $\theta = 0.9$) is next examined The parameter that measures the relative power of unions *vis-a-vis* the firms in Nash-bargaining for the wage rate is ϕ. As discussed earlier, our base calibration above is based on a value for ϕ that is effectively in the middle of the range of the empirically relevant values. In what follows, the changes in ϕ that encompass the entire range used in the literature, (see e.g. Domeij (2005))

[56]Not that for all cases considered, the spending shares are re-calibrated so that the base in all cases is an economy with 7% unemployment and 27% labour tax rate. Otherwise, the parameters used are as in Table 2.1. Also note that the results in Tables 2.7 and 2.8 for consumption are in percent differences, whereas the unemployment rate is in percentage point differences.

are examined. As in Table 2.7, for each case considered, the results are reported as changes from a tax reform that eliminates the capital tax relative to the current tax policy.

The results in Table 2.8 suggest that the welfare gains for all agents from a tax reform are increasing in ϕ. As ϕ increases, the unemployment benefit needs to fall in the pre-reform economy so that the new calibration implies the same labour tax and unemployment rate as in the base case.[57] This suggests that the relative power of the union in the labour market derives more from the institutional features associated with ϕ and less from the outside option or else, that $\overline{G}^u$ has a smaller effect on determining unemployment. As discussed above, after the capital tax cut and the concurrent increase in labour tax, the negative effect on unemployment takes place via the increase in the labour tax relative to the unemployment benefit, as they determine the relative weight of the outside option for unions. Therefore, given that the importance of this outside option has been reduced for a combination of higher ϕ and lower $\overline{G}^u$, the effect of the increase in the labour tax relative to $\overline{G}^u$ after the tax reform exerts a smaller negative effect on unemployment.

Table 2.8: Union power (ϕ) in the labour market (changes relative to current policy)

	0.250	**0.375**	**0.500**	**0.625**	**0.750**
C^k	0.392	0.396	0.400	0.403	0.406
C^w	0.005	0.007	0.009	0.011	0.013
$(1-e)$	-0.060	-0.062	-0.064	-0.066	-0.068
ζ^k	0.268	0.271	0.273	0.276	0.278
ζ^w	0.006	0.008	0.010	0.012	0.014
ζ	0.052	0.055	0.057	0.059	0.060

2.5 Model dynamics

In line with most DGE literature (see e.g. Judd (1989), McGrattan (1994), and Chari *et al.* (2007b)), the time path of capital income tax is assumed to be an $AR(1)$ process which is approximated by the data series. The stochastic exponential $AR(1)$ process is given by:

$$\tau_{t+1}^k = \left(\tau_0^k\right)^{\left(1-\rho^k\right)} \left(\tau_t^k\right)^{\rho^k} e^{\varepsilon_{t+1}^k} \tag{98}$$

[57] As before, the spending shares are re-calibrated so that the base in all cases is an economy with 7% unemployment and 27% labour tax rate. Otherwise, the parameters used are as in Table 2.1.

or

$$\ln(\tau_{t+1}^k) = \left(1-\rho^k\right)\ln\left(\tau_0^k\right) + \rho^k \ln\left(\tau_t^k\right) + \varepsilon_{t+1}^k \tag{99}$$

where τ_0^k is constant which gives the steady-state value of τ_t^k; $0 < \rho^k < 1$ is the first-order autoregressive parameter which measures the persistence of the $AR(1)$ process; and ε_t^k represents a random shock to the capital income tax which is characterised by a normal distribution with zero mean and standard deviation of σ_k. The estimation results show that the persistence parameter ρ^k is 0.956 and the variance σ_k^2 is 0.24% for the UK economy.

To solve the model, all the non-linear DE conditions are first log-linearised around the steady-state values of variables. The log-linearised system is then solved by applying Klein's (2000) generalised Schur or QZ decomposition algorithm. We find that, in all cases, the model exhibits saddle path stability. This implies a unique convergent solution to the model. The solution for each model can be represented in the form of first-order linear difference equations. The variable in the solution is expressed as log-difference from its corresponding steady-state which approximates the percent deviation from its respective steady-state value.[58]

Next, we study the dynamic behavior of the economy in response to a temporary, unanticipated 1% decrease in the tax rate on capital income while keeping all the other policy variables fixed to their respective steady-states. We discuss the principal channels via which an exogenous change to the capital income tax can influence the economy under four different model structures. The dynamic results are compared with those obtained in the steady-state analysis of capital tax reforms. In this section, we keep the same sequence of models as in the steady-state analysis. We first discuss the results under the competitive model without markets distortions and then analyse the effects in the model with labour market distortion or product market distortion, respectively. The model with distortions in both labour and product markets are finally studied. The results emphasize the relevance of labour and product market distortions in determining the dynamics of the model. Our analysis also highlights the similarities and differences in the transmission mechanisms across four models. In addition, the cumulative effects of the tax changes on unemployment, the distribution of income and welfare of agents are evaluated during the transition by comparing the post-shock deviations relative to the steady-state. In particular, the present discounted values of

[58] See Chapter 1 for the mathematical details. The linear solution for models in the chapter are not presented in this chapter to save space. But they are available upon request.

variables in years after the shock are compared with their respective present discounted values if variables are assumed to stay at the steady state and never deviate from it.

The post-shock economy is simulated 100 years following the temporary, unanticipated capital tax cut. The impulse responses are calculated as percent deviations from the steady-state values. The percent deviations in the present discounted values are calculated using the following formula:

$$D_j^X = \frac{\sum_{t=1}^{j} \beta^{t-1} X_t}{\sum_{t=1}^{j} \beta^{t-1} X} - 1 \tag{100}$$

where j denotes years after the shock; X is the steady-state value of variable; X_t is the value of variable for year t; and D_j^X is the percent deviation after j years. All the $t > 1$ values are discounted by the discount factor, $0 < \beta < 1$.

Figure 2.1 below provides a full picture of the complete paths of percent deviations in the post-shock economy in all four models. The results of different models are pooled in different rows in Figure 2.1. This figure clearly shows the cumulative effects of the temporary negative shock on the whole economy.

[Figure 2.1 about here]

Figures 2.2-2.5 in the following sub-sections, for each model, plot the impulse responses of endogenous variables, which show the time paths of variables in response to the temporary and unanticipated tax change. In addition, in Tables 2.9-2.12 which follow, for each case of comparison, the numeric results of the percent deviations calculated in (100) for eight different values of j are reported, i.e. $j = 1, 3, 5, 10, 20, 50, 80, 100$.

2.5.1 The competitive model

The impulse responses of variables in the competitive model without market distortions are plotted in Figure 2.2. The numeric results of percent deviations of variables after j years calculated using the formula (100) are reported in Table 2.9 and we can also refer to Figure 2.1a above for the complete paths of percent deviations.[59]

[Figure 2.2 about here]

Table 2.9: Percent deviation from steady-state in the competitive model

	Years after negative τ_t^k shock							
	1	3	5	10	20	50	80	100
C_t^k	-0.031	-0.020	-0.012	0.005	0.024	0.036	0.034	0.033
C_t^w	-0.078	-0.070	-0.062	-0.047	-0.029	-0.011	-0.008	-0.007
Y_t^k	0.004	0.010	0.014	0.023	0.033	0.036	0.033	0.032
$(1-e)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Y_t^f	0.000	0.007	0.013	0.024	0.036	0.040	0.037	0.036
K_t^k	0.000	0.020	0.037	0.069	0.102	0.115	0.106	0.103
τ_t^w	0.315	0.300	0.286	0.256	0.212	0.152	0.132	0.127
τ_t^k	-0.995	-0.953	-0.914	-0.829	-0.702	-0.514	-0.450	-0.432
U_t^k	-0.031	-0.020	-0.012	0.005	0.024	0.036	0.034	0.033
U_t^w	-0.078	-0.070	-0.062	-0.048	-0.029	-0.011	-0.008	-0.007
U_t	-0.067	-0.058	-0.050	-0.035	-0.016	0.001	0.002	0.003

The temporary capital income tax cut results in an increase in the labour income tax. This is because additional labour tax revenue is required to finance the government spending when the capital income tax decreases. The decrease in capital income tax increases the investment of capitalists. I_t^k rises dramatically following the shock and this is transformed into higher capital accumulation. As the shock dies away, the increase in investment weakens although it is still above the steady-state level over time. The increase in investment generates a negative income effect crowding out the consumption of capitalists. Hence, C_t^k falls short of its steady-state in the first 5 years (i.e. -0.012%). After then, C_t^k exceeds its initial steady-state and the present discounted value of utility, U_t^k, is also above its steady-state as can be seen in Table 2.9. The capital stock increases and it results in increase in output and labour productivity. This is transformed into the higher wage rate, w_t. The net return to work, $\widetilde{w}_t$, falls short of its steady-state in the first 20 years. This is because the labour income tax increases by more than the wage rate. In turn, income, consumption and welfare of workers decrease. After 20 years, the net return to work exceeds its initial steady-

[59] Y_t^k denotes the income of capitalist at period t. The worker does not save, so that $Y_t^w = C_t^w$.

state and thus income, consumption and welfare of workers go above their steady-state levels. However, as can be seen in Table 2.9, the present discounted value of workers' utility is smaller that it would be at steady state. This implies that workers are worse off in terms of welfare in the transition to steady-state although the welfare losses are diminishing over time. Yet in light of the capitalist, after 10 years, the present discounted value of welfare is bigger than it would be at steady state. The negative shock generates welfare losses at the aggregate level in the first 20 years (i.e. -0.016%), but after 50 years, the aggregate welfare improves (i.e. 0.001%). This implies that the temporary capital tax cut, in the long run, improves the aggregate welfare. The income inequality increases in the post-shock economy. The above findings are consistent with the steady-state results of capital tax reforms shown in Table 2.3.

2.5.2 The union model

The impulse responses in the model with only labour market distortion are illustrated by Figure 2.3 and Table 2.10 shows the percent deviations of different variables. In addition, the paths of percent deviations of variables are shown in Figure 2.1a.

[Figure 2.3 about here]

Table 2.10: Percent deviation from steady-state in the union model

	Years after negative τ_t^k shock							
	1	3	5	10	20	50	80	100
C_t^k	-0.017	-0.004	0.006	0.023	0.038	0.040	0.036	0.035
C_t^w	0.019	0.016	0.013	0.009	0.004	0.001	9E-04	8E-04
Y_t^k	0.026	0.032	0.036	0.043	0.046	0.041	0.036	0.035
$(1-e)$	-1.720	-1.459	-1.124	-0.838	-0.410	-0.109	-0.077	-0.072
Y_t^f	0.085	0.084	0.082	0.078	0.070	0.053	0.047	0.045
K_t^k	0.000	0.033	0.060	0.105	0.141	0.137	0.123	0.119
τ_t^w	-0.124	-0.073	-0.032	0.039	0.104	0.122	0.111	0.107
τ_t^k	-0.995	-0.953	-0.914	-0.829	-0.702	-0.514	-0.450	-0.432
U_t^k	-0.017	-0.004	0.006	0.023	0.038	0.040	0.036	0.035
U_t^w	0.019	0.016	0.013	0.009	0.004	0.001	9E-04	8E-04
U_t	0.010	0.011	0.012	0.013	0.014	0.011	0.010	0.009

In the model with unionised labour market, the negative τ_t^k shock generates reductions in the labour income tax in the first 5 years (i.e. -0.032%). There are two opposite effects in determining the direction of this change. On one hand, more labour tax revenue is needed to make up the revenue losses from capital tax cuts given that all the other policy variables are unchanged in the government budget constraint. On the other hand, unemployment rate, $(1-e)$, falls dramatically initially and this change reduces the required government spending and in turn the labour income tax decreases. The latter effect dominates and the labour income tax falls initially, but quickly goes above its steady-state afterwards. As in the competitive model, the capital tax cut can increase the investment of capitalists resulting from the higher net return to capital. Therefore, investment and capital stock increase in response to capital tax cut. In turn, individual firm's output, Y_t^f, goes up. This intends to increase the desire of firms to have a successful outcome of wage bargaining and the firms' power relative to the unions decreases. This effect leads to an increase in unemployment. Meanwhile, the initial decrease in labour income tax increases the net return to work. This implies that the relative attractiveness of the unions' outside option has decreased. This tends to increase the desire of unions to have a successful outcome of wage bargaining and the union's power relative to the firms decreases. so that unemployment falls. These two opposite effects, on the whole, generate an initial decrease in unemployment which is beneficial to the workers. The negative shock does not influence the net turn to work, $\widetilde{w}_t$, over time. The income, consumption and welfare of workers increase as a result of the increase in employment. After 5 years, the labour income tax goes above its initial steady-state. Unemployment exceeds its steady-state after 10 years and in turn income, consumption and welfare of workers are lower in the post-shock economy.

The income of capitalists increases as lower capital tax results in higher interest income. The consumption of capitalists falls short of its steady-state in the first 3 years (i.e. -0.004%) and then goes above it. The initial decrease in consumption is due to the crowding-out effect of investment. The present discounted value of capitalists' utility increases monotonically over time and the welfare of capitalists improves compared to the steady state after 5 years (i.e. 0.006%). The present discounted value of workers' utility is bigger relative to its steady-state in all periods of post-shock economy. But the benefits get smaller over time. The temporary cut in capital income tax generates welfare gains at the aggregate level than at the steady state.

2.5.3 The profits model

The importance of distortion in the product markets in determining the model dynamics is now examined. Figure 2.4 gives the impulse responses of variables to the negative τ_t^k shock. Table 2.11 and Figure 2.1b above show the percent deviations of variables from their steady-state values quantitatively and qualitatively.

[Figure 2.4 about here]

Table 2.11: Percent deviation from steady-state in the profits model

	Years after negative τ_t^k shock							
	1	3	5	10	20	50	80	100
C_t^k	0.032	0.042	0.050	0.065	0.080	0.081	0.074	0.071
C_t^w	-0.138	-0.125	-0.114	-0.092	-0.062	-0.032	-0.026	-0.025
Y_t^k	0.041	0.047	0.052	0.061	0.069	0.065	0.060	0.057
$(1-e)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Y_t^f	0.000	0.009	0.017	0.031	0.046	0.052	0.048	0.046
K_t^k	0.000	0.026	0.048	0.089	0.133	0.148	0.137	0.132
τ_t^w	0.596	0.566	0.538	0.479	0.395	0.279	0.243	0.233
τ_t^k	-0.995	-0.953	-0.914	-0.829	-0.702	-0.514	-0.450	-0.432
U_t^k	0.032	0.041	0.050	0.065	0.080	0.081	0.074	0.071
U_t^w	-0.138	-0.126	-0.114	-0.092	-0.063	-0.032	-0.026	-0.025
U_t	-0.102	-0.090	-0.079	-0.058	-0.032	-0.008	-0.005	-0.004

Table 2.11 shows that, in the model with only product market distortion, the increase in labour income tax is bigger than in the competitive model (see Table 2.9). This is because the labour tax has to increase by more to make up for revenue losses of taxing profits. The cut in capital income tax raises the net return to capital. The higher net return to capital boosts the investment of capitalists. As a result, the accumulation of capital stock increases and in turn, firms' output and labour productivity increase resulting in higher wage rate. However, the net return to work decreases after the shock. This occurs because the increase in labour income tax outweighs the increase in the wage rate. The income, consumption and welfare of workers decrease by more in the profits model relative to the competitive model. This is due to the resulting higher labour income tax in the profits model. In the model with only product market distortion, the consumption of capitalists increases. This happens because the positive

income effects of profits on consumption outweighs the negative crowding-out effect of investment.

As can be seen in Table 2.11, the welfare of capitalists improves in the post-shock economy despite welfare losses for the workers. The welfare losses for the workers get smaller as the shock dies out. There are welfare losses at the aggregate level since the welfare gains for the capitalists cannot make up for the welfare losses for the workers. By comparing the paths of percent deviations of variables in Figure 2.1a with Figure 2.1b , we can see that variables behave similarly over time in the competitive model and profits model. However, the effects of capital tax cut are bigger in the profits model. The dynamic results are in accordance with those have been found in the steady-state analysis (see Table 2.5).

2.5.4 The full model

The transitional dynamics of the model with distortions in both labour and product markets in response to the temporary and unanticipated decrease in tax rate on capital income are finally studied. The results of impulse responses and percent deviations are shown in Figure 2.5 and Table 2.12 as follows.

As in the union model above, the cuts in capital tax rate, at first, result in reductions in the labour income tax (i.e. -0.077%). But after 5 years, the labour income tax is higher than it would be at the steady state. The direction of change in τ_t^w is determined by two effects shown above in the union model. However, the changes after j year are larger, compared to those in the union model (see Table 2.10) due to the existence of profits. The net return to work remains the same in the post-shock economy. In the unionised labour market, the higher labour income tax reduces the return to work. This implies that the relative attractiveness of the unions' outside option has increased and therefore the desire of the unions to have a successful outcome of wage bargaining has decreased. In this case, the unions' power relative to the firm increases and unemployment rate rises. Meanwhile, the distortion in the intermediate goods market generates an opposite effect on unemployment rate. The decrease in capital tax rate has a positive income effect on the investment of capitalists. Investment increases dramatically after the negative shock and this positive effect is transformed into an increase in the capital stock. In turn, output, Y_t^f, goes up. In the production of intermediate goods, the firms take the final output, Y_t, as given and hence the increase in output results in a fall in price, P_t^f. As a result, the expected revenue of the firms will decrease. Therefore, the monopolistic competition reduces the positive revenue effects of the increase in firms' output. This tends to decrease the desire of firm to have a successful outcome of wage bargaining and therefore the relative attractiveness of the outside option for the firm increases. This implies that the firms' power relative to the unions increases and unemployment falls. Figure 2.5 shows that these two opposite effects, on the whole, result in a fall in unemployment rate which is beneficial to the worker. The wage rate quickly rises above its steady-state in response to the negative shock although it falls short of the steady-state in the first few years. The net wage rate remains the same over time in the post-shock economy, so that the income and consumption of workers are totally determined by employment. As a result, they rise in the post-shock economy. Both consumption and investment of capitalists increase due to the positive income effect of lower capital income tax. The magnitude is greater in the full model relative to the union model due to the positive profits.

[Figure 2.5 about here]

Table 2.12: Percent deviation from steady-state in the full model

	Years after negative τ_t^k shock							
	1	3	5	10	20	50	80	100
C_t^k	0.070	0.085	0.095	0.112	0.120	0.103	0.091	0.088
C_t^w	0.036	0.031	0.026	0.019	0.011	0.005	0.004	0.003
Y_t^k	0.087	0.094	0.098	0.104	0.103	0.085	0.075	0.072
$(1-e)$	-3.094	-2.658	-2.295	-1.627	-0.919	-0.396	-0.320	-0.305
Y_t^f	0.154	0.150	0.147	0.137	0.120	0.091	0.080	0.077
K_t^k	0.000	0.052	0.094	0.162	0.214	0.203	0.182	0.175
τ_t^w	-0.223	-0.142	-0.077	0.036	0.137	0.164	0.149	0.144
τ_t^k	-0.995	-0.953	-0.914	-0.829	-0.702	-0.514	-0.450	-0.432
U_t^k	0.070	0.085	0.095	0.112	0.120	0.103	0.091	0.088
U_t^w	0.036	0.031	0.026	0.019	0.011	0.005	0.004	0.003
U_t	0.043	0.042	0.041	0.039	0.034	0.026	0.022	0.021

Table 2.12 shows that all the agents are better off in terms of welfare in the post-shock economy. In other words, the present discounted values of utilities are bigger than they would be at the steady state. This implies that adding more frictions can lead to improved outcomes, given that one distortion, effectively, corrects the other one,

although the welfare inequality increases. These findings are consistent with results from the steady-state analysis (see Table 2.6).

2.6 Summary and concluding remarks

This chapter examined the long-run welfare effects of tax reforms under heterogeneity in capital holdings assuming imperfect competition in product and labour markets. Given the relevance of these market failures, our analysis could help to inform current policy discussions regarding the potential impacts of capital tax reforms. Finally, the transitional paths of model variables in response to a temporary capital tax cut were studied to investigate the importance of market distortions in determining model dynamics. Using a calibrated model whose steady-state reproduced the main features of the UK economy, our main findings are as follows.

First, in the presence of the labour market distortion only, a tax reform that implements a zero capital tax implies welfare losses for the workers, whereas capitalists' and aggregate welfare increases. We find that the unemployment channel is the critical link that modifies the results from the benchmark model with competitive labour markets. In particular, although labour productivity and the after-tax wage increase, the workers cannot capture the full benefit of this as unemployment also increases.

Second, under competitive labour markets but non-competitive product markets, a zero capital tax leads to welfare gains for the capitalists and losses for the workers as well as the aggregate economy. This occurs since, the government has to forego revenue from a non-distortionary tax base comprised of "pure profits" so that the required increase in the labour tax is larger and thus the net wage falls after the tax reform.

Finally, monopolistic competition in the product market, under unionised labour markets, introduces a market failure that works to correct the negative implications of imperfect labour markets. In particular, monopolistic competition tends to reduce the positive revenue effects of the increase in output after a capital tax cut and thus reduce the benefits to the firm for a successful outcome of wage bargaining. In turn, this implies that the relative attractiveness of the firm's outside option in bargaining and its power relative to the union increases. This tends to increase employment and the benefits to all agents after a tax cut.

Given that most modern economies are characterised by imperfect competition in both product and labour markets, we would consider the final set of results to be potentially the most useful. However, the above analysis also makes clear that combining market failures that, independently, have similar welfare effects, does not necessarily lead to total effects which work in the same direction. Instead, the welfare implications can be completely reversed. Thus, our results also imply that omission of relevant market and policy failures may bias the results in ways that cannot always be predicted *ex ante*.

2.7 Figures

Figure 2.1a: Paths of percentage deviations after the shock

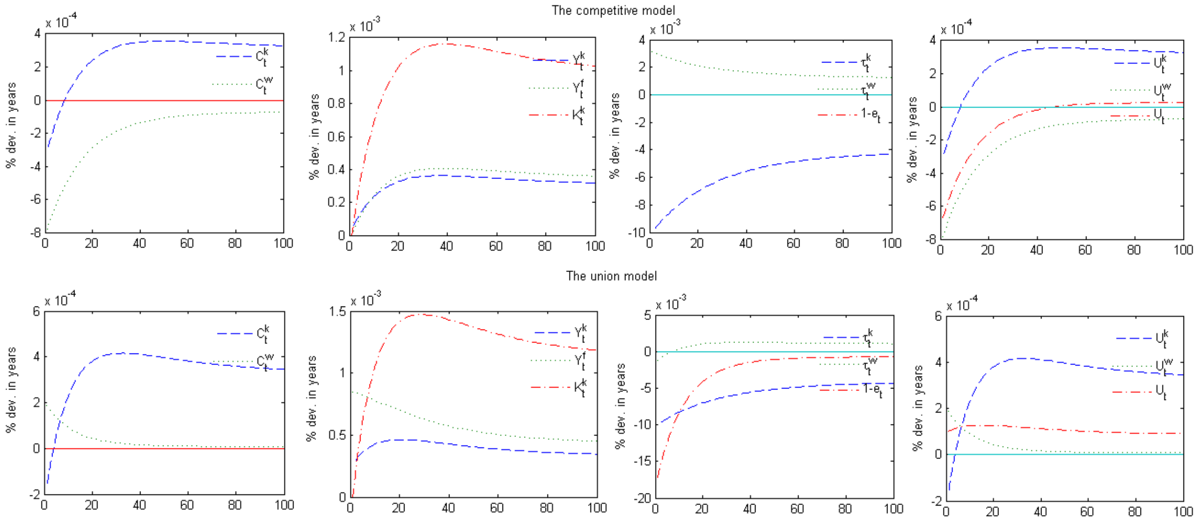


Figure 2.1b: Paths of percentage deviations after the shock

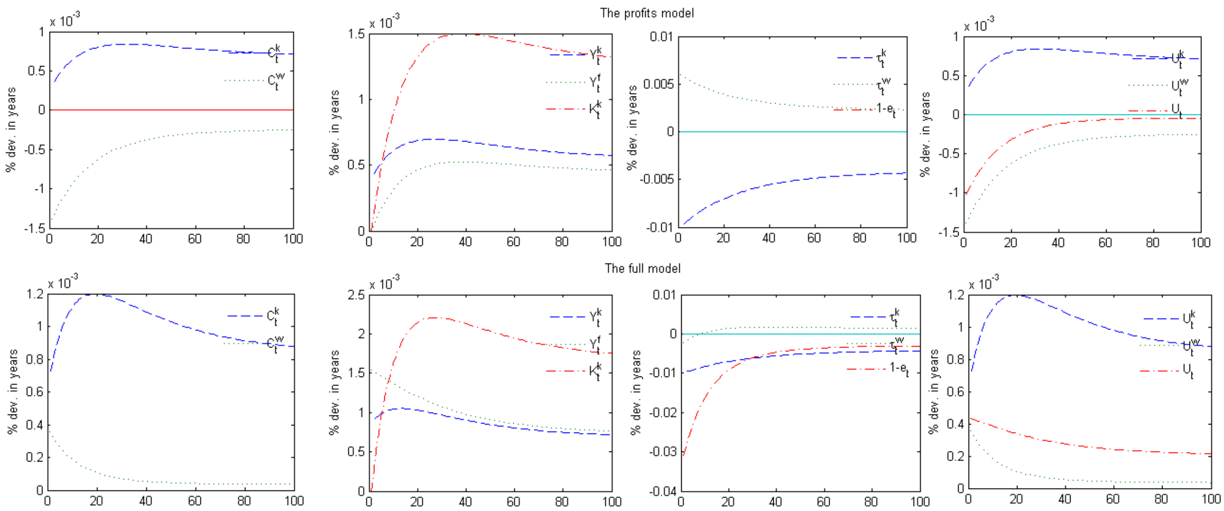


Figure 2.2: Impulse responses to a negative one-period 1% shock to τ_t^k in the competitive model

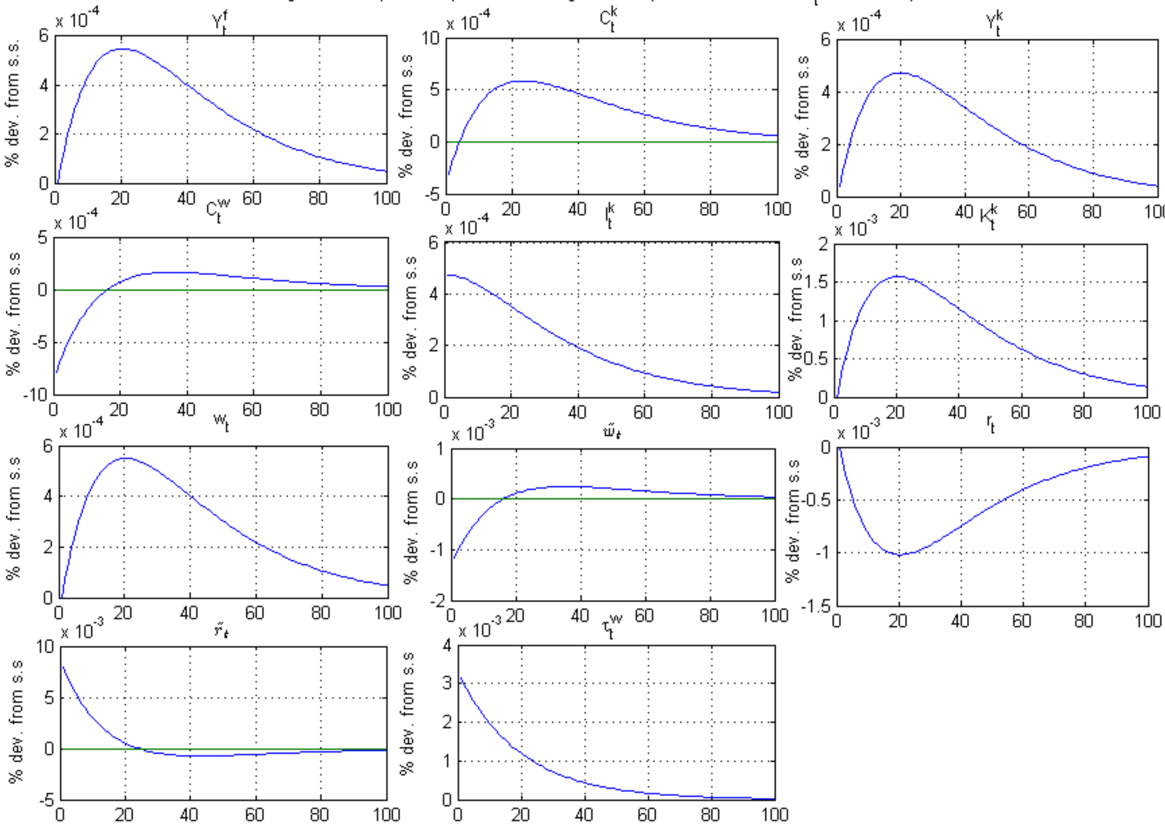


Figure 2.3: Impulse responses to a negative one-period 1% shock to τ_t^k in the union model

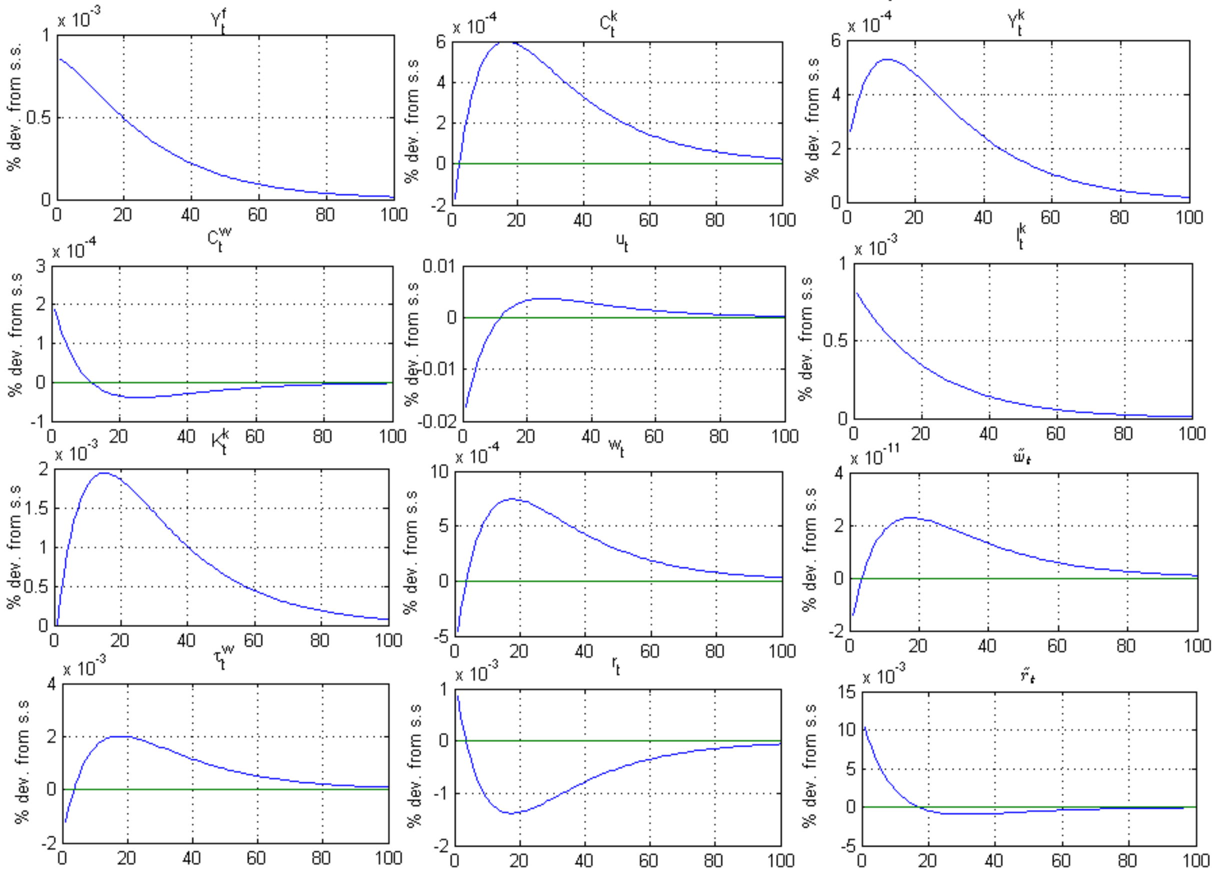


Figure 2.4: Impulse responses to a negative one-period 1% shock to τ_t^k in the profits model

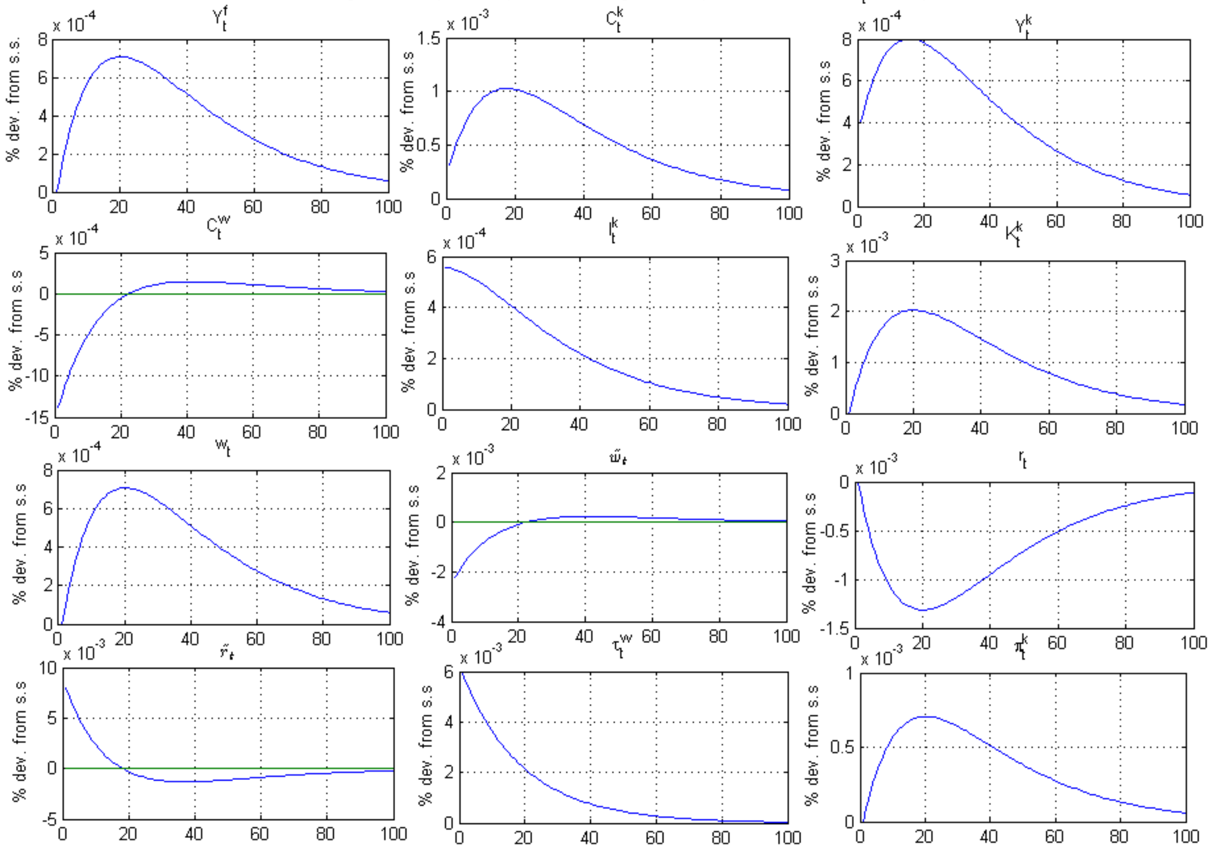
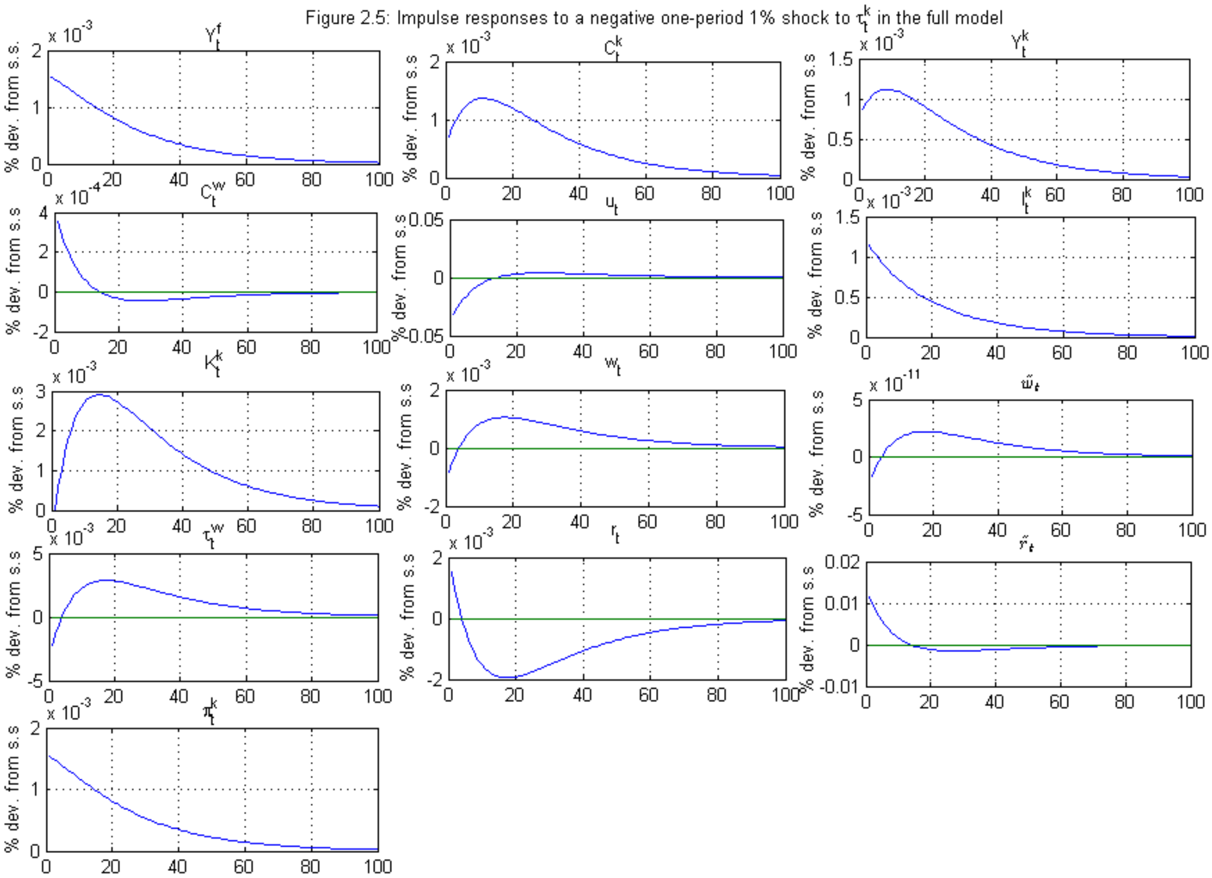


Figure 2.5: Impulse responses to a negative one-period 1% shock to τ_t^k in the full model



2.A Appendix

2.A.1 DE conditions

$$C_t^k : \lambda_t^k = \left(C_t^k\right)^{-\sigma}$$

$$K_{t+1}^k : \lambda_t^k = \beta\lambda_{t+1}^k \left[r_{t+1} - \delta + \tau_{t+1}^k \left(\delta - r_{t+1}\right) + 1\right]$$

$$K_t^f : r_t = \theta\alpha \frac{Y_t^f}{K_t^f}$$

$$L_t^f : w_t = \theta\left(1-\alpha\right)\frac{Y_t^f}{L_t^f}$$

$$BC^w : C_t^w = \left(1-\tau_t^w\right) w_t e_t + \overline{G}_t^u \left(1-e_t\right) + \overline{G}_t^t$$

$$BC^g : \overline{G}_t^t + \overline{G}_t^u \left(1-e_t\right) = n^k \tau_t^k \left(r_t - \delta\right) K_t^k + n^k \tau_t^k \pi_t^k + \tau_t^w w_t e_t$$

$$RC : n^k Y_t^f = n^k C_t^k + n^w C_t^w + n^k \left[K_{t+1}^k - \left(1-\delta\right) K_t^k\right]$$

$$MC_K : K_t^k = K_t^f$$

$$MC_\pi : \pi_t^k = \pi_t^f$$

$$MC_L : e_t = n^k L_t^f$$

$$w_t : \lambda_t^u = \frac{e_t\left(\phi-1\right)\left[-e_t\Omega_t\right]^\phi}{n^k\left[\left(n^k\right)^{1-\theta} Y_t^f - w_t L_t^f\right]^\phi} - e_t\phi[\Psi\left(\tau_t^w - 1\right)\left[-e_t\Omega_t\right]^{(\phi-1)}$$

$$\begin{aligned} e_t \quad : \quad & \lambda_t^u \left[\frac{\alpha\left(\alpha-1\right)A\left(K_t^k\right)^\alpha}{n^k\left(L_t^f\right)^{\alpha+1}}\right] + \frac{\left(2\alpha-2\right)\left(\alpha-1\right)\left(\theta-1\right)A\left(K_t^k\right)^\alpha\left(L_t^f\right)^{-\alpha-1}}{n^k} = \\ = \quad & \frac{\left(\alpha-2\right)\left(\alpha-1\right)\left(\theta-1\right)A\left(K_t^k\right)^\alpha\left(L_t^f\right)^{-\alpha-1}}{n^k} + \phi\Psi^{(1-\phi)}\Omega_t\left[-e_t\Omega_t\right]^{(\phi-1)} - \\ & -\left(\phi-1\right)\left[\frac{w_t}{n^k} + \frac{\left(\alpha-1\right)\left(n^k\right)^{-\theta}A\left(K_t^k\right)^\alpha}{\left(L_t^f\right)^\alpha} + \frac{\left(\alpha-1\right)\left(\theta-1\right)A\left(K_t^k\right)^\alpha\left(L_t^f\right)^{-\alpha}}{\left(n^k\right)^\theta}\right]\frac{\left[-e_t\Omega_t\right]^\phi}{\Psi^\phi} \end{aligned}$$

where λ_t^k and λ_t^u refer to the Lagrangian multipliers from the capitalist's and union's problems respectively; $Y_t^f = A\left(K_t^f\right)^\alpha\left(L_t^f\right)^{1-\alpha}$; $\pi_t^k = (1-\theta)Y_t^f$; $\Psi \equiv \left(n^k\right)^{1-\theta} Y_t^f - w_t L_t^f$; and $\Omega_t \equiv \overline{G}_t^u + w_t\left(\tau_t^w - 1\right)$.

Chapter 3: Tax policy in search-and-matching model with heterogeneous agents

Abstract: Using a Mortensen-Pissarides search-and-matching framework, this chapter investigates the importance of search frictions in determining the welfare and distributional effects of tax reforms which re-allocate the tax burden from capital to labour income assuming agent heterogeneity in their economic roles. With a realistic calibration to the current UK economy, we find that the tax reforms will be Pareto improving but increase inequality in the long run, despite welfare losses for at least one segment of the population in the transition period. The results are robust to the variations in the relative bargaining power of workers and different specifications of unemployment benefits. But the welfare gains are higher for all agents by reducing workers' relative bargaining power or assuming unemployment benefits depending on past wages. We finally show that the long-run optimal policy implies a negative capital tax which helps to correct the market frictions caused by the forward-looking behavior of firms. Meanwhile, the labour tax is increased and unemployment benefits are reduced to finance public spending. The optimal policy increases the number of vacancies and employment. As a result, there are welfare gains for all agents.

3.1 Introduction

Over the last two decades, the Mortensen-Pissarides search-and-matching framework has become a powerful tool for the analysis of unemployment, job vacancies and labour market behavior.[60] The theory focuses on the interaction between unemployment and job creation. It builds on the idea that matching in the labour markets takes time and is costly. Frictions originate from lack of coordination between unemployed workers and vacant jobs that disrupts the ability to form employment relationships. A significant body of literature has applied the search-and-matching models to explain the cyclical fluctuations in unemployment (see e.g. Yashiv (2007) for a review of this literature). For example, Rogerson *et al.* (2005) discuss the usefulness of a range of search-theoretic models for analyzing the unemployment dynamics, job turnover and wages. There is also a debate on whether or not a calibrated matching model can quantitatively account for observed aggregate fluctuations in the labour markets (see e.g. Shimer (2005), Hall

[60] See Mortensen and Pissarides (1994) and Pissarides (2001).

and Milgrom (2008) and Pissarides (2009)).

Another strand of the literature has studied the effects of factor income taxation on welfare and improving labour market efficiency. For instance, Shi and Wen (1999) show that labour income taxation is more costly than capital income taxation with realistic parameter values by computing the marginal deadweight losses associated with capital and labour income tax. Boone and Bovenberg (2002) explore the optimal role of the tax system in alleviating labour market imperfections and raising revenue. They suggest that the optimal tax system should not distort the labour market tightness and only the ad valorem component of the wage tax should be employed to raise revenue. Domeij (2005) examines the condition under which a long-run optimal zero capital tax can be obtained by assuming that the government has access to a commitment technology and optimally chooses capital and labour income taxation. He finds that when the Hosios parameter restriction[61] is satisfied or the government can use subsidies to vacancies or unemployment, the optimal capital tax in the long run is always zero.[62] If these conditions do not hold, a non-zero capital tax can then works to correct the distortions arising from search externalities.

This chapter contributes to the growing literature on tax policy study in search-and-matching models. It aims to shed some light on the welfare and distributional issues of re-allocating the tax burden from capital to labour income. We study the importance of search frictions in determining the effects of such tax reforms in both the long- and short-run. To the best of our knowledge, this question is not answered yet in the search-and-matching literature. In our setup, the government taxes capital income, including interest on savings and profits, and labour income by using two different tax rates to finance its spending requirements. We stay as close as possible to the standard search-and-matching model with capital accumulation (see e.g. Merz (1995) and Andolfatto (1996)), but incorporate household heterogeneity. Following Judd (1985), Lansing (1999) and Ardagna (2007), the households are divided into capitalists and workers in terms of economic roles. Only capitalists have access to the asset markets and only workers can work. The setup of household heterogeneity allows us to examine the distributional effects of income taxation and inequality issues.

Pissarides (1998) suggests that unemployment benefits specification is one of the key

[61] This refers to the condition that the elasticity of search in job matches coincides with the relative bargaining power of workers in the wage bargaining. We will discuss this condition in more detail in sub-section 3.2.8.

[62] In contast, Klein and Rios-Rull (2003) show that the optimal capital tax rate tends to be high when the government cannot commit to future tax rates.

influences on the effects of tax reforms on unemployment and wages. In this chapter, we further examine an alternative specification of unemployment benefits by assuming that they are proportional to past wages but are constant in the long run. As Chéron and Langot (2010) point out, this setup for unemployment benefits introduces a feedback effect of distribution of wages on the distribution of unemployment benefits and is important for the predictions of models with search frictions. Therefore, we intend to analyse the importance of the specification of unemployment benefits in determining the effects of tax reforms and explore the different mechanisms of tax reforms driving the results.

Finally, we study optimal taxation under commitment. In our case, a Benthamite government optimally chooses all the tax rates with the aim of maximising social welfare. We particularly focus on the optimal capital tax rate in the long run. Our results are compared against those in Domeij (2005). Our model setup differs from his in several ways.[63] We intend to examine whether the Chamley-Judd result survives in our model.

We calibrate the model so that its steady-state solution can reflect the main features of the current UK economy, in particular, its tax structure and labour market characteristics. The UK is chosen to illustrate the quantitative analysis since its tax structure stands in stark contrast with other European countries, by having a very high effective capital to labour income tax ratio. Since the effects of tax reforms that reallocate the tax burden from capital to labour income are monotonic in our model, we focus on the reform of eliminating the capital income tax which is widely investigated in optimal taxation literature.

Our main findings are summarized as follows. First, in a model with search and matching frictions, the tax reform considered is Pareto improving in the long run although it increases inequality between agents. In other words, all the agents are better off, despite higher welfare gains for the capitalists compared to the workers. This happens because, in the new, post-reform economy, the increase in labour income tax and labour productivity due to higher capital accumulation leads to an increase in the bargained wage rate. Unemployment benefit increases as it is assumed to be proportional to the wage rate. As a result, the search-unemployment of workers increases. The higher wage rate reduces the expected profits of firms even if the higher labour productivity has a positive profits effect. Thus, the firms open less job vacancies. The net

[63] More details are provided in Section 3.6.

wage rate rises in the new, post-reform economy. This is because the productivity gains outweigh the increase in labour income tax which is beneficial to the workers. Overall, the income, consumption and welfare of workers increase in the long run. Capitalists can directly benefit from the zero capital tax and the interest income from capital increases. The capital income effect is bigger than the labour income effect. Capitalists benefit more from this tax reform and therefore inequality increases. However, the capital tax cut met with the labour tax increase hurts the workers and also worsens the aggregate welfare over the transition. This is because the positive effects resulting from higher capital accumulation take time to be realized. As a result, the combination of an initially lower net wage rate and higher search-unemployment creates short-run losses for the workers and aggregate economy, which are reversed in the long run. We also show that our results are robust to variations in the relative bargaining power of workers in the Nash bargain. Increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improving.

Second, when we assume that unemployment benefits depend on past wages, the model can generate similar welfare results in both the long- and short-run although the mechanism driving results is different. The tax reform is still Pareto improving run but generates higher welfare gains for all agents in the long run. This happens because, there are larger positive effects resulting from higher capital accumulation. On one hand, unemployment benefits remain the same at the steady-state of post-reform economy so that search-unemployment goes down when the net wage rate rises. On the other hand, the firms open more job vacancies as higher capital accumulation increases firms' expected profits from a successful match. In turn, the tightness of labour market is reduced which can lead to the increase in employment. Thus, the long-run welfare gains for the workers are higher in the post-reform economy since working can generate higher labour income for them. Similar to the long-run welfare implications, the tax reform has higher welfare effects for all agents in the transition period. In other words, the lifetime welfare gains for capitalists are higher and welfare losses for workers are lower. The lifetime aggregate welfare improves. The wage rate increases by more in the short run since the capital tax cuts have larger initial effects on boosting capital accumulation. In turn, the net wage increases over time. The unemployment benefits depend on past wages and their path follows the path of wage rate which causes the inertia in the increases in unemployment benefits during the transition. This tends to weaken the increase in search-unemployment which is beneficial to the workers.

The welfare of workers is raised more quickly in the new, post-reform economy. To summarise, these imply that the long- and short-run welfare gains of tax reforms are higher for all agents by assuming unemployment benefits depending on past wages.

Finally, we find that the optimal tax policy under commitment implies a negative optimal capital income tax in the long run which is accomplished by the labour tax increase and unemployment benefits reduction. The negative capital income tax helps to correct the market frictions due to the forward-looking behavior of firms. As a result, the investment is boosted and this is transformed into higher capital accumulation. The income, consumption and welfare of capitalists increase. The labour income tax is increased and the unemployment benefits are reduced to make up for the tax revenue losses from capital income. The lower unemployment benefits reduce the search-unemployment of workers. The firms open more job vacancies resulting from higher labour productivity. As a result, the probability at which job seekers can be matched with job vacancies is increased. In turn, employment increases which is beneficial to the workers. There are welfare gains for the workers and aggregate welfare improves in the long run.

The rest of this chapter is organised as follows: Section 3.2 describes the model structure. Section 3.3 discusses the calibration of the model to the current UK economy and gives the old, pre-reform steady-state. The results of tax reforms are analysed in Section 3.4. Section 3.5 studies an alternative specification of unemployment benefits and investigates the effects of tax reforms. The optimal tax policy under commitment is examined in Section 3.6 and Section 3.7 summarizes this chapter and concludes.

3.2 The model

3.2.1 Economic environment

The features of the economy are summarized as follows. Infinitely lived households, homogenous firms, and a government populate the economy. Households are divided into capitalists and workers in terms of whether they can save or work. Only capitalists can save and own firms. Their income is comprised of interest income from physical capital and dividends of firms. Workers cannot save and consume all their disposable income in each period. The workers can engage in one of three activities: working to obtain wage income, searching for a job or enjoying leisure. If employed, they sell one unit labour endowment to only one firm at a time. The labour supply is thus indivisible. At any given period of time, the workers searching for jobs are randomly matched with job vacancies open by firms through a matching function. The search frictions can generate unemployment. Unemployment arises as the job seekers are not successful in their search for new employment. The unemployed workers receive unemployment benefits from the government. When there is a successful match, the matched worker and firm bargain over the wage rate to maximise a weighted average of worker's and firm's surpluses. Furthermore, we assume that two workers who are hired at different times must be paid the same wage at any given time. If the bargaining is successful, the worker will be employed by the firm in the following period. In this sense, employment at a given period of time is predetermined. It changes as unemployed workers get new jobs and employed workers separate from old jobs at an exogenous rate of separation. Following Andolfatto (1996), Merz (1995) and Pissarides (1998), we assume that workers are identical in the labour market. Individual unemployment risk is completely smoothed by using employment lotteries[64] and all workers have equal employment and income.[65] Hence, one worker can be thought of as being endowed with one unit of time in each period, which can be split into: working, search and leisure. The firms produce final goods by employing capital and labour. As discussed above, since employment is predetermined, the firms open job vacancies at a constant resource cost to target their next-period employment. The government taxes interest income from physical capital, profits and labour income to finance its spending requirements.

3.2.2 Population composition

Total population is given by N which is exogenous and constant over time with the population of capitalists and workers respectively being denoted by N^k and N^w. The population shares of capitalists and workers are assumed to be: $N^k/N \equiv n^k$, and $N^w/N \equiv n^w = 1 - n^k$, respectively. We further assume that each capitalist owns one single firm, so that $N^f = N^k$.

[64] The technique is introduced in models with indivisible labour, e.g. Hansen (1985) and Rogerson (1988).

[65] For a model with price taking firms and uninsured idiosyncratic employment risk due to incomplete financial markets and search frictions (see e.g. Krusell *et al.* (2010)).

3.2.3 Matching technology

As in the standard search-and-matching literature,[66] the matching technology is represented by the Cobb-Douglas (CD) function:[67]

$$M_t = mS_t^{\eta}V_t^{1-\eta} \tag{101}$$

where M_t is the new matches at t; $S_t = N^w s_t$ denotes the aggregate number of workers who are looking for a job ; $V_t = N^f v_t$ denotes the aggregate number of job vacancies created by firms in the labour market; $m > 0$ represents the constant efficiency of matching; $0 < \eta < 1$ denotes the elasticity of matches to searching. In addition, we define the ratio: $z_t = V_t/S_t$, as the tightness of the labour market. The smaller the ratio of z_t, the tighter the labour market and therefore the harder for an unemployed worker to match with a job vacancy.

The probability at which aggregate job searches lead to a new job match is given by:

$$p_t = \frac{M_t}{S_t} = mS_t^{\eta-1}V_t^{1-\eta} = mz_t^{1-\eta} \tag{102}$$

and its inverse, $1/p_t$, measures the duration of a search.

The probability at which a job vacancy can be matched with an unemployed worker is calculated by:

$$q_t = \frac{M_t}{V_t} = mS_t^{\eta}V_t^{-\eta} = mz_t^{-\eta} \tag{103}$$

and its inverse, $1/q_t$, measures the duration of a job vacancy.

3.2.4 Utility function

The objective of household $i \equiv k, w$, is to maximise his discounted lifetime utility. A representative capitalist has preferences represented by the following lifetime utility function:

$$U_t^k = \sum_{t=0}^{\infty} \beta^t u_t^k \tag{104}$$

[66] See Pissarides (1986) and Blanchard and Diamond (1989).

[67] Mortensen and Wright (2002) discuss results for increasing-returns-to-scale matching function and find that equilibria are unlikely to be efficient.

where $0 < \beta < 1$ stands for the constant rate of time preference. The instantaneous utility function of the capitalist is given by:

$$u_t^k = \frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma} \tag{105}$$

where C_t^k is the capitalist's private consumption; and $\sigma > 1$ is the coefficient of relative risk aversion.

The lifetime utility of a representative worker is as follows:

$$U_t^w = \sum_{t=0}^{\infty} \beta^t u_t^w \tag{106}$$

with the instantaneous utility function given by:

$$u_t^w = \frac{\left(C_t^w\right)^{1-\sigma}}{1-\sigma} - \xi \frac{\left(e_t + s_t\right)^{\mu}}{\mu} \tag{107}$$

where C_t^w is the worker's private consumption; e_t is the employment; s_t is the search activities of worker which effectively belongs to unemployment; $e_t + s_t$ is therefore the labour force participation rate and $s_t / \left(e_t + s_t\right)$ gives the unemployment rate; $\xi > 0$ is a disutility parameter attached to the non-leisure activities; $\frac{1}{\mu-1} > 0$ measures the wage elasticity of labour force participation. Different from the capitalist, exerting work or search effort in the labour market generates disutility for the worker.

3.2.5 Capitalists

The within-period budget constraint of each capitalist is described as:

$$C_t^k + I_t^k = r_t K_t^k - \tau_t^k \left(r_t - \delta\right) K_t^k + \left(1 - \tau_t^k\right) \pi_t^k + \overline{G}_t^t \tag{108}$$

where I_t^k is the capitalist's private investment; K_t^k is the physical capital held by the capitalist at the beginning of t; r_t is the gross return to physical capital; π_t^k is profits received from firms which are taxed at the same rate as interest income from savings; $\overline{G}_t^t$ is per capita transfer from the government; $0 \leq \tau_t^k < 1$ is the tax rate on capital income; and $0 < \delta < 1$ is the constant depreciation rate of physical capital.

The capital stock evolves according to:

$$K_{t+1}^k = \left(1 - \delta\right) K_t^k + I_t^k. \tag{109}$$

We then rewrite the budget constraint of the capitalist by making use of equation (109). This yields:

$$C_t^k + K_{t+1}^k = R_t K_t^k + \left(1 - \tau_t^k\right) \pi_t^k + \overline{G}_t^t. \tag{110}$$

Here, we have defined a new variable, $R_t = 1 - \delta + r_t - \tau_t^k (r_t - \delta)$, as 1+the net return to physical capital after taxation and depreciation.

The capitalist's optimization problem is to choose $\left\{C_t^k, K_{t+1}^k\right\}_{t=0}^{\infty}$ to maximise (104) subject to the constraint (110) taking the return to capital $\{r_t\}_{t=0}^{\infty}$, profits $\left\{\pi_t^k\right\}_{t=0}^{\infty}$, policy variables $\left\{\tau_t^k, \overline{G}_t^t\right\}_{t=0}^{\infty}$ and the initial condition for K_0^k as given.[68]

3.2.6 Workers

Each worker's within-period budget constraint is given by:

$$C_t^w = (1 - \tau_t^w) w_t e_t + \overline{G}_t^u s_t + \overline{G}_t^t \tag{111}$$

where w_t is the gross wage rate; $0 \leq \tau_t^w < 1$ is the tax rate on labour income; and $\overline{G}_t^u$ denotes per capita unemployment benefits offered by the government. Following Pissarides (1998), Shi and Wen (1999) and Ardagna (2007), we assume that unemployment benefits are proportional to the wage rate, i.e. $\overline{G}_t^u = \overline{r}_t w_t$, where $\overline{r}_t$ is defined as the replacement ratio. Unemployment benefits are less than the net wage rate, i.e. $\overline{r}_t w_t < (1 - \tau_t^w) w_t$, searching is costly to the worker.

Employment evolves according to:

$$e_{t+1} = p_t s_t + (1 - \gamma) e_t \tag{112}$$

where $0 < \gamma < 1$ is the constant exogenous rate of job separation. The worker chooses $\{C_t^w, s_t, e_{t+1}\}_{t=0}^{\infty}$ to maximise (106) subject to the constraints (111) and (112), taking the gross wage rate $\{w_t\}_{t=0}^{\infty}$, the matching probability $\{p_t\}_{t=0}^{\infty}$, policy variables $\left\{\tau_t^w, \overline{r}_t, \overline{G}_t^t\right\}_{t=0}^{\infty}$ and the initial condition for e_0 as given.[69]

3.2.7 Firms

A representative firm produces the final goods with a constant-returns-to-scale technology in two productive inputs: capital, K_t^f, and labour, L_t^f. The production function

[68] The utility-maximization of the capitalist is given in the Appendix 3.A.1.

[69] The utility-maximization of the worker is given in the Appendix 3.A.2.

is given by:

$$Y_t^f = A\left(K_t^f\right)^{\alpha}\left(L_t^f\right)^{1-\alpha} \tag{113}$$

where $0 < \alpha, 1-\alpha < 1$ denote the constant output elasticities of capital and labour, respectively.

Two remarks follow. First, employment is pre-determined at any given period of time prior to the production taking place. The firm takes the number of workers currently employed as given. It opens new vacancies in order to employ the desired number of workers next period. There is an exogenous resource cost from creating a new vacancy. The firm also needs to decide on the size of the capital stock that it needs for production. Second, the firm can earn positive profits. This is because the firm can influence its future employment by controlling the currently created job vacancies. This forward-looking decision making results in a marginal product of labour which is higher than the marginal cost of labour, in other words, the gross wage, so that the search frictions result in positive profits in the product markets.

The job transition function which links the future number of filled jobs to the net hiring plus the current stock of filled jobs is given by:

$$L_{t+1}^f = q_t v_t + (1-\gamma) L_t^f \tag{114}$$

where the old jobs dissolve at a constant rate of $0 < \gamma < 1$.

The profits function of the firm is given by:

$$\pi_t^f = Y_t^f - r_t K_t^f - w_t L_t^f - \nu v_t \tag{115}$$

where $\nu > 0$ stands for the constant resource cost of opening a new vacancy.

It is worth noting that the profit-maximisation problem of the firm is intertemporal, in the sense that the firm can influence its future employment by choosing the number of vacancies created in contemporaneous period. The firm takes the factor prices $\{r_t, w_t\}_{t=0}^{\infty}$, the matching probability $\{q_t\}_{t=0}^{\infty}$ and an initial condition for L_0^f as given, and chooses $\left\{K_t^f, v_t, L_{t+1}^f\right\}_{t=0}^{\infty}$ to maximise the present value of a stream of profits:

$$\sum_{t=0}^{\infty}\prod_{i=0}^{t} R_i^{-1}\pi_t^f \tag{116}$$

subject to the constraints (113), (114), and (115).[70]

The profit-maximising conditions of the firm can be summarized by two equations in the following:

$$r_t = \alpha \frac{Y_t^f}{K_t^f} \tag{117}$$

which states that the rate of return on capital is equal to the marginal product of capital.

The second condition is given by:

$$\frac{\nu}{q_t} = R_{t+1}^{-1} \left[(1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} + \frac{\nu(1-\gamma)}{q_{t+1}} \right] \tag{118}$$

which states that the average vacancy creation costs of a successful match at time t are equal to the discounted expected value of profits brought about at time $t+1$.

3.2.8 Wage determination

The wage rate is determined by a Nash bargaining between a pair of matched worker and firm with the aim of maximising the weighted product of their surpluses resulting from a successful match. The worker's utility can be increased by $(1-\tau_t^w)\, w_t u_{1,t}^w - u_{2,t}^w$ if he is employed.[71] We rewrite the worker's surplus as $w_t - \frac{u_{2,t}^w}{(1-\tau_t^w)u_{1,t}^w}$ by separating out the wage rate, w_t. The quantity, $\frac{u_{2,t}^w}{(1-\tau_t^w)u_{1,t}^w}$, is then interpreted as the worker's reservation wage. By employing one additional unit of labour, the firm can increase its profits by, $Y_{2,t}^f - w_t$, which is the firm's marginal profitability from hiring. To summarise, the worker and the firm bargain over the wage rate to maximise the following weighted average of surpluses:

$$\left[w_t - \frac{u_{2,t}^w}{(1-\tau_t^w)\, u_{1,t}^w} \right]^{\phi} \left[Y_{2,t}^f - w_t \right]^{1-\phi} \tag{119}$$

where $0 \le \phi \le 1$ represents the constant relative bargaining power of the worker.

The wage rate after a successful bargaining is given by:[72]

$$w_t = (1-\phi) \frac{u_{2,t}^w}{(1-\tau_t^w)\, u_{1,t}^w} + \phi Y_{2,t}^f. \tag{120}$$

It shows that the wage rate is a weighted average of the reservation wage (the outside

[70] The profit-maximization of the firm and the derivation of its expected profits are provided in the Appendix 3.A.3 and 3.A.4, respectively.

[71] We assume that an unemployed worker is not entitled to the unemployment benefits if he does not accept a potential job. If this is the case, the worker leaves the labour force and he is then counted as part of $(1-e_t-s_t)$.

[72] The derivation of the wage rate in Nash bargain is given in the Appendix 3.A.5.

factor) and labour productivity (the inside factor). Since we do not study heterogeneity of labour, all the workers who are hired will receive the same wage from the firms at any given time. It further implies that we work with a symmetric equilibrium.

As in Domeij (2005) and Arseneau and Chugh (2008), we assume that the households cannot affect the reservation wage via the marginal rate of substitution between consumption and leisure. This assumption is standard in the literature. As such, only the labour tax changes affect the reservation wage and therefore the equilibrium wage rate.

There exist externalities in the matching models which arise from the fact that one additional job seeker can increase the probability that a firm is matched with a worker, i.e. a positive externality, but decrease the probability of a job seeker already existing in the markets to be matched with a firm, i.e. a negative externality. As pointed out by Mortensen (1982) and Hosios (1990), the workers and firms ignore the externalities created by their choices and therefore search inefficiency arises. But in a model with no distortionary taxes, Hosios (1990) shows that two opposite externalities will be balanced and the search inefficiency vanishes when the elasticity of searches in the matching is equal to the relative bargaining power of the workers, i.e. $\eta = \phi$.

3.2.9 Government and market clearing conditions

The per-capita government budget constraint equating spending and revenues is given by:

$$\overline{G}_t^t + n^w \overline{G}_t^u s_t = n^k \tau_t^k \left(r_t - \delta\right) K_t^k + n^k \tau_t^k \pi_t^k + n^w \tau_t^w w_t e_t. \tag{121}$$

To ensure that the government budget is balanced in each period, we allow the labour income tax, τ_t^w, to be residually determined.

The capital markets clear when the supply is equal to the demand for capital per capita:

$$K_t^k = K_t^f. \tag{122}$$

All the profits of firms are equally distributed to the capitalists which gives the following per capita market clearing condition for the dividends:

$$\pi_t^k = \pi_t^f. \tag{123}$$

In the labour markets, the equality of per capita labour supply and demand is given by:

$$n^w e_t = n^k L_t^f. \tag{124}$$

Finally, in the goods markets, the economy's per capita resource constraint is satisfied:

$$n^k Y_t^f = n^k C_t^k + n^w C_t^w + n^k \left(K_{t+1}^k - (1-\delta) K_t^k \right) + n^k \nu v_t. \tag{125}$$

3.2.10 Decentralised equilibrium (given policy)

We summarize the decentralised equilibrium conditions in real terms in the following. Given the paths of prices $\{w_t, r_t\}_{t=0}^{\infty}$, the paths of policy instruments $\left\{\tau_t^k, \overline{r}_t, \overline{G}_t^t\right\}_{t=0}^{\infty}$ and initial conditions for K_0^k and e_0, a decentralized equilibrium (DE) is defined to be an allocation $\left\{C_t^k, K_{t+1}^k, C_t^w, s_t, e_{t+1}, v_t\right\}_{t=0}^{\infty}$ and one residually determined policy instrument, τ_t^w, such that (i) capitalists, workers, and firms undertake their respective optimization problems outlined above; (ii) wage rate is determined by a Nash bargaining between a pair of matched worker and firm; (iii) all budget constraints are satisfied; and (iv) all markets clear.

Thus, the DE consists of the capitalist's optimality conditions for C_t^k and K_{t+1}^k; the worker's optimality conditions for C_t^w, s_t and e_{t+1}; the firm's first-order conditions for K_t^f, v_t and L_{t+1}^f; the optimality condition for the wage rate in the Nash-bargain, w_t; the evolution of employment, e_t; the budget constraints of worker and government, i.e. BC^w and BC^g; the aggregate resource constraint, RC; and the market clearing conditions in the capital, dividends and labour markets, i.e. MC_K, MC_π and MC_L.[73]

3.3 Calibration and steady-state solution

The structural parameters of the model are next calibrated so that the model's steady-state solution reflects the main empirical characteristics of the UK economy, particularly the features of its labour market. Table 3.1 below reports the structural parame-

[73] Note that relying on Walras's law, we drop the budget constraint of the capitalist from the DE. The full DE conditions are provided in the Appendix 3.A.6.

ters in the model.

Table 3.1: Calibration ($\overline{G}_t^u = \overline{r}_t w_t$)

Parameter	Definition	Value
$0 < \beta < 1$	rate of time preference	0.970
$\sigma > 1$	relative risk aversion	2.000
$\mu > 1$	elasticity parameter in utility	5.000
$0 < \delta < 1$	depreciation rate on capital	0.100
$\xi > 0$	utility parameter	12.000
$0 < \gamma < 1$	job separation rate	0.200
$0 < \alpha < 1$	capital's share	0.350
$\nu > 0$	job posting cost	0.316
$0 \leq \phi \leq 1$	worker's bargaining power	0.500
$m > 0$	efficiency of matching	2.800
$0 < \eta < 1$	elasticity of unemployment	0.500
$0 < n^k < 1$	population share of capitalists	0.115
$A > 0$	TFP level	1.000
$0 \leq \tau^k < 1$	tax rate on capital income	0.442
$0 < \overline{r} < 1$	replacement ratio	0.500
$\overline{G}^t > 0$	per capita government transfer	0.164

Preferences: Time is measured in years. The annual rate of time preference is set to 0.97, i.e. $\beta = 0.97$ (see e.g. Angelopoulos *et al.* (2011b) and references therein). In the utility function, we use a value for σ that is common in DGE literature, i.e. $\sigma = 2$. The value of μ is set at 5 to obtain the labour participation elasticity of $\frac{1}{\mu - 1} = 0.25$. This elasticity value falls in the range in Killingsworth (1983). The value of ξ is calibrated to get the labour participation rate of 63.1%.[74]

Production: We use a standard value for the capital productivity parameter, i.e. $\alpha = 0.35$. The annual depreciation rate of physical capital is 10%, which is consistent with the quarterly depreciation rate of 2.5%. These two parameters imply a realistic steady-state capital-to-output ratio of 2.25 on an annual basis. The unit cost of opening a vacancy of ν is calibrated to get the steady-state unemployment rate of 7%. The unemployment rate corresponds to the data average from 1970 to 2010 from the UK Office for National Statistics (ONS). The productivity parameter, A, is normalised to

[74] See Schweitzer and Tinsley (2004) for the empirical evidence.

1.

Labour market: We assume that the capitalists do not work in the model. But they can save in the form of capital, own firms and receive profits. Following Ardagna (2007), we treat the self-employed as capitalists to calibrate the population share of capitalists. The data of self-employment becomes available from 1992 for the UK economy in the Labour Force Statistics (LFS) database. The population shares of capitalists is set to the data averages of 0.115.[75] With regards to the bargaining process, we set the worker's bargaining power to 0.5 which features a symmetric Nash bargaining solution. In later tax policy analysis, we allow this parameter to take a range of different values as a robustness test, i.e. $\phi \in [0.25, 0.375, 0.5, 0.625, 0.75]$. The elasticity of unemployment in the matching function is set to 0.5. The exogenous job separation rate γ is set to 0.2 as in Pissarides (1998). This is also consistent with a quarterly job separation rate of 0.05 as in, e.g. Shi and Wen (1997 and 1999) and Domeij (2005). The inverse of γ gives the average duration of a job. Our calibration implies that the average duration of a job is five years which is consistent with the data average from 1992 to 2010 from OECD.Stat database. The matching technology is represented by a homogenous of degree one function and characterized by the efficiency parameter, m. We calibrate, m, to obtain the duration of unemployment of 4.5 months at the steady-state when the tightness of labour market is 0.9. The unit cost of creating a vacancy yields the duration of a job vacancy of 4 months, similar to Pissarides (2006).

Policy instruments: Effective average tax rates for capital and labour income from 1970 to 2005 are constructed by following the approach in Conesa *et al.* (2007), i.e. $\tau^k = 0.442$, and $\tau^w = 0.27$. We then calibrate the per capita government transfers, $\overline{G}_t^t$, to obtain the steady-state τ^w of 0.27. The replacement ratio is set to 0.5 which is comparable with the data for industrialised countries (see e.g. Nickell and Nunziata (2001)) and between the values used in previous studies, ranging from 0.45 (Shi and Wen (1999)) to 0.6 (Pissarides (1998)).

The parameters imply the steady-state solution which is reported in Column (1) of Table 3.2 below. The net returns to labour and capital, $\widetilde{w} = (1 - \tau^w)\, w$ and $\widetilde{r} = \left(1 - \tau^k\right)(r - \delta)$ are useful for the policy analysis which follows. The steady-state disposable income of capitalists and workers is given by Y^k and Y^w, respectively. The lifetime welfare of agent is obtained using the formula $U_{ss}^i = \frac{\left(1-\beta^T\right)}{1-\beta} u^i$, for $i = k, w$,

[75] This value differs from the one calibrated in Chapter 2. It is due to the different model setup for agent heterogeneity in these two cases. In the other case, capitalists are assumed to work and save in the economy.

where u^i is the welfare of i calculated at the steady-state using (105) and (107) and $T = 1000$. The aggregate or social welfare, U_{ss}, is defined in the Benthamite fashion as the average welfare of all agents in the economy.[76]

The steady-state solution for the above parameterisation implies the following shares of public spending in GDP: $\frac{G^t}{Y} = 0.203$ and $\frac{G^u}{Y} = 0.024$, which further implies that the government spending in transfers is about 23% of GDP consistent with UK data from the OECD.Stat database.

3.4 Distributional effects of tax reforms

3.4.1 The long-run effects

We first examine the long-run effects of reducing the tax burden on capital income in the model. In all cases, we find that the effects of capital tax reductions are monotonic and increase with the magnitude of the capital tax cut. Hence, we particularly analyse the effects of abolishing capital taxation which is associated with a concurrent increase in labour income tax rate, τ^w, to generate the required tax revenues to finance public spending. The zero capital tax has been intensively examined in the optimal taxation literature. We evaluate the effects of the tax reform by comparing the post- with the pre-reform steady-states with main focuses on the labour market and the distribution of welfare. The steady-state allocations together with welfare of agents after the tax reform are shown in Column (2) of Table 3.2.

As can been seen in Table 3.2, the implementation of a zero capital income tax will be Pareto improving in the long run (see Us), even if it increases inequality (see $\frac{Y^k}{Y^w}$). In other words, all the agents are better off after the tax reform, although the gains for the capitalists compared to the workers are higher. This is consistent with Judd's (1985) results that it is optimal for both capitalists and workers to choose a zero capital tax in the long run.

The trade-off for the workers after implementing the zero capital tax can be seen by noting that, the labour tax, τ^w, increases (i.e. from 0.27 to 0.334) to make up for the tax revenue losses, due to the elimination of the capital tax. In turn, the workers' reservation wage, $\frac{u^w_{2,t}}{(1-\tau^w_t)u^w_{1,t}}$, increases. Meanwhile, the labour productivity, $Y^f_{2,t}$, increases which induced by higher capital accumulation. Both changes intend to increase the wage rate which can be seen in the optimality condition in the bargaining

[76] The aggregate welfare is given by: $U_{ss} = n^k U^k_{ss} + n^w U^w_{ss}$.

(90). Therefore, the bargained wage rate, w, increases at new, post-reform steady-state (i.e. from 0.982 to 1.081). The net return to labour increases as well (i.e. from 0.717 to 0.720), since the increase in the wage rate outweighs the increase in the labour tax.

Table 3.2: Long-run effects of tax reform ($\overline{G}_t^u = \overline{r}_t w_t$)

	Pre-reform (1)	Post-reform (2)
$\frac{C^k}{Y}$	0.094	0.106
$\frac{C^w}{Y}$	0.667	0.615
$\frac{C}{Y}$	0.761	0.721
$\frac{Y^k}{Y^w}$	3.687	4.671
$\frac{I^k}{Y}$	0.225	0.267
$\frac{K^k}{Y}$	2.252	2.673
$\frac{G^t}{Y}$	0.203	0.187
$\frac{G^u}{Y}$	0.024	0.026
e	0.587	0.583
s	0.044	0.047
v	0.306	0.283
τ^w	0.270	0.334
w	0.982	1.081
$\widetilde{w}$	0.717	0.720
r	0.155	0.131
$\widetilde{r}$	0.031	0.031
z	0.900	0.782
p	2.657	2.475
q	2.951	3.168
U^k	-50.586	-41.261
U^w	-63.029	-62.687
U	-61.598	-60.223

The increase in the wage rate leads to an increase in unemployment benefits. This is because unemployment benefits are proportional to the wage rate, i.e. $\overline{G}^u = \overline{r}w$. One one hand, the number of workers looking for jobs at the steady-state, s, increases (i.e. from 0.044 to 0.047). On the other hand, the increase in the wage rate reduces firms' expected profits from a successful match, despite the higher labour productivity due to higher capital accumulation. Thus, the firms reduce the number of vacancies open for unemployed workers (i.e. from 0.306 to 0.283). These changes, in turn, imply a tighter labour market which can be seen that z falls from 0.9 to 0.782. A tighter labour market implies a lower probability for an unemployed worker to match with a job vacancy (i.e. from 2.657 to 2.475), and a higher probability at which a job vacancy can be matched with an unemployed worker (i.e. from 2.951 to 3.168). According to the employment evolution equation (112), steady-state employment falls (i.e. from 0.587 to 0.583). The income, consumption and welfare of workers rise resulting from two positive effects. On one hand, unemployment benefits are higher resulting from the increasing wage rate so that income from search is higher. On the other hand, the increased net wage rate raises the income from working. The tax reform can also benefit the capitalists since the elimination of capital tax boosts investment and capital. The pre- and post- reform investment-to- and capital-to-output ratios are (0.225, 2.252) and (0.267, 2.673), respectively. As a result, the income, consumption and welfare of capitalists increase. Thus, all agents benefit from the reform that implements a zero capital tax in the long run. Capitalists directly benefit from the zero capital tax and also the increased capital. The capital income effect is bigger than the labour income effect. Hence, capitalists benefit more from this tax reform and inequality increases despite the Pareto superiority of the reform.

3.4.2 The transitional effects

In contrast to the above steady-state analysis, we now investigate the welfare effects of the tax reform in the transition period. The literature suggests that during the transition period, capital tax cuts met by labour tax increases will hurt the agents whose income rely on labour income, even if there are benefits to them in the long run (see e.g. Garcia-Milà *et al.* (2010) and Angelopoulos *et al.* (2011b)). To assess the implications of the transition period in our model, in Table 3.3 as follows, we present the post-reform lifetime welfare, U_{lt}^{i}, for each type of agent, which measures the lifetime welfare of agent along the transition period. We assume that the tax reform is implemented at period 0. The post-reform lifetime welfare is then computed by using the discounted lifetime utility expression in (104) and (106) from period 1 until period

1000. It is given as follows:

$$U_{lt}^{i} = \sum_{j=1}^{1000} \beta^{j} u_{j}^{i} \tag{126}$$

where $i = k, w$; j denotes the number of years after the tax reform; u_{j}^{i} is the j'th period welfare for agent i in the post-reform economy. The aggregate post-reform lifetime welfare is computed as:

$$U_{lt} = n^{k} U_{lt}^{k} + n^{w} U_{lt}^{w}.$$

We also report the relevant welfare gains/costs measured by the compensating consumption supplement, ζ^{i}. [77] The measures of U_{lt}^{i} and ζ_{lt}^{i} can therefore capture the importance of the timing of the benefits and costs of eliminating the capital tax.

In contrast to our findings in Table 3.2, the results in Table 3.3 suggest that there are welfare losses for the workers (i.e. from -63.029 to -64.231) and at the aggregate level (i.e. from -61.598 to -61.765) over the lifetime, although this tax reform is Pareto improving in the long run. It predicts that the elimination of capital tax will hurt the workers and also worsen social welfare during the transition.

Table 3.3: Welfare effects of tax reform ($\overline{G}_{t}^{u} = \overline{r}_{t} w_{t}$)

	Pre-reform	Post-reform
U_{ss}^{k}	-50.586	-41.261
U_{ss}^{w}	-63.029	-62.687
U_{ss}	-61.598	-60.223
U_{lt}^{k}	-50.586	-42.784
U_{lt}^{w}	-63.029	-64.231
U_{lt}	-61.598	-61.765
ζ_{ss}^{k}	n.a.	0.184
ζ_{ss}^{w}	n.a.	0.005
ζ_{ss}	n.a.	0.022
ζ_{lt}^{k}	n.a.	0.154
ζ_{lt}^{w}	n.a.	-0.019
ζ_{lt}	n.a.	-0.003

[77] In particular, it is the additional level of consumption to give to the agent so that he is equally well off before and after the tax reform (see e.g. Baier and Glomm (2001) and Ardagna (2007)). For more detail, see Section 1.7 in Chapter 1.

To understand the underlying transmission mechanism driving these results, we first plot the transitional paths of variables to the new steady state. We assume that, at initial period, the capital tax unexpectedly and permanently shifts from 0.442 to 0. In response to the permanent policy change, the responses of variables are illustrated in Figure 3.1 as follows. In other words, Figure 3.1 shows how the economy gradually coverages to the new steady-state. These paths are generated by simulating the model as it converges to the new, post-reform steady-state, starting from the pre-reform steady-state (see e.g. Giannitsarou (2006)).

[Figure 3.1 about here]

Table 3.4 further presents the effects of the zero capital tax on the key variables in the short-run, i.e. one, two and three years after the zero capital tax has been implemented, as well as the long-run, i.e. fifty years after the reform and also at the new steady-state.

Table 3.4: Transitional effects of zero τ^k ($\overline{G}_t^u = \overline{r}_t w_t$)

	Pre-reform	Post-reform				
	Steady-state	Year 1	Year 2	Year 3	Year 50	Steady-state
C^k	0.659	0.715	0.722	0.728	0.806	0.808
C^w	0.606	0.556	0.564	0.566	0.608	0.609
K^k	15.746	16.032	16.335	16.605	20.278	20.383
τ^w	0.270	0.367	0.361	0.360	0.334	0.334
w	0.982	0.991	1.001	1.005	1.079	1.081
$\widetilde{w}$	0.717	0.629	0.641	0.644	0.718	0.720
e	0.587	0.583	0.585	0.584	0.584	0.583
s	0.044	0.055	0.053	0.053	0.047	0.047
v	0.306	0.249	0.251	0.254	0.283	0.283
z	0.900	0.588	0.620	0.622	0.777	0.782
p	2.657	2.147	2.205	2.209	2.468	2.475
q	2.951	3.652	3.555	3.549	3.177	3.168

We note that the labour tax initially goes above its new, post-reform steady-state (i.e. 0.367 versus 0.334). This intends to increase the bargained wage rate via its positive effect on the reservation wage as discussed above. However, the labour productivity does not increase much initially, as the higher capital accumulation due to elimination of capital tax is not yet realized at first which weakens the increase in wage rate. These two effects, on the whole, result in slightly rising wage rate for the first period (i.e. from 0.982 to 0.991). This, in turn, leads to higher unemployment benefits. The net wage rate falls short of its old, pre-reform steady-state (i.e. from 0.717 to 0.629), as the initial increase in labour tax exceeds the increase in wage rate. The search-unemployment overshoots its post-reform steady-state (i.e. 0.055 versus 0.047). This result is driven by the higher unemployment benefits and lower net wage rate in the post-shock economy. The firms cut job vacancies in the short-run, since prior to the new steady-state, the positive higher capital accumulation effect on labour productivity is not strong enough to outweigh the negative profits effect induced by rising wage rate. As capital accumulates and this is transformed into higher labour productivity, the firms begin to open more job vacancies over time, although the number is less than the old, pre-reform steady-state. Labour market tightness is increasing as search-unemployment falls and the number of vacancies increases over time. As can be seen in Figure 3.1a, employment at the first period falls, but remains almost unchanged over time. This is because the increase in p and decrease in s effectively net out over time, which leaves no effect on employment. The combination of lower net wage rates and higher search-unemployment creates short-run losses for the workers and also aggregate welfare worsens, which are reversed in the long run, similar to Domeij and Heathcote (2004).[78]

3.4.3 Changes in bargaining power of workers

As have discussed earlier, the choice of worker's bargaining power, ϕ, is crucial in the models with search frictions due to the existence of externalities. We now illustrate the degree to which our results are robust to variations in this parameter and examine the importance of worker's bargaining power on the welfare effects of elimination of capital income tax. Our calibration above is based on the Hosios condition, $\eta = \phi$. In what follows, we examine changes in ϕ that encompass the entire range used in the literature, see e.g. Domeij (2005). In Table 3.5, for each value of ϕ, we report the differences in the long run between the pre- and post-reform steady-state for the key

[78] They find that in the heterogeneous agent economy capital tax cuts are supported only by a minority of households during the transition.

economic variables.[79] We also report the compensating consumption supplement for each agent and the aggregate economy at the steady-state.[80]

Table 3.5: Changes in workers' bargaining power for tax reforms (difference from pre-reform policy)

	0.250	**0.375**	**0.500**	**0.625**	**0.750**
$\%\Delta C^k$	0.2364	0.2297	0.2260	0.2237	0.2220
$\%\Delta C^w$	0.0066	0.0060	0.0058	0.0057	0.0057
$\%\Delta\left(\frac{C^k}{Y}\right)$	0.1346	0.1279	0.1243	0.1220	0.1205
$\%\Delta\left(\frac{C^w}{Y}\right)$	-0.0764	-0.0773	-0.0776	-0.0778	-0.0779
$\%\Delta\left(\frac{Y^k}{Y^w}\right)$	0.2683	02675	0.2669	0.2665	0.2662
$\%\Delta K^k$	0.2937	0.2942	0.2945	0.2946	0.2947
$\Delta\tau^w$	0.0683	0.0651	0.0635	0.0626	0.0619
Δw	0.1098	0.1038	0.1009	0.0992	0.0981
$\Delta\widetilde{w}$	0.0060	0.0053	0.0051	0.0050	0.0049
ζ^k_{ss}	0.1912	0.1868	0.1843	0.1828	0.1817
ζ^w_{ss}	0.0061	0.0056	0.0055	0.0054	0.0053
ζ_{ss}	0.0227	0.0225	0.0224	0.0223	0.0222

As can be seen in Table 3.5, the welfare gains for all agents from elimination of the capital tax are decreasing in ϕ. In other words, for fixed η, increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improvement. As discussed above, the wage rate is rising after the tax reform via the higher labour productivity and concurrent increase in labour tax channel. The results in Table 3.5 show that, as ϕ increases, the tax reform has a smaller effect on labour tax but bigger effect on labour productivity due to higher capital accumulation. As the relative bargaining power of worker increases, the importance of the increase in labour tax has been improved relative to the labour productivity. Hence, the tax reform exerts a smaller effect on wage rate and in turn on unemployment benefits. The effect on net wage rate has also been reduced for a combination of smaller effects on labour tax and wage rate. Therefore, given that the effects on net wage rate and unemployment benefits

[79] Note that for all cases considered, the parameters, ν and $\overline{G}^t$ are re-calibrated so that the base in all cases is an economy with 7% unemployment and 27% labour tax rate. The remaining parameters used are as in Table 3.1.

[80] Note that the results in Table 3.5 are in percent differences, except that the values starting from τ^w until q are in percentage point differences.

have been reduced, the income, consumption and welfare for the workers increase by less as ϕ goes up.

As ϕ increases, the job posting cost needs to fall in the pre-reform economy such that the new calibration can yield the same unemployment rate as in the base case. This implies that the costs of posting vacancies have been reduced. As a result, the expected profits of firms are getting bigger and the firms increase production. It implies that the tax reform has a bigger effect on boosting investment of capitalists and therefore a smaller effect on consumption increase. The welfare gains for the capitalists become smaller as ϕ goes up. Finally, it is worth noting that inequality improves as can be seen from decreasing relative income of capitalists and workers, $\frac{Y^k}{Y^w}$. It indicates that increasing the workers' bargaining power can help to reduce the income gap between capitalists and workers.

3.5 Alternative specification of unemployment benefits

3.5.1 The new specification

In this section, we employ an alternative specification of unemployment benefits. Pissarides (1998) and Koskela and von Thadden (2008) have discussed the importance of the specification of unemployment benefits in the wage bargaining. We now assume that unemployment benefits depend on past wages due to some institutional features in the labour market, see e.g. Blanchard and Katz (1999) and Chéron and Langot (2010). Thus, unemployment benefit, $\overline{G}_t^u$, is specified as follows:

$$\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1} \tag{127}$$

where w is the steady-state wage rate. As can be seen, unemployment benefits are proportional to past wages by the constant $\frac{z}{w} > 0$ in the transition period. However, in the steady-state, they are constant and equal to $z > 0$. When the wage rate rises after the tax reform, unemployment benefits remain the same. Hence, this new specification of unemployment benefits is important in determining both the long- and short-run results of the tax reforms.

The parameter, z, is re-calibrated to obtain the steady-state τ^w of 0.27. All the other parameters are used as in Table 3.1.

3.5.2 The long-run effects of tax reforms

We first examine the importance of this new specification of unemployment benefits in determining the long-run effects of the tax reforms. Column (1) and Column (2) of Table 3.6 present the pre- and post-reform steady-states, respectively. We report the steady-state allocations and welfare of agents.

The results in Table 3.6 show that there are welfare gains for all agents in the long run if the government chooses a zero capital tax and increases labour tax to make up for the tax revenue losses although this tax reform increases inequity. It implies that the tax reform is still Pareto improving despite increasing income difference (see $\frac{Y^k}{Y^w}$). In the new unemployment benefits setup, the tax reform has different effects on labour market which can be seen from the changes in labour market variables. The unemployment benefits remain the same in the post-reform economy. In this case, the search-unemployment only depends on net wage rate.

As discussed before, the net wage increases after the tax reform and therefore search-unemployment falls (i.e. from 0.044 to 0.041). The firms create more vacancies at the post-reform steady-state (i.e. from 0.306 to 0.328). This is because, on one hand, the increase in wage rate is relatively smaller (i.e. from 0.982 to 1.081 versus from 0.982 to 1.077), which implies smaller negative revenue effect. On the other hand, the higher in labour productivity due to higher capital accumulation increases the firms' expected profits from a successful match. The production is more profitable at the post-reform steady-state which can be seen in equation (118) since the discounted expected value of profits at $t+1$ are higher. The labour market tightness is rising (i.e. from 0.9 to 1.029) when v increases and s decreases. Therefore, the probability at which unemployed workers can be matched with job vacancies increases (i.e. from 2.657 to 2.841), and the probability at which job vacancies can be matched with job seekers decreases (i.e. from 2.951 to 2.760). In turn, employment goes up at the post-reform steady-state (i.e. from 0.587 to 0.589). This creates one additional channel for the increases in income, consumption, and welfare of workers as working can generate higher income relative to searching. The steady-state welfare gains for workers are therefore when unemployment benefits are assumed to depend on past wages.

Table 3.6: Long-run effects of tax reform ($\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1}$)

	Pre-reform	Post-reform
	(1)	(2)
$\frac{C^k}{Y}$	0.094	0.106
$\frac{C^w}{Y}$	0.667	0.613
$\frac{C}{Y}$	0.761	0.719
$\frac{Y^k}{Y^w}$	3.687	4.686
$\frac{I^k}{Y}$	0.225	0.267
$\frac{K^k}{Y}$	2.252	2.673
$\frac{G^t}{Y}$	0.203	0.185
$\frac{G^u}{Y}$	0.024	0.020
e	0.587	0.589
s	0.044	0.041
v	0.306	0.328
τ^w	0.270	0.324
w	0.982	1.077
$\widetilde{w}$	0.717	0.729
r	0.155	0.131
$\widetilde{r}$	0.031	0.031
z	0.900	1.029
p	2.657	2.841
q	2.951	2.760
U^k	-50.586	-40.867
U^w	-63.029	-62.332
U	-61.598	-59.864

As explained before, a zero capital tax boosts investment and capital. The income, consumption and welfare of capitalists increase at the post-reform steady-state. Furthermore, the capitalists can gain more from the tax reform due to the increase in firms' profits in the production. To summarise, if unemployment benefits depend on past wages, the tax cuts met by the labour tax increases can result in higher welfare gains for all agents and the tax reform is still Pareto improving in the long run. Hence, the formation of unemployment benefits only influences the magnitude of steady-state welfare effects of tax reforms. But as discussed above, the tax reforms have different effects on labour market variables so that the mechanism driving the results is different.

3.5.3 The transitional effects of tax reforms

We then analyse how the results will change during the transition period. We report the same variables in Table 3.7 in order to compare with those in Table 3.3.

As can be seen, our main result that the capital tax cuts will hurt the agents whose income rely on labour income during the transition period stands in the model with new specification of unemployment benefits. As in the long run, the tax reform has higher welfare effects for all agents in transition period. It is worth noting that the aggregate welfare losses turn into the welfare gains over the lifetime. Our results show that the tax reforms imply short-run welfare losses only for the workers, similar to Ardagna (2007).[81]

Table 3.7: Welfare effects of tax reform ($\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1}$)

	Pre-reform	Post-reform
U_{ss}^k	-50.586	-40.867
U_{ss}^w	-63.029	-62.332
U_{ss}	-61.598	-59.864
U_{lt}^k	-50.586	-42.388
U_{lt}^w	-63.029	-63.853
U_{lt}	-61.598	-61.385
ζ_{ss}^k	n.a.	0.192
ζ_{ss}^w	n.a.	0.011
ζ_{ss}	n.a.	0.028
ζ_{lt}^k	n.a.	0.169
ζ_{lt}^w	n.a.	-0.008
ζ_{lt}	n.a.	0.009

To understand what drives these results, we evaluate the transitional dynamics between steady-states. Table 3.8 reports the effects of tax reform in the short-run and

[81] She employs a model with unionised labour market to examine exogenous changes in fiscal instruments accomodated by changes in government debt and finds that workers' welfare goes down after the increase in labour tax.

Figure 3.2 plots the transitional paths of variables.

[Figure 3.2 about here]

Table 3.8: Transitional effects of zero τ^k ($\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1}$)

	Pre-reform	Post-reform				
	Steady-state	Year 1	Year 2	Year 3	Year 50	Steady-state
C^k	0.659	0.740	0.745	0.750	0.814	0.816
C^w	0.606	0.571	0.576	0.578	0.612	0.613
K^k	15.746	16.951	17.207	17.440	20.483	20.564
τ^w	0.270	0.347	0.344	0.343	0.324	0.324
w	0.982	1.005	1.012	1.016	1.076	1.077
$\widetilde{w}$	0.717	0.657	0.664	0.668	0.727	0.729
e	0.587	0.590	0.590	0.590	0.589	0.589
s	0.044	0.046	0.045	0.045	0.042	0.042
v	0.306	0.299	0.301	0.303	0.328	0.328
z	0.900	0.844	0.868	0.873	1.025	1.029
p	2.657	2.573	2.609	2.617	2.835	2.841
q	2.951	3.047	3.005	2.996	2.765	2.760

In contrast to Table 3.4, the wage rate increases by more (i.e. from 0.982 to 1.005 versus from 0.982 to 0.991) for the first period although at the new, post-reform steady-state, it is smaller (i.e. 1.077 versus 1.081). This is because the capital tax cuts have larger initial effect on boosting capital accumulation in this model (i.e. 16.951 versus 16.032). This is transformed into higher labour productivity and thus higher wage rate as can be seen in the wage condition (90). The wage rate is rising over time since capital accumulation is increasing which results in increasing labour productivity. The net wage, $\widetilde{w}$, initially falls short of the old, pre-reform steady-state, due to the large increase in labour tax as discussed before. Thus, the search-unemployment at the first period overshoots its old steady-state (i.e. 0.046 versus 0.044). The net wage is rising over time which causes the decrease in search-unemployment. Besides, unemployment benefits depend on past wages and their path follows the path of wage rate which causes the inertia in the increases in unemployment benefits in the transition. This tends to weaken the increase in search-unemployment. As a result, the increase in net wage dominates and search-unemployment falls during the transition. The firms cut job vacancies at the first period (i.e. from 0.044 to 0.046) as the positive profits effect due to the higher capital accumulation is not yet realized and the increase in wage rate makes the production less profitable. As more capital is built up, the firms open more job vacancies in the labour market. The labour market tightness is reduced for the first period (i.e. from 0.9 to 0.844) due to less available vacancies and more search-unemployment. In turn, employment increases for the first period (i.e. from 0.587 to 0.590), but there are small fluctuations of employment over time, which can be seen in Figure 3.2a.

We see in Figure 3.2, that the tax cuts have larger effects on increasing capital accumulation and raising the net wage rate during the transition. As a result, the income, consumption and welfare of capitalists increase by more over time, and the income, consumption, and welfare of workers is raised more quickly relative to the model with old specification of unemployment benefits. Thus, the lifetime welfare gains for capitalists are higher and the lifetime welfare losses for workers are lower. The lifetime social welfare improves in the aggregate economy.

3.6 Optimal tax policy under commitment

In this section, the optimal design of steady-state factor taxes is studied. We intend to investigate the relevance of search frictions in determining the optimal taxation of government. Domeij (2005) shows that when the Hosios parameter restriction holds, the optimal capital tax in the long run is always zero. In our case, we still assume that the Hosios parameter restriction is satisfied, i.e. $\eta = \phi$, but our optimal policy set-up is is different from Domeij's in several ways.[82] As a result, we will show later, this model implies different optimal steady-state factor taxes.

In the optimal policy set-up, the tax rates are no longer fixed to their data averages, but optimally chosen by the government with the aim of maximising the aggregate welfare subject to the DE conditions given in sub-section 3.2.10.[83] We assume that the government is Benthamite in the sense that the weight attached to the welfare of agent in its objective function corresponds to its population share. Furthermore,

[82] Our model differs from his model in that: (i) household heterogeneity is introduced; (ii) government debt is not present; (iii) government optimally chooses all the tax rates; (iv) profits are taxed.

[83] We have also checked the optimal tax policy in the model with unemployment benefits depending on past wages, and find that the steady-state results are qualitatively the same as those in the baseline model. This implies that different unemployment benefits assumption can influence the effects of tax reforms, but not the optimal choice of policy.

following for example, Guo and Lansing (1999) and Domeij (2005), the government is assumed to have a commitment technology to avoid any issue of time-inconsistency problem. It assumes that the government commits to a set of fiscal policies announced at initial period and never reoptimizes. This is referred to as the government's Ramsey problem. We apply the dual approach to solve the Ramsey problem. In this sense, the government optimally chooses both tax rates and allocation of resources.[84]

Column (1) and Column (2) in Table 3.9, respectively, give the steady-state of decentralised equilibrium with given policies and the steady-state of Ramsey equilibrium when all the tax rates, τ_t^k, τ_t^w and $\overline{r}_t$ are optimally chosen by the government.

First, the long-run optimal capital tax is negative, i.e. -0.108. This result is different from that in Domeij (2005). He shows that the optimal capital tax is zero in the long run as in Judd (1985) and Chamley (1986). However, this finding is consistent with Judd (2002), Schmitt-Grohe and Uribe (2005), and Chugh (2006), who suggest that negative optimal capital tax rates should be used to indirectly affect some types of market frictions.[85] In our case, the market frictions arise as the forward-looking behavior of firms results in a higher marginal product of labour than the marginal cost of labour. Therefore, the investment subsidy helps to correct the search frictions. Due to the investment subsidy, the investment is boosted and this is transformed into higher capital accumulation (i.e. from 15.746 to 21.907). The income, consumption and welfare of capitalists increase as the negative capital boosts capital accumulation and therefore brings them more capital income.

Second, the government increases the labour tax to make up for the tax revenue losses from reducing capital tax (i.e. from 0.27 to 0.3). The steady-state wage rate as the outcome of bargaining goes up due to higher labour productivity resulting from higher capital accumulation and increase in labour tax (i.e. from 0.982 to 1.071). The net wage rate is increased in Ramsey (i.e. from 0.717 to 0.750). This is because the increase in gross wage outweighs the labour tax increase. The optimal replacement rate, $\overline{r}$, falls substantially in the long-run of Ramsey (i.e. from 0.500 to 0.009). Thus, unemployment benefits are much lower in Ramsey. The steady-state search-unemployment is less than it would be in the model with exogenous tax rates (i.e. 0.024 versus 0.044) as the net wage rate increases and unemployment benefits decrease. The firms create more job vacancies (i.e. from 0.306 to 0.595) since the positive effect of higher labour

[84]The government's optimization in Ramsey problem is described in the Appendix 8.7.

[85]In these papers, a capital subsidy boosts output via encouraging capital accumulation, which is inefficiently low due to the presence of a monopolistic distortion.

productivity on profits outweighs the negative effect of increased wage rate and therefore expected profits from a successful match are higher. The steady-state employment increases since the probability at which unemployed workers can be matched with job vacancies increases (i.e. from 2.657 to 5.006). This is beneficial to the workers as working can generate higher labour income and therefore the consumption and welfare of workers increase.

Table 3.9: Optimal tax policy under commitment

	Exogenous (1)	Optimal (2)
C^k	0.659	0.873
C^w	0.606	0.618
K^k	15.746	21.907
τ^k	0.442	-0.108
τ^w	0.270	0.300
τ^p	0.500	-0.108
$\overline{r}$	0.500	0.009
w	0.982	1.071
$\widetilde{w}$	0.717	0.750
e	0.587	0.605
s	0.044	0.024
v	0.306	0.595
z	0.900	3.197
p	2.657	5.006
q	2.951	1.566
U^k	-50.586	-38.168
U^w	-63.029	-61.848
U	-61.598	-59.125

3.7 Summary and concluding remarks

This chapter investigated the effects of tax policy on unemployment, distribution of income and the welfare of agents assuming household heterogeneity. The households were divided into capitalists and workers. Only workers worked and only capitalists had access to the asset market. The analysis was conducted in a search and matching model. Unemployed workers sought potential job opportunities and firms opened new job vacancies to employ the desired number of workers in the following period. The wage rate was determined in a Nash bargaining between a pair of worker and firm once they were matched through a Cobb-Douglas matching function. If the bargaining was successful, the worker was employed by the firm in the following period and the firm produced employing capital and labour. In this sense, employment was pre-determined at any given period of time. The government taxed interest income from physical capital, profits and labour income to finance its spending. The model was calibrated to match the main characteristics of the UK economy, with particular focuses on its labour market. In the tax reform experiments, we analysed the effects of capital tax cuts associated with concurrent labour tax increases. This allowed us to examine the productivity-tax burden trade-off and different impacts on heterogeneous households. Finally, optimal tax policy of the government was examined. In this case, the government chose all the tax rates in order to maximise the lifetime social welfare and it had access to the commitment technology in the policy making. Our main findings are summarized as follows.

First, in a model with search and matching frictions, the tax reform considered is Pareto improving in the long run although it increases inequality between agents. In other words, all the agents are better off, despite higher welfare gains for the capitalists compared to the workers. However, the capital tax cut met with the labour tax increase hurts the workers and also worsens the aggregate welfare in the transition period. This is because the positive effects resulting from higher capital accumulation take time to be realized. As a result, the combination of an initially lower net wage rate and higher search-unemployment creates short-run losses for the workers and aggregate economy, which are reversed in the long run. We also show that our results are robust to variations in the relative bargaining power of workers in the Nash bargain. Increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improving.

Second, when we assume that unemployment benefits depend on past wages, the model can generate similar welfare results in both the long- and short-run although the mechanism driving results is different. The tax reform is still Pareto improving in the long run but generates higher welfare gains for all agents. Similar to the long run results, the tax reform has higher welfare effects for all agents in the transition period. In other words, the lifetime welfare gains for capitalists are higher and welfare losses for workers are lower. As a result, the lifetime aggregate welfare improves. The wage rate increases by more in the short run since the capital tax cuts have larger initial effects on boosting capital accumulation. In turn, the net wage increases in the transition period. The unemployment benefits depend on past wages and their path follows the path of wage rate which causes the inertia in the increase in unemployment benefits over the transition. The welfare of workers is raised more quickly in the new, post-reform economy. To summarise, the long- and short-run welfare gains of tax reforms are higher for all agents by assuming unemployment benefits depending on past wages.

Finally, we find that the optimal tax policy under commitment implies a negative optimal capital income tax in the long run which is accomplished by the labour tax increase and unemployment benefits reduction. The negative capital income tax helps to correct the market frictions caused by the forward-looking behavior of firms. As a result, the investment is boosted and this is transformed into higher capital accumulation. The income, consumption and welfare of capitalists increase. The labour income tax is increased and the unemployment benefits are reduced to make up for the tax revenue losses from capital income. The lower unemployment benefits reduce the search-unemployment of workers. The firms open more job vacancies resulting from higher labour productivity. As a result, the probability at which job seekers can be matched with job vacancies is increased. In turn, employment increases which is beneficial to the workers. There are welfare gains for the workers and aggregate welfare improves in the long run.

Our analysis makes clear that the tax reform of reducing capital tax and a concurrent labour tax increase can increase the welfare of all agents but with the sacrifice of inequality under different specifications of unemployment benefits. We further see that, in the short run, the tax reform will hurt the agents who rely on labour income. Thus, our analysis adds to the tax policy studies in the search-and-matching literature and offers new results about the redistributional effects of the tax policy.

3.8 Figures

Figure 3.1a: Transition dynamics when unemployment benefits do not depend on past wages (τ^k=0)

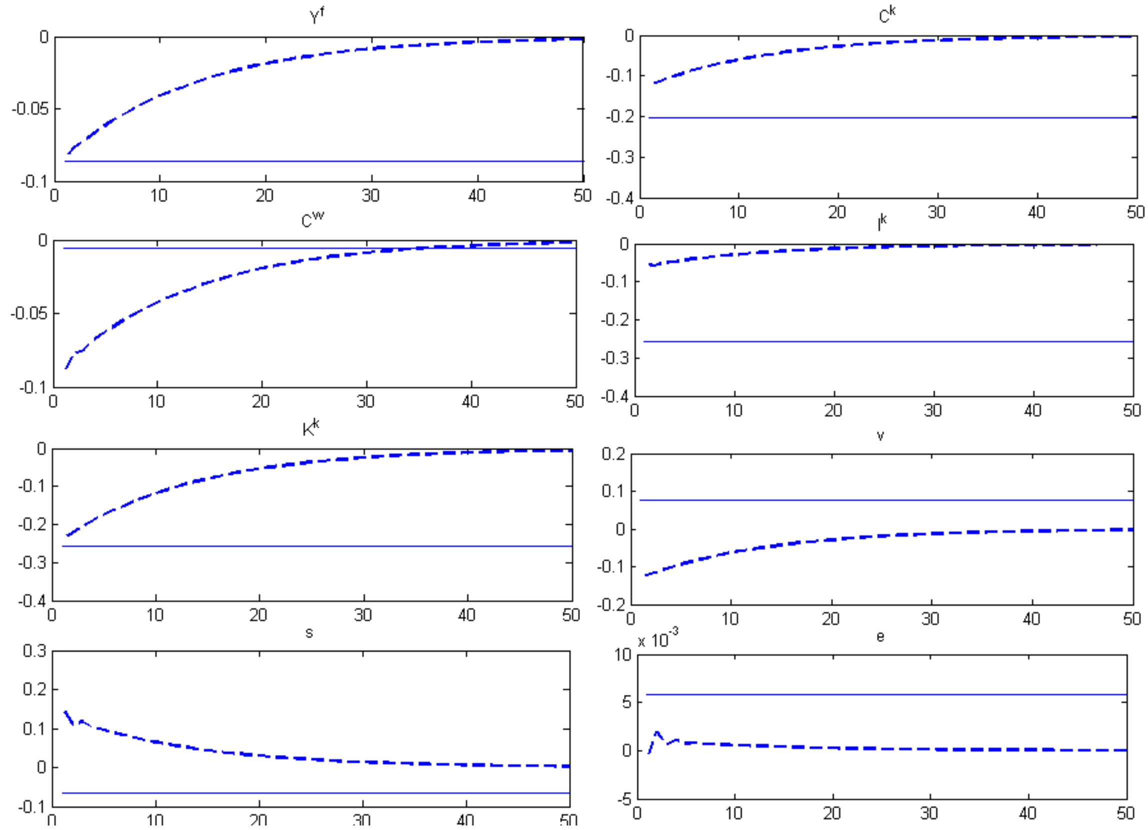


Figure 3.1b: Transition dynamics when unemployment benefits do no depend on past wages (τ^k=0)

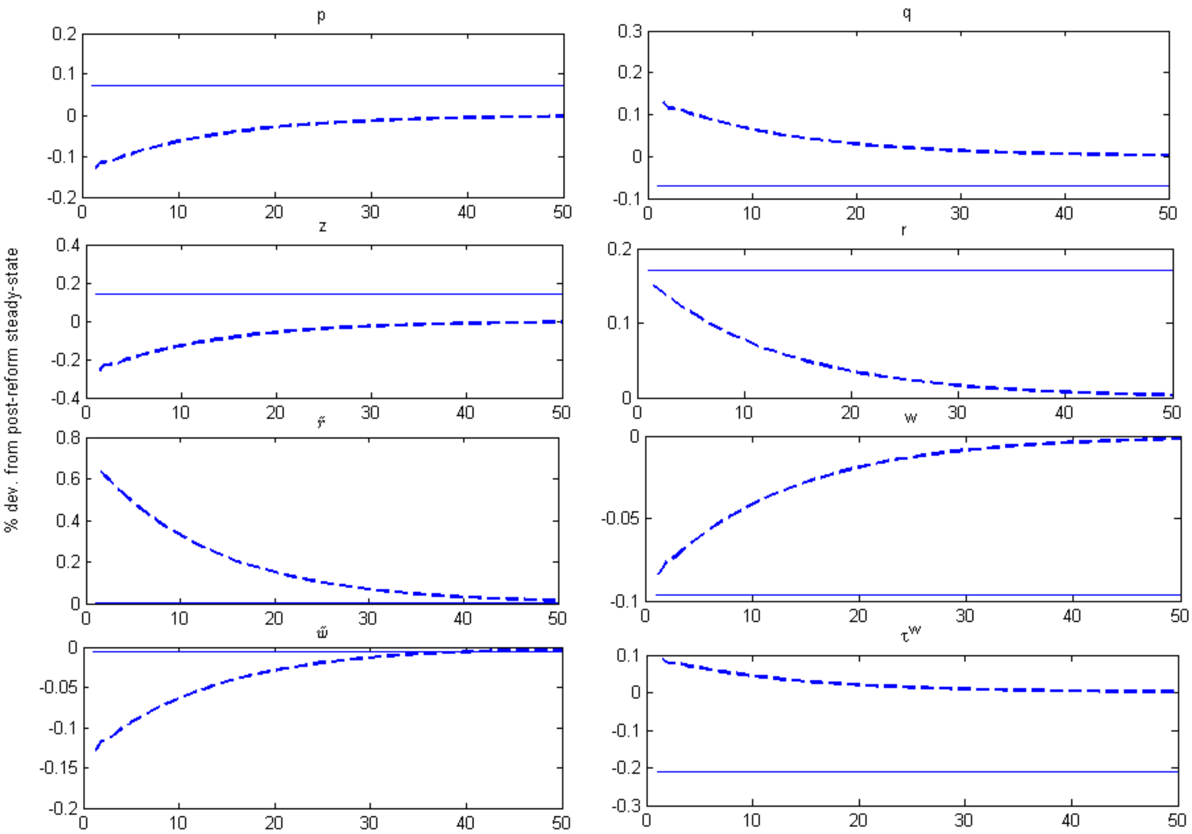


Figure 3.2a: Transition dynamics when unemployment benefits depend on past wages (τ^k=0)

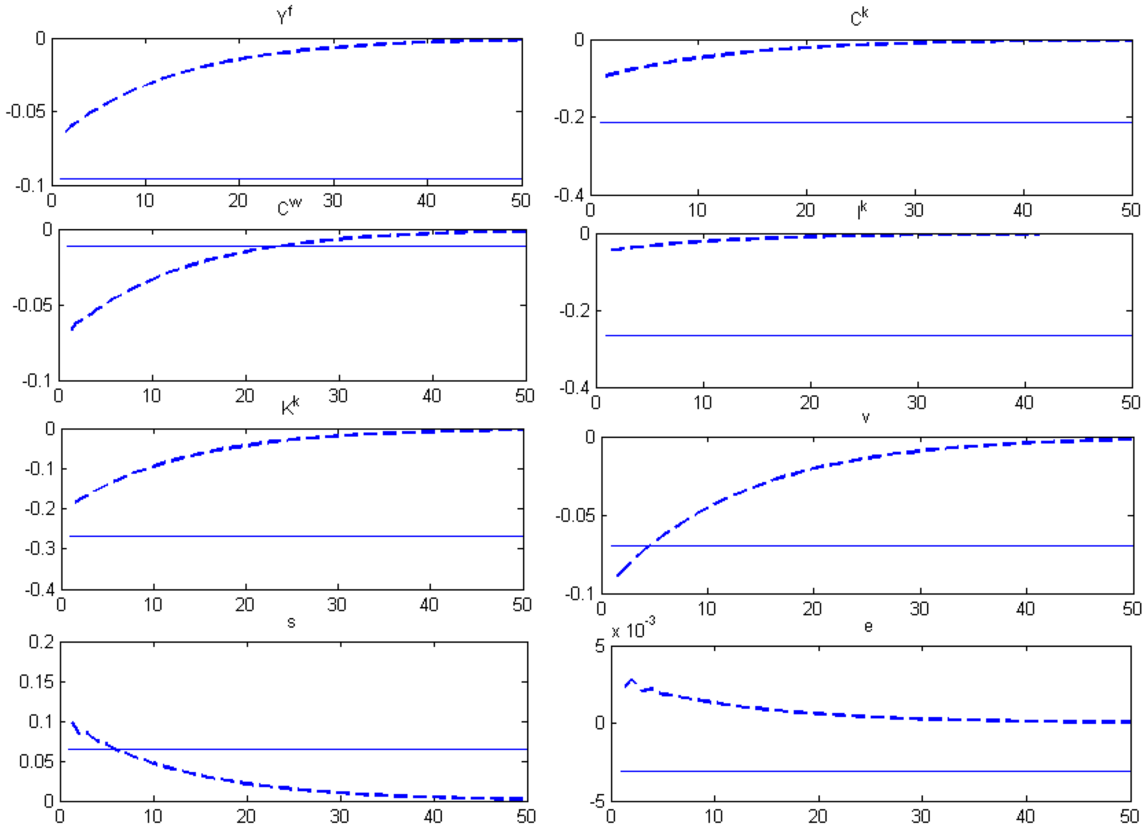
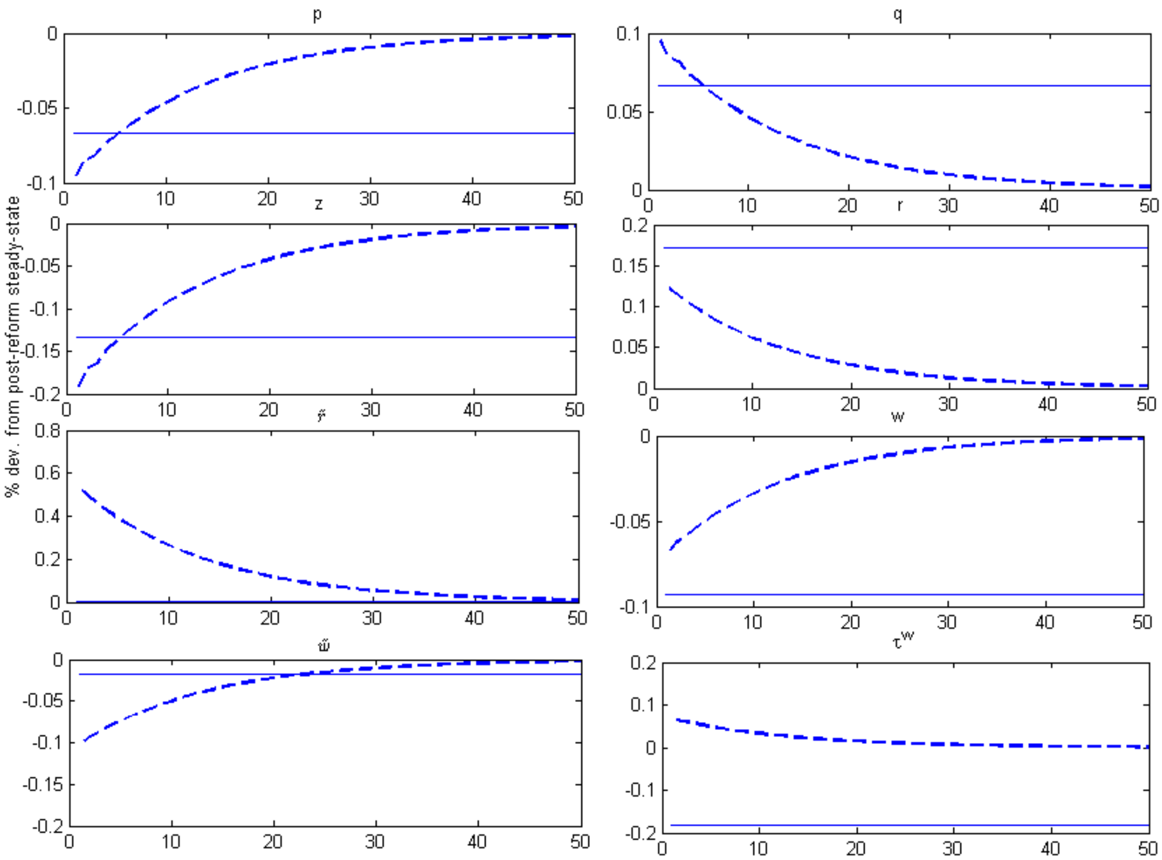


Figure 3.2b: Transition dynamics when unemployment benefits depend on past wages (τ^k=0)



3.A Appendix

3.A.1 Optimization of capitalists

The Lagrangian function of the capitalist is written as:

$$L^k = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma} + \lambda_t^1 \left[R_t K_t^k + \left(1-\tau_t^k\right) \pi_t^k + \overline{G}_t^t - C_t^k - K_{t+1}^k \right] \right\}$$

where λ_t^1 is the Lagrangian multiplier on the capitalist's budget constraint.

The first-order condition (FOC) for C_t^k is:

$$(1-\sigma) \frac{\left(C_t^k\right)^{-\sigma}}{1-\sigma} - \lambda_t^1 = 0$$

$$\frac{1}{\left(C_t^k\right)^{\sigma}} = \lambda_t^1.$$

The FOC for K_{t+1}^k is:

$$\beta \lambda_{t+1}^1 R_{t+1} - \lambda_t^1 = 0$$

$$\beta \lambda_{t+1}^1 R_{t+1} = \lambda_t^1.$$

3.A.2 Optimization of workers

The Lagrangian function of the worker is written as:

$$\begin{aligned} L^w &= \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t^w)^{1-\sigma}}{1-\sigma} - \xi \frac{(e_t + s_t)^{\mu}}{\mu} + \right. \\ & +\lambda_t^2 \left[p_t s_t + (1-\gamma) e_t - e_{t+1} \right] + \\ & \left. +\lambda_t^3 \left[(1-\tau_t^w) w_t e_t + \overline{G}_t^u s_t + \overline{G}_t^t - C_t^w \right] \right\} \end{aligned}$$

where λ_t^2 is the Lagrangian multiplier on the evolution equation of employment and λ_t^3 is the Lagrangian multiplier on the worker's budget constraint.

The FOC for C_t^k is:

$$(1-\sigma) \frac{(C_t^w)^{-\sigma}}{1-\sigma} = \lambda_t^3$$

$$\frac{1}{(C_t^w)^{\sigma}} = \lambda_t^3.$$

The FOC for s_t is:

$$-\xi\mu \frac{(e_t + s_t)^{\mu-1}}{\mu} + \lambda_t^2 p_t + \lambda_t^3 \overline{G}_t^u = 0$$

$$\xi (e_t + s_t)^{\mu-1} = p_t \lambda_t^2 + \lambda_t^3 \overline{G}_t^u.$$

The FOC for e_{t+1} is:

$$-\beta\xi\mu \frac{(e_{t+1} + s_{t+1})^{\mu-1}}{\mu} - \lambda_t^2 + \beta\lambda_{t+1}^2 (1-\gamma) + \lambda_{t+1}^3 (1-\tau_t^w) w_{t+1} = 0$$

$$\beta\xi (e_{t+1} + s_{t+1})^{\mu-1} = -\lambda_t^2 + \beta \left[\lambda_{t+1}^2 (1-\gamma) + \lambda_{t+1}^3 (1-\tau_t^w) w_{t+1} \right].$$

3.A.3 Optimization of firms

The Lagrangian function of the firm is written as:

$$
\begin{aligned}
L^f &= \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_i^{-1} \left\{ A \left(K_t^f\right)^{\alpha} \left(L_t^f\right)^{1-\alpha} - r_t K_t^f - w_t L_t^f - \nu v_t + \right. \\
&\quad \left. + \lambda_t^4 \left[q_t v_t + (1-\gamma) L_t^f - L_{t+1}^f \right] \right\}
\end{aligned}
$$

where λ_t^4 is the Lagrangian multiplier on the evaluation of firm's labour input.

The FOC for K_t^f is:

$$
\prod_{i=0}^{t} R_i^{-1} \alpha A \left(K_t^f\right)^{\alpha} \left(L_t^f\right)^{1-\alpha} - \prod_{i=0}^{t} R_i^{-1} r_t = 0
$$

$$
r_t = \alpha A \left(K_t^f\right)^{\alpha} \left(L_t^f\right)^{1-\alpha}
$$

$$
r_t = \alpha \frac{Y_t^f}{K_t^f}.
$$

The FOC for v_t is:

$$
-\prod_{i=0}^{t} R_i^{-1} \nu + \prod_{i=0}^{t} R_i^{-1} \lambda_t^4 q_t = 0
$$

$$
\nu = \lambda_t^4 q_t. \tag{128}
$$

The FOC for L_{t+1} is:

$$
\prod_{i=0}^{t+1} R_i^{-1} \left[(1-\alpha) A \left(K_{t+1}^f\right)^{\alpha} \left(L_{t+1}^f\right)^{-\alpha} - w_{t+1} \right] -
$$

$$
-\prod_{i=0}^{t} R_i^{-1} \lambda_t^4 + \prod_{i=0}^{t+1} R_i^{-1} \lambda_{t+1}^4 (1-\gamma) = 0
$$

$$
R_{t+1}^{-1} \left[(1-\alpha) A \left(K_{t+1}^f\right)^{\alpha} \left(L_{t+1}^f\right)^{-\alpha} - w_{t+1} \right] = \lambda_t^4 - R_{t+1}^{-1} \lambda_{t+1}^4 (1-\gamma)
$$

$$
R_{t+1}^{-1} \left[(1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} \right] = \lambda_t^4 - R_{t+1}^{-1} \lambda_{t+1}^4 (1-\gamma). \tag{129}
$$

We then solve for λ_t^4 in condition (128) and substitute the expression into condition (129):

$$
R_{t+1}^{-1} \left[(1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} \right] = \frac{\nu}{q_t} - R_{t+1}^{-1} \frac{\nu}{q_{t+1}} (1-\gamma)
$$

which can be simplified to:

$$\frac{\nu}{q_t} = R_{t+1}^{-1}\left[(1-\alpha)\,\frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} + \frac{\nu(1-\gamma)}{q_{t+1}}\right].$$

3.A.4 Derivation of firm's expected profits

We first rewrite the present value of the stream of firm's profits in (116) starting from time 1 by making use of two first-order conditions of firms set out above, the profits equation (88), the law of motion for the firm's employment (114), and the properties of the production function:

$$
\begin{aligned}
\sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\pi_t^f &= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left(Y_t^f - r_t K_t^f - w_t L_t^f - \nu v_t\right) \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left(Y_t^f - \alpha Y_t^f - w_t L_t^f - \nu v_t\right) \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left[(1-\alpha) Y_t^f - w_t L_t^f\right] - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left(f_{L_t^f} L_t^f - w_t L_t^f\right) - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left(f_{L_t^f} - w_t\right) L_t^f - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=0}^{\infty}\prod_{i=0}^{t} R_{i+1}^{-1}\left(f_{L_{t+1}^f} - w_{t+1}\right) L_{t+1}^f - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left(Y_t^f - \alpha Y_t^f - w_t L_t^f - \nu v_t\right) \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left[(1-\alpha) Y_t^f - w_t L_t^f\right] - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left[(1-\alpha)\frac{Y_{t+1}^f}{L_{t+1}^f} L_t^f - w_t L_t^f\right] - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\left[(1-\alpha)\frac{Y_{t+1}^f}{L_{t+1}^f} - w_t\right] L_t^f - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t \\
&= \sum_{t=0}^{\infty}\prod_{i=0}^{t} R_{i+1}^{-1}\left[(1-\alpha)\frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1}\right] L_{t+1}^f - \sum_{t=1}^{\infty}\prod_{i=1}^{t} R_i^{-1}\nu v_t.
\end{aligned}
$$

Then, we substitute out $\left((1-\alpha)\frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1}\right)$ by making use of the condition

(118). The r.h.s. of above equation can be rewritten as:

$$
\begin{aligned}
& \sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}L_{t+1}^{f}\left[\frac{\nu}{q_t}R_{t+1}-\frac{\nu(1-\gamma)}{q_{t+1}}\right]-\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\nu v_t \\
= & \sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}L_{t+1}^{f}\frac{\nu}{q_t}R_{t+1}-\sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}L_{t+1}^{f}\frac{\nu(1-\gamma)}{q_{t+1}}-\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\nu v_t \\
= & \sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}\left(q_t v_t+(1-\gamma)L_t^{f}\right)\frac{\nu}{q_t}R_{t+1}- \\
& -\sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}L_{t+1}^{f}\frac{\nu(1-\gamma)}{q_{t+1}}-\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\nu v_t \\
= & \left(\sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}R_{t+1}\nu v_t-\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\nu v_t\right)+ \\
& +\left(\sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}(1-\gamma)L_t^{f}\frac{\nu}{q_t}R_{t+1}-\sum_{t=0}^{\infty}\prod_{i=0}^{t}R_{i+1}^{-1}L_{t+1}^{f}\frac{\nu(1-\gamma)}{q_{t+1}}\right) \\
= & \ \nu v_0+(1-\gamma)L_0^{f}\frac{\nu}{q_0}.
\end{aligned}
$$

Making use of the evolution equation of employment, $L_1^f = q_0 v_0 + (1-\gamma)L_0^f$, we can rewrite the final expression above as follows:

$$
\nu v_0+(1-\gamma)L_0^{f}\frac{\nu}{q_0}=L_1^{f}\frac{\nu}{q_0}
$$

so that

$$
\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\pi_t^{f}=L_1^{f}\frac{\nu}{q_0}
$$

or

$$
\nu=q_0\frac{\sum_{t=1}^{\infty}\prod_{i=1}^{t}R_i^{-1}\pi_t^{f}}{L_1^{f}}. \tag{130}
$$

It states that, in equilibrium, the costs of posting a vacancy today should equalize the discounted value of stream of profits brought about by each filled vacancy tomorrow. This implies that the marginal cost of a vacancy is equal to the marginal benefit of filling it in the next period.

3.A.5 Solution to Nash bargain

The FOC with respect to w_t is given by:

$$\phi \left[w_t - \frac{u_{2,t}^w}{\left(1-\tau_t^w\right) u_{1,t}^w} \right]^{\phi-1} \left[Y_{2,t}^f - w_t \right]^{1-\phi} -$$

$$-\left(1-\phi\right) \left[w_t - \frac{u_{2,t}^w}{\left(1-\tau_t^w\right) u_{1,t}^w} \right]^{\phi} \left[Y_{2,t}^f - w_t \right]^{-\phi} = 0$$

$$\phi \left[Y_{2,t}^f - w_t \right] = \left(1-\phi\right) \left[w_t - \frac{u_{2,t}^w}{\left(1-\tau_t^w\right) u_{1,t}^w} \right]$$

$$\phi Y_{2,t}^f - \phi w_t = \left(1-\phi\right) w_t - \left(1-\phi\right) \frac{u_{2,t}^w}{\left(1-\tau_t^w\right) u_{1,t}^w}$$

$$w_t = \left(1-\phi\right) \frac{u_{2,t}^w}{\left(1-\tau_t^w\right) u_{1,t}^w} + \phi Y_{2,t}^f.$$

3.A.6 DE conditions

The DE conditions consist of:

$$C_t^k : \frac{1}{\left(C_t^k\right)^\sigma} = \lambda_t^1$$

$$K_{t+1}^k : \beta\lambda_{t+1}^1 R_{t+1} = \lambda_t^1$$

$$C_t^w : \frac{1}{\left(C_t^w\right)^\sigma} = \lambda_t^3$$

$$s_t : \xi\left(e_t + s_t\right)^{\mu-1} = p_t\lambda_t^2 + \lambda_t^3 \overline{G}_t^u$$

$$e_{t+1} : \beta\xi\left(e_{t+1} + s_{t+1}\right)^{\mu-1} = -\lambda_t^2 + \beta\left[\lambda_{t+1}^2\left(1-\gamma\right) + \lambda_{t+1}^3\left(1-\tau_t^w\right)w_{t+1}\right]$$

$$K_t^f : r_t = \alpha\frac{Y_t^f}{K_t^f}$$

$$v_t : \nu = \lambda_t^4 q_t$$

$$L_{t+1}^f : R_{t+1}^{-1}\left[\left(1-\alpha\right)\frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1}\right] = \lambda_t^4 - R_{t+1}^{-1}\lambda_{t+1}^4\left(1-\gamma\right)$$

$$w_t : w_t = \left(1-\phi\right)\frac{U_{2,t}^w}{\left(1-\tau_t^w\right)U_{1,t}^w} + \phi Y_{2,t}^f$$

$$e_t : e_{t+1} = p_t s_t + \left(1-\gamma\right)e_t$$

$$BC^w : C_t^w = \left(1-\tau_t^w\right)w_t e_t + \overline{G}_t^u s_t + \overline{G}_t^t$$

$$BC^g : \overline{G}_t^t + n^w\overline{G}_t^u s_t = n^k\tau_t^k\left(r_t - \delta\right)K_t^k + n^k\tau_t^k\pi_t^k + n^w\tau_t^w w_t e_t$$

$$RC : n^k Y_t^f = n^k C_t^k + n^w C_t^w + n^k\left(K_{t+1}^k - \left(1-\delta\right)K_t^k\right) + n^k\nu v_t$$

$$MC_K : K_t^k = K_t^f$$

$$MC_\pi : \pi_t^k = \pi_t^f$$

$$MC_L : n^w e_t = n^k L_t^f$$

where λ_t^1 refers to the Lagrangian multiplier from the capitalist's problem; λ_t^2 and λ_t^3 refer to the Lagrangian multipliers from the worker's problem; λ_t^4 refers to the Lagrangian multiplier from the firm's problem; $Y_t^f = A\left(K_t^f\right)^\alpha\left(L_t^f\right)^{1-\alpha}$; $\pi_t^f = Y_t^f - r_t K_t^f - w_t L_t^f - \nu v_t$; $R_{t+1} = 1 - \delta + r_{t+1} - \tau_{t+1}^k\left(r_{t+1} - \delta\right)$; and $\overline{G}_t^u = \overline{r}_t w_t$.

3.A.7 Ramsey approach to optimal taxation

The optimization of Ramsey problem can be summarized as follows:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[n^k u_t^k + n^w u_t^w \right] \tag{131}$$

subject to the DE conditions above, taking $\left\{ \overline{G}_t^t \right\}_{t=0}^{\infty}$, and initial conditions for K_0^k, e_0 and L_0^f as given.[86] We substitute out multipliers, $\lambda_t^1 - \lambda_t^4$, by making use of some DE conditions. As a result, the Lagrangian function of the government can be written as:

$$\begin{aligned}
L^g = & E_0 \sum_{t=1}^{\infty} \beta^t \left\{ \left[n^k \left(\frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma} \right) + n^w \left(\frac{\left(C_t^w\right)^{1-\sigma}}{1-\sigma} - \xi \frac{(e_t + s_t)^{\mu}}{\mu} \right) \right] + \right. \\
& + \psi_t^1 \left[\frac{1}{\left(C_t^k\right)^{\sigma}} - \beta \frac{1}{\left(C_{t+1}^k\right)^{\sigma}} \left[1 - \delta + r_{t+1} - \tau_{t+1}^k \left(r_{t+1} - \delta \right) \right] \right] + \\
& + \psi_t^2 \left[\beta \xi \left(e_{t+1} + s_{t+1} \right)^{\mu-1} + \frac{\xi \left(e_t + s_t \right)^{\mu-1}}{p_t} - \right. \\
& - \frac{\overline{r}_t w_t}{\left(C_t^w\right)^{\sigma} p_t} - \beta \left(\frac{(1-\gamma) \xi \left(e_{t+1} + s_{t+1} \right)^{\mu-1}}{p_{t+1}} - \right. \\
& \left. \left. - \frac{(1-\gamma) \overline{r}_{t+1} w_{t+1}}{\left(C_{t+1}^w\right)^{\sigma} p_{t+1}} + \frac{1}{\left(C_{t+1}^w\right)^{\sigma}} \left(1 - \tau_{t+1}^w \right) w_{t+1} \right) \right] + \\
& + \psi_t^3 \left[\frac{\nu}{q_t} - R_{t+1}^{-1} \left((1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} + \frac{\nu (1-\gamma)}{q_{t+1}} \right) \right] + \\
& + \psi_t^4 \left[w_t - (1-\phi) \frac{\xi \left(e_t + s_t \right)^{\mu-1}}{\left(1 - \tau_t^w \right) \left(C_t^w\right)^{\sigma}} - \phi (1-\alpha) \frac{Y_t^f}{L_t^f} \right] + \\
& + \psi_t^5 \left[p_t s_t + (1-\gamma) e_t - e_{t+1} \right] + \\
& + \psi_t^6 \left[\left(1 - \tau_t^w \right) w_t e_t + \overline{r}_t w_t s_t + \overline{G}_t^t - C_t^w \right] + \\
& + \psi_t^7 \left[n^k \tau_t^k \left(r_t - \delta \right) K_t^k + n^k \tau_t^k \pi_t^k + n^w \tau_t^w w_t e_t - \overline{G}_t^t + n^w \overline{r}_t w_t s_t \right] + \\
& \left. + \psi_t^8 \left[n^k Y_t^f - n^k C_t^k - n^w C_t^w - n^k \left(K_{t+1}^k - (1-\delta) K_t^k \right) - n^k \nu v_t \right] \right\}
\end{aligned}$$

where $Y_t^f = A \left(K_t^f\right)^{\alpha} \left(L_t^f\right)^{1-\alpha}$; $K_t^f = K_t^k$; $L_t^f = \frac{n^w}{n^k} e_t$; $r_t = \alpha \frac{Y_t^f}{K_t^f}$; $\pi_t^k = Y_t^f - r_t K_t^f - w_t L_t^f - \nu v_t$; and ψ_t^i, $i = 1, 2, \cdots, 8$, denotes the multiplier associated with each constraint. The choice variables of the government are $\left\{ C_t^k, K_{t+1}^k, C_t^w, s_t, e_{t+1}, v_t, w_t, \tau_t^k, \tau_t^w, \overline{r}_t \right\}_{t=0}^{\infty}$. Note that we consider the Ramsey problem starting from time 1 and assume that time 0 optimality conditions do not alter the results in equilibrium. This is because some FOCs of the government at time 0 are different from the same rules governing behav-

[86] $\overline{G}_t^t$ is fixed at 0.164 as in the case with policy given.

ior from time 1 on. Specifically, these include the FOCs of values appearing in the forward-looking intertemporal constraints. The FOCs of the government should also include the eight constraints to the Ramsey problem.[87]

[87] We did not show the FOCs of the government in the text to preserve space but they are available upon request.

Summary and conclusions

The thesis is composed of three inter-related chapters studying the effects of both exogenous and optimal fiscal policy in heterogeneous agent models incorporating unemployment. These include models of equilibrium unemployment, right-to-manage union bargaining, and search and matching in three chapters, respectively. Households in the thesis are distinguished by their different economic roles, namely capitalists and workers. Specifically, capitalists are assumed not to work and workers by assumption cannot save in Chapter 1 and Chapter 3. In Chapter 2 both capitalists and workers can work and save, but they differ in their capital holdings. The heterogeneous agent setup allows us to investigate the distributional effects of policy, and to study inequality issue which arises from the conflict of interests between agents. In addition, the models in three chapters incorporate different labour and product market failures. The analysis undertaken informs us of the importance of market failures in fiscal policy studies.

Chapter 1 studied optimal taxation under commitment and its effects on unemployment, the distribution of income and the welfare of agents. Unemployment was generated in perfectly competitive labour markets as the outcome of optimal choices made by the workers. The workers did not save and the capitalists did not work in the economy. Competitive firms produced by using a constant-returns-to-scale technology by employing capital and labour. The equilibrium profits were zero. The government taxed labour income and interest income from capital and profits. The government expenditures included utility-enhancing public consumption and unemployment benefits. As a model extension, productive public investment was introduced in the modified model. In this case, equilibrium profits were strictly positive. The relevance of model profits in determining the optimal burden of taxes was also examined.

First, it is found that, in the model with zero profits, it is optimal for the government to implement a zero tax rate on capital income in the long-run. The long-run optimal replacement rate becomes negative. It is equivalent to a subsidy to labour supply of workers, while the explicit tax rate on labour income increases. The optimal taxation eliminates the labour wedge and labour supply increases in the long run. There are welfare gains for all agents. It implies that the optimal taxation is Pareto-improving. All these results hold when the government attaches different weights to welfare of agents, so that there is no conflict of interests between agents. Second, in the model with profits as a result of the presence of productive public investment, it is optimal for the government to subsidize capital income of capitalists and tax the leisure of workers in the long run. The long-run labour wedge cannot be eliminated. As a result, the welfare of workers worsens despite welfare gains for the capitalists. Finally, the optimal taxation has the redistributional effects when the weight attached to the welfare of capitalists in government's objective function exceeds a critical value, so that there is a trade-off between efficiency and equity for the government.

Chapter 2 examined the relevance of distortions in labour and product markets in determining the welfare effects of capital tax cut associated with labour tax increase. The households were distinguished by differences in their capital holdings. The labour market distortion appeared in a right-to-manage bargaining setup in which unions and firms bargained over the wage rate with the aim of maximising a weighted average of labour income and profits. In the monopolistically competitive product market, intermediate goods producers earned non-zero economic profits due to their market power. The final goods producers, in contrast, were perfectly competitive. The government taxed capital income, including interest on savings and profits, and labour income by using two different tax rates to finance unemployment benefits and non-employment related public transfers.

We first find, in the model with labour market distortion only, a tax reform that implements a zero capital tax results in welfare losses for the workers despite welfare gains for the capitalists and the aggregate economy as well. The unemployment channel is found to be the critical link that modifies the results from the model without market distortions. In particular, although the labour productivity and after-tax wage increase, the workers cannot capture the full benefit of this as unemployment also increases. Second, in the model with product market distortion only, the tax reform leads to welfare gains for the capitalists but welfare losses for the workers and the aggregate economy. This happens because the government has to forego revenue from a non-distortionary tax base comprised of profits. Hence, the required increase in labour tax is larger and thus the net wage falls. However, when these two distortions are combined in a realistic calibration to the current UK economy, employment increases and there are welfare gains for all agents. These results imply that the tax reform becomes Pareto improving. Consistent with the theory of second-best, the two distortions work to correct the negative distributional effects of a capital tax cut that each other, on its own, creates.

Chapter 3 augmented a heterogeneous agent general equilibrium model with a Mortensen-Pissarides search-and-matching technology. The wage paid in any given job was determined via a Nash bargaining between a pairing of matched worker and firm. The matching was of Cobb-Douglas form. The bargain was over the wage rate to maximise a weighted average of worker's and firm's surpluses. If the bargaining was successful, the worker was employed in the following period. Thus, employment, at any given period of time, was predetermined. The capitalists did not work and the workers did not save. The time endowment of workers was split into working, seeking working opportunities and leisure. The firms, in each period, opened new job vacancies and employed capital and labour to produce goods using a constant-returns-to-scale technology. Finally, the government taxed capitalists and workers to finance the public spending. Using this framework, the importance of search frictions in determining the welfare and distributional effects of tax reforms which re-allocated the tax burden from capital to labour income was examined. In addition, the optimal policy under commitment by assuming that the government optimally chose all the tax rates in the economy was studied.

Using a realistic calibration to the UK economy, we find that the tax reform is Pareto improving but increases inequality in the long run, despite welfare losses for at least one segment of the population over the transition. These results also hold when the relative bargaining power of workers and the unemployment benefit structure are varied, respectively. However, the welfare gains will be higher for all agents as workers' bargaining power falls or when unemployment benefits positively depend on past wages. It is finally demonstrated that optimal policy under commitment implies a negative capital tax in the long run which can help to correct the market frictions caused by the forward-looking behavior of firms. Meanwhile, the government increases the labour tax and reduces the unemployment benefits in order to generate enough revenues to finance public spending. As a result, the firms open more job vacancies and employment increases as well. In this case, there are welfare gains for all agents in the long run.

This thesis contributes to the growing literature on the general equilibrium analysis of fiscal policy. It makes clear that the structure of the economy, especially the distortions in the markets, are important in determining not only the effects of tax reforms but also the choice of optimal policy. Therefore, the relevant market failures should be taken into account in policy decision making. Each of the three chapters explains a different channel through which the fiscal policy can impact on the allocation of resources and therefore welfare of agents in the economy. The analysis is constructed by assuming different labour and product failures. This thesis further analyses the distributional effects of fiscal policy by assuming the household heterogeneity. Household are grouped with regards to their working and saving propensities. We intend to explore the redistributive potential of fiscal policy in the economy. The results show that the tax reform considered is Pareto improving in the long run although it has redistributional effects in the transition period. Our studies suggest that all the agents will benefit from the capital tax cut which is accomplished by the labour tax increase despite welfare losses for the agents relying on labour income in the transition period. This result stands even if we assume different market distortions in the economy.

Closing remarks

This thesis employed several models with different unemployment setups to study the effects of tax reforms and the determination of optimal taxation. Although the analysis undertaken contributed to overcoming some of the theoretical gaps existing in the literature, there are further areas where more research could be conducted. These are summarised point by point as follows.

Chapters 2 and 3 investigated the welfare and distributional effects of cutting capital income tax. In the absence of public debt, the government needs to increase labour income tax concurrently in order to retain a balanced budget in each period. This tax reform assumption makes our analysis comparable with the optimal taxation literature by Judd (1985) and Chamley (1986). In their studies, the government optimally chooses the capital and labour income tax rates. However, given that the government has two other balancing instruments in its budget constraint, i.e. government transfer and unemployment benefit, one interesting exercise in the future research could be to reduce the unemployment benefit to fulfil the capital tax cut. Since the change in unemployment benefit would have a big effect on the employment. In Chapter 2, it represents the outside option for the union in the bargaining. If unemployment benefit is reduced by the government, the outside option for the union becomes less attractive and therefore the relative bargaining power of the union is reduced. This in turn generates a positive effect on the employment. In Chapter 3, the reduction in unemployment benefit has a direct effect on the search-unemployment of workers which can be seen in the optimization problem of workers. Specifically, search-unemployment falls as unemployment benefit is reduced.

In the future research, the expenditure tax can be introduced as the model extension. Instead of applying a tax based on the income earned, the expenditure tax is levied based on the amount of spending of households. Its advantage is that the tax can eliminate the adverse effects of the income tax on the investment and saving incentives of households. This new tax form is expected to bring many interesting findings to the models in terms of studies on the effects of tax reforms and the determination of optimal fiscal policy.

The agent heterogeneity setup in this thesis was only restricted to the different working and saving abilities of agents. The heterogeneity in the labour markets was ignored. In practice, many other types of agent heterogeneity can be considered in the future research. For example, the workers are assumed to have heterogeneous preferences towards working and leisure. This is reflected by the different elasticity of labour force participation rate in the utility function. This new assumption is particularly useful for the analysis of welfare distribution within the working population. Another extension is to introduce the skill heterogeneity of the working population. In particular, the workers are assumed to be able to provide either skilled labour or unskilled labour to the firms. These two different labours can contribute to the production of firms. But they are paid different wages. Therefore, the issue of wage premium can be analysed in the research.

In Chapter 1, the government consumption acted as a perfect substitute for the private consumption and it provided direct utility for the households. One alternative specification of the utility function of households could be the imperfect substitutability between private and government consumption. In other words, these two components no longer have a linear relationship in the utility function. This in effect could generate some interesting results regarding the impacts of TFP and fiscal policy shocks, as the government consumption is assumed to be the residual variable for the government to ensure that the government budget constraint is balanced at any given period of time. The change in any other exogenous variable will be fulfilled by the change in government consumption. Different substitutabilities with private consumption create different channels via which private consumption is crowed out in the post-shock economy. This is crucial for the analysis of welfare of agents.

When the optimal policy was analysed in the thesis, the government was assumed to only choose the tax rates. Another experiment would be to allow the government to optimise the public expenditure alongside the tax instruments in its Ramsey setup. We should expect the government to pick a different best policy which can yield the highest aggregate welfare in the economy when more policy variables are allowed to be optimally chosen. In turn, the new optimal policy implies a different equilibrium allocation in the private sectors. One interesting thing to be looked at would be the changes in the welfare of different agents compared to the previous Ramsey setup.

In Chapter 3, we studied both the long-run and transitional effects of eliminating capital income tax accompanied by a concurrent labour tax increase. We found that the tax reform considered was Pareto-improving in the long-run but it hurt the agents whose income reply on labour income in the short-run. However, the focus of Chapter 2 in this thesis was the study of welfare and distributional effects of the tax reform in the long-run. We aimed to analyse the relevance and importance of labour and product market distortions in the tax reform across four different models. It would be interesting to investigate the transitional effects on the welfare of agents as an extension to Chapter 2. It is expected that the tax reform will hurt the workers in the short-run as in Chapter 3.

In Chapter 1, the utility function of households exhibited constant elasticity of substitution (CES) between capital and labour. The constant elasticity of substitution was set to be 2 in the calibration. But we can consider other values for this parameter. For example, if it approaches 1 in the case of unit elasticity of substitution, in the limit we get the Cobb-Douglas utility function which is the special case of the CES utility function. This in effect will affect the transitional path of labour supply in response to the exogenous shocks. In this case, the substitution effect resulting from a wage rate change should get closer to the income effect as the elasticity of substitution approaches 1. In turn, the dynamics of wage income and welfare of workers are affected.

In this thesis, we assumed that the government could not distinguish between return to capital stock and profits of firms so that both types of income were taxed at the same rate. One interesting exercise to be tried is to release this assumption. In other words, the government is assumed to be able to tax the return to capital stock separately from the profits of firms. In this case, a new tax instrument of profit tax will be introduced in the model. This new assumption is expected to deliver some new results in terms of optimal policy study. Specifically, the government would have the incentive to confiscate all the profits of firms, as the profit tax is non-distortionary. As a result, the optimal tax rate on capital stock could become even smaller compared to the previous findings. It is because the government gains a great amount of tax revenue from a non-distortionary tax base comprised of firm profits.

A final possible extension could be to broaden the model to an open economy model. A recent paper by Correia (2010) shows that the assumption of the open economy is crucial for the analysis of distributional consequences of abolishing capital income taxation. Thus, we can assume that the capitalists can invest in both domestic and foreign assets markets when assessing the effects of cutting capital income tax. This would create another income resource for the capitalists and also more tax revenue for the government. In this case, when the government reduces the capital income tax, there could be higher addition to the labour income tax. Therefore, the tax reform considered implies a larger effect on the unemployment.

References

[1] Aghion, P. and Howitt, P. (2009). *The Economics of Growth*, The MIT Press.

[2] Ahmed, S. (1986). 'Temporary and permanent government spending in an open economy: some evidence for the UK', *Journal of Monetary Economics*, vol. 17, pp. 197-224.

[3] Ai, C. and Cassou, S. (1995). 'A normative analysis of public capital', *Applied Economics,* vol. 27, pp. 1201-1209.

[4] Ambler, S. and Paquet, A. (1996). 'Fiscal spending shocks, endogenous government spending, and real business cycles', *Journal of Economic Dynamics and Control*, vol. 20, pp. 237-256.

[5] Andolfatto, D. (1996). 'Business cycles and labor-market search', *American Economic Review*, vol. 86, pp. 112-132.

[6] Angelopoulos, K., Malley, J. and Philippopoulos, A. (2011a). 'Time-consistent fiscal policy under heterogeneity: Conflicting or common interests?', CESifo Working Paper Series: 3444.

[7] Angelopoulos, K., Fernandez, B. and Malley, J. (2011b). 'The distributional consequences of supply-side reforms in general equilibrium', CESifo Working Paper Series: 3504.

[8] Ardagna, S. (2001). 'Fiscal policy composition, public debt, and economic activity', *Public Choice*, vol. 109, pp. 301-325.

[9] Ardagna, S. (2007). 'Fiscal policy in unionized labor markets', *Journal of Economic Dynamics and Control*, vol. 31, pp. 1498-1534.

[10] Arseneau, D. and Chugh, S. (2008). 'Optimal fiscal and monetary policy with costly wage bargaining', *Journal of Monetary Economics*, vol. 55, pp. 1401-1414.

[11] Arseneau, D. and Chugh, S. (2009). 'Tax smoothing in frictional labor markets', *International Finance Discussion Papers*: 2009-965.

[12] Aschauer, A. (1985). 'Fiscal policy and aggregate demand', *American Economic Review*, vol. 75, pp. 117-127.

[13] Aschauer, A. (1989). 'Is public expenditure productive?', *Journal of Monetary Economics*, vol. 23, pp. 177-200.

[14] Aschauer, A. and Greenwood, J. (1985). 'Macroeconomic effects of fiscal policy', *Carnegie-Rochester Conference Series on Public Policy*, vol. 23, pp. 91-138.

[15] Atkinson, A. and Stiglitz, J. (1980). *Lectures on Public Economics*, Mc-Graw Hill, London.

[16] Baier, S. and Glomm, G. (2001). 'Long-run growth and welfare effects of public policies with distortionary taxation', *Journal of Economic Dynamics and Control*, vol. 25, pp. 2007-2042.

[17] Barro, J. (1981). 'Output effects of government purchases', *Journal of Political Economy*, vol. 89, pp. 1086-1121.

[18] Barro, J. (1989). 'The neoclassical approach to fiscal policy', *Modern Business Cycle Theory*, Harvard University Press, Cambridge, Massachusetts, pp. 178-235.

[19] Benabou, R. (1996). 'Heterogeneity, stratification, and growth: macroeconomic implications of community structure and school finance', *American Economic Review*, vol. 86, pp. 584-609.

[20] Benhabib, J. and Farmer, R. (1994). 'Indeterminacy and increasing returns', *Journal of Economic Theory*, vol. 63, pp. 19-41.

[21] Blanchard, O. and Diamond, P. (1989). 'The beveridge curve', *Brookings Papers on Economic Activity*, vol. 1, pp. 1-60.

[22] Blanchard, O. and Katz, L. (1999). 'Wage dynamics: reconciling theory and evidence', *American Economic Review*, vol. 89, pp. 69-74.

[23] Blanchard, O. and Giavazzi, F. (2003). 'Macroeconomic effects of regulation and deregulation in goods and labor markets', *The Quarterly Journal of Economics*, vol. 118, pp. 879-907.

[24] Boone, J. and Bovenberg, L. (2002). 'Optimal labour taxation and search', *Journal of Public Economics*, vol. 85, pp. 53-97.

[25] Burnside, C., Eichenbaum, M., and Fisher, J. (2003). 'Fiscal shocks and their consequences', *Journal of Economic Theory*, vol. 115, pp. 89-117.

[26] Chamley, C. (1986). 'Optimal taxation of capital income in general equilibrium with infinite lives', *Econometrica*, vol. 54, pp. 607-622.

[27] Chari, V., Lawrence, C and Kehoe, P. (1994). 'Optimal fiscal policy in a business cycle model', *Journal of Political Economy*, vol. 102, pp. 617-652.

[28] Chari, V. and Kehoe, P. (1999). 'Optimal fiscal and monetary policy', in J.B. Taylor & M. Woodford (ed.), *Handbook of Macroeconomics*. Elsevier 1(26), 1671-1745.

[29] Chari, V., Kehoe, P. and McGrattan, E. (2002). 'Accounting for the great depression', *American Economic Review*, vol. 92, pp. 22-27.

[30] Chari, V., Kehoe, P. and McGrattan, E. (2007a). 'Business cycle accounting', *Econometrica*, vol. 75, pp. 781—836.

[31] Chari, V., Kehoe, P. and McGrattan, E. (2007b). 'Comparing alternative representations and alternative methodologies in business cycle accounting', Federal Reserve Bank of Minneapolis Staff Report 3

[32] Chéron, A. and Langot, F. (2010). 'On-the-job search equilibrium with endogenous unemployment benefits', *Labour Economics*, vol. 17, pp. 383-391.84.

[33] Chugh, S. (2006). 'Optimal fiscal and monetary policy with sticky wages and sticky prices', *Review of Economic Dynamics*, vol. 9, pp. 683-714.

[34] Conesa, J., Kehoe, T. and Ruhl, K. (2007). 'Modeling great depressions: the depression in Finland in the 1990s', *Quarterly Review*, Federal Reserve Bank of Minneapolis, Nov., pp. 16-44.

[35] Conesa, J., Kitao, S. and Krueger, D. (2009). 'Taxing capital? not a bad idea after all!', *American Economic Review*, vol. 99, pp. 25-48.

[36] Correia, I. (2010). 'Taxation and globalization?', Bank of Portugal Working Paper Series: 201020.

[37] Cooley, T. and Hansen, G. (1992). 'Tax distortions in a neoclassical monetary economy', *Journal of Economic Theory*, vol. 58, pp. 290-316.

[38] Daveri, F. and Maffezzoli, M. (2001). 'A numerical approach to fiscal policy, unemployment, and growth in Europe', Econometrics and Applied Economics Working Paper, Istituto di Economia Politica, Bocconi University, No. 4.

[39] Dixit, A. and Stiglitz, J. (1977). 'Monopolistic competition and optimum product diversity', *American Economic Review*, vol. 67, pp. 297-308.

[40] Domeij, D. and Heathcote, J. (2004). 'On the distributional effects of reducing capital taxes', *International Economic Review*, vol. 45, pp. 523-554.

[41] Domeij, D. (2005). 'Optimal capital taxation and labour market search', *Review of Economic Dynamics*, vol. 8, pp. 623-650.

[42] Farber, H. (1986). 'The analysis of union behavior', NBER Working Papers Series: 1502.

[43] Finn, M. (1998). 'Cyclical effects of government's employment and goods purchases', *International Economic Review*, vol. 39, pp. 635-657.

[44] Gali, J., López-Salido, J. and Vallés, J. (2007). 'Understanding the effects of government spending on consumption', *Journal of the European Economic Association*, vol. 5, pp. 227-270.

[45] Galor, O. and Zeira, J. (1993). 'Income distribution and macroeconomics', *Review of Economic Studies*, vol. 60, pp. 35-52.

[46] Garcia-Milà, T., Marcet, A. and Ventura, E. (2010). 'Supply side interventions and redistribution', *Economic Journal*, vol. 120, pp. 105-130.

[47] Giannitsarou, C. (2006). 'Supply-side reforms and learning dynamics', *Journal of Monetary Economics*, vol. 53, pp. 291-309.

[48] Guo, J. and Lansing, K (1999). 'Optimal taxation of capital income with Impefectly competitive product markets', *Journal of Economic Dynamics and Control*, vol. 23, pp. 967-995.

[49] Hagedorn, M (2010). 'Ramsey tax cycles', *Review of Economic Studies,* vol. 77, pp. 1042-1077.

[50] Hall, R. and Milgrom, P. (2008). 'The limited influence of unemployment on the wage bargain', *American Economic Review*, vol. 98, pp. 1653–1674.

[51] Hansen, G. (1985). 'Indivisible labor and business cycles', *Journal of Monetary Economics*, vol. 16, pp. 309-327.

[52] Hosios, A. (1990). 'On the efficiency of matching and related models of search and unemployment', *Review of Economic Studies*, vol. 57, 279-298.

[53] Judd, K. (1985). 'Redistributive taxation in a simple perfect foresight model', *Journal of Public Economics*, vol. 28, pp. 59-83.

[54] Judd, K. (1989). 'Optimal taxation in dynamic stochastic economies: theory and evidence', manuscript, Hoover Institution, Stanford University.

[55] Judd, K. (1997). 'The optimal tax on capital income is negative', NBER Working Paper, No. 6004.

[56] Judd, K. (1999). 'Optimal taxation and spending in general competitive growth models', *Journal of Public Economics*, vol. 71, pp. 1-26.

[57] Judd, K. (2002). 'Capital income taxation with imperfect competition', *American Economic Review*, vol. 92, pp. 417-421.

[58] Kass, L. and von Thadden, L. (2004). 'Budgetary policy and unemployment dynamics in an OLG model with collective bargaining', *Economic Journal*, vol. 114, pp. 867-889.

[59] Klein, P. (2000). 'Using the generalized schur form to solve a multivariate linear rational expectations model', *Journal of Economic Dynamics and Control*, vol. 24, pp. 1405-1423.

[60] Klein, P. and Rios-Rull, J.-V. (2003). 'Time-consistent optimal fiscal policy', *International Economic Review*, vol. 44, pp. 1217-1245.

[61] Klein, P., Krusell, P. and Ríos-Rull, J. (2008). 'Time-consistent public policy', *Review of Economic Studies*, vol. 75, pp. 789-808.

[62] Kormendi, R. (1983). 'Government debt, government spending, and private sector behavior', *American Economics Review*, vol. 73, pp. 994-1010.

[63] Koskela, E. and von Thadden, L. (2008). 'Optimal factor taxation under wage bargaining: a dynamic perspective', *German Economic Review*, vol. 9, pp. 135-159.

[64] Krusell, P. (2002). 'Time-consistent redistribution', *European Economic Review*, vol. 46, pp. 755-769.

[65] Krusell, P., Mukoyama, T. and Sahin, A. (2010). 'Labour-market matching with precautionary savings and aggregate fluctuations', *Review of Economic Studies*, vol. 77, pp. 1477-1507.

[66] Lansing, K. (1998). 'Optimal fiscal policy in a business cycle model with public capital', *Canadian Journal of Economics*, vol. 31, pp. 337-364.

[67] Lansing, K. (1999). 'Optimal redistributive capital taxation in a neoclassical growth model', *Journal of Public Economics,* vol. 73, pp. 423-453.

[68] Leith, C. and Malley, J. (2005). 'Estimated general equilibrium models for the evaluation of monetary policy in the US and Europe', *European Economic Review,* vol. 49, pp. 2137–2159.

[69] Ljungqvist, L. and Sargent, T. (2006). 'Do taxes explain European employment? Indivisible labor, human capital, lotteries, and savings', *NBER Macroeconomics Annual*, vol. 21, pp. 181-246.

[70] Lucas, R. (1990). 'Supply-side economics: an analytical review', *Oxford Economic Papers*, vol. 42, pp. 293-316.

[71] Maffezzoli, M. (2001). 'Non-walrasian labor markets and real business cycles', *Review of Economic Dynamics,* vol. 4, pp. 860-892.

[72] Malley, J., Philippopoulos, A. and Woitek, U. (2009) 'To react or not? Technology shocks, fiscal policy and welfare in the EU-3. *European Economic Review*, vol. 53, pp. 689-714.

[73] Martinez-Mongay, C. (2000). 'ECFIN's effective tax rates, properties and comparisons with other tax indicators', European Economy - Economic Papers Series: 146.

[74] McGrattan, E. (1994). 'The macroeconomic effects of distortionary taxation', *Journal of Monetary Economics*, vol. 33, pp. 573-601.

[75] Mirrlees, J., Adam, S. , Besley, T., Blundell, R., Bond, S., Chote, R., Gammie, M., Johnson, P., Myles, G. and Poterba, J. (eds) (2010). *Dimensions of Tax Design: the Mirrlees Review*, Oxford University Press.

[76] Mirrlees, J., Adam, S. , Besley, T., Blundell, R., Bond, S., Chote, R., Gammie, M., Johnson, P., Myles, G. and Poterba, J. (2011). *Tax by Design: the Mirrlees Review*, Oxford University Press.

[77] Merz, M. (1995). 'Search in the labor market and the real business cycle', *Journal of Monetary Economics*, vol. 36, pp. 269-300.

[78] Mortensen, D. (1982). 'Property rights and efficiency in matching, racing, and related games', *American Economic Review*, vol. 72, pp. 968-979.

[79] Mortensen, D. and Pissarides, C. (1994). 'Job creation and job destruction in the theory of unemployment', *Review of Economic Studies*, vol. 61, pp. 397-415.

[80] Mortensen, D. and Wright, R. (2002) 'Competitive pricing and efficiency in search equilibrium', *International Economics Review*, vol. 43, pp. 1-20.

[81] Munnell, A. (1990). 'Why has productivity growth declined? productivity and public investment', Federal Reserve Bank of Boston, *New England Economic Review*, January/February, 3-22.

[82] Nickell, S. and Andrews, D. (1983). 'Unions, real wages, and employment in Britain 1951-79', *Oxford Economic Papers*, vol. 35, pp. 183-206.

[83] Nickell, S. and Nunziata, L. (2001). *Labour Market Institutions Database.*

[84] OECD (2004). 'Wage-setting institutions and outcomes', *Employment Outlook*, Chapter 3, pp. 127-181.

[85] Ohanian, L. (1997). 'The macroeconomic effects of war finance in the United States: World War II and the Korean War', *American Economic Review*, vol. 871, pp. 23-40.

[86] Pissarides, C. (1986). 'Vacancies and unemployment in Britain', *Economic Policy*, vol. 3, pp. 499-559.

[87] Pissarides, C. (1998). 'The impact of employment tax cuts on unemployment and wages; the role of unemployment benefits and tax structure', *European Economic Review*, vol. 42, pp. 155-183.

[88] Pissarides, C. (2001). *Equilibrium unemployment theory*, 2nd ed. Cambridge, MA: MIT Press.

[89] Pissarides, C. (2006). 'Unemployment in Britain: a European success story', in: Werding, Martin, (ed.) Structural unemployment in Western Europe: reasons and remedies, CESifo seminar series, MIT Press, Cambridge, Mass., pp. 209-236.

[90] Pissarides, C. (2009). 'The unemployment volatility puzzle: is wage stickiness the answer?', *Econometrica*, vol. 77, pp. 1339-1369.

[91] Prescott, E. (2002). 'Prosperity and depression', *American Economic Review*, vol. 92, pp. 1-15.

[92] Prescott, E. (2004). 'Why do Americans work so much more than Europeans?', *Quarterly Review*, Federal Reserve Bank of Minneapolis, issue July, pp. 2-13.

[93] Rogerson, R. (1988). 'Indivisible labor, lotteries and equilibrium', *Journal of Monetary Economics*, vol. 21, pp. 3-16.

[94] Rogerson, R., Shimer, R. and Wright, R. (2005). 'Search-theoretic models of the labor market: a survey', *Journal of Economic Literature*, vol. 43, pp. 959-988.

[95] Schmitt-Grohe, S. and Uribe, M. (2006). 'Optimal fiscal and monetary policy in a medium-scale macroeconomic model', *NBER Macroeconomics Annual 2005*, vol. 20, MIT Press.

[96] Schweitzer, M. and Tinsley, D. (2004), 'The UK labour force participation rate: business cycle and trend influences', *Bank of England Quarterly Bulletin*, vol. 44, pp. 329.

[97] Shi, S. and Wen, Q. (1997). 'Labour market search and capital accumulation: some analytical results', *Journal of Economic Dynamics and Control*, vol. 21, pp. 1747-1776.

[98] Shi, S. and Wen, Q. (1999). 'Labour market search and dynamic effects of taxes and subsidies', *Journal of Monetary Economics*, vol. 43, pp. 457-495.

[99] Shimer, R. (2005). 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review*, vol. 95, pp. 25–49.

[100] Shimer, R. (2009). 'Convergence in Macroeconomics: the labor wedge', *American Economic Journal: Macroeconomics*, vol. 1, pp. 280-297.

[101] Yashiv, E. (2007). 'Labor search and matching in macroeconomics', *European Economic Review,* vol. 51, pp. 1859-1895.