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A Survey of the Methods on Fingerprint Orientation Field Estimation

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ABSTRACT Fingerprint orientation field (FOF) estimation plays a key role in enhancing the performance of the automated fingerprint identification system (AFIS): accurate estimation of FOF can evidently improve the performance of AFIS. However, despite the enormous attention on the FOF estimation research in the past decades, the accurate estimation of FOFs, especially for poor-quality fingerprints, still remains a challenging task. In this paper, we devote to review and categorization of the large number of FOF estimation methods proposed in the specialized literature, with particular attention to the most recent work in this area. Broadly speaking, the existing FOF estimation methods can be grouped into three categories: gradient-based methods, mathematical models-based methods, and learning-based methods. Identifying and explaining the advantages and limitations of these FOF estimation methods is of fundamental importance for fingerprint identification, because only a full understanding of the nature of these methods can shed light on the most essential issues for FOF estimation. In this paper, we make a comprehensive discussion and analysis of these methods concerning their advantages and limitations. We have also conducted experiments using publically available competition dataset to effectively compare the performance of the most relevant algorithms and methods.

INDEX TERMS Fingerprint identification, fingerprint orientation field estimation, sparse coding, dictionary learning, convolutional neural networks.

I. INTRODUCTION

A number of biometric technologies have been developed and several of them have been successfully deployed commercially. Among these, fingerprint is the most commonly used approach. In fact, fingerprints and biometrics are often considered as synonyms! Fingerprints recognition can be tracked back over 100 years when they were first introduced as a method for person identification [1]. With the continuous growing demand on security, fingerprint recognition technology has been deployed in a wide range of applications, such as smartcards, commercial services, airport security and automated banking. In recent decades, the research has been focused mainly on development of automatic fingerprint identification systems (AFIS). In general, the AFIS works by extracting and matching fingerprint minutiae [2], [3]. A typical AFIS consists of fingerprint acquisition, fingerprint preprocessing (such as fingerprint segmentation, orientation field estimation and enhancement), fingerprint classification, minutiae detection and matching [4].

There is a common misconception that automatic fingerprint recognition is a fully solved problem since the AFIS have been around for decades. However, fingerprint recognition remains as a challenging and important pattern recognition problem due to its large intra-class variability and inter-class similarity in fingerprint patterns [1]. Furthermore, automatic fingerprint identification has to acquire reliable matching features from fingerprint images with poor quality, which are degraded by various factors such as dirt, scar, greasing and moisture on the surface of fingertips.

Fingerprints features as alternated ridges and valleys where the former are foreground and the latter are background.

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In general, ridge and valley flows are of a local constant orientation. So, the orientations, formally fingerprint orientations, are often used to describe the ridge flow patterns, providing useful features for further fingerprint processing and recognition [5], [6]. Although significant advances have been achieved for extracting fingerprint orientation field (FOF) in literatures, it is still challenging to reliably estimate the orientations of poor quality and latent fingerprints, which are usually caused by unclear ridge structure and various overlapping patterns. Due to its inherent global and reliable nature, the FOF plays a very important role in the areas of fingerprint segmentation [7]–[10], enhancement [11]–[20], singularity detection [21]–[25], classification [26]–[32], and matching [33]–[42]. Errors in computing the FOF propagate through all the stages of the AFIS. In particular, errors in estimation of the FOF will affect enhancement, feature extraction and as a consequence the accuracy of the recognition. So, an increasing number of researches have been focused on the study of the reliability of an FOF estimation, and a wealth of methods has been proposed for extracting the FOF in the literatures.

To present a unified understanding of the state of the art research in FOF, avoid duplicated efforts and address the lack of existing empirical comparisons, this survey endeavors to cover most emerging FOF estimation methods, and reflect the research trend toward the extraction of FOF. Table 1 lists in a compact manner the most relevant works for fingerprint orientation field estimation. We will classify the pertinent FOF estimation methods presented in the literatures into three classes according to the nature of the characteristics considered: gradient-based, mathematical models-based and learning-based methods. Furthermore, the different classes of algorithms and their evolved counterparts in the literatures will be analyzed. Finally, we conduct an empirical study to analyze the most important FOF algorithms in terms of accuracy and time cost when they are applied to AFIS. We will make some performance evaluations of the algorithms by the relevant experimental results.

The remainder of this paper is organized as follows. Section 2 gives a glance at popular gradient-based methods that are widely accepted as basic FOF estimation approaches in this field. Section 3 summarizes various mathematical models-based methods. The learning-based methods are reviewed in Section 4. The inherent properties and differences among them are investigated, and the advantages and limitations of them are discussed and analyzed within each sections. Performance evaluations are reported and analyzed in Section 5. Finally, some conclusions of this paper are drawn in Section 6.

II. GRADIENT-BASED METHODS

The aim of FOF estimation is to determine the global structure and orientation of the ridges in the fingerprint image. The simplest and most natural method for extracting ridge orientation is based on gradients in the fingerprint image. The gradient-based method was introduced in [43] and adopted by many researchers [44]–[61], where the gradient vectors were calculated in a fingerprint image by taking the partial derivatives of gray intensity at each pixel.

$$\mathbf{G}(x, y) = \begin{bmatrix} G_x(x, y) \\ G_y(x, y) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{I}(x, y)}{\partial x} \\ \dfrac{\partial \mathbf{I}(x, y)}{\partial y} \end{bmatrix} \tag{1}$$

where $\mathbf{I}$ represents the gray-scale fingerprint image, $G_x(x, y)$ and $G_y(x, y)$ components are the derivatives of $\mathbf{I}$ at pixel$[x, y]$ with respect to the *x-axis* and *y-axis* directions, respectively. The local ridge orientation at pixel $[x, y]$ is defined as the angle $\theta(x, y)$. It is noteworthy that fingerprint ridges are not directed, and $\theta(x, y)$ is an unoriented direction lying in $[0, \pi)$. For notational convenience, the gradient vector is often rewritten as $\mathbf{G}(x, y) = \left[G_x, G_y\right]^T$. The ridge orientation $\theta(x, y)$ is perpendicular to the gradient orientation $\varphi(x, y)$. Except in the region of singularities such as core and delta as shown in Fig 2, the ridge orientation varies very slowly across the fingerprint image. Therefore the FOF is seldom computed at pixel level. Most of the AFISs compute the local ridge orientation at discrete positions based on patch of pixels. The problem that needs to be solved is how we can find the optimal orientation for the gradient vectors in a patch. One method that is easy to come up with is to average these vectors and estimate the local optimal orientation by minimizing the mean square error. However, the ridge orientation is undirected, and the opposite gradient vectors actually indicate the same ridge orientation [86]. So, this method will not work for the reason that gradients cannot be directly averaged in local neighborhood since the opposite gradient vectors will cancel rather than strengthen each other [57], [86]. Kass and Witkin [43] proposed a simple yet elegant solution to map two ridge directions that are over 180 degrees into a single direction by doubling them. This achieves a single direction representation of the ridge directions and allows local gradient estimations to be averaged. In their algorithm, the orientation of a pixel gradient can be computed by the vector $\mathbf{J}(x, y) = \left[G_x^2 - G_y^2, 2G_xG_y\right]^T$. The magnitude of $\mathbf{J}(x, y)$ is just the square of the magnitude of $\mathbf{G}(x, y)$ and the angle between $\mathbf{J}(x, y)$ and the *x-axis* is twice as the angle between $\mathbf{G}(x, y)$ and the *x-axis*, and the gradient orientation $\varphi(x, y)$ can be estimated by:

$$\varphi(x, y) = \frac{1}{2}\tan^{-1}\left(\frac{2G_xG_y}{G_x^2 - G_y^2}\right) \tag{2}$$

Based on the above idea, some feasible methods can be derived for estimating the FOF. Rao and Schunck [44] and Rao and Jain [45] presented another method based on the gradient of the Gaussian, and it was related directly to results concerning the mean direction and dispersion in statistics of directional data. It is an improvement over Kass and Witkin's scheme in [43] as it removes the assumptions of the nature of the oriented texture. In their method, the gradient vector $[G_x, G_y]^T$ is represented as polar expression of $R_{mn}e^{i\varphi_{mn}}$,

TABLE 1. List of the most relevant fingerprint orientation field estimation works.

Categories	Authors	Years	Main properties	References
Gradient-based methods	Kass et al.	1987	Double angle averaging	[43]
	Rao et al.	1991	The gradient of Gaussian	[44]
	Rao et al.	1992		[45]
	Lee et al.	1997		[46]
	Awad	2016		[47]
	Ratha et al.	1995	Averaging squared gradients and its variants	[48]
	Jain et al.	1997		[4]
	Bazen et al.	2002		[21]
	Wang et al.	2007		[51]
	Mei et al.	2009		[53]
	Mei et al.	2012		[55]
	Saparudin et al.	2016		[56]
	Bian et al.	2018		[58]
	Li et al.	2018		[59]
	Bazen et al.	2002	Principal component analysis	[21]
	Hong et al.	1998	Low-pass filtering smooth	[2]
	Chikkerur et al.	2007		[52]
	Babatunde et al.	2012		[54]
	Jiang et al.	2005	Modulus handling	[50]
	Bian et al.	2014	Linear projection analysis	[86]
	Bian et al.	2017		[57]
Mathematical models-based methods	Sherlock et al.	1993	Heuristic knowledge-based methods	[63]
	Vizcaya et al.	1996		[64]
	Gu et al.	2003		[66]
	Zhou et al.	2004		[65]
	Zhou et al.	2004		[67]
	Gu et al.	2004		[68]
	Li et al.	2006		[69]
	Li et al.	2007		[70]
	Huckemann et al.	2008		[71]
	Gottschlich et al.	2016		[72]
	Wang et al.	2007	Orthogonal polynomials-based methods	[73]
	Ram et al.	2008		[78]
	Tashk et al.	2009		[75]
	Tao et al.	2010		[76]
	Ram et al.	2010		[79]
	Tashk et al.	2010		[80]
	Wang et al.	2011		[74]
	Jirachaweng et al.	2011		[81]
	Liu et al.	2014		[82]
	Bian et al.	2014		[86]
	Gupta et al.	2015		[83]
	Gupta et al.	2016		[84]
	Gupta et al.	2018		[85]
Learning-based methods	Nagaty	2003	Neural network	[90]
	Zhu et al.	2006		[91]
	Ji et al.	2008		[92]
	Sahasrabudhe et al.	2013		[94]
	Cao et al.	2015		[106]
	Schuch	2017		[108]
	Schuch	2017		[109]
	Tang et al.	2017		[110]
	Cao et al.	2018		[107]
	Qu et al.	2018		[111]
	Ram et al.	2009	Ridge orientation model fitting	[95]
	Zhang et al.	2014		[96]
	Zhang et al.	2014		[97]
	Feng et al.	2013	Dictionary	[98]
	Yang et al.	2014		[99]
	Cao et al.	2014		[102]
	Jain et al.	2015		[100]
	Chen et al.	2016		[101]
	Liu et al.	2017		[105]
	Lee et al.	2008	Markov random field	[93]
	Turroni et al.	2011	A combination of local and learning-based models	[112]

where m and n are the x and y co-ordinates of pixels in local patch, and the estimation of the optimal ridge orientation Θ in a local patch of size $N \times N$ pixels can be given by

$$\Theta = \tan^{-1}\left(\frac{\sum_{m=1}^{N}\sum_{n=1}^{N} R_{mn}^{2}\sin(2\varphi_{mn})}{\sum_{m=1}^{N}\sum_{n=1}^{N} R_{mn}^{2}\cos(2\varphi_{mn})}\right) + \frac{\pi}{2} \tag{3}$$

The method based on the gradient of the Gaussian is also used to estimate FOF in [46] and [47]. Unlike the above method, Jain *et al.* [4], Bazen and Gerez [21], and Ratha *et al.* [48] estimated the local orientation of each fingerprint patch based on averaging squared gradients method (ASGM). The optimal gradient orientation of fingerprint patch is computed by

$$\Phi = \frac{1}{2}\tan^{-1}\left(\frac{\sum_{h=1}^{N}\sum_{w=1}^{N} 2G_x(h,w)G_y(h,w)}{\sum_{h=1}^{N}\sum_{w=1}^{N}\left(G_x^2(h,w) - G_y^2(h,w)\right)}\right) \tag{4}$$

And then the ridge orientation can be computed by:

$$\Theta = \Phi + \frac{\pi}{2} \tag{5}$$

The FOF estimated by ASGM still contains inconsistencies caused by creases, adhesions and ridge breaks. Utilizing the regularity property of the fingerprint, each patch orientation is smoothened with its neighborhood average by low-pass filtering, and a Gaussian window [2], [52], [54] is employed to smooth patch orientation according to Eq. (6).

$$\hat{\Theta}(i,j) = \frac{1}{2}\tan^{-1}\left(\frac{G(x,y) * \sin(2\Theta(i,j))}{G(x,y) * \cos(2\Theta(i,j))}\right) \tag{6}$$

Here $G(x, y)$ represents a Gaussian smoothening kernel. Low-pass filtering is very effective when the patch orientations of miscalculation are relatively isolated in the local region as shown in Fig. 1 row 1 (a1)-(c1). However, if the falsely estimated patch orientations dominate the local region, the correctly estimated orientation will be falsely impacted by low-pass filtering as shown in Fig. 1 row 2(a2)-(c2). Besides, the location of the singular points (see section 2) will shift slightly away from the true location after low-pass filtering as shown in Fig. 1 row 3 (a3)-(c3) [91].

Averaging the squared gradients can only reduce the estimation errors caused by noise that has no dominant orientation in the averaging window. Obviously, the essence of the ASGM is computing the weighted average of all squared gradient vectors within a patch, where the weights for all squared gradient vectors are one. In fact the anti-noise ability of this method varies in relation to the patch size. It is weakened when the patch is small and is strengthened when the patch is large. On the other hand, the smaller the size of a patch, the higher the accuracy of the orientation estimation. In other words, the distortion of the patch orientation becomes more noticeable when the patch size increases. To overcome the contradiction, methods based on composite window were employed in [55], [57], and [58] to achieve the balances between accuracy and anti-noise ability. The composite window integrates the robustness of a large outer window and the accuracy of a small inner window. However, we have to face a challenge: the selection of the appropriate scales for the composite window. Aiming at this problem, Li *et al.* [59] used a weighted multi-scale composite window to adapt the scales of the patches based on the squared gradient coherence.

Bazen and Gerez [21] showed another effective averaging concept for computing the orientation field. Their derived method is found to be mathematically equivalent as taking the principal component analysis (PCA) of autocorrelation matrix of a group of the gradient vectors within a local neighborhood. The method assumes that the expectations of gradient vectors are zeros. However, Bian *et al.* [86] challenged this assumption and showed that it might not be well established. They estimated the local ridge orientation by the linear projection analysis (LPA) based on the gradient vectors within a local neighborhood, and the LPA method can reveal the inner nature and rationality of the algorithm more comprehensively. The estimation of the dominant ridge orientation Θ in a patch can be given by the LPA:

$$\Theta = \tan^{-1}\left(\frac{2G_{xy}}{(G_{xx}-G_{yy}) + \sqrt{(G_{xx}-G_{yy})^2 + 4G_{xy}^2}}\right) + \frac{\pi}{2} \tag{7}$$

In this expression,

$$G_{xx} = \sum_{j=1}^{N\times N} G_{jx}^2, \quad G_{yy} = \sum_{j=1}^{N\times N} G_{jy}^2, \; G_{xy} = \sum_{j=1}^{N\times N} G_{jx}G_{jy}$$

After analyzing the robustness of the orientation and anisotropy estimation methods and the effect of the modulus normalization on the estimation performance, Jiang [50] proposed a two stage averaging framework with patch-wise modulus handling to extract the FOF. And this method inherits the merits of both linear and normalized averaging methods for the de-noising. Wang *et al.* [51] improved the performance of gradient-based methods and proposed an enhanced gradient-based algorithm for extracting the FOF. The enhanced algorithm choose the best orientation estimated from four overlapping neighborhoods of every image patch, where the voting scheme was based on the reliability measures. Mei *et al.* [53] presented a gradient-based combined method for extracting the FOF. To overcome inherent weaknesses of using just one single size patch, they tried to extract the FOF by combining the orientation fields calculated by using multiple sized patches. The orientations within large noisy regions are predicted by iteration using the already estimated orientations. Inspired by Wang *et al.* [51] and Saparudin and Sulong [56] proposed a new scheme of enhanced FOF based on minimum variance of squared gradients to find the minimum variance among the four directions of squared gradients. This method only focuses on the

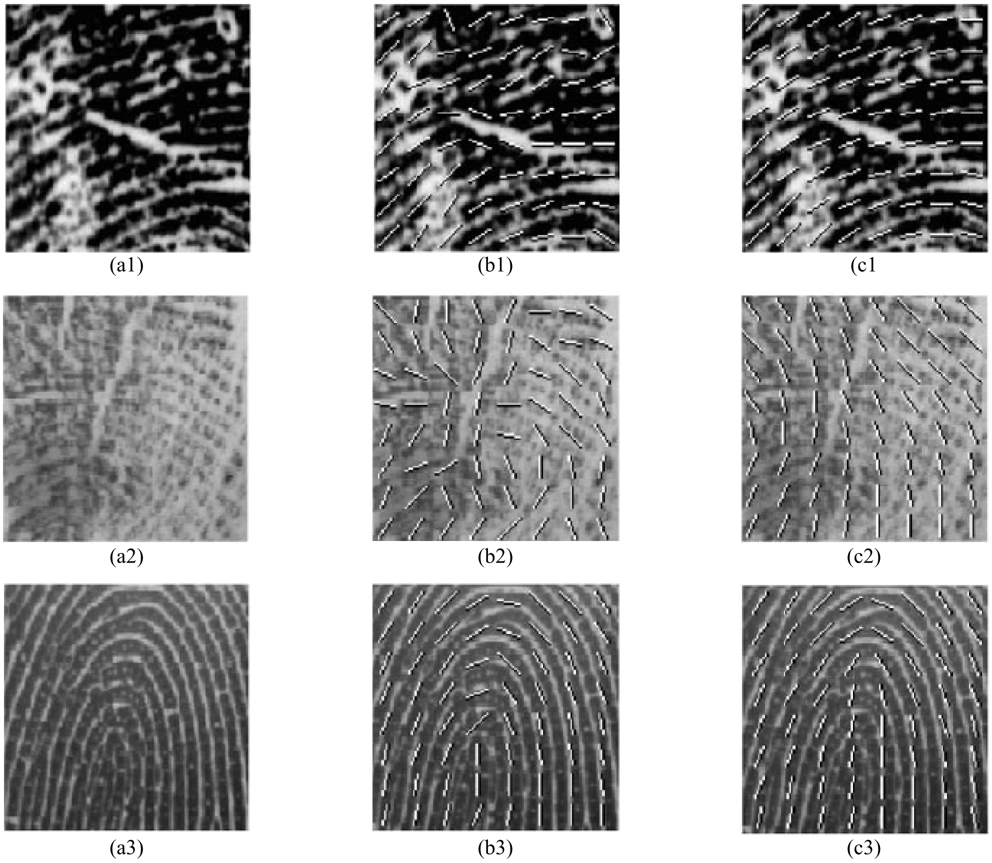

FIGURE 1. FOF estimation based on gradient and FOF smooth by low-pass filtering. Column 1: original fingerprints; Column 2: FOF by gradient-based method; Column 3: FOF filtered by low-pass filter.

noise regions without encroaching rest of clean area of the fingerprint.

In the gradient-based methods, there are different opinions on whether or not the modulus of the gradient vector should be normalized. In [21] and [43]–[45], not only the angle of the gradients is doubled, but also the length of the gradient vectors is squared. The authors thought that the gradient orientation was closely related to the modulus of gradient. In their methods, the stronger gradients have higher votes than the weaker ones in computing the local orientation. On the contrary, some researchers [60], [61] believe that we are purely interested in the orientation and the modulus only reflects the image contrast, therefore the modulus should be normalized. Jiang [50] analyzed the effect of the modulus normalization of gradients on estimation performance, and showed that the gradient modulus normalization has both advantages and disadvantages. In order to compare the two viewpoints directly, the FOF is calculated with the classical gradient-based method using normalized and unnormalized point gradient vectors respectively in [55]. The comparison of the experimental results indicate that the effective point gradient vectors (which are located at the edges between ridges and valleys) often take positive effects and the ineffective point gradient vectors (which are often located inside ridges or valleys with small modulus, or located at the edges of abrupt noise with distinctly large modulus) often take side effects.

In order to assess the effectiveness of point gradient vector, Bian et al. [57] proposed a weighted LPA algorithm based on the similarity of point gradient orientations. The effectiveness of point gradient vector is judged according to the similarity of point gradient orientations. The similarity measure $R(m, n)$ is computed by

$$R(m,n)=\frac{1}{N\times N}\sum_{h=1}^{N}\sum_{w=1}^{N}\frac{\cos\left(\theta(h,w)-\theta(m,n)\right)+1}{2} \tag{8}$$

Bian et al. [57] reconstruct the FOF by combining weighted LPA with orientation diffusion. The accuracy and reliability of lower quality patch orientations can be improved by the feedback of the estimated patch orientations with higher quality.

However, there are drawbacks in gradient-based methods. Gradient-based methods always consist of two components: gradient computation and orientation estimation. There are sensitive to noise due to the gradients computed at the pixel level and not robust against large scale noises. Another shortcoming with gradient-based methods is the orientation bias caused by the discrete operators used to approximate the differentiation operation.

III. MATHEMATICAL MODELS-BASED METHODS

In fact, the FOF is sufficiently smooth except for a few regions with singularities, so it is a very promising approach to infer local structure using more global information. In the meanwhile, the most attractive feature of the mathematical model-based methods is that they have capability to give a global constraint to each point in a FOF. When reconstructing one orientation in a FOF, all the other orientations also contribute. We coarsely divide the model-based methods into two categories: heuristic knowledge based methods and orthogonal polynomials based methods.

A. HEURISTIC KNOWLEDGE BASED METHODS

Singular points (SPs) are points of discontinuity of the FOF. The two types of SPs (cores and deltas) were defined in terms of the ridge structures [62]. They have often been considered as prior or heuristic knowledge and used to refine the FOF. The core point is the end point of the innermost curving ridge while the delta point is the confluence point of three different flow directions (see Fig. 2).

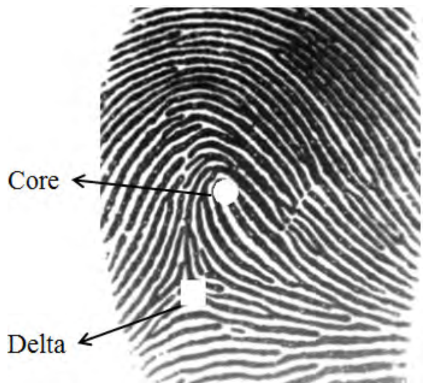

FIGURE 2. Singular regions in fingerprint image. Core: white circle; Delta: white box.

To reconstruct the FOF robustly, some global models based on heuristic knowledge (GMHK) are developed in the past years by many researchers. The pioneering research on GMHK was presented by Sherlock and Monro [63]. In [63], a simple mathematical model is developed which computes fingerprint local ridge orientation from core and delta positions, which take core as zero and delta as pole in complex plane. This method is named as zero-pole model and modeled the FOF by using:

$$O_m(z) = O_0 + \frac{1}{2}\left[\sum_{l=1}^{L} \arg(z - z_c^l) - \sum_{k=1}^{K} \arg(z - z_d^k)\right] \tag{9}$$

where O_0 is an initial FOF, z_c^l, z_d^k are the positions of the cores and deltas. Quite evidently, the influence of a core z_c is 0.5arg(z-z_c) for a point z, and that of a delta z_d, is -0.5arg(z-z_d). The orientation at z is the sum of the influences of all cores and deltas. The main contribution of this model is to provide a powerful approach to interpolate the orientation at any point, which can be used to reconstruct the FOF with noise.

However, it is an inescapable fact that any two fingerprints with the same SPs will both be modeled by the same function even though their local ridge orientation values may differ significantly. In other words, the zero-pole model cannot model all possible fingerprint orientation patterns accurately. Vizcaya and Gerhardt [64] modify the zero-pole model to a non-linear orientation model by using a piecewise linear approximation model around SPs to adjust the zero and pole's behavior. The orientation at any point z is given by:

$$O_m(z)=O_0+\frac{1}{2}\left[\sum_{k=1}^{K} g_d^k(\arg(z-z_d^k))-\sum_{l=1}^{L} g_c^l(\arg(z-z_c^l))\right] \tag{10}$$

where g is the correction term, which are some family of nonlinear functions that preserve the singularity (Poincare index) at the given point. This additional correction is not precise enough to approximate all orientation fields. These two models mentioned above cannot handle the fingerprints where there are no SPs.

Zero-pole based methods are aiming at obtaining the local orientation patterns near the singularities followed by a series of nonlinear functions to correct the global patterns. Zhou and Gu [65] built another complex model, which was based on zero-pole model but with a high-order rational function used as the non-linear correction instead. It can estimate the FOF for all types of fingerprints at regions near or far from SPs. This model can be defined as

$$\phi(z) = \frac{1}{2}\arg\left[\frac{f(z)}{g(z)} \cdot \frac{P(z)}{Q(z)}\right] \tag{11}$$

where $P(z) = \prod_{l=1}^{s_0}(z - z_c^l)$, $Q(z) = \prod_{k=1}^{s_1}(z - z_d^k)$, $\{z_c^l\}_{1\le l<s_0}$ and $\{z_d^k\}_{1\le k<s_1}$ are the cores and deltas of the fingerprint in the known region. The zeros of $f(z)$ and $g(z)$ are outside the known region. The FOF can be reconstructed by this model regardless of the existence of singular points in fingerprint.

Gu and Zhou [66], Zhou and Gu [67], and Gu *et al.* [68] observed that the power-series polynomials worked well for smooth FOF but had difficulty in the regions with singularities, and proposed a combination model to reconstruct the FOF. On the one hand, from the global pattern consideration, the fingerprint ridge orientation is quite smooth and continuous except at singular point regions, so they apply a polynomial model to approximate the global orientation field. On the other hand, at each singular point, a point-charge model is used to describe the local region. These two models are combined together by a weight function.

$$\begin{pmatrix} R(x,y) \\ I(x,y) \end{pmatrix} = \alpha_{PM} \cdot \begin{pmatrix} PR \\ PI \end{pmatrix} + \sum_{k=1}^{K} \alpha_{PC}^k \cdot \begin{pmatrix} H_1^k \\ H_2^k \end{pmatrix} \tag{12}$$

where PR and PI are the real and imaginary parts of the polynomial model respectively, α_{PM} is the weighting factor for polynomial model, k is the number of SPs, α_{PC}^{k} is the weighting factor of the kth singular point in point-charge model, H_1^k and H_2^k are the real and imaginary parts of the point-charge model for the kth singular point. This combination model gives a better estimation of the FOF compared to other previous models. However, in this method, the partition of each effective region relies on a trial and error means. There appears to be no solid rules for the seamless integration of different models in the combination method.

A similar work was presented by Li *et al.* [69], which combined the piecewise linear model and the high order phase portrait model for the local and global descriptions of the FOF, respectively. They first divide the fingerprint into several regions and predict the orientation in regions where there are no ridge information or the coherence of the orientation fields are low by using piecewise first-order phase portrait model. And then their algorithm computes a global model using a constrained nonlinear phase portrait algorithm. It should be noted that the prediction model is heavily dependent on the detection of SPs. If the SPs cannot be detected correctly due to bad image quality, the prediction model may fail. Furthermore, the model could do nothing in some cases, especially when the two SPs are close to each other. Li *et al.* [70] presented a modified algorithm and gave a thorough stability analysis to deal with such cases. However the model still demands constraints for each of the singularities.

Although adding substantial flexibility, these models [64]–[70] do not explicitly take more of the geometrical structure of fingerprints into account. Led by the analytic properties of quadratic differentials (QDs), Huckemann *et al.* [71] built the model honoring the special geometry of fingerprints while keeping the model as simple as possible. In fact, the model of Sherlock and Monro [63] can be viewed as the simplest QDs respecting the observed singularities. The QD model better fits real FOFs especially for the fingerprint of the type arch. To apply the quadratic differential based models, the fingerprints have to be aligned with respect to the coordinate systems defined by SPs. However, it is not feasible to establish the required coordinate systems when some of the SPs are missing. Gottschlich *et al.* [72] introduced a locally adaptive global model called the extended quadratic differential (XQD) model to model the FOF. The major advantage of the XQD model lies in its small number of parameters, each of which has a simple and obvious geometric meaning. This method adds a variable number of local correction points (manually marked) as anchor points thereby obtaining an extended quadratic differential (XQD) model. With these points the local orientation field modeled by a QD can be corrected to better match with the ridge flow of a fingerprint. One important application of XQD model is to semi-automatically mark a FOF by an expert.

The modeling methods mentioned above have one common limitation, i.e., they all require the heuristic knowledge such as SPs or anchor points in the input fingerprints in order to refine model descriptions and predict the fingerprint global orientation field. These methods mostly depend on the precise detection of SPs. However the accurate detection of SPs, in turn, relies on a good estimation of the FOF. As a result, the problem turns this task into the paradoxical chicken-egg problem. An accurate extraction of SP is a hard problem for bad quality fingerprints, and thus these algorithms are not much useful. For most of aforementioned methods, SPs or anchor points are often marked manually, which evidently limits their application in the AFIS.

B. ORTHOGONAL POLYNOMIALS BASED METHODS

To overcome inherent weaknesses of the methods mentioned above, some global FOF modeling algorithms without any heuristic knowledge are reported in the literatures. We will review this category of work in this section.

The problem of FOF modeling can also be solved as a data fitting problem where FOF obtained from a local method is fitted on some well-defined basis functions. Optimization algorithms are first applied to obtain the parameters of the model to fit the data. Then the basis functions and the modeling parameters are used to reconstruct the FOF. For example, Wang *et al.* [73] proposed a fingerprint orientation model based on 2D Fourier expansions (FOMFE) in the phase plane. It first uses a series of 2D Fourier expansion basis functions to model the FOF. It can be expressed in the following general form,

$$f(x, y) = \sum_{m=0}^{k}\sum_{n=0}^{k} \lambda_{mn} \left[a_{mn} b_{mn} c_{mn} d_{mn}\right] \times \begin{bmatrix} \cos(m\nu x)\cos(n\omega y) \\ \sin(m\nu x)\cos(n\omega y) \\ \cos(m\nu x)\sin(n\omega y) \\ \sin(m\nu x)\sin(n\omega y) \end{bmatrix} + \varepsilon(x, y) \quad (13)$$

where $-l \leq x \leq l$, $-h \leq y \leq h$, $m, n, k \in N$ are the order of the Fourier expansion, $\varepsilon(x, y)$ is the residual, $\nu = \pi/l$, $\omega = \pi/h$, $(a_{mn}, b_{mn}, c_{mn}, d_{mn})$ are the Fourier coefficients and λ_{mn} is a constant scalar. Then in order to reconstruct the FOF using the FOMFE, the coarse FOF needs to be estimated by using the classic gradient-based method. And the FOF is to be mapped into a new vector field where each element is denoted as a 2D vector $\mathbf{v} = (v_s, v_c)$ with v_s, v_c being the phase functions of $\cos 2\theta$ and $\sin 2\theta$, respectively, and θ is the ridge orientation angle. And then the Fourier coefficients are computed by solving a linear least square problem below,

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\Lambda}\boldsymbol{\alpha} - \mathbf{b}\| \quad (14)$$

where the matrix $\boldsymbol{\Lambda}$ is constructed from the $(2k+1)^2$ basis functions evaluated at the total M sampling points in the phase space, the observation vector $\mathbf{b}$ is a single column vector weighted from the initially estimated FOF, and $\boldsymbol{\alpha}$ is the Fourier coefficient vector. The FOF is reconstructed by the evaluated parameters using FOMFE. This method starts from the noisy initial estimation, so the orientation fields cannot be recovered in very poor quality areas. Wang and Hu [74] further improved the modeling methodology by exploiting the predictive capability of mathematical modeling to reconstruct the full orientation fields from partial fingerprints.

Tashk *et al.* [75] pointed out that the FOMFE did not work perfectly in fingerprints areas with high curvature and poor quality. They proposed a modification of fingerprint orientation model based on 2D Fourier expansion (M-FOMFE) to address these drawbacks. The squared gradient matrix named coherence matrix is used to tackle the uncertainty of each ridge orientation and improve the accuracy of FOMFE. The elements of coherence matrix are large for ridges of clear ridge pattern and are small for noisy regions and non-ridge regions. The coherence matrix elements are considered as weight values of the uncertainty of ridge orientations, and they are introduced into original FOMFE to correct the error of ridge orientation estimation. Similar to [75], Tao *et al.* [76] pointed out that the FOMFE was sensitive to abrupt changes in orientation field and it did not differentiate the impacts from patches that were of different quality. Hence they built a FOF model based on weighted 2D Fourier expansion (W-FOMFE) to deal with these two drawbacks. They use the Harris-corner strength (HCS) [77] to remove abrupt changes in FOF, and then incorporate the normalized HCS as weighted value into original FOMFE. The weighted 2D W-FOMFE model demonstrates better results to reconstruct FOF by assigning higher weights to uniform and linear ridge valley flow and lower weights to the areas near SPs. However it cannot well handle the poor quality areas.

Fourier approximations substantially reduce the problem of ill conditioned equation systems, but are slow to compute and evaluate. Furthermore, they are still subject to error for higher terms. On the other hand, the trigonometric functions naturally obey the property of being orthogonal. The trigonometric polynomial can be used to fit the discrete orientation data obtained by the local estimation [73]. Ram *et al.* [78], [79] presented a fingerprint ridge orientation model based on Legendre polynomials. The authors propose to independently model the sine and cosine data of the given original orientation field $\mathbf{O}$ computed by gradient-based method. Let $\mathbf{a}$ and $\mathbf{b}$ be the parameters of a Legendre expansion of the sine and cosine data respectively. Then, the orientation field can be computed in the following:

$$\tilde{\mathbf{O}} = \frac{1}{2}\arctan\left(\frac{\boldsymbol{\varphi}\mathbf{a}^T}{\boldsymbol{\varphi}\mathbf{b}^T}\right) \tag{15}$$

where $\boldsymbol{\varphi}$ represents the set of basis functions based on Legendre polynomials. The optimal parameters $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ for the sine and cosine approximation can be computed by minimizing the following non-linear function:

$$(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \arg\min_{\mathbf{a},\mathbf{b}} \omega\left(\sin\left(\frac{1}{2}\arctan\left(\frac{\boldsymbol{\varphi}\mathbf{a}^T}{\boldsymbol{\varphi}\mathbf{b}^T}\right) - \mathbf{O}\right)\right)^2 \tag{16}$$

where $\boldsymbol{\omega}$ is the diagonal weighting matrix containing the weights for every coordinates. It is computed using fingerprint segmentation, $\omega = 0$ for background and $\omega = 1$ for foreground pixels. To minimize the cost function, Ram *et al.* [78] employed a simple non-linear optimization algorithm based on line search. In addition, a more standard algorithm called the Levenberg Marquard algorithm (LMA) is adopted to solve the problem described in [79, eq. (16)].

Finally, the reconstructed FOF can be obtained using

$$\hat{\mathbf{O}} = \frac{1}{2}\arctan\left(\frac{\boldsymbol{\varphi}\hat{\mathbf{a}}^T}{\boldsymbol{\varphi}\hat{\mathbf{b}}^T}\right) \tag{17}$$

To smooth the original FOF, Tashk *et al.* [80] employed a combination of filtering- and model-based orientation smoothing methods to reconstruct FOF. They employ a Gaussian filter to smooth the original FOF first, and then reconstruct the FOF by using one of the orthogonal polynomials such as Legendre and Chebyshev type I or II, based on the results obtained at the filtering-based stage. The algorithm adaptively selects the best orthogonal polynomials (Legendre, Chebyshev type I or II) for reconstructing FOF using its basis functions according to the following criteria:

$$\boldsymbol{\varphi} = \begin{cases} \text{Chebyshev I}, & \textit{if } |Con \cdot Coh| \geq \text{UB} \\ \text{Chebyshev II}, & \textit{if } \text{UB} > |Con \cdot Coh| \geq LB \\ \text{Legendre polynomials}, & \textit{if } |Con \cdot Coh| < \text{LB} \end{cases} \tag{18}$$

where *Coh* and *Con* are the coherence and consistency parameters, respectively. The lower bound (LB) and the upper bound (UB) can be determined through trial and error by experimenting on fingerprint image databases. The authors did not explain why these different orthogonal polynomials were considered. They only simply tested it, without giving any theoretical explanation. In addition, it should be noted that this is a very computationally complex algorithm.

In [79], the LMA was used to minimize the modeling cost function in order to preserve SPs. This method is positively advantageous in preserving SPs, but at a cost of high computation load. In addition, false SPs could be kept unintentionally. A major problem in FOF modeling is to remove the noise but preserve the singularity points in the meantime. A higher order polynomial model is usually advantageous in singularity points preservation, but tends to over-fit the data and keeps some noisy structures. In contrast, a lower order polynomial model performs better in noise removal, but at the cost of larger error in singularity points preservation, Fig. 3 depicts this situation as an example [82]. Aiming at this problem, Jirachaweng *et al.* [81] modeled the preliminary FOF first using a lower order Legendre polynomial to capture the global orientation pattern, then the regions with SPs were dynamically refined using a higher order Legendre polynomial. The singular point regions are automatically detected through the analysis on the orientation residual field between the original FOF and the preliminary FOF reconstructed by the lower order Legendre polynomial. In fact, this method has two models, one for smooth region and the other for region with singularities. A good balance between noise suppression and SPs preservations has been achieved by the method.

Liu *et al.* [82] chose weighted discrete cosine transforms (DCT) as basis functions to reconstruct FOF. Instead of using a fixed order, they combine the DCT basis atoms of low and high orders with weights evaluated by singularity measurements. Large weights are assigned to the DCT atoms of high orders for reconstructing the orientations in singular regions. The weighted DCT model is further extended for partial fingerprints with a large amount of noisy or incomplete information, by gradually and iteratively expanding the orientations from known to unknown regions. By making full use of the basis atoms of low and high orders, the proposed model is advantageous in preserving the true orientations of singular regions during the process of noise removal. The DCT basis functions at the frequencies (u, v) are computed as follows:

$$\varphi_{u,v}(m, n) = \frac{C_u \times C_v}{\sqrt{PQ}} \cos\left(\frac{(2m+1)u\pi}{2P}\right) \times \cos\left(\frac{(2n+1)v\pi}{2Q}\right) \tag{19}$$

in this expression,

$$C_u = \begin{cases} 1, & if\ u = 0 \\ \sqrt{2}, & otherwise \end{cases} \quad C_v = \begin{cases} 1, & if\ v = 0 \\ \sqrt{2}, & otherwise \end{cases} \tag{20}$$

where $0 \leq m \leq P-1, 0 \leq n \leq Q-1, 0 \leq u \leq U-1, 0 \leq v \leq V-1$, U and V denote the order of DCT model. The DCT basis vectors are generated from the evaluations of basis functions at all coordinates of FOF. The FOF can be reconstructed by weighted DCT model:

$$\hat{O}(m, n) = \hat{O}_l(m, n) + \omega \hat{O}_h(m, n) \tag{21}$$

where $\hat{O}_l(m, n)$ and $\hat{O}_h(m, n)$ represent the orientation elements of patch (m, n) in FOF reconstructed by the DCT basis functions of low and high orders, respectively; ω is the weight assigned to $\hat{O}_h(m, n)$. The weight ω related to a singularity measurement which evaluates how much the local orientations are close to singular regions. It can be computed using complex filters.

Weights are assigned based on probable locations of SPs in [82]. However, it cannot handle the situations when SPs are absent (e.g. in an arch type fingerprint) or spurious SPs are obtained due to lager noise. In order to preserve the genuine FOF in the areas containing uniform flow and to accurately reconstruct the FOF in poor quality areas, Gupta and Gupta [83], [84] proposed a weighted FOF modeling algorithm based on orthogonal basis. In [83], the weighted FOF model was built based on Legendre basis, and it was built based on Fourier basis in [84]. However, the authors did not give a convincing explanation why the basis was selected. The weights are assigned according to the patch quality evaluated by symmetric filters. Gupta and Gupta [83], [84] used a variational approach to regularize the FOF in first. It can successfully preserve the genuine FOF in the areas containing uniform flow and accurately reconstruct the FOF in bad quality areas. However, sometimes it may generate spurious FOF in SPs close-by areas. So, next they employed weighted FOF modeling based on Legendre/Fourier basis to resolve this problem. Also, Gupta and Gupta [85] used the similar method to estimate the FOF for vivificating the slap fingerprint. It is slightly different from FOF model described in [83] and [84], because it reconstructs FOF only using polynomial basis without the use of any other weights.

The Legendre and Chebyshev are both classical continuous orthogonal polynomials, and their basis functions do not exactly satisfy the orthogonal properties under the condition of the finite discrete data set. Therefore, the coefficient matrix of the normal equations used in these papers [78]–[81], [83] were actually not standard orthogonal matrix, and they were likely to be nonsingular matrix in some cases. If this is the case, the method of a weighted pseudo-inverse technique for least squares approximation proposed in these papers will not be applicable. In addition, the high computational cost of these methods limits their usage in large data sets because the authors try to fit directly the original point-gradient vectors by using polynomial and non-linear technology to approximate the parameter vectors of the polynomial. There are also no information of execution time reported by these papers. To address these issues, Bian *et al.* [86] modeled the FOF by using orthogonal polynomials in two discrete variables. The authors build the set of basis functions based on the 2D discrete orthogonal polynomials which are composed by two sets of 1D discrete basis functions that satisfy the orthogonal property under the condition of the finite discrete data set. A 1D discrete complete orthogonal basis is denoted as the set of polynomials $P_i(x)\,(i = 0, 1, 2, \cdots, k)$, and the basis function satisfies

$$\sum_{m=0}^{n} P_i(x_m) P_j(x_m) \begin{cases} \neq 0, & i = j \\ = 0, & i \neq j \quad for\ i, j \leq k < n \end{cases} \tag{22}$$

where $x_m (m = 0, 1, 2, \cdots, n)$ is the finite 1D point set, and the discrete orthogonal polynomials are computed using the so-called three-term recurrence relation, as described below:

$$\begin{cases} P_0(x) = 1 \\ P_1(x) = (x - \alpha_1) P_0(x) \\ P_{i+1}(x) = (x - \alpha_{i+1}) P_i(x) - \beta_i P_{i-1}(x) \\ \qquad (i = 1, 2, \cdots, k-1) \end{cases} \tag{23}$$

The coefficients α_{i+1} and β_i can be obtained as follows:

$$\alpha_{i+1} = \frac{\sum_{m=0}^{n} x_m P_i^2(x_m)}{\sum_{m=0}^{n} P_i^2(x_m)}, \quad \beta_i = \frac{\sum_{m=0}^{n} P_i^2(x_m)}{\sum_{m=0}^{n} P_{i-1}^2(x_m)} \tag{24}$$

Considering the 1D discrete polynomials $P_i(x)$ and $Q_j(y)$ in the two variables x and y, the 2D discrete orthogonal basis functions can be obtained [86]:

$$T_{ij}(x, y) = P_i(x) Q_j(y) \begin{cases} 0 \leq i \leq M \\ 0 \leq j \leq N \end{cases} \tag{25}$$

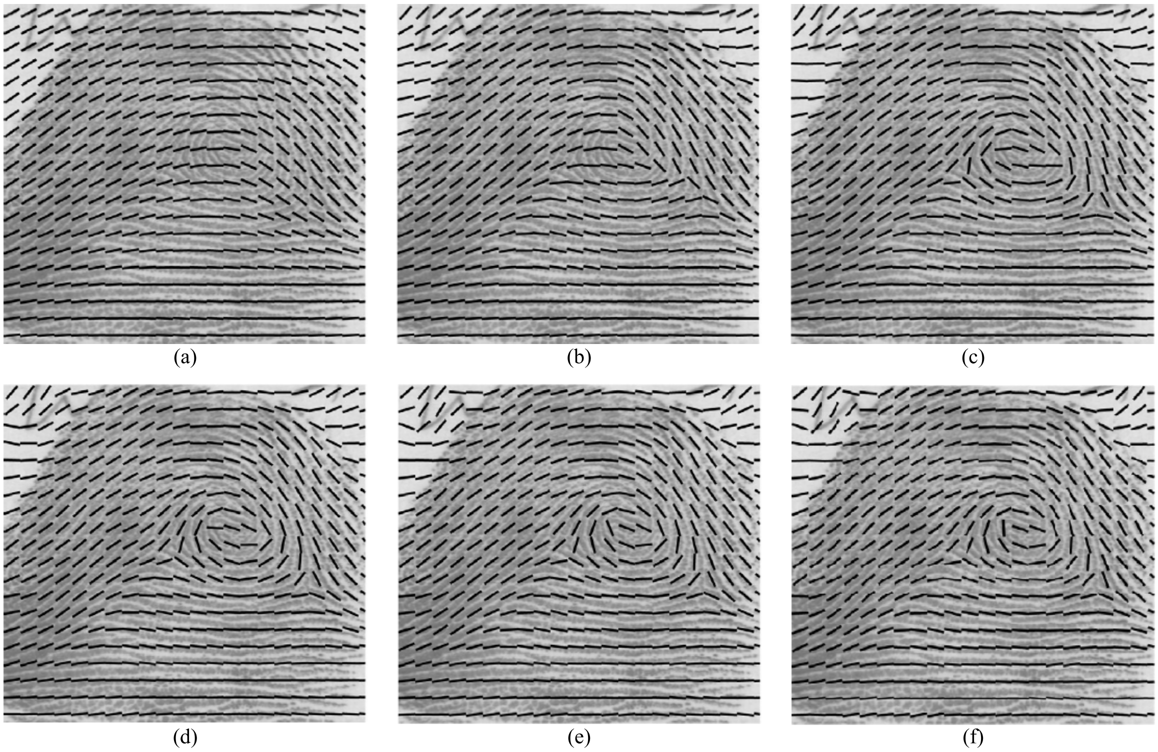

FIGURE 3. The orientation fields reconstructed with the discrete cosine transforms models of different orders: (a) order = 2, (b) order = 4, (c) order = 6, (d) order = 8, (e) order = 10 and (f) order = 12.

In this expression, $P_i(x)$ and $Q_j(y)$ are 1D discrete orthogonal polynomials of order M and N respectively. The reconstructed FOF can be obtained using

$$\hat{\mathbf{O}}(\mathbf{x}, \mathbf{y}) = \sum_{i=0}^{M} \sum_{j=0}^{N} \hat{a}_{ij} P_i(\mathbf{x}) Q_j(\mathbf{y}) \tag{26}$$

The optimal parameters $\hat{a}_{ij}$ can be computed by the best quadratic approximation.

In addition to the above work, there are also some other FOF models which do not require prior knowledge. For example, papers [87], [88] used a class of spatial smoothing priors and a novel Bayesian formulation. Dass [87] proposed a Markov random field (MRF) model for the reliable computation of FOF. The FOF is modeled in a Bayesian framework and fingerprint SPs are described using SPs template models, and parametric template models are used to extract SPs. Joint extraction of the FOF and SPs has the advantage of dynamic updating of features and the ability to detect previously missed SPs. However, a false singular point detected in the first iteration of the joint update will be reinforced in the subsequent iterations. In addition, this approach only contains the quality and smooth pairwise priors. Therefore, it cannot recover the FOF in those areas with bad quality. Hou *et al.* [88] proposed a framework for modeling the fingerprint orientation field based on the variational principle, where the orientation pattern can be estimated through solving the associated Euler–Lagrange equation. The presented model included the term of data smoothness and the term of data fidelity for singularity preservation. Similar to [81], weights are assigned in this algorithm using the saliency of SPs. It assigns low weights in smooth areas and high weights in SPs areas. Different from the methods that approximate the FOF using some function through regression, the variational method needs no explicit form of the approximated function.

As a summary, the success of these methods heavily relies on the accurate local FOF estimations and proper weight assignment. Various approaches and strategies are proposed for improving the accuracy and reliability of local FOF estimation and weights computation. These proposed algorithms do not require any prior knowledge of SPs and they are found to be more practical in AFIS than those algorithms based on heuristic knowledge.

IV. LEARNING-BASED METHODS

Despite the many research efforts made on the estimation of FOF using mathematical and model building methods, there are still many challenges. One of the challenges is to simultaneously smooth out noise and preserve the orientation information in SPs regions. The learning-based methods have demonstrated their advantages to balance the contradictions by adequately utilizing the prior knowledge of various orientation patterns.

Nagaty [90] proposed a learning based FOF estimation method using a hierarchical neural network. Two separately trained neural networks are connected in series: a back propagation neural network (BPNN) is used for computing feature vectors of fingerprint patches with the same size of 16×16, then these feature vectors of fingerprint patches are grouped into distinct orientation classes by a self-organized feature map (SOM) neural network using an unsupervised learning strategy. The patches that submit to the first network have to be binarized. It should be noted that the smaller the granularity of orientation divisions (e.g. from four of a 180 degree circle to eight of that circle), the more accurate the representation of a FOF, but the higher the computation cost. These may influence the performance of the proposed algorithm. To correct falsely estimated orientations in FOF, Zhu *et al.* [91] designed a systematic scheme for estimating FOF. The fingerprint is firstly divided into unoverlapped patches of size 15×15 and the original FOF is estimated by a gradient-based method. And then a neural network is utilized to correct the falsely estimated patch orientations according to their surrounding correct orientations. In their systematic scheme the correct orientations and incorrect orientations are distinguished by neural network and the orientation correctness is learned by a BPNN with a 11-dimensional feature vector. Unlike [90] and [91], Ji and Yi [92] used neuron pulses of pulse coupled neural network (PCNN) to estimate the FOF. Similar to [90], this method also firstly pre-processes the original image so as to obtain a series of valid binary fingerprint patches with the same size of 16×16, and then these patches are submitted to PCNN to determine their optimal ridge orientation. By comparing the projective distance variances along four orientations of the optimal ridge orientation, the orientation related to the minimal variance is determined as the initial patch orientation. Finally, the initial FOF is improved by a dual correction scheme. Moreover, Sahasrabudhe and Namboodiri [94] attempted to use two continuous restricted Boltzmann machines (CRBMs) to learn various types of ridge patterns by training a neural network. The trained CRBMs contain representations of the data used for training and are used to approximate the input FOF that have been estimated by a gradient method. The corresponding output can be interpreted as the best to the learned representations. They demonstrated the advantage of using continuous restricted Boltzmann machines in correcting initial FOF.

Both of the methods discussed above evaluate the correctness of ridge orientation based on neural network. Their performances depend heavily on the data used in the learning phase. A MRF model with small neighborhood or context is able to exploit only limited prior knowledge about fingerprint structure [89] and thus cannot deal with fingerprints of poor quality. To improve FOF estimation by MRF model [87], a global mixture model of orientation fields learned from training fingerprint examples and the gradient information of fingerprint was incorporated into a MRF [93]. Lee and Prabhakar [93] firstly constructed a MRF and then inferred the FOF from the MRF model. This method bears some resemblance to [87] in a sense, it also admits a Markovian interpretation. Put slightly differently, their algorithm incorporates a global mixture model of FOF learned from training examples and thus yields more accurate FOF. Ram *et al.* [95] proposed another statistical model called active fingerprint ridge orientation model (AFROM) that iteratively fits the FOF. This approach consists of an off-line subspace learning stage and an on-line stage for feature projection. The FOF is represented by a vectorially linear regression using Legendre Polynomials. The variability among the global orientation patterns is learned using a training set. Its idea is to perform PCA over a dataset of vectors corresponding to feasible parameters for a given global model and then, starting from an original FOF, find the nearest legal patterns through projection, optimization and back-projection. In proposed model, the parameters are limited to a previously learned linear subspace during the optimization procedure, and the objective function is optimized in each iterative process and results in a high time cost. Zhang *et al.* [96], [97] employed an adaptive orientation model to separate latent overlapped fingerprints. In their applications the reconstructed FOF is used to compare and then adjust the initial FOF, and it only needs to reconstruct the approximate fingerprint ridge flow. So they only derived the rough FOF models and did not optimize the objective function during the iterative process to obtain the accurate model parameters, and only close form solutions were used. Both these strategies reduce the computation time cost.

Inspired by spelling correction techniques used in natural language processing, Feng *et al.* [98] proposed a novel FOF estimation algorithm based on prior knowledge of fingerprint structure. We name it as global dictionary (GlobalDic) as compared to localized dictionaries (LocalizedDic) proposed later by Yang *et al.* [99]. The proposed algorithm consists of an offline dictionary construction stage and an online orientation field estimation stage. In the offline stage, a set of good quality fingerprints of various pattern types is manually selected and their orientation fields are used to construct an orientation patch dictionary that represent the prior knowledge of real fingerprints. In the online stage, the original FOF is estimated by local Fourier analysis method and then noisy orientation patches in original FOF are replaced by the closest orientation patches in the dictionary by dictionary lookup and context-based correction, and the FOF is automatically estimated. To correct the errors in the initial FOF, for each initial orientation patch, a list of candidates from the dictionary which might be the true orientation patch is found by a dictionary lookup operator, and then the contextual information is used to determine a single candidate for each patch. The final FOF is obtained by finding the combination of candidates that minimizes an energy function $E(\mathbf{r})$:

$$E(\mathbf{r}) = E_s(\mathbf{r}) + \omega_c E_c(\mathbf{r}) \tag{27}$$

where $E_s(\mathbf{r})$ denotes the similarity term, $E_c(\mathbf{r})$ denotes the compatibility term, and ω_c is the weight of compatibility term. Two factors are considered in the design of the energy function: one is the similarity between the reference orientation patches and the corresponding initial orientation patches, and the other is the compatibility between neighboring reference orientation patches. In this method, the initial FOF was obtained by a conventional approach which itself was sensitive to noise. Therefore, this approach may not be useful when the latent is very noisy or overlaps with strong background noise. The use of prior knowledge in the GlobalDic is helpful for correcting many non-word errors. However, it cannot correct the real-word errors because the spatial distribution of orientation patches is not taken into account, i.e., the orientation patch is real but its presence at that location is impossible, as shown in Fig. 4(c). The FOF estimated by short time Fourier transform (STFT) [52] in Fig. 4(b) contains many non-word errors as marked by the red box, while the FOF estimated by GlobalDic in Fig. 4(c) contains real-word errors as marked by the yellow box.

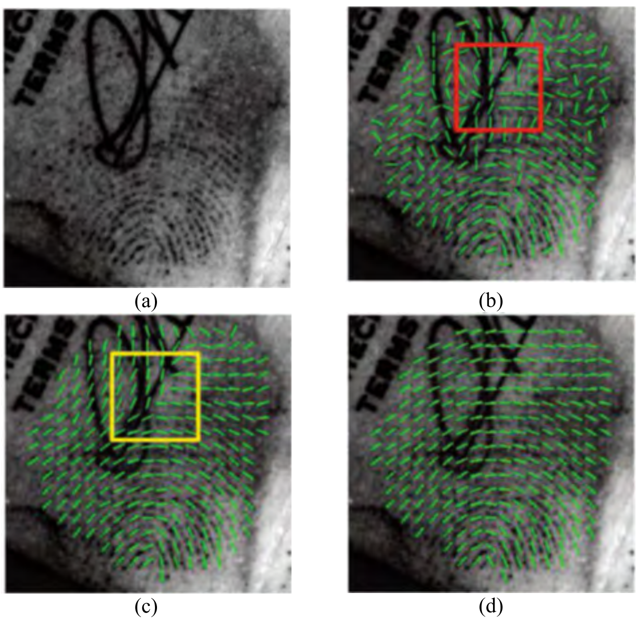

FIGURE 4. Comparison of FOF estimation extracted by three different algorithms; (a) original latent fingerprint; (b) STFT in [52]; (c) GlobalDic in [98]; and (d) LocalizedDic in [99].

To overcome the limitations in [98], instead of a single dictionary, Yang *et al.* [99] constructed a set of localized dictionaries based on the fact that ridge orientations in different regions of fingerprints have different characteristics. Therefore, it is improper to estimate the whole FOF using a single dictionary. They introduced a stronger prior knowledge of fingerprints in [99]. Each dictionary in LocalizedDic contains only orientation patches which are likely to appear at the corresponding location. It aims to reduce both non-word errors and real-word errors in estimating FOF, as shown in Fig. 4(d), neither of the two types of errors is present in the FOF. Obviously, The FOF estimated by LocalizedDic is much better than that reconstructed by GlobalDic due to the introduction of position-dependent dictionaries. The procedure of the algorithm in [99] is the same as the one in [98] except that the pose of the fingerprint needs to be known in [99]. It is a big obstacle to the LocalizedDic. A probabilistic voting algorithm is used to estimate fingerprint pose. But it is inevitable that the performance of this method highly depends on the fingerprint pose estimation. In addition, similar to [98], the registration step again is based on the initial FOF which is not reliable.

Jain and Cao [100] summarized the role of the two types of dictionaries, i.e. GlobalDic and LocalizedDic, as representations of prior knowledge about fingerprint patterns, and showed how these dictionaries can be used in latent segmentation and enhancement. GlobalDic and LocalizedDic algorithms both show promising performance in correcting and smoothing FOFs. However, both GlobalDic and LocalizedDic may encounter inaccurate estimations around singularities, as shown in Fig. 5 (orientations different from ground truth by more than 20 degrees are marked in red). Chen *et al.* [101] extended the GlobalDic to a multi-scale version, we call it as MultiSDic. The motivation is that small scale dictionary is more accurate while large scale dictionary is more robust against noise. The proposed algorithm also includes an off-line dictionary construction module and an on-line estimation module. In off-line stage, the separate dictionaries for each scale are learnt by using the corresponding orientation patches of the same scale, and the process is similar to [98]. In the on-line stage, similar to [98] and [99], the FOF is estimated by three main steps, namely, initial FOF estimation, multi-scale dictionaries lookup and context-based correction. But unlike [98] and [99], after dictionary lookup, the FOF reconstruction is formulated as a multi-layer MRF model and then it can be optimized by minimizing an energy function $E(\mathbf{r})$ defined as

$$E(\mathbf{r}) = E_s(\mathbf{r}) + \omega_{cn} E_{cn}(\mathbf{r}) + \omega_{cl} E_{cl}(\mathbf{r}) \tag{28}$$

where $E_s(\mathbf{r})$ denotes the similarity term, $E_{cn}(\mathbf{r})$ denotes the compatibility between neighboring candidate patches and $E_{cl}(\mathbf{r})$ refers to the compatibility across different layers. ω_{cn} and ω_{cl} are the weights of compatibility terms. The proposed method integrates the different scales of orientation patches information into multi-scale dictionaries and thus improves the accuracy of orientation estimation.

GlobalDic, LocalizedDic and MultiSDic are based on orientation patches and they ignore the ridge structure information. Instead of using orientation patch dictionaries, the ridge structure dictionary (RidgeSDic) are learnt from a set of high quality fingerprint patches directly and then used to recover the fingerprint ridge structure from noisy patches [102]. In order to construct a reliable and robust ridge structure dictionary, a large number of high quality fingerprint patches are selected to build up a training set. Constructing the RidgeSDic involves the solution of following optimization problem

$$\begin{aligned} \mathbf{D} = \arg\min_{\mathbf{D},\Gamma} \|\mathbf{P} - \mathbf{D}\Gamma\|_F^2 \\ \text{s.t } \forall k \ \|\gamma_k\|_0 \leq L, \quad k = 1, 2, \cdots K \end{aligned} \tag{29}$$

where $\mathbf{P}$ is a training set, K is the number of atoms of dictionary, Γ is the sparse representation matrix, γ_k is the kth column of Γ. The K-SVD algorithm [103] is employed to

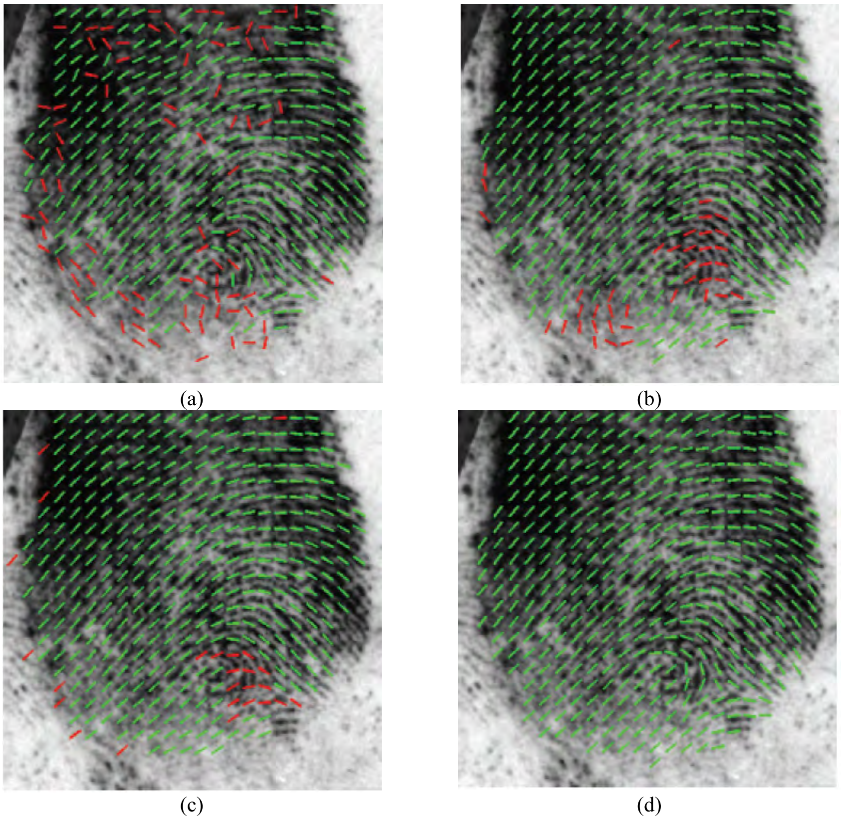

FIGURE 5. FOF of a latent fingerprint obtained by (a) STFT in [52]; (b) GlobalDic in [98]; and (c) LocalizedDic in [99]; (d) manual method (ground truth), respectively.

solve the optimization problem. Once the dictionary $\mathbf{D}$ is constructed, the reconstructed patch $\hat{\mathbf{p}}$ for a given texture image patch $\mathbf{p}$ can be obtained by solving the following optimization problem using orthogonal matching pursuit [104].

$$\hat{\gamma} = \arg\min_{\gamma} \|\mathbf{p} - \mathbf{D}\gamma\|_2^2 \quad \text{s.t } \|\gamma\|_0 \leq L \tag{30}$$

Generate the reconstructed patch $\hat{\mathbf{p}}$ according to the optimal solution $\hat{\gamma}$ to Eq. 30

$$\hat{\mathbf{p}} = \mathbf{D}\hat{\boldsymbol{\gamma}} \tag{31}$$

The orientation field of reconstructed patch $\hat{\mathbf{p}}$ can be obtained by using the approach described in [2]. Quite evidently, the proposed algorithm in [102] is a fingerprint enhancement technology based on dictionary learning over sparse representation in essence. It firstly reconstructs the fingerprint patch using RidgeSDic, and then estimates the orientation field using traditional method. Liu *et al.* [105] proposed a similar FOF estimation algorithm based on dictionary learning and sparse coding. But the algorithms in [102] and [105] are different in nature from dictionary learning to orientation estimation. In dictionary learning stage, the approach used in [105] is similar to [98]. A large number of high quality fingerprint patches are selected, and all patch orientations are estimated by the traditional gradient based method and they are used to build the training set. An initial redundant dictionary is constructed, a greedy algorithm is employed to select a set of reference orientation patches from training set, and thus construct the dictionary $\mathbf{D}$. The orientation estimation procedure in [105] is similar to [102], where the fingerprint texture component is obtained by decomposing the original fingerprint with the total variation model. Instead of searching the similar orientation patch from the dictionary [98], Liu *et al.* [105] modeled the fingerprint orientation estimation as a sparse coding and reconstruction problem over the orientation dictionary, we call it as SparseCod. It can be described as the following optimization problem

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\gamma} \left\{ \|\mathbf{p} - \mathbf{D}\boldsymbol{\gamma}\|_2^2 + \lambda \|\boldsymbol{\gamma}\|_1 \right\} \tag{32}$$

where λ is a regularization parameter which balances the trade-off between the sparsity of the solution and the fidelity of the approximation to $\mathbf{P}$. The estimated orientation patch $\hat{\mathbf{p}}$ can be obtained according to Eq. 31. To capture the prior knowledge of various orientation patterns, multi-scale dictionaries are learned for iterative estimation of local ridge orientations on the corresponding scale texture image.

However, the above dictionary-based methods have a common drawback that the dictionaries learnt from high quality fingerprints may not work well on poor quality fingerprints (see Fig. 6 (b) and (c)). Therefore, an intuitive idea is to directly learn orientation fields on large number of fingerprint

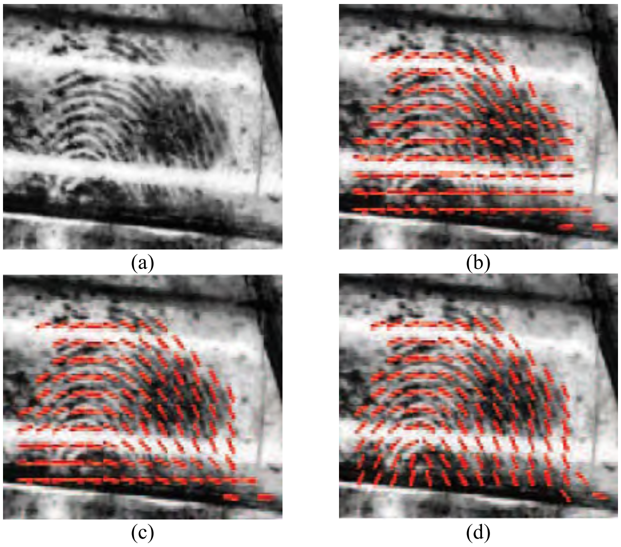

FIGURE 6. A comparison of latent FOF estimated by different algorithms. (a) a latent fingerprint; (b) GlobalDic in [98]; (c) RidgeSDic in [102]; (d) CNN in [106] and [107].

patches of poor quality. Cao and Jain [106], [107] proposed a convolutional neural network (CNN) based FOF estimation algorithm by posing orientation field estimation of a latent patch as a classification task. They classify the orientation field of a latent patch as one of a set of representative orientation patterns using a CNN. CNN has the capability to learn distinctive features directly from the input images. The FOF estimated from CNN generally performs better than dictionary based methods (see Fig. 6(d)). In the proposed method, target labels for the classes are a selection of 128 characteristic orientation fields, which have to be carefully selected in advance. However, the quality of pattern of target labels is directly determined by the quality of training database, so the reliability of dictionary largely depends on the training database. Schuch *et al.* [108] proposed to train CNNs as a regression to estimate the FOF, namely the ConvNetOF. Compared to Cao *et al.*'s classification approach [106], regression is a more natural approach for the estimation of continuous values (The local ridge orientation in fingerprint image is a continuous value). In addition, the proposed method doesn't need to select the target patterns of characteristic orientation fields. Further, Schuch *et al.* [109] incorporated Deep Expectation into CNN and trained a CNN to estimate FOF called DEX-OF. Compared to LocalizedDic and ConvNetOF, DEX-OF has better performance. Tang *et al.* [110] first transformed traditional FOF estimation method to convolutional kernels and integrated it as shallow network, and then expanded it to the deeper version using some convolutional layers to enhance its representation ability. Qu *et al.* [111] recently proposed a deep regression neural network (DRNN) to estimate FOF. Outputs of the network are directly the predicted orientations. The authors claimed that the proposed algorithm had exhibited a higher accuracy and faster training speed. Turroni *et al.* [112] implemented and tested several well-known FOF estimation methods and proved that parameter optimizations, pre- and post-processing stages could remarkably improve accuracy of the baseline methods on bad quality fingerprints. The authors proposed an adaptive method which selectively exploited accuracy of local-based analysis and learning-based global methods, thus could obtain more reliable FOF from poor quality fingerprint.

Learning-based methods have shown improvements of the performance in FOF estimation, especially when the traditional methods fail to work in the cases of poor quality fingerprints (such as latent fingerprints). However these methods also have inherent limitations: 1) the dictionaries are learnt from the initial orientation fields which themselves are not reliable; 2) most of algorithms rely on the high quality fingerprints to learn dictionaries, and they may fail on poor quality fingerprints; 3) Human interventions are often required during the process of algorithms executions; 4) the approaches normally have a high computational complexity.

V. PERFORMANCE EVALUATIONS

This section is devoted to performing experimental evaluations on the most important FOF estimating algorithms. We expend a great deal of effort trying to provide a comparative study between these methods. The FOFs estimated by these algorithms are evaluated via SPs detection, fingerprint matching, and accuracy of FOF estimation.

A. SINGULAR POINTS DETECTION

The SPs are an important feature in characterizing the fingerprint structure, thus SPs detection is an important indicator to measure FOF estimation performance. The public fingerprint verification competition database FVC2004 DB1_A is used to analyze the performance of the selected algorithms. It contains 800 fingerprint images acquired by an optical sensor under uncontrolled environment that may result in poor quality fingerprints. A Poincare index algorithm is used to detect the SPs. Performance of SPs detection can be evaluated by the following metrics:

$$\text{Precision} = \frac{TP}{TP + FP} \tag{33}$$

$$\text{Recall} = \frac{TP}{TP + FN} \tag{34}$$

$$\text{F-measure} = \frac{2 \times \text{Recall} \times \text{Precision}}{\text{Recall} + \text{Precision}} \tag{35}$$

where TP, FP, and FN represent true positive (the number of correctly detected SPs), false positive (the number of spuriously detected SPs), and false negative (the number of lost SPs), respectively.

Precision measures the rate of detected SPs that are the correct ones. Recall describes the proportion of correctly detected SPs. F-measure is the trade-off between Precision and Recall. Well performed FOF estimation will be indicated by good SPs detection where high values of Precision, Recall and F-measure are achieved. In fact, the average localization

FIGURE 7. **A comparison of SPs detection by various algorithms. (a) Precision; (b) Recall; (c) F-measure; (d) ALError.**

error (ALError) is also used for evaluating the performance of SPs detection, and the smaller the better. The localization error of a singular point is given by the Euclidean distance between its detected location and the true location, i.e. the manually annotated location.

Fig. 7 shows a group of comparison between the selected methods, and they are all based on mathematical models. It can be found that the performance of the core point detection is better than delta point detection in all algorithms. Tao *et al.* [76] used the HCS to remove isolated noises in orientation field, and further incorporate the normalized HCS as weighted value into FOMFE [73] to take into account the quality difference of the patches. We can see from Fig. 7 that the performance of the method presented in [76] is better than that in [73]. Hou *et al.* [88] used the saliency of singular points as weights and its value increases with respect to the increase of the saliency. It can obtain better performance than the methods based on orthogonal polynomials in most cases. Further, we can find that the method in [83] has a better performance than the two methods in [73] and [76]. It illustrates that the higher weights should be assigned to the areas that is closer to the SPs. In addition, the lower weights should be assigned to the areas with lower quality, and thus improve the reliability of FOF estimation and reduce the number of false positives. The algorithm proposed in [84], by contrast, shows a better performance. The main reason may be that both the regularization and weighted modeling are used in [84].

B. FINGERPRINT MATCHING

As described in Section 1, FOF estimation plays a key role in AFIS. The accuracy of an FOF estimation can largely affect the final fingerprint matching performance. The final performance of an AFIS is assessed by two indices: false match rate (FMR) and the false non-match rate (FNMR). The FMR index is defined as the general percentage of an imposter being falsely matched by the system. The performances of some FOF estimation algorithms have been reported in terms of the corresponding equal error rate (EER), where the FMR is equal to the FNMR. The lower the EER, the better the performance by the AFIS. In real applications, the AFIS often operate far from the EER point by decreasing the FMR in order to assure a high level of security. However, decreasing the FMR will cause the FNMR to increase. It is therefore not surprising that the performance of an AFIS is often evaluated using the indicator FMR100, which is defined as the value of the FNMR when FMR is 1%. Table 2 summarizes the matching performance for various FOF estimation algorithms.

Mathematical models-based methods in general are superior to gradient-based methods from the fingerprint matching point of view. Mathematical models-based methods have gained significant improvements over the FOF estimated by gradient-based methods. For the FVC2000 DB2-A database, we observe that there is significant improvement to fingerprint matching using proposed method [80] in comparison to competing methods in [43]. This is due to the fact that this algorithm in [80] can select adaptively the best orthogonal polynomials (Legendre, Chebyshev type I or II) for reconstructing FOF. For the FVC2002 DB1-A, the model-based algorithms can obtain significant improvements from the coarse FOF estimated by gradient-based. The algorithms in [67] and [68] has reduced the FMR100 to only a quarter of that with gradient-based methods [21], [43]. The algorithm in [73] further reduces the FMR100 by about 60%. For the FVC2004 DB2-A the proposed method [81] is better at delivering the matching performance than the method proposed in [79]. The EER is 6.280% by the method proposed in [79]. By using the approach proposed in [81], this measure could be reduced to 5.410%, achieving a relative improvement of 14%. The FOF estimation method [81] play a more effective role in preserving true SPs. For the FVC2004 DB3-A, the EER is 2.28% using the approach [73]. The EER is reduced to 1.81% by using the method proposed in [79], giving a relative improvement of 21%. There is a significant improvement on SPs preservation in the region with presence of true SPs [79], but it could fail to distinguish some false SPs due to noise from the true ones. Consequently, some structures around these false SPs are recovered as well, leading to the degradation of the matching accuracy. For the FVC2006 DB2-A, since the algorithm proposed in [112] has both characteristics of the local-based analysis technique and the learning-based global methods, this method achieves better performance than others.

C. ACCURACY OF FOF ESTIMATION

The accuracy of an orientation field estimation algorithm can be measured by average root mean square deviation (RMSD) [112] which is defined as

$$\mathrm{RMSD}\left(\mathbf{D},\mathbf{G}\right)=\sqrt{\frac{\sum_{(i,j)\in\mathbf{F}}\Delta\varphi\left(\theta_{i,j}^{d},\theta_{i,j}^{g}\right)^{2}}{|\mathbf{F}|}} \tag{36}$$

where $\mathbf{G}$ is the ground truth FOF, $\mathbf{F}$ is the valid orientation elements of $\mathbf{G}$ which belongs to fingerprint foreground, $|\mathbf{F}|$ is the total amount of elements in $\mathbf{F}$, and $\Delta\phi\left(\theta_1,\theta_2\right)$ is the difference between angles θ_1 and θ_2.

TABLE 2. Performance comparison of different FOF estimation algorithms on FVC databases.

Database		Algorithm	EER(%)	FMR100(%)
FVC2000	DB2-A	[43]	9.480	-
		[80]	6.330	-
FVC2002	DB1-A	[21, 43]	-	42.40
		[67, 68]	-	10.40
		[73]	-	3.60
FVC2004	DB1-A	[73]	3.920	6.78
		[76]	3.680	6.57
	DB2-A	[79]	6.280	-
		[81]	5.410	-
	DB3-A	[43]	8.850	-
		[73]	2.280	-
		[79]	1.810	-
		[81]	3.630	-
FVC2006	DB2-A	[49]	0.796	-
		[53]	0.374	-
		[73]	0.680	-
		[95]	0.722	-
		[79]	0.664	-
		[112]	0.206	-

Given a dataset $\mathbf{X}$ of n fingerprints with the corresponding ground truth FOFs, the average RMSD can be denoted by

$$\mathrm{AvgErr}\left(\mathbf{X}\right)=\frac{1}{n}\sum_{(\mathbf{D},\mathbf{G})\in X}\left(\mathrm{RMSD}(\mathbf{D},\mathbf{G})\right) \tag{37}$$

The latent database NIST SD27 [114] is used for comparison. This database contains 258 latent fingerprints which are classified into three different qualities, i.e. Good, Bad and Ugly. The number of latent fingerprints in three categories are 88, 85 and 85, respectively. The ground truth FOFs of the NIST SD27 and manually marked region of interest (ROI) were provided in [98]. The performances of FOFs estimation on the NIST SD27 latent fingerprint database are reported. Average RMSDs of the nine algorithms (STFT [53], FOMFE [73], LocalizedDict [99] GlobalDict [98], manually marked pose (ManuMark) [99], MultiSDic [101], RidgeSDic [102], SparseCod [105], and CNN [106], [107]) are computed both for the overall NIST SD27 database and three subsets belonging to three quality levels (Good, Bad and Ugly). Fig. 8 gives the experimental results of the various methods for visual inspection. Table 3 summarize the accuracy of FOF for various methods.

From Fig. 8 and Table 3, we can find that the learning-based algorithms have obvious advantages in estimating bad quality fingerprints orientation fields. The Localized-Dic (manually marked pose) [99] obtains a slightly better result than LocalizedDic (automatically marked pose) [99]. Unsurprisingly, MultiSDic [101] achieves better accuracy than GlobalDic [98] because it integrates information from different scale orientation fields. SpareseCod [105] achieves lower RMSD than the other algorithms in [52], [73], [98], and [102]. The LocalizedDic [99], which applied the localized dictionaries to make full use of the prior knowl-

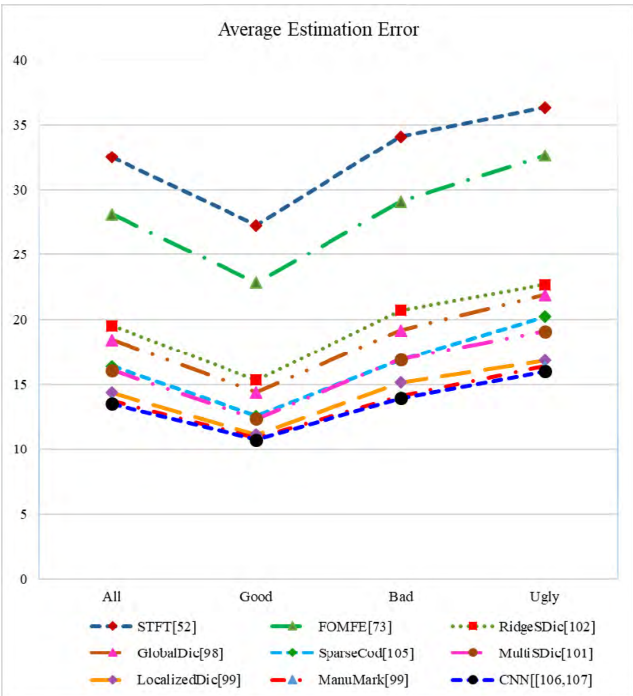

FIGURE 8. A comparison of RMSDs of various FOF estimation algorithms on NIST SD27 latent database.

TABLE 3. RMSDs of different FOF estimation algorithms on NIST SD27 latent database.

Methods	All	Good	Bad	Ugly
STFT[52]	32.51	27.27	34.1	36.36
FOMFE[73]	28.12	22.83	29.09	32.63
RidgeSDic[102]	19.53	15.34	20.7	22.68
GlobalDic[98]	18.44	14.4	19.18	21.88
SparseCod[105]	16.38	12.57	16.88	20.22
MultiSDic[101]	16.09	12.35	16.94	19.11
LocalizedDic[99]	14.35	11.15	15.15	16.85
ManuMark[99]	13.76	10.87	14.12	16.4
CNN[106, 107]	13.51	10.76	13.94	16

edge of fingerprints, obtains lower RMSD than SpareseCod. However, the pose of latent fingerprint needs to be reliably estimated in LocalizedDic. As reported in [99], the pose estimation is computationally expensive in both the offline and online stages. SpareseCod does not require the estimation of fingerprint pose, which can save more computation cost. CNN [106], [107] outperforms the other eight algorithms on latents of different quality levels, and it is even slightly better than LocalizedDic [99], where manually marked pose is used.

VI. CONCLUSIONS

In this paper, we have given a comprehensive survey of the methods on FOF estimation. We have collected and carefully studied the most relevant works in the scientific literature about algorithms and methods used in fingerprint orientation field estimation. We first described the background in this field. Then, we studied the main technologies of the FOF estimation algorithms, as well as the underlying strategies which fundamentally characterize the distinction between different methods: gradient-based methods, mathematical models-based methods and learning-based methods. Finally, we gave an insight into various methods and also highlighted their advantages and limitations. In our opinion, discovering and explaining benefits and limitations of the currently used FOF estimation methods is of essential importance in fingerprint identification, because without a full understanding of the nature of these methods, it is difficult to identify the most essential FOF estimation issues that remain to be solved.

These literatures discussed in this paper contain many good ideas related to FOF estimation, some of them are quite similar and even overlapped. In this paper, we have made an in-depth investigation and analysis, together with empirical studies, to some commonly known methods, trying to explain and summarize the differences observed in the related literatures. However, this task is difficult due to the fact that there lack detailed descriptions to the relevant algorithms, and it is extremely difficult to evaluate a given method fully, especially when the information relating to the values of the parameters involved have not been clearly described by the authors. However, the systematic quantitative evaluations of different methods is essential, and a continued research activity in this area will eventually introduce the FOF estimation benchmark, which will provide both the database and the testing protocol. If we can achieve the target, it has great significance for the development of FOF estimation techniques and technologies.

Most of the FOF estimation methods introduced in the last decade are learning-based. One of the reasons to expect learning-based algorithms to perform well is the sheer amount of research done on this approach. Some of them have exhibited a good robustness against poor-quality fingerprints, such as latent fingerprints, where none of the traditional algorithms, such as gradient-based and mathematical models-based can compete in this aspect. Of course, there does not exist a perfect solution. The traditional algorithms are sufficient enough to be embedded into an AFIS for many real applications when the fingerprints taken by the AFIS are of satisfactory quality. But, most learning-based algorithms are too computational intensive to be fitted into an existing AFIS for real applications. We believe this could be the next challenge for developing future FOF estimation techniques, and substantial research still needs to be done on this subject.

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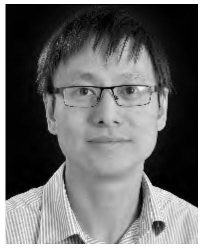

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