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Bayesian parameter inference with stochastic solar dynamo models

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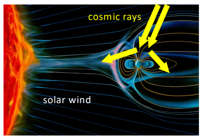
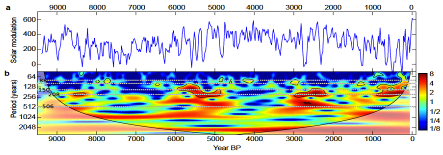
Solar magnetic activity: cosmogenic radionuclides open up new frontiers



- **Sunspots** provide the longest DIRECT record of solar magnetic activity (covering **only 400 years!**)
- Time-series of **cosmogenic radionuclides** (^{10}Be, ^{14}C) stored in natural archives such as ice cores and tree rings are a **proxy** for solar magnetic activity on **multi-millennial time-scales**
- **New data** have uncovered **exciting features** (e.g., intermittent long-period stable cycles, Grand Minima)
- Solar physicists have put great efforts into the development of sound physically based **stochastic solar dynamo models** that can qualitatively reproduce the observed features



Solar activity based on ^{10}Be and ^{14}C records

[1] J. A. Abreu et al., A&A **548** (2012).

- **OUR GOAL**: carry out the first quantitative calibration of the models to the data (**Bayesian inference**) and run model performance comparisons to shed light on the underlying mechanisms leading to the observed features.

Bayesian inference with nonlinear stochastic models can become computationally extremely expensive

ABC (Approximate Bayesian Computation)

Basic concept:

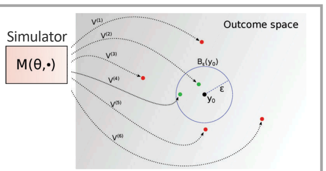
- Instead of evaluating the **expensive likelihood function** of a stochastic model, many datasets are generated from the corresponding probabilistic model, for different parameter sets, and compared with the measured data. If the match is within a certain **tolerance**, parameter sets are accepted as posterior samples.
- For computational feasibility, data needs to be compressed into few **summary statistics**.

Pros & Cons:

- ✗ Provides only **approximate results**, due to summary stats.
- ✓ Plug-n-run: only (black box) forward simulator needed.
- ✗ Computation-intense, many simulations required that need to be discarded.
- ✓ Population method, **well suited for parallel computing**.

From [3]:

- Stochastic simulator M is run multiple times with fixed parameters θ
- The proportion of outcomes less than ε away from observed data (y_0) provides an approximation to the likelihood of θ



[2] C. Albert et al., Stat. Comput. **25** (2015). [3] Lintusaari et al., Syst. Biol. **66** (2017).

HMC (Hamiltonian Monte Carlo)

Basic concept:

- The Bayesian posterior is re-interpreted as the **partition function** of an **interacting particle system**, and the **state of a stochastic model**, defined by its parameters and dynamic stochastic variables, is thus interpreted as the **configuration of a statistical mechanics** system.
- **Molecular Dynamics** algorithms are used to explore the space of **configurations compatible with measured data, model and prior**.

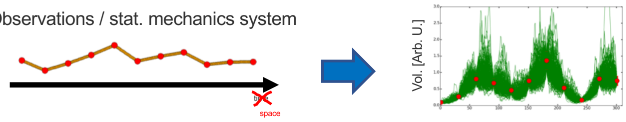
Pros & Cons:

- ✓ Provides **exact results**, no summary statistics needed.
- ✗ Requires encoding the **probability density of model realizations**, not just a forward simulator.
- ✓ Very efficient (it exploits the local shape of the posterior landscape to suppress the random-walk nature of standard MCMC algorithms).
- ✓ Local interactions, **well suited for parallel computing**.

Simple hydrological application from [4]:

Observations / stat. mechanics system

System realizations



[4] C. Albert, S. Ulzega and R. Stoop, Phys. Rev. E, **93** (2016).

Solar dynamo models: a "simple" stochastic iterative map

- Magnetic field dynamics exhibit a well-known 11y cycle that can be captured by a **stochastic iterative map**:

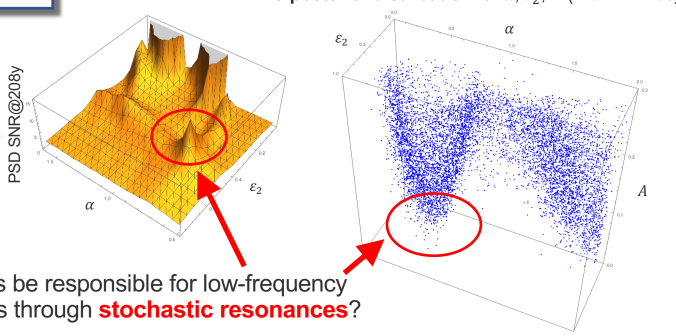
$$p_{n+1} = \alpha \cdot f(p_n) + \varepsilon_n \qquad \varepsilon_n \in [0, \varepsilon_2]$$

- **External periodic forcing** representing **planetary gravitational effects** can be included in the iterative map:

$$\tau = \sum_i A_i \sin(2\pi \frac{t}{T_i})$$

- Power spectral density at the frequencies of interest is used as **summary statistics**

ABC posterior distribution for α, ε_2, A (with $T = 208y$)



Can planets be responsible for low-frequency modulations through **stochastic resonances**?