

# SMART MANUFACTURING USING CONTROL AND OPTIMIZATION

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To Mom, Dad and Shalini

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## ABBREVIATIONS

DSM	Demand Side Management
DR	Demand Response
JSP	Job Shop Scheduling
HVAC	Heating, Ventilation, Air
LP	Linear Programming
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
IP	Integer Programming
MPC	Model Predictive Control
NIST	National Institute of Standards and Technology
LQR	Linear Quadratic Regulator
OLS	Optimal Load Scheduler
CLS	Compressor Load Scheduler
PSO	Particle Swarm Optimization
LS	Load Shifting
VF	Valley Filling
PAR	Peak to Average Ratio
PID	Proportional Integral Derivative
PLC	Programmable Logic Control
Min	Minimum
Max	Maximum
DOE	Department of Energy
RC	Resistance Capacitance
VFD/VSD	Variable Frequency/Speed Drive
HCL	Hydrochloric Acid

EM	Electro Magnetic
CFM	Cubic Foot per Minute
SEU	Significant Energy Use

## ABSTRACT

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Energy management has become a major concern in the past two decades with the increasing energy prices, overutilization of natural resources and increased carbon emissions. According to the department of Energy the industrial sector solely consumes 22.4% of the energy produced in the country [1]. This calls for an urgent need for the industries to design and implement energy efficient practices by analyzing the energy consumption, electricity data and making use of energy efficient equipment. Although, utility companies are providing incentives to consumer participating in Demand Response programs, there isn't an active implementation of energy management principles from the consumer's side. Technological advancements in controls, automation, optimization and big data can be harnessed to achieve this which in other words is referred to as "Smart Manufacturing" . In this research energy management techniques have been designed for two SEU (Significant Energy Use) equipment HVAC systems, Compressors and load shifting in manufacturing environments using control and optimization.

The addressed energy management techniques associated with each of the SEUs are very generic in nature which make them applicable for most of the industries. Firstly, the loads or the energy consuming equipment has been categorized into flexible and non-flexible loads based on their priority level and flexibility in running schedule. For the flexible loads, an optimal load scheduler has been modelled using Mixed Integer Linear Programming (MILP) method that find carries out load shifting by using the predicted demand of the rest of the plant and scheduling the loads during the low demand periods. The cases of interruptible loads and non-interruptible have

been solved to demonstrate load shifting. This essentially resulted in lowering the peak demand and hence cost savings for both “Time-of-Use” and Demand based price schemes.

The compressor load sharing problem was next considered for optimal distribution of loads among VFD equipped compressors running in parallel to meet the demand. The model is based on MILP problem and case studies was carried out for heavy duty ( $>10\text{HP}$ ) and light duty compressors ( $\leq 10\text{HP}$ ). Using the compressor scheduler, there was about 16% energy and cost saving for the light duty compressors and 14.6% for the heavy duty compressors.

HVAC systems being one of the major energy consumer in manufacturing industries was modelled using the generic lumped parameter method. An Electroplating facility named Electro-Spec was modelled in Simulink and was validated using the real data that was collected from the facility. The Mean Absolute Error (MAE) was about 0.39 for the model which is suitable for implementing controllers for the purpose of energy management. MATLAB and Simulink were used to design and implement the state-of-the-art Model Predictive Control for the purpose of energy efficient control. The MPC was chosen due to its ability to easily handle Multi Input Multi Output Systems, system constraints and its optimal nature. The MPC resulted in a temperature response with a rise time of 10 minutes and a steady state error of less than 0.001. Also from the input response, it was observed that the MPC provided just enough input for the temperature to stay at the set point and as a result led to about 27.6% energy and cost savings. Thus this research has a potential of energy and cost savings and can be readily applied to most of the manufacturing industries that use HVAC, Compressors and machines as their primary energy consumer.

## 1. INTRODUCTION

With the growing population and incessant demands, energy management and conservation has become a major challenge in the smart grid. Demand side management programs are being initiated around the globe so as to reduce the overall energy load and emissions that pose a threat to the non-renewable forms of energy and environment respectively. As a result, in the recent years, there has been an exponential increase in the interest for energy management research. According to energy.gov, the Department of Energy of United States spends approximately \$5.9 on energy research for clean and better utilization of energy resources. Besides the United States, South Korea and Germany have been actively implementing “smart” manufacturing techniques to optimize production, energy consumption and cost in response to this activity [2]. Process and other energy intensive industries have already resorted to smart systems to run plants in an economical and productive manner. The purpose of this thesis is to investigate and implement the potential overlooked energy saving practices for major energy consuming systems in manufacturing industry using optimization and control.

### 1.1 Smart Manufacturing

Smart manufacturing is a type of manufacturing where the optimized techniques and processes are used to obtain maximum yield while keeping the energy footprint and costs low. This is made possible with the advanced modelling, controls, optimization, and big data that has been on rise in the past decade. In fact smart manufacturing is regarded as the industrial revolution 4.0 as a result of this. According to The National Institute of Standards and Technology (NIST) [3], Smart Manufacturing systems are fully integrated, collaborated manufacturing systems that

respond in real time to meet changing demands and conditions in the factory in the supply network and customer needs. This is exactly what this thesis is attempts to achieve by using tapping the energy management techniques using control and optimization.

## **1.2 Demand Side Management**

Demand Side Management refers to the energy measures taken from the demand side (consumer) to reduce the electricity bills and utility infrastructure costs. This is usually done by shifting or scheduling the consumption of energy from high demand periods to low demand ones. For example costumers could use renewable resources or energy storage devices like batteries for their energy needs during the high demand periods. Another simple yet effective way would be to prioritize the energy needs and schedule the low priority energy needs during the off peak periods. DSM can also be implemented at subsystem level by carrying out energy audits to find out potential energy saving methods, installing energy efficient equipment like VFDs, improving the schedule of machines, upgrading the control systems of the energy demanding systems such as the HVAC. The following figure shows how load shifting can be used to smoothen the peak demand and hence the demand based charges which is one of the most commonly used DSM techniques.,

## **1.3 Research Milestones**

Problem Statement: The HVAC systems together with the air compressors and electric motors consume more than half of the total energy in the manufacturing sector. This significant share of energy consumption is a result of inefficient energy management practices which in turn strain the utility companies and increase the utility bills and carbon footprint. Hence, there is a dire need of optimizing the energy consumption through energy management and energy efficient control and optimization systems

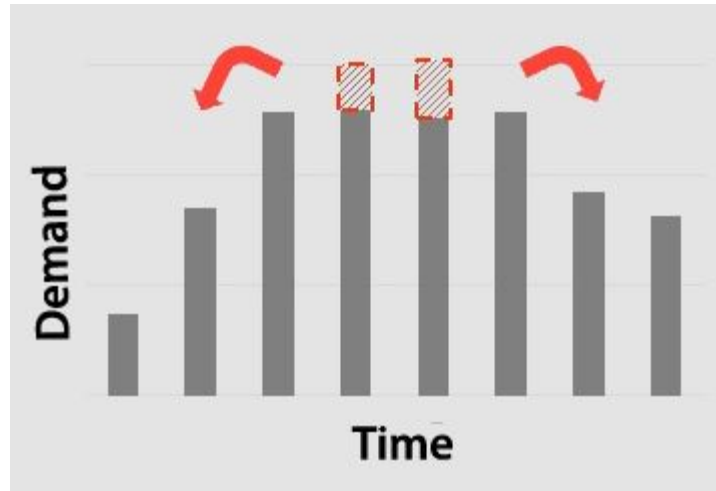


Figure 1.1. Demand side management by load shifting [4]

Major objectives in this research are,

- Identification and selection of potential energy management techniques and processes to increase energy efficiency and reduce costs
- Development of a mathematical model that can schedule the flexible machines with the help of the Demand Side Management.
- Development of a compressor scheduler that can distribute the loads between compressors so that the demand is satisfied in a cost effective manner
- Generic modelling of a manufacturing facility using lumped parameter modelling and validation using real data
- Using state-of-the-art MPC to reduce the total energy consumption by the HVAC fans while meeting the temperature requirements of the manufacturing facility

As per the U.S Energy Information Administration, the manufacturing sector consumes about 75% of all the energy used by the industrial sector which translate

to 20,008 TBtu and 1064 MMT Co<sub>2</sub> emissions [5]. In both the energy intensive and non energy intensive manufacturing, the primary consumers of energy in the nation are HVAC and manufacturing systems. In fact, HVAC alone consumes 62% of the total non-manufacturing energy [6]. Manufacturing systems mainly include assembly line or production line machines that are powered by electric motors.

Compressed air systems that are widely used for drying, power tools and painting applications account for almost 10% of all the electricity used in the country according to energy.gov [7]. Smart manufacturing and demand response strategies together can improve the energy efficiency of HVAC, compressors and production lines. Systems can be optimized at system and component level using control and optimization which are a subset of smart manufacturing. Manufacturing industries want high quality and energy efficiency to keep their throughput and profits high and costs low. When compared to residential and commercial consumers, industrial consumers are less flexible to scheduling due to throughput and time constraints. Therefore there is a need to consider problems that are flexible and non-flexible in terms of scheduling.

Optimally scheduling the production line machines can reduce the peak energy consumption in manufacturing. Abdul et al [8] have developed an energy management model that adjusts the set points of controllable equipment in response to real time pricing i.e demand shifting. However, the pricing has to be notified to the consumer 1 hour in advance and a complex optimization problem is solved every two hours which is computationally expensive. Also, the forecasted data has been assumed to be accurate enough. In [6], a demand response model was used to schedule production line by taking into account its heat transfer characteristics and thereby controlling the HVAC system. The outcome is optimal schedule for the production line and total reduction in peak production without the violation of production and comfort constraints. Although, the heat transfer was well modelled, the HVAC system was vaguely modelled by converting the temperature to heat load and without considering other temperature disturbances. Also, this implementation requires temperature forecast of the building which is quite challenging in a manufacturing environment.



Another commonly used technique for production line scheduling is job shop scheduling where the objective is to assign  $n$  jobs to  $m$  machines of different processing times such that the make span is minimum. This has been extended to achieve minimum energy use by et al [9], by adding a secondary cost function that takes into account the idle time, processing time and startup time of machines. A similar approach has been taken by the authors in [10], to optimize the energy consumption and make span using a biologically inspired Particle Swarm Optimization (PSO). Both of these works lack a generic nature in the problem formulation which is essential for production line problems that can be different types. Also non-flexible scheduling was not considered in either of these works. Authors in [11] used a Discrete event model to model machines and buffer in an manufacturing environments and suggested Model Predictive Control as a method to reduce the energy consumption while meeting the make span requirement. However they didn't show the how the effective processing time can be calculated and any of the results obtained by the MPC implementation.

Some works have tried to integrate smart manufacturing and demand response. In the smart manufacturing paradigm, demand side management is one of the major energy saving measure that has been actively researched and implemented in the past decade. The most commonly proposed solution to the decrease the peak demand is the penetration of renewable resources. For example, in [12] the authors have used a hierarchical Model Predictive Control system for the utilization of renewable resources when there is peak demand as a part of active demand side management. They have carried out load shifting for this purpose using the weather forecast, price signals and the MPC for a case study involving a residential building. In [13], a Demand Response (DR) model has been proposed for cases where there are additional renewable power generation units like solar generator and wind generator. Using a model that is based on Mixed Integer Linear Programming (MILP), they were able to optimally shift loads to reduce peak demand and use renewable energy to meet demands that cannot be compromised. In both these works, the high costs of installation and maintenance of these renewable resources and their uncertainty in the delivering power outweighs

the energy saving cost. Particle Swarm Optimization was used by Hashemi et al in [14] to solve for optimal schedule of loads in residential, commercial and industrial sectors. This work lacks the consideration of non-flexible loads which are the most common loads in the manufacturing sector.

The design of HVAC systems is much more complicated in manufacturing sector than in commercial and residential sectors due to the uncertainties and stochastic temperature disturbances in manufacturing make the modelling and control of the thermal system quite intricate. Therefore, there needs to a system level modelling approach that is generic enough to be applied in all the manufacturing environments yet be able to capture the necessary heat transfer dynamics for the control system design. One such commonly used model is the “Resistor-Capacitor lumped parameter” model that is simple yet robust enough to handle disturbances [15]. The application of Economic-MPC (MPC with custom cost function involving price and energy) in building HVAC system was explored in [16] where a RC thermal model was established and energy and pricing were minimized. Though, the Particle Swarm Optimization (PSO) and EMPC were quite effective in avoiding suboptimal solutions (energy optimal schedules) and converging quickly as per these works, the objective function that was minimized by the PSO was dependent on the time varying prices rather than the demand which is not the case of states like Indiana where the consumers are charged based off their peak demand. Real time optimization based on economy and MPC were combined together to create E-MPC for a chemical plant in [17] with storage units. Again The E-MPC accounted for the product market pricing rather than the power usage of the energy consuming subsystems. Black box modelling (input-output data based modelling) was used extensively with the help of system identification tool in MATLAB to builds thermal state space models for MPC in [18]. Though this may seem like a plausible option, only the energy data is readily available in most the industries. A supervisory architecture was used to facilitate demand side management by using the plant wide optimization in conjunction with MPC in HVAC and refrigeration systems to reduce the peak demand [19]. The above implementations are

just focused on one of the systems instead of consider the entire manufacturing plant. In [20], MILP was used in conjunction with MPC for energy management in micro grids. However this is only applicable for small size micro grids with storage systems. Most of the works related to MPC for building energy management in literature have been done on residential and commercial buildings rather than manufacturing facilities where there is a huge potential for energy savings. This has been addressed in this thesis by modelling a real manufacturing facility using a generic modelling method, validating it and then implementing MPC for energy and cost savings.

In manufacturing, different types of air compressors are used for a variety of purposes. Compressors have various control schemes such as Start/Strop, Load/Unload, Modulating, Variable displacement and Variable speed. Out of these, Variable displacement control (Variable Speed Drive, VSD) is the most efficient control as it varies the motor operation based on the load and avoids unwanted motor loads. As per GAMBICA (UK's Trade Association for Instrumentation, Control, Automation and Laboratory Technology) [21], VSD's can help lower the energy consumption by 50% for air compressors. A further step is to optimally schedule the already efficient VSD compressors in parallel to further lower the energy footprint. Guise et al [22] proposed a novel control scheme based on MPC that lowers the upper and lower pressure set points and reduces the energy consumption for a VSD screw compressor. The shortcoming of this implementation is that the air flow requirements need to be forecasted for a given time horizon which was left out as a future scope. In [23], the authors have devised a framework that can optimally distribute the load between parallel compressors using MILP formulation. In this work, compressor maps were used to make sure that compressor are run at optimal conditions and MILP was used to optimally switch between compressor with minimal surge. Though this implementation yielded positive results, this is only suitable for applications involving only centrifugal compressors. In [24] authors have used such an approach for compressor scheduling problem. Given N number of compressors, their objective was to schedule the compressors to different clients based on their flowrate and pressure limits such

that the total cost of installing and operating the compressors is minimized. The compressors that have been considered in this work were based on Start/Stop control which is one of the most energy inefficient control methods for compressors. Hanh et al [25] used genetic algorithms for scheduling compressors to different customers for gas pipeline operations. The idea is to select the optimal set of compressor to operate so that the fuel cost, start-up cost and gas cost is minimal. Neural networks were used for demand forecasting and genetic algorithms were used to find the minimum of the objective function. The study however uses penalties on the objective functions to include constraints and genetic algorithms always don't provide optimal solutions like MILP. To the author's knowledge, there exists no work that has been done on the scheduling of VSD compressors in parallel setting.

From the previous works on energy management and energy efficiency for smart manufacturing, it can be clearly seen that the previous works lack the implementation of generic MPC in manufacturing, demand based scheduling systems and compressor load distribution systems. In the thesis, this gap has been addressed by taking an energy oriented control and optimization approach that is well suited for a manufacturing environment.

## 2. SCHEDULING PROBLEM

### 2.1 Linear Programming

The purpose of this chapter is to introduce the readers to the concept of scheduling. In a typical scheduling problem, the goal is to find a set of assignments to machines so that a given objective is minimized or maximized. This is a common problem that is solved in the field of Computer Science and is referred to as Job Shop Scheduling (JSP). In such a problem, there are “m” jobs that need to be completed by “n” machines of different processing times and powers. The objective is the assignment of these jobs to the machines such that the jobs are completed in the least amount of time and effort (power consumption).

Scheduling problems are usually solved using a mathematical optimization technique called Linear Programming (LP) [26]. In a LP problem, a cost that is function of a set of variables is minimized or maximized and is subjected to variable constraints. The constraints are linear and can be inequalities or equalities or both. The feasible region is determined by the constraints. The solution to the linear programming problem is the values of the variables provide the “best” possible value of the cost. Example problem 1 illustrates how LP works

$$\begin{aligned}
 &\text{Maximize } Z = 3x + 2y \\
 &\text{Subject to } 4x + 2y \leq 15 \\
 &\qquad\qquad x + 2y \leq 8 \\
 &\qquad\qquad x - y \leq 2 \\
 &\qquad\qquad x \geq 0 \\
 &\qquad\qquad y \geq 0
 \end{aligned} \tag{2.1}$$

There can be variations in the LP problem such as Integer Programming and Mixed Integer Programming. As the name suggests, integer programming is a LP problem with variables that can take only integer values and the MILP problem is one where the variables can be integers of variables. Figure 2.1 shows the feasible solution space for the given problem and the optimum solution for MIP, LP and IP problems.

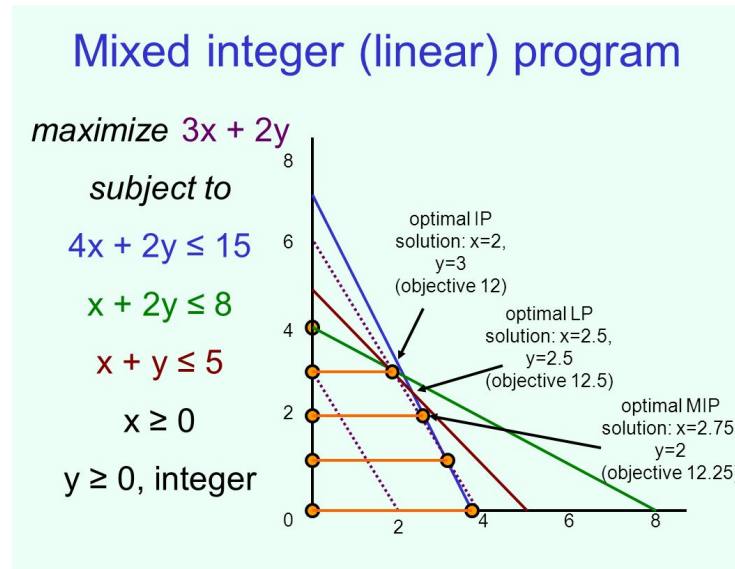


Figure 2.1. MIP, LP and IP problems and their solutions

The maximum  $Z = 34$  which occurs at point  $(6, 4)$  and the minimum  $Z = 34$  occurs at  $(-1, -3)$ . The above example is a simple illustration of LP, however when there are 3 or more variables LP problems cannot be simply solved by plotting as higher dimension spaces are difficult to visualize. Such problems are solved using simplex, Big M and other advanced LP techniques. MIP has been used in this thesis for the scheduling problems as it is required to find the status of machines (integer) and the power/capacity assignments (variable).

### 2.1.1 Brand and Bound Algorithm

The complexity of solving MILP problems is NP-hard which means that they cannot be solved by any known algorithm in polynomial time and the complexity of the problem increases exponentially with time [27]. This type of problems are mostly solved using Branch and Bound which searches for the solution by dividing the relaxed problem it into smaller sets of problems and without actually enumerating all the possible solutions which significantly reduces the time complexity of the problem. This algorithm is similar to how decision trees that have nodes and branches work. The algorithm starts with an initial computation of the relaxed solution (solution with only equalities) at the root node. From the root node, more nodes (or sub problems) are branched out (or explored) by increasing or decreasing the value of the decision variables. Whenever, a node is found to lead to an unfeasible solution (less optimal than parent node) or violate the bounds of the decision variables, that node is fathomed (the children nodes of that node are not explored and other adjacent nodes is explored). This process continues until the optimum solution (min or max) is found. This way the algorithm can search through the solution space without actually listing out all the possible solutions by pruning. Branch and Bound has been illustrated using the following example [28] and figures 2.2 and 2.3. Example problem 2 further illustrates the working of the algorithm.

$$\begin{aligned}
 &\text{Maximize } 13x_1 + 8x_2 \\
 &\text{Subject to } x_1 + 2x_2 \leq 8 \\
 &\qquad\qquad x_1 \leq 8 \\
 &\qquad\qquad x_2 \leq 8
 \end{aligned} \tag{2.2}$$

As explained earlier, the solution starts with the relaxed problem  $IP^0$  and then branches out by looking at the possible values  $x_1$  can take. In the next level, the nodes  $IP^1$  and  $IP^2$  are the relaxed solutions for  $x_1 \geq 3$  and  $x_1 \leq 2$  without the remaining constraints violated respectively.  $IP^2$  is fathomed as it is worse than  $IP^1$  and the children nodes(  $IP^3$  and  $IP^3$  ) of  $IP^1$  are explored for better solutions.  $IP^3$

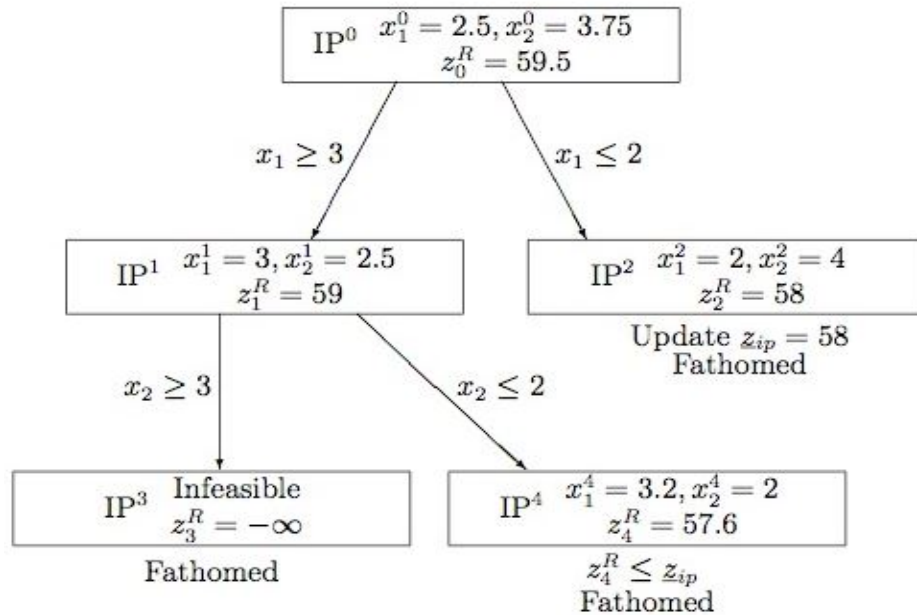


Figure 2.2. Solution to example problem 2 using branch and bound

is fathomed as it leads to a infeasible solution that violates constraints 2a and 2b(  $x_1 \geq 3$  and  $x_1 \geq 3$  ). On the other hand,  $IP^4$  is fathomed as it yields a cost that is worse than its parent node  $IP^1$  .  $IP^0$  is the best solution that maximizes the cost function in case of inequality constraints. This way the Brand and Bound algorithm is able to find optimal solution efficiently without looking into all the possible solutions. The MIP problems in MATLAB can be solved using the `intlinprog()` function that uses brand and bound algorithm. The inputs to this function are the cost function, inequalities, equalities. All of these are input as matrices and the position of each matrix element corresponds to the respective decision variable that is involved in the equation/inequality/cost function. Branch and Bound algorithm is then used to find the optimal solution to the problem. Major steps in this algorithm has been shown in the flow chart below.



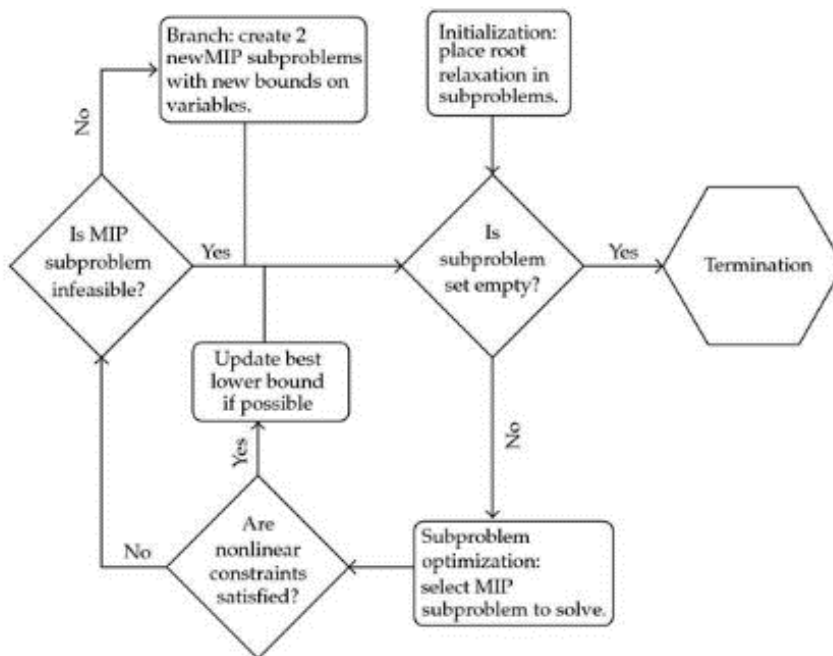


Figure 2.3. Branch and Bound algorithm flowchart

### 3. ENERGY ORIENTED CONTROL

#### 3.1 Optimal Control

This chapter serves to provide a basic understanding about optimal control and the Model Predictive Control that has been used for the energy efficient control of HVAC systems in manufacturing industries. There are several control techniques that are used in industries such as the Programmable Logic Control (PLC), Proportional Integral Derivative (PID) Control and ON/OFF controller. Although these control techniques satisfy the performance requirements but they are not the most optimum input to the actuators. In case of ON/OFF controller, the input goes from zero to full and keeps running the machine until the set point is reached. Although this type of control has its roots from optimal control, the response is oscillatory about the dead band which is undesirable and it starts and stops the machine quite frequently. As for the PID control, it may be able to continuously adapt and follow the set point, however it cannot handle Multi Input Multi Output System (MIMO) and constraints and is susceptible to integral windup. PLC control is purely based on logic for very simple systems like valves and cannot be used for complex systems altogether. Optimal control techniques like the MPC provide the best possible input to the system with respect to the objective function. If the objective function requires minimum energy effort, then the inputs provided by the MPC is energy efficient.

For any optimal control problem the idea is to find a maximum or minimum of a given functional which is basically a function of function as shown below,

$$J = \int_{t_0}^{t_1} F(t, x(t), \dot{x}(t)) dt \quad (3.1)$$

Where  $F(t, x(t), \dot{x}(t))$  could represent a nonlinear system of states  $x(t)$  and  $\dot{x}(t)$ . This problem is usually solved by the Pontryagin's maximization principle.

or Hamilton Jacobi equation [29]. This cost function is usually subject to certain constraints that could be bounds that are placed on the system. Also, depending on what needs to be minimized the cost function can be modified, for example for a system defined by  $\dot{x}(t) = Ax(t) + B(u)$ ,  $y(t) = Cx(t)$  for which the objective is to drive the states from  $x(t_0)$  to  $x(t)$  while expending the least amount of energy, the cost function becomes,

$$J = \int_{t_0}^{t_1} u^2 dt \quad (3.2)$$

Solving this problems yields a solution with the least input effort or energy. Continuous systems are converted to their counterparts using zero order hold that replicates the evolution of the state through time, by holding each value for the given sample time period.

### 3.1.1 Discrete Linear Quadratic Regulator(LQR) Controller

For a discrete systems defined by

$$x(n+1) = Ax(n) + Bu(n), y(n) = Cx(n) \quad (3.3)$$

The LQR problem is to find inputs  $u(0) \dots u(N-1)$  that makes the following cost as small as possible

$$J(u) = x(N)^T P x(N) + \sum_{k=0}^{N-1} x(k)^T Q x(k) + u(k)^T R u(k) \quad (3.4)$$

Where P is the final state cost, Q (diagonal matrix) is the cost related to the states and R is the cost related to input. The Q and R matrices are used for tuning the LQR controller. The controller becomes more aggressive or reaches the states faster at the expense of oscillations when the elements in Q are large. By making R large, the controller is forced to move the states with the least possible input effort. A balance needs to be set between these two matrices to achieve the desired performance

without high overshoot (oscillations) or large rise time. The solution to the above problem can be found using Dynamic Programming or Backward Riccati Recursion

### 3.2 Model Predictive Control

Model Predictive Control is one of the most robust multivariate control system primarily that has been used in process and manufacturing industries since the 1980s. It is essentially an optimal control system wherein the past and current states and outputs are used to optimize a cost function along a definite horizon (also known as receding horizon) to track the reference trajectory. One of the major strengths of MPC is its ability to handle constraints of MIMO systems. The following figure depicts the ideology behind the functionality of the MPC.

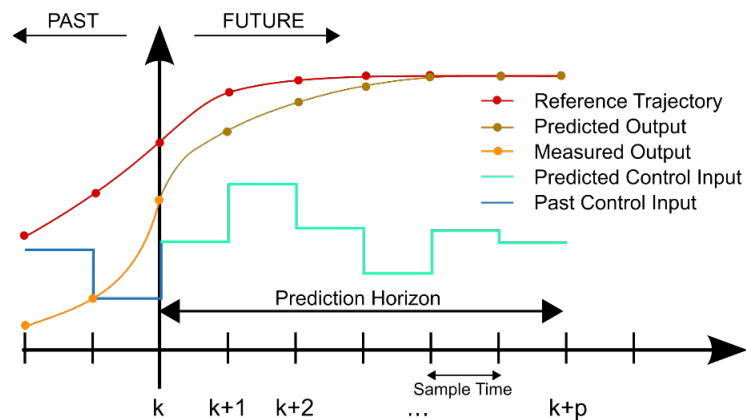


Figure 3.1. Receding horizon control of MPC showing the manipulated and controlled variables

The most common form of the cost function that is used for MPC is the Quadratic cost function based on that of Linear Quadratic Regulator. This cost function is minimized to obtain predict the outputs and inputs until the end of the horizon ( $k$  to  $k+p$ ). Then the first control input is implemented and the outputs and the states are used as a feedback for optimizing the cost function the next horizon ( $k+1$  to  $k+p+1$ ).

This process is repeated for every horizon. The typical cost function of MPC is as follows

$$J = \sum_{k=0}^{N_p} (\hat{y} - r)^T Q (\hat{y} - r) + \sum_{k=0}^{N_p} \Delta u^T R \Delta u \quad (3.5)$$

In the above cost function,  $\hat{y}$  is the predicted output and  $r$  is the reference output.  $\Delta u$  is the difference in the between the predicted input. These values are summed until the end of the prediction horizon in the cost function.  $Q$  and  $R$  are the weight matrices (similar to the LQR controller) that can be used to control the penalty on the inputs and outputs. Starting with the discrete system state space in equation 1. The dimensions of  $x$ ,  $A$ ,  $B$ ,  $u$ ,  $C$  and  $y$  are  $n \times 1$ ,  $n \times n$ ,  $n \times 1$ ,  $n \times m$ ,  $m \times n$ . The state space is first augmented by performing a difference operation on equation 4 and matrix manipulations [30],

$$\Delta x(k+1) = x(k+1) - x(k), \quad (3.6)$$

$$\Delta u(k) = u(k) - u(k-1) \quad (3.7)$$

$$\Delta x(k) = x(k) - x(k-1) \quad (3.8)$$

$$\begin{bmatrix} \Delta x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A & O^T \\ CA & 1 \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix} \Delta u(k) \quad (3.9)$$

$$y(k) = \begin{bmatrix} \Delta & 1 \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix} \quad (3.10)$$

$\begin{bmatrix} \Delta x(k+1) \\ y(k+1) \end{bmatrix}$ ,  $\begin{bmatrix} A & O^T \\ CA & 1 \end{bmatrix}$ ,  $\begin{bmatrix} B \\ CB \end{bmatrix}$ ,  $\begin{bmatrix} \Delta & 1 \end{bmatrix}$  and  $\begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix}$  are shorthanded as  $x(k+1)$ ,  $A$ ,  $x(k)$ ,  $C$  and  $B$  respectively. The next step which the prediction part is done as follows,

$$x(k+1|k) = Ax(k) + B\Delta u(k) \quad (3.11)$$

$$x(k+2|k) = Ax(k+1|k) + B\Delta u(k+1)$$

Substituting & in 7a in 7b

$$x(k+2|k) = A^2x(k) + AB\Delta u(k) + B\Delta u(k+1) \quad (3.12)$$

For a prediction horizon that is of length  $N_p$  and the control horizon of length  $N_c$ , the state vector for the last timestep is

$$x(k+N_p|k) = A^{N_p}x(k) + A^{N_p-1}B\Delta u(k) + A^{N_p-2}B\Delta u(k) + \dots + A^{N_p-N_c}B\Delta u(k+N_c-1)$$

Putting it together  $Y = Fx(k) + \Phi\Delta U$  where

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}$$

$$F = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ \vdots & & & & \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_c}B \end{bmatrix}$$

and  $\Delta U$  is a vector of the future inputs. After minimizing the cost function in 6 without any constraints,

$$\Delta u(k) = [IO \dots O] (\Phi^T \Phi + R)^{-1} (\Phi^T r(k) - \Phi^T Fx(k)) \quad (3.13)$$

To minimize the same cost function with respect to the state space model, input, input rate and output bounds, quadratic programming methods are used which is

basically an extension of linear programming discussed in the previous chapter. Figure 3.2 shows the major steps of the MPC.

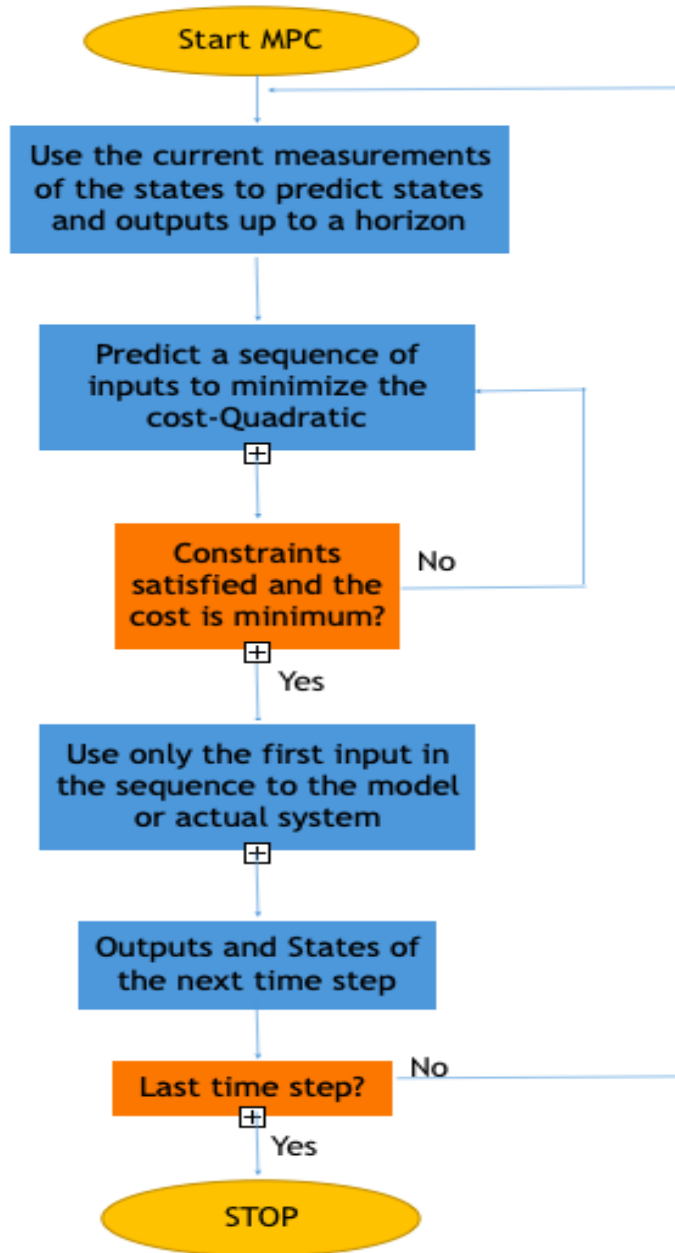


Figure 3.2. Flowchart for the basic Model Predictive Control Design

### 3.3 State Estimation

As mentioned above, for the MPC to make predictions about the states and find the optimal inputs sequence, the present states should be available. In practice, it may be difficult or expensive to measure all the states. In such cases, an observer that mimics the actual model is used to make estimates of the states. The most commonly used observer is the Luenberger observer that is defined as follows for a discrete system in equation 3.3,

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k)) \quad (3.14)$$

Taking the difference between  $x(k+1)$  and  $\hat{x}(k+1)$

$$\tilde{x}(k+1) = (A - LC)\tilde{x}(k) \quad (3.15)$$

As long as the poles of  $A-LC$  are within the unit circle the, observer error is bound to decrease exponentially. An extension to this discrete observer is the Kalman filter which is an optimal state estimator that can be used for stochastic systems that have uncertainty in the inputs and states. It can be used for systems that have process noise (model mismatch) and measurement noise. The Kalman filter essentially uses the measurements of the outputs and the model predictions of the state to estimate the states with higher confidence (less variance). This way even if the inputs and outputs have disturbances, model has mismatches and all states except the outputs aren't available for measurement which is the case in most of the real systems, the MPC in conjunction with a Kalman filter can steer the states along the desired trajectory. Major steps of the Kalman filter are shown in figure 3.3.



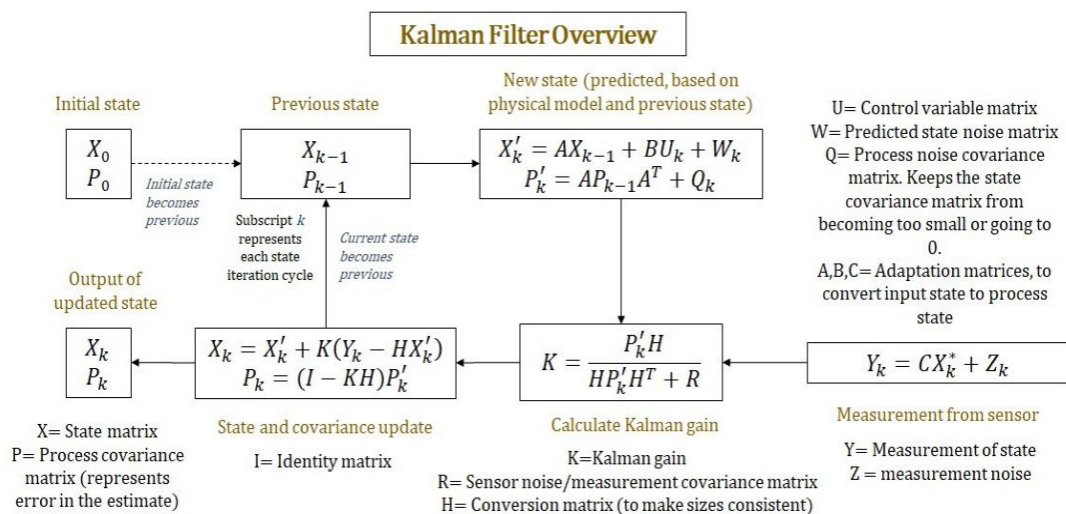


Figure 3.3. Kalman filter process flow chart [31]

## 4. MODELLING

In this chapter, the modelling of the three main problems in this thesis is discussed. The Optimal Load Shifter (OLS) is first formulated and modelled, followed by the Compressor Load Scheduler (CLS) and the lumped parameter modelling of thermal systems. These models and formulations are then used for the model validation, case studies and results in the upcoming chapters. The subsystems that have been targeted for energy management using control and optimization are HVAC system, compressor and machines that are driven by electrical motors. These have been chosen as these together are integral part of most of the manufacturing plants. Energy and cost saving can be done at different levels while meeting the demand constraints as follows (figure 4.1).

The process flow starts (figure 4.1) with the forecasted demand which is the input to both the compressor systems and the flexible machine.

Mixed Integer Linear Programming is proposed to schedule the machines and the compressors, while Model Predictive Control is proposed for the control of the HVAC system. The energy usage is minimized by shifting the flexible loads to time intervals when the demand rate is decreasing or low (load shifting) which is done by the Branch and Bound Algorithm (figure 3.3). This makes sure that only the non-flexible or non-shift able loads are not used at the same time as the flexible ones resulting in overall reduction in the peak demand thus potential cost savings.

The loads can be divided into flexible and non-flexible loads based on their usage priority and time of use flexibility. Flexible loads are the ones that can be scheduled during the low demand periods or in other words can be flexible in terms of when they are used during the day. Non-flexible loads are the opposite i.e they need to be run at a certain time thus being non-flexible in terms of scheduling. HVAC systems and Compressors are the ones most of the industries and these need to be run

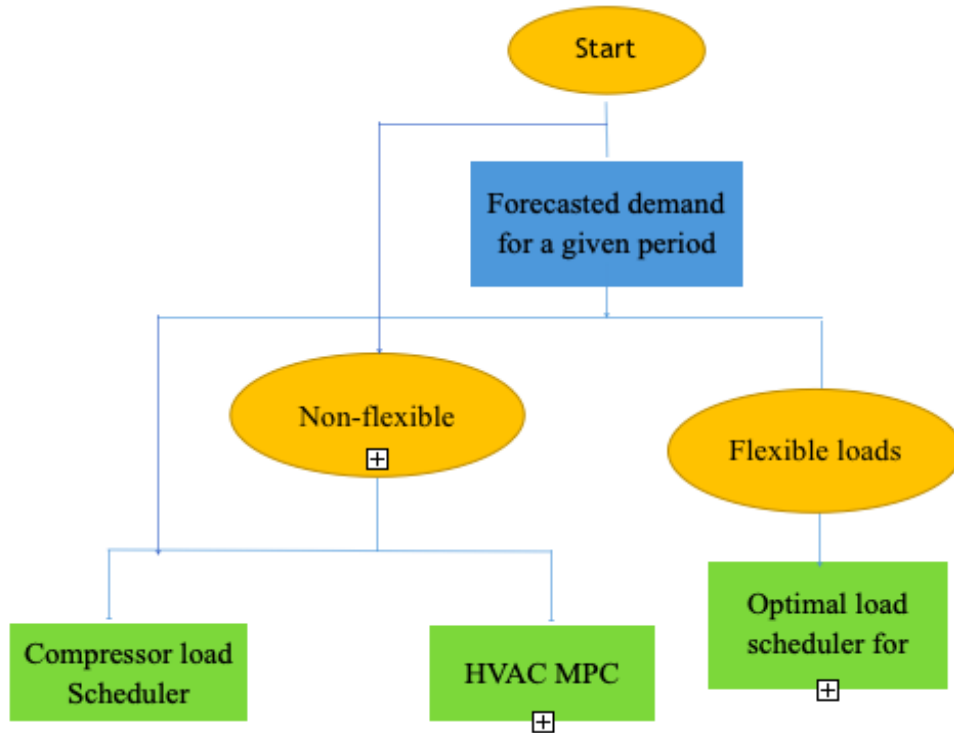
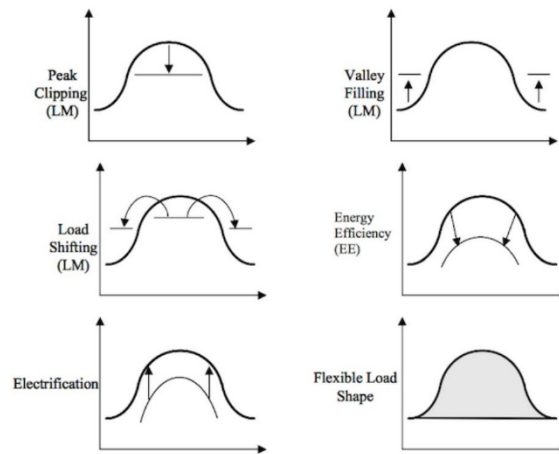


Figure 4.1. Proposed energy management framework using control and optimization

continuously sometimes without any interruption. Therefore HVAC systems can only be made energy efficient by deploying control system that operates it just enough to meet the room requirements and saves energy. The same can be said for the compressors, however there are cases where a group of compressors run together in parallel configuration to meet the demand of the plant. In such cases, there is a need to optimally distribute the loads among the compressors so that the compressors run in their efficient conditions and less costs can be incurred. Flexible loads are machines that are mostly powered by electrical motors and are of lower priority when compared to the non-flexible. Due to their lower priority these loads or machines can be scheduled during low demand periods.

## 4.1 Optimal Load shifter

As mentioned previously, to avoid peak demand charges the Peak to Average Ratio (PAR) should be reduced. Demand side management techniques includes load shifting, peak clipping, valley filling, electrification, energy efficiency and flexible load shape (figure 4.2). In the figure below, each of these DSM techniques illustrate how the power peak is modified to reduce energy consumption and/or costs.



Source: Primer on Demand-Side Management, World Bank Document CRA No. D06090, 02/2005

Figure 4.2. Commonly used Demand Side Management techniques; Peak Clipping, Valley Filling, Load Shifting, Energy Efficiency, Electrification, Flexible Load Shape

Considering the peak demand charges in Indiana where the consumer is charged the peak demand cost throughout the period, load shifting and valley filling (figure 4.2) are the suitable ones in reducing peak demand as it helps in rescheduling the flexible loads such that they are used only during the low peak periods. If at least half of the plant/consumer demand can be predicted, the rest of it can be efficiently scheduled such that the peak demands of the both the halves do not occur at the same time thus shifting the load. The forecasted demand is the input the optimal load shifter and the output is the start and stop sequences (scheduling) of the flexible

loads for a given time horizon. The below formulation is valid for two assumptions 1) it can be applicable for machines running for more than 1 time step and are flexible and interruptible 2) if the loads are only flexible then it is applicable for machines whose minimum run time is at least 1 time step . Finally, this model also assumes that the minimum runtime of each machine is greater than or equal to the sample time of the predicted demand.

$P_i$  : Average power consumption of machine i

$D_t$  : Predicted demand at time t

$n$  : Number of machines

$\varepsilon_i$  : Number of starts for each machine

$x_{i,t}$  : Binary decision variable that indicates the state of the  $i^{\text{th}}$  energy system at time t

The cost function f is as follows

$$\text{Min } f = \sum_{t=1}^T \sum_{i=1}^N x_{i,t} \frac{-1}{\text{avg}(D_{t+1}, D_t) P_i} \quad (4.1)$$

S.T:

Demand Constraint:

$$\sum_{t=1}^{24} \sum_{i=1}^n x_{t,i} = \varepsilon_i \text{ where } i = 1, 2 \dots n$$

Binary Constraints:

$$x_i \in \{0, 1\}$$

For flexible and uninterruptable loads two more constraints need to be added to the formulation to make sure that the machine that is uninterruptable runs for the prescribed period continuously.

$$\sum_{t=1}^T \sum_{i=1}^N x_{i,t} + My_k = \gamma_i$$

where  $\gamma_i$  is the number o intervals machine i is supposed to run continuously for . k is the number of combinations of the decision variables for this to occur.

Where  $M$  is a very large number and  $y$  is a slack variable that is used to select the optimal combination from the possibilities above. This is done using another constraint involving  $y$

$$\sum_1^k y_k = k - 1$$

The above cost function when minimized gives the solution that moves most of the loads to the lower demand region of the curve. This helps in reducing the overall demand which is caused by both flexible and non-flexible loads. This flexible load scheduling problem is solved in MATLAB using the inbuilt function called `intlinprog()` that handles MILP using branch and bound optimization algorithm.

#### 4.1.1 Examples for illustration

Load Shifting test case 1: Flexible and Interruptible loads

Problem: A hypothetical demand (shown in blue in figure 4.3) has been predicted for the next 5 hours and it is required to schedule the start of two machines that run for two hours during this time horizon in an energy efficient manner. These machines are flexible and interruptible i.e they can be started and stopped at any time to reduce excessive demand and cost. The figure below shows the default schedule of the two machines

Blue line is the predicted demand profile and the rectangular blocks represent the schedule of the two machines. Machine one lasts for the first two time steps and Machine starts at timestep 3 and continues until time step 5.

Solution

Input: Predicted demand profile, number of machines and time horizon

Find: Start status of each machine at every timestep i.e  $x_{i,t}$  where  $t = 1,2,3,4,5$  and  $i = 1,2$

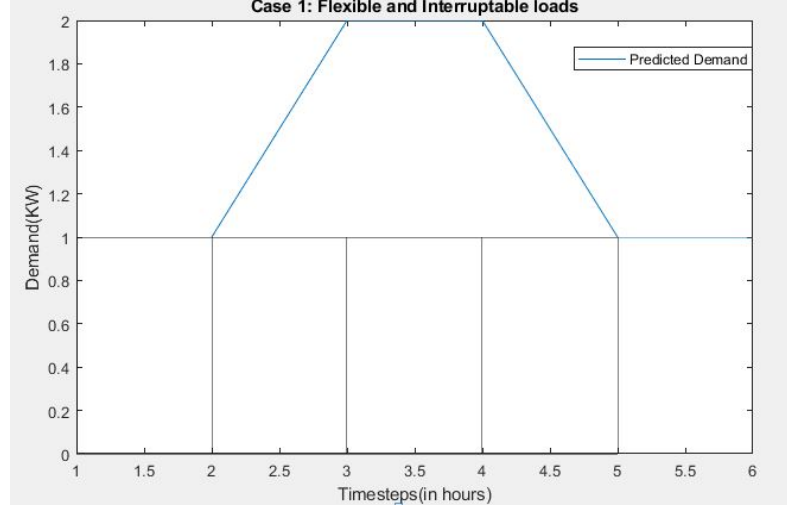


Figure 4.3. Default load schedule for flexible and interruptible loads

Where  $x_{i,t}$  is the status of machine  $i$  at time  $t$ ,  $D_t$  is the predicted demand at time  $t$ , and  $P_i$  is the average power consumed by machine  $i$ . The cost function  $f$  is as follows

$$\begin{aligned}
Max f &= \sum_{t=1}^T \sum_{i=1}^N x_{i,t} \frac{-1}{avg(D_{t+1}, D_t) P_i} \\
&= \frac{1}{P_1} \left( \frac{-1}{avg(D_2, D_1)} x_{1,1} + \frac{-1}{avg(D_3, D_2)} x_{1,2} + \frac{-1}{avg(D_4, D_3)} x_{1,3} \right. \\
&\quad \left. + \frac{-1}{avg(D_5, D_4)} x_{1,4} + \frac{-1}{avg(D_6, D_5)} x_{1,5} \right) + \frac{1}{P_2} \left( \frac{-1}{avg(D_2, D_1)} x_{2,1} \right. \\
&\quad \left. + \frac{-1}{avg(D_3, D_2)} x_{2,2} + \frac{-1}{avg(D_4, D_3)} x_{2,3} + \frac{-1}{avg(D_5, D_4)} x_{2,4} \right. \\
&\quad \left. + \frac{-1}{avg(D_6, D_5)} x_{2,5} \right) \quad (4.2)
\end{aligned}$$

Constraints:  $\sum_{t=1}^T \sum_{i=1}^N x_{t,i} = \varepsilon_i$  where  $\varepsilon_i$  is the number of times  $i$ th machine needs to be started. Here  $\varepsilon_i = 2$  for  $i=1,2$ .

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} = 2$$

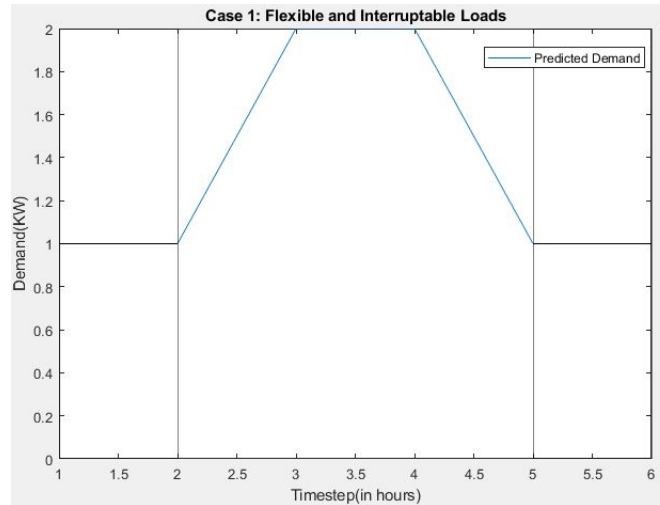


Figure 4.4. Optimal schedule of the two machines found by the algorithm

$$x_{1,2} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} = 2$$

After using `intlinprog` to solve the problem using branch and bound in MATLAB, the following result was obtained (figure 4.4).

From the above figure, it can be noted that, the algorithm has scheduled the machines to only run during the off demand time steps (1 to 2 and 5 to 6) which is desired for energy efficiency.

#### Load Shifting test case 2: Flexible and Uninterruptible loads

Problem: The test demand has been predicted for the next 5 hours and it is required to schedule the start of two machines that run for two hours during this time horizon in an energy efficient manner. Machine 2 is flexible and interruptible while machine 1 is flexible but not interruptible i.e once started it cannot be stopped until it completes its job.

Figure 4.5 shows the default status of the two machines. Machine 1 and 2 run continuous for two time steps and stop one after the other. This default configuration is non-optimal as the machines are running during the high demand periods as well.

#### Solution



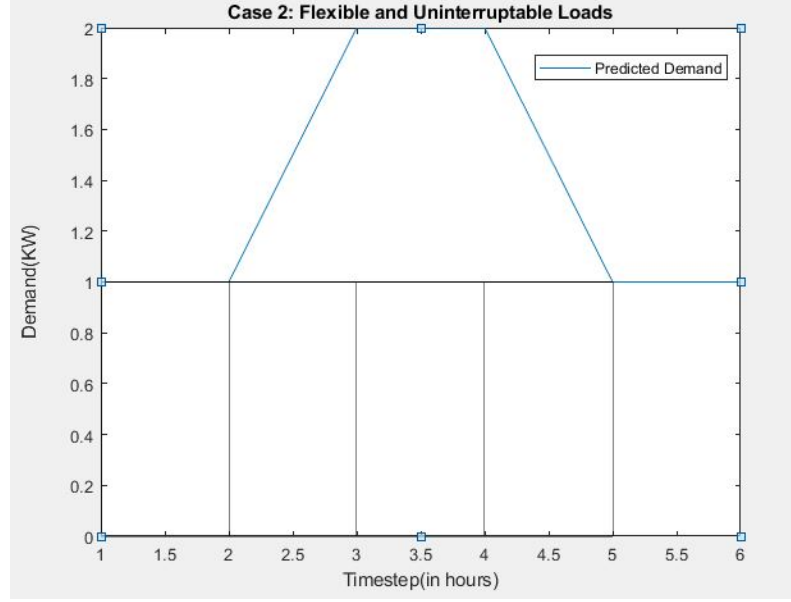


Figure 4.5. Default sequence of the two machines

Input: Predicted demand profile, number of machines and time horizon

Find: Start status of each machine at every timestep i.e  $x_{i,t}$  where  $t = 1,2,3,4,5$  and  $i = 1,2$

Where  $x_{i,t}$  is the status of machine  $i$  at time  $t$ ,  $D_t$  is the predicted demand at time  $t$ , and  $P_i$  is the average power consumed by machine  $i$ . The cost function  $f$  is as follows

$$\begin{aligned}
 Min f &= \sum_{t=1}^T \sum_{i=1}^N x_{i,t} \frac{-1}{avg(D_{t+1}, D_t) P_i} \\
 &= \frac{1}{P_1} \left( \frac{-1}{avg(D_2, D_1)} x_{1,1} + \frac{-1}{avg(D_3, D_2)} x_{1,2} + \frac{-1}{avg(D_4, D_3)} x_{1,3} \right. \\
 &\quad \left. + \frac{-1}{avg(D_5, D_4)} x_{1,4} + \frac{-1}{avg(D_6, D_5)} x_{1,5} \right) + \frac{1}{P_2} \left( \frac{-1}{avg(D_2, D_1)} x_{2,1} \right. \\
 &\quad \left. + \frac{-1}{avg(D_3, D_2)} x_{2,2} + \frac{-1}{avg(D_4, D_3)} x_{2,3} + \frac{-1}{avg(D_5, D_4)} x_{2,4} \right. \\
 &\quad \left. + \frac{-1}{avg(D_6, D_5)} x_{2,5} \right)
 \end{aligned}$$

Constraints:  $\sum_{t=1}^T \sum_{i=1}^N x_{t,i} = \varepsilon_i$  where  $\varepsilon_i$  is the number of times ith machine needs to be started. Here  $\varepsilon_i = 2$  for  $i=1, 2$ .

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} = 2$$

$$x_{1,2} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} = 2$$

In addition to above constraint, the following constraints makes sure that machine 1 runs continuous for 2 time periods

$\sum_{t=1}^T \sum_{i=1}^N x_{i,t} + My_k = \gamma_i$  where  $\gamma_i$  is the number o intervals machine i is supposed to run continuously for . k is the number of combinations of the decision variables for this to occur.

In this problem, since only machine 1 is uninterruptable,

$$x_{1,1} + x_{1,2} + My_1 = 2$$

$$x_{1,2} + x_{1,3} + My_2 = 2$$

$$x_{1,3} + x_{1,4} + My_3 = 2$$

$$x_{1,4} + x_{1,5} + My_4 = 2$$

Where M is a very large number and y is a slack variable that is used to select the optimal combination form the possibilities above. This is done using another constraint involving y

$$\sum_{1}^k y_k = k - 1$$

$$y_1 + y_2 + y_3 + y_4 = 3$$

The above constraint allows only one of the possible combinations (start sequences of machine 1) to be allowed while computing the cost. The following result (figure 4.6) was obtained when this problem was solved using Intlingprog in MATLAB,. In the results section the similar cases have been solved for more complex scenarios using the real demand from Compressors.

From figure 4.6, it can be seen that machine 1 runs continuously without interruption from time step 1 to 3 and machine two runs from time step 1 to 2 and then

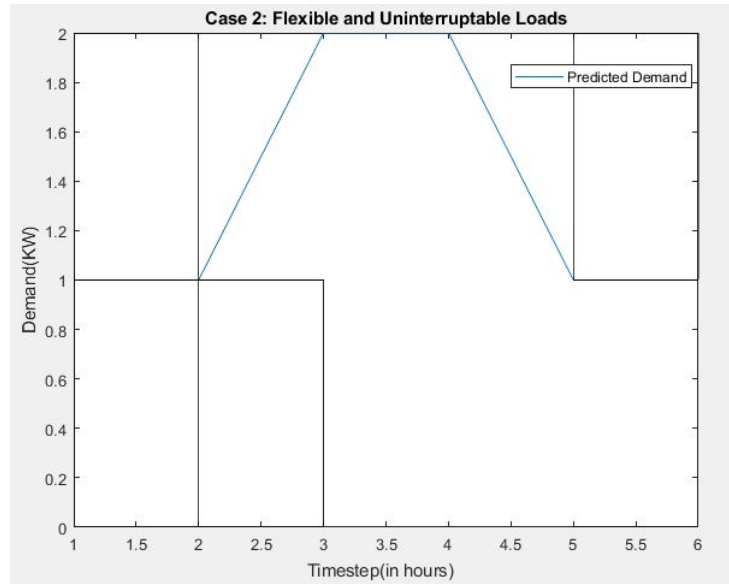


Figure 4.6. Optimal schedule was found for case 2 by operating the machines during the low demand periods

from 5 to 6. Clearly, machine 1 constraint of running without interruption has not been violated and the most energy efficient schedule (machines are running during periods of low demand) has been found by the algorithm.

## 4.2 HVAC Modelling

In this thesis, for the HVAC system white box modelling has been used. In the white box modelling itself, there are many approaches to modelling a HVAC systems which involves nonlinear Partial differential equations or simple lumped parameter model. Lumped parameter model has been chosen due to its simplicity that later translates into smaller dimensional state space. Also, it is more intuitive and easier to deal with the heat transfer in terms of conduction, convection and radiation in 1-D (heat transfer in one of the dimension dominates the others). In the lumped parameter model, every system is analogous to a Resistance-Capacitance (RC) circuit. In this case, from the first law of thermodynamics

$$\Delta U = Q - W \quad (4.3)$$

Which can be re-written as  $\Delta E = E_{in} - E_{out}$  where E terms can be either Heat (Q) or Work (W). RC model for HVAC systems assumes that the temperature is uniform in the room and only has gradient in a single dimension. The three modes of heat transfer namely conduction, convection and radiation can be as shown in table 4.1.

Figure 4.7 shows the zones(zone 1 is the target zone) and thermal circuit of the zones(adapted from [16]). In a typical RC modelling, the space is basically divided into n nodes. These nodes represent walls or zones(adjacent rooms of the target zone). The walls are treated as capacitors that store some of the heat that is being transferred through them. Between two nodes there can be one or more thermal resistance that dictates the rate of change of temperature between the nodes. The central node which usually is the node representing the target zone temperature can have external heat sources like heat generation in the zone, radiation and the HVAC input flow rate. For models that do not have windows radiation( $Q_{abs}$  in the figure 4.7) can is considered negligible, as the convection and conduction are the more dominant modes of heat transfer when there is a thick medium for heat transfer like a wall . This also helps in keeping the model from being highly non-linear(radiation is 4<sup>th</sup> power of temperature) which makes it easier to implement control system design.

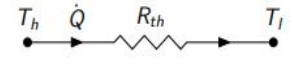
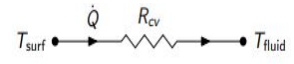
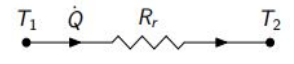
The differential equations of the lumped parameter model of zone 1 are as follows

$$\text{Bottom} : C_{w2} \frac{dT_{w4}}{dt} = \frac{T_{z3} - T_{w4}}{R_{z3,w4}} - \frac{T_{w4} - T_{z1}}{R_{z1,w4}} \quad (4.4)$$

$$\text{Center} : C_{z1} \frac{dT_{w4}}{dt} = Q_{int} + Q_{HVAC} + \frac{T_{w1} - T_{z1}}{R_{in1} + R_{\frac{w1}{2}}} + \frac{T_{w2} - T_{z1}}{R_{in2} + R_{\frac{w2}{2}}} - \frac{T_{z1} - T_{w4}}{R_{z1,w4}} - \frac{T_{z1} - T_{w3}}{R_{z1,w3}}$$

$$\text{left} : C_{w3} \frac{dT_{w3}}{dt} = \frac{T_{z2} - T_{w3}}{R_{z2,w3}} - \frac{T_{w3} - T_{z1}}{R_{z1,w3}}$$

Table 4.1. Conduction, Convection and Radiation in terms of the circuit model

Sl.No	Conduction	Convection	Radiation
Definition	Heat transfer through a solid body.	Heat transfer between solid surface and fluid	Heat transfer between bodies due to EM waves
Equation	$\dot{Q} = kA \frac{T_h - T_l}{l}$ $\dot{Q} = \frac{T_h - T_l}{R_{th}}$	$\dot{Q} = hA\Delta T$ $\dot{Q} = \frac{\Delta T}{R_{cv}}$	$\dot{Q} = \epsilon h_r A (T_1 - T_2)$ $\dot{Q} = \frac{\Delta T}{R_r}$
Circuit Analogue			

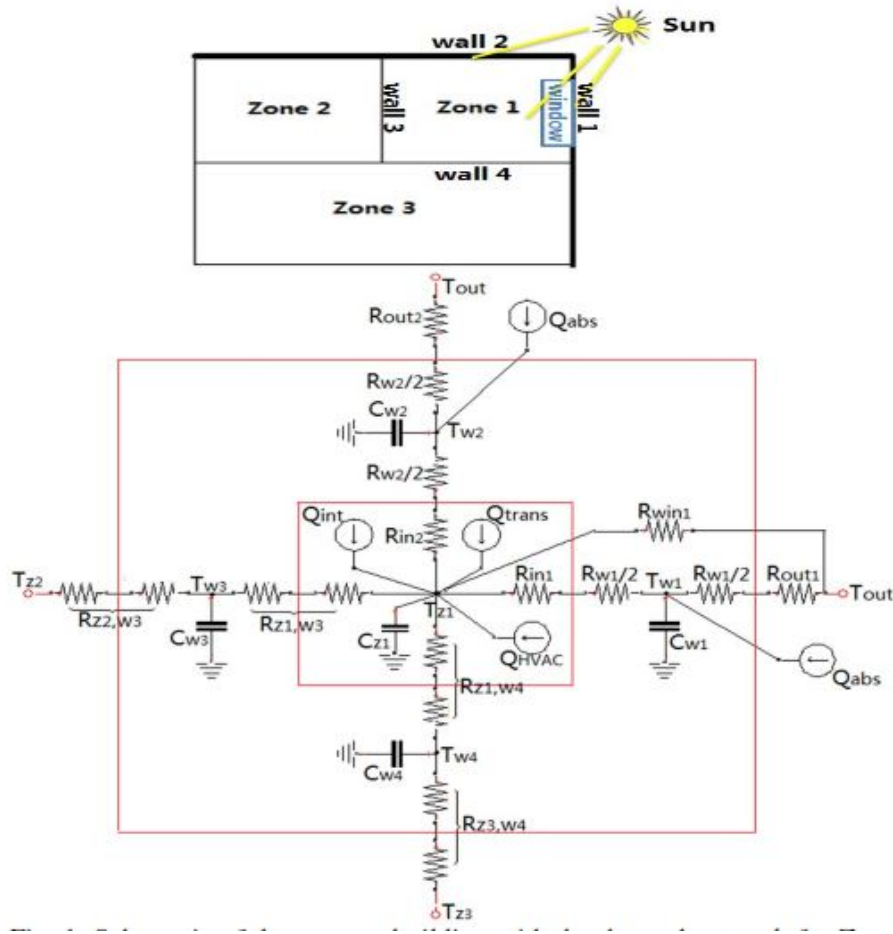


Figure 4.7. Zone model of the room (top), Lumped parameter model of zone 1(bottom) [16]

$$\text{Top : } C_{w2} \frac{dT_{w2}}{dt} = \frac{T_{z1} - T_{w2}}{R_{in2} + R_{\frac{w2}{2}}} - \frac{T_{w2} - T_{out}}{R_{out2} + R_{\frac{w2}{2}}}$$

$$\text{Right : } C_{w1} \frac{dT_{w1}}{dt} = \frac{T_{z1} - T_{w1}}{R_{in1} + R_{\frac{w1}{2}}} - \frac{T_{w1} - T_{out}}{R_{out1} + R_{\frac{w1}{2}}}$$

$$Q_{HVAC} = m_{zi} c_a (T_{si} - T_{zi})$$

Where mass flow rate  $m_{zi}$  is the input and  $Q_{radi}$ ,  $Q_{int}$  are the disturbance input. The states include all the temperature variables and the output is the target zone

temperature. For VAV systems where the temperature is fixed (cooling/heating) and the air mass flow is varied as per the temperature requirement, the problem becomes nonlinear due to the  $Q_{HVAC}$  term that has the product of the input mass flow rate and the target zone temperature. However, the model is linear for CAV systems where input mass flow rate is fixed and the temperature is the variable.

### 4.3 Compressor Load Scheduler

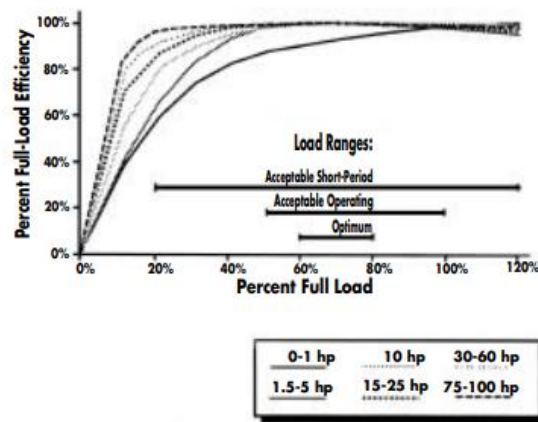


Figure 4.8. Efficiency vs % full load of light duty and heavy duty motors [32]

The main purpose of load scheduling is to save energy in a by optimally distributing load among a set of energy systems that are working towards meeting the demand additively. Since, the motor in a compressor consumes most of the energy, this study has been focused on running the compressor motors at their most efficient points depending on the size and load. In most of the motor driven systems like compressors efficiency drastically decreases with part loads or lower. This is more pronounced in compressors of 10HP or lower (figure 4.8) [31]. Given two such equally sized compressor, it is optimal to run one at full load and keep the other one switched off rather than splitting the load. This makes sure that the energy systems are not run at their inefficient points and are turned off or idles when needed. This can be

extended to more than two compressors or systems of similar nature where it is best to keep them running at loads greater than part load. However, this may not be the case for compressors with higher HP as the cost of running a bigger compressor solely outweighs the relatively higher efficiency at full loads as seen from figure 4.8 [32]. Thus the two modes have been considered for the load scheduler. One for the light duty compressors with HP equal to or less than 10 and the other for heavy duty compressors with more than 10HP. Figure 4.8 from the DOE shows how the efficiency decreases with the decrease in the load for three different electric motors. The same trend can be seen from table 4.2 [33] for the VFDs associated with the motors. Thus it can be concluded that low duty compressors should be run at at least 70% load to keep them efficient and heavy duty compressors are uniformly efficient above 20% loads.

Table 4.2. Efficiency vs load of different compressor VFDs [33]

Variable Drive hp Rating	Efficiency (%)						
	Load, Percent of Drive Rated Power Output						
	1.6	12.5	25	42	50	75	100
5	35	80	88	91	92	94	95
10	41	83	90	93	94	95	96
20	47	86	93	94	95	96	97
30	50	88	93	95	95	96	97
50	46	86	92	95	95	96	97
60	51	87	92	95	95	96	97
75	47	86	93	95	96	97	97
100	55	89	94	95	96	97	97
200	61	81	95	96	96	97	97

The concept of job shop scheduling can be applied for optimal scheduling of energy systems in order to reduce the energy and electricity consumption. In a typical job shop problem, there are “n” jobs and “m” machines that can perform these jobs operation by a sequence of “k” operations. Constraints are imposed when the machine is available to run in the time horizon. The solution to the Travelling Salesman Problem(TSS)problem is scheduling the tasks of the jobs on each of the devices such



that the total time of processing is minimized. In this work, this type of formulation has been generalized and modified so that it can be applied to compressors that have pumps equipped with VFDs. This model assumes that the compressors with VFDs are running in parallel to meet the CFM together at a certain pressure set point. The variables used for the formulation are very generic and are as follows,

Input: Total demand that needs to be met by the energy systems through load distribution

$$\text{Minimize } f = \sum_{i=1}^N \text{Constant}_i * \text{State}_i + \sum_{i=1}^N \text{Output}_i$$

Constraints:

- Total output from the systems must be greater than or equal to the demand
- The output of the  $i^{\text{th}}$  system is within the output bounds (between 0 and some value)
- States are binary i.e they can only take the values of 1 (system is “ON” ) and 0 (system is “OFF” )
- The output is greater than 0 iff the system is “ON” and 0 iff the system is “OFF”

Expected Solution: The systems shall be run in their most efficient points i.e high loads and as many systems as possible should be shut off to save energy. The formulation is as follows,

$d$  : Total demand

$q_i$  : Output capacity of the  $i^{\text{th}}$  energy system

$q_{i,max}$  : Maximum Output capacity of the  $i^{\text{th}}$  energy system

$q_{j,min}$  : Minimum Output capacity of the  $i^{\text{th}}$  energy system

$x_i$  : Binary decision variable that indicates the state of the  $i^{\text{th}}$  energy system

$C_i$  : Cost associated with operating the  $i^{\text{th}}$  energy system.

$$\text{Min } f = \sum_{i=1}^N C_i x_i + \sum_{i=1}^N q_i \quad (4.5)$$

S.T:

$$\sum_{i=1}^N q_i x_i \geq d(\text{non linear})$$

$$q_{i,\min} \leq q_i \leq q_{i,\max}, q_{i+1,\min} \leq q_{i+1} \leq q_{i+1,\max}, \dots \dots \dots q_{i,\min} \leq q_i \leq q_{i,\max}$$

$$x_i \in \{0, 1\}$$

From the above, it can be seen that the formulation is a modified and simpler version of the work in [25] which makes it easier to solve. The inputs to this problem are total demand, minimum and maximum output capacities of the N energy systems. The cost function consists of two terms; the first is the cost of running the  $i^{\text{th}}$  energy system and the second is the cost of having low output capacity for the energy system.  $C_i$  is a constant that depends on the size of the subsystem and it has been used so that the more energy intensive systems are chosen less frequently . 4.4a ensures that the total output capacity of the running energy systems is at least as much as the demand and this constraint is non-linear. 4.4b is used for keeping the output capacities bounded between the full load and no load. 4.4c shows that  $x_i$  is a binary variables that can take either 0 or 1. The outputs variables are  $x_i$  and  $p_i$  provide with the number of energy systems that are running and their respective assigned output capacities.

The above scheduling problem is solved using as a MILP problem. Though this formulation is expected to give optimal results (illustrated in the below example), the model still needs to be validated so that the solution is not sub optimal and the feasible solution exist for every problem. Also, MINLP gets computationally expensive as the number of decision variables increase. This is not the case with MILP, therefore it is better to convert the model into a MILP problem. The cost function in

the formulation is linear however constraint (4.5a) that involves the product of decision variables is what makes the formulation non-linear. This constraint essentially ensures that  $q_i$  is 0 only when  $x$  is zero(off) and is greater than zero when  $x$  is 1(on) besides fulfilling the demand constraint. Therefore this constraint can be linearized and the problem is reformulated as a MILP as follows,

$$\text{Min } f = \sum_{i=1}^N C_i x_i + \sum_{i=1}^N q_i \quad (4.6)$$

S.T:

$$\text{Demand Constraint: } \sum_{i=1}^N q_i \geq d \quad (4.7)$$

$$\text{Bounds: } q_{i,\min} \leq q_i \leq q_{i,\max} \dots \dots q_{N,\min} \leq q \leq q_{N,\max}$$

$$\text{Logical Constraints: } q_i - q_{i,\max} x_i \leq 0 \dots \dots \dots q_N - q_{N,\max} x_N \leq 0$$

$$\text{Binary Constraints: } x_i \in \{0, 1\}$$

Thus the model is a linear integer programming problem that basically splits the CFM among the VFD compressors in an optimal fashion (energy efficient) by making sure that the compressors stay away from part loads. Case studies further prove how the proposed energy efficient load sharing strategy can be effective saving the overall energy consumption and costs.

## 5. MODEL VALIDATION

The purpose of this chapter is to prove that the proposed modelling generic technique for the HVAC system is realizable and applicable. This has been done by modelling a real manufacturing facility using the modelling technique and validating it with the input and output data collected at the actual facility. Since the Optimal Load Shifter and the Compressor Load Scheduler are just mathematical formulations rather than models they have been directly tested using the case studies (Results Chapter).

### 5.1 HVAC Model Validation

To represent small scale manufacturing industries, a company named Electro-Spec that specializes in Electroplating, Passivation and Heat treating services [33] has been chosen. To validate the model, the inputs (HVAC mass flow rate and disturbances) and output (Temperature in the plant) of the actual plant were logged for 5 days. Table 5.1 shows the details of the sensor and loggers that were used to log the input and output parameters for model validation.

For heating and cooling purposes, Electrospec uses 3 Rooftop Units and 2 Packaged that are based on On/OFF control with a maximum Volumetric flow rate of  $47m^3/s$  and minimum volumetric flow rate of  $4.7m^3/s$ . Since the actual plant has numerous disturbances affecting the temperature, at least the major temperature disturbances had to be accounted for model validation accuracy. This includes the 1500 T5s light bulbs that are rated 35 W each and are about 10% efficient (90% of the power consumed is dissipated as heat), an oven that releases exhaust gases at 116°F (46.67°C) and 40 different chemical tanks (of similar dimensions) that keep releasing heat at an average temperature of 150°F (65.56°C). Since the material properties of the chemicals in the tanks were not available, these tanks were approximated

Table 5.1. Sensors and Loggers used for Model Validation

#	Sensor	Qty	Purpose	Placement	Specifications	Sampling rate
1	Temperature Sensor and Datalogger	8	Measure and log surrounding rooms temperatures	Close to the thermostat in the room	-40° to 122° F(-40 ° C to 50 ° C) $\pm$ 0.45° F from 32° to 122° F ( $\pm$ 0.25° C from 0° to 50° C)	1 sample/min
2	Temperature Sensor and Datalogger	5	Measure average plant temperature	Close to the thermostats in the plant	-40° to 122° F(-40 ° C to 50 ° C) $\pm$ 0.45° F from 32° to 122° F ( $\pm$ 0.25° C from 0° to 50° C)	1 sample/min
3	Current Sensor	5	Measure the 3 phase current of the HVAC blower fans	Hooked to the one of the 3 phase wires of the HVAC blower fans	10-100Amps $\pm$ 4.5% of full scale	1 sample/min

as 40 tanks containing HCl (being the most common chemical in the tanks). The wall surrounding the plants have been named based on the material properties and the zones that they cover (figure 5.1). Each zone temperature is the disturbance and its associated wall temperature is the state that affects the temperature of the system. Figure 5.1 shows the plant layout (provided by Electro-Spec personnel) with zone names and sensor locations. Table 5.2 shows the common parameters values used to model the Electro-Spec plant using the RC HVAC method. Table 5.3 shows the material properties of the zone walls.

There are other uncertain disturbances such as the loading/unloading area that is open at irregular intervals to the outside temperature for operating the forklifts and other equipment that haven't been accounted for in this model.

Table 5.2. Parameters used for Electro-Spec plant model

Parameter	Definition	Value
$C_p$	Specific heat capacity of air	$1005 \frac{J}{kg} \cdot K$
$h_i$	Convection coefficient of inner walls	$5 \frac{W}{m^2} \cdot K$
$h_o$	Convection coefficient of outer wall	$20 \frac{W}{m^2} \cdot K$
$V_{min}$	Minimum Volumetric flow rate of air from HVAC	$4.7 m^3/s$
$V_{max}$	Maximum Volumetric flow rate of air from HVAC	$47 m^3/s$

Table 5.3. Heat Disturbances at Electro-Spec

Disturbance	Quantity	Temperature
Oven Exhaust	1	320°F (160°C)
Chemical Tanks	40	~ 150°F (65.56°C)
T5 lamps	1500	95°F (35°C)

Then using the exact heat transfer and mechanical properties of the materials in the plant for the model, Electro-Spec plant was modelled using Simulink as shown in figure 5.2. To use the apply the same temperature disturbances as that of the actual plant that were logged in the plant using a sampling time of 1min, the whole system was converted from continuous to discrete with the same sample time (clearly illustrated in figure 5.4) . Figure 5.4 shows the exploded view of the plant and one of its subsystem.

The discrete model was linearized about the initial conditions of the plant . The state space model was found to be Controllable and Observable. The final model had 34 states, 1 input, 1 output and 32 disturbances. Figure 5.5 shows the root locus of the Electro-Spec model which can be used to assess the stability of the system.. From the root locus, it can be seen that all the poles are within the unit circle. This means that the system is stable and can be used for implementing control systems.

# ESI Facility Map

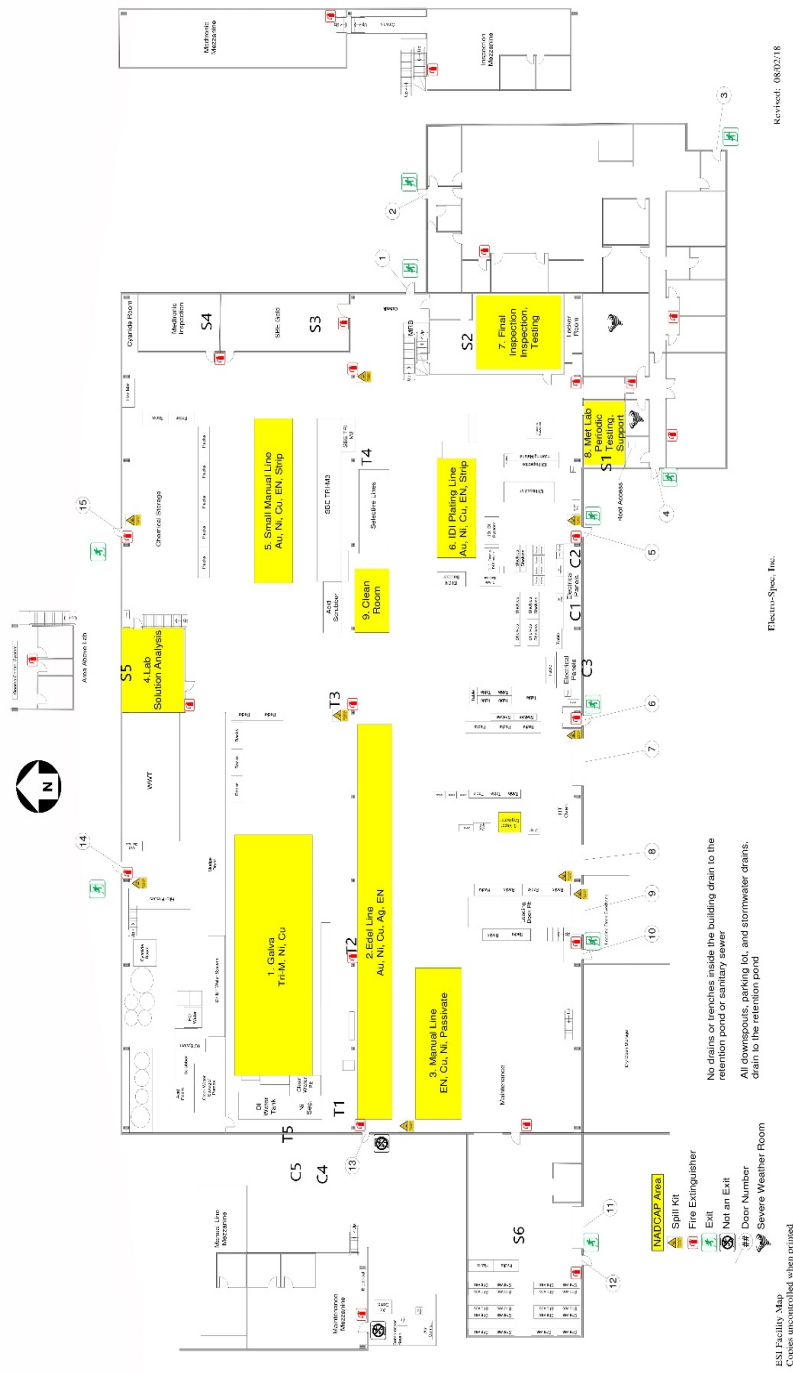


Figure 5.1. ElectroSpec Plant Material Specifications. S1 to S6 are the surrounding zones with temperature sensors at every zone, T1 to T5 are the temperature sensors used for the measurement of the average plant temperature. C1 to C5 are the current sensors used to measure the HVAC fan currents

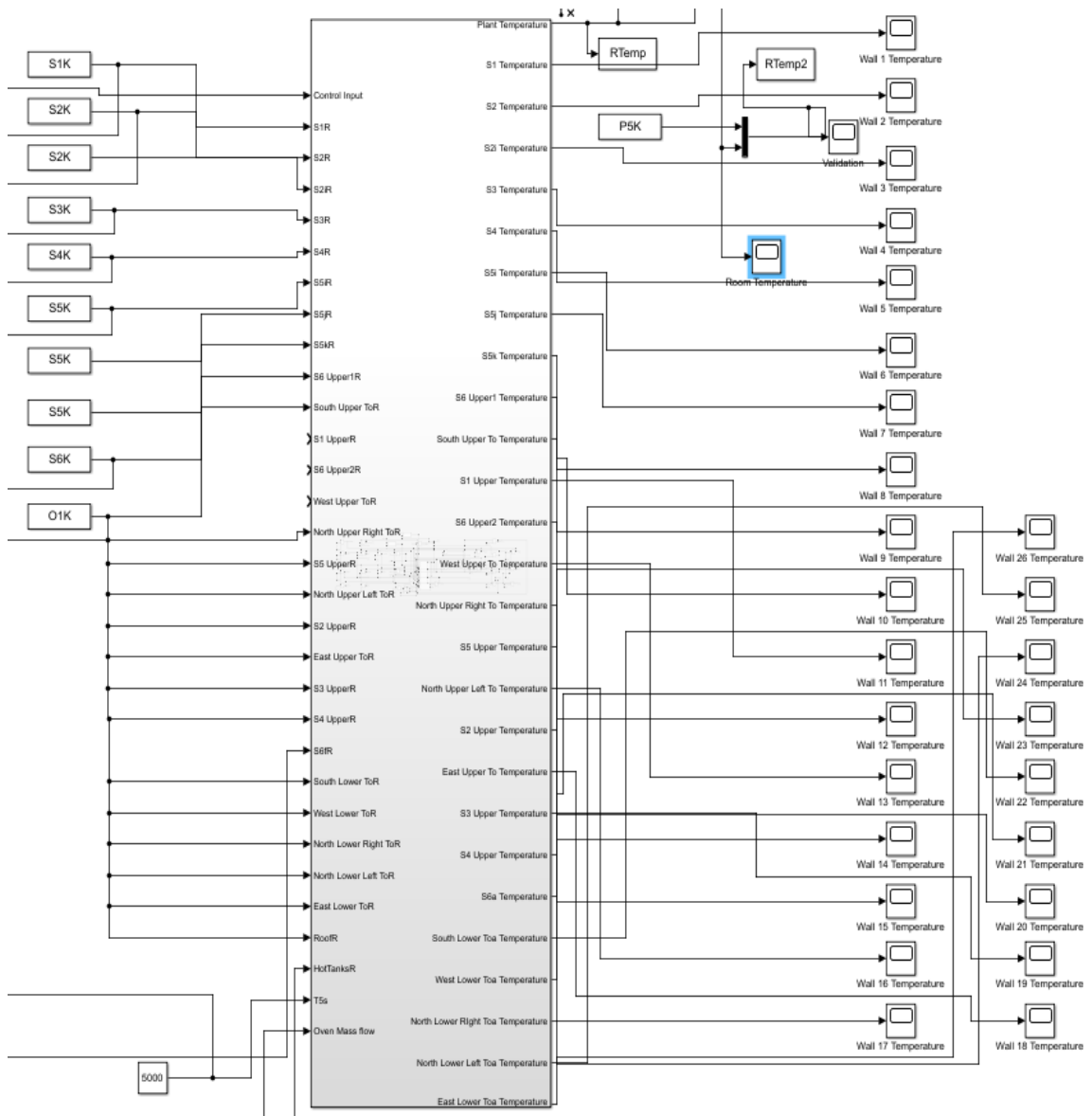


Figure 5.2. Simulink block diagram model of the Electro-Spec Plant. The inputs are the HVAC flow rate and the surrounding room temperature data (S1K, S2K, S3K, S4K, S5K, S6K and O1K)



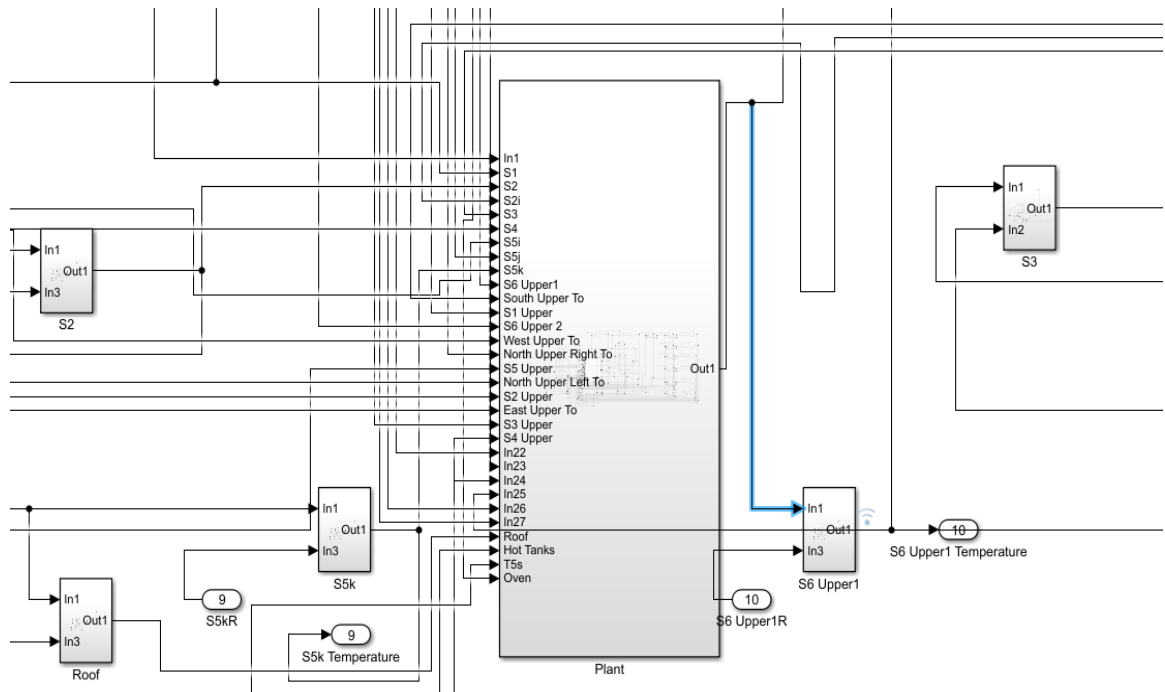


Figure 5.3. Exploded view of the Electro-Spec Plant model showing all the subsystems (zone walls) and disturbances

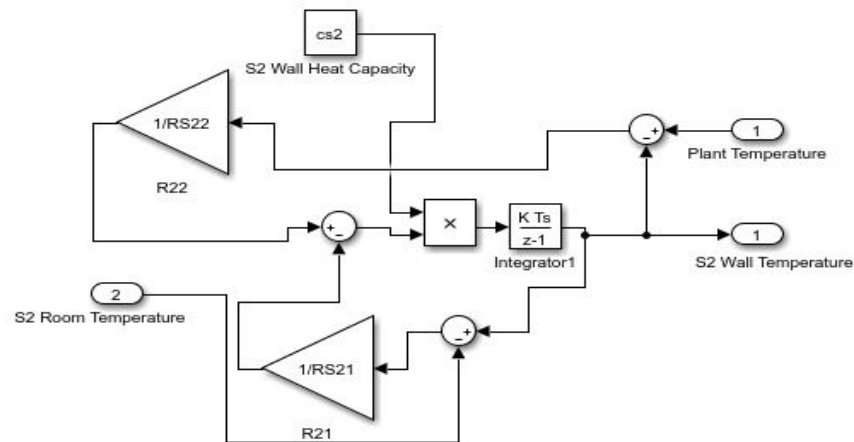


Figure 5.4. Exploded view of the S2 zone wall subsystem. The inputs to this block are the plant temperature and S2 room temperature and the output is the S2 zone wall temperature.  $R21$  and  $R22$  are the thermal resistances,  $cs2$  is the thermal capacity of S2 zone wall

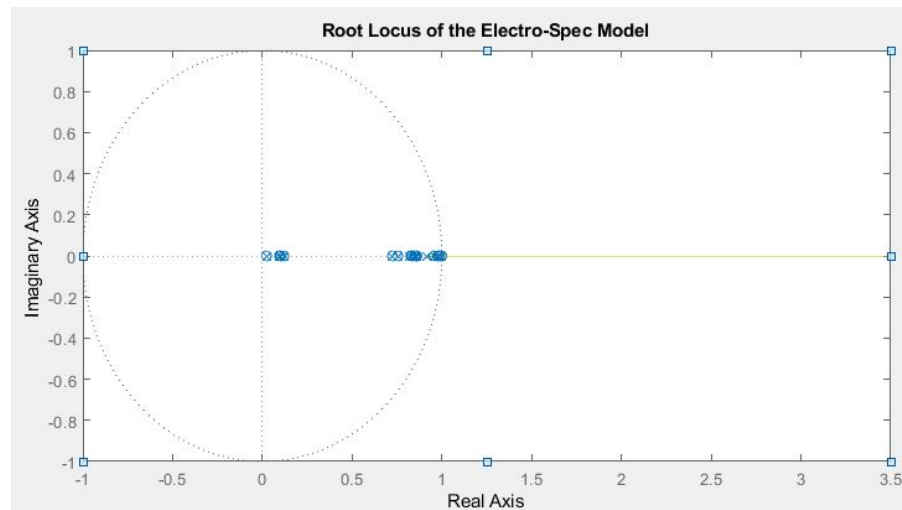


Figure 5.5. Root locus of the Electro-Spec plant model

Then the temperature response of the model and that of actual plant were compared by applying the same input flow rate to the model as shown in figure 5.6

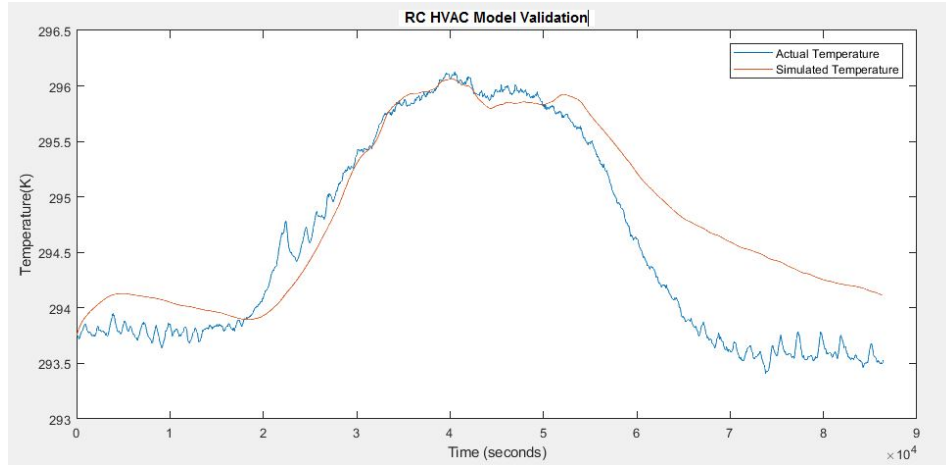


Figure 5.6. Comparison between the Actual Temperature of the plant and the model temperature for 1 day (86400 seconds) with the same input volumetric flow rate of  $47 \text{ m}^3/\text{s}$

From figure 5.6, it can be clearly seen that the model is quite accurate in terms of mimicking actual plant. Deviations in the temperature are mainly due to the assumptions based on which the RC HVAC model for Electro-Spec was built and other stochastic temperature variation factors that have been previously mentioned. To find the model validation accuracy, the Mean Absolute Error [34] test was carried out using the following equation

$$MAE = \frac{1}{n} \sum_{j=1}^n |y_j - \hat{y}_j| \quad (5.1)$$

The MAE for this model is 0.39 which means that there is an average error of 0.39 Kelvins between the actual manufacturing facility and model. This model was further used for applying the energy efficient Model Predictive Control as shown in the results section of this report.

Table 5.4. Zone Wall properties

Zone Wall Name	Material	Area in ( $m^2$ )	Volume ( $m^3$ )
S1	Concrete	23.980	4.872
S2	Gypsum	52.84	5.73
S2 Other side	Gypsum	27.31	2.772
S3	Polystyrene	53.821	5.468
S4	Polystyrene	36.391	5.468
S5i	Gypsum	25.966	2.638
S5j	Gypsum	22.97	2.334
S5k	Gypsum	22.97	2.334
S6	Fiberglass, Air, Steel	29.729	7.55
S6 Upper 1	Fiberglass	29.729	1.507
South To lower	Fiberglass, Air, Steel	149.015	30.847
South To Upper	Fiberglass	298.03	15.139
S1 Upper	Fiberglass	17.477	0.866
S6 Upper 2	Fiberglass	31.586	1.601
West To lower	Fiberglass, Air, Steel	18.084	22.604
West To Upper	Fiberglass	10.234	10.234
North To Upper Right	Fiberglass	183.146	9.326
North To Lower Right	Fiberglass, Air, Steel	73.57	18.67
S5 Upper	Fiberglass	41.67	2.112
North To Upper Left	Fiberglass	232.256	11.775
North To Lower Left	Fiberglass, Air, Steel	92.902	23.595
S2 Upper	Fiberglass	55.912	2.834
East To Upper	Fiberglass	45.874	2.326
East To Lower	Fiberglass, Air, Steel	20.810	5.287
S3 Upper	Fiberglass	53.735	2.78
S4 Upper	Fiberglass	39.567	2.006

## 6. COMPUTATIONAL RESULTS

### 6.1 Case Study: Optimal load shifter

As mentioned in the modelling section of the optimal load shifter, real data from a compressor running at Electro-Spec has been used to show how the predicted demand can be useful in energy efficient scheduling of the flexible machines.

The demand data of the compressor is defined as follows

$$\text{Compressor power} = \sqrt{3} I^*V \quad (6.1)$$

Where  $I$  is the current and  $V$  is the voltage The data had been logged at a sample time of 1 min. This was adequate enough to capture the changes in the 3 phase current of the compressor and any lower sampling times would result in the inclusion of high frequency noise. Since the model requires the sample time of the predicted data to be at least as much as the minimum run time( the minimum amount of time the machine has to run once it has been started)of the machine, it may sound feasible to use existing data. Without down sampling the number of decision variables increase with the number of machines and scheduling horizon. For example, for a scheduling horizon of 2.5 hours and two machines, the number of variables would be at least 300 which is undesirable. However, down sampling too much has its drawbacks in that some of the surges of the demand would not be recorded due to the loss of the data points. Thus it is crucial to first analyze the data in terms of the surge times and minimum run time of the machine. Figures 6.1 shows the actual demand data (1 minute sample time) and down sampled (5 minute sample time) demand data for of a compressor for 2.5 hours

The objective was to schedule all the machines in the low demand period while not violating the machine runtime constraints(figure 6.2). As discussed earlier, flexible



Figure 6.1. Compressor demand data sampled after down sampling

machines can be interruptible or uninterruptible. Interruptible machines just need to run for a minimum run time that is equal to the sample time of the demand data before they can be stopped or continued. Uninterruptible machines can have a minimum run time that is equal to  $n$  times the sample time of the demand [35].

Load Shifting case 1: Flexible and Interruptible loads

**Problem Statement:** Two machines both of which have a minimum runtime of 5 minutes need to be scheduled for a total runtime of 75 minutes and 50 minutes respectively.

The cost function formulation is as follows

$$Maxf = \sum_{t=1}^{30} \sum_{i=1}^2 x_{i,t} \frac{-1}{avg(D_{t+1}, D_t) P_i}$$

Constraints:  $\sum_{t=1}^{30} \sum_{i=1}^2 x_{t,i} = \varepsilon_i$  where  $\varepsilon_i$  is the number of timesteps the machine should be in running state

$$\sum_{t=1}^{30} x_{t,2} = 15$$

$$\sum_{t=1}^{30} x_{t,1} = 10$$

The above cost function and constraints were input as matrices in `intlinprog` function. Figure 6.3 shows the optimal run time sequence generated by the optimization model.

Blue line is the predicted demand profile and the rectangular blocks represent the schedule of the two machines. The bottom rectangular blocks correspond to the sequence of machine runs for machine 1 and the top ones correspond to that of machine 2. Similar notation will be followed for the rest of the figures related to optimal machine scheduling. From visual inspection in figure 6.3 it can be found that the algorithm makes sure that the machines are run only in the low demand regions. Also, since the minimum runtime of the machine is same as that of the sample time, there are no additional mathematical constraints (equations 5.1 c and d) associated with it which makes it easier for the algorithm to find the global minimum.

Load Shifting case 2: Flexible and Uninterruptible loads

Problem: Two scenarios have been tested for this case. In both the scenarios, machine 1 is flexible and uninterruptable (minimum run time is not same as demand sample time) for a certain period while machine 2 is flexible and interruptible.

For the first scenario machine 1 minimum runtime is 10minutes and its total run time is 20 and the total runtime for machine 2 is 50 minutes. Machine 1 minimum runtime is 15 minutes for both machines and the total run time is 30 minutes for machine 1 and 50 minutes for machine 2 in the second scenario. The formulation is as follows,

$$Min f = \sum_{t=1}^{30} \sum_{i=1}^2 x_{i,t} \frac{-1}{avg(D_{t+1}, D_t) P_i}$$

Constraints:

$$\sum_{t=1}^{30} x_{t,2} = 4$$

$$\sum_{t=1}^{30} x_{t,1} = 10$$

$$x_{1,1} + x_{1,2} + x_{1,3} + My_1 = 2$$

$$x_{1,2} + x_{1,3} + x_{1,3} + My_2 = 2$$

$$\vdots$$

$$x_{1,34} + x_{1,35} + My_{35} = 2$$

$$x_{2,1} + x_{2,2} + My_{36} = 2$$

$$x_{2,2} + x_{2,3} + My_{37} = 2$$

$$\vdots$$

$$x_{1,33} + x_{1,34} + My_{70} = 2$$

$$\sum_1^{35} y_k = 3$$

$$\sum_{36}^{70} y_k = 9$$

The formulation for the second scenarios is quite similar to the first except for the change in the minimum time parameters for both machines.

As can be seen from figures 6.4 and 6.5 the algorithm schedules the machines such that the machines are run only during the low demand time steps. Also, the constraints for machine 1 is satisfied for both the scenarios i.e uninterruptable runtime and the total runtime. This is made possible through the additional constraints that force the algorithm to consider all the possible combinations of the machine sequence and selects the ones that have the lease cost. The total number of variables for scheduling 2 machines flexible and uninterruptable over a period of 2.5 hours is



at least 60 depending on the minimum runtime. Intlingprog() which is the integer programming solver in MATLAB solves for 89 variables within 0.2 seconds which means that more than 2 machines and longer schedule periods are feasible using the load shifter. For detailed problem formulation in MATLAB refer to Appendix A or the illustrative examples.

## 6.2 Case Study: Compressor Scheduler

In this case study, 6 industrial grade compressors with VSDs have been chosen to test the heavy-duty mode and light duty mode of the load scheduler. The specifications of the compressors are detailed in table 6.1. The objective here is to schedule in an energy efficient way while meeting the demand thus reducing costs [25].

Table 6.1. Light duty and heavy duty compressors used for the case study

#	Name	Max CFM at 100 PSI	HP
1	Quincy QT-54	16	5
2	Emax ERVK070003	29	7.5
3	CPVsd 10 BM	43	10
4	Atlas GA37 125 AFF	229	50
5	J75 Mohawk VSD	341	75
6	EMAX 100-HP Rotary Screw Air Compressor	423	100

The first three compressors would be tested using the light duty mode and the last three will be tested using the heavy duty mode as per their HP ratings.

Case 1: A total demand of 25cfm at 100 Psi has to be met by 3 light duty compressors with Max CFMs  $q_{1,max} = 229$  CFM,  $q_{2,max} = 341$  CFM ,  $q_{3,max} = 423$  CFM at the same pressure.

Find:  $x_1$  ,  $x_2$  ,  $x_3$  (systems states) and  $q_1$  ,  $q_2$  ,  $q_3$  (Compressor CFM)

As mentioned in the modelling section only the constants corresponding to the states of the compressors are chosen to follow the triangle law of sides i.e C1=3,C2=4 and C3=5

$$Minf = \sum_{i=1}^N C_i x_i + \sum_{i=1}^N q_i$$

$$Minf = 3 x_1 + 4 x_2 + 5 x_3 + q_1 + q_2 + q_3$$

Compressor Scheduler Solution:  $x_1 = 0, x_2 = 1, x_3 = 0, q_1 = 0, q_2 = 25, q_3 = 0$  .

In, other words only compressor 2 should run at 25CFM (100Psi).

The flow of the algorithm is as follows,

Figure 6.7 shows the results obtained using the MATLAB code that uses `intlinprog()`(branch and bound algorithm) to find the optimal solution for scenario 1.

To show the effectiveness of the light duty mode, the results obtained using this mode have been compared to a scheduling system that loads the compressors in the increasing order of their capacities. The following equation is used to find the cost of running each of the compressors over a period of 5 hours for both the schedulers being compared,

$$Cost = \frac{(bhp) \times (0.746) \times (\#of\ operating\ hours) \times \left(\frac{\$}{kWh}\right) \times (\%time) \times (\%full\ load\ bhp)}{Moto\ Efficiency}$$

Using \$ 0.10 per KWh(Indiana) and the specs of each of the compressor in table 6.1 the cost incurred without using the proposed scheduler for scenario 1 is \$ 3.07 and that when the scheduler is used is \$ 2.58 which translates to about 16% energy and cost savings.

Case 2: A total demand of 400cfm at 100 Psi has to be met by the 3 heavyduty compressors with Max CFMs  $q_{1,max} = 229$  CFM,  $q_{2,max} = 341$  CFM ,  $q_{3,max} = 423$  CFM at the same pressure. Find:  $x_1$  ,  $x_2$  ,  $x_3$  (systems states) and  $q_1$  ,  $q_2$  ,  $q_3$  (Compressor CFM)

In this case only the constants corresponding to the capacities of the compressors are chosen i.e i.e D1=1, D2=4 and D3=5

$$Minf = \sum_{i=1}^N x_i + \sum_{i=1}^N D_i q_i$$

$$Minf = x_1 + x_2 + x_2 + 1q_1 + 2q_2 + 3q_3$$

Compressor Scheduler Solution:  $x_1 = 1, x_2 = 1, x_3 = 0, q_1 = 229, q_2 = 271, q_3 = 0$  which means that compressor 1 should run at full load and and 2 should run at 271 CFM. Figure 6.8 shows the results obtained using the MATLAB code for scenario 2.

A parallel compressor system without the heavy duty mode would not switch to each of the compressor in the increasing order of their capacities. To show the effectiveness of the proposed compressor scheduler in the above scenario, it has been compared to a system where the compressor whose capacity is closest to the total capacity is used first and the rest of the demand is fulfilled by the remaining compressor in the decreasing order of their capacity. Using equation and 0.10\$ /Kwh for Indiana, the cost incurred without using the proposed scheduler for scenario 1 is \$ 50.6 and that when the scheduler is used is \$ 43.18. Therefore, the scheduler can result in about 14.6% lesser costs if used for scheduling heavy duty air compressor systems.

### 6.3 HVAC MPC

The validated Simulink model representing the Electro-Spec plant was used to show the effectiveness of the MPC in decreasing energy consumption hence the costs. In terms of the practical implementation of such a control system, the blower fans need to equipped with Variable Frequency Drives(VFDs) so that the fans can run just enough to meet the heating or cooling loads without wasting energy unnecessarily. The model that was built using the RC modelling technique is first the model was first linearized about the equilibrium points (294K) and converted into continuous state space form. Then using the zero order hold and sampling time, it is converted to discrete state space form. This has to be done for two reasons 1)MPC works with only discrete models and 2)the disturbances and the inputs that were measured at the facilities are obviously discrete in time(sampled at 1 sample per second). Then

as discussed in the MPC section, the matrix was converted to augmented form and corresponding matrixes  $Y, F$  and  $\Phi$  were found. Since, the MPC requires, the past and present states of the systems Kalman filter based state estimator of MATLAB was used to achieve this. The MPC controller was implemented in Simulink which has a cost function as follows (as per MATLAB Documentation),

$$J(u) = \sum_{i=1}^p \left\{ \frac{w_i^y}{s^y} [r_j(k+i|k) - y_j(k+i|k)] \right\}^2 + \sum_{i=1}^{p-1} \left\{ \frac{w_i^{\Delta u}}{s^u} [u_j(k+i|k) - y_j(k+i-1|k)] \right\}^2 + \rho_\epsilon \epsilon_k^2 \quad (6.2)$$

Which is subject to the following constraints,

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k)$$

$$\frac{y_{min(i)}}{s^y} - \epsilon_k V_{min}^y(i) \leq \frac{y(k+i|k)}{s^y} \leq \frac{y_{max(i)}}{s^y} - \epsilon_k V_{max}^y(i), \quad i = 1:p, j = 1:n$$

$$\frac{u_{min(i)}}{s^u} - \epsilon_k V_{min}^u(i) \leq \frac{u(k+i-1|k)}{s^u} \leq \frac{u_{max(i)}}{s^u} - \epsilon_k V_{max}^u(i), \quad i = 1:p, j = 1:n$$

$$\frac{\Delta u_{min(i)}}{s^y} - \epsilon_k V_{min}^{\Delta u}(i) \leq \frac{\Delta u(k+i-1|k)}{s^y} \leq \frac{\Delta u_{max(i)}}{s^y} - \epsilon_k V_{max}^{\Delta u}(i), \quad i = 1:p, j = 1:n$$

Where  $y$  is the output,  $u$  is the input,  $\Delta u$  is the input rate, and  $w_i^y$  and  $w_i^{\Delta u}$  are the weights associated with the  $i$ th tracking and input rate terms respectively.  $\epsilon_k$  is used as slack variable to soften the constraints whenever possible to keep the controller from stopping without finding the solution. The above cost function is responsible for reference tracking and controlling the change in the input (reducing the energy). This is subject to the discrete state space model and bounds placed on the output, input and input rate. Figure 6.9 shows simplified block diagram of how the controller and Electro-Spec plant are connected together and figure 6.10 shows its Simulink model. At the facility the HVAC fans are running at full speed to cool the space.

Before starting the simulation, the input which is HVAC flow rate was constrained between 4.7 and 47 m<sup>3</sup>/s as these were the physical limitations of the HVAC systems at the actual facility and the rate of flow rate was constrained between 5 m<sup>3</sup>/s<sup>2</sup> and

-5 m<sup>3</sup>/s<sup>2</sup> which is quite reasonable for most fans with VFDs. To get acceptable performance with less overshoot or oscillations or more rise time, the MPC was tuned by changing the weights  $w_i^y$  and  $w_i^{\Delta u}$ . The MPC was tested on the plant model for a setpoint of 296K as shown in figure 6.11 and 6.12. In figure 6.12, the blue plot represents the actual temperature at the facility with the fans running at full speed and providing a maximum CFM of 47 m<sup>3</sup>/s. The red plot on the other hand is the simulated temperature response of the plant with the same inputs and disturbances.

From figure 6.11, it can be seen that the output response has a slight % overshoot (50%) in the beginning, but has a very good tracking (steady state error of 0.001), a rise time of 16 minutes and a settling time of 46 minutes despite the input and input rate constraints. In figure 6.12, the difference between the red line and the blue line represent the potential energy savings associated with the MPC implementation. This performance is reasonable since in a plant environment temperature fluctuations of about 2°F over a period of 1 hour are tolerable than a large rise time that leads to discomfort for longer periods of time. The total HVAC fan power consumption (as per the measured average 3 phase current at the facility) in case of the existing ON/OFF control (3 phase) is given by

$$P = \sum_1^5 V * \sqrt{3}$$

$$= 460V * \sqrt{3} * (24.61 + 24.42 + 17.38 + 14.22 + 13.83) \text{ Amps} = 75.260 \text{ KW}$$

The average Input rate as per figure 31 is 33.996 m<sup>3</sup>/s. The power consumption of the HVAC fans with the proposed controller is 54.436KW. This is about 27.6% reduction in the power and energy consumption. The assumption here is that the current varies almost linearly with the air flow rate which was used to linearly interpolate the power consumption using the average input rate of the Controller.

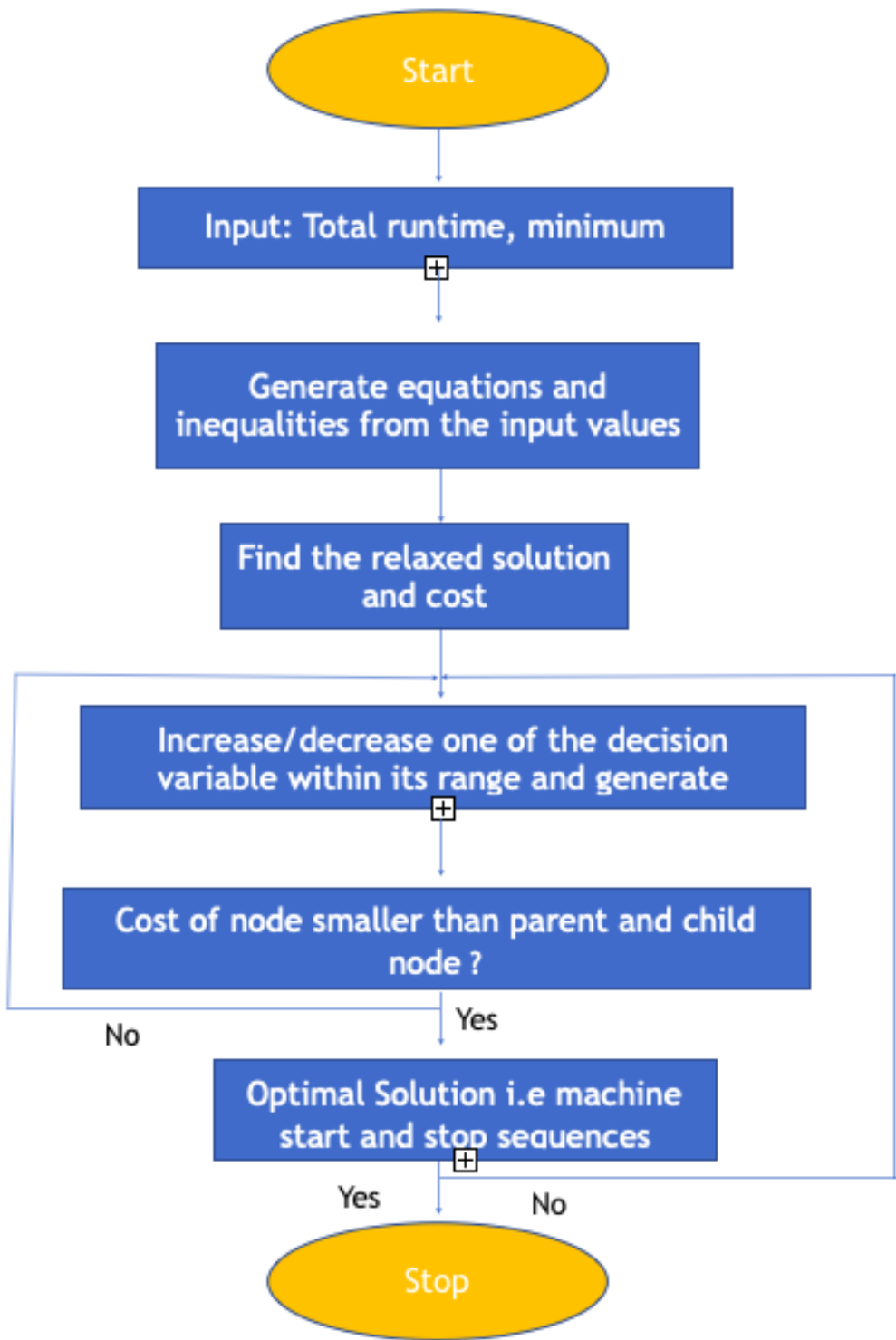


Figure 6.2. Flow chart of the Optimal Load Shifting problem

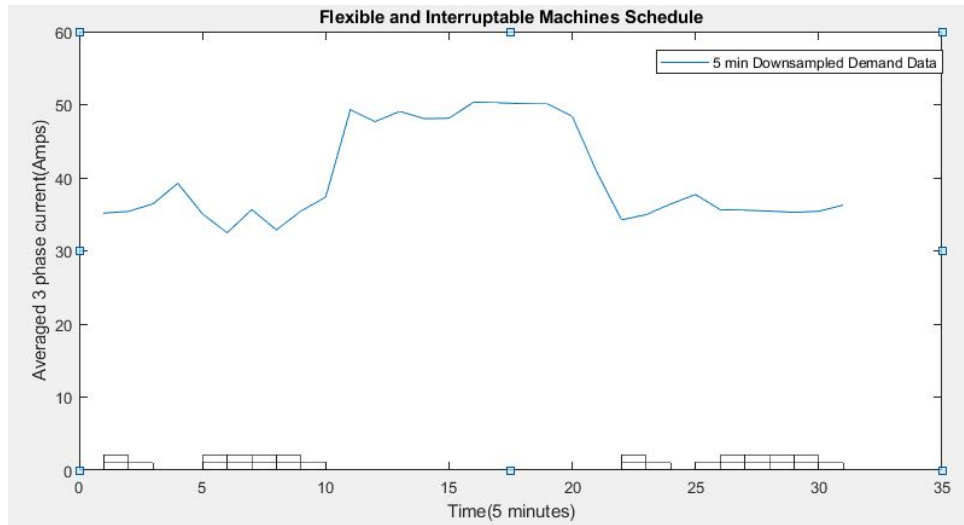


Figure 6.3. Optimal machine schedule for case 1. Each block in the x axis is the 5 minute timestep which is down sampled timestep from figure 6.1

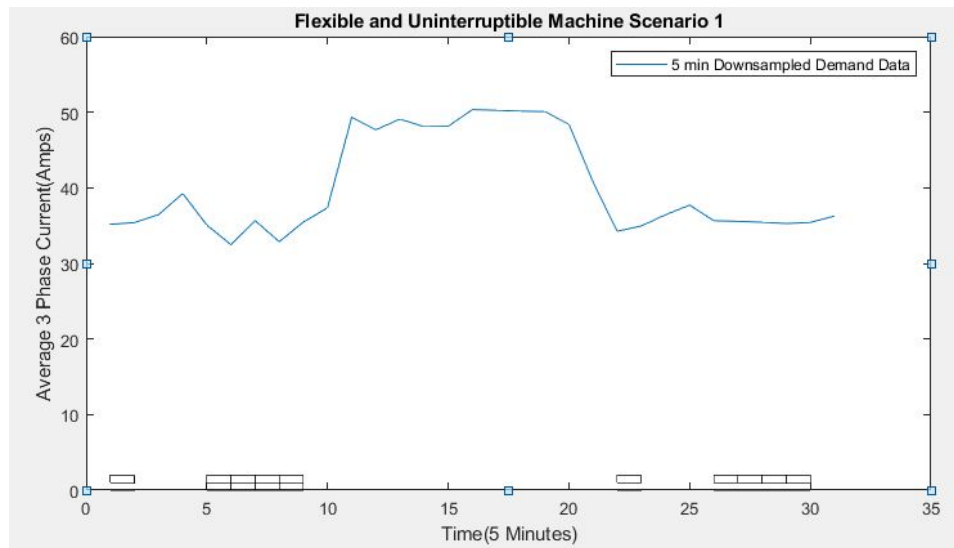


Figure 6.4. Optimal machine schedule for scenario 1 of Flexible and Uninterruptible machines case

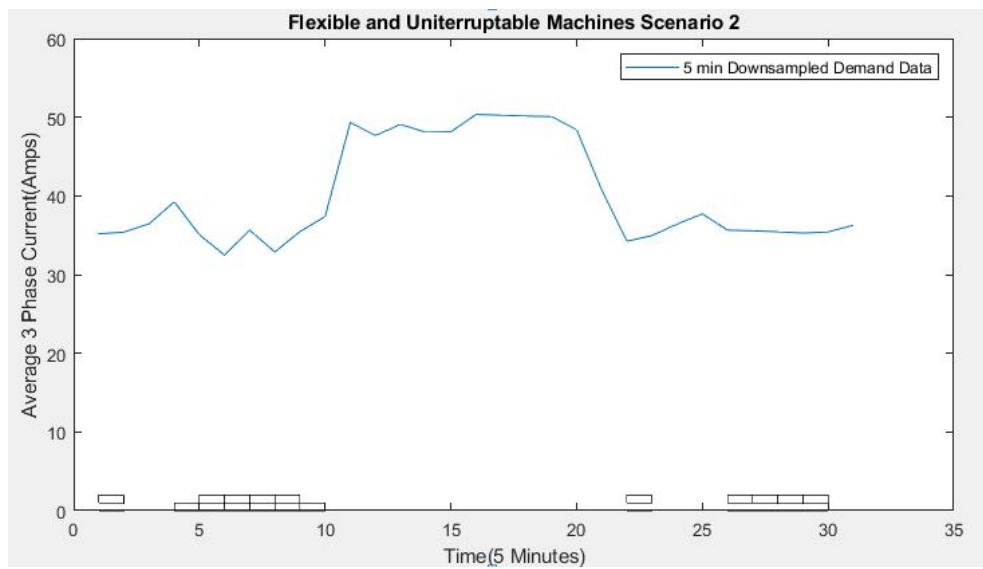


Figure 6.5. Optimal machine schedule for scenario 2 of Flexible and Uninterruptible machines case



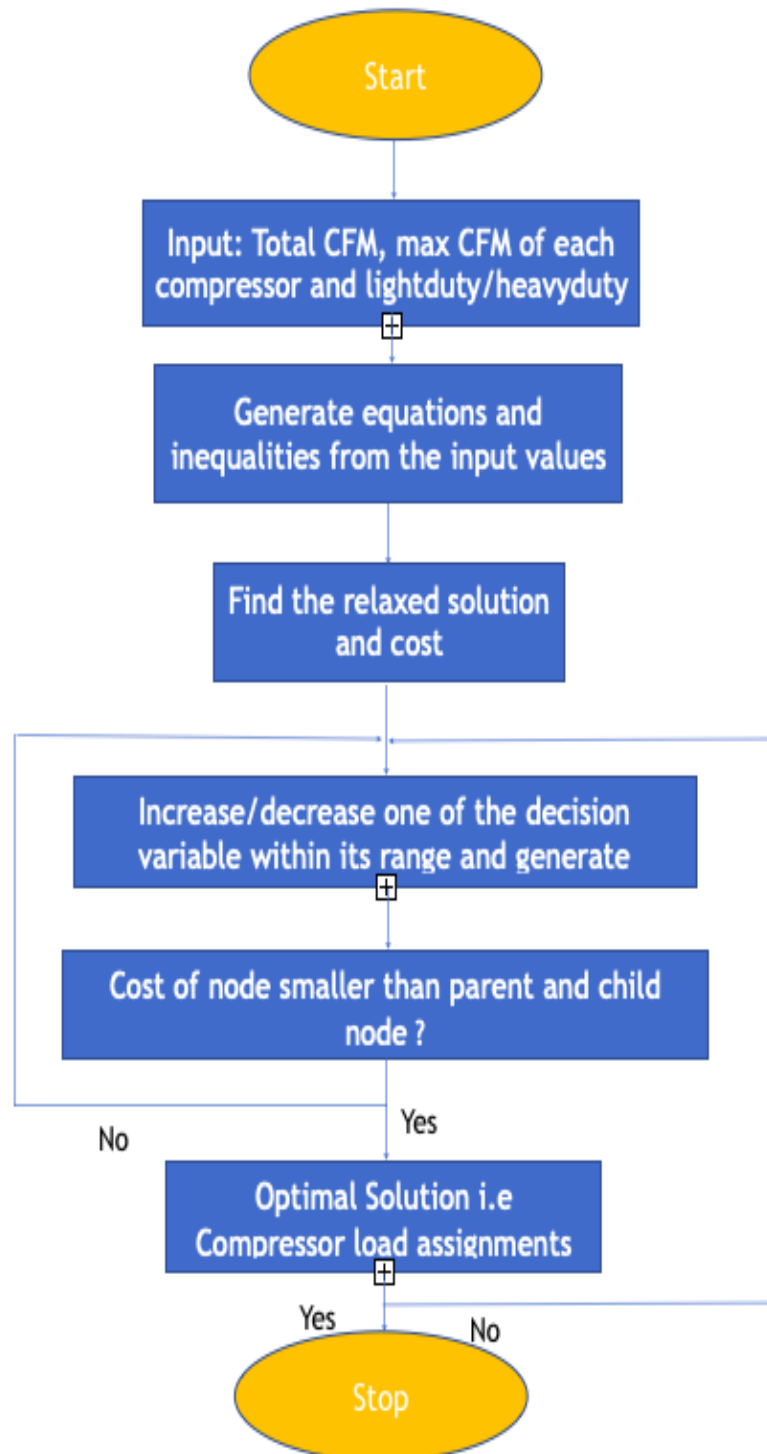


Figure 6.6. Flow chart for the Compressor Load Shifting problem

Intlinprog stopped at the root node because the [objective value is within a gap tolerance](#) of the optimal value, options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are [integer within tolerance](#), options.IntegerTolerance = 1e-05 (the default value).

```
x =
     0
    1.0000
     0
     0
   300.0000
     0
```

Figure 6.7. The solution for compressor case 1. X is a vector of the decision variables that indicates the status of the compressors (first three values) and the CFMs assigned to the compressors (last three values)

Intlinprog stopped at the root node because the [objective value is within a gap tolerance](#) of the optimal value, options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are [integer within tolerance](#), options.IntegerTolerance = 1e-05 (the default value).

```
x =
    1.0000
    1.0000
     0
   229.0000
   271.0000
     0
```

Figure 6.8. The solution for compressor case 2. X is a vector of the decision variables that indicates the status of the compressors (first three values) and the CFMs assigned to the compressors (last three values)

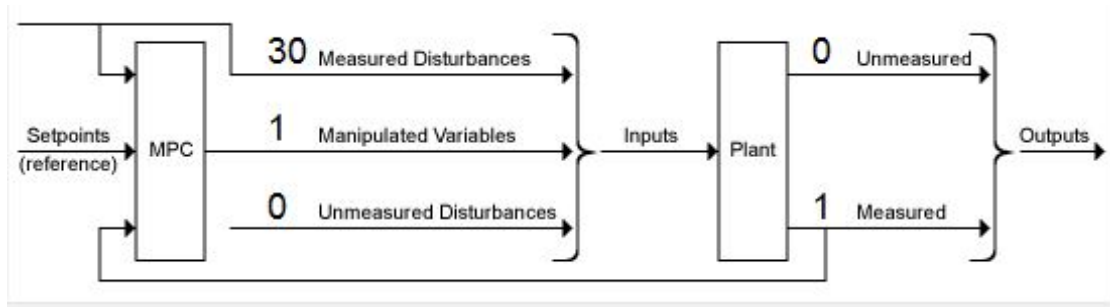


Figure 6.9. Simulink Schematic for the Electrospec Model with MPC

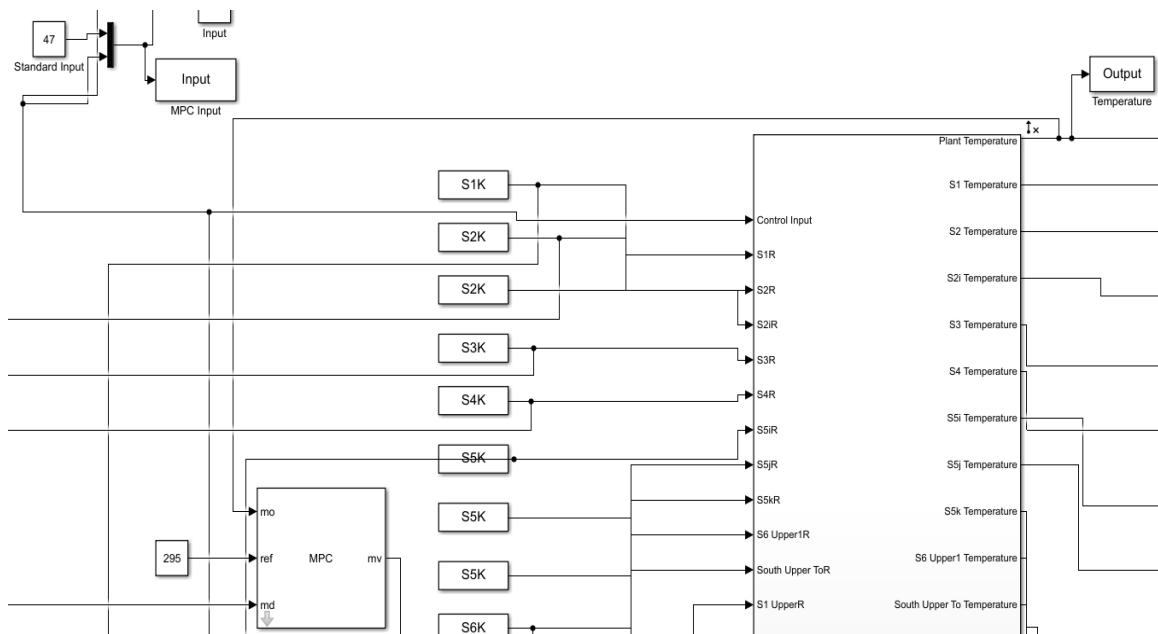


Figure 6.10. Simulink block diagram of the HVAC model with MPC (block on the left)

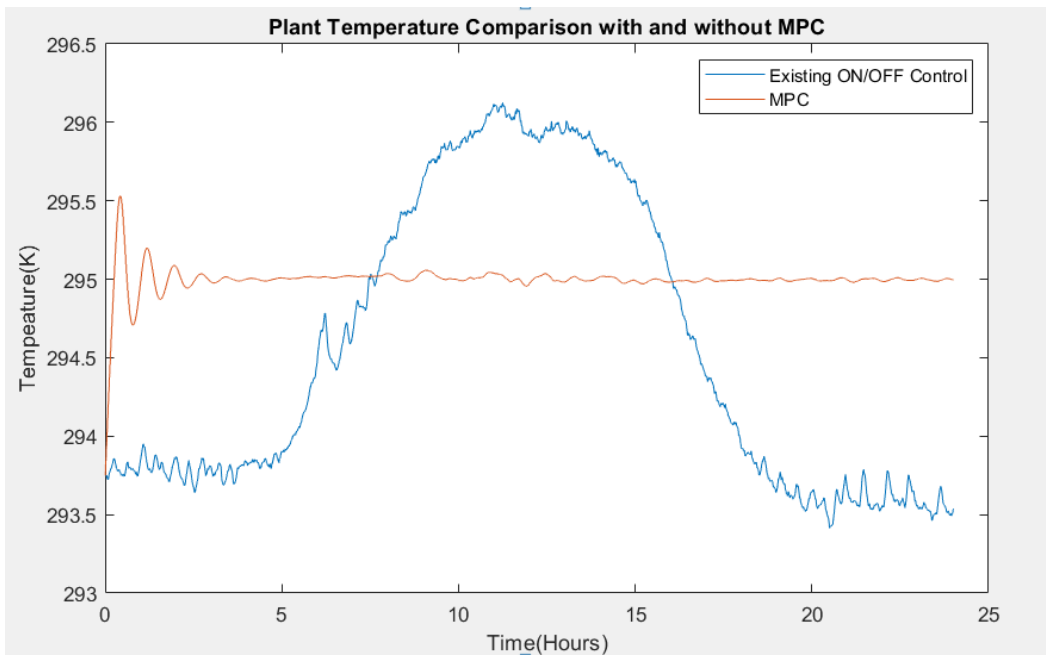


Figure 6.11. Controlled plant temperature using the MPC for a set point of 295K

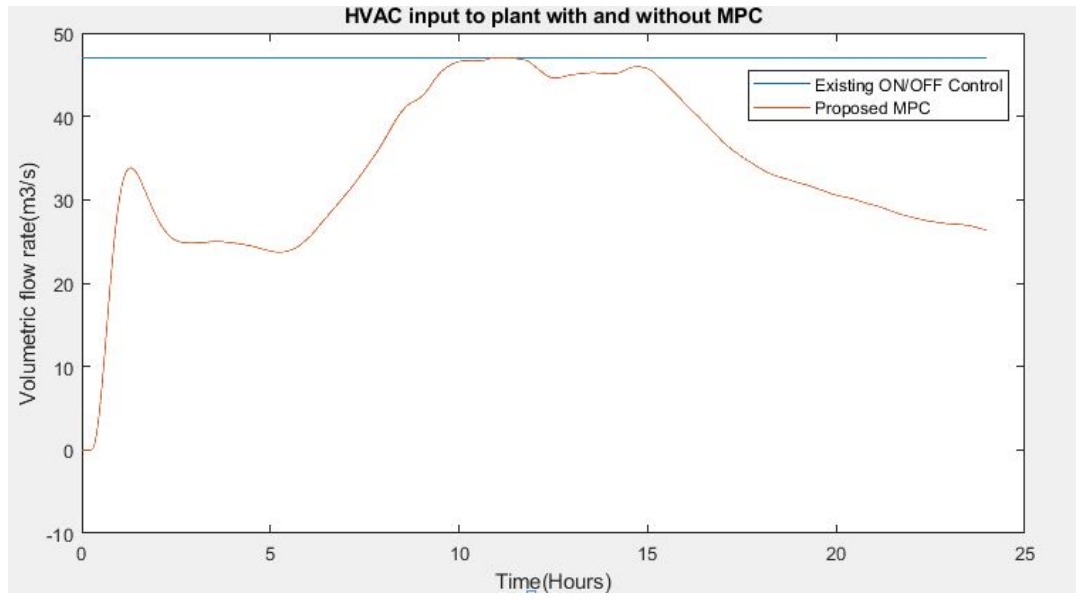


Figure 6.12. Controlled plant input flow rate using the MPC for a set point of 296K

## 7. SUMMARY AND CONCLUSION

In this thesis, the potential energy saving strategies have been explored for the SEU systems which are the HVAC, Compressors and machines driven by electrical motors. The proposed energy management techniques have been proved to be effective in reducing the energy and costs. The OLS was used to achieve load shifting and valley filling which resulted in lower peak demand and costs. The CLS was used to distribute loads among compressors such that all the compressor run in their most efficient conditions for energy efficiency. The lumped parameter HVAC model was used to model a manufacturing facility and the MPC was implemented as the HVAC control for the validated model to improve the energy savings.

In case of the optimal shifter, there is potential for cost saving even though the energy consumption is same due to the load shifting that reduces the peak demand. This is usefully manufacturing industries that are located in regions with utilities that have TOU and Peak Demand schemes. The Compressor scheduler was able to reduce the energy and cost for both the heavy and light duty compressors and this is a generic formulation that can be used regardless of the compressor type as long as the compressor is VFD type and works with other compressor in parallel configuration. The MPC pertaining to its optimal nature was able to reduce the overall energy consumption by running the fan only when needed. The total framework when implement has a potential of saving upto 40% of energy and costs as summarized in the below table. Table 7.1 summarizes the cost and energy savings of the proposed energy management framework.

Table 7.1. Estimated energy savings with the proposed framework

#	Energy Management Technique	Type	Savings
1	Optimal Load Shifter	Optimization	27.6% Energy and Cost saving
2	Compressor Scheduler	Optimization	14.6% (Heavy duty) and 16% (Light duty)Energy and Cost saving
3	HVAC MPC	Control System	Cost Savings depending on the peak demand price or TOU price

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## REFERENCES

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## APPENDICES

## A. COMPRESSOR SCHEDULER CODE

```

1 d=25;
2 f = [3;4;5 ,;1;1;1];
3 intcon = [1 2 3 4 5 6];
4
5 %linear inequality constraints.
6 % A = [-1,-1,0,0;0,0,-1,-1;-50,0,1,0;0,-100,0,1];
7 % b = [-1,-100,0,0];
8 A = [-1 -1 -1 0 0 0;-16 0 0 1 0 0;0 -29 0 0 1 0;0 0 -43 0 0 1;0 0 0
      -1 -1 -1];
9 b = [-1 0 0 0 -d];
10 Aeq=[];beq=[];
11
12 %bound constraints.
13 lb = [0;0;0;0;0;0];
14 ub = [1;1;1;16;29;43]; % Enforces x(3) is binary
15
16 %Calling intlinprog.
17 x = intlinprog(f,intcon,A,b,Aeq,beq,lb,ub)

```

## B. LOAD SHIFTING CODE

```

1 function [] = loadshiftvalmod2f(n,ta,T,ST,nT,M2)
2 tic
3 close all;
4 nd=n-1;
5 ta=ta(1:n)';
6 tad=mean([ta(1:end-1);ta(2:end)]);
7 tad=1./tad;
8 ndT=T/ST-1
9 f1=[-1/p1;-1/p2];
10 f2= repmat(f1,nd,1);
11 intcon = 1:2*nd+nd-ndT;
12 tad=repelem(tad,2);
13 tadd=tad';
14 f=f2.*tadd;
15 f=[f;zeros(nd-ndT,1)]
16 %linear inequality constraints.
17 zo=[0 -1];
18 zone=repmat(zo,1,ndT)
19 Azone=[-1 zone];
20 A0(1:2*nd-length(Azone))=zeros;
21 A1=[Azone A0];
22 A2=repmat(A1,nd-ndT,1);
23 % id = [0:2:2*nd-4];
24 id = [0:2:2*(nd-ndT)-2];
25 A3=cell2mat(arrayfun(@(x) circshift(A2(x,:),[1 id(x)]),(1:numel(id)
    )','un',0));
26 Aeye=(-10)*eye(nd-ndT);
27
28 A=[A3 Aeye]
29 Ae=eye(2);

```

```

30 Aeq1= repmat(Ae, nd, 1) . ' ;
31 Aeq2=[Aeq1 zeros(2, nd-ndT) ] ;
32 Aeqzo=[zeros(1, 2*nd) ones(1, nd-ndT) ] ;
33
34 Aeq=[Aeq2; Aeqzo]
35 %Aeq=Aeq2
36 beq = [nT/ST M2 nd-nT/ST
37
38 %bound constraints.
39 lb=zeros(2*nd+nd-ndT, 1) . '
40 ub=ones(2*nd+nd-ndT, 1) . '
41
42 %Calling intlinprog.
43 [x, fval] = intlinprog(f, intcon, A, b, Aeq, beq, lb, ub) ;
44
45 t=1:n;
46 plot(t, ta)
47 % plot(t, t25)
48 hold on
49 %default flexible load schedule
50
51 rectangle('Position', [1 0 1 1])
52 rectangle('Position', [2 0 1 1])
53 rectangle('Position', [3 0 1 1])
54 rectangle('Position', [4 0 1 1])
55 %load shifted schedule
56
57 figure
58 % plot(t, t25)
59 plot(t, ta)
60 hold on
61
62 k=0;
63 flag=0;
64 i=1;

```

```
65 k=1;
66 toc
67 while k<=nd
68     if x(2*k)~=0 || x(2*k-1)~=0
69         rectangle('Position',[k 0 1 x(2*k-1)])
70         rectangle('Position',[k 1 1 x(2*k)])
71     end
72     k=k+1;
73 end
74 disp(x);
75 end
```