Wideband Cyclostationary Spectrum Sensing with Receiver Constraints and Optimization

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Abstract

Spectrum sensing is one of the important functions in the context of cognitive radio systems. It determines the presence or absence of free channels in the spectrum and makes them available for the secondary users. Cyclostationary spectrum sensing is one of the spectrum sensing techniques which involves the detection of signals based on their features such as cyclic frequencies, symbol rates, carrier frequencies and modulation types. It detects signals at very low signal-to-noise ratios. Cyclostationary spectrum sensing involves the use of large number of samples for detection resulting in high complexity, cost and low efficiency. In addition there are performance degrading constraints such as cyclic and sampling clock offsets that can occur at the receiver end. These offsets are caused by local oscillator frequency offsets, Doppler effects and jitter.

In order to address some of these issues in the absence of the constraints, the thesis proposes an efficient low complexity multi-slot cyclostationary spectrum sensing technique that uses small number of samples to detect small spectral components made possible by the use of fast Fourier transform and slots of small lengths. Statistical and simulation tests are performed to verify the functionality of the model to offer low complexity and consequently low cost and efficiency.

The thesis also proposes another multi-slot cyclostationary spectrum sensing model that included the receiver constraints such as cyclic frequency offset and sampling clock offset in the test statistic. This model is analysed statistically and uses small lengths of fast Fourier transform and slots to effect significant reduction of these constraints which is also verified with Matlab simulation results.

In order to have a non ad hoc systematic way of detecting and optimizing the sizes of fast Fourier transform and slots, the thesis also proposes step by step algorithms that can be applied to any set of total number of samples representing a wideband channel. This will result in getting the appropriate sizes of slots and fast Fourier transform that will produce low complexity, cost and efficient detection. Matlab simulations are also used to verify this.

Finally, the proposed models are able to address the issues previously mentioned which are associated with the cyclostationary spectrum sensing.

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Dedication

This thesis is dedicated to my lovely wife Mrs Ijeoma Janet Anyim and our great children, Mr Winner Onyedikachi Anyim and Mr David Chukwudi Anyim.

Declaration

Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.

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Contents

Li	st of	Tables		xiii
Li	st of	Figure	S	xiv
GI	ossa	ry		xxv
1	Intro	troduction		1
	1.1	Motiva	ation and State of the Art	1
	1.2	Cogni	tive Radio System	3
	1.3	Cogni	tive Radio Functional Blocks	3
	1.4	Spect	rum Awareness	4
		1.4.1	SNR Estimation Technique	5
		1.4.2	Database Technique	5
		1.4.3	Spectrum Sensing	5
	1.5	Resea	arch Gaps	6
	1.6	Resea	arch Objectives	7
	1.7	Resea	arch Contributions	8
	1.8	Resea	arch Methodology	9
	1.9	Thesis	organization	10
2	Spe	ctrum	Sensing	12
	2.1	Coope	erative Sensing System	13
		2.1.1	Centralized Cooperative Method	13
		2.1.2	Distributed Cooperative Method	14
	2.2	Interfe	erence Based Sensing System	14

2.3	Non-c	ooperative	14
	2.3.1	Energy Detector	15
	2.3.2	Implementation of Energy Detector	15
	2.3.3	Advantages	16
	2.3.4	Disadvantages	16
	2.3.5	Energy Detection Test Statistic	17
2.4	Match	ed Filter	17
	2.4.1	Implementation of Matched Filter	18
	2.4.2	Advantages of Matched Filter	19
	2.4.3	Disadvantages of Matched Filter	19
	2.4.4	Cyclostationary Feature Detection	19
	2.4.5	Advantages of Cyclostationary Feature Detectors	20
	2.4.6	Disadvantages of Cyclostationary Feature Detectors	20
	2.4.7	Compressive Sensing	21
	2.4.8	Wideband Compressive Sensing	21
	2.4.9	Drawbacks of Compressive sampling	23
2.5	Fourie	r Analysis	24
	2.5.1	The Sampling process	24
	2.5.2	Fourier Transform	26
		2.5.2.1 Properties of Fourier Transform	27
	2.5.3	Fourier series	29
	2.5.4	Short-Time Fourier Transform	29
	2.5.5	Discrete-Time Fourier Transform	30
	2.5.6	Discrete Fourier Transform	30
		2.5.6.1 Properties of Discrete Fourier Transform	31
	2.5.7	Fast Fourier Transform	32
		2.5.7.1 Decimation-in-Time	32
		2.5.7.2 The FFT Complex Multiplications and Additions	35
	2.5.8	DFT Errors	35
		2.5.8.1 Aliasing	35
		2.5.8.2 Leakage	36

		2.5.9	Bandpass and Equivalent Lowpass Signals	36
	2.6	Windo	w Functions	39
		2.6.1	The Effects of Windowing	42
	2.7	Chapt	er Two Summary	43
3	Сус	lostatio	onary Spectrum Sensing	44
	3.1	Cyclic	Autocorrelation Function and Cyclostationarity	45
	3.2	Spectr	ral Correlation Function	48
	3.3	Cyclos	stationary Spectrum Sensing model	50
		3.3.1	Feature Detection	51
	3.4	Narrow	wband Spectrum Sensing	52
	3.5	Wideb	and Spectrum Sensing	53
		3.5.1	Non-Blind Wideband Cyclostationary Sensing	54
		3.5.2	Blind Wideband Cyclostationary Feature Sensing	55
	3.6	Cyclic	Frequency and Sampling Clock Offsets	56
	3.7	Energ	y Detection and Cyclostationary Feature Detection in Simulir	ık 58
	3.8	Chapt	er Three Summary	61
4	Mul	ti-slot \	Nideband Cyclostationary Feature Detector	62
	4.1	Syster	n Model	63
		4.1.1	Threshold and Detection	67
			4.1.1.1 Detection Algorithm	69
		4.1.2	The Means and Variances of the Test Statistic $\ensuremath{\mathrm{TS}}_1$ at both	
			H_0 and H_1	70
		4.1.3	Probability of Detection and Probability of False Alarm	71
		4.1.4	Frequency and Time Resolutions trade-off	72
	4.2	Signal	to Noise Ratio of the Test Statistics	73
	4.3	Result	s and Discussion	74
		4.3.1	Verifications of the Effects of Slot Size N and FFT size L	
			on Probability of Detection under noiseless conditions	75
		4.3.2	Section Summary	79

		4.3.3	The Effect of FFT sizes on the Spectral Correlation with	
			the Test Statistic for different bandwidths	79
		4.3.4	Section Summary	84
		4.3.5	Relationship between bandwidth, slot size N and number P	85
		4.3.6	Section Summary	86
		4.3.7	Correlation Performance of the Test Statistic $\ensuremath{TS}\xspace_1$ under	
			low SNR values	87
		4.3.8	Robustness of Test Statistic to noise	88
		4.3.9	Comparison with Energy Detection and Conventional Spec-	
			tral Correlation Density Function	89
		4.3.10	Detection performance in terms of real signal features	91
	4.4	Chapte	er four Summary	93
5	Wide	eband	Cyclostationary Feature Detector under Receiver Con-	
	strai	ints		95
	5.1	Syster	n Model with Receiver Constraints	98
	5.2	Effects	of Sampling Clock Offset on the Test Statistic	99
	5.3	Combi	ned Effects of Sampling clock and Cyclic Frequency Off-	
		sets in	noiseless condition	102
	5.4	Thresh	nold, Detection and Probabilities of False Alarm and De-	
		tection		104
		5.4.1	P_d and $P_f a$ under Receiver Constraints	104
	5.5	Signal	to Noise Ratio of the Test Statistics under CFO and SCO	106
	5.6	Result	s and Discussion	106
		5.6.1	Effects of FFT and Slot sizes with SCO and CFO under	
			signal with noise H_1	106
		5.6.2	Comparison between with and without receiver constraints	
			for multi-SNRs.	110
		5.6.3	Performance of the FFTs and slots under multi-SNR con-	
		0.0.0		

		5.6.4	Comparison with Energy Detection and SCF Conven-	
			tional Test Statistics	113
		5.6.5	Application of test statistic to real signal in wideband chan-	
			nel	116
	5.7	Chapt	er Five Summary	116
6	Algo	orithmi	c Optimization of Cyclostationary Spectrum Sensing un	-
	der	Receiv	er Constraints	118
	6.1	Comp	utational Complexity	119
	6.2	Optim	izing the FFT Size L and Number M	119
	6.3	Optim	izing the Slot Size N and Number P	120
	6.4	Algorit	thm for Optimizing the FFT and slot sizes and numbers	121
		6.4.1	Flowcharts for optimizing FFT and slot sizes	123
	6.5	The op	otimized N, P, L and M for a given distribution of CFO and	
		SCO		127
	6.6	Result	s and Discussion	127
		6.6.1	Optimizing L and M with the Optimization Algorithm	127
		6.6.2	Optimizing N and P with the Optimization Algorithm	132
	6.7	Effect	of using FFT size 64 with signals of low SNR	133
	6.8	Applyi	ng the Optimized L and N under SCO and CFO \ldots	134
		6.8.1	Optimized ${\it L}$ under different values of SCO and CFO	135
		6.8.2	Effect of the optimized L with signals of low SNR	135
		6.8.3	Comparison with Conventional test statistic	138
		6.8.4	Performance in 2-dimensions of carrier frequency f and	
			cyclic frequency α	139
		6.8.5	The Impact of FFT Complexities	140
	6.9	Chapt	er Six Summary	143
7	Con	clusior	ns and Future Work	145
	7.1	Conclu	usions	145
	7.2	Future	9 Work	148

Co	ontents	xii	
Ap	Appendices		
A	Derivation of Cyclic Autocorrelation Function	151	
В	Derivation of Spectral Correlation Function	153	
С	Spectral Correlation Function OF M-ary PSK Signals	155	
D	Derivation of the Effect of Phase Offset on the Test Statistic.	157	
Е	Calculations of the FFT Complexities	160	
F	FFT Resolutions and Complexities	163	
G	Time domain Window Functions Comparisons	165	
Н	Certificate of Ethics Review	166	
I	Form UPR16	168	
8	Bibliography	170	

List of Tables

2.1	Types of Fourier transforms.	25
2.2	Properties of Fourier Transform	28
2.3	Properties of Discrete Fourier Transform	32
3.1	Cyclic features for some modulation types [1–5]	50
4.1	Simulation Parameters with QPSK signal	75
4.2	Simulation Parameters for different Bandwidths	80
E.1	Calculations of the FFT Complexities with different slot sizes, P	
	= 16 and $f = 5 \text{ MHz}$	161
E.2	Calculations of the FFT Complexities with fixed slot size $N = 256$	
	samples, $f_s = 4 \times f$, $T_{acq} = 0.15$ millisecond	162
F.1	FFT Resolutions and Complexities	164
G 1	Comparison of Window Functions	165

List of Figures

1.1	Spectrum occupation in time domain.	2
1.2	Illustration of the concepts involved in Cognitive Radio	4
1.3	Spectrum Awareness Techniques.	4
2.1	Spectrum Sensing Techniques.	12
2.2	Energy detector in the time domain.	15
2.3	Energy detector in the frequency domain	16
2.4	Conventional Matched Filter	19
2.5	Fourier Transform of $x(t)$	26
2.6	Fourier Transform of $x(t)$	27
2.7	Fourier Transform of $x(t)$	34
2.8	Bandpass AM Modulated signal	37
2.9	Fourier transform of complex-valued analytic signal $x_a(t)$	38
2.10	Fourier transform of complex-valued analytic signal $x_a(t)$	39
2.11	Triangular Window	40
2.12	Hanning Window	41
3.1	Conventional Cyclostationary Feature Detector.	51
3.2	Energy Detection in Simulink	59
3.3	Cyclostationary Feature Detector in Simulink	60
3.4	Energy and Cyclostationary Feature Detectors comparison	61
4.1	Wideband Multi-slot Cyclostationary Feature Detector	63

4.2	Probability of detection simulation see (4.14)) for QPSK signal	
	at -5dB SNR, with a range of different sizes of FFT, $N = 512$	
	samples, $P = 8$, $f_c = 5$ MHz, $f_s = 20$ MHz and total samples N_B	
	= 4096	76
4.3	Probability of detection (simulation see (4.14)) for QPSK signal	
	at -5dB SNR, with multi-FFTs, $N = 256$ samples, $P = 16$, $f = 5$	
	MHz, $f_s = 20$ MHz and total samples $N_B = 4096$	77
4.4	Probability of detection (simulation see (4.14)) for QPSK signal	
	at -5dB SNR, with multi-FFTs, $N = 128$ samples, $P = 16$, $f = 2.5$	
	MHz, $f_s = 10$ MHz and total samples $N_B = 2048$	78
4.5	Probability of detection (simulation see (4.14)) of 5MHz QPSK	
	signal for different slot sizes using TS_1 , at -5dB noise, FFT size	
	16, $P = 16$ samples, $f = 5$ MHz and $f_s = 20$ MHz	78
4.6	Effects of FFT size vs Number of slots (analytic, see (4.12)) with	
	N = 256 and P = 8, f = 5 MHz , f_{s} = 20 MHz and N_{B} = 2048	
	samples	81
4.7	Effects of FFT size vs Number of slots (analytic, see (4.12)),	
	$f = 10$ MHz, $N = 256$, $P = 16$, $f_s = 40$ MHz and $N_B = 4096$ samples.	82
4.8	Effects of FFT size vs Number of slots (analytic, see (4.12)),	
	$f = 20$ MHz, $N = 512$, $P = 16$, $f_s = 80$ MHz and $N_B = 8192$ samples.	82
4.9	Effects of FFT size vs Number of slots (analytic, see (4.12)),	
	$f = 10$ MHz, $N = 512$, $P = 8$, $f_s = 40$ MHz and $N_B = 4096$ samples.	83
4.10	Effects of FFT size vs Number of slots (analytic, see (4.12)) with	
	$f = 2.5$ MHz, $N = 128$, $P = 8$, $f_s = 10$ MHz and $N_B = 1024$ samples.	83
4.11	Effects of FFT size vs Number of slots (analytic, see (4.12)) with	
	$f = 1.25$ MHz, $N = 64$, $P = 8$, $f_s = 10$ MHz and $N_B = 512$ samples.	84
4.12	Probability of detection vs Number of slots (simulation, see(4.14))	
	for 10/20 MHz bandwidths at $N = 256$, $N_B = 4096/8192$ samples.	85
4.13	Probability of detection vs Number of slots (simulation, see(4.14))	
	for 20 MHz bandwidth at $N = 512/256$ samples, $N_B = 8192$ sam-	
	ples.	86

4.14 Magnitude of Test Statistic TS_1 vs Number of slots (analytic,	
see(4.12)) for QPSK signal only with $f = 5$ MHz, $N = 128$, P	
= 16, f_s = 20 MHz	87
4.15 Magnitude of Test Statistic TS_1 vs Number of slots (analytic,	
see(4.12)) for QPSK signal plus noise with $f = 5$ MHz, $N = 128$,	
$P = 16, f_s = 20 \text{ MHz}, \text{ SNR} = -3 \text{dB}.$	88
4.16 Magnitude of Test Statistic TS_1 vs Number of slots (analytic,	
see(4.12)) for QPSK signal plus noise with $f = 5$ MHz, $N = 128$,	
$P = 16, f_s = 20 \text{ MHz}, \text{ SNR} = -5 \text{dB}.$	88
4.17 Receiver Operating Characteristics (analytic, see (4.29, 4.25))	
under low SNR values with FFT size 8, $f = 5$ MHz, $N = 128$	
samples, $P = 16$, $f_s = 20$ MHz and $N_B = 2048$ samples	89
4.18 Detection Performance vs Total Number of samples (simulation,	
see (4.14) under low SNR values with FFT size 8, $f = 5$ MHz, N	
= 128, P = 16, f_s = 20 MHz	90
4.19 Comparison between the CFD Test Statistic $\ensuremath{\mathrm{TS}}_1$ and Energy De-	
tection (ED) (simulation, see (2.2, 4.7)) under signals for low	
SNR values with $f = 5$ MHz, $N = 128$ samples, FFT = 32 sam-	
ples, $P = 16$, $f_s = 20$ MHz	90
4.20 Comparison between the discrete conventional SCF and model	
non-conjugate test statistic (simulation, see (4.9, 4.7)) TS_1 for	
multi-SNR values with 5 MHz QPSK signal using TS_1 , N = 256,	
P =16, f_s = 20 MHz and FFT szes 4-64	91
4.21 Detection in terms of real signal features (simulation, see (4.7))	
with FFT size 16, $f = 10$ MHz, $N = 256$ samples, $P = 16$, $f_s = 40$	
MHz and N_B = 4096 samples	92
4.22 Contour plot of Fig. 20 (simulation, see (4.7)) with FFT size 16,	
$f = 10 \text{ MHz}, \alpha = 2f, N = 256, P = 16, f_s = 40 \text{ MHz}. \dots$	92
4.23 Contour plot (simulation, see (4.7)) for two BPSK signals with	
FFT size 16, $f = 10,5$ MHz, $\alpha = 2f$, $N = 256$, $P = 16$, $f_s = 40$ MHz.	93

4.24	Contour plot (simulation, see (4.7)) for BPSK signals with FFT	
	size 32, $f = 10$ MHz, $\alpha = 2f$, $N = 256$, $P = 16$, $f_s = 40$ MHz	93
5.1	Conventional Cyclic Autocorrelation Function with and without	
	CFO (simulation, see (4.9))	97
5.2	Conventional Spectral Correlation Function with and without CFO	
	(simulation, see (4.9))	98
5.3	Probability of detection using non-conjugate Test Statistic with	
	constraints (simulation, see (5.18)) for QPSK Signal of -5 dB	
	SNR, SCO/CFO = 0.09, slot size = 256 samples, number of	
	slots = 8, frequency = 2.5 MHz and f_s = 10 MHz	107
5.4	Probability of detection using $TS_{1 \delta,\Delta\alpha}$ (simulation, see (5.18)) for	
	Signal with -5 dB SNR, SCO/CFO = 0.09, multi-FFTs, slot size	
	= 128 samples, number of slots = 16, frequency = 2.5 MHz and	
	$f_s = 10 \text{ MHz.}$	108
5.5	Probability of detection of 3.5MHz QPSK signal for different slot	
	sizes using $TS_{1 \delta,\Delta\alpha}$ (simulation, see (5.18)), at -5dB noise, FFT	
	size 16, $P = 12$ samples and $f_s = 14$ MHz	108
5.6	Probability of detection (analytic, see (5.32)) for QPSK Signal of	
	-5dB SNR under multiple SCO/CFO with FFT/slot size = 32/256	
	samples, Number of slots = 8, frequency = 2.5 MHz, f_s = 10 MHz	.109
5.7	Probability of detection (analytic, see (5.32)) for QPSK Signal of	
	-5dB SNR under multiple SCO/CFO with FFT/slot size = 16/128	
	samples, Number of slots = 8, frequency = 5 MHz, f_s = 20 MHz.	110
5.8	Probability of detection (analytic, see (5.32)) for QPSK Signal	
	with Noise at different SNRs, slot size/number = 256/8 samples,	
	FFT size = 128 samples, SCO/CFO = 0.08, frequency = 2.5 MHz	
	and $f_s = 10 \text{ MHz}$	111

5.9	Probability of detection (analytic, see (5.32)) for QPSK Signal	
	with Noise at different SNRs, slot size/number = 256/8 samples,	
	FFT size = 64 samples, SCO/CFO = 0.08, frequency = 2.5 MHz	
	and $f_s = 10 \text{ MHz}$	112
5.10	Probability of detection (analytic, see (5.32)) for Signal with Noise	
	at different SNRs, slot size/number = 64/32 samples, FFT size	
	= 16 samples, SCO/CFO = 0.08, frequency = 2.5 MHz and f_s =	
	10 MHz	112
5.11	Receiver Operating Characteristic with (analytic, see (5.32, 5.28))	
	for multi-SNR values with 5 MHz QPSK signal, N = 256, P = 16,	
	N_B = 4096 samples, FFT size 64, f_s = 20 MHz and SCO/CFO =	
	0.09	113
5.12	Receiver Operating Characteristic with (analytic, see (5.32, 5.28))	
	for multi-SNR values with 5 MHz QPSK signal using $TS_{1 \delta,\Delta\alpha}$, N	
	= 256, P =16, N_B = 4096 samples, FFT size 32, f_s = 20 MHz	
	and SCO/CFO = 0.09	114
5.13	Receiver Operating Characteristic with (analytic, see (5.32, 5.28))	
	for multi-SNR values with 5 MHz QPSK signal using $TS_{1 \delta,\Delta\alpha}$, N	
	= 256, P =16, N_B = 4096 samples, FFT size 16, f_s = 20 MHz	
	and SCO/CFO = 0.09	114
5.14	Performance of Test Statistic with Offset against Energy Detec-	
	tion with (simulation, see (2.2, 5.18)) for FFT size = 16 sam-	
	ples, SCO/CFO = 0.02, slot size/number = 256/16 samples, fre-	
	quency = 5 MHz, f_s = 20 MHz	115
5.15	Comparison between the Conventional Test Statistic and model	
	non-conjugate test statistic with (simulation, see (4.9, 5.18)) for	
	multi-SNR values with 2.5 MHz QPSK signal, N = 64, P = 32, N_B	
	= 2048 samples, f_s = 10 MHz, FFT sizes 8-64 and SCO/CFO =	
	0.1	115

5.16	Contour plot with $TS_{1 \delta,\Delta\alpha}$ (simulation(5.18)) for 2 BPSK signals,	
	SCO/CFO = 0.05, with FFT size 16, $f = 5,2.55$ MHz, $\alpha = 2f$,	
	$N = 256, P = 16 \text{ and } f_s = 20 \text{ MHz.}$	116
5.17	Contour plot with ${\rm TS}_{1 \delta,\Delta\alpha}$ (simulation(5.18)) for BPSK signals,	
	SCO/CFO = 0.5, with FFT size 16, f = 2-12 MHz, α = 2 f , N =	
	256, $P = 38$ and $f_s = 48$ MHz	117
6.1	Flowchart for optimizing FFT size.	124
6.2	Flowchart for optimizing slot size	126
6.3	Probability of detection of 5MHz QPSK signal for multi-FFTs with	
	P_d in (simulation, see(4.14)) at -5dB noise, $N = 256$ samples, P	
	= 8 and f_s = 20 MHz	128
6.4	Probability of detection of 5MHz QPSK signal for for multi-FFTs	
	with P_d in (simulation, see(4.14)) at -5dB noise, $N = 128$ sam-	
	ples, $P = 16$, $f_s = 20$ MHz	129
6.5	Probability of detection of 2.5MHz QPSK signal for for multi-	
	FFTs with P_d in (simulation, see(4.14)) at -5dB noise, $N = 128$	
	samples, $P = 8$ and $f_s = 10$ MHz	129
6.6	Detection in terms of real signal features using $\ensuremath{\mathrm{TS}}_1$ in (simula-	
	tion, see (4.7)) with FFT sizes 4, 8, 16, 32 $f = 1$ MHz, $N = 256$	
	samples, $P = 16$, $f_s = 4$ MHz and $N_B = 4096$ samples at SNR =	
	-5dB	130
6.7	Detection in terms of real signal features using \ensuremath{TS}_1 in (simu-	
	lation, see (4.7)) with FFT sizes 16, 32 $f = 1$ MHz, $N = 256$	
	samples, $P = 16$, $f_s = 4$ MHz and $N_B = 4096$ samples at SNR =	
	-5dB	131
6.8	Comparison of FFT Frequency Resolution and Complexity (an-	
	alytic, see Appendices E and F) for 1-4KHz BPSK signal for Slot	
	Sizes of 64-512 samples, $f_s = 4 \times f$.	131

6.9	Probability of detection of 10MHz QPSK signal for different Slot	
	Sizes with P_d in (simulation, see(4.14)) at -5dB noise, FFT size	
	16, $P = 16$ and $f_s = 40$ MHz	132
6.10	Probability of detection under multi-SNR values with 5 MHz QPSK	
	signal with $P_{d \delta,\Delta\alpha}$ in (analytic, see (5.32)) at f_s = 20 MHz, N =	
	256, <i>P</i> = 8, FFT64, SCO/CFO = 0.04	134
6.11	Probability of detection under multi-SNR values with 2.5 MHz	
	QPSK signal with $P_{d \delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 10$ MHz,	
	N = 128, P = 8, FFT64 and SCO/CFO = 0.04	134
6.12	Probability of detection with multi-offsets for 2.5MHz QPSK sig-	
	nal with $P_{d \delta,\Delta\alpha}$ in (analytic, see (5.32)) of -5dB noise at $f_s = 10$	
	MHz, $N = 128$, $P = 8$ and FFT 16	135
6.13	Probability of detection under multi-SNR values with 5 MHz QPSK	
	signal with $P_{d \delta,\Delta\alpha}$ in (analytic, see (5.32)) at f_s = 20 MHz, N =	
	256, <i>P</i> = 8, FFT16 and SCO/CFO = 0.04	136
6.14	Probability of detection under multi-SNR values with 2.5 MHz	
	QPSK with $P_{d \delta,\Delta\alpha}$ in (analytic, see (5.32)) at f_s = 10 MHz, N =	
	128, <i>P</i> = 8, FFT16 and SCO/CFO = 0.04	136
6.15	Performanace under SCO/CFO = 0.04 for P = 32, multi-SNR	
	values with 5 MHz QPSK signal with $P_{d \delta,\Deltalpha}$ in (analytic, see (
	5.32)) at f_s = 20 MHz, N = 64, N_B = 2048 samples and FFT16.	137
6.16	Performanace under SCO/CFO = 0.04 for $P = 64$, multi-SNR	
	values with 10 MHz QPSK signal with $P_{d \delta,\Delta\alpha}$ in (analytic, see (
	5.32)) at f_s = 40 MHz, N = 64, N_B = 4096 samples and FFT16.	138
6.17	Receiver Operating Characteristic for multi-SNR values with 10	
	MHz QPSK signal with (analytic, see (5.32, 5.28)), N = 64, P	
	=32, N_B = 2048 samples, FFT16 and SCO/CFO = 0.1	138
6.18	Comparison between Model and Conventional TS in (simulation,	
	see (4.9, 5.18)) for multi-SNR values with 5 MHz QPSK signal	
	at f_s = 20 MHz, N = 64, P =32, N_B = 2048 samples, FFT16 and	
	SCO/CFO = 0.1.	139

6.19	Contour plot with (simulation (5.18)) for BPSK signals, SCO/CFO	
	= 0.2, with FFT size 16, f = 1-10 MHz, α = 2 f , N = 64, P = 64	
	and $f_s = 40 \text{ MHz}$	140
6.20	Complexity for different slot sizes and FFTs Table E.1 of Ap-	
	pendix E	141
6.21	Complexity for different slot numbers and FFTs Table E.2 of Ap-	
	pendix E	141
6.22	Complexity for increasing number of FFTs Table E.2 of Appendix	
	Ε	142
6.23	Comparison between Model Statistic and Conventional SCF Com-	
	plexities Table E.1 of Appendix E	143
H.1	Ethics Review Certificate.	167
I.1	Ethical Checklist	169

Glossary

- AC Alternating Current
- AF Autocorrelation function
- AWGN Additive White Gaussian Noise
- BCWCSS Blind Compressive Wideband Cyclostationary Spectrum Sensing
- BCWCSS Blind Compressive Wideband Cyclostationary Spectrum Sensing
- **BEEWCCSS** Blind Energy Efficient Wideband Compressive Cyclostationary Spectrum Sensing
- BPSK Binary Phase Shift Keying
- **BW** Bandwidth
- **BWCSS** Blind Wideband Cyclostationary Spectrum Sensing
- **CAF** Cyclic autocorrelation function
- **CDP** Cycle frequency Domain Profile
- **CFD** Cyclostationary Feature Detector
- **CFO** Cyclic frequency offset
- **CoRaSat** Cognitive Radio Satellite
- **CR** Cognitive Radio
- **CS** Compressed or Compressive Sensing

- **CSD** Cyclic Spectral Density
- CU Cognitive User
- **DB** Database
- **DC** Direct Current

DINWC Digital Information, Networking, and Wireless Communications

- **DT** Discrete Time
- **DFT** Discrete Fourier Transform
- EAI European Alliance Innovation
- FCC Federal Communications Commission
- FD Feature Detector
- FFT Fast Fourier Transform
- FRESH Frequency Shift
- f Frequency
- **GDB** Geolocation database
- GNG Gaussian Noise Generator
- Hz Hertz
- **IF** Intermediate Frequency
- **IFFT** Inverse Fast Fourier Transform
- in Inch
- **ITU** International Telecommunications Union
- LR-LS Regularised Least Squares
- m Metre

NB Narrowband

NBWCSS Non-Blind Wideband Cyclostationary Spectrum Sensing

NRFFE Narrowband RF Front-End

OFCOM Office of Communications

PU Primary User

QAM Quadrature Amplitude Modulation

RCTF Raised Cosine Transmit Filter

RF Radio Frequency

SatCom Satellite Communication

SCF Spectral Correlation Function

SCD Spectral Correlation Density

SCO Sampling clock offset

SCSD Squared Cyclic Spectral Density

SNR Signal to Noise Ratio

SS Spectrum Sensing

SSPU Small Scale Primary User

SU Secondary User

UK United Kingdom

USA United States of America

WB Wideband

WCS Wideband Compressed Sensing

- WCSS Wideband Cyclostationary Spectrum Sensing
- WGN White Gaussian Noise
- WRFFE Wideband RF Front-End
- WSS Wideband Spectrum Sensing

Chapter 1

Introduction

1.1 Motivation and State of the Art

Radio frequency spectrum is a physical resource that is naturally limited in availability. This calls for its efficient utilization in order to provide users with different services at higher data rates. There is increasing demand for the spectrum due to rapidly expanding markets of wireless broadband, multimedia users and applications which require high data rates. The radio frequencies are therefore becoming scarce due to spectrum segmentation and dedicated frequency allocation of the standardized wireless systems. As the demand for broadband services and higher data rates continue to increase, efficient spectrum usage is now a critical issue. On the other hand, several spectrum occupancy measurement campaigns carried out in different parts of the world indicate that a significant amount of the wireless spectrum are under-utilized over a wide range of radio frequencies [6]. The Federal Communications Commission (FCC) of the United States of America (USA) survey measurements indicated that several licensed frequency bands are unused up to ninety percent of the time [7].

Currently, wireless networks are regulated by fixed spectrum assignment policies, where the spectrum is regulated by governmental agencies such as the Office of Communications (OFCOM) in the United Kingdom (UK). Frequencies are allocated to license holders mostly on a long term basis for large geographical regions. A large portion of the assigned spectrum is being used sporadically [8]. The spectrum usage is concentrated on certain portions of the spectrum while a significant amount is either under-utilized or not utilized at all as illustrated in Fig.1.1. As the demands of services continue to increase, the scarcity of communication spectrum has become one of the major issues for the development of new communication systems. It is either because the spectrum is not available in some places or not sufficient in others. In this context, Cognitive Radio (CR), a context-aware intelligent radio communication system, has emerged as a promising solution to address the spectrum scarcity by exploring spectral opportunities and deliver a more efficient utilization of the available spectral resources [9]. The CR, which was first proposed as a solution in mobile communications is sometimes considered as making software defined radios more significant as discussed in [10]. The United States (US) FCC and UK OFCOM have already opened up significant parts of the Television (TV) spectrum for unlicensed use in order to encourage the realisation of Cognitive Radio implementation based on geo-location database spectrum sensing approach, see [11], [12], [13] and [14].



Figure 1.1: Spectrum occupation in time domain.

1.2 Cognitive Radio System

This section will explain the principles of Cognitive Radio (CR) and the building blocks. Cognitive Radio is a context-aware intelligent radio capable of reconfiguring itself by learning from the surrounding communication environment and adapting to it. CR results in flexible and dynamic spectrum access. The basic underlying principle is to permit unlicensed (secondary) users to access opportunistically and without interference some licensed bands which are temporarily and/or spatially unoccupied by the licensed users, see e.g [15]. In order to meet increasing demand for services, Cognitive Radio is a promising solution to address the spectrum scarcity by exploring spectral opportunities and to deliver a more efficient utilization of the available spectral resources, see e.g [9].

1.3 Cognitive Radio Functional Blocks

The concept of CR can be divided into four main blocks as shown in Fig. 1.2. Namely: spectrum awareness, spectrum management, spectrum sharing and spectrum mobility. Spectrum Awareness aims at determining spectrum availability and the presence or absence of licensed users (primary or incumbent users). Spectrum management predicts the length of time the spectrum holes (unused bandwidth or vacancies as in 1.1) will be available for use between when the primary user has need of it [16]. Spectrum sharing allocates the spectrum holes among the secondary users according to demand. Spectrum mobility maintains hitless or error free (seamless) communication during frequency allocation to or from the primary and secondary users thus producing better spectrum usage transition [17].



Figure 1.2: Illustration of the concepts involved in Cognitive Radio.

1.4 Spectrum Awareness

Spectrum awareness is divided into three main sections namely: spectrum sensing techniques, database estimation and Signal-to-Noise-Ratio (SNR) techniques as shown in Fig. 1.3



Figure 1.3: Spectrum Awareness Techniques.

1.4.1 SNR Estimation Technique

This technique enables the Secondary User to have some information about the SNR and channels of the Primary signals. The purpose is to help the secondary user in controlling its transmission power towards the co-channel it shares with the primary user, thereby radiating at an acceptable interference level. Non-Data-Aided (NDA) SNR estimators predict the SNR values directly from the unknown information-bearing part of the received PU signal [18]. This method is not very useful in cognitive networks because it is not opportunitybased. This research will explore the spectrum sensing functional block of cognitive radio.

1.4.2 Database Technique

A geolocation database contains the spectrum usage information of the primary users in several places. This information can include frequency, place, time, location, coverage areas, transmission powers, SNR, radio technologies, etc. These databases are being provided by independent operators, regulated in the UK by the Office of Communications (OFCOM) [13, 14, 19], [20]. The database is expected to serve large number of requests daily. It may require significant amount of data to service each user [21]. However, the challenge is in keeping the updating of the databases active in real-time communication environments to minimize interference to users . The geo-location spectrum approach is being proposed by OFCOM [14, 21] pending when the full cognitive radio concept will be implemented without harmful interference to the primary users. The full cognitive radio requires sensing the spectrum without the use of databases [22–24].

1.4.3 Spectrum Sensing

This is an important method to determine spectrum occupancy. From the concept of cognitive radio, spectrum sensing is one of the crucial tasks in establishing cognitive radio networks. It determines the presence or absence of the primary users and spectrum availability [17]. It looks at spectral holes in different parametric bases such as time, frequency, space, polarization and angular domains. It is expected that the secondary user radio or terminal has the ability to sense the presence or absence of the primary user through a signal processing based technique. Spectrum sensing for cognitive radio uses some unique characteristics such as: no prior knowledge of the signal statistics or waveforms and noise form. The detection of the primary user signal has to be active and immediate; processing of very low signals which may be due to fading and multi-path phenomenon and noise and interference levels will vary with time and uncertainty [25].

1.5 Research Gaps

The research gaps identified in this research are presented below.

Detection of low signal-to-noise ratio (SNR) signals in a wideband channel (channel with multiple carriers) with cyclic frequency and sampling clock offsets.

Cyclic Frequency Offset (CFO) is due to the imperfect knowledge of the cyclic frequency or symbol rate while Sampling Clock Offset (SCO) is due to the imperfect knowledge of the sampling rate or sampling period at the receiver end of the radio system. These are receiver constraints and affect the accuracy of detection using cyclostationary spectrum sensing method. From literatures, little attention has been given to the reduction of these constraints.

Low computational complexity, cost and high efficiency.

The approach of using small windows in the implementation of wideband cyclostationary spectrum sensing has not been fully explored. This results in low computational complexity, cost and high efficiency.

Model optimization in frequency domain

Little attention has so far been given to the best of the author's knowledge with regards to the optimization of cyclostationary spectrum sensing models in the frequency domain which is necessary in order to evaluate the effectiveness of an implementable hardware system.

1.6 Research Objectives

The objectives are to develop statistical and test-based wideband cyclostationary spectrum sensing models that will perform the following.

- The detection of the presence of RF signals of very low SNR values in order to demonstrate the robustness of the models to noise.
- The ability to offer low computational complexity will be analysed and implemented through achieving significant sensing of the RF spectrum with small number of samples.
- Efficiency in terms of the number of samples used will be verified using the model.
- Low cost sensing of the RF spectrum in terms of resources such as Fast Fourier Transform (FFT) will be done by comparing with different sizes of FFT and slots.
- The reduction of receiver constraints such as cyclic frequency offset and sampling clock offset in a wideband communication environment will be analysed and implemented. This will be demonstrated by comparing the cases of with and without the constraints.
- Algorithms for both detection and optimization of the performance of the models will be considered, in order for the models to be applicable to different sample sets. This will consider different sizes and numbers of FFT and slots. The optimization will also offer low computational complexity.

1.7 Research Contributions

The contributions of this research are as follow.

- Design and implementation of multi-slot wideband Cyclostationary Feature Detection (WCFD) model which offers low computational complexity and robustness to noise.
- Design and implementation of wideband Cyclostationary Spectrum Sensing model for the reduction of receiver constraints namely: CFO and SCO. It also offers low computational complexity and robustness to noise.
- The development of generic optimization algorithm and code for the WCFD for the scenarios with or without the receiver constraints.

The above research contributions were achieved in the chapters as follow.

Chapter Four

This chapter addresses the design and implementation of a Wideband Cyclostationary Spectrum Sensing using a multi-slot test statistic without the constraints namely CFO and SCO. The combinations of small size Fast Fourier Transforms (FFTs) and time-based slots are used to produce a higher detection of signals as against the use of large size FFTs with and without slots. Time averaging is quicker with small FFTs and slots because fewer samples are needed which result in a quicker exit point when the signal is detected. The use of small FFTs and slots for spectral correlation also results in lower computational complexity. The detection of signals of low signal-to-noise ratios (SNRs) are shown. The use of appropriate sizes of FFTs can save in terms of the resources required by the FFTs such as processor time. Small FFTs will also prevent the leakage of processors typical of large FFTs over a longer time. Matlab simulations were used to verify the performance of the test statistic in achieving these contributions. This chapter has been published in [26].

Chapter Five

This chapter addresses the issue of receiver constraints: cyclic frequency offset and sampling clock offset. It includes the development of the design and implementation of a Wideband Cyclostationary Spectrum Sensing system using a multi-slot test statistic incorporating the aforementioned receiver constraints. The effects of these constraints on the test statistic are analysed. The results indicate that small sizes of FFTs in combination with multi-slots are capable of significantly reducing the effects of cyclic frequency offset and sampling clock offset and helping to detect signals at very low SNRs. It also offers low complexity and high efficiency. Matlab simulations were used to verify these contributions. A part of this work has been published in [27].

Chapter Six

The issue of algorithms for cyclostationary signal detection and optimization are examined in more detail in this chapter for signals with and without receiver constraints. A step by step algorithm is described which is capable of producing the right sizes of FFTs and slots. These can be applied to any number of samples covering potentially any wideband channel which could serve as a non ad hoc approach. Matlab simulations were used to verify these contributions. A part of this chapter has been published in [28] while the full chapter will be published in the paper to be submitted to the Institute of Electrical and Electronic Engineers (IEEE) Transactions on Cognitive Communications and Networking.

1.8 Research Methodology

Theoretical analysis of the various stages of the proposed wideband cyclostationary spectrum sensing models are presented. The applicable mathematical equations such as cyclic autocorrelation function (CAF), Spectral correlation function (SCF), computation Complexity and Detection Hypothesis are considered. The models considered both scenarios when there are cyclic frequency and sampling clock offsets and when there are not. The models are robust to noise and detects signals of low SNR. algorithms for the selection of the parameters of the models in order to optimize the performance of the models have been developed. This will result in the use of the parameters such as small sizes of FFT and slots which will offer low computational complexity. Also, it makes it possible for the models to be adapted to different sizes of sample sets for various communication scenarios.

Matlab has been used for the simulations. This software is extensively used in academic research, particularly for the fields of signal processing and communications. It is user-friendly and can be adapted for use in many real world communication systems scenarios. All the sections that make up the wideband cyclostationary spectrum models were analysed and the overall models tested. Matlab codes were also used extensively in analysing and testing the proposed models. The results obtained, which include figures, graphs and tables were then interpreted and analysed to demonstrate each achievement of the research contributions. More details of the methodology are presented in chapters four, five and six.

1.9 Thesis Organization

This chapter has introduced the subject of cognitive radio systems and the building blocks which will progress to a more detailed discussion on spectrum sensing techniques in the next chapter.

Chapter two presents the different types of spectrum sensing techniques. The discussions on them end up in cyclostationary spectrum sensing which is the main topic of this research.

Chapter three focuses on the principles of cyclostationarity such as cyclic autocorrelation function and spectral correlation function. It also looks at the different adaptations of the cyclostationary spectrum sensing such as narrowband and wideband channels. The issue of receiver constraints such as sam-
pling clock offset and cyclic receiver offset which are some of the important considerations of this research are discussed.

Chapter four presents the multi-slot wideband cyclostationary spectrum sensing model in the absence of the receiver offsets. The model investigates the use of small sizes of fast Fourier transform and slots to obtain efficient detection.

Chapter five focuses on the reduction of sampling clock offset and cyclic frequency offset. The test statistics were derived to reflect the presence of these constraints. It also considers the issue of computational complexity, op-timization and the robustness to noise of the model.

Chapter six concentrates on the production of detection and optimization algorithms. These algorithms were followed in the simulations to show low complexity with and without receiver constraints at the selected combinations of fast Fourier and slot sizes. It also considers the performance of the optimized sizes of fast Fourier transform and slots.

Chapter seven presents the thesis conclusion and the suggested future work.

Chapter 2

Spectrum Sensing

Spectrum sensing is an important process in Cognitive Radio. Spectrum sensing determines the presence or absence of the primary users and spectrum availability [17]. It looks at spectral holes in different parametric bases such as time and frequency. Spectrum Sensing can be classified according to the categories shown in Fig. 2.1.



Figure 2.1: Spectrum Sensing Techniques.

2.1 Cooperative Sensing System

A Cooperative sensing system is when multiple cognitive radios share their local sensing information for more accurate primary signal detection. This information includes the statuses of certain carrier frequencies within their local environments [17], [29]. Cooperative sensing can be implemented in either a centralized or a distributed method.

Cooperative sensing methods can also be categorized as a soft or hard combination, according to the nature of the information being shared among the cognitive radios. The soft combination is when each radio or node senses a certain frequency band and then sends the results of the received signal to the central node for decision on the presence or absence of a user, see [30–32]. On the other hand, in a hard combination approach, each user takes an extra step of deciding whether a primary user is present or not and then reports the results to the central unit.

2.1.1 Centralized Cooperative Method

Here a group of Secondary users sense the radio environment locally and report to the nominated central unit or controller for managing the sharing of the free spectrum as discussed in [33–35]. The central unit also called the Fusion Centre (FC) collects the sensing information from the cognitive radios, identifies the available spectrum bands and makes broadcasts to other cognitive radios in the networks. A centralized sensing system is more accurate and effectively mitigates both the multi-path fading and shadowing effects as a result of the shared information. The central unit can also assign a specific weight to each spectrum sensing result in order to mitigate fading. However, the centralized sensing system requires a backbone infrastructure which may be costly [36].

2.1.2 Distributed Cooperative Method

In the distributed system there is no central unit, rather the secondary users individually sense the radio environment and send out the information to other cognitive users in the network as discussed in [37]. It is simpler to implement and does not require a huge backbone infrastructure but falls short of mitigating against multi-path fading and shadowing effects [17].

Several algorithms have been used in distributed sensing to coordinate the sensed data at different cognitive nodes. A discrete time protocol was discussed in [38] where a secondary user senses a band of interest during a certain time slot, and later sends its results to a set of randomly selected neighbouring cognitive radios. Similarly, another approach has been discussed in [39], where a small group of cognitive radios exchange their local decisions during a particular time slot. After which a cognitive user within this group sends all the received information to a randomly selected neighbour that will then act as the designated user of the free frequency in the next time slot. This process is repeated until all the cognitive users receive the sensing information.

2.2 Interference Based Sensing System

This method considers the leakage radiation from a primary receiver's local oscillator. An external sensor is attached near the Primary User's (PU) receiver which can transmit a signal back to the Secondary User (SU) for the purpose of identifying the PU [16].

2.3 Non-cooperative

Each of the detection methods under this category independently detects the PU signals and acquires vacant channels for the SU. This approach has an advantage of requiring less operational bandwidth. These methods are as

follow.

2.3.1 Energy Detector

This is a non-coherent type of detector requiring no prior knowledge of PU signal waveform detection method. Energy detectors detect the primary signal based on sensing the energy, see e.g [17, 40]. An energy detector evaluates the received signal's energy at the antenna input.

2.3.2 Implementation of Energy Detector

Energy Detection can be implemented in either the time or frequency domain with similar results. However, in practice it is more flexible to implement Energy Detection in the frequency domain than in the time domain due to the use of fast Fourier transform, see e.g [41]. In the time-domain, as in Fig. 2.2, the received radio signal from the RF front end or receiver is passed through a band pass filter with a particular bandwidth and converts it to a digital signal in the Analogue/Digital (A/D) block. This is then squared and summed up. The output is then compared to a predefined threshold. This comparison is to ascertain the existence or absence of the primary user, see e.g [40,42]. In the frequency domain, the Fast Fourier Transform (FFT) is also used to implement ED as shown in Fig. 2.3.



Figure 2.2: Energy detector in the time domain.



Figure 2.3: Energy detector in the frequency domain.

2.3.3 Advantages

There are two main advantages of Energy Detection when compared against other detection techniques such as the cyclostationary feature detection.

- Energy detection requires less computational complexity than some other spectrum sensing methods such as the cyclostationary feature detection. It relies on the overall energy of the signal including noise and does not require much re-sampling of the signal for the detection.
- Energy Detection requires no prior knowledge of the Primary User's signal in order to detect the signal. This is possible since it does not distinguish the signal content from the noise.

2.3.4 Disadvantages

The detection performance is subject to the uncertainty of noise power since it evaluates the signal's energy which is made up of noise as well. This will result in arriving at incorrect detection levels. Other disadvantages can include:

- Energy Detection cannot be used to distinguish the primary user's signals from the secondary user's signals in a cognitive radio system because it does not process the features of the signal such as the cyclic frequency.
- Energy Detection is not capable of detecting the signals of a spread spectrum system which usually are noise-like.
- It is not effective in handling signals of low SNR values because of the inability to distinguish noise from signal.

2.3.5 Energy Detection Test Statistic

Let the discrete received signal be represented by,

$$x[n] = s[n] + w[n]$$
(2.1)

where n, s[n] and w[n] are the sample index, signal only and Additive White Gaussian Noise (AWGN) respectively. The test statistic for energy detection can be stated as,

$$S_{ED} = \sum_{n=0}^{N-1} |x[n]|^2$$
(2.2)

where *N* is the total number of samples during the observation or sensing period. The decision on the occupancy status of a channel is achieved by comparing the magnitude of the test statistic Y_{ED} (2.1) against a fixed threshold λ_{ED} as expressed in (2.3) and (2.4) and discussed in [43], [44], [45] and [41]. This will result in two hypotheses H_0 and H_1 which are defined to correspond to the following,

$$S_{ED} < \lambda_{ED}$$
 for $H_0: s[n] = \eta[n]$, noise only (2.3)

and

 $S_{ED} \ge \lambda_{ED}$ for $H_1: s[n] = x[n] + \eta[n]$, of signal and noise. (2.4)

2.4 Matched Filter

A Matched Filter (MF) is obtained by correlating a known signal or reference with an unknown signal in order to detect the presence of the reference signal in the unknown signal. This is a coherent detection method where the secondary user has a priori knowledge of the primary user signal. The operation of matched filtering is equivalent to the correlation in which the unknown signal is convolved with the filter whose impulse response is the mirror and time shifted version of a reference signal. In other words, MF is equivalent to convolving the unknown signal with a conjugated time-reversed version of the reference signal. The matched filter is one of the optimal linear filters for optimizing the SNR in the presence of additive stochastic noise as discussed in [46,47]. The test statistic for MF as used in [46–48] can mathematically be expressed as,

$$S_{MF} = \left| \sum_{n=0}^{N-1} h^*[n] x[n] \right|^2$$
(2.5)

where $h^*[n]$ and x[n] are the conjugated (*) time-reversed version of the known or reference signal and the unknown signal respectively.

Matched filters are commonly applied in radar systems where a known signal is transmitted, and the reflected signal which is the received signal is then examined for common features of the transmitted signal. This is difficult to apply in real radio communication systems where the full nature of the waveform or signal is not constant at all times or where the radio environment is not predictive. However, it can be used in data communications where binary messages are being sent out from the transmitter to the receiver across a noisy channel. MF can be used to detect the transmitted pulses in the noisy received signal.

Matched filter uses fewer samples to achieve acceptable detection such as a low probability of missed detection or false alarm [49] in a short time. However, more samples are needed as the SNR increases which results in SNR walls [50].

2.4.1 Implementation of Matched Filter

According to Fig. 2.4, the received signal, x(t) is passed through a band-pass filter (BPF). This measures the energy in the selected band. The output signal of the BPF is convolved with the match filter whose impulse response is the same as the reference signal. The output of the Matched Filter is then compared to a threshold of a known signal (reference) for detecting the existence or absence of the primary user as was used in [46].



Figure 2.4: Conventional Matched Filter.

2.4.2 Advantages of Matched Filter

- Matched Filter (MF) is an optimal detection method when the there is prior knowledge of the received signal, thereby maximizing the SNR in the presence of noise [41], [51], [52].
- Matched Filter requires short time to reach a significant and acceptable detection performance such as low probability of false alarm [49].
- MF uses fewer received samples to achieve acceptable detection performance [51].

2.4.3 Disadvantages of Matched Filter

- Matched Filter (MF) requires full prior knowledge of every received signal for detection. Therefore, it requires receivers with their corresponding algorithms for different signals.
- MF has implementation complexity and requires high power consumption due to the different receiver to be implemented [53], [51].
- MF is difficult to apply in real communication environments where the full nature of the waveform is not always known.

2.4.4 Cyclostationary Feature Detection

A process is said to be Cyclostationary if the signal statistics vary periodically with time [2]. Modulated signals on their own are not truly periodic but have features with in-built periodicity such as carrier, symbol rate and modulation type such as Quadrature Phase Shift Keying (QPSK) [54]. These features are used to detect the presence or absence of primary users [5, 33, 55]. Also, a process shows cyclostationarity if the Auto Correlation function (ACF) and mean are periodic. A Cyclostationary Feature Detector (CFD) which can be used to detect the features of the signal such as frequency or cyclic frequency is robust to noise [54]. More details of CFD will be given in chapter 3 as this technique is the focus of this research.

2.4.5 Advantages of Cyclostationary Feature Detectors

Cyclostationary Feature Detectors have some advantages over Energy Detection Methods (most widely used form of spectrum sensing).

- It has discriminatory capability, see [56]. CFD is able to differentiate between the features of signals such as carrier frequency and cyclic frequency.
- CFD can function in lower SNR better than Energy detectors because of the mentioned discriminatory ability [33]
- It can detect signals without classification and without its prior knowledge.

2.4.6 Disadvantages of Cyclostationary Feature Detectors

- CFD has high computational complexity when compared with energy detection. This is because, it requires a greater number of samples for the detection of the signal's features such as the carrier and cyclic frequencies.
- CFD has a relatively long sensing time which is comparable to the number of samples used for the detection as discussed in [33], [56].
- CFD suffer from receiver offsets such as Cyclic Frequency Offset (CFO) and Sampling Clock Offset (SCO), see [57, 58]

2.4.7 Compressive Sensing

Compressed or compressive sensing is a signal processing technique that is used to sufficiently acquire and reconstruct a signal with fewer samples than the theoretical Shannon-Nyquist sampling limit. This will reduce the complexity of analogue-to-digital converter (ADC) and offer efficient detection if the signal is properly recovered. It is applied to signals that have sparse representation in the bandwidth of interest. Compressed sensing exploits the sparsity of a signal for its recovery [59, 60]. In order for signals to be correctly reconstructed in Compressive sensing (CS), a condition called the Restricted Isometry Property (RIP) will be met. RIP defines the limit of effective compressive sampling [61]. Conventional spectrum sensing techniques sample at higher rates about or above the Nyquist rate. Compressive sampling reduces the amount of spectrum to be sampled before sensing. CS replaces samples with a general linear measurement [61, 62] and can fundamentally be expressed as,

$$y = \Phi x \tag{2.6}$$

where *y* is the measurement, Φ is the sensing function and *x* is the original signal vector. When *y* and Φx are considered as vectors, it follows that *y* is the sum of all the instantaneous measurements.

2.4.8 Wideband Compressive Sensing

It was illustrated in [63,64] and recently in [61] that if signal x is *S*-sparse, it can be compressed by taking some measurements M as shown in (2.7), where N is the number of Nyquist samples and C is some positive constant.

$$M \ge CSlogN. \tag{2.7}$$

The choice of M depends on the sparsity factor S, since the occupation rate of the wideband channel is unknown. In [65], it was proved that a sparse signal can be reconstructed with fewer samples. This became the basis for the first study on compressed sensing in [62, 66] where Basis Pursuit (BP) was used

for signal recovery. Let the signal vector *x* be represented by,

$$x = \sum_{y} \alpha_{y} \Phi_{y}$$
(2.8)

where Φ_y is the waveform due to the parameter y and α_y represents all the coefficients expressed as a column vector. BP finds the signal representation whose coefficients have minimal L_1 -Norm as in (2.9). That is, min|| α ||₁ subject to,

$$\Phi \alpha = x. \tag{2.9}$$

Compressive Sampling (CS) in Wideband was first studied in [59]. CS was used with a Stationary Wavelet Transform (SWT) to implement Wideband Spectrum Sensing (WSS) in [67–69]. It requires a large number of samples as discussed in [70]. Another CS approach used Non-Gaussian Testing to detect a Primary User (PU) signal from the compressed samples without reconstructing the cyclic spectrum of the sampled signal [71]. It compares the frequency response distribution of the compressed signal against the Gaussian Noise distribution. A multi-antenna receiver was suggested in [72]. The accuracy of synchronization of all signals from different antennas may affect the sampling. The use of an ultra-wideband (UWB) channel was explored in [73]. The use of a Matched-filter with compressive sampling and interference immunity was studied in [74]. This assumed that the spectral shapes of the PU signals were known which were compared with the Power Spectral Density (PSD) of the received signal. In compressive sampling, the recovery of the compressed signal can be done with L_1 – Norm Regularized Least Squares minimization (LR-LS) before the detection or estimation of the spectral occupancy for low complexity at reduced SNR. The L_1 – Norm Regularized Least Squares minimization (LR-LS) is expressed mathematically as, Let R^N represent the received signal such that $x \in \Re^N$, where x is a vector. Considering noise vector e in (2.10) we have,

$$y = \Phi x + e \tag{2.10}$$

where $||e||_2^2 \le \epsilon$ while *e* is assumed to have a value less than ϵ [61, 66].

$$\underset{\hat{x} \in \Re^N}{\operatorname{argmin}} ||x||_1 \tag{2.11}$$

where \hat{x} is the recovered signal, such that

$$||y - \Phi x||_2^2 \le \epsilon.$$
 (2.12)

Equation 2.12 represents L_2 – Norm which is the constraint that is needed to shrink the required measurements for recovering the signal vector *x* in (2.10). Therefore, the LR-LS can be expressed as,

$$\hat{x} = \operatorname{argmin} ||x||_1 + ||y - \Phi x||_2^2 \le \epsilon.$$
 (2.13)

2.4.9 Drawbacks of Compressive sampling

Some of the constraints of CS as discussed in [75–77] are as follow.

•Structured sensing matrices. The sensing matrix is often dictated by the physical properties of the sensing process (e.g., the laws of wave propagation) and constraints associated with its practicability.

•The requirement for structured sparsity.

This is important because sparsity patterns may not be equally the same along the entire signal duration.

Application-specific prior information

Application information such as the likelihood of a certain minimum distance between sparse coefficients may be a constraint as this is not always available.

Hardware design

The hardware design of a compressive sensing device is still a challenge despite the progress already made. Noise is a major limiting factor as well as calibration of CS devices [75, 76].

Accuracy of spectral location.

The locations of the spectral components may not be accurate due to the sub-Nyquist sampling rate and uncertainty in the amount of spectrum estimation adequate for effective sensing of the wideband of interest.

2.5 Fourier Analysis

Fourier analysis describes the method of representing general functions by the sums of simpler trigonometric functions such as sines and cosines. In Fourier analysis, functions are decomposed into oscillatory components while the process of rebuilding the function from these components is known as Fourier synthesis. There are different types of Fourier transforms which depend on the types of signals as shown in table 2.1, see [78].

2.5.1 The Sampling process

Consider a signal x(t) shown in Fig. 2.5(a), whose spectrum is band-limited to *B* Hz. Let x(t) be sampled at a rate of f_s (samples per second) by multiplying x(t) by an impulse train p(t)) shown in Fig. 2.5(b), which consists of unit impulses repeating periodically every *T* seconds, where $T = \frac{1}{T}$, see [79, 80]. The sampled signal or discrete x(n) shown in Fig. 2.5(c) consists of impulses spaced every sampling interval *T* seconds. The value of x(t) at t = nT is x(nT)which is the strength of the n^{th} impulse at t = nT. Therefore,

$$x(n) = x(t)p(t)$$

= $\sum_{n=-\infty}^{\infty} x(nT)p(t-nT)$ (2.14)

where p(t) is the pulse train or pulse shape filter and defined by

$$p(t) = \sum_{n=-\infty}^{\infty} p(t - nT).$$
(2.15)

Type of signal	Type of Fourier Transform	
Aperiodic-Continuous		
This signal goes up to both positive	Fourier Transform (FT)	
and negative infinities without periodi-		
cally repeating itself. These include de-		
caying exponentials and the Gaussian		
curve		
Periodic-Continuous		
The signal repeats itself in a regular or	Fourier Series (FS)	
periodic pattern from negative to pos-		
itive infinity such as sine waves and		
square waves.		
Aperiodic-Discrete		
The signals is only defined at discrete	Discrete Time Fourier Transform	
or definite points between positive and	(DTFT)	
negative infinity, and is not repeated in		
a periodic patttern		
Periodic-Discrete		
The signal repeats itself in a periodic or	Discrete Fourier Transform (DFT)	
regular pattern from negative to positive	Short-time Fourier Transform (STFT)	
infinity.	Time-Dependent Fourier Transform	
	(TDFT), Fast Fourier Transform (FFT)	

Table 2.1: Types of Fourier transforms.

This can also be expressed as discrete complex samples by

$$x[n] = \sum_{n=-\infty}^{\infty} x(nT)p(t-nT)e^{j2\pi f_c t} \cos(2\pi f_c t)$$

=
$$\sum_{n=-\infty}^{\infty} x(nT)p(t-nT)e^{j2\pi f_c t}$$
 (2.16)

where f_c is the carrier frequency as was used in [81].



Figure 2.5: Fourier Transform of x(t)

2.5.2 Fourier Transform

The Fourier transform (FT) decomposes a time-series signal x(t) in Fig. 2.6(a) or function of time into the frequencies that make it up as shown in Fig. 2.6(b). FT of a signal x(t) is a complex-valued function of frequency, whose absolute value represents the amount of that frequency present in the original function of time x(t) while the complex argument is the phase offset of the basic sinusoid in that frequency, see, [78–80, 82]. FT is also known as the frequency domain

representation of the original time-series signal. The Fourier Transform of the



Figure 2.6: Fourier Transform of x(t)

original pulse signal x(t), would be represented as

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$
(2.17)

and the inverse Fourier Transform is

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega,$$
(2.18)

where $\omega = 2 \pi f$ and *f* is the frequency of the signal.

2.5.2.1 Properties of Fourier Transform

The properties of the Fourier Transforms will be described in this section.

Linearity If x(t) and y(t) have the Fourier transform H(f) and Y(f), respectively, then the sum x(t) + y(t) has the Fourier transform X(f) + Y(f).

$$\int_{-\infty}^{\infty} \left[x(t) + y(t) \right] e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt + \int_{-\infty}^{\infty} y(t) e^{-j2\pi f t} dt$$

$$= X(f) + Y(f).$$
(2.19)

Therefore,

$$x(t) + y(t) = X(f) + Y(f).$$
 (2.20)

Symmetry If h(t) and H(f) are a Fourier transform pair, then

$$H(t) \Leftrightarrow h(-f) \tag{2.21}$$

•

The FT pair above can be shown by rewriting

$$h(-t) = \int_{-\infty}^{\infty} H(f) e^{-j2\pi f t} df$$
 (2.22)

and by interchanging t and f,

$$h(-f) = \int_{-\infty}^{\infty} H(t) e^{-j2\pi f t} dt$$
 (2.23)

The FT properties can be summarized as in Table 2.2, see [83].

Table 2.2: Pro	perties of Fourie	er Transform
----------------	-------------------	--------------

Time D	omain	Frequenc	y Domain
Property	Expression	Equivalent Prop-	Fourier Trans-
		erty	form
Linearity	x(t)+y(t)	Linearity	X(f) + Y(f)
Symmetry	H(t)	Symmetry	h(-f)
Time Scaling	h(kt)	Inverse Scale	$\frac{1}{ k }H(\frac{f}{k})$
		Change	
Inverse Scale	$\frac{1}{ k }h(\frac{t}{k})$	Frequency Scal-	H(kf)
change		ing	
Time shifting	$h(t-t_0)$	Phase shifting	$H(f)e^{-j2\pi ft_0}$
Modulation	$h(t)e^{j2\pi t f_0}$	Frequency shifting	$H(f - f_0)$
Even function	$h_e(t)$	Real function	$H_e(f) = R_e(f)$
Odd function	$h_o(t)$	Imaginary	$H_o(f) = j I_o(f)$
Real function	$ht = h_r(t)$	Real part even,	$H(f) = R_e(f) +$
		Imaginary part	$jI_o(f)$
		odd	
Imaginary function	$ht = jh_i(t)$	Real part odd,	$H(f) = R_o(f) +$
		Imaginary part	$jI_e(f)$
		even	

2.5.3 Fourier series

Fourier series (FS) is a way of representing a function as the sum of waves. Technically, it decomposes any periodic function or signal x(t) into the weighted sum of sines and cosines or equivalent complex exponentials as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jkwt}$$
(2.24)

where C_n is the Fourier coefficient.

2.5.4 Short-Time Fourier Transform

Short-Time Fourier Transform (STFT) which is also known as Time-Dependent Fourier Transform (TDFT) can be defined in both continuous and discrete time instants.

Continuous-time STFT

The signal x(t) to be transformed is multiplied by a non-zero window function w(r) for only a short period of time t. The Fourier transform of the signal is taken as the window w(t) is slid along the time axis t, resulting in a two-dimensional representation of the signal, see [80, 84]. This can be expressed mathematically as

$$X(t',\omega) = \int_{-\infty}^{\infty} x(t) w(t-t') e^{-j\omega t} dt$$
(2.25)

where $X(t', \omega)$ is the continuous-time STFT for each window w(t) centred at t = t' with the continuous frequency variable ω .

Discrete-time STFT

In the discrete case, the STFT can be represented as

$$X(k,\omega) = \sum_{n=-\infty}^{\infty} x(n) w(n-k) e^{-j\omega n}$$
(2.26)

where the window w(n) is of discrete length k while the frequency parameter ω is still continuous [80].

2.5.5 Discrete-Time Fourier Transform

In Discrete-Time Fourier Transform (DTFT), a discrete signal x[n] or a sampled version of continuous time signal which is not periodic is transformed into the frequency domain. DTFT relates an aperiodic, discrete signal, with a periodic, continuous frequency spectrum, see [78]. It can be expressed as

$$X[\Omega] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$
(2.27)

where Ω is between 0 and 2π and is the discrete time frequency parameter. DTFT is not suitable for Digital Signal Processing (DSP) applications because in DSP, the spectrum is only computed at discrete values of the frequenies ω , see [78]. Also, in DSP applications, a signal is measured only at discrete points.

2.5.6 Discrete Fourier Transform

The Discrete Fourier Transform (DFT) is the equivalent of the continuous Fourier Transform (FT) of discrete signals separated by samples at finite times or period *T* (i.e a finite sequence of data). Consider a finite sequence x[n] defined as $0 \le n \le (N-1)$. Let *N* samples of the signal x(t) be denoted by x[0], x[1], ..., x[k], ..., x[N-1]. The Fourier transform of x(t) is given in (2.17). Let each sample x[k] be regarded as an impulse having an area x[k]. Since the integrand exists only at the sample points for a DFT as explained previously, the Fourier Transform in (2.17) becomes,

$$F(\omega) = \int_{0}^{(N-1)T} x(t) e^{-j\omega t} dt$$
 (2.28)

$$\begin{aligned} F(\omega)) &= \int_0^{(N-1)T} x(t) e^{-j\omega t} dt \\ &= x[0] e^{-j0} + x[1] e^{-j\omega T} + x[2] e^{-2j\omega T} + \dots + x[k] e^{-j\omega kT} + \dots + x[N-1] e^{-j\omega(N-1)T} \\ &= \sum_{k=0}^{(N-1)T} x[k] e^{-j\omega kT} \end{aligned}$$

(2.29)

Note that can be evaluated for any ω , but with *N* data points only *N* final outputs will be significant, see [80, 82]. Since there are finite samples, the DFT treats the data as periodic such that x(N) to x(2N-1) is the same as x(0) to x(N-1). Therefore, the DFT equation in (2.29) will be evaluated for the fundamental frequency, i.e one cycle per sequence ($\frac{1}{NT}$ Hz or $\frac{2\pi}{NT}$ rad/sec), its harmonics and the dc component at $\omega = 0$. Therefore we set,

 $\omega = 0, \frac{2\pi}{NT}, \frac{2\pi}{NT} \times 2, \dots \frac{2\pi}{NT} \times n, \dots \frac{2\pi}{NT} \times (N-1),$

and substitute ω into (2.29) to have the general DFT equation,

$$X[n] = \sum_{k=0}^{(N-1)} x[k] e^{-j\frac{2\pi}{N}nk} \quad \text{for } 0 \le n \le (N-1).$$
(2.30)

From where the Inverse Discrete Fourier Transform (IDFT) becomes

$$x[k] = \frac{1}{N} \sum_{n=0}^{(N-1)} X[n] e^{+j\frac{2\pi}{N}nk} \quad \text{for } 0 \le k \le (N-1),$$
(2.31)

while equations (2.30) and (2.31) are known as the DFT pair.

2.5.6.1 Properties of Discrete Fourier Transform

The properties of the Fourier transform given in Table 2.2 can be extended to the Discrete Fourier Transform.

Linearity If x(k) and y(k) have discrete Fourier transforms X(n) and Y(n), respectively, then the sum x(k) + y(k) has the Discrete Fourier transform X(n) + Y(n).

$$x(k) + y(k) \Leftrightarrow X(n) + Y(n).$$
(2.32)

This follows directly from the DFT pair in (2.30) and (2.31).

Symmetry If x(k) and X(k) are a discrete Fourier transform pair, then

$$\frac{1}{N}X(k) \Leftrightarrow x(-n) \tag{2.33}$$

The DFT pair of equation (2.33) is established by rewriting equation (2.31)

$$x[-k] = \frac{1}{N} \sum_{k=0}^{(N-1)} X[n] e^{+j\frac{2\pi}{N}n(-k)}$$
(2.34)

and by interchanging k and n,

$$x[-n] = \frac{1}{N} \sum_{n=0}^{(N-1)} X[k] e^{-j\frac{2\pi}{N}nk}.$$
 (2.35)

Some of the properties of the DFT can be summarized as in Table 2.3, see [83].

Property	Discrete Fourier Transform
Linearity	$x(k)+y(k) \Leftrightarrow X(n) + Y(n)$
Symmetry	$\frac{1}{N}X(k) \Leftrightarrow x(-n)$
Time shifting	$x(k-i) \Leftrightarrow X(n)e^{-j2\pi ni/N}$
Frequency shifting	$x(k)e^{j2\pi ni/N} \Leftrightarrow X(n-i)$
Even functions	$x_e(k) \Leftrightarrow R_e(n)$
Odd functions	$x_o(k) \Leftrightarrow j I_o(n)$
Correlation	$y(k) = \sum_{i=0}^{(N-1)} x(i)h(k+i)$

 Table 2.3: Properties of Discrete Fourier Transform

2.5.7 Fast Fourier Transform

There are different methods of calculating the Discrete Fourier Transform (DFT) such as solving simultaneous linear equations. The Fast Fourier Transform (FFT) is a more efficient algorithm for the computation of the DFT. It is also known as the Cooley-Tukey algorithm to reflect the developers, see [78,83,85]. It reduces significantly the computation time. The FFT algorithm popularly known as Cooley-Tukey algorithm can be grouped into two basic types namely: Decimation-in-Time (DIT) and Decimation-in-Frequency (DIF).

2.5.7.1 Decimation-in-Time

For the Decimation-In-Time (DIT), the FFT is computed by dividing up, or decimating, the samples x[n] into sub-sequences until only 2-point DFT's remain for a radix 2 FFT, see [79,83]. This algorithm is called the decimation-in- time since the division is on the input time samples, see [83]. Let us consider the basic computational structure associated with the DIT algorithm. For convenience, we define,

$$W_N = e^{-j\frac{2\pi}{N}}$$
(2.36)

and the DFT equation (2.30) can be re-written as,

$$X[k] = \sum_{n=0}^{(N-1)} x[n] W_N^{nk} \quad \text{for } 0 \le k \le (N-1).$$
 (2.37)

and the IDFT becomes

$$x[n] = \frac{1}{N} \sum_{k=0}^{(N-1)} X[k] W_N^{-nk} \quad \text{for } 0 \le n \le (N-1).$$
(2.38)

For simplicity as discussed in [79], let us choose *N* to be a power of 2. The *N*-point data sequence x[n] will be divided into two $\frac{N}{2}$ -point of even-and-odd-numbered sequences, *g*[*n*] and *h*[*n*], respectively as follow.

 $\underbrace{X_0, X_2, X_4, ..., X_{N_{0-2}}}_{X_1, X_3, X_5, ..., X_{N_{0-1}}}.$

even sequence g[n] odd sequence h[n]From (2.37),

$$X[k] = \sum_{n=0}^{(N/2)-1} x[2n] W_N^{2nk} + \sum_{n=0}^{(N/2)-1} x[2n+1] W_N^{(2n+1)k}.$$
 (2.39)

It can be shown from (2.36) that

$$W_{N/2} = W_N^2. (2.40)$$

Therefore, we have

$$X[k] = \sum_{n=0}^{(N/2)-1} x[2n] W_{N/2}^{nk} + W_N^k \sum_{n=0}^{(N/2)-1} x[2n+1] W_{N/2}^{kn}$$

= $G_k + W_N^k H_k$ for $0 \le k \le (N-1)$ (2.41)

where G_k and H_k are the $\frac{N}{2}$ -point DFTs of the even-and-odd-numbered sequences, g[n] and h[n], respectively. Since G_k and H_k are the $\frac{N}{2}$ -point DFTs, they are periodic in N/2. Therefore,

$$G_{k+(N/2)} = G_k$$

 $H_{k+(N/2)} = H_k$
 $X_{k+(N/2)} = X_k$
 $W_N^{k+(N/2)} = W_N^k$
(2.42)

Note that

$$W_{N}^{k+(N/2)} = W_{N}^{N/2} W_{N}^{k}$$

= $e^{-j\pi} W_{N}^{k}$ (2.43)
= $-W_{N}^{k}$

From equations (2.41), (2.42) and (2.43), we obtain

$$X[k + (N/2)] = G_k - W_N^k H_k.$$
 (2.44)

The linearity property of the DFT demonstrated above is useful in reducing the number of computations. The first N/2 points ($0 \le n \le (N/2) - 1$ of X_k by using equation (2.41) and the last N/2 points by using equation (2.44) as

$$X[k] = G_k + W_N^k H_k \quad \text{for } 0 \le k \le (N/2 - 1)$$
(2.45)

and

$$X[k + (N/2)] = G_k - W_N^k H_k \quad \text{for } 0 \le k \le (N/2 - 1)$$
(2.46)

Therefore, an *N*-point DFT can be computed by combining the two (*N*/2)-points DFTs as in equations (2.45) and (2.46). These equations are being shown in a flow or process graph in Fig. 2.7. This structure is known as the Butterfly and is so called because of its criss-cross appearance. where G_k and H_k are



Figure 2.7: Fourier Transform of x(t)

the $\frac{N}{2}$ -point DFTs of the even-and-odd-numbered sequences, g[n] and h[n], respectively. The second FFT algorithm, the decimation-in-frequency (DIF), is similar to the decimation-in-time algorithm. The only difference is that instead

of dividing x_n into two sequences of even-and odd-numbered samples, we divide x_n into two sequences formed by the first N/2 and the last N/2 points. The decimation continues the same way as with DIT, until a single-point DFT is reached in log *N* steps. The total number of computations is the same for both DIT and DIF algorithms.

2.5.7.2 The FFT Complex Multiplications and Additions

The DFT requires N^2 complex multiplications and N(N-1) complex additions in order to compute an *N*-point DFT. This can be reduced by the use of FFT. At each steps of the FFT, N/2 complex multiplications are needed to combine the results of the previous steps, see [79, 82, 83]. Since there are log*N* steps, in order to compute an *N*-point DFT with the FFT, a total of $N/2 \log N$ complex multiplications and *N* log *N* complex additions will be required. Therefore, the FFT reduces the number of computations from the order of N^2 to *N* log *N*. This has facilitated the use of the FFT in computing the DFT in digital signal processing. More details on FFT complexities can be found in Table F.

2.5.8 DFT Errors

There are two main types of errors that affect the accuracy of Discrete Fourier Transform namely: Aliasing and Leakage.

2.5.8.1 Aliasing

Aliasing can be described as an overlapping in the frequency domain of two different samples continuous signals. Aliasing occurs when samples of continuoustime signals or sinusoids of at least two different frequencies produce the same discrete-time signal see [80, 82]. That is, the different analogue signals generate the same discrete-time identity. It happens when the sampling rate f_s is lower than the Nyquist criterion of $2 \times f_h$ where f_h is the highest signal frequency. Since f_s is the reciprocal of the sampling interval T_s , in order to avoid aliasing, the condition in (2.47) will be met.

$$f_s = \frac{1}{T_s} \ge 2f_h \tag{2.47}$$

Equation (2.47) satisfies the Nyquist criterion. this is important since the discrete time signal processing considers the limit on the highest frequency that can be processed, see [82]. Aliasing will result in incorrect frequency value. Aliasing can also be reduced by pre-filtering or down-converting the signal in order to minimise its high frequency spectral content.

2.5.8.2 Leakage

Recall from sections 2.5.2.1, 2.5.6 and equations (2.17), (2.30), that the Fourier transform of a periodic signal requires the integration to be performed over the interval $-\infty$ and $+\infty$ while the DFT computes over an integer number of cycles of the signal or expects the signal to be period. If the DFT is computed over a non-periodic waveform then the transform may be corrupted. It is equivalent to computing DFTs of a signal with major discontinuities. hence, other frequency components. This effect is known as Leakage and arises because the DFT appears to be for a signal with different frequencies, see [79, 82, 86, 87]. The leakage can in turn cause aliasing. Leakage can be reduced by using a tapered window function for signal truncation.

2.5.9 Bandpass and Equivalent Lowpass Signals

A Bandpass (BP) signal $x(t) = A(t)\cos 2\pi f_c t + \theta$ is defined as having its Fourier transform X(f) to be non-zero only in some small band around a central frequency f_c , see [79,88]. It follows that a Bandpass signal has most of its energy centred around some frequency f_c . As an illustration, consider a bandpass modulated signal $z(t) = A(t) \cos 2\pi f_c t + \theta$ in Fig. 2.8, whose spectrum is centred at f_c . The carrier signal is $\cos 2\pi f_c t$ while A(t) is the modulating signal of bandwidth W and is also the real-valued envelope of x(t). The bandwidth B of the BP signal is equal to the width of the positive-frequency interval on which the signal is non-zero as seen in Fig. 2.8b. The BP signal can be expressed as

$$x(t) = A(t)\cos(2pif_{c}t + \theta(t))$$

$$= \frac{A(t)}{2}e^{j(2\pi f_{c}t + \theta(t))} + \frac{A(t)}{2}e^{-j(2\pi f_{c}t + \theta(t))}$$
(2.48)

and

$$X(f) = 0$$
 for $|f - f_c| > W$ where $W < f_c$ (2.49)

as can be seen in Fig. 2.8.



Figure 2.8: Bandpass AM Modulated signal

Bandpass signals are classified as real signals and are widely used in Radio Frequency (RF) communication and radar signals. In the analysis and processing of BP signals, it is convenient to use its related equivalent signals called the Equivalent Lowpass (ELP) signals, see [88].

Firstly, in order to get the FT of the complex-valued analytic signal $x(t) = A(t)\cos 2\pi f_c t + \theta(t)$, we suppress the negative frequency part of the BP signal,

$$x_{a}(t) = A(t)e^{j(2pif_{c}t+\theta(t))}$$

$$= A(t)cos(2\pi f_{c}t+\theta(t)) + jA(t)sin(2\pi f_{c}t+\theta(t))$$
(2.50)

and is shown in Fig. 2.9.



Figure 2.9: Fourier transform of complex-valued analytic signal $x_a(t)$

Secondly, in order to get the FT of the Equivalent Lowpass signal, we frequency-shift the positive frequency part down by f_c to get

$$x_l = e^{(-j2pif_ct)x_a(t)}$$

= $A(t)e^{j\theta(t)}$ (2.51)

where $A(t)e^{j\theta(t)}$ is the Equivalent Lowpass signal that represents the bandpass signal $A(t)\cos 2\pi f_c t + \theta(t)$ shown in Fig. 2.8 while the FT of the ELP signal is shown in Fig. 2.10. Note that $|X_l(f)|$ in Fig.2.10 does not necessarily have even symmetry as a complex signal whereas the bandpass signal it represents is real valued with symmetry about the zero frequency, see [79].



Figure 2.10: Fourier transform of complex-valued analytic signal $x_a(t)$

In radio communications, It is more convenient to use the complex-valued equivalent lowpass signal to analyse bandpass signals. This gives the opportunity of dealing with low frequencies rather than high frequencies associated with bandpass signals.

2.6 Window Functions

A window function is a mathematical function that has zero values outside the chosen period. It is symmetrical about the centre which is usually about the maximum value and tapers away from the centre [80, 82]. When another function or data sequence is multiplied by a window function, the product is also of zero value outside the chosen window period. There are different types of window functions with different shapes. For example an *N*-point the Triangle window is given as,

for N odd,

$$w(n) = \begin{cases} \frac{2n}{N+1} & 1 \le n \le (N+1)/2 \\ \\ 2 - \frac{2n}{N+1} & (N+1)/2 + 1 \le n \le N \end{cases}$$
(2.52)

and for N even,

$$w(n) = \begin{cases} \frac{2n-1}{N} & 1 \le n \le N/2 \\ & & . \\ 2 - \frac{2n-1}{N} & (N/2+1) \le n \le N \end{cases}$$
(2.53)

It has nonzero values at points 1 and N, see [80], as shown in Fig. 2.11



The Hanning window is a cosine-sum window is similar to the Hamming window except that its end points touch zero value as shown in Fig. 2.12 This is unlike the Hamming window where the end points do not just touch the zero value. The Hanning window is given as

$$w(n) = 0.5 \left[1 - \cos\left(\frac{2\pi n}{N-1}\right) \right] \quad 0 \le n \le N-1$$
 (2.54)



The general expression of a windowed signal v[n] can be expressed as follow [79, 80]. Considering a discrete-time signal,

$$x[n] = A_0 \cos(\omega_0 n + \theta_0) + A_1 \cos(\omega_1 n + \theta_1) \quad -\infty < n < \theta \tag{2.55}$$

where $\omega_0 = \Omega_0 T$ and $\omega_1 = \Omega_1 T$. The windowed sequence is

$$v[n] = A_0 w[n] w \cos(\omega_0 n + \theta_0) + A_1 w[n] \cos(\omega_1 n + \theta_1)$$
(2.56)

where v[n] is the window sequence. In order to obtain the DFT of v[n], equation 2.56 can be expressed in terms of the complex exponentials and using the frequency shifting property of the DFT in Table 2.3,

$$v[n] = \frac{A_0}{2} w[n] e^{j\theta_0} e^{j\omega_0 n} + \frac{A_0}{2} w[n] e^{-j\theta_0} e^{-j\omega_0 n} + \frac{A_1}{2} w[n] e^{j\theta_1} e^{j\omega_1 n} + \frac{A_1}{2} w[n] e^{-j\theta_1} e^{-j\omega_1 n}$$
(2.57)

from the Time and frequency shifting properties of the Fourier Transform in Table 2.2, the Fourier transform of the windowed sequence is

$$V(e^{j\omega}) = \frac{A_0}{2} e^{j\theta_0} W\left(e^{j(\omega-\omega_0)}\right) + \frac{A_0}{2} e^{-j\theta_0} W\left(e^{j(\omega+\omega_0)}\right) \\ \times + \frac{A_1}{2} e^{j\theta_1} W\left(e^{j(\omega-\omega_1)}\right) + \frac{A_1}{2} e^{-j\theta_1} W\left(e^{j(\omega+\omega_1)}\right).$$
(2.58)

According to equation 2.58, the Fourier Transform of the windowed signal consists of the Fourier transform of the window, reproduces at the frequencies $\pm \omega_0$ and $\pm \omega_1$ and scaled by the complex amplitudes of each complex exponential that make up the signal.

2.6.1 The Effects of Windowing

There are two main primary effects on the spectrum as a result of applying a window function to the signal [79, 80, 89].

Spectral Spreading and Frequency resolution

The choice of window especially tapered window increases spectral spreading, smearing, or broadening of the impulses in the theoretical Fourier representation [79, 80]. That is, it increases the bandwidth by an amount of the window function. This reduces the ability to resolve sinusoidal signals that are closely spaced in frequency. Spectral spreading can be reduced by increasing the window size N. This is equivalent to increasing the signal period T and therefore reduces the spectral component f (i.e increases spectral or frequency resolution). Signal bandwidth is inversely proportional to the signal duration (width). Thus, the wider the window, the smaller is its bandwidth, and the smaller is the spectral spreading. A wider window accepts more data and results in closer resolution or approximation, therefore causing smaller distortion or smaller spectral spreading, see [80]. Smaller window width results in poorer resolution and therefore causes more spectral spreading (more distortion). However, the wider window may adversely cause Spectral leakage.

Spectral Leakage

Since the bandwidth of the window is not really bandlimited, then its spectrum only reduces asymptotically to zero. This in turn causes the spectrum or Fourier transform of the signal multiplied with the window to asymptotically at the same rate rerduce to zero, even if the spectrum of the signal may be bandlimited, see [79,80]. Therefore windowing causes the spectrum of the signal to leak into the band where ideally it is expected to be zero. This effect is called Spectral Leakage. Spectral Leakage can be reduced by using a tapered window to truncate the signal such as Hanning, Welch, Blackman and Kaiser-Bessel windows, see Appendix G. This will essentially reduce the amount of data allowed by the

window. Therefore, there is a trade-off between the Spectral spreading and Spectral Leakage in the choice of the right window to use. Both the type of window and the window size are considered.

Windowing smears or broadens the impulses in the Fourier transform and results in not defining the exact signal frequency sharply

For more details, a comparison of different windows and their characteristics are presented in Appendix G. The choice of the application of a window depends on the compared characteristics.

2.7 Chapter Two Summary

In this chapter we have described some of the conventional spectrum sensing techniques that can be applied in communication systems. The basic implementation of some of them were explained while highlights were given of their advantages, disadvantages and applications in communication systems.

From the drawbacks of the discussed spectrum sensing techniques and in order to detect signals of very low SNR, cyclostationary feature detection can be an option. However, the issue of receiver constraints which affects the detection accuracy of cyclostationary spectrum sensing technique will have to be considered. This research will focus on the development of the design and implementation of wideband cyclostationary spectrum sensing method with emphasis on the reduction of the receiver constraints while achieving low computational complexity, high efficiency and robustness to noise.

Chapter 3

Cyclostationary Spectrum Sensing

In this chapter we will describe more in details the cyclostationary spectrum sensing technique which is within the research interest of this work. As mentioned in the previous section, modulated signals have built-in periodic features such as the sinewave component of the modulated signal (carrier), symbol rate and modulation type as mentioned in [54]. These features can be used to detect the presence of primary users as in [5], [55] and [33]. These components have statistical characteristics which vary periodically and unlike noise whose statistical distribution does not vary with time or said to be stationary. Also, a process shows cyclostationarity if the Auto Correlation function (ACF) and mean are periodic. Simply put, a cyclostationary signal is one which has correlation or similarities between areas in its spectrum. It exhibits both periodic and stationary characteristics. A Cyclostationary Feature Detector (CFD) capitalizes on these features of the modulated signal which exhibit cyclostationarity to deliver good signal sensing. CFD is non-coherent because, in order to detect the primary user's signal, it does not require the prior knowledge of the primary user's modulated signal envelope. It considers the features of the received signal and in this case, the exact types of signal envelopes are not necessary even with the ability to detect the presence of the primary user's signal. However, it exhibits an element of coherency in detecting the types of signal features. Coherency is involved, when it is necessary to classify the features under detection, for example, the modulation type as discussed in [54].

For the purpose of spectrum sensing applications, the subject of interest is the ability to detect the presence of a primary user signal and not the classification of features in the signal. The CFD can detect signals with low signal-to-noise such as a spread spectrum signal. This makes cyclostationary feature detection unique.

3.1 Cyclic Autocorrelation Function and Cyclostationarity

From [5, 90, 91], a process or signal x(t) is said to have first-order periodicity when it is periodic in *t* with a period *T*,

$$x(t) = x(t+T)$$
 (3.1)

It can be represented using Fourier series coefficients as ,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t},$$
(3.2)

where

$$w_0 = \frac{2\pi}{T},\tag{3.3}$$

is the fundamental frequency and Fourier coefficient a_k , given as,

$$a_k = \frac{1}{T} \int_T x(t) e^{-jkw_0 t} dt.$$
 (3.4)

Therefore, a process x(t) is said to have first-order cyclostationarity, if its mean M_x is periodic in t with a period T,

$$M_x(t) = M_x(t+T)$$
 (3.5)

On the other hand, a process x(t) is said to have second-order periodicity if its quadratic transformation y(t) (3.6) is periodic in t with a period T,

$$y(t) = x(t)^2$$
 (3.6)

Therefore, a process x(t) exhibits second-order cyclostationarity in the widesense when its mean and autocorrelation are periodic with some period, say, *T*, see [5, 90–92]. That is, second-order cyclostationarity exhibits both the characteristics of first and second order periodicities. Autocorrelation function (AF) of the signal x(t) is the similarity with a time-lagged version of itself , i.e., $x(t+\frac{\tau}{2})$ and $x(t-\frac{\tau}{2})$ where τ is the relative time difference between two instances of time t_1 and t_2 . For an appropriate model, the signal is assumed to be a cycloergodic process which means that the long run behaviour of the time-averaged measurements or samples of the signal can be predicted from the calculated expectations based on the probabilistic model of the process, see [2,5,93].

Similarly, the probabilistic autocorrelation of the random process X, at two time instances, t_1 and t_2 can be defined as the correlation of two random variables $X(t_1)$ and $X(t_2)$ and is expressed as,

$$R_{x}(t_{1}, t_{2}) = \mathbb{E} \left[X(t_{1}) X(t_{2}) \right]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_{1} x_{2} f_{x(t_{1})x(t_{2})}(x_{1}, x_{2}) dx_{1} dx_{2}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(t_{1}+nT) x(t_{2}+nT)$$

$$= R_{x}(t, \tau).$$
(3.7)

It follows as discussed in [4, 5, 90], that the autocorrelation function (AF) for a signal x(t) with periodic T in t is the mean of the lag products which is synchronized to T as shown in (3.7)

Therefore, the autocorrelation function of the signal x(t) can be expressed in terms of the expected value \mathbb{E} as used in [2] as,

$$R_x\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right) = \mathbb{E}\left[x\left(t+\frac{\tau}{2}\right)x\left(t-\frac{\tau}{2}\right)\right].$$
(3.8)

A process exhibits second-order periodicity wide-sense cyclostationarity when its mean and autocorrelation are periodic with some period, say, *T*, see [5,91, 92].

Autocorrelation Function can be represented by Fourier series as in (3.9)

$$R_x\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{-j2\pi\alpha t}$$
(3.9)
for α over all integer multiples *m* of the fundamental frequency of periodicity $\frac{1}{T_0}$ i.e. $\alpha = \frac{m}{T_0}$ where α is the Fourier or cyclic frequency, T_0 is the period and $R_x^{\alpha}(\tau)$ is the Fourier-series coefficient as discussed in [4].

The Fourier coefficient which is a function of the lag product τ is given as,

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x \left(t + \frac{\tau}{2}, t - \frac{\tau}{2} \right) e^{-j2\pi\alpha t} dt.$$
(3.10)

, see [2,5,90,94]. This is a time-based Cyclic Autocorrelation Function (CAF) which depends on the time-difference or lag parameter τ . The CAF computes the correlation of the received signal with a delayed, frequency shifted, version of itself. The cycle frequency or the frequency of autocorrelation is $\alpha = m/T$. The CAF R_x^{α} which is periodic with α becomes the conventional autocorrelation function (CoAF) R_x^0 when $\alpha = 0$ and represents the direct current (dc) component of the lag-product waveform $x(t + \tau/2)x(t - \tau/2)$ for each value of τ where as R_x^{α} is the alternating current (ac) component corresponding to sinewave frequency, α . The CAF at fundamental periodic frequency 1/T is shown in (3.10). However, for a cyclostationary signal which has more than one fundamental frequency or periodicity such as $(1/T_1, 1/T_2, 1/T_3, \ldots, 1/T_n)$, α will cover all integer multiples of all fundamental frequencies i.e $(m_1/T_1, m_2/T_2, m_3/T_3, \ldots, m_n/T_n)$.

It should be mentioned that the limit in (3.10) will be omitted if the signal x(t) has only one period. For an appropriate model, the signal is assumed to be a cycloergodic process (limit is retained) and we can substitute (3.8) into (3.10) to obtain the generalised Cyclic Autocorrelation Function (CAF)

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x\left(t + \frac{\tau}{2}\right) x\left(t - \frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt.$$
(3.11)

This results in representing the generalised non-conjugate Cyclic Autocorrelation Function for a cyclostationary signal as,

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x\left(t + \frac{\tau}{2}\right) x^* \left(t - \frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt.$$
(3.12)

The expression of non-conjugate refers to the term on the left of (3.12). Simi-

larly, the conjugate CAF is expressed as,

$$R_{x^*}^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x \left(t + \frac{\tau}{2} \right) x \left(t - \frac{\tau}{2} \right) e^{-j2\pi\alpha t} dt.$$
(3.13)

Notice that none of the factors on the right in (3.13) is conjugated as discussed in [55, 95] except for the term on the left. The conjugation is to accommodate complex-valued signals. A more detailed description for the CAF can be found in Appendix A.

It should be mentioned that every cyclostationary process is an Asymptotically Mean Stationary (AMS) process, see [5, 93, 96]. These are processes that have time-variant probabilistic parameters such as mean and autocorrelation and are not identically zero. That is the time-variant distributions have asymptotic behaviour and in this case will not be exactly zero, see [96, 97].

3.2 Spectral Correlation Function

The generalised CAF in (3.12) can be expressed as the conventional crosscorrelation of the two complex-valued frequency-shifted versions of x(t). It can also be shown that periodicity in time-based autocorrelation can also be observed in the frequency domain, see e.g [5]. According to Cyclic Wiener relation, the time-based CAF can be expressed in the frequency domain by the Fourier Transform of CAF (3.12) as,

$$S_x^{\alpha}(f) = FT\left\{R_x^{\alpha}(\tau)\right\} = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau.$$
(3.14)

It is called the cyclic spectrum, Spectral Correlation Function (SCF) or Cyclic Spectral Density (CSD) function for a given cyclic frequency α . The Wiener relationship is also called the Wiener-Khintchine theorem and it states the Fourier transform relation between the conventional power spectral density and the autocorrelation

$$S_x^0(f) = FT\{R_x^0(\tau)\} = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau.$$
 (3.15)

where $S_x^0(f)$ is the conventional power spectrum and $R_x^0(\tau)$ is the conventional autocorrelation function.

It is shown in [2, 4, 5, 90, 98] that the spectral correlation function can be measured by the limiting average of the cyclic periodogram which is defined as,

$$I_T^{\alpha}(f) = \frac{1}{T} X_T \left(t, f + \frac{\alpha}{2} \right) X_T^* \left(t, f - \frac{\alpha}{2} \right).$$
(3.16)

As the temporal correlation increases without bound, the spectral resolution 1/T' is allowed to decrease to zero. The limit SCF can be expressed as,

$$S_x^{\alpha}(f) = \lim_{1/T' \to 0} \lim_{T \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} \frac{1}{T} X_T \left(t, f + \frac{\alpha}{2} \right) X_T^* \left(t, f - \frac{\alpha}{2} \right) \mathrm{d}t$$
(3.17)

where $X_T(t, f)$ is the short time Fourier transform or complex envelope of the narrow-band spectral component of x(t) with centre frequency f, bandwidth on the order of 1/T with T as the signal period for the observation time T'. The complex envelope is expressed as,

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi f u} du.$$
 (3.18)

The limit SCF in (3.17) is also known as temporally or time smoothed cyclic periodogram. Therefore the SCF (3.17) shows the limit as spectral resolution 1/T' becomes infinitesimal (i.e $1/T' \rightarrow 0$) of the limit of the temporal correlation (i.e $T \rightarrow \infty$) of the two spectral components of x(t) with frequencies $f + \alpha/2$ and $f - \alpha/2$.

The SCF in (3.17) is a function in 2-dimensional (2-D) representation of signal frequency f and cyclic frequency α . When $\alpha = 0$ it becomes the conventional Power Spectral Density Function (PSDF), which is the spectral density of average power in time domain [2]. However, for $\alpha \neq 0$, it shows that $S_x^{\alpha}(f)$ is the density of spectral correlation, that is, the density of correlation between spectral components at the frequencies $f + \alpha/2$ and $f - \alpha/2$. It then follows that a signal that has periodic autocorrelation is cyclostationary and will characteristically have spectral correlation. A cyclostationary signal has peaks at the cyclic frequencies [96], [4] and [54]. In summary, Fourier analysis of the autocorrelation function of the cyclostationary signal produces the cyclic autocorrelation function. Then, Fourier transform of the cyclic autocorrelation function results in the Spectral Correlation Function or Spectral Density Function (SDF) which is the basis for cyclostationary feature detection. More detailed descriptions can be respectively found in Appendices A, B and C for the CAF, SCF and SCF of Phase-Shift Keying signals such as Binary Phase Shift Keying (BPSK), Quadrature PSK (QPSK) and Offset QPSK (OQPSK) signals. As previously mentioned, the peaks of cyclostationary signals occur at α and in relation to the modulation type as shown in Table 3.1 as discussed in [1,3,54,55,95].

Modulation type	Peaks at (α, f)
BPSK	$\left(\frac{1}{T}, f_c\right), \left(2f_c, 0\right), \left(2f_c \pm \frac{1}{T}, 0\right)$
QPSK	$\left(\frac{1}{T}, f_c\right)$
MSK	$\left(\frac{1}{T}, f_c\right), \left(2f_c \pm \frac{1}{2T}, 0\right)$
QAM	$\left(\frac{1}{T}, f_c\right)$
AM	$(2f_c,0)$

Table 3.1: Cyclic features for some modulation types [1–5].

In Table 3.1, α is the cyclic frequency, f is the spectral frequency at which there is correlation while f_c is the carrier signal. The cyclic frequency α is a function of the symbol rate and carrier frequency. The conjugate CAF is used to detect the features at a signal's symbol rate $(\frac{1}{T})$ while the non-conjugate is used to detect features at the carrier frequency and is applicable in cyclostationary feature detection (CFD) as discussed in [2,99].

3.3 Cyclostationary Spectrum Sensing model

A Cyclostationary Spectrum Sensing (CSS) or Cyclostationary Feature Detection (CFD) model operates on the principles of cyclostationarity, correlation and detection. As discussed earlier, the spectral correlation density function (SCDF) shows the variations of intensities of the spectral components (peaks) in the frequency spectrum which distinguishes points of spectral energy concentration of the modulated signals from noise where the SCDF is just flat. This character of spectral redundancy makes it possible for signal selectivity in CFD. As previously mentioned in sections 3.1 and 3.2, cyclic spectrum domain preserves the signal's features such as carrier frequency and cyclic frequency, therefore the overlapping features in the power spectrum density are non-overlapping features in the cyclic spectrum domain which is the domain of cyclostationary feature detection, see [56], [100].

It should be mentioned that Cyclostationary Spectrum Sensing functions at the receiver front end of a cognitive radio system. The blcock diagram of the conventional CFD is shown in Fig. (3.1 with all the associated blocks, see [101, 102].



Figure 3.1: Conventional Cyclostationary Feature Detector.

The received signal x(t) is sampled and converted to a discrete data set in the Analogue-to-Digital Converter (ADC). The Hanning window produces a finite data set to reduce spectral leakage inherent with the fast Fourier Transform (FFT) especially when the signal is not completely periodic as discussed in [87]. The FFT decomposes the signal into component frequencies or spectral components which are then correlated at $(f + \alpha, f - \alpha)$ to give the Spectral correlation function $S_x^{\alpha}(f)$. This is averaged over the period T_{acq} (selected time period) to get the peaks and then passed through a detection algorithm that compares the peaks with a reference threshold.

3.3.1 Feature Detection

The detection of the presence or absence of signals is a function of the periodic peaks in the considered spectrum. These peaks distinguish signal from noise which has flat PSD and no spectral peaks, see [103]. In a low SNR condition, detection will still occur because of the spectral peaks or redundancy due to the periodicity in the signal which will be extracted during correlation. As discussed in [104, 105], the detection algorithm compares the peaks of a Test Statistic *TS* of the spectral correlation function density against a reference threshold λ

to determine the presence or absence of signal at the spectrum frequency fand cyclic frequency α of the received signal, i.e

$$|TS| < \lambda$$
, for when the signal is absent, H_0 ; (3.19)

and

$$|TS| \ge \lambda$$
, for when the signal is present, H_1 . (3.20)

3.4 Narrowband Spectrum Sensing

In Narrowband Spectrum Sensing the signal band of interest is usually small. The channel frequency response or characteristics are considered to be flat within the band. Gardner in his papers [2,55] first gave the fundamental framework for feature detection to overcome the problems of unknown and varying levels of noise and interference activity. Further work by [106] on CFD dealt with multi-cycle detection [91, 99, 107, 108]. In [109], a blind narrowband cyclostationary feature detector based on a sparsity hypothesis was presented. Further works in [110–112] elaborated on narrowband sensing. Universal Software Radio Peripheral (USRP) with GNU Radio was used in [111] while in [112] the PU signal parameters were estimated. In [113], Cyclic Spectrum leakage due to FFT was exploited. However, spectral leakage can be reduced by windowing as discussed in [86] and [87]. The sensitivity of cyclic frequency mismatch was discussed in [58] in a single cycle carrier narrowband CFD. Sampling clock offset in narrowband single cycle CFD was studied in [114–116]. The research in [113, 117] looked at blind narrowband CFD. Frequency Shift (FRESH) Filters in CSS was used in [91, 118] and applied Cycle frequency Domain Profile (CDP) to detect the signals. However, narrowband spectrum sensing although applicable in lower frequency spectrum is not very useful in communications where the band is often wide.

3.5 Wideband Spectrum Sensing

Wideband spectrum sensing (WSS) requires the sampling of a received signal at very high rates usually exceeding 1 Giga symbols per second (GSps) and which is very difficult to realize by state of art analogue to digital converters (ADCs) as discussed in [119, 120]. Note that the term wideband is used to respresent wideband channel which describes a channel with multiple frequency carriers. WSS is defined in [120] as simultaneous sensing of a wider range of frequencies beyond the single user's bandwidth such as Satellite C Band downlink (3.4-4.2 GHz). It increases the probability of finding unused frequencies. WSS can be achieved using Wideband RF Front-End (WBRFF) or multichannel Narrowband RF Front-End (NBRFF). WBRFF covers a wider range of frequencies at a time while multi-channel NBRFF handles a single sub-band at a time which is aggregated to equate a wider range. Sensing with NBRFF (multi-channel) involves adjusting or tuning the local oscillator in the downconversion chain to respond to different frequencies which are consequently sensed. In [121], multiple narrowband channels are sensed simultaneously and aggregated to give the total available channels for transmission. On the other hand, WBRFF down-converts to a baseband signal. The down-converted baseband signal is passed through a BandPass Filter (BPF). Sensing a wideband spectrum entails changing the filtering done by the BPF whilst keeping the RF Front-end constant, see [120].

There are contributions in implementing WSS in cooperative CR networks with prior knowledge of the PU signal [122] using generalized likelihood ratio detector where emphasis was on knowing some modulation parameters of the PU signal. More cooperative approaches can be found in [123, 124]. The use of Spectral Analysis of Randomized Sampling (SARS) in wideband sensing was investigated in [125]. The Spectral Analysis tool used was the Random Sampling on Grid (RSG) which utilizes a sampling rate lower than the conventional Nyquist rate. It is affected with performance degradation due to the non-stationary nature of a communication signal. Further works as in [126], adopted Complex valued Power Spectrum Density (CPSD) using a sub-Nyquist rate. It considers the signal to be a complex value rather than as a magnitude as is usually done in energy detection. It was not clear how this method would perform with a noise-like Spread Spectrum signal. In the same vein, [127] adopted a wideband spectrum sensing approach based on a timedomain coprime sampling and sub-band-bin energy detector. This relied on the energy detection of PSD obtained from the Fourier transform of the estimated autocorrelation of the samples obtained from time-based coprime sampling. In [128], the combination of Energy and Cyclostationary Feature Detectors was studied to blindly detect Orthogonal Frequency-Division Multiplexing (OFDM) signals.

3.5.1 Non-Blind Wideband Cyclostationary Sensing

Spectrum sensing is said to be non-blind when some known parameters of the PU signal are applied during a sensing process. Conventionally, detection processes involve the detection of a PU signal relying on the signal power which obviously are impaired in the face of a low signal to noise (SNR) spectrum which may be due to e.g. multipath, spread spectrum and interference. Non-Blind Wideband Cyclostationary Sensing (NWCS) detects the presence of the signal through the features. This makes it more robust to noise and low SNR. This has been studied using different methods. Sub-Nyquist samples were adopted in [3] to implement Non-blind Wideband Cyclostationary Sensing (NBWCS) assuming known modulation parameters. The use of a low complexity approach for reconstructing the Nyquist Spectral Correlation Function (SCF) from the sub-Nyquist samples was researched in [129]. The paper optimized the sparsity of SCF with known signal parameters. Partial interception of a signal was used in [130] in conjunction with some unknown features for the rest of the band. A new approach in [1] combined CFD with Compressive Signal Processing (CSP) without the need for signal reconstruction. The prior knowledge of some of the parameters of the received signal helps to

produce a more perfect detection of the signals considering the sensitivity and importance of spectrum sensing in a cognitive radio system.

3.5.2 Blind Wideband Cyclostationary Feature Sensing

Blind Wideband Cyclostationary Spectrum Sensing (BWCSS) performs spectrum sensing with no knowledge of the signal. The first study on BWCSS was in [131] where the recovery of a 2D Cyclic Spectrum of the compressed samples was estimated using L_1 – Norm minimization. The spectrum occupancy testing was on a band-by-band basis to reduce computational complexity at the expense of increased detection time. It should be mentioned that most of the works in blind cyclostatioanry detection considered compressed sampling or sensing. This is because blind detection involves the estimation of spectral holes or vanacies without going through the full spectrum, therefore, fewer number of samples can be appropriate. Following this was a study in [132] that used Jittered Random Sampling (JRS) to get the compressed samples before applying L_1 – Norm minimization to recover the 2D Cyclic Spectrum of the compressed samples. Energy Detection and Cyclostationary Feature Detector methods were combined in [133]. This method although blind involved more samples as there was no compression. Further work adopted compressed sensing as in [134]. This approach performs better with a limited number of samples. Sampling Clock Offset (SCO) and Cyclic Frequency Offset (CFO) which may arise as a result of frequency offsets, jitter and incorrect knowledge of the sampling rate were not considered. Also, [135] quantified the improvement in energy efficiency of Cyclostationary Feature Detection in CR by modifying the narrowband compressed sensing method already adopted in [110] by using Basis Pursuit (BP) rather than Orthogonal Matching Pursuit (OMP). In [136], multiple receive antennas were adopted. From these works, the success of blind Cyclostationary wideband sensing depends on the number of samples and the accuracy of estimation to achieve maximum detection and reduce false alarms.

3.6 Cyclic Frequency and Sampling Clock Offsets

Cyclostationary Feature Detectors require the knowledge of both the carrier frequency f_c and symbol rate 1/T for correctly sensing the signal. The imperfect knowledge of them will result in constraints or challenges or impairments which will impact on the performance of the CFD. These constraints are the Cyclic Frequency Offset (CFO) and Sampling Clock Offset (SCO). CFO is caused by the imperfect knowledge of the carrier frequency, local oscillator mismatch or Doppler shifts as discussed in [57, 81]. Zeng in his paper [58] included transmitter clock error in addition to the oscillator error. It is known that any clock or oscillator could produce errors. The transmitter clock will produce a symbol period T with offset and the oscillator produces a carrier frequency f_c will be repeated by the receiver clock and oscillator. These will affect the cyclic frequency α which is a function of the symbol rate 1/T and for some modulation types is also a function of the carrier frequency multiples, i.e. $2f_c$ as shown in table 3.1. Let the CFO be represented by Δ_{α} . Given that,

$$\alpha' = \alpha \times (1 + \Delta_{\alpha}) \tag{3.21}$$

where α' and α are the actual and ideal cyclic frequencies at the Receiver and transmitter respectively. When Δ_{α} is zero, then the actual and ideal cyclic frequencies are the same. Zeng in his paper [58] showed that in the presence of a non-zero CFO the performance of the CFD was degraded and was not improved even by increasing the number of samples for the detection. However, the solution to the problem of CFO was not presented in this paper.

Sampling Clock Offset (SCO) occurs from the frequency error produced by the oscillator affecting the sampling period T_s at the Analogue to Digital (A/D) stage of the receiver as discussed in [116]. Sampling frequency or rate $1/T_s$ is often in multiples of the symbol rate 1/T. Therefore, imperfect knowledge of the symbol rates at the Analogue to Digital (A/D) stage of the receiver results in a SCO δ which in turn affects the sampling rate 1/Ts. SCO can be stated as,

$$T'_s = (1+\delta) \times T_s \tag{3.22}$$

where T'_s is the actual sampling period used at the receiver, T_s is the ideal sampling period with good knowledge of the symbol rate at the transmitter. SCO will result in a drift in sampling times or points within the symbol time and this time-shift varies as the number of samples increases. A constant time offset produces a phase offset in both frequency and SCF domain as mentioned in [137].

The impairments associated with A/D stage were discussed in [114]. Arash Zahedi-Ghasabeh in his papers [116], [138] presented a method to compensate for performance degradations in the presence of SCO and CFO for a pilotaided Cyclostationary detector for Orthogonal Frequency Division Multiplexing (OFDM). Rebeiz in his paper [81], proposed a new multi-frame test statistic that reduces the degradation due to cyclic frequency offsets and sampling clock offset using the discrete Fourier transform (DFT) and cyclic autocorrelation function. He developed an offline optimization framework and determined the best frame length that maximizes the average detection performance of the proposed cyclostationary detection method given the statistical distributions of the receiver impairments. This was also demonstrated in a Ricean channel.

However, there are some gaps that call for more research in terms of providing a closer guide to hardware realisation. It was not clearly shown in that work how the approach offers the much needed reduction in computational complexity and improvement in efficiency in terms of the number of samples required for effective detection. Computational complexity should also be extended to the Fast Fourier Transform (FFT) which is used in practice. This has not been explored to the best of the author's knowledge.

There are no detection algorithms associated with the research either generic or conventional that are designed to make it easier to serve as a hardware or practical implementation guide. Algorithms are needed to provide systematic steps necessary for the practical realisation of e.g frame length for other sample sets outside the maximum fixed 5000 samples that was the choice of the work in [81]. Cyclostationary Feature Detectors should also be able to show more robustness to noise at values of CFO or SCO up to 0.01. A more recent work used complex exponential basis model to reduce CFO only through the estimation of the Doppler frequency shift which could cause the CFO in cognitive vehicle, see [139].

In order to address the problems mentioned above, we propose a multi-slot wideband cyclostationary feature detection with test statistics that will be analysed using the Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT) and Spectral Correlation Function (SCF). This will enable us to operate in the domain of computation and perform analysis closer to practical realisation. We will derive recommended FFT sizes, slot sizes, number of slots that will offer low computational complexity, efficiency, low computational cost at low signalto-noise ratios. Consideration will be given to the application of this model in order to reduce the effects of CFO and SCO and demonstrate its effectiveness in a Ricean channel. A Ricean channel is chosen because of its bi-variate normal random characteristic and non-zero mean. It is applicable to SCF which is a function of cyclic frequency and spectrum frequency with a non-zero mean. This will be followed with generic optimization and detection algorithms that enhance a hardware implementation. To the best of our knowledge there has not been any work that looked into the use of small FFTs and slot sizes to reduce the effects of SCO and CFO. Furthermore, there is a need to develop algorithms for the detection and optimization, at low computational complexity, to help make a practical realisation more possible.

3.7 Energy Detection and Cyclostationary Feature Detection in Simulink

A Simulink-based experiment was developed to compare the energy detection in Fig. 3.2 with the cyclostationary feature detection in Fig. 3.3 and the results shown in Fig. 3.4. Figure 3.2 shows the arrangement of Simulink blocks for the implementation of energy detection model. The Random Integer Generator (RIG) block randomly generates data. This is fed into the Buffer which converts it to frames. M-PSK Baseband Modulator produces QPSK output of the input to the Raised Cosine Transmit Filter (RCTF). The RCTF output is added to noise from the Gaussian Noise Generator (GNG). The resultant noisy signal is fed to the Hanning Window block which filters through the wanted band. A Welch Periodogram (equivalent to FFT, magnitude squared and average) was set to produce mean square output of the signal, equivalent to its energy. The MinMax outputs maximum values only. This is necessary since detection is based on peaks. The Threshold compares the input with the operand inside the block and outputs digit 1 if condition is true otherwise digit 0. The Detector output on Threshold switch can be monitored on Scope as 1 or 0. Adjusting the RCTF and GNG will alter the signal-to-noise (SNR) received at the Raised Cosine Receive Filter (TCRF). The combination of RCTF and TCRF is equivalent to Up and Down conversion process in a real communication channel.



Figure 3.2: Energy Detection in Simulink

The Cyclostationary Feature Detection in Fig. 3.3 is similar to the previous Energy Detector except with the additions of FFT, Autocorrelation Function (ACF), Absolute and Average. These additional blocks are necessary for the implementation of ACF to produce the Spectral Correlation function of the signal. The output of the Threshold switch can be monitored on Scope as 1 or 0.



Figure 3.3: Cyclostationary Feature Detector in Simulink

The results of the tests for both ED (in blue colur) and CFD (in orange colour) were monitored on the Scope and presented in Fig. 3.4. The graph shows that at low SNR below -1.3 dB, the Energy Detection (blue) does not recognize the presence of signal contrary to CFD result (orange). The CFD is able to detect the signal down to -22 dB. This demonstrates that CFD is more robust to noise than ED and this makes the CFD better than ED.



Figure 3.4: Energy and Cyclostationary Feature Detectors comparison

Note that digit 1 in Fig. 3.4 when expressed in terms of statistical probability of detection is a value between 0 and 1 (exclusive).

3.8 Chapter Three Summary

In this chapter the descriptions of the principles or processes of the cyclostationary spectrum sensing method were given in more details. Explanations and mathematical details were given for the fundamental principles of cyclic autocorrelation, spectral correlation functions and cyclic features such as cyclic frequency, carrier frequency and modulation types. The applications of the cyclostationary spectrum sensing in both narrowband and wideband were also considered.

The impact of receiver constraints such as sampling clock offset and cyclic frequency offset were discussed. The adaptation of cyclostationary spectrum sensing with and without prior knowledge were discussed. Also, the advantages and dis-advantages of cyclostationary spectrum sensing over other spectrum sensing methods were discussed.

The cyclostationary feature detection was compared with the optimal energy detection. It was found to show more robustness to noise than the energy detection by detecting signals of low-signal-to-noise values.

Chapter 4

Multi-slot Wideband Cyclostationary Feature Detector

Consideration will be given to a wideband radio frequency (RF) spectrum of multiple spectrally non-overlapping signals. In the context of spectrum sensing for Cognitive Radio (CR) systems, interest will be on the processing of a received RF wideband signal or a wideband channel of bandwidth B centred at any carrier frequency f_c and down-converted or demodulated to baseband of bandwidth B. Note that the term wideband signal or wideband channel represents a signal or channel with multiple frequency carriers. Downconversion to baseband is important in order to meet the requirement of Analogue to Digital Conversion (ADC) stage and also because spectral correlation between baseband spectral components is high compared with bandpass as supported in [96]. This analysis will also apply to other channel bandwidths. Given the time and power limitations in the sensing stage, it is important to consider a limited sensing time of length T_{acq} during which the radio receiver acquires a total number of N_B complex samples at a sampling rate f_s and symbol period T. The objective is to implement an efficient low complexity model that detects the presence of the signals in the wideband spectrum. Spectrum sensing takes the discrete samples of the wideband signal and outputs the signals occupying the wideband channel. It should be mentioned that the work done in [81] using a multi-frame cyclic autocorrelation function (CAF) statistic did not consider the

size of the FFT in arriving at the best possible frame size which is relevant in determining the computational complexity and efficiency of the approach. By adopting a frequency domain test statistic, spectral correlation function (SCF) it is possible to associate the length of the FFT in samples and arrive at more implementable conclusions concerning the sizes and numbers of FFT and slot. In addition, our analysis is focused on a more practical approach.

4.1 System Model

Let us represent the wideband signal s(t) that occupies the wideband spectrum or channel over a time variable $t \in [0, T_{acq}]$ as $s_u(t) \forall u \in [1, ..., U]$ consisting of U Primary User (PU) signals with residual carrier frequencies f_{cu} as was used in [140]. We propose a wideband multi-slot window-based Fast Fourier Transform (FFT) Cyclostationary Feature Detection (CFD) model that detects the wideband signal $s_u(t)$ by correlating in the frequency domain with the spectral correlation function (SCF) as defined in (3.17).



Figure 4.1: Wideband Multi-slot Cyclostationary Feature Detector.

In Fig. 4.1, the received high frequency wideband signal in GHz is first down-converted to baseband in MHz and sampled at Nyquist rate to give a complex-valued sequence of length of samples N_B . This sequence of complex samples is segmented into non-overlapping blocks of N samples for correlation

which in this research are called slots. The relationship can be stated as,

$$N_B = \frac{T_{\text{acq}}}{T_s}$$

$$= P \times N$$
(4.1)

where T_{acq} , T_s , P and N are the total sensing or acquisition time for the wideband channel, the sampling period, the number of slots and samples per slot respectively. Since we are detecting across the entire wideband channel after it has been down-converted to baseband, the sampling period T_s is applicable to the baseband channel being detected. Note that the entire down-converted baseband channel is sampled at a single sampling rate. This is different from a type of multi-channel wideband approach where the wideband is not downconverted but first divided into individual channels. Each channel would then need to be sampled with a different oscillator and detection applied to individual signal bandwidths of interest which is not cost effective because channelization occurs before the sampling. Also, the use of a window in the FFT computation within the slots reduces spectral leakage as we sample through the baseband channel, see section 2.6.1. As was described in section 2.5.1 and adopted in [81, 140], we assume the received signal s(t) is down-converted and sampled with sampling period T_s giving a complex-valued discrete time signal as,

$$s(n) = \sum_{u=1}^{U} x_u(n)$$
 (4.2)

and

$$x_{u}(n) = \left\{ \sum_{l=-\infty}^{\infty} \alpha_{u}(lT_{u}) p_{u}(nT_{s} - lT_{u}) \right\} e^{j2\pi f_{cu}nT_{s}} + w_{u}(n)$$
(4.3)

where T_u , $\alpha_u(lT_u)$ and $p_u(nT_s)$ are the symbol period, transmitted information symbols and the pulse shaping filter of the u^{th} transmitted signal respectively while $w_u[n]$ is the complex AWGN in the band occupied by the u^{th} transmitted signal. Note that the transmitted information symbols with unit average power is σ_a^2 ; a pulse shape filter is $p_u(nT_s)$ of unit energy; Signal to Noise Ratio (SNR) is given by $\sigma_a^2/\sigma_w^2 = 1/\sigma_w^2$ where σ_w^2 is the noise variance in the channel occupied by the PU signal s_u . Applying a windowed Short-Time Fourier Transform (STFT) to the computation of the complex envelope in (3.18) and as the transform of length L slides along the signal, it produces a number of down-converted spectral components with approximate bandwidth 1/L given as,

$$X(L,f) = \sum_{n=-\infty}^{\infty} x(n) w(n-L) e^{-j2\pi f n}$$
(4.4)

where w(n) is a data tapering window of length *L* of the narrow-band spectral component of the received discrete-time signal x(n) at the continuous frequency *f*, see section 2.5.4 on STFT. Notice that time is discrete *n* while the frequency is continuous *f* which makes it difficult to be computed. Therefore for the purpose of computation, we will use the measurement of the spectral correlation which is given by the limiting average of the cyclic periodogram discussed in [103, 141] and defined by,

$$\bar{I}^{\alpha}(n,k) = \frac{1}{L} X_L\left(n,k+\frac{\alpha}{2}\right) X_L^*\left(n,k-\frac{\alpha}{2}\right)$$
(4.5)

where $X_L(n, k)$ is the L-point window-based FFT around the n^{th} sample for integer cyclic frequency α . It allows fine control over the spectral resolution of the SCF estimate through the choice of window.

When the cyclic periodogram is applied in a slot of size N and as the amount of processed data increases in the slot in a sensing time T_{acq} , the average estimate of the non-conjugate SCF based on the received samples for the p^{th} slot from (4.5) becomes,

$$S_x^{\alpha}(k,p) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p}\left(n,k+\frac{\alpha}{2}\right) X_{L,p}^*\left(n,k-\frac{\alpha}{2}\right)$$
(4.6)

Note that we assume that α is the integer multiples of 1/L, therefore the multiple cyclic periodograms given in (4.6) will converge without additional phase compensation, see [141]. Note that AWGN is a widesense stationary (WSS) process and exhibits no cyclic correlation and has no spectral features at $\alpha \neq 0$, see [103]. When we consider a multi-slot wideband channel and as the number of slots *P* of slot size *N* increases as the sensing time T_{acq} approaches the maximum value, the estimate of the non-conjugate SCF for the wideband channel from (4.6) becomes,

$$\bar{S}_{x}^{\alpha}(k) = \frac{1}{P} \sum_{p=1}^{P} \left(\frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p} \left(n, k + \frac{\alpha}{2} \right) X_{L,p}^{*} \left(n, k - \frac{\alpha}{2} \right) \right)$$
$$= \frac{1}{P} \sum_{p=1}^{P} S_{x}^{\alpha}(k, p)$$
$$\mathbb{R} \left\{ \bar{S}_{x}^{\alpha}(k) \right\} = \Re \left\{ \frac{1}{P} \sum_{p=1}^{P} \left(\frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p} \left(n, k + \frac{\alpha}{2} \right) X_{L,p}^{*} \left(n, k - \frac{\alpha}{2} \right) \right) \right\}$$
(4.7)

where $\mathbb{R}{\{S_x^{\alpha}(k)\}}$ is the non-conjugate multi-slot test statistic, denoted as TS₁. Note that $\mathbb{R}{.}$ denotes the real part of a complex expression or number. As the sensing time T_{acq} is finite, P also has to be finite. It follows that the conjugate multi-slot test statistic denoted as TS₂ is given as,

$$\mathbb{R}\left\{\bar{S}_{x^{*}}^{\alpha}(k)\right\} = \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{L}X_{L,p}\left(n,k+\frac{\alpha}{2}\right) \times X_{L,p}\left(n,\frac{\alpha}{2}-k\right)\right)\right\}$$
(4.8)

and indicated by the absence of conjugation (*) in the right side of the equation as discussed in [95, 141]. The multi-slot non-conjugate Test Statistic TS_1 is used to detect features at the carrier frequency as in Table 3.1 while the multislot conjugate Test Statistic TS_2 is used to detect the features at a signal's symbol rate 1/T as discussed in [95]. These test statistics in the frequency domain will enable us to make analysis towards the reduction of computational complexity by considering the sizes and numbers of FFTs and slots. This will eventually lead to a reduced number of samples necessary for hardware implementation of cyclostationary feature detection. The test statistics will make it possible to consider the trade-off between frequency resolution and the number of samples for computation by using FFTs for analysis. In addition it leads to the avoidance of the use of large FFTs which may result in processor bleeding. To the best of our knowledge, these have not been similar considerations of the analysis and implementation of the cyclostationary feature detection involving the combinations FFT and slot sizes or the application of this combination in the reduction of computational complexity.

Equations (4.7) and (4.8) show that the TS is the correlation of the FFT of the received wideband signal with itself as discussed in [26,105,137,142–144]

and in this research the correlation is done in the down-converted baseband channel on slot by slot basis. The Test Statistic results in a two-dimensional (2-D) plot with cyclic frequency α and carrier frequency f of the SCF which can also can be presented in a redone-dimensional (1-D) plot of SCF versus the number of samples for the purpose of analysis. Recall that the objective is to implement an efficient, low complexity robust wideband CFD. The introduction of slots in effect is similar to reducing the total number of samples from N_B to N and the applicable FFTs will correlate between the separate small spectral components across a smaller number of samples for the slot. This will enhance the probability of detecting across small spectral components instead of detecting across large spectral components applicable without slots. However, the frequency resolution of the FFT given as f_s/L should also be considered in the analysis for more efficient detection.

Note that the discrete form of the continuous time conventional SCF (3.17) is a form of the proposed model Test Statistics in (4.7) and (4.8) where *P* in (4.7) = 1 and *N* in (4.7) = N' in (3.17) and given as

$$\hat{S}_{x}^{\alpha}(k) = \frac{1}{N'} \sum_{n=0}^{N'-1} \frac{1}{L} X_{L}\left(n, k + \frac{\alpha}{2}\right) X_{L}^{*}\left(n, k - \frac{\alpha}{2}\right)
\mathbb{R}\left\{\hat{S}_{x}^{\alpha}(k)\right\} = \mathbb{R}\left\{\frac{1}{N'} \sum_{n=0}^{N'-1} \frac{1}{L} X_{L}\left(n, k + \frac{\alpha}{2}\right) X_{L}^{*}\left(n, k - \frac{\alpha}{2}\right)\right\}$$
(4.9)

where $\mathbb{R}\{\dot{S}_x^{\alpha}(k)\}\$ is the discrete conventional SCF test statistic and N' is the total samples available for the SCF without slots.

4.1.1 Threshold and Detection

Fundamentally, Additive White Gaussian Noise (AWGN) is a wide-sense stationary process and has no cyclic correlation of any order [103]. The Spectral Correlation Function (SCF) of noise asymptotically has no spectral features at $\alpha \neq 0$. The SCF is expected to be flat in the presence of AWGN. From the Central limit theorem (CLT) as in [104], [105], [145], [146] and [90], the SCF $S_x^{\alpha}(f)$ distribution is Gaussian and for the total samples \mathcal{N} is given as,

• $S_x^{\alpha}(f) \sim \mathcal{N}(\mu_0, \sigma_0^2)$ under null hypothesis H_0 ;

• $S_x^{\alpha}(f) \sim \mathcal{N}(\mu_1, \sigma_1^2)$ under alternative hypothesis H_1 ;

where $\mu_{(.)}$ and $\sigma_{(.)}^2$ represent the mean and variance of the SCF at H_0 and H_1 respectively. For the two hypotheses H_0 and H_1 , they are defined to correspond to the following,

$$H_0: s(t) = \eta(t), \qquad \text{for noise only} \tag{4.10}$$

and

$$H_1: s(t) = x(t) + \eta(t)$$
, for presence of signal and noise (4.11)

where $\eta(t)$ is Additive White Gaussian Noise (AWGN) and x(t) is the received signal without noise. A binary decision rule of the two hypotheses H_1 and H_0 will be adopted in order to detect the absence or presence of the signals after the computation of the spectral correlation of the process. The second moment or power of the non-conjugate Test Statistic can be expressed from (4.7) as,

$$\mathbb{E}\left[|TS_1|^2\right] = \mathbb{E}\left[\mathbb{R}\left\{\left|\bar{S}_x^{\alpha}(n,k)\right|^2\right\}\right]$$
(4.12)

We will also investigate the detection performance of the test statistic in a Ricean channel environment. Considering the non-conjugate Test Statistic TS_1 and given that the SCF has non-zero mean at H_0 , the TS_1 will have Ricean distribution over an AWGN channel. Let λ be the selected threshold that is approximately at the level of the test statistic magnitude at noise only condition H_0 and gives a Constant False Alarm Rate (CFAR) during the detection. Since a cyclostationary feature detector (CFD) collects the energy of the received signal at a given cyclic frequency α corresponding to the modulation type, the power of the cyclic feature determines the detection performance of the detector. The energy of the received signal is concentrated at α . Given that the second moment of the generalized SCF corresponds to the power of the cyclic feature, the analysis of the detection performance in a Ricean channel will be done using the non-conjugate second moment in (4.12).

It is mathematically intractable to the best of the author's knowledge, to find the threshold λ for the Probability of False Alarm P_{fa} and Probability of

Detection P_d for Cyclostationary feature detector due to the limited knowledge about the received signal as discussed in [3], Monte Carlo simulations will be used to evaluate the performance and identify the optimal threshold that gives Constant False Alarm Rate (CFAR) as was also adopted with other Test Statistics in [3], [143], [104], [106], [147]. Since CFD is robust to noise, it makes sense to select the threshold level that is approximately at the level of the test statistic magnitude at noise only condition H_0 . The non-conjugate Test Statistic will then be compared against the detection threshold to determine the probability of false alarm and probability of detection of the signal during the 10³ Monte Carlo iterations as used in [144] to give,

$$P_{\text{fa}} = \operatorname{Prob} \{TS_1 > \lambda \mid H_0\}, \text{ for when the signal is absent, } H_0;$$
 (4.13)

and

$$P_{d} = Prob\{TS_{1} > \lambda \mid H_{1}\}, \text{ for when the signal is present}, H_{1}.$$
 (4.14)

4.1.1.1 Detection Algorithm

The detection algorithm can be summarized as follow. Note that this algorithm is able to quit at any time a signal is detected. If multiple bands are being detected then the algorithm can continue until all bands have been searched.

Algorithm 1 Algorithm for the Detection Process

- 1: Inputs: FFT Size *L*, slot size *N*, slot number *P*, Test statistic TS_1 and Threshold λ .
- 2: Output: |S| = Magnitude of TS_1 simulation
- 3: Select *N*, *P*, *L*,
- 4: **for** p = 0 to *P*-1 **do**
- 5: Run simulation with TS_1 , N, P and L.
- 6: **if** $|S| \ge \lambda$ **then**

- 7: Signal is present.
- 8: Therefore exit the algorithm.
- 9: **else**
- 10: signal is absent.
- 11: end if
- 12: end for

4.1.2 The Means and Variances of the Test Statistic TS_1 at both H_0 and H_1

As mentioned in section 4.1.1 in order to investigate the detection performance in a Ricean channel, the mean and variance at both H_0 and H_1 will be established prior to the calculation of the probability of false alarm P_{fa} and the probability of detection P_{d} . The mean at H_1 is expressed as,

$$\mu_1 = \mathbb{E}[\bar{S}_x^{\alpha}(k)]. \tag{4.15}$$

The test statistic at H_0 can be expressed from (4.7) as,

$$\mathbb{R}\left\{\bar{S}^{\alpha}_{\eta}(k)\right\} = \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{L}X_{\eta,L,p}\left(n,k+\frac{\alpha}{2}\right) \times X^{*}_{\eta,L,p}\left(n,k-\frac{\alpha}{2}\right)\right)\right\}$$
(4.16)

where η is the notation for noise. It then follows that the mean at H_0 and H_1 from (4.16) and (4.7) are given as,

$$\mu_0 = \mathbb{R}\left\{\bar{S}^{\alpha}_{\eta}(k)\right\}$$
(4.17)

and

$$\mu_1 = \mathbb{R}\left\{\bar{S}_x^{\alpha}(k)\right\} \tag{4.18}$$

respectively. Similarly, the mean for signal only (not H_1) μ_s can be derived from (4.17) and (4.18) as,

$$\mu_s = \mu_1 - \mu_0. \tag{4.19}$$

The second order moment of the non-conjugate SCF for the multi-slot test statistic at H_1 can be expressed from (4.6) as,

$$\mathbb{E}\left[\mathbb{R}\left\{\left\{S_{x}^{\alpha}(k,p)\right\}^{2}\right\}\right] = \mathbb{E}\left[\mathbb{R}\left\{\left\{\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{L}X_{L,p}\left(n,k+\frac{\alpha}{2}\right)X_{L,p}^{*}\left(n,k-\frac{\alpha}{2}\right)\right\}^{2}\right\}\right].$$
 (4.20)

Following the expression of variance as discussed in [148, 149] for different test statistics, the variance at H_1 can be given from (4.20) and (4.7) as,

$$\sigma_1^2 = \mathbb{E}\left[\mathbb{R}\left\{\left\{S_x^{\alpha}(k,p)\right\}^2\right\}\right] - \left\{\mathbb{E}\left[\mathbb{R}\left\{\bar{S}_x^{\alpha}(k)\right\}\right]\right\}^2.$$
(4.21)

Similarly, from (4.20) and (4.21), the second order moment and variance at H_0 can be expressed as,

$$\mathbb{E}\left[\mathbb{R}\left\{\left\{S_{\eta}^{\alpha}(k,p)\right\}^{2}\right\}\right] = \frac{1}{P}\sum_{p=1}^{P}\mathbb{R}\left\{\left\{S_{\eta}^{\alpha}(k,p)\right\}^{2}\right\}$$
(4.22)

and

$$\sigma_0^2 = \mathbb{E}\left[\mathbb{R}\left\{\left\{S_\eta^{\alpha}(k,p)\right\}^2\right\}\right] - \left\{\mathbb{E}\left[\mathbb{R}\left\{\bar{S}_\eta^{\alpha}(k)\right\}\right]\right\}^2 \tag{4.23}$$

respectively. The variance at signal only σ_s^2 can be derived from (4.21) and (4.23) as,

$$\sigma_s^2 = \sigma_1^2 - \sigma_0^2.$$
 (4.24)

4.1.3 Probability of Detection and Probability of False Alarm

As stated in section 4.1.1 since the SCF has non-zero mean at H_0 , the TS₁ will have Ricean distribution over an AWGN channel as was used in [81]. The Ricean parameters can be estimated using moments and variances of the variables as was discussed in [149–152] for different test statistics. Therefore, we will use the moments and variances for the test statistic already derived in the previous section 4.1.2. We will use these parameters to express both the probability of false alarm and probability of detection with the generalized Marcum Q function as was discussed in [145, 150, 153, 154] for different test statistics. Therefore, the probability of false alarm $P_{\rm fa}$ at the detection threshold λ can be expressed as,

$$P_{\text{fa}} = \operatorname{Prob} \{TS_1 > \lambda \mid H_0\} = \mathbb{Q}\left(\frac{\mu_0}{\sigma_0}, \frac{\lambda}{\sigma_0}\right), \qquad (4.25)$$

where $\mathbb{Q}(.)$, σ_0 and μ_0 are the generalized Marcum-Q function, noise deviation and mean of the test statistic at noise only. (4.17). Under H_1 the Ricean parameters can be estimated as,

$$\mu_1 = \sqrt{\mu_s^2 + \mu_0^2} \tag{4.26}$$

and

$$\sigma_1^2 = \frac{\sigma_s^2}{2} + \frac{\sigma_0^2}{2}$$
(4.27)

where σ_s^2 and σ_0^2 have already been derived in (4.24) and (4.23). From where,

$$\sigma_1 = \sqrt{\frac{\sigma_s^2}{2} + \frac{\sigma_0^2}{2}}.$$
 (4.28)

Note that σ_1^2 is from the conventional Ricean parameter $2\sigma^2$. Therefore, the expression for the probability of detection can be given as,

$$P_{d} = Prob\{TS_{1} > \lambda \mid H_{1}\} = \mathbb{Q}\left(\frac{\mu_{1}}{\sigma_{1}}, \frac{\lambda}{\sigma_{1}}\right)$$
(4.29)

where μ_1 and σ_1 are the mean and deviation of TS₁ at signal plus noise H_1 as derived above.

4.1.4 Frequency and Time Resolutions trade-off

Cyclic frequency α is a function of symbol rate 1/T and signal frequency f while the cyclic frequency resolution Δ_{α} is determined by the windowed FFT size Land in a multi-slot wideband by the combination of slot size N and number of slots P. It should be mentioned that in this research, the FFTs are implemented as windowed FFTs, but for simplicity will be referred as FFT. Similarly, FFT frequency resolution Δ_f is a function of the FFT size L and sampling frequency f_s as discussed in [82] and can be expressed for an L-point FFT as,

$$\Delta_f = \frac{1}{T} = \frac{f_s}{L} = \frac{1}{LT_s}$$
(4.30)

where T_s , L, f_s and T are the sampling period, FFT length in samples, sampling rate and signal period respectively. Small FFTs require less points for computation and offer lower complexity according to the complexity formula given conventionally in [155,156] as $L \log L$. This takes less processing power and increases the temporal resolution Δ_t , but reduces the frequency resolution Δ_f (4.30) as discussed in section 2.6.1. This will result in better efficiency. However, the increase in time resolution reduces the frequency resolution Δ_f which is made better with larger FFTs (4.30) as discussed in section 2.6.1. There is obviously a trade-off between Δ_t and Δ_f which are needed for efficient and low cost spectrum sensing. The selection of larger *L* will mean to take more samples to arrive at the frequency domain result, which also means that more samples will be taken for a longer time, loosing temporal resolution. This makes it more important to find the trade-off between Δ_f and Δ_t by which an FFT size can be selected. The total number of FFTs required for one slot of size *N* can be seen to be,

$$M = \frac{N}{L}.$$
 (4.31)

Since a Cyclostationary Feature Detector (CFD) collects the energy of the received signal at a given cyclic frequency α according to the modulation type, the power of the cyclic feature determines the detection performance of the detector since the energy of the received signal is concentrated at α . Given that the second moment of the generalized SCF corresponds to the power of the cyclic feature, analysis of the detection performance will be done using the non-conjugate second moment in (4.12).

The issue of spectrum sensing in cognitive radio and the use of FFT for correlation has much practical significance and is the subject of the next section. One of the drawbacks of cyclostationary spectrum sensing is the use of a large number of samples for analysis. Among the objectives of this research is to statistically analyse a model that utilises fewer samples for correlation and detection of individual channels in a wideband scenario.

4.2 Signal to Noise Ratio of the Test Statistics

The SNR for the test statistic SNR_{TS} can be defined as the ratio of the test statistic power under the signal only condition at a given cyclic frequency α , to that under noise only H_o and expressed from (4.20) and (4.22) as,

$$SNR_{TS} = \frac{\mathbb{E}\left[\mathbb{R}\left\{\left\{S_{x}^{\alpha}(\mathbf{k}, \mathbf{p})\right\}^{2}\right\}\right]}{\mathbb{E}\left[\mathbb{R}\left\{\left\{S_{\eta}^{\alpha}(\mathbf{k}, \mathbf{p})\right\}^{2}\right\}\right]}.$$
(4.32)

For the SNR_{TS} , the discrete variable *x* represents the signal only (signal without noise) condition.

4.3 Results and Discussion

In this section, consideration was given to signals of linear modulation types such as QPSK and BPSK whose features were discussed in [1], [2], [3] and presented in this report in Table 3.1. Mathworks Matlab programming software application was used for simulations. This software has been extensively used in academic research, particularly for the fields of signal processing and communication. It is user-friendly and can be adapted for use in many real world communication systems scenarios. Matlab codes were used more extensively in analysing and testing the proposed models. The results obtained were quantified to demonstrate the implementation of the research objectives. These included figures, graphs and tables. The results were obtained analytically and through simulations using Monte Carlo iterations, [157] in order to determine the sizes of FFT and slot that will give efficient and low complexity detection. The simulations were based on the proposed non-conjugate multi-slot test statistic TS₁ in (4.7), (4.13), (4.14), discrete conventional SCF in (4.9), energy detection test statistic in (2.2), while the analytical results were based on second moment of TS_1 in (4.12), proposed non-conjugate multi-slot test statistic TS₁ in (4.7), probability of false alarm P_{fa} in (4.25) and probability of detection $P_{\rm d}$ in (4.29). The section provides some results showing the performance of TS₁ to correlate the spectral components of the waveforms in the wideband and to detect the frequencies using different slot sizes N and Fast Fourier transform (FFT) sizes L.

It should be mentioned that all the plots involving the probability of detection in this section asymptotically approach the maximum detection mark of 1. It follows that as a cyclostationary process, its mean and autocorrelation are asymptotically not zero, see [5,96,97] and section 3.1.

4.3.1 Verifications of the Effects of Slot Size *N* and FFT size *L* on Probability of Detection under noiseless conditions

Some of the simulation parameters for this subsection are listed in Table 4.1. In order to satisfy Nyquist criterion, sampling was done at $4 \times$ bandwidth (B) of the carrier frequency f_c of the analytical signal.

Variables	Specifications
Frequency f_c (MHz)	2.5, 5
Modulation type	QPSK
Channel Bandwidth B (MHz)	2.5, 5
Sampling rate fs	$4 \times B$
Slot size N (samples)	128, 256, 512
Number of slots P (samples)	8, 16
FFT size <i>L</i> (samples)	4, 8, 16, 32, 64, 128
Sensing time T _{acq}	2×10^{-4}
Total samples N _B	2048, 4096
SNR	-5dB

Table 4.1: Simulation Parameters with QPSK signal.

The effects of varying the slot size N and FFT size L were investigated by considering their probabilities of detection (4.14) in terms of number of samples (sample size) using 10^3 Monte Carlo iterations. The results are cumulatively displayed following the correlation and detection done on a per slot basis. For the purpose of correlation, the FFTs must be in the powers of 2. In Fig. 4.2, a 5 MHz QPSK baseband modulated signal was used with different FFT sizes of 4, 8, 16, 32, 64, 128 for slot size N of 512 samples and the number of slots P 8 at 0.2 milliseconds (ms) sensing time T_{acq} . This was sampled at Nyquist sampling rate f_s of 20 MHz for a total samples N_B of 4096. Note that N_B was rounded up to the nearest powers of 2. The results in Fig. 4.2 are cumulatively displayed following the correlation and detection done on a per slot basis.



Figure 4.2: Probability of detection simulation see (4.14)) for QPSK signal at -5dB SNR, with a range of different sizes of FFT, N = 512 samples, P = 8, $f_c = 5$ MHz, $f_s = 20$ MHz and total samples $N_B = 4096$.

slot basis. It is shown that 512 samples, the larger FFT sizes of 64 and 128 require more samples to asymptotically reach the maximum detection probability mark of 1 than the smaller ones. For instance FFT with sizes of 128 and 64 samples required more than 4000 samples compared against approximately 3000 for an FFT with size of 32 samples. This can be understood because a smaller non-overlapping FFT will have a greater number of opportunities to detect cyclostationary behaviour in comparison to a larger FFT in a slot. From this it can be noted that when detection in a wideband scenario is done on a time slot basis as in a multi-slot model rather than using just a single slot for the entire wideband channel, the process of detecting the signals earlier is enhanced. From this, it can be noted that the process of detecting a signal earlier is enhanced, in a wideband scenario when using a multiple slot model in comparison to a single slot applied to the entire wideband. Detection time is reduced as the whole wideband channel will not have to be correlated before applying the detection algorithm expressed in section 4.1.1.1. This is further shown in Figs. 4.3 and 4.4.

When N was reduced to 256 samples as shown in Fig. 4.3 for the same 2.5 MHz frequency baseband signal, the samples needed for each FFT to reach

the maximum detection probability mark were reduced, for instance, FFT of 64 samples then required approximately 2500 samples while FFT of 32 samples needed 2000 samples. Further reduction of *N* to 128 samples for a different 2.5



Figure 4.3: Probability of detection (simulation see (4.14)) for QPSK signal at -5dB SNR, with multi-FFTs, N = 256 samples, P = 16, f = 5 MHz, $f_s = 20$ MHz and total samples $N_B = 4096$.

MHz QPSK signal in Fig. 4.4 shows more reductions in the samples needed to asymptotically reach the maximum detection probability mark. Therefore, the change in carrier frequency value does not affect the observed response of the slot size. For instance FFT of 32 samples now requires approximately 1000 samples as against 2000 samples in Fig. 4.3. Analysing these results it can be stated that the smaller the slot size the less the samples required for correlation and detection. It should be pointed out that the same analysis can be applied to other linear digital modulation types such as Binary Phase Shift Keying (BPSK). Furthermore, the P_d can be compared across different slot sizes for the same P as shown in Fig. 4.5. It shows that the smaller slots get to the maximum detection mark with less number of samples than the larger slots. Therefore, it can be stated that the smaller the slot size, the less number of samples required for detection which can result in faster processing of the spectral components. It can be observed that the smaller the FFT size, the smaller the number of samples required for detection which can result in faster processing of the spectral components.



Figure 4.4: Probability of detection (simulation see (4.14)) for QPSK signal at -5dB SNR, with multi-FFTs, N = 128 samples, P = 16, f = 2.5 MHz, $f_s = 10$ MHz and total samples $N_B = 2048$.



Figure 4.5: Probability of detection (simulation see (4.14)) of 5MHz QPSK signal for different slot sizes using TS_1 , at -5dB noise, FFT size 16, P = 16 samples, f = 5 MHz and $f_s = 20$ MHz.

detection mark. These differences in the number of samples required become even more significant when we consider that a greater number of the samples will be needed for larger bandwidths. This will require more resources such as more processing power for the processors and result in more complexity as more FFTs will be involved. It will also cause more delays since a greater number of samples will mean more processing time and an increase in resource cost. Such a system may be less efficient and effective when compared with a system that maximizes the sample requirements.

Notice that Figs. 4.2, 4.3 and 4.4 show the capability of the FFTs to correlate the spectral components and detect the signal. However, they do not completely indicate the best option of which FFT size to be selected. Further analysis is required to consider the capability of each FFT in terms of frequency resolution which is important for more accurate detection and that is the focus of the next subsection.

4.3.2 Section Summary

From the figures above, it can be noted that the detection of the spectral components of the signal is quicker with the smaller FFTs reaching the peak detection with small number of samples. Therefore, the adoption of small FFTs in a 1 slot process feature detection entails correlating all the samples of the slot before applying the detection hypothesis comparison described in section 4.1.1.1. This may result in delay of the discovery of spectral holes in the spectrum and therefore less efficient. The multi-slot method offers quicker correlation and consequent detection and quicker discovery of spectral holes.

4.3.3 The Effect of FFT sizes on the Spectral Correlation with the Test Statistic for different bandwidths

The aim of this section is to gain insight into how the different lengths or sizes of the FFTs affect the magnitude of the spectral correlation density using the non-conjugate test statistic TS_1 or its second moment $|TS_1|^2$ during the correlation process. This is prior to the TS_1 magnitude being compared against the selected threshold λ as described in the detection algorithm in section 4.1.1.1. Some of the simulation parameters for this section are listed in Table 4.2. Since the Cyclostationary Feature Detection (CFD) collects the energy or power of the received signal at a specific cyclic frequency α , the power of the cyclic feature gives the detection performance of the CFD. Therefore, the second moment of the non-conjugate test statistic $|TS_1|^2$ (4.12) corresponds to the power of the cyclic feature. Note that in this section, the results of Figs. 4.6 to 4.11 are computed and displayed on a slot by slot basis without the accumulation of the slots. These show the spectral correlation densities of the spectral components of the signal on a per slot basis for an FFT size depending on the signal distribution. This is a process prior to the application of the correlation magnitude to the detection algorithm described in section 4.1.1.

Variables	Specifications
Frequency f_c (MHz)	5, 10, 20
Modulation type	QPSK
Bandwidth B (MHz)	5, 10, 20
Sampling rate f_s	$4 \times B$
Slot size N (samples)	256, 512
Slot number P (samples)	8, 16, 32
FFT size <i>L</i> (samples)	4, 8, 16, 32, 64
Sensing time T_{acq} (seconds)	1×10^{-4}

Table 4.2: Simulation Parameters for different Bandwidths.

We will use the Hanning window which produces a finite data set to reduce spectral leakage inherent with the fast Fourier Transform (FFT) especially when the signal is not completely periodic as discussed in section 2.6 and presented in Appendix (G), see [86,87,89]. Let us examine the effect of the fast Fourier Transform (FFT) size *L* based on the Hanning window for different signal bandwidths using the second moment of the non-conjugate Test statistic $|TS_1|^2$ (4.12). Given that the received signal will be down-converted to baseband of bandwidth *B* MHz, the maximum signal bandwidth is the same as the highest signal frequency. The sampling rate f_s is $4 \times B$ for a sensing time T_{acq} of 0.1 ms. For a 5 MHz QPSK signal in Fig. 4.6, the use of different sizes of FFT produced Spectral Correlation Density (SCD) peaks or envelopes specific to that FFT. These peaks represent the concentration or spread of the cyclic feature power of the received signal. It shows that the smaller the FFT, the higher the correlation peaks. This is expected since the smaller the FFT length (size) the more the number of FFTs that will be contained in any slot according to (4.31). Also, since the process of spectral correlation involves the correlation of the FFTs, then having more available FFTs for correlation results in higher spectral correlation density (SCD). Fig. 4.6 shows that the smaller the FFT size the less the number of slots required to have high spectral correlation density or form the correlation peaks or envelope. The correlation peaks shown in all the plots for the $|TS_1|^2$ represent the expected spectral peaks due to the QPSK modulated signal similar to the peaks described in Table 3.1. When the



Figure 4.6: Effects of FFT size vs Number of slots (analytic, see (4.12)) with N = 256 and P = 8, f = 5 MHz, $f_s = 20$ MHz and $N_B = 2048$ samples.

bandwidth is increased to 10 MHz together with the slot number *P* as shown in Fig. 4.7, the magnitude of the Test Statistic TS_1 increased significantly for some FFTs. This is due to the increased sample size N_B (4096) for the wideband (4.1) channel. For instance, for FFT of 16 samples, it increased from 35×10^1 in Fig. 4.6(c) to 14×10^2 in Fig. 4.7(c). Apparently, it can be observed that because of the high peaks obtained with small FFTs, correlation based on them will cover more small spectral components than the larger FFTs. In Fig. 4.8, the bandwidth *B* is expanded to 20 MHz with 8192 samples for *N* = 512 samples. The FFTs still show similar behaviour with increased magnitude due to increased number of FFTs per slot. For the same *N* and with 10



Figure 4.7: Effects of FFT size vs Number of slots (analytic, see (4.12)), f = 10 MHz, N = 256, P = 16, $f_s = 40$ MHz and $N_B = 4096$ samples.



Figure 4.8: Effects of FFT size vs Number of slots (analytic, see (4.12)), f = 20 MHz, N = 512, P = 16, $f_s = 80$ MHz and $N_B = 8192$ samples.

MHz signal, when the slot number *P* is reduced to 8 with 4096 samples as shown in Fig. 4.9, the FFTs show reduced magnitude compared with Fig. 4.8 due to the reduction in *P* and consequent total samples. It can be stated that across different bandwidths, each FFT shows this similar performance where the smaller FFTs produced higher magnitude of the test statistic TS_1 in (4.7). The test statistic magnitude increased as the bandwidth was increased due to the fact that a greater number of samples are usually associated with higher bandwidths. It should be mentioned that the same results will be obtained using the conjugate second moment of the unconstrained Test Statistic TS_2 in (4.8).


Figure 4.9: Effects of FFT size vs Number of slots (analytic, see (4.12)), f = 10 MHz, N = 512, P = 8, $f_s = 40$ MHz and $N_B = 4096$ samples.

The slot size *N* can further be reduced to ascertain specifically the effects on the non-conjugate unconstrained magnitude of the Test Statistic $|TS_1|^2$. Consider a 2.5 MHz frequency QPSK signal in a 2.5 MHz bandwidth (1024 samples) at $f_s = 4 \times B$, sensing time of 0.1 ms and *N* reduced to 128 samples as in Fig. 4.10. It shows significant reductions in the magnitude of $|TS_2|^2$ for each FFT compared with previous figures. The reduction in magnitude is con-



Figure 4.10: Effects of FFT size vs Number of slots (analytic, see (4.12)) with f = 2.5 MHz, N = 128, P = 8, $f_s = 10$ MHz and $N_B = 1024$ samples.

tinued in Fig. 4.11 when N is further reduced to 64 samples for the same signal parameters as for Fig. 4.10. As expected, the reduction is due to the reduced number of FFTs in each slot (4.31) and given that spectral correlation involves

the correlation of the FFTs within themselves in the slots.

However, this decreasing magnitude of the spectrally correlated peaks may affect the detection of the signal when compared against the selected reference threshold λ . On the other hand, with the increased bandwidth (4096 samples) in Fig. 4.7 and compared with Fig. 4.8 in previous section 4.3.3 for the same 16 slot numbers, the result is increased magnitude of the spectral peaks due to the increased number of FFTs following the increase in *N* to 512 in Fig. 4.8 as against 256 samples in Fig. 4.7.



Figure 4.11: Effects of FFT size vs Number of slots (analytic, see (4.12)) with f = 1.25 MHz, N = 64, P = 8, $f_s = 10$ MHz and $N_B = 512$ samples.

4.3.4 Section Summary

It can be concluded that the FFT size and number of FFTs in a slot, L and M respectively determine the magnitude of the peaks and also that the combination of small slot and FFT sizes produces more correlation peaks. This confirms that the smaller FFTs are able to correlate small narrow-band spectral components with fewer number of samples than the larger FFTs.

4.3.5 Relationship between bandwidth, slot size *N* and number *P*

This section is concerned with investigating the relationship between bandwidth, slot size and number of slots. In what follows, highest frequency or bandwidth of a baseband signal is *B* MHz, sampling rate is given by $f_s = 4 \times B$ MHz and the sensing time is 0.1 ms for the wideband. Figs. 4.12(a) and 4.12(b) show the probability of detection for two different bandwidths *B* of 10 MHz (4096 samples) and 20 MHz (8192 samples) respectively with the same slot size. Note that the probability of detection asymptotically approaches 1. Also, the variation of bandwidth is expressed by the total number of samples N_B which in turn is a function of frequency, sampling frequency and sensing time T_{acq} . In both Figs. 4.12(a) and 4.12(b), given a fixed slot size *N* of 256



Figure 4.12: Probability of detection vs Number of slots (simulation, see(4.14)) for 10/20 MHz bandwidths at N = 256, $N_B = 4096/8192$ samples.

samples, the expected ideal performance for an individual FFT is to have the maximum detection probability with a small number of slots or samples in order to achieve the objective of a low-complexity and effective detection model. FFT sizes 32 and 64 require more slot numbers P than FFT sizes 8 and 16 to reach the maximum detection probability. As expected, increasing the bandwidth results in a greater number of samples in order to cover the entire bandwidth and for fixed N, more slots will be created and required for correlation as expressed

in (4.1) and (4.31). FFT size 64 takes 5 slots in larger bandwidth in Fig. 4.12(b) as against 3 in F.g 4.12(a).

Similar results can be seen at larger signal bandwidth of 20 MHz as in Fig. 4.13. In Fig. 4.13(a) it shows that 2 slots of 512 samples are required for FFT sizes of 8, 16, 32 samples while an FFT of size 64 samples requires 3 slots for maximum detection. However, this is different in Fig. 4.13(b) when N is reduced to 256 samples with more slots giving lower P_d per slot as supported in (4.1) and (4.31). Although more slots are required in Fig. 4.13 (b), representing



Figure 4.13: Probability of detection vs Number of slots (simulation, see(4.14)) for 20 MHz bandwidth at N = 512/256 samples, $N_B = 8192$ samples.

the results of Fig. 4.13 in terms of total samples shows that the reduction of N from 512 to 256 samples still gave fewer total samples for peak detection.

4.3.6 Section Summary

It follows that for the same bandwidth, the smaller the N, the more the slots created, see (4.1). Consequently, the more the number of slots required for detection while the effective total samples for peak detection will be reduced as in Fig. 4.13(b). Therefore using smaller N reveals the effective number of samples required for peak detection and therefore reduces redundant or excessive samples resulting with the use of larger N.

4.3.7 Correlation Performance of the Test Statistic TS₁ under low SNR values

The performance of the test statistic TS_1 in terms of its robustness can be investigated by using a QPSK modulated signal at different values of Signalto-Noise Ratio (SNR). Fig. 4.14 is for signal only and will be compared against Figs. 4.15 and 4.16 which have the presence of noise. Fig. 4.15 is for a QPSK



Figure 4.14: Magnitude of Test Statistic TS_1 vs Number of slots (analytic, see(4.12)) for QPSK signal only with f = 5 MHz, N = 128, P = 16, $f_s = 20$ MHz.

signal at -3 dB SNR. As expected noise raises the levels of the peaks as it usually spreads at the signal base. This raise is more evident in Fig. 4.16 at -5 dB SNR where the levels show more significant increases.



Figure 4.15: Magnitude of Test Statistic TS_1 vs Number of slots (analytic, see(4.12)) for QPSK signal plus noise with f = 5 MHz, N = 128, P = 16, $f_s = 20$ MHz, SNR = -3dB.



Figure 4.16: Magnitude of Test Statistic TS_1 vs Number of slots (analytic, see(4.12)) for QPSK signal plus noise with f = 5 MHz, N = 128, P = 16, $f_s = 20$ MHz, SNR = -5dB.

4.3.8 Robustness of Test Statistic to noise

The performance and robustness to noise can also be verified in terms of the Receiver Operating Characteristic (ROC) as shown in Fig. 4.17. For the ROC curves, the threshold level is kept constant, while the probability of detection P_d (4.29) is performed for each value of the probability of false alarm P_{fa} (4.25).

It shows good performance up to -26 dB with FFT length 16. Note that similar results will be obtained with other smaller FFTs. The robustness to

noise of the Test Statistic can be further verified by expressing Fig. 4.17 in terms of the number of total samples as shown in Fig. 4.18. This is able to achieve lower SNR values gith small slot and FFT sizes than other works such as discussed in [81]. It is shown in Fig. 4.18 that the lower the SNR values



Figure 4.17: Receiver Operating Characteristics (analytic, see (4.29, 4.25)) under low SNR values with FFT size 8, f = 5 MHz, N = 128 samples, P = 16, $f_s = 20$ MHz and $N_B = 2048$ samples.

of the signal, the more samples are required to reach peak detection which is expected. Similar results were obtained with other total number of samples. It should be mentioned that detection of such low level signals is not possible with some spectrum sensing methods such as Energy Detection.

4.3.9 Comparison with Energy Detection and Conventional Spectral Correlation Density Function

In Fig. 4.19 simulations were carrierd out to compare the detection performance of the model test statistic TS_1 (4.7) against the Energy Detection (ED) test statistic (2.1) as given in [44, 158]. The detection magnitude due to TS_1 is significantly higher than that the ED statistic at low values of SNR. Note that similar results will be obtained with other FFT lengths such as 16 and 64. It shows the robustness to noise of the proposed test statistic more than the Energy Detection which is one advantage of cyclocstationary feature detection



Figure 4.18: Detection Performance vs Total Number of samples (simulation, see (4.14) under low SNR values with FFT size 8, f = 5 MHz, N = 128, P = 16, $f_s = 20$ MHz.





Figure 4.19: Comparison between the CFD Test Statistic TS_1 and Energy Detection (ED) (simulation, see (2.2, 4.7)) under signals for low SNR values with f = 5 MHz, N = 128 samples, FFT = 32 samples, P = 16, $f_s = 20$ MHz.

statistic TS_1 (4.7) and the discrete conventional spectral correlation function without slots (4.9) and FFT size of 256 samples. Note that the conventional discrete SCF is simulated on the basis of the maximum-likelihood estimates over the total available samples for the wideband N', see 4.9). The multi-slot test statistic TS_1 with multi-FFT sizes as shown in Fig. 4.20 shows higher probability of detection at lower SNR values which is significant with smaller FFT sizes such as 4, 8 and 16. Other combinations of FFT and slot size will produce similar results.



Figure 4.20: Comparison between the discrete conventional SCF and model non-conjugate test statistic (simulation, see (4.9, 4.7)) TS_1 for multi-SNR values with 5 MHz QPSK signal using TS_1 , N = 256, P = 16, f_s = 20 MHz and FFT szes 4-64.

4.3.10 Detection performance in terms of real signal features

In this section, the detection performance will be verified in terms of the locations of the signal frequency f and cyclic frequency α using the test statistic TS₁ in 3-dimensional plots. In Fig. 4.21, the two extreme right and left peaks or envelopes show the accurate locations of the real-valued signal frequency f at \pm 10 MHz with the fundamental at 0 Hz. Note that BPSK signal has the cyclic frequency α as twice the carrier frequency as given in Table 3.1. For clarity, this can be shown from aerial view in a contour plot as in Fig. 4.22. Here, the locations of both f and α are more clearly shown. Similar results can be obtained using other FFTs. This is expanded to include two BPSK signals as in Fig. 4.23. The locations of both the signal and cyclic frequencies are shown for both BPSK signals. Note that this can also be expanded to include



Figure 4.21: Detection in terms of real signal features (simulation, see (4.7)) with FFT size 16, f = 10 MHz, N = 256 samples, P = 16, $f_s = 40$ MHz and $N_B = 4096$ samples.



Figure 4.22: Contour plot of Fig. 20 (simulation, see (4.7)) with FFT size 16, f = 10 MHz, $\alpha = 2f$, N = 256, P = 16, $f_s = 40$ MHz.

more signals in the wideband channel as shown in Fig. 4.24. As previously mentioned, BPSK signals have their cyclic frequencies α as twice the carrier frequency as given in Table 3.1.



Figure 4.23: Contour plot (simulation, see (4.7)) for two BPSK signals with FFT size 16, f = 10,5 MHz, $\alpha = 2f$, N = 256, P = 16, $f_s = 40$ MHz.



Figure 4.24: Contour plot (simulation, see (4.7)) for BPSK signals with FFT size 32, f = 10 MHz, $\alpha = 2f$, N = 256, P = 16, $f_s = 40$ MHz.

4.4 Chapter four Summary

In this chapter the wideband multi-slot window-based Fast Fourier Transform Cyclostationary Feature Detection model was analysed and implemented in Matlab software. Descriptions of the Test Statistics were given for both conjugate and non-conjugate processes. Considerations were also given to the two conditions of signal only and signal with noise. The trade-off between frequency and time resolutions were considered in the investigations. As a statistical model, the probabilities of detection and false alarm were implemented analytically and simulations and both showed good performances. From the results, the effects of the size of the fast Fourier transform and the number slots on the second moment of the Test Statistic were investigated for different values of signal-to-noise ratios and bandwidths.

The detection performance of the model in terms of real value signal features such as signal frequency and cyclic frequency with specific location detection showed that the model can be applied to real communications settings. It has also been shown that smaller fast Fourier transforms give better detection using reduced sample size than the larger ones. Also, the smaller slot size is of advantage depending on the fast Fourier transform size adopted. Additionally, the smaller slot size will enable correlation of the spectral components at shorter time and eventual shorter detection time which will compensate in terms of temporal resolution. This is in contrast with correlating the entire wideband channel once for a longer time prior to the detection algorithm which is the case for a 1 slot model. Detection of low level signals were shown with the simulations which is expected of a cyclostationary feature detector. Also, the simulations show that the model can be adopted in both narrowband and wideband environments.

Chapter 5

Wideband Cyclostationary Feature Detector under Receiver Constraints

Cyclostationary feature detectors (CFDs) require the knowledge of the signal's carrier frequency and symbol rate for spectral correlation. It has been discussed in [159] that under perfect knowledge of the signal's symbol rate and carrier frequency, CFDs can theoretically suppress noise at all SNRs with increasing sensing time or samples by averaging out the stationary noise. Therefore, cyclostationary detectors are more reliable detectors at low SNRs since they do not suffer from the known SNR wall phenomenon. However, there are some factors which may constrain the knowledge of a signal's cyclic features such as the cyclic frequency offset (CFO) Δ_{α} and sampling clock offset (SCO) δ . Cyclic Frequency offsets may occur due to the mismatch of local oscillator, Doppler shifts and insufficient knowledge of the actual carrier frequency of the signal or channel as studied in [57]. In this condition, the radio receiver is said to have a non-zero CFO which is a concern. The presence of non-zero CFO will impede the CFD detection performance resulting in the costly option of a large number of samples for detection. This will affect the detection performance more in low SNR scenarios requiring a large number of samples in order to suppress the noise as studied in [160], [161] and [162]. It was also discussed in [58] that under a non-zero CFO, the cyclic frequency used for detection decays with increased sensing time and even at low SNRs. This behaviour is analogous in effect to the SNR wall issue significant in energy detection-based methods [159]. Therefore, reliable signal detection becomes difficult to achieve under CFOs below a certain SNR threshold and consequently, cyclostationary feature detectors lose their advantages over energy-based detectors.

However, none of these works proposed a solution to overcome the performance degradation due to these constraints with cyclostationary detection based methods. Further work in [81] proposed a multi-frame Test Statistic using a cyclic autocorrelation function to reduce the constraints in the time domain otherwise involving the use of relatively large frames with a large number of samples and thus greater computational complexity. It should be mentioned that cyclic frequency offset is different from carrier frequency offset which is conventionally being investigated in other spectrum detection techniques in many literatures. The latter is the subject of this research. The cyclic frequency offset is applicable to cyclostationary feature detectors and not applicable to some detectors such as energy and matched filter detectors.

Another constraint is the Sampling Clock Offset (SCO) δ which occurs from the frequency offset produced by insufficient knowledge of the symbol rates 1/*T* at the Analogue to Digital (A/D) stage of the receiver as discussed in [114], [137], [163] where *T* is the symbol period. Every symbol period is generated by the transmitter clock and it is well known that a clock will have a certain accuracy or error expressed in parts per million (ppm). A digital modulated communication signal such as QPSK with *T* has a cyclic frequency of $\alpha = 1/T$. This shows that SCO affects the CFO in terms of the symbol rate. Therefore, the imperfect knowledge of the symbol rate produces an SCO which results in a drift in sampling times. This time-shift varies as the number of samples increases; producing phase shifts further which can affect the cyclic frequency offset. SCO can be formulated in terms of *T* as,

$$\delta = \frac{1}{T} - \frac{1}{T}(1+\epsilon) \tag{5.1}$$

where ϵ is the clock error in ppm.

The presence of SCO impacts the detection performance of cyclostationary feature detection as shown in [115] using spectral correlation density (SCD) and convolution processes; [116, 138, 164] used the spectral correlation function for a pilot-aided cyclostationary feature detection in orthogonal frequency division multiplexing (OFDM) where the phase offset due to SCO was estimated and compensated from one frame to the next. A blind solution approach to the SCO problem was adopted in [140] where the symbol rate of the incoming signal was estimated and sampled so that the resulting samples were interpolated at the correct rate. The drawbacks of these methods are: interpolation is costly in terms of power and the interpolation rate has to be modified for each signal of interest in the wideband, which is a computationally expensive solution. The effects of CFO can be seen in Fig. 5.1 where the magnitude of the Cyclic Autocorrelation Function (CAF) of the affected QPSK signal is very low compared with the CAF of the unaffected signal and reduces at an increasing number of samples.



Figure 5.1: Conventional Cyclic Autocorrelation Function with and without CFO (simulation, see (4.9)).

In Fig. 5.2 (b), the effect of CFO is shown as a shift on the location of the signal frequency (away from \pm 0.2) compared with the same signal in Fig. 5.2 (a) without CFO.



Figure 5.2: Conventional Spectral Correlation Function with and without CFO (simulation, see (4.9)).

Since the effects of both the CFO Δ_{α} and SCO δ get intensified as the number of samples are increased, our proposed multi-slot cyclostationary feature detection approach will remedy these constraints looking at relatively small number of samples in conjunction with window-based fast Fourier transform. More so, in a practical implementation, the SCF is estimated from the frequency domain processing of the received signal, by taking the average of the autocorrelation or cross correlation of the FFT coefficients. It will be seen that it can achieve significant gains over the conventional cyclic detection in the presence of CFOs and SCOs. The CFO and SCO with the model will now be analysed followed by simulations to verify the analysis.

5.1 System Model with Receiver Constraints

This model integrates the effects of both the SCO and CFO and how to reduce them utilizing the previous model described in section (4.1) where the nonconjugate and conjugate Test Statistics TS_1 and TS_2 without any constraints were derived and given as in (4.7) and (4.8). The received wideband signal is down-converted to baseband and sampled at Nyquist rate to produce complex samples. Let us assume that *U* Primary Users' (PUs) signals could occupy the wideband channel with time variable $t \in [0, T_{acq}]$ and be represented as $x_u(t) \forall u \in [1, ..., U]$. Therefore the received signal can be discretely expressed under signal plus noise H_1 condition and in the presence of SCO δ and CFO Δ_{α} as,

$$s[n|\delta,\Delta_{\alpha}] = \sum_{u=1}^{U} x_{u}[n|\delta,\Delta_{\alpha}]$$
(5.2)

where from (4.3)

$$x_{u}[n|\delta,\Delta_{\alpha}] = \sum_{l=-\infty}^{\infty} \alpha_{u}(lT_{u}) p_{u}(nT_{s}(1+\delta) - lT_{u}) e^{j2\pi f_{c_{u}}(1+\Delta_{\alpha})nT_{s}(1+\delta)} + \eta_{u}[n].$$
(5.3)

The noise only condition H_0 is given as,

$$x_u[n|\delta,\Delta_\alpha] = \eta_u[n] \tag{5.4}$$

where $\alpha_u(lT_u)$ and $p_u(nT_s(1+\delta))$ are the transmitted information symbols and the pulse shaping filter of the u^{th} transmitted signal respectively while $\eta_u[n]$ is the complex AWGN in the band occupied by the u^{th} transmitted signal. Let the transmitted information symbols with unit average power σ_a^2 , a pulse shape filter $p_u(nT_s(1+\delta))$ of unit energy, Signal to Noise Ratio (SNR) = $\sigma_a^2/\sigma_\eta^2 =$ $1/\sigma_\eta^2$ where σ_η^2 is the noise variance in the channel occupied by the PU signal $x_u(nT_s(1+\delta))$.

5.2 Effects of Sampling Clock Offset on the Test Statistic

Although the received signal may contain noise but since noise is not affected by sampling clock offset (SCO) due to its stationary statistical characteristic and eventual flat Spectral Correlation density, it may be ignored at this stage in the analysis. Therefore, we first consider the effect of SCO on the proposed model in noiseless conditions assuming a zero CFO. SCO can then be stated as,

$$\hat{T}_s = (1+\delta) \times T_s \tag{5.5}$$

where \hat{T}_s is the actual sampling period used at the receiver, T_s is the ideal sampling period with good knowledge of the symbol rate at the transmitter and

 δ is the SCO. In order to adequately represent a signal, the sampling rate is in multiples of the symbol rate 1/T, where *T* is the symbol period. The objective of the model is to remedy these effects so as to improve the possibility of detection through a statistical model using the SCF and FFT-based slot in a wideband scenario [27]. Once we select the sensing period for the wideband channel T_{acq} and the sampling frequency f_s or sampling period T_s , the total samples for the wideband channel is given as,

$$N_B = \frac{T_{\rm acq}}{\hat{T}_s} \tag{5.6}$$

where \hat{T}_s is the actual sampling period used at the receiver as given in (5.5) and substituting it in (5.6) we can express N_B as,

$$N_B = \frac{T_{\rm acq}}{T_s(1+\delta)}.$$
(5.7)

If $\delta = 0$, then N_B is unchanged and if $\delta \neq 0$, N_B is affected. The denominator $T(1+\delta)$ should also be an integer for us to have an integer number of samples. Otherwise, we have a non-integer N_B which is an indication of time offset or drift. It is well known that as the number of samples are increased, a sampling clock offset will result in progressive drift in the real positions of sample points or the number of samples (N_B) within a sensing time T_{acq} . From the time shifting property of the Fourier Transform (FT), this drift in time produces phase offset in the frequency domain and consequently affects the positions of the cyclic features such as the cyclic frequency α in the SCF. A time shifted received wideband signal x(t) can be represented as

$$\bar{x}(t) = x(t - t_s) \tag{5.8}$$

where t_s is the time shift in the time domain and varies as the number of samples increases. It can be expressed in terms of sampling clock offset δ and for all the time samples in the p^{th} slot of size N as,

$$t_p = N\delta p. \tag{5.9}$$

It results in a phase offset $e^{-j2\pi\alpha t_p}$ for the p^{th} slot and is accumulated over all the slots in the wideband channel. Therefore, SCF estimate for the p^{th} slot

that is affected by the SCO after substituting for t_p is given as,

$$\bar{S}_{x}^{\alpha}(k,p|\delta) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p}\left(n,k+\frac{\alpha}{2}\right) \times X_{L,p}^{*}\left(n,k-\frac{\alpha}{2}\right) e^{-j2\pi\alpha N\delta p}.$$
(5.10)

Consider a total P number of slots, the total phase offset is given as,

$$\phi = \sum_{p=1}^{P} e^{-j2\pi\alpha N\delta p}$$
(5.11)

where $e^{-j2\pi\alpha N\delta p}$ is the phase offset for the p^{th} slot as explained above. It can be shown as used in [137] that the analysis of ϕ in (5.11) is,

$$\sum_{p=1}^{P} e^{-j2\pi\alpha N\delta p} = \frac{\sin(\pi\alpha PN\delta)}{\sin(\pi\alpha N\delta)} e^{-j\pi\alpha N\delta(P-1)}.$$
(5.12)

More details of the analysis of the phase offset ϕ can be found in Appendix D. The non-conjugate Test Statistic TS₁ derived in (4.7) of section 4.1 in chapter 4 is a function of the slots and FFTs. The SCF estimates for the p^{th} slot given in (5.10) are non-coherently added and the resulted test statistic has the attenuated feature, see [137]. This can be represented by substituting the total phase offset ϕ given in (5.11) into the non-conjugate SCF for the wideband channel (4.7) giving a non-conjugate Test Statistic under SCO only as,

$$\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta)\right\} = \Re\left\{\frac{1}{P}\sum_{p=1}^{P}\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{L}X_{L,p}\left(n,k+\frac{\alpha}{2}\right)X_{L,p}^{*}\left(n,k-\frac{\alpha}{2}\right)\right)\sum_{p=1}^{P}e^{-j2\pi\alpha N\delta p}\right\}$$
(5.13)

and can be further expressed as,

$$\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta)\right\} = \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}S_{x}^{\alpha}(k,p)\sum_{p=1}^{P}e^{-j2\pi\alpha N\delta p}\right\}$$
(5.14)

where $S_x^{\alpha}(k,p) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p} \left(n, k + \frac{\alpha}{2}\right) \times X_{L,p}^* \left(n, k - \frac{\alpha}{2}\right)$ as derived in (4.6) of section 4.1 in chapter 4. Note that $\bar{S}_x^{\alpha}(k|\delta)$ represented in (5.13) is a representation of the sum of the effects of the phase offset for the total wideband of *P* slots. After substituting (5.12) into (5.14), see Appendix D, the Test Statistic with SCO effect only is given as,

$$\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta)\right\} = \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}S_{x}^{\alpha}(k,p)\frac{\sin(\pi\alpha PN\delta)}{\sin(\pi\alpha N\delta)}e^{-j\pi\alpha N\delta(P-1)}\right\}.$$
(5.15)

Note that if P = 1 (no multi-slot) with zero SCO , (5.15)is equivalent to the conventional SCF as given in (3.17). If $P \ge 1$ and $\alpha N\delta$ is an integer, then looking at the pure real (fraction) in (5.15) we can observe that numerator and denominator are zero for any integral value of $\alpha N\delta$. Consequently, the SCO will have no effect on the TS₁. It follows also, that for any non-integral value of $\alpha N\delta$, the TS₁ will be affected. Usually, in practice under the presence of SCO $\alpha N\delta$ is non-integral.

5.3 Combined Effects of Sampling clock and Cyclic Frequency Offsets in noiseless condition

The effects of both the SCO and CFO will now be considered. Since noise is not affected by CFO due to it's stationary property, the CFO will be analysed in a noiseless condition. Let the CFO be represented by Δ_{α} . Given that,

$$\alpha' = \alpha \times (1 + \Delta_{\alpha}) \tag{5.16}$$

where α' and α are the actual and ideal cyclic frequencies (CF) at the receiver and transmitter respectively. The CFO will also be represented exponentially by substituting α' in the offset summation (right) of (5.14 without affecting the previous analysis of SCO. We then have the non-conjugate Test Statistic with the receiver constraints, SCO and CFO analysed as,

$$\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta,\Delta_{\alpha})\right\} = \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x}^{\alpha}(k,p)\sum_{p=1}^{P}e^{-j2\pi N\delta\alpha(1+\Delta_{\alpha})p}\right\}$$
$$= \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x}^{\alpha}(k,p)\frac{\sin(\pi N\delta P\alpha(1+\Delta_{\alpha}))}{\sin(\pi N\delta\alpha(1+\Delta_{\alpha}))}e^{-j\pi N\delta\alpha(1+\Delta_{\alpha})(P-1)}\right\}$$
(5.17)

where $\bar{S}^{\alpha}_{s}(k,p)$ is the spectral correlation for the p^{th} slot for signal only as derived in (4.6). It can be observed that when both SCO and CFO are zero, we are left with the summation on the left which is the test statistic without offsets derived in (4.7) and if either of them is non-zero, the Test statistic is constrained. Also, the highest achievable gain using the multi-slot test statistic under receiver constraints (5.17) is whenever *N* is chosen such that $N\alpha\delta$ is an

integer. The non-conjugate and conjugate test statistics with both SCO and CFO are expressed from (5.17) as,

$$TS_{1|\delta,\Delta\alpha} = \mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta,\Delta_{\alpha})\right\}$$
$$= \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x}^{\alpha}(k,p)\frac{\sin(\pi N\delta P\alpha(1+\Delta_{\alpha}))}{\sin(\pi N\delta\alpha(1+\Delta_{\alpha}))}e^{-j\pi N\delta\alpha(1+\Delta_{\alpha})(P-1)}\right\}$$
(5.18)

and

$$TS_{2|\delta,\Delta\alpha} = \mathbb{R}\left\{\bar{S}_{x^*}^{\alpha}(k|\delta,\Delta_{\alpha})\right\}$$
$$= \mathbb{R}\left\{\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x^*}^{\alpha}(k,p)\frac{\sin(\pi N\delta P\alpha(1+\Delta_{\alpha}))}{\sin(\pi N\delta\alpha(1+\Delta_{\alpha}))}e^{-j\pi N\delta\alpha(1+\Delta_{\alpha})(P-1)}\right\}$$
(5.19)

respectively. It follows that the second order moments of the non-conjugate and conjugate Test Statistics with both SCO and CFO are given as,

$$\mathbb{E}\left[\left(TS_{1|\delta,\Delta\alpha}\right)^{2}\right] = \mathbb{E}\left[\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta,\Delta\alpha)^{2}\right\}\right]$$
$$= \mathbb{E}\left[\mathbb{R}\left\{\left(\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x}^{\alpha}(k,p)\frac{\sin(\pi N\delta P\alpha(1+\Delta_{\alpha}))}{\sin(\pi N\delta\alpha(1+\Delta_{\alpha}))}e^{-j\pi N\delta\alpha(1+\Delta_{\alpha})(P-1)}\right)^{2}\right\}\right]$$

and

$$\mathbb{E}\left[\left(TS_{2|\delta,\Delta\alpha}\right)^{2}\right] = \mathbb{E}\left[\mathbb{R}\left\{\bar{S}_{x^{*}}^{\alpha}\left(k|\delta,\Delta\alpha\right)^{2}\right\}\right]$$
$$= \mathbb{E}\left[\mathbb{R}\left\{\left(\frac{1}{P}\sum_{p=1}^{P}\bar{S}_{x^{*}}^{\alpha}\left(k,p\right)\frac{\sin(\pi N\delta P\alpha(1+\Delta_{\alpha}))}{\sin(\pi N\delta\alpha(1+\Delta_{\alpha}))}e^{-j\pi N\delta\alpha(1+\Delta_{\alpha})(P-1)}\right)^{2}\right\}\right]$$

(5.21)

respectively. They will be used to determine the probability of detection shortly. Since the cyclic frequency α is a function of symbol rate 1/T and signal frequency f, the resolution of the cyclic frequency Δ_{α} is determined by the FFT size and in a multi-slot wideband by the combination of slot size N and number of slots P.

5.4 Threshold, Detection and Probabilities of False Alarm and Detection

A binary decision rule of two hypotheses will be adopted after the spectral correlation of the baseband signal in order to detect the absence or presence of the signals.

• Null Hypothesis, H₀ for noise only,

$$H_0: s(t) = \eta(t);$$

• Alternative Hypothesis, *H*₁ for signal plus noise,

 $H_1: s(t) = x(t) + \eta(t);$

where $\eta(t)$ is Additive White Gaussian Noise (AWGN) and x(t) is the received signal without noise. The detection threshold λ that is approximately at the level of the test statistic magnitude at H_0 and gives a Constant False Alarm Rate (CFAR) during the detection. The non-conjugate Test Statistic with the constraints (5.18) will then be compared against the detection threshold to determine the probability of false alarm and probability of detection of the signal during the 10^3 Monte Carlo iterations to give,

$$P_{\text{fa}|\delta,\Delta\alpha} = \text{Prob}\left\{TS_{1|\delta,\Delta\alpha} > \lambda \mid H_0\right\}$$
, for when the signal is absent, H_0 ; (5.22)

and

 $P_{d|\delta,\Delta\alpha} = Prob\{TS_{1|\delta,\Delta\alpha} > \lambda \mid H_1\}$, for when the signal is present, H_1 . (5.23)

5.4.1 P_d and $P_f a$ under Receiver Constraints

In order to calculate analytically the probability of false alarm P_{fa} and probability of detection P_d , the mean μ and variance σ^2 at both H_0 and H_1 under receiver constraints will be derived. The mean noise only H_0 and signal H_1 under CFO and SCO are expressed from (5.17) as,

$$\mu_{0|\delta,\Delta\alpha} = \mathbb{R}\left\{\bar{S}^{\alpha}_{\eta}(k|\delta,\Delta\alpha)\right\}.$$
(5.24)

and similarly, the mean for H_1 is,

$$\mu_{1|\delta,\Delta\alpha} = \mathbb{R}\left\{\bar{S}_x^{\alpha}(k|\delta,\Delta\alpha)\right\}$$
(5.25)

It can be shown similar in (4.23) and (4.24) that the variances for H_o and signal H_1 under receiver constraints can be expressed as,

$$\sigma_{0|\delta,\Delta\alpha}^{2} = \mathbb{E}\left[\mathbb{R}\left\{\left\{S_{\eta}^{\alpha}(k,p|\delta,\Delta\alpha)\right\}^{2}\right\}\right] - \left\{\mathbb{E}\left[\mathbb{R}\left\{\bar{S}_{\eta}^{\alpha}(k|\delta,\Delta\alpha)\right\}\right\}\right\}^{2}$$
(5.26)

and

$$\sigma_{1|\delta,\Delta\alpha}^{2} = \mathbb{E}\left[\mathbb{R}\left\{\left\{S_{x}^{\alpha}(k,p|\delta,\Delta\alpha)\right\}^{2}\right\}\right] - \left\{\mathbb{E}\left[\mathbb{R}\left\{\bar{S}_{x}^{\alpha}(k|\delta,\Delta\alpha)\right\}\right]\right\}^{2}$$
(5.27)

respectively.

Recall, as stated in section 4.1.3 that the SCF has non-zero mean at H_0 , the TS₁ and is Ricean distributed over an AWGN channel as was used in [81]. From the derivations of the probability of false alarm P_{fa} and the probability of detection P_d in section 4.1.3, the P_{fa} at the detection threshold λ under SCO and CFO is given by,

$$P_{\mathrm{fa}|\delta,\Delta\alpha} = \mathbb{Q}\left(\frac{\mu_{0|\delta,\Delta\alpha}}{\sigma_{0|\delta,\Delta\alpha}}, \frac{\lambda}{\sigma_{0|\delta,\Delta\alpha}}\right)$$
(5.28)

where $\mathbb{Q}(.)$, $\sigma_{0|\delta,\Delta\alpha}^2$ and $\mu_{0|\delta,\Delta\alpha}$ are the generalized Marcum-Q function, noise deviation and the mean of the test statistic under noise. Similarly, from section 4.1.3 we can express the Ricean parameters under constraints for H_1 as,

$$\mu_{1|\delta,\Delta\alpha} = \sqrt{\mu_{s|\delta,\Delta\alpha}^{2} + \mu_{0|\delta,\Delta\alpha}^{2}}$$

= $\sqrt{\left\{ \mathbb{E} \left[\mathbb{R} \left\{ \bar{S}_{s}^{\alpha}(k|\delta,\Delta\alpha) \right\} \right] \right\}^{2} + \left\{ \mathbb{E} \left[\mathbb{R} \left\{ \bar{S}_{\eta}^{\alpha}(k|\delta,\Delta\alpha) \right\} \right] \right\}^{2}}$ (5.29)

and

$$\sigma_{1|\delta,\Delta\alpha}^2 = \frac{\sigma_{s|\delta,\Delta\alpha}^2}{2} + \frac{\sigma_{0|\delta,\Delta\alpha}^2}{2}.$$
(5.30)

From where,

$$\sigma_{1|\delta,\Delta\alpha} = \sqrt{\frac{\sigma_{s|\delta,\Delta\alpha}^2}{2} + \frac{\sigma_{0|\delta,\Delta\alpha}^2}{2}}.$$
(5.31)

Note that $\sigma_{1|\delta,\Delta\alpha}^2$ is from the conventional Ricean parameter $2\sigma^2$ as was discussed in [28, 149–152] for different test statistics. Therefore, the probability of detection under SCO and CFO is given by,

$$P_{\mathbf{d}|\delta,\Delta\alpha} = \mathbb{Q}\left(\frac{\mu_{1|\delta,\Delta\alpha}}{\sigma_{1|\delta,\Delta\alpha}}, \frac{\lambda}{\sigma_{1|\delta,\Delta\alpha}}\right)$$
(5.32)

where $\mu_{1|\delta,\Delta\alpha}$ and $\sigma_{1|\delta,\Delta\alpha}$ are the mean and deviation of the non-conjugate test statistic under the receiver constraints $TS_1|\delta,\Delta\alpha$.

5.5 Signal to Noise Ratio of the Test Statistics under CFO and SCO

In this section the Signal to Noise Ratio (SNR) of the Test Statistic under both considered constraints will be derived and is useful as a performance metric in maximising the detection performance. The SNR can be defined as the ratio of test statistic power under the signal only condition at a given cyclic frequency α , to that under noise only H_o for a given distribution of CFO and SCO. From the derivations of the SNR without constraints given in section 4.2, the SNR under SCO and CFO can be expressed as,

$$\operatorname{SNR}_{\operatorname{TS}|\delta,\Delta\alpha} = \frac{\mathbb{E}\left[\mathbb{R}\left\{\left\{\bar{S}_{x}^{\alpha}(\mathbf{k}|\delta,\Delta\alpha)\right\}^{2}\right\}\right]}{\mathbb{E}\left[\mathbb{R}\left\{\left\{\bar{S}_{\eta}^{\alpha}(\mathbf{k}|\delta,\Delta\alpha)\right\}^{2}\right\}\right]}.$$
(5.33)

Note that if both SCO and CFO are zero, we are left with the SNR of the test statistic without constraints SNR_{TS} as derived in (4.32). For the $\text{SNR}_{\text{TS}|\delta,\Delta\alpha}$, the discrete variable *x* represents the signal only (signal without noise) condition.

5.6 Results and Discussion

5.6.1 Effects of FFT and Slot sizes with SCO and CFO under signal with noise *H*₁

The results were obtained analytically and through simulations using Monte Carlo iterations, [157] in order to determine the sizes of FFT and slot that will give efficient and low complexity detection under the combinations of SCO and CFO. The simulations were based on the proposed non-conjugate multi-slot test statistic with offsets $TS_1|\delta,\Delta\alpha$ in (5.18), discrete conventional SCF in (4.9), energy detection test statistic in (2.2), while the analytical results were based

on proposed non-conjugate multi-slot test statistic with constraints $TS_1|\delta, \Delta \alpha$ in (5.18), probability of false alarm $P_{fa}|\delta, \Delta \alpha$ in (5.28) and probability of detection $P_d|\delta, \Delta \alpha$ in (5.32). It should be mentioned that all the plots involving the probability of detection in the results section asymptotically approach the maximum detection mark of 1.

Some examples of CFO and SCO will be selected from the typically known values as discussed in [48, 58, 165]. Under the presence of 0.09 SCO/CFO, the non-conjugate Test Statistic with SCO and CF0, $TS_{1|\delta,\Delta\alpha}$ (5.18) was used together with the detection algorithm in (4.1.1.1) where the spectral correlation density (magnitude of the correlated spectral components) were compared against the selected threshold λ . Fig. 5.3, shows that more samples are needed for larger FFTs. The slot size *N* was reduced from 256 to 128 samples



Figure 5.3: Probability of detection using non-conjugate Test Statistic with constraints (simulation, see (5.18)) for QPSK Signal of -5 dB SNR, SCO/CFO = 0.09, slot size = 256 samples, number of slots = 8, frequency = 2.5 MHz and $f_s = 10$ MHz.

for 16 slots in Fig. 5.4. As expected according to the effect of slot size N discussed in section 4.3.1 reducing it from 256 to 128 samples improved the detection performance of the FFTs even under the receiver constraints as shown in Fig. 5.4. For instance, FFT size of 16 samples now require approximately 800 samples for maximum detection in Fig. 5.4 as against a little over 1000



Figure 5.4: Probability of detection using $TS_{1|\delta,\Delta\alpha}$ (simulation, see (5.18)) for Signal with -5 dB SNR, SCO/CFO = 0.09, multi-FFTs, slot size = 128 samples, number of slots = 16, frequency = 2.5 MHz and f_s = 10 MHz.

samples in Fig. 5.3. Further comparison of the probability of detection across different slot sizes at 0.1 SCO and CFO is shown in Fig. 5.5. As expected, the smaller N quickly attains the maximum level of probability of detection.



Figure 5.5: Probability of detection of 3.5MHz QPSK signal for different slot sizes using $TS_{1|\delta,\Delta\alpha}$ (simulation, see (5.18)), at -5dB noise, FFT size 16, P = 12 samples and $f_s = 14$ MHz.

For consistency a single combination of CFO and SCO was in Figures 5.3, 5.4 and 5.5. These figures show that in the presence of the constraints CFO

and SCO, the smaller FFTs still asymptotically approach the perfect detection of 1 with less number of samples than the larger ones. This was the same behaviour in the previous chapter where there were no receiver constraints.

In order to understand the effects of different combinations of SCO and CFO, further investigations were carried out with different values of both the SCO and CFO using $P_{d|\delta,\Delta\alpha}$ in (5.32) with any of the FFT sizes and in this case FFT size 32 was used as in Fig. 5.6. It is shown that the Probability of detection under the receiver offsets (5.32) reduced as the values of SCO and CFO were increased. This can also be verified with another FFT size as in Fig. 5.7 where



Figure 5.6: Probability of detection (analytic, see (5.32)) for QPSK Signal of -5dB SNR under multiple SCO/CFO with FFT/slot size = 32/256 samples, Number of slots = 8, frequency = 2.5 MHz, $f_s = 10$ MHz.

FFT size 16 was used with N of 128 samples and P of 8 samples giving 1024 total samples. In Fig. 5.7 the larger the constraints (SCO & CFO), the more is the number of samples required for detection. Also, due to reduced slot size from 256 to 128 samples, the number of samples required for correlation and detection is reduced. For instance in Fig. 5.7, approximately 700 samples are required for maximum probability of detection at constraints of 0.1 while approximately 1800 samples are required in Fig. 5.6 for the same constraints of 0.1.

From Figures 5.6 and 5.7, it can be observed that higher values of SCO and



Figure 5.7: Probability of detection (analytic, see (5.32)) for QPSK Signal of -5dB SNR under multiple SCO/CFO with FFT/slot size = 16/128 samples, Number of slots = 8, frequency = 5 MHz, f_s = 20 MHz.

CFO require more samples for detection. This is consistent with the requirement of cyclostationary feature detection where more samples are normally required for correlation.

5.6.2 Comparison between with and without receiver constraints for multi-SNRs.

The detection performance of the Test Statistic under receiver constraints $TS_{1|\delta,\Delta\alpha}$ (5.18) will now be computed with different FFT sizes and for different values of the SNR. In consideration of the disparity in the performance of different sizes of FFT and slots, it makes sense to compare their performances with and without the receiver constraints starting from the larger ones. In Fig. 5.8, a larger FFT of 128 samples was used to perform the spectral correlation of the received QPSK signal under the conditions of with and without the receiver offsets of SCO and CFO. The separation between the lines for the two conditions of with and without is very significant. The closer the two lines are for a specific SNR value, the smaller the error of detection between the conditions of with and without the constraints. Therefore, it translates into differences in

performance when comparing the results of the test statistic with no receiver constraints against the results with receiver constraints. It should be noted that this separation increases as the SNR is reduced down to -17 dB.



Figure 5.8: Probability of detection (analytic, see (5.32)) for QPSK Signal with Noise at different SNRs, slot size/number = 256/8 samples, FFT size = 128 samples, SCO/CFO = 0.08, frequency = 2.5 MHz and f_s = 10 MHz.

Another FFT size such as 32 samples can be used for the same number of samples. As shown in Fig. 5.9, the gaps between the curves of no constraints and with constraints get reduced implying a better detection when compared against the previous results in Fig. 5.8.

When *N* is reduced to 64 samples and *P* increased to 32 for the same total number of samples, detection using FFT size 32 shows improved performance under SCO/CFO as can be seen in Fig. 5.10. Here, the gaps between the curves with constraints are much more closed up against the curves without the constraints. The plots representing the non-zero receiver offsets follow approximately the same paths as the plots for the zero offsets. Note that the closer the gap between the curves of with constraints and the one without constraints for any SNR value, the better the effects of the constraints are reduced. Figure 5.10 shows significant improvement when compared with the results in Figs. 5.8 and 5.9 where larger FFT sizes of 128 and 64 samples were used. Therefore, it demonstrates the robustness of the Test Statistic to



Figure 5.9: Probability of detection (analytic, see (5.32)) for QPSK Signal with Noise at different SNRs, slot size/number = 256/8 samples, FFT size = 64 samples, SCO/CFO = 0.08, frequency = 2.5 MHz and f_s = 10 MHz.

both noise and receiver constraints which is improved further using smaller FFT sizes and slot sizes.



Figure 5.10: Probability of detection (analytic, see (5.32)) for Signal with Noise at different SNRs, slot size/number = 64/32 samples, FFT size = 16 samples, SCO/CFO = 0.08, frequency = 2.5 MHz and f_s = 10 MHz.

5.6.3 Performance of the FFTs and slots under multi-SNR conditions

The detection performance of the test statistic can be verified for different values of SNR by considering the Receiver Operating Characteristic (ROC) with different FFT sizes such as 64, 32 and 16 as shown in Figs. 5.11, 5.12 and 5.13 respectively. It can be observed that the performance is progressive with



Figure 5.11: Receiver Operating Characteristic with (analytic, see (5.32, 5.28)) for multi-SNR values with 5 MHz QPSK signal, N = 256, P = 16, N_B = 4096 samples, FFT size 64, f_s = 20 MHz and SCO/CFO = 0.09.

the relative higher detection achieved with the smaller FFTs. For instance at -11 dB SNR, FFT size 64 achieves a P_{fa} of 1 in Fig. 5.11 as against 0.9 and 0.5 for FFT sizes 32 and 16 as shown in Figs. 5.12 and 5.13 respectively.

5.6.4 Comparison with Energy Detection and SCF Conventional Test Statistics

In the presence of noise and SCO/CFO of 0.02 for the proposed CFD test statistic with offsets $TS_{1|\delta,\Delta\alpha}$, the performance of this Test Statistic was compared against the Test Statistic for Energy Detection (ED) given in [44,158] as shown in Fig. 5.14. As expected, the proposed test statistic shows relatively more robustness to signals of low SNR values down to -25 dB with perfect de-



Figure 5.12: Receiver Operating Characteristic with (analytic, see (5.32, 5.28)) for multi-SNR values with 5 MHz QPSK signal using $TS_{1|\delta,\Delta\alpha}$, N = 256, P =16, N_B = 4096 samples, FFT size 32, f_s = 20 MHz and SCO/CFO = 0.09.



Figure 5.13: Receiver Operating Characteristic with (analytic, see (5.32, 5.28)) for multi-SNR values with 5 MHz QPSK signal using $TS_{1|\delta,\Delta\alpha}$, N = 256, P = 16, N_B = 4096 samples, FFT size 16, f_s = 20 MHz and SCO/CFO = 0.09.

tection starting at -17 dB. Note that the ED simulation has a zero CFO since ED is not affected by CFO.

The model $TS_{1|\delta,\Delta\alpha}$ can be compared against the discrete conventional SCF (4.9) as in Fig. 6.18 at SCO/CFO value of 0.1 for different FFT sizes. It can be observed that the model test statistic gives higher detection performance



Figure 5.14: Performance of Test Statistic with Offset against Energy Detection with (simulation, see (2.2, 5.18)) for FFT size = 16 samples, SCO/CFO = 0.02, slot size/number = 256/16 samples, frequency = 5 MHz, f_s = 20 MHz.



Figure 5.15: Comparison between the Conventional Test Statistic and model non-conjugate test statistic with (simulation, see (4.9, 5.18)) for multi-SNR values with 2.5 MHz QPSK signal, N = 64, P =32, N_B = 2048 samples, f_s = 10 MHz, FFT sizes 8-64 and SCO/CFO = 0.1.

even at low values of SNR. Also, the smaller FFTs show higher probability of detection at low SNR than the larger ones. There is a significant improvement over the conventional SCF even in the presence of the receiver constraints.

5.6.5 Application of test statistic to real signal in wideband channel

The application of the test statistic can be shown in a 2-dimensional plot of the signal's carrier frequency f and the cyclic frequency α . For clarity, the contour plots will be shown. In Fig. 5.16 two signals at -5dB SNR of BPSK modulation type are shown. When looking at the plot from the vertical axis, the carrier frequency under consideration is seen to match the centre of the line which represents the signal. Note that the cyclic frequency for BPSK signal as previously stated is $2 \times f$ and shown on the x-axis. The wideband capability can



Figure 5.16: Contour plot with $TS_{1|\delta,\Delta\alpha}$ (simulation(5.18)) for 2 BPSK signals, SCO/CFO = 0.05, with FFT size 16, f = 5,2.55 MHz, $\alpha = 2f$, N = 256, P = 16 and $f_s = 20$ MHz.

be demonstrated by extending the number of channels as in Fig. 5.17 where six channels are shown. Note that the frequency values are at the center of the appropriate line when looking through the carrier frequency axis.

5.7 Chapter Five Summary

In this chapter the receiver offsets namely, SCO and CFO were integrated into the analysis of the model. Simulations were carried out using the test



Figure 5.17: Contour plot with $TS_{1|\delta,\Delta\alpha}$ (simulation(5.18)) for BPSK signals, SCO/CFO = 0.5, with FFT size 16, f = 2-12 MHz, $\alpha = 2f$, N = 256, P = 38 and $f_s = 48$ MHz.

statistics under receiver offsets. It was shown that the presence of receiver offsets impact the level of probability of detection and requires more samples to reduce the effects, which adds to the computational cost. It was also shown that the smaller the FFT is in size, the more robust the model performs under the receiver offsets. Simulations showed that for signals with low SNR, the performance under the receiver offsets was improved with the use of small fast Fourier Transform sizes. When compared against the Energy Detection test statistic it showed significant improvement even in the presence of the receiver offsets and low SNR conditions. The model test statistic was also compared with the conventional spectral correlation function test statistic and the multi-slot model showed improvement over the conventional one. This was also made more significant at smaller FFT sizes. The detection of signals with the test statistic under the receiver constraints was also shown in terms of 2-D plot of the carrier and cyclic frequencies. It was also shown to have wideband capability and was demonstrated with six QPSK signals of different carrier frequencies.

Chapter 6

Algorithmic Optimization of Cyclostationary Spectrum Sensing under Receiver Constraints

In this chapter the term Optimization is used in the context of its meaning discussed in [27,81,166] as a process of maximizing or minimizing some function relative to some set, often representing a range of choices available in a certain situation. In this research, the generic algorithm will adopt the method of brute force search where the available variables will be processed with a view of selecting the one that best meets the expected goal.

The generic algorithm for optimizing the different parameters such as slot size N, number of slots P, fast Fourier transform (FFT) size L and number of FFTs M needed to achieve efficient and low computational complexity is described. This will be applied to the scenario when the receiver constraints cyclic frequency offset (CFO) and sampling clock offset (SCO) are present in the received signal. The optimized parameters of the model test statistic will also be applied to a Ricean distribution communication Channel. The performance will be verified under low RF signal levels while the computational complexity will be compared against the conventional spectral correlation test statistic.
6.1 Computational Complexity

From section 2.5.7.2, Fast Fourier Transform (FFT) reduces the number of computations involved in the application of the Discrete Fourier Transform (DFT) from the order of N^2 to $N\log N$. Recall, the DFT requires N^2 complex multiplications and N(N-1) complex additions while the FFT requires a total of N/2 log N complex multiplications and $N \log N$ complex additions to compute N-point DFT. It makes sense to use the complex additions to analyse the impact on computational complexity since it is higher than the complex multiplications for any FFT. It also follows that the proposed multi-slot model is based on windowed FFT computation.

It is given that the total FFTs in a slot is,

$$M = \frac{N}{L},\tag{6.1}$$

where *L* and *M* are the FFT and slot sizes in samples. The total complex additions FFT_{cx} for 1 slot is in the order of,

$$O_1(M,L) = M(L\log L) \tag{6.2}$$

where *M* is the total number of FFTs in a slot (6.1) and LLogL is the complex additions for one conventional radix 2 FFT as discussed in section 2.5.7.2, see [155, 156]. From (4.1), (6.1) and (6.2), we derive the total computational complexity FFT_{tc} for the wideband channel of *P* the total number of slots,

$$O_T(M,L) = P \times M \times L \log L.$$
(6.3)

See Appendices E and F for examples of the calculation of the FFT computational complexities and resolutions that were used in this chapter.

6.2 Optimizing the FFT Size L and Number M

We will obtain the optimum L and M from the first optimization problem in (6.4) which will be used to further solve the second optimization (6.5) within

the requirement of computational complex additions and the minimum number of samples L_{min} required in a slot to represent the information in a channel. FFT size should be chosen for maximum P_d , correct frequency resolution and reduced complexity. The objective is to maximize P_d subject to the values of Land M.

$$(\bar{L}, \bar{M}) = \underset{L,M}{\operatorname{argmax}} P_d$$

such that $LM \le N$, (6.4)
 $L \ge L_{min}$ and
 N, P (fixed)

where \bar{L} and \bar{M} are the optimized L and M. From (4.1), (6.3) and we can verify the effect of computational complexity or complex additions.

6.3 Optimizing the Slot Size N and Number P

According to (4.1), the number of slots P is affected by the total N_B and slot size N. Therefore, it makes sense to optimize N and P for maximum P_d for a given number of samples for the wideband channel. N should be optimized so that it covers the minimum samples N_{min} required for the information symbols within the slot. The objective is to maximize P_d subject to the values of N and P. The optimization problem can be formulated as,

$$(\bar{N}, \bar{P}) = \underset{N,P}{\operatorname{argmax}} P_d$$

such that $NP = N_B$,
 $N \le N_{min}$ and
 L, M (fixed) (6.5)

where \bar{N} and \bar{P} are the optimized N and P and P_d is the probability of detection.

There will be an increase in P for small N as in (4.1) which will subsequently produce an increase in the number of FFTs which could impact on the computational complexity. In order to further solve the optimization problem

in (6.5), consideration will be given to the overall computational complexity (in terms of the complex additions) from combining multiple FFTs which is given in (6.3). The choice of N and P should be made for maximum Pd and reduced complexity.

6.4 Algorithm for Optimizing the FFT and slot sizes and numbers

In this section, the generic algorithm will adopt the method of brute force search where the different available parameters of L, M, N and P will be processed with a view of selecting the optimized combination of them that best meets the expected goal of producing an efficient and low computational complexity wideband cyclostationary feature detection. The optimized combination set of L, M, N and P will then be applied to sample sets for the detection of different cyclostationary signals without repeating the search method.

The derivations of test statistics with and without receiver offsets TS_1 (4.7) and $TS_{1|\delta,\Delta\alpha}$ (5.18) respectively will be used. The threshold level λ that is approximately at the level of the test statistic magnitude at noise only condition H_0 obtained by simulation will be selected. The FFT and slot sizes will be selected in terms of the following three metrics; probability of detection P_d against total number of samples N_B ; the clarity of the two dimensional simulations of second moment of the test statistic $E[(TS_1)^2]$ in cyclic frequency α and carrier frequency f_c and low computational complexity offered by the FFT size.

Algorithm 2 Optimizing FFT Size and Number

- 1: Let i = 1:6, j = 1:4
- 2: Inputs: Total samples = N_B , FFT sizes: $L_1 = 4$, $L_2 = 8$, $L_3 = 16$, $L_4 = 32$, $L_5 = 64$, $L_6 = 128$, Slot sizes: $N_1 = 256$, $N_2 = 128$, $N_3 = 64$, $N_4 = 32$, Test statistics TS_1 , $TS_{1|\delta,\Delta\alpha}$ and Threshold λ ;
- 3: Outputs: $|S_i| =$ Magnitude of TS_{11} , $|S_{\delta,\Delta\alpha}|, i =$ Magnitude of $TS_{1|\delta,\Delta\alpha}$, Probability of detection P_d , Optimized $L = \overline{L}$, $M = \overline{M}$, $N = \overline{N}$, $P = \overline{P}$,
- 4: Slot number $P_j = \frac{N_j}{L_i}$;
- 5: for j = 1:2 do
- 6: $N = N_j$; { $N \ge 2 \times L_i$ to support correlation between at least 2 FFTs};
- 7: **for** *i* = 1:6 **do**
- 8: Run simulations of P_d against N_B with the second moment of the test statistic $E[(TS_1)^2]$ and the parameters N_B , N_1 , P_i and L_i , M_i ;
- 9: Then $P_d = P_{d|i}$ {Probability of detection for the i^{th} FFT.};
- 10: end for
- 11: Sort $P_{d|i}$ in descending order;
- 12: $A = P_{d|i}$ in descending order;
- 13: Select four FFT sizes (FFT1, FFT2, FFT3, FFT4) with high $P_{d|i}$ and small number of samples;
- 14: end for
- 15: FFT1, FFT2, FFT3, FFT4 have high $P_{d|i}$ { first selection metric for *L*};
- 16: Use same N_1 (step 5) and FFT1, FFT2, FFT3, FFT4 to run two-dimensional (2-D) simulations of $E[(TS_1)^2]$ in dimensions of cyclic frequency α and the carrier frequency f_c ;
- 17: Select two FFT sizes that have more distinguishable correlation peaks from the 2D plots {second selection metric for FFT size};
- 18: Calculate the total complexity according to (6.3) and the frequency resolution (4.30) for each of the two FFTs with N_j ; {Calculate with different slot sizes, see Appendix F.1}.
- 19: Select the FFT with the lowest computational complexity {third metric};
- 20: $L = \overline{L}$ {Optimized FFT size};
- 21: $\overline{M} = N_j / \overline{L}$ {Optimized *M* for N_j }

Algorithm 3 Optimizing Slot Size and Number

- 1: Let j = 1:4
- 2: Inputs: Total samples = N_B , FFT size $L_1 = 4$, Slot sizes: $N_1 = 256$, $N_2 = 128$, $N_3 = 64$, $N_4 = 32$, Test statistics TS_1 , $TS_{1|\delta,\Delta\alpha}$ and Threshold λ ;
- 3: Outputs: $|S_i| =$ Magnitude of TS_{11} , $|S_{\delta,\Delta\alpha}|$, i = Magnitude of $TS_{1|\delta,\Delta\alpha}$, Probability of detection P_d , Optimized $N = \overline{N}$, $P = \overline{P}$,
- 4: Slot number $P_j = \frac{N_j}{L_i}$;
- 5: for j=1:4 do
- 6: $N = N_j;$
- 7: $M_j = N_j / \overline{L}$ {Number of \overline{L} in N_j };
- 8: $P_j = N_B/N_j;$
- 9: Run simulations of P_d against N_B with $E[(TS_1)^2]$ and the parameters N_B , N_j , P_j and \bar{L} , M_j ;
- 10: $P_d = P_{d|i}$ {Probability of detection for the $j^{th} N$.};
- 11: end for
- 12: Sort $P_{d|i}$ in descending order;
- 13: $A = P_{d|j}$ in descending order;
- 14: Select N_i with high $P_{d|i}$ with small number of samples;
- 15: $N_i = \bar{N}$; { Optimized N}
- 16: $\bar{P} = \bar{N}_B / \bar{N}$; {Optimized *P* fot total number of samples}

6.4.1 Flowcharts for optimizing FFT and slot sizes

For more clarity, the optimization algorithms 2, 3 can be represented with two flowchart diagrams for the optimized FFT size L and N.



Figure 6.1: Flowchart for optimizing FFT size. (a) Optimizing FFT size part 1.



(b) Optimizing FFT size continued.



Figure 6.2: Flowchart for optimizing slot size.

6.5 The optimized *N*, *P*, *L* and *M* for a given distribution of CFO and SCO

We will use the optimized values of N, P, L and M from section 6.4 obtained with a zero CFO and SCO and apply the derivations of P_{fa} (5.28) and P_d (5.32) from section 5.4.1 in order to optimize the values of N, P, L and M over a Ricean channel for a given distribution of CFO and SCO [28]. In order to model the CFO and SCO, we adopt a zero mean Gaussian probability distribution function (pdf) as used in [167, 168]. Therefore, the pdf of the CFO and SCO will be represented by,

$$P_{\Delta\alpha} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\Delta_{\alpha}^2}{2\sigma^2}}$$
(6.6)

and

$$P_{\delta} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\delta^2}{2\sigma^2}}$$
(6.7)

respectively. For simplicity of comparison, the $\frac{\Delta \alpha}{\sigma}$ ratios of 1 or 0.1 will be used to represent the CFO and SCO from a typical range of CFO and SCO values [-0.1,0.1] as was used in [124, 165].

6.6 Results and Discussion

In this section, for simplicity FFT sizes of 4, 8, 16, 32, 64 and 128 samples will be referred to as FFT4, FFT8, FFT16, FFT32, FFT64 and FFT128. Also slot sizes 64, 128 and 256 samples as slot64, slot128 and slot256. It should be mentioned that all the plots involving the probability of detection in the results section asymptotically approach the maximum detection mark of 1.

6.6.1 Optimizing *L* and *M* with the Optimization Algorithm

In this section the test statistic TS_1 (4.7) will be used in the simulation. During the 10³ Monte Carlo iterations, different combinations of *L* and *M* will be processed while keeping the *N* and *P* constant with a view of selecting the



Figure 6.3: Probability of detection of 5MHz QPSK signal for multi-FFTs with P_d in (simulation, see(4.14)) at -5dB noise, N = 256 samples, P = 8 and $f_s = 20$ MHz.

best combination of L and M that achieves maximum P_d with small number of samples. References are made to the algorithms 2 and 3. In order to select an FFT, consideration should be given to the total samples that will cover the bandwidth to be sensed. Secondly, N shall be selected to cover the certain minimum samples required by any L to effectively correlate as in step 6 of the algorithm 2 and derived in (4.8). For instance FFT length 64 requires minimum of 128 samples to properly correlate considering that FFTs are required in pairs for correlation. Given a total number of samples N_B and slot size N of 2048 and 256 samples respectively as in Fig. (6.3), P can be calculated from step 4 of the algorithm (2) as derived in (4.1) as 2048/256 = 8 slots. From Fig. 6.3 FFT lengths 4, 8 and 16 use the smallest sample sizes to reach the peak detection. Since we were interested in optimization, N was further reduced to 128 as in Fig. 6.4 where FFT of size 16 shows more clearly the effect of the reduction in the number of samples required for correlation and detection. The reduction of N from 256 to 128 samples results in the use of fewer samples for correlation. For the same N of 128 samples and the total samples reduced from 2048 to 1024 as in Fig. 6.5, it shows that the same numbers of samples required to reach the detection peak for the FFTs were maintained. These



Figure 6.4: Probability of detection of 5MHz QPSK signal for for multi-FFTs with P_d in (simulation, see(4.14)) at -5dB noise, N = 128 samples, P = 16, $f_s = 20$ MHz.



Figure 6.5: Probability of detection of 2.5MHz QPSK signal for for multi-FFTs with P_d in (simulation, see(4.14)) at -5dB noise, N = 128 samples, P = 8 and $f_s = 10$ MHz.

show that the FFT and slot sizes determine the number of samples required for peak detection. According to step 13 of algorithm 2, from Figs. 6.3, 6.4, 6.5 the four FFTs with the highest level of P_d are FFT4, FFT8, FFT16 and FFT32. Apart from the maximum P_d requirement in step 13 and according to step 16 of algorithm 2, the representation of the signal in 2-dimensions (2-D) of f_c and



Figure 6.6: Detection in terms of real signal features using TS_1 in (simulation, see (4.7)) with FFT sizes 4, 8, 16, 32 f = 1 MHz, N = 256 samples, P = 16, $f_s = 4$ MHz and $N_B = 4096$ samples at SNR = -5dB.

 α will also be considered in choosing an FFT size which is the second metric of FFT selection. In Fig. 6.6, it can be observed that FFT sizes 16 and 32 have more distinguishable correlation peaks of the 1 MHz BPSK carrier signal than others. Therefore both FFT sizes 16 and 32 are selected according to step 17. For the purpose of clarity both FFT sizes 16 and 32 are plotted as in Fig. 6.7

According to step 18, further investigation are carried out as shown in Fig. 6.8 in terms of complexity and frequency resolution of the FFTs Δ_f , f_s/L derived in (4.30), where f_s is the sampling rate. We will use the FFT complex additions in calculating the complexities, see section 2.5.7.2 and Appendices E and F for more details. For the purpose of clarity we used a sensing time T_{acq} of 1 second and different slot sizes of 64 to 512 samples with f_s of $4 \times f$ to calculate the Δ_f and complexity as stated in (4.30) and (6.3) respectively. More details of the calculations of complexity and Δ_f can be found in Appendix F. In Fig. 6.8, it can be observed that FFT sizes 16 and 32 compensate for both Frequency resolution Δ_f and complexity when compared with others. Note that similar results will be obtained with other f_s . However, FFT size 16 gives a lower complexity than FFT size 32. Since complexity is one of the key objection.



Figure 6.7: Detection in terms of real signal features using TS_1 in (simulation, see (4.7)) with FFT sizes 16, 32 f = 1 MHz, N = 256 samples, P = 16, $f_s = 4$ MHz and $N_B = 4096$ samples at SNR = -5dB.



Figure 6.8: Comparison of FFT Frequency Resolution and Complexity (analytic, see Appendices E and F) for 1-4KHz BPSK signal for Slot Sizes of 64-512 samples, $f_s = 4 \times f$.

tives of this research, the optimized FFT size will be 16 samples. This value gives a trade-off between large and small FFTs. The number of optimized FFTs can simply be derived by dividing N by the optimized L according to step 21 of the algorithm 2.

6.6.2 Optimizing N and P with the Optimization Algorithm

According to step 9 of algorithm 3, simulations to get the probability of detection P_d with different slot sizes are carried out. For a given total number of samples N_B of 4096 as in Fig. 6.9, using different N with FFT of length 16. In Fig. 6.9 it shows that the smaller the size of the slot, the smaller the number of samples required to get to the peak detection.



Figure 6.9: Probability of detection of 10MHz QPSK signal for different Slot Sizes with P_d in (simulation, see(4.14)) at -5dB noise, FFT size 16, P = 16 and $f_s = 40$ MHz.

From step 6 of the algorithm 2, the choice of *N* is affected by the *L*. It should be noted that the minimum *N* for FFT size 16 is 32 samples according to step 3 of the optimization section 6.4 due to the minimum *N* required for acceptable correlation since the FFTs are required in pairs to effectively correlate as in TS_1 (4.7). In dimensioning the test statistic TS_1 (4.7) the product of correlating the Fourier transform is L^2 and gives the maximum computable slots *P* for any *L*. Therefore, the maximum *P* for FFT length 16 is 256. According to the optimization requirements in (6.5), we should minimize *N* to achieve a maximum P_d which in turn creates more *P*. However in order to achieve this we will consider not exceeding the maximum computable slots *P* of L^2 as previously mentioned.

Consequently, another concern in selecting an N is to consider the total

P that will cover the total samples N_B while not exceeding the maximum of L^2 . It can be reasoned that the selection be based on a slot size that is not at the minimum of 32 samples but satisfies both the maximum P_d with small number of samples and maximum computable slots which is the square of the FFT size, i.e L^2 . Therefore, *N* of 64 samples will be selected as the optimized value since it satisfies both requirements. Given the optimized value of *N*, the optimized number of slots *P* depends on N_B .

In summary, considering the simulation results in Figs. 6.3-6.8 and calculations from (4.1) and (6.1), given a bandwidth (BW) of total samples N_B of 4096, the optimized values of *L*, *N*, *P*, *M* were found to be 16, 64, 64 and 4 (per slot) respectively giving 256 total FFTs. Therefore, with algorithms 2, 3, the optimized values of *M*, *N*, *P* and *L* can be obtained. This can be applied to other total sample sizes.

6.7 Effect of using FFT size 64 with signals of low SNR

Before considering the effects of applying the small optimised *L* and *N* in the next section 6.8, it makes sense to observe the effects of using a larger FFT such as FFT64 using the P_d (5.32) and P_{fa} (5.28) derived in section 5.4.1 of chapter 5. This will be useful in making comparisons. In Fig. 6.10 the use of FFT64 at SCO & CFO of 0.04 could not bridge the gap between the curves of the results under SCO and CFO and those when there were no constraints. This shows that the presence of the constraints will have more impact on the accuracy of detecting the signals. The slot size was reduced from 256 to 128 samples in Fig. 6.11 and it shows no significant improvement in the detection.



Figure 6.10: Probability of detection under multi-SNR values with 5 MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 20$ MHz, N = 256, P = 8, FFT64, SCO/CFO = 0.04.



Figure 6.11: Probability of detection under multi-SNR values with 2.5 MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 10$ MHz, N = 128, P = 8, FFT64 and SCO/CFO = 0.04.

6.8 Applying the Optimized *L* and *N* under SCO and CFO

In this section, the performance of the non-conjugate test statistic second moment $\mathbb{E}[{TS_1}^2]$ is being investigated for different FFTs and fixed combinations of SCO and CFO values. This will compare the performance of the model in the presence of these constraints using the P_d in (5.32) and P_{fa} in (5.28) with receiver constraints in section 5.4.1 of chapter 5.

6.8.1 Optimized L under different values of SCO and CFO

As expected, in Fig. 6.12, the response of the model using FFT size 16 to a distribution of the combinations of CFO and SCO shows that the lower the SCO and CFO constraints the higher the detection probability. It shows it is able to deal with different values of the constraints.



Figure 6.12: Probability of detection with multi-offsets for 2.5MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) of -5dB noise at $f_s = 10$ MHz, N = 128, P = 8 and FFT 16.

6.8.2 Effect of the optimized *L* with signals of low SNR

The use of the optimized FFT16 in Fig. 6.13 at N of 256 samples improved the detection significantly and especially at moderately low signals such as -4dB and 1 dB. The gaps get narrower than the previous results in Fig. 6.11. The reduction of N to 128 samples for the same FFT16 as in Fig. 6.14 shows a significant closing of the gaps between the constrained and non-constrained curves at lower signal levels. However, note that the slight reduction in the magnitude



Figure 6.13: Probability of detection under multi-SNR values with 5 MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 20$ MHz, N = 256, P = 8, FFT16 and SCO/CFO = 0.04.



Figure 6.14: Probability of detection under multi-SNR values with 2.5 MHz QPSK with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 10$ MHz, N = 128, P = 8, FFT16 and SCO/CFO = 0.04.

of detection is as a result of the reduced slot size to 128 samples which invariably produces less number of FFTs for correlation. Further reduction of N to 64 samples 64 as in Fig. 6.15 for 32 slots to cover for the total samples for the wideband channel showed significantly improved detection. The constrained curves are closely matched by the unconstrained curves. This shows that the



Figure 6.15: Performanace under SCO/CFO = 0.04 for P = 32, multi-SNR values with 5 MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at f_s = 20 MHz, N = 64, N_B = 2048 samples and FFT16.

effect of the SCO and CFO have been significantly reduced by using the optimized values of L and N. Therefore this combination of FFT size 16, slot size 64 and slot number 32 is optimized for the total samples. This approach can also be applied to other sample sets.

For instance, the reduction of the impact of SCO and CFO is shown in Fig. 6.16 with the optimized L, N, M and P for N_B of 4096 samples. Here, the gaps between the curves of with and without the constraints of SCO and CFO are quite closely matched indicating a similar performance when the optimized L, M, N and P are applied with or without the constraints. It reduces the concern that a large number of samples are needed for the Cyclostationary Feature Detection under receiver constraints.

The performance can be further investigated by considering the receiver operating characteristic for a fixed constraints value of 0.1 for 2048 total samples using the optimized L and N as shown in Fig. 6.17 The model can detect signals significantly down to -11 dB SNR.



Figure 6.16: Performanace under SCO/CFO = 0.04 for P = 64, multi-SNR values with 10 MHz QPSK signal with $P_{d|\delta,\Delta\alpha}$ in (analytic, see (5.32)) at $f_s = 40$ MHz, N = 64, $N_B = 4096$ samples and FFT16.



Figure 6.17: Receiver Operating Characteristic for multi-SNR values with 10 MHz QPSK signal with (analytic, see (5.32, 5.28)), N = 64, P = 32, N_B = 2048 samples, FFT16 and SCO/CFO = 0.1.

6.8.3 Comparison with Conventional test statistic

The model test statistic $TS_{1|\delta,\Delta\alpha}$ can be compared against the conventional SCF without slots (4.9) with FFT16 and slot size of 64 samples as shown in Fig. 6.18 at SCO/CFO value of 0.1. It shows significant improvement over the conventional SCF even in the presence of receiver constraints.



Figure 6.18: Comparison between Model and Conventional TS in (simulation, see (4.9, 5.18)) for multi-SNR values with 5 MHz QPSK signal at f_s = 20 MHz, N = 64, P = 32, $N_B = 2048$ samples, FFT16 and SCO/CFO = 0.1.

6.8.4 Performance in 2-dimensions of carrier frequency fand cyclic frequency α

The performance of the second moment of the test statistic can be investigated in terms of the cyclic features of the signal such as carrier and cyclic frequencies. The optimized slot size 64 and FFT size 16 were used for five different BPSK signals at a combination of SCO and CFO value of 0.2 in a 2-dimensional contour plot shown in Fig. 6.19. Note that the carrier frequency is read off the centre of each legend line that represents it.



Figure 6.19: Contour plot with (simulation (5.18)) for BPSK signals, SCO/CFO = 0.2, with FFT size 16, f = 1-10 MHz, $\alpha = 2f$, N = 64, P = 64 and $f_s = 40$ MHz.

6.8.5 The Impact of FFT Complexities

There is an associate increase in the number of overall FFTs when small FFTs and small slot sizes are used to cover a wideband channel. Next the optimized FFT size 16 will be investigated in the form of comparisons with other larger FFTs in terms of slot size and number. The FFT complexities due to the multi-slot approach will be compared against the conventional SCF using the calculations derived in (4.1), (6.2), (6.3). Note that the FFT complexity in this research is expressed in terms of the FFT complex multiplications, see section 2.5.7.2 and Appendix E.

The comparison of the complexities between the different sizes of FFTs and slot sizes are shown in Fig. 6.20. For smaller slot sizes N, the lower the complexity with fixed FFT size 16. This shows lower complexity for the wideband FFT_{tc} (6.3) even as the slot size N is increased. The lower complexity performance using FFT size 16 is also supported in Fig. 6.21 at N of 256 samples even as the number of slots is increased. As previously mentioned, for details of the calculations of the FFT complexities, refer to Table E.2 of Appendix E. In Fig. 6.22, although more FFTs are involved using FFT size 16 for the same



Figure 6.20: Complexity for different slot sizes and FFTs Table E.1 of Appendix E.



Figure 6.21: Complexity for different slot numbers and FFTs Table E.2 of Appendix E.

slot size as other FFTs, the complexity is still lower in comparison to the cases with larger FFTs. Therefore, from Figs 6.20, 6.21 and 6.22, it is shown that the use of small FFTs and slot sizes for the correlation does not impact the model in terms of computational complexity. As previously mentioned, it is understood that using FFTs of smaller sizes results in an increased number of FFTs M for any given bandwidth. Although there is an increased number of FFTs using the smaller FFTs, the total computational complexity due to a small



Figure 6.22: Complexity for increasing number of FFTs Table E.2 of Appendix E.

FFT is lower, compared with the cases when larger FFTs are used. Applying this to the optimization problem in (6.4), means that the increase in M is not a disadvantage as long as L is small. Therefore, the use of small N and L satisfies the optimization requirements in (6.4) and (6.5).

The validity of the research approach in terms of small complexity can also be compared against the conventional SCF. The gain in complexity of the Test Statistic over the conventional SCF increases more with small size FFTs as in Fig. 6.23. The smaller the FFT size the more the difference in the complexities between the conventional SCF and the Test Statistic. It offers lower complexity,and as a result faster speed and improved accuracy for low SNRs in the presence of receiver constraints due to the small size FFT and slot. It shows significant complexity gain.



Figure 6.23: Comparison between Model Statistic and Conventional SCF Complexities Table E.1 of Appendix E.

6.9 Chapter Six Summary

In this chapter, the procedures for the optimization of the parameters of the multi-slot wideband cyclostationary feature detection were given and demonstrated through simulations. The optimized values of the Fast Fourier Transform and slot sizes in samples were obtained taking into consideration the trade-off in frequency and temporal resolutions. These values were applied for use with the test statistic under sampling clock and cyclic frequency offset receiver constraints. It was shown that the effects of these offsets were reduced in comparison to the case of when just the conventional statistics of the spectral correlation density function were applied.

The performance of the test statistic under low signal-to-noise ratio has shown significant improvement over the conventional statistic. It was also shown that the use of small fast Fourier transform sizes together with the use of multiple slots which are features of the multi-slot model test statistics do not result in an increase in the computational complexity when compared with the use of the conventional spectral correlation function.

The use of 2-dimensional plots of carrier and cyclic frequencies showed the

precision of detecting the presence of different signals in a wideband channel which is prevalent in real communication environments. It should also be mentioned that this algorithm and associated procedures can be applied to other sample sets to achieve similar outcomes.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

In spectrum sensing, it has been a research challenge to have the technique that can combine robustness to noise with low complexity, cost and efficiency. Conventional cyclostationary spectrum sensing is robust to noise but at the cost of a large number of samples which result in high complexity, cost and efficiency. In this research, multi-slot wideband cyclostationary spectrum sensing models have been deployed in the frequency domain using test statistics. These models will serve as guides to the hardware or physical implementation of wideband cyclostationary spectrum sensing models.

The test statistic was statistically analysed and investigated with different sizes of fast Fourier transforms in combination with different slot sizes. Different plots were produced which included the probability of detection, receiver operating characteristics using the linearly modulated signals such as Quadrature Phase Shift Keying and Binary Phase Shift Keying signals without receiver constraints. The combinations that produced high detection levels with small numbers of samples were selected for other tests. Such combinations will produce lower computational complexities in terms of complex additions associated with the computation of the fast Fourier transform. These combination were also used to observe the performance of the cyclostationary feature detection with signals under low values of signal-to-noise ratios in order to test its

robustness to noise. The results were obtained analytically or by applying the test statistic in simulations.

It has been shown that smaller fast Fourier transforms give better detection using reduced sample size than the larger ones. Also, the smaller slot size is of advantage depending on the fast Fourier transform size that is adopted. Additionally, the smaller slot and fast Fourier transform sizes will enable the correlation of spectral components at shorter time and eventual shorter detection time which will compensate in terms of temporal resolution.

From the calculations of computational complexity, it was shown that the use of small sizes of FFTs and slots will result in low complexity, cost and efficiency in terms of the number of samples that will be used for processing the signals. This is in line with the objectives of the research. It was also shown that the benefits of cyclostationary features which are robust to signals with low signal-to-noise ratios were achieved using these models and fulfilling one of the research objectives. The model was compared against the energy detection and conventional cyclostationary feature detection that uses the conventional discrete spectral correlation function. It showed significant benefits over them. The detection performance of the model in terms of real value signal features such as signal frequency and cyclic frequency with specific location detection showed that the model can be applied to real communications settings.

One of the important drawbacks of cyclostationary spectrum sensing is the issue of receiver constraints. We investigated the reduction of these constraints using these models to obtain the optimum sizes of FFTs and slots for better signal detection. The test statistic was analysed for signals with receiver constraints such as cyclic frequency offset and sampling clock offset. Different combinations of the fast Fourier transform and slots were used in investigating the test statistic in the presence of the receiver constraints. The combinations that produce higher levels of probability of detection with small number of samples were similar to the combinations obtained previously for the conditions of no receiver constraints.

This shows that even in the presence of receiver constraints of cyclic fre-

quency offset and sampling clock offset the same computational complexity of the fast Fourier transform will be produced which makes the models applicable in both circumstances of receiver constraints and no receiver constraints and fulfilling one of the research objectives. The model was compared against the conventional cyclostationary feature detection under receiver constraints and showed significant benefits over it.

The detection of signals with the test statistic under the receiver constraints was also shown in terms of 2-dimensional plot of the carrier and cyclic frequencies. It was also shown to have wideband capability and was demonstrated with six QPSK signals of different carrier frequencies fulfilling one of the objectives.

Generic algorithms that can be applied for the detection and optimization of the use of this test statistic were developed. These algorithms can be applied to any number of total samples for the desired wideband channel in order to obtain the optimized combination of FFTs and slots. Generic algorithms were developed in order for the detection procedures already discussed to be applicable to different sample sets that represent different signal types and frequencies.

Optimization problems were formulated considering the constraints of cyclic frequency offset and sampling clock offset. The objective of the optimization problem was to produce high probability of detection with small number of samples. This was investigated with the test statistic under the receiver constraints and the method of brute force search was applied. The specific combination of sizes and numbers of fast Fourier transform and slots was obtained. This particular combination was tested with different sample sets and the performance under the receiver constraints were significant. The algorithms provided early exit points if signal is detected. This was made possible by detection with small number of samples. Therefore, the entire wideband need not be processed resulting in faster detection which satisfies one of the objectives.

The performance of the test statistic with the optimized sizes and numbers of fast Fourier transform and slot in the presence of the receiver constraints and under low signal-to-noise ratio has shown significant improvement over the conventional statistic. It was also shown that with the optimized small sizes of fast Fourier transform and slots together with the use of multiple slots which are features of the multi-slot model test statistics do not result in an increase in the computational complexity when compared with the use of the conventional spectral correlation function.

7.2 Future Work

This research adopted the principle that cyclostationary spectrum sensing requires prior knowledge of some of the signals' features for effective detection. An extension of this work can be done by carrying out the analysis of the conventional spectral correlation function using other signal types such as orthogonal frequency division multiplexing. this will follow similar approach adopted in this research but emphasis will be put on the multiple carriers which is synonymous of orthogonal frequency division multiplexing. This will involve carrier frequency offset as an additional receiver offset because of the multiple carriers associated with this type of modulation.

Another extension could be to study a blind cyclostationary feature detection when the signal parameters such as modulation type, carrier frequency are not known. This will involve the estimation of these parameters. The analysis can also be done using the idea of small sizes of fast Fourier transform and slots. This is expected to reduce the uncertainty of estimation which will be large if correlation is wholly applied on large number of sample sets. Such a model can also be compared with the non-blind approach adopted in this research.

A hybrid spectrum sensing method can be investigated by combining the non-blind cyclostationary feature detection and the energy detection method. Such a model will benefit from one of the characteristics of energy detection which is not having prior knowledge of the signal parameters such as modulation type. It will also exhibit robustness to noise associated with cyclostationary spectrum sensing. The hybrid model can be compared with the model in this research for conditions under receiver constraints and for computational complexity.

Further extension of this work could be to study compressive sensing under no prior knowledge of the signal's features. It can then be evaluated to observe how effective the detection and complexity could be using appropriate sizes of FFTs and slots for cases of known and unknown signal features could also be done. Appendices

Appendix A

Derivation of Cyclic Autocorrelation Function

Let us consider a signal x(t) with a fundamental period T and mean m_x ,

$$x(t) = x(t+T) \tag{A.1}$$

$$m_x(t+T) = m_x(t). \tag{A.2}$$

As discussed in [91], periodic signals can be represented using Fourier series coefficients as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t},$$
(A.3)

where

$$w_0 = \frac{2\pi}{T_0},\tag{A.4}$$

is the fundamental frequency and Fourier coefficient a_k , given as,

$$a_{k} = \frac{1}{T} \int_{T} x(t) e^{-jkw_{0}t} dt.$$
 (A.5)

According to [5,91], the autocorrelation function (AF) is periodic in t with period T and expressed as,

$$R_{x}(t,\tau) = R_{x}(t+T,\tau)$$

$$= R_{x}\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right)$$
(A.6)

and can further be expressed as,

$$R_x\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right) = \mathbb{E}\left[x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\right]$$
(A.7)

where \mathbb{E} is the Expectation. The AF is time-based and periodic function of the variable *t*. That is, equation (A.7) is periodic in *t* with period *T* and can also be represented by Fourier series as,

$$R_x\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{-j2\pi\alpha t}$$
(A.8)

for α over all integer multiples *m* of the fundamental frequency of periodicity $\frac{1}{T}$ i.e. $\alpha = \frac{m}{T}$ where α is the Fourier or cyclic frequency, *T* is the period and $R_x^{\alpha}(\tau)$ is the Fourier-series coefficient as discussed in [4],

$$R_x^{\alpha}(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} R_x(t,\tau) e^{-j2\pi\alpha t} dt$$
(A.9)

and it is the Cyclic Autocorrelation Function (CAF). From (A.1) the CAF can be expressed as,

$$R_{x}^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{T} R_{x} \left(t + \frac{\tau}{2}, t - \frac{\tau}{2} \right) e^{-j2\pi\alpha t} dt.$$
 (A.10)

Also, substituting (A.7) into (A.10),

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T \mathbb{E}\left[x\left(t + \frac{\tau}{2}\right)x^*\left(t - \frac{\tau}{2}\right)\right] e^{-j2\pi\alpha t} dt$$
(A.11)

where (*) represents a conjugation and as previously mentioned \mathbb{E} is the expected value of the autocorrelation. The CAF depends on the time-difference or lag τ . Therefore, the generalised non-conjugate CAF can simply be written as,

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x\left(t + \frac{\tau}{2}\right) x^* \left(t - \frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt.$$
(A.12)

The expression of non-conjugate refers to the term on the left of equation A.12. Similarly, the conjugate CAF is expressed as,

$$R_{x^*}^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x\left(t + \frac{\tau}{2}\right) x\left(t - \frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt$$
(A.13)

and notice that none of the factors is conjugated as discussed in [55,95] except the term on the left of equation A.13. The conjugation is to accommodate complex-valued signals.

Appendix B

Derivation of Spectral Correlation Function

The generalised CAF in equation (A.12) can be expressed as the conventional cross-correlation of the two complex-valued frequency-shifted versions of x(t). Recall that multiplying a signal by $e^{\pm j\pi\alpha t}$ shifts the spectral content of the signal by $\pm \alpha/2$, see [55]. This can be illustrated as,

$$u(t) = x(t)e^{-j\pi\alpha t}$$

and (B.1)

 $v(t) = x(t)e^{j\pi\alpha t}$

and the generalised cross-correlation for u(t) and v(t) is given as

$$R_{uv}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u\left(t + \frac{\tau}{2}\right) v^* \left(t - \frac{\tau}{2}\right) dt$$
(B.2)

where v * (t) is the complex conjugate of v(t) [95]. This reverses the sign of the imaginary part of v(t) as in $v^*(t) = x(t)e^{-j\pi\alpha t}$. When (B.1) is substituted into (B.2),

$$R_{uv}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x\left(t + \frac{\tau}{2}\right) e^{-j\pi\alpha(t+\tau/2)} x^* \left(t - \frac{\tau}{2}\right) e^{-j\pi\alpha(t-\tau/2)} dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x\left(t + \frac{\tau}{2}\right) x^* \left(t - \frac{\tau}{2}\right) dt$$

$$= R_x^{\alpha}(\tau).$$
 (B.3)

This shows that CAF can be described as time-averaged cross-correlation between two frequency-shifted versions of the process x(t). The time-based CAF is expressed in equations (A.12) and (B.3) and can be represented in an equivalent frequency domain by taking the Fourier Transform of (B.3). This expresses the Cyclic Wiener relation [55,91] as,

$$S_x^{\alpha}(f) = FT\left\{R_x^{\alpha}(\tau)\right\} = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau)e^{-j2\pi f\tau}d\tau.$$
 (B.4)

where *FT* is the Fourier Transform. It is called the Cyclic Spectrum, spectral correlation Function (SCF) or Spectral Correlation Density Function (SCDF) for a given cyclic frequency α . In [4,5], it is shown that (B.4) is obtainable from the operations in (B.5),

$$S_x^{\alpha}(f) = \lim_{1/T' \to 0} \lim_{T \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} \frac{1}{T} X_T \left(t, f + \frac{\alpha}{2} \right) X_T^* \left(t, f - \frac{\alpha}{2} \right) \mathrm{d}t \tag{B.5}$$

where $X_T(t, f)$ is the short time Fourier transform or complex envelope of the narrow-band spectral component of x(t) with centre frequency f, bandwidth on the order of $\frac{1}{T}$ with T as the period for the observation time T'. The complex envelope is expressed as,

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(t) e^{-j2\pi f t} dt.$$
 (B.6)

The generalised non-conjugate Spectral Correlation Function (SCF) (B.5) shows the limit as spectral resolution becomes infinitesimal $(1/T \rightarrow \infty)$ of the limit $(T \rightarrow \infty)$ temporal correlation of the two spectral components of x(t) with frequencies $f + \alpha/2$ and $f - \alpha/2$. Similarly, from (B.5), the conjugate SCF is given as,

$$S_{x^*}^{\alpha}(f) = \lim_{1/T' \to 0} \lim_{T \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} \frac{1}{T} X_T \left(t, f + \frac{\alpha}{2} \right) X_T \left(t, f - \frac{\alpha}{2} \right) \mathrm{d}t.$$
(B.7)
Appendix C

Spectral Correlation Function OF M-ary PSK Signals

Phase-Shift Keying

A Phase-Shift Keying (PSK) signal x(t) is a Phase-Modulated (PM) in which the phase time series is a digital Pulse Amplitude Modulated (PAM) signal $\phi(t)$ and expressed as,

$$x(t) = \cos(2\pi f_c t + \phi(t)) \tag{C.1}$$

and

$$\phi(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT - t_0)$$
(C.2)

where g(.) is the shaping pulse, a_n is the pulse amplitude, t_0 is the timing parameter for the phase keying. Also, f_c and T are the carrier frequency and period respectively. It can be shown that,

$$x(t) = \frac{1}{2}e^{j2\pi f_c t + j\phi} + \frac{1}{2}e^{-j2\pi f_c t - j\phi}.$$
 (C.3)

After Substituting (C.3) in (16) and applying Fouriet transform as discussed in [4, 5, 55, 98, 169], it can be shown that the spectral correlation function for

BPSK signal is given by,

$$S_{x}^{\alpha}(f) = \frac{1}{4T} \left\{ \left[G\left(f + f_{c} + \frac{\alpha}{2}\right) G\left(f + f_{c} - \frac{\alpha}{2}\right) + G\left(f - f_{c} + \frac{\alpha}{2}\right) G\left(f - f_{c} - \frac{\alpha}{2}\right) \right] e^{-j2\pi\alpha t_{0}} + G\left(f + \frac{\alpha}{2} + f_{c}\right) G\left(f - \frac{\alpha}{2} - f_{c}\right) e^{\left[-j(2\pi[\alpha + 2f_{c}]t_{0} - 2\phi_{0})\right]} + G\left(f + \frac{\alpha}{2} - f_{c}\right) G^{*}\left(f - \frac{\alpha}{2} + f_{c}\right) e^{\left[-j(2\pi[\alpha - 2f_{c}]t_{0} - 2\phi_{0})\right]} \right\}$$
(C.4)

for all integers k, $\alpha = \frac{k}{T}$, where the rectangular pulse is given by,

$$G(f) = \frac{\sin(\pi f T)}{\pi f}.$$
 (C.5)

G(f), 1/*T* and ϕ_0 are the Fourier Transform of the shaping pulse g(t) given in (C.2), the symbol rate and carrier phase respectively. PSK signal is also described as a binary Amplitude-Shift Keying (ASK) signal for M-ary (M) = 2 and for M > 2, it is called a Quadrature Amplitude Modulation (QAM).

$$x(t) = c(t)\cos(2\pi f_c t + \phi_0) = s(t)\sin(2\pi f_c t + \phi_0)$$
(C.6)

where c(t) and s(t) are the time-aligned in-phase and quadrature binary digital PAM signals respectively. Similarly, as discussed in [4, 98, 169], it can be shown that the spectral correlation function for QPSK signal is given as,

$$S_{x}^{\alpha}(f) = \frac{1}{2T} \left[G \left(f + \frac{\alpha}{2} + f_{c} \right) G \left(f - \frac{\alpha}{2} + f_{c} \right) S_{c}^{\alpha}(f + f_{c}) + G \left(f + \frac{\alpha}{2} - f_{c} \right) G \left(f - \frac{\alpha}{2} - f_{c} \right) \right] e^{-j2\pi\alpha t_{0}}$$
(C.7)

for all integers of k, $\alpha = \frac{k}{T_0}$. For more details of the derivations, see [4,5,55,98, 169].

Appendix D

Derivation of the Effect of Phase Offset on the Test Statistic.

A time shifted received wideband signal x(t) can be represented as

$$\bar{x}(t) = x(t - t_s) \tag{D.1}$$

where t_s is the time shift in the time domain and varies as the number of samples increases. It can be expressed in terms of sampling clock offset δ and for all the time samples in the p^{th} slot of size N as,

$$t_p = N\delta p. \tag{D.2}$$

It results in a phase offset $e^{-j2\pi\alpha N\delta p}$ for the p^{th} slot and is accumulated over all the slots in the wideband channel. Therefore, SCF estimate for the p^{th} slot that is affected by the SCO after substituting for t_p is given as,

$$\bar{S}_{x}^{\alpha}(k,p|\delta) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p}\left(n,k+\frac{\alpha}{2}\right) \times X_{L,p}^{*}\left(n,k-\frac{\alpha}{2}\right) e^{-j2\pi\alpha N\delta p}.$$
 (D.3)

Consider a total *P* number of slots, the total phase offset is given as,

$$\phi = \sum_{p=0}^{P-1} e^{-j2\pi\alpha N\delta p} \tag{D.4}$$

where $e^{-j2\pi\alpha N\delta p}$ is the phase offset for the p^{th} slot as explained above. The non-conjugate Test Statistic TS₁ derived in (4.7) of section 4.1 in chapter 4 is a function of the slots and FFTs. The SCF estimates for the p^{th} slot given in (D.3)

are non-coherently added and the resulted test statistic has the attenuated feature, see [137]. This can be represented by substituting the total phase offset ϕ given in (D.4) into the non-conjugate SCF for the wideband channel (4.7) giving a non-conjugate Test Statistic under SCO only as,

$$\bar{S}_{x}^{\alpha}(k|\delta) = \frac{1}{P} \sum_{p=1}^{P} \left(\frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p}\left(n,k+\frac{\alpha}{2}\right) \times X_{L,p}^{*}\left(n,k-\frac{\alpha}{2}\right) \right) \sum_{p=0}^{P-1} e^{-j2\pi\alpha N\delta p}$$
(D.5)

and can be further expressed as,

$$\bar{S}_{x}^{\alpha}(k|\delta) = \frac{1}{P} \sum_{p=1}^{P} S_{x}^{\alpha}(k,p) \sum_{p=0}^{P-1} e^{-j2\pi\alpha N\delta p}$$
(D.6)

where

$$S_x^{\alpha}(k,p) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{L} X_{L,p}\left(n,k+\frac{\alpha}{2}\right) \times X_{L,p}^*\left(n,k-\frac{\alpha}{2}\right)$$
(D.7)

represents the non-conjugate SCF without phase shifts for the p^{th} slot as derived in (4.6). As was used in [137], it can be shown that the analysis of ϕ in (D.4) is,

$$\sum_{p=0}^{P-1} e^{-j2\pi\alpha N\delta p} = \frac{\sin(\pi\alpha PN\delta)}{\sin(\pi\alpha N\delta)} e^{-j\pi\alpha N\delta P-1}.$$
 (D.8)

We can adopt the geometric series discussed in [170], [171] and stated in (D.9), to analyse the phase offset ϕ (D.4) which is same as the summation part (right) in (D.6), i.e, using,

$$\sum_{n=0}^{N-1} e^{jnx} = \frac{1 - e^{jNx}}{1 - e^{jx}}.$$
 (D.9)

Therefore applying the geometric series rule expressed in (D.9) the phase offset ϕ (D.4) becomes,

$$\begin{split} \phi &= \sum_{p=0}^{P-1} e^{-j2\pi\alpha N\delta p} \\ &= \frac{1 - e^{-j2\pi\alpha(PN\delta)}}{1 - e^{-j2\pi\alpha(N\delta)}} \\ &= \frac{-e^{-j\pi\alpha(PN\delta)} \left(e^{j\pi\alpha(PN\delta)} - e^{-j\pi\alpha(PN\delta)} \right)}{-e^{-j\pi\alpha(N\delta)} \left(e^{j\pi\alpha(N\delta)} - e^{-j\pi\alpha(N\delta)} \right)} \\ &= \frac{\sin(\pi\alpha PN\delta)}{\sin(\pi\alpha N\delta)} e^{-j\pi\alpha N\delta(P-1)}. \end{split}$$
(D.10)

Substituting (D.10) in (D.6) the test statistic with SCO effect becomes,

$$\bar{S}_{x}^{\alpha}(k|,\delta) = \frac{1}{P} \sum_{p=1}^{P} S_{x}^{\alpha}(k,p) \frac{\sin(\pi\alpha PN\delta)}{\sin(\pi\alpha N\delta)} e^{-j\pi\alpha N\delta(P-1)}.$$
 (D.11)

Note that if P = 1 (no multi-slot) with zero SCO , (D.11)is equivalent to the conventional SCF as given in (3.17). If $P \ge 1$ and $\alpha N\Delta_s$ is an integer, then looking at the pure real (fraction) in (D.11) we can observe that numerator and denominator are zero for any integral value of $\alpha N\Delta_s$. Consequently, the SCO will have no effect on the TS₁. It follows also, that for any non-integral value of $\alpha N\Delta_s$, the TS₁ will be constrained. Usually, in practice under the presence of SCO $\alpha N\delta$ is non-integral.

Appendix E

Calculations of the FFT Complexities

The following tables were based on the formula of the complex additions in section 2.5.7.2 and the following equations (4.1), (6.2), (6.3) to calculate the complexities of the FFTs.

Given the following,

Total number of FFTs $\frac{NP}{L}$

Complex Additions per FFT

 $N \log N$

Model Complexity (with slots)

 $\frac{NP}{L} \times N \log N$

Conventional Complexity (without slots)

 $N \times P \log N \times P$

Slot	Size	FFT Size L	Total	Complex	Model	Convent.
N			Number of	Additions	Complex-	Complex-
			FFTs	per FFT	ity (6.3)	ity
512		16	512	64	32768	106496
256		16	256	64	16384	49152
128		16	128	64	8192	22528
64		16	64	64	4096	10240
512		32	256	160	40960	106496
256		32	128	160	20480	49152
128		32	64	160	10240	22528
64		32	32	160	5120	10240
512		64	128	384	49152	106496
256		64	64	2384	24576	49152
128		64	32	384	12288	22528
64		64	16	384	6144	10240

Table E.1: Calculations of the FFT Complexities with different slot sizes, P = 16 and f = 5 MHz

Number of	FFT Size L	Total	Complex	Model	Convent.
slots P		Number of	Additions	Complex-	Complex-
		FFTs		ity (6.3)	ity
24	16	384	64	24576	77322
12	16	192	64	12288	35589
6	16	96	64	6144	16258
3	16	48	64	3072	7361
2	16	32	64	2048	4608
24	32	192	160	30720	77322
12	32	96	160	15360	35589
6	32	48	160	7680	16258
3	32	24	160	3840	7361
2	32	16	160	2560	4608
24	64	96	384	36864	77322
12	64	48	384	18432	35589
6	64	24	384	9216	16258
3	64	12	384	4608	7361
2	64	8	384	3072	4608

Table E.2: Calculations of the FFT Complexities with fixed slot size N = 256 samples, $f_s = 4 \times f$, $T_{acq} = 0.15$ millisecond.

Appendix F

FFT Resolutions and Complexities

The objective is to establish the trade-off between the effects of spectral spreading and spectral leakage caused by the use of window and FFT as discussed in section 2.6.1 and frequency resolutions in section 4.1.4. The total complexity is based on the FFT complex multiplications only. The following parameters and equations were used to calculate the total FFT complxities and frequency resolutions in Table F. The number of slots P = 16, sensing time $T_{acq} = 1$ second, frequency f = 1, 2, 3 and 4 KHz, sampling frequency $f_s = 4 \times (f)$ and eqs. 6.4, 6.5, 6.2, 6.1, 6.3 and 4.30.

Slot Size	Sampling	FFT Siz L	Total	Total	Frequency
N	Fre-		Number	Com-	Resolu-
	quency		of FFTs	plexity	tion
512	4000	4	2048	16384	1000
256	8000	4	1024	8192	2000
128	12000	4	512	4096	3000
64	16000	4	256	2048	4000
512	4000	8	1024	24576	500
256	8000	8	512	12288	1000
128	12000	8	256	6144	1500
64	16000	8	128	3072	2000
512	4000	16	512	32768	250
256	8000	16	256	16384	500
128	12000	16	128	8192	750
64	16000	16	64	4096	1000
512	4000	32	256	40960	125
256	8000	32	128	20480	250
128	12000	32	64	10240	375
64	16000	32	32	5120	500
512	4000	64	128	49152	62.5
256	8000	64	64	24576	125
128	12000	64	32	12288	187.5
64	16000	64	16	6144	250

Table F.1: FFT Resolutions and Complexities.

Appendix G

Time domain Window Functions Comparisons

For more details on the window functions, see [86, 87, 89].

Window	Best for these signal	Frequency Resolution	Spectral Leakage	Amplitude Accuracy
Barlett	Random	Good	Fair	Fair
Blackman	Random or mixed	Poor	Best	Good
Flat top	Sinusoids	Poor	Good	Best
Hanning	Random	Good	Good	Fair
Hamming	Random	Good	Fair	Fair
Kaiser-Bessel	Random	Fair	Good	Good
None(boxcar)	Transient and Synchronous Sampling	Best	Poor	Poor
Tukey	Random	Good	Poor	Poor
Welch	Random	Good	Good	Fair

Table G.1: Comparison of Window Functions

Appendix H

Certificate of Ethics Review



Certificate of Ethics Review

Project Title:	Wideband Cyclostationary Spectrum Sensing with
	Receiver Constraints and Optimization
User I D:	376711
Name:	Ikedieze Gabriel Anyim
Application Date:	20/07/2018 18:58:06

You must download your certificate, print a copy and keep it as a record of this review.

It is your responsibility to adhere to the University Ethics Policy and any Department/School or professional guidelines in the conduct of your study including relevant guidelines regarding health and safety of researchers and University Health and Safety Policy.

It is also your responsibility to follow University guidance on Data Protection Policy:

General guidance for all data protection issues
University Data Protection Policy

You are reminded that as a University of Portsmouth Researcher you are bound by the UKRIO Code of Practice for Research: any breach of this code could lead to action being taken following the University's Procedure for the Investigation of Allegations of Misconduct in Research.

Any changes in the answers to the questions reflecting the design, management or conduct of the research over the course of the project must be notified to the Faculty Ethics Committee. Any changes that affect the answers given in the questionnaire, not reported to the Faculty Ethics Committee, will invalidate this certificate.

This ethical review should not be used to infer any comment on the academic merits or methodology of the project. If you have not already done so, you are advised to develop a clear protocol/proposal and ensure that it is independently reviewed by peers or others of appropriate standing. A favourable ethical opinion should not be perceived as permission to proceed with the research; there might be other matters of governance which require further consideration including the agreement of any organisation hosting the research.

GovernanceChecklist A1-BriefDescriptionOfProject: The Statistical development of cyclostationary spectrum sensing in radio communication systems. A2-Faculty: Technology A3-VoluntarilyReferToFEC: No A5-AlreadyExternallyReviewed: No B1-HumanParticipants: No HumanParticipantsDefinition B2-HumanParticipantsConfirmation: Yes

Certificate Code: 4176-B971-6811-7CD6-206A-41FA-DDE4-D76F Page 1

Figure H.1: Ethics Review Certificate.

Appendix I

Form UPR16

FORM UPR16

Postgraduate R	esearc	h Stud	ent (PGRS	S) Information	1	Student ID:	376711	PORTS	MOUTH
PGRS Name:	IKE	DIEZE	GABRIEL	ANYIM					
Department:	EN ELE	IERGY ECTRO GINEEI	AND NIC RING	First Supervisor:		DR JOHN CHIVERTON			
Start Date: or progression date	for Prof	Doc stud	lents)	01/02/2015					
Study Mode and Route:):	Part-time Full-time	MPhil			MD Professional Doctor		ate
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Figure I.1: Ethical Checklist.

Chapter 8

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