STATISTICAL ANALYSIS OF INTERCROPPING DATA USING A CORRELATED ERROR STRUCTURE*

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1. INTRODUCTION

Farmers, in the semi-arid tropics, have practised for generations the growing of two or more species in conjunction with one another (intercropping) and experimenters have recognized the need to investigate such systems, (Willey, 1979a, b). Statistical techniques for intercropping have been reviewed by Mead and Riley (1981).

A model, for a single factor design, to study two species grown as sole and as an intercrop is discussed in section 2, with a competition coefficient derived in terms of the land equivalent ratio (LER). (Anon, 1972). Estimation of the models' parameters and tests of hypotheses of treatments are discussed in section 3 with an optimum choice of treatments considered in section 4.

2. MODEL STRUCTURE

For an investigation on a two species intercrop, let there be *p, q* and *pq* treatments, tested on each single species and intercrop respectively, in a randomized block design for simplicity. The technique however, can be generalized to other designs.

Let Y^{o}_{1ij} and $Y^{o}_{2i'j}$ be the yields from the two species when grown as sole crops in the jth block ($j=1,\ldots,r$) with treatments i and i' ($i=1,\ldots,p$; $i'=1,\ldots,q$) applied to the sole crops. The model to study treatment behaviour in sole crops can be taken as the widely used one.

$$Y^{o}_{1ij} = \zeta_{1i} + \beta_{1j} + \varepsilon_{1ij}$$

$$Y^{o}_{2i'j} = \zeta_{2i'} + \beta_{2j} + \varepsilon_{2i'j} \qquad (1)$$

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where ζ_{1i} and $\zeta_{2i'}$ are the effects of ith and i'th treatments and β_{1j} and β_{2j} are jth block effects for the two species respectively.

ε_{1ij} and $\varepsilon_{2i'j}$ are assumed independent random errors with mean zero and variances σ^2_{10} and σ^2_{20}.

Further on the intercrop, let $Y_{1ii'j}$ and $Y_{2ii'j}$ be the yields of two species with treatment (i,i') applied in jth block. We consider the following model for intercrop yields.

$$Y_{1ii'j} = \zeta_{1i} + \delta_{1ii'} + \beta_{1j} + \varepsilon_{1ii'j}$$

$$Y_{2ii'j} = \zeta_{2i'} + \delta_{2ii'} + \beta_{2j} + \varepsilon_{2ii'j} \qquad (2)$$

$\delta_{1ii'}$ measures the influence of the second species on the first with treatments (i,i') applied. Similarly, $\delta_{2ii'}$ is the measure of the influence of the first species on the second i.e. the competition effects between the two species in the presence of treatments (i,i'). With no competition $\delta_{1ii'}$ and $\delta_{2ii'}$ are zero. Further $\delta_{1ii'}$ and $\delta_{2ii'}$ may relate to treatments in the following way.

$$\delta_{1ii'} = \zeta_{1i}.\alpha_{1ii'}$$

$$\delta_{2ii'} = \zeta_{2i'}.\alpha_{2ii'}$$

where $\alpha_{1ii'}$ and $\alpha_{2ii'}$ are the competition coefficients defined as the LER by

$$\alpha_{1ii'} = (E(\bar{Y}_{1ii'j})/E(\bar{Y}^{0}_{1ij}))-1 = \text{LER1}-1$$

with LER1, the LER of the first species. Similar relationships can be obtained for the second species.

$\varepsilon_{1ii'j}$ and $\varepsilon_{2ii'j}$ are random errors with means zero and variances σ^2_1 and σ^2_2 respectively, assumed to be different from (1) because the variability in species may differ in the intercrop compared to the sole crop situation (K.Sakai, 1955, McGilchrist, 1965). We further assume that the correlation $\rho_{ii'}$ between $\varepsilon_{1ii'j}$ and $\varepsilon_{2ii'j}$ is a function of the treatments. This is an extension of the correlation structure given by Pearce and Gilliver (1978) who analysed intercropping data assuming constant ρ between errors.

The estimation of $\rho_{ii'}$ for each combination of treatments requires many replications to obtain efficient estimates of correlation coefficients. However, if the interactions of treatments have an insignificant effect on the correlation coefficients, the model $\rho_{ii'} = \rho_0\rho_{1i}\rho_{2i'}$ *may represent the* structure and require estimates of only $1+p+q$ correlation parameters. However, the testing of these correlation models are under study and will be reported later.

The number of parameters to be estimated from models (1) and (2) are $p+q+3pq+2r+4$ from $r(p+q+pq)$ observations. Thus we require

$$r>(p+q+3pq+4)/(p+q+pq-2) - 1$$

3. ESTIMATION OF PARAMETERS AND TESTING OF HYPOTHESES

Maximum likelihood estimates (MLE) of the parameters and likelihood ratio testing for various hypotheses on competition and treatment effects can be obtained assuming that the errors in models (1) and (2) have univariate and bivariate normal distributions with previously structured parameters. (Kendall & Stuart, 1979). The MLE of ζ's, δ's and α's are

$$\hat{\zeta}_{1i} = \bar{Y}^{0}_{1i} \qquad \hat{\delta}_{1ii'} = \bar{Y}_{1ii'} - \bar{Y}^{0}_{1i} \qquad \hat{\alpha}_{1ii'} = (\bar{Y}_{1ii'}/\bar{Y}^{0}_{1i}) - 1$$

$$\hat{\zeta}_{2i'} = \bar{Y}^{0}_{2i'} \qquad \hat{\delta}_{2ii'} = \bar{Y}_{2ii'} - \bar{Y}^{0}_{2i'} \qquad \hat{\alpha}_{2ii'} = (\bar{Y}_{2ii'}/\bar{Y}^{0}_{2i'}) - 1$$

MLE's of other parameters and variance-covariance matrix of MLE's , we refer to Singh and Gilliver (1983). An approximation to the bias (B(.)) and mean squared error (MSE(.)) are below, retaining only terms up to $(1/r)$.

$$B(\hat{\alpha}_{1ii'}) = (1+\alpha_{1ii'})\sigma^2_{10}/(r\zeta^2_{1i})$$

$$B(\hat{\alpha}_{1ii'}) = (1+\alpha_{2ii'})\sigma^2_{20}/(r\zeta^2_{2i'})$$

$$MSE(\hat{\alpha}_{1ii'}) = (\sigma^2_1 + (1+\alpha_{1ii'})^2\sigma^2_{10}/\zeta^2_{1i})/r$$

$$MSE(\hat{\alpha}_{2ii'}) = (\sigma^2_2 + (1+\alpha_{2ii'})^2\sigma^2_{20}/\zeta_{2i'})/r$$

$$B(\hat{\alpha}_{1ii'}, \hat{\alpha}_{2ii'}) = \rho_{ii'}\sigma_1\sigma_2/(r\zeta_{1i}\zeta_{2i'})$$

Experimenters may wish to examine competition between species arising from various combinations of treatments. Thus for a treatment combination (i,i') $i=1,\ldots p$; $i'=1,\ldots q$ the hypothesis of no competition is $\alpha_{1ii'} = \alpha_{2ii'} = 0$ and the test statistic

$$W_{ii'} = (\hat{\alpha}_{1ii'}, \hat{\alpha}_{2ii'})D^{-1}(\hat{\alpha}_{1ii'}, \hat{\alpha}_{2ii'})'$$

can be used, where D is the (estimated) dispersion matrix of $(\hat{\alpha}_{1ii'}, \hat{\alpha}_{2ii'})$. The asymptotic distribution of $W_{ii'}$ is chi square with two degrees of freedom. Further $W_{ii'}$ can be taken as a measure for choosing treatment combinations leading to maximum or minimum competition, if they exist in the region of the treatments under study.

4. OPTIMUM TREATMENT

If treatments are qualitative then min $W_{ii'}$ or max $W_{ii'}$ may give the optimal treatment. When the treatments are quantitative, denoted by P_i, $i=1,\ldots p$; $Q_{i'}$, $i'=1,\ldots q$ then a (second degree) surface in the model given below can be fitted with $\sqrt{W_{ii'}}$ and P_i, $Q_{i'}$ with pq treatment combinations.

$$\sqrt{W_{ii'}} = a_0 + a_1 P_i + a_2 P_i^2 + b_1 Q_{i'} + b_2 Q_{i'}^2 + c P_i Q_{i'} + \epsilon_{ii'}$$

The coefficients a_0, a_1, a_2, b_1, b_2, and c can be estimated using least squares technique. Let the fitted surface be

$$\sqrt{W} = \hat{a}_0 + \hat{a}_1 P + \hat{a}_2 P^2 + \hat{b}_1 Q + \hat{b}_2 Q^2 + \hat{c} PQ$$

Then the optimum treatment combination P_{opt} & Q_{opt} are given by

$$P_{opt} = (\hat{c}\hat{b}_1 - 2\hat{b}_2\hat{a}_1)/(4\hat{a}_2\hat{b}_2 - \hat{c}^2)$$

$$Q_{opt} = (\hat{c}\hat{a}_1 - 2\hat{a}_2\hat{b}_1)/(4\hat{a}_2\hat{b}_2 - \hat{c}^2)$$

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SUMMARY

Intercropping has been widely recognized as an important farming system in the Semi-Arid Tropics. The analysis of sole and intercrop data from two species has been suggested using a statistical model with competition coefficients and correlated error structure. A procedure for obtaining optimum combination of treatments has also been advocated.

RESUME

Dans les zones tropicales semi-arides, l'association des cultures represente un important système de production agricole. Les données sur les cultures pures et associèes de deux espèces sont analysées á l'aide d'un modèle statistique basé sur des coefficients de concurrence et la corrélation des erreurs. Un processus permettant une combinaison optimale des differents traitements est proposé.