NONLINEAR ANALYSIS OF LOUDSPEAKER

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Abstract

This project developed a measurement system to investigate the behavior of a loudspeaker on a frequency band. This system works with Network-analyzer (MU-Box manufactured by Mimosa Acoustics), a breakout box that handles CH-OUT and CH-IN, and a circuit board driving the impedance load via an amplifier and 7 Volts voltage source. The purpose is to determine characterization and to model nonlinearity of audio speaker by using the speaker harmonic distortion, representation of nonlinearity, to evaluate hypotheses that the pressure of speaker should be proportional to the current passing through speaker. Theories used in the measurement include Op-Amp amplifier, Thevenin Equivalent, harmonic observation, Maxwell Equation and 2-port Network. During several experiments, the existing problems include limitation of amplifier and power supply, communication between hardware and software. Chirp and pure tones are used as stimulus to measure impedance and harmonic distortion respectively. The final result verifies the stated hypotheses by the pattern of second harmonic for both pressure and current, which provides significant foundation of future work.
Subject Keywords: Impedance; linearity; 2-port Network; Transducers; Admittance; Harmonic Distortion; Maxwell Equation; Thevenin Equivalent

Contents

1. Introduction

1.1 Hypotheses, Purpose and Task

The basic hypotheses for this entire work is the statement that pressure of a speaker is proportional to current. Additionally, the purpose of this project was to characterize and build model for nonlinearity of a loudspeaker, via self-built measurement system, and also to use the nonlinearity to evaluate hypotheses. The primary task is to view nonlinearity correlation of the target speaker by analyzing the pressure and current harmonic distortion given pure tones stimulus, or by visualizing impedance in increasing gain levels given chirp stimulus signal. The plot of ratio in impedance at different levels should be generated and used to characterize the impedance, while the plot of harmonic for both pressure, current and voltage will show the nonlinear correlation.

1.2 Outline of Work

During the first semester, the introduction step is to review concepts of transducer via Noori's paper (2013) "Two-port network analysis and modeling of a balanced armature" [1] and get familiar with MU-box, which is network analyzer and will be explained and introduced in section 3.1.1, by making some basic measurement on merely resistors and capacitors. Because of the inconsistence of amplifier and reliability of the power supply, much time had been spent to determine the correct circuit that was able to precisely perform the measurement.

For the second half of the semester, some problems with amplifiers have occurred. For example, no matter which value of gain level, the plot of impedance always had massive ripples and noise. The reason why it happened is that all the amplifiers used had burned out, probably due to unexpected misconnection. The result became smooth after replacing all of them with new ones. In order to achieve the calibration of impedance, chirp signal was fed into speaker, giving a response of speaker at wide band frequencies. Ripples have been observed at low frequencies probably due to distortion inside chirp. Therefore, harmonic distortions are measured through pure tones, which only have value at one frequency per tone. Harmonic for pressure and current has been plotted and compared to that of voltage.

An initial MATLAB code was provided to generate a chirp passing through a MU-box, feeding into audio speaker, and then taking the response back for analysis. This version is used to simply view impedance of audio speaker in magnitude and phase. In order to further investigate the nonlinearity, some portion of code has been modified and the final version can plot impedance in different gain levels and correspond ratios to the normalized level. For the pure tones and harmonics, the signal is made in FFT frequencies and then take the Inverse Fast Fourier Transform to get pure tones in time domain. 2^{nd} and 3^{rd} harmonics are pulled out and plotted as a function of fundamental frequency for both pressure and current. Furthermore, to process the given measurement of a hearing aid, a more completed version has been implemented to a plot polar plot and remove group delay, which will be mentioned in Section 3.3.

2. Methods

2.1 Hardware

The essential component of this project is network-analyzer, which was called the "MU-box" manufactured by Mimosa Acoustics, Inc. in Champaign, IL. It has two output channels, CH0-OUT and CH1-OUT, 2 input channels, CH0-IN and CH1-IN. To sample the input signal in a time-synchronous manner, MU-box used a full-duplex codec. The response blocks are simultaneously captured and time-averaged block by block, which is called overlap add averaging (OLA), preventing random noise. There is a 24-bit build-in audio codec (Cirrus Logic CS4272) in MU-Box performing the job of communication between input and output. To measure the linear and nonlinear characteristics, MU-box, a circuit with op-amps driving the loudspeaker to MU-Box, which assembled on a breadboard (Figure 1 and Figure 2) and power supply are needed as basic components. Figure 1 is the measurement circuit taking input from CH0-OUT and passing stimulus signal through a loudspeaker, and then returning to CH1-IN. The purpose of CH0-IN is used to calculate the impedance of loudspeaker via voltage division method, which will be mention in Section 3.3.

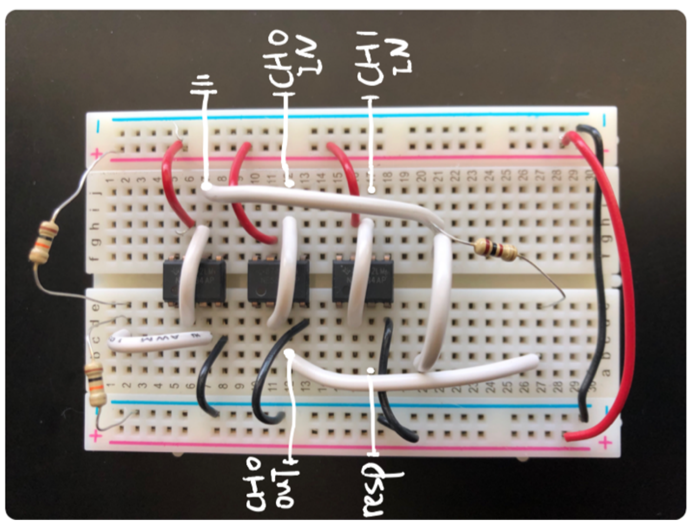


Figure 1: The assembled circuit board

For the selection of power supply, it was found that batteries are unsafe to use, because they easily die out and are unstable, so that DC power supply was chosen to drive the op-amps. Firstly, two A23 batteries with voltage +12 were used to power all the amplifiers. From Figure 1, the upper two 100 omegas resistors and the first op-amp are used as voltage divider. To be specific, looking from positive side, voltage is 6V; from negative side, voltage is -6V; the output of op-amp1 is the virtual ground.

However, from the output displayed on the oscilloscope, power with batteries has two potential problems: batteries are not long-lasting, since there is high demand of measurement, and second, amplifiers draw too much current and power so that it is highly possible the result would be

distorted. After consideration of these limitation, the discrete DC power supply has been selected finally. In addition, one of convenience is that +-6V can be easily generated.
Besides the problem of power supply, the selection of op-amp is also significant. It amplifies the output of the signal generator, in order to deliver sufficient power to drive the loudspeaker.
Before choosing for the perfect amplifier, NE5534, the ones generated mass of oscillation ripples, which is not expected. The reason for the ripples probably that the maximum input voltage and current are so small that amplifier had burn out, incurring an occurrence of oscillation ripples.

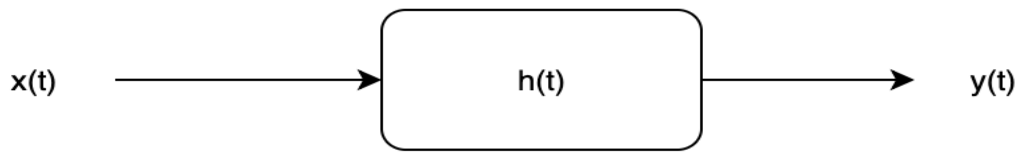


Figure 2 Linear Loudspeaker System

2.2 Derivation and Theory

2.2.1 Linear System

Of the most common practices to specify the characteristics of a loudspeaker is frequency response. However, frequency response is purely a linear measurement, so that I made measurement under the assumption that a loudspeaker is a linear system. For a linear system, showed in Figure 2, where x(t) is an input signal, a chirp signal in this project, h(t) is the impulse response of the ideal loudspeaker, and y(t) is output, a sound pressure.

$$y(t) = x(t) \times \mathrm{h(t)} \tag{1}$$

Denoting X(f), Y(f), and H(f) as the frequency representation of x(t), y(t) and h(t)

$$H(f) = \frac{Y(f)}{X(f)} \tag{2}$$

To visualize the nonlinear distortion, admittance of a loudspeaker and pressure wave will be calibrated at different gain levels. From the plot of change of these two parameters, it is more obvious to show the relationship of nonlinearity with increasing gain levels and frequency levels. The methods and theory that used to calibrate admittance and pressure are stated in Section 3.2.2.

2.2.2 Thevenin Equivalent

To achieve impedance of audio speaker, the theory of KVL has been utilized. The current going through Z and R_{ref} is the same, so that when the voltages drop across these two components and reference resistor are known, impedance under measurement can be obtained by its voltage and current. Therefore, there are two inputs CH0-IN and CH1-IN, which represent V1 and V0 respectively. The impedance Z can be determined by:

$$I = \frac{V_1}{R_{ref}} = \frac{V_0 - V_1}{Z} \; [A] \tag{3}$$

$$Z = R_{ref} \times \frac{V_0 - V_1}{V_1} \; [\Omega] \tag{4}$$

With multimeter, the average magnitude of the studio speaker is measured at around 40 Ω. To maintain the similar voltage drop across the test impedance and reference resistor, 100 Ω resistor has been chosen as the reference resistor.

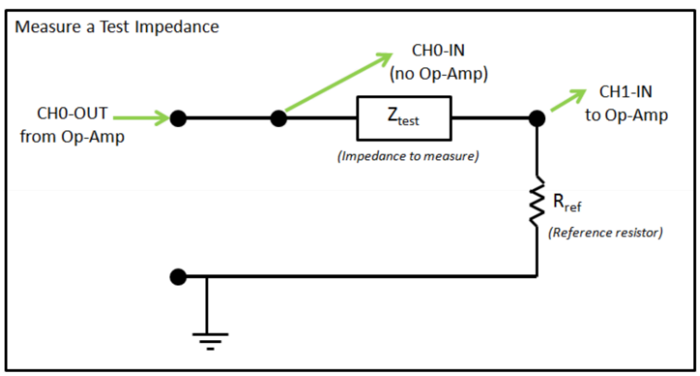
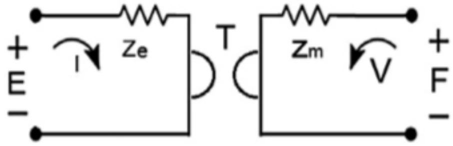


Figure 3: Thevenin circuit of impedance measurement

2.2.3 Transducer and Maxwell Equation

A transducer converts energy among electrical, mechanical and acoustics components. All three parts contribute to the frequency-dependent impedance. The specific hearing-aid receiver is an electromagnetic loudspeaker that converts voltage and current into force and velocity and then converts force and velocity into pressure and volume velocity. An electromagnetic transducer is mainly made by two significant components: semi-inductor and gyrator. As Noori mentioned in her Hearing Research Paper in 2013 [1], Hunt's 1945 publication [2] stated first implementation of anti-reciprocity in electromagnetic transducer model using a gyrator. The basic two-port impedance matrix converts voltage and current into mechanical force – pressure and velocity.



$$\begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} 1 & Z_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & T \\ T^{-1} & 0 \end{bmatrix} \begin{bmatrix} 1 & z_m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F \\ -V \end{bmatrix}$$

Figure 4: Schematic circuit of electromagnetic transducer and equivalent ABCD matrix, where E is the voltage, I is the current, F is the force or pressure and V is the velocity. Adapted from [1].

According to the Maxwell Equation, force is proportional to current and magnetic field [4]. Since magnetic field is constant, what needed to prove is the relationship of mechanical force, which is pressure and current by harmonic distortion, the representation of nonlinearity. Furthermore, the nonlinearity of current comes from impedance, the pattern of voltage should be different from that of current and pressure, so that there should be distinctive relationship between voltage and pressure.

2.2.4 Total Harmonic Distortion

Nonlinear distortion of speakers is typically described with a simple percentage [3], %THD (Total Harmonic Distortion), which represents the percentage of total output energy is devoted to harmonic distortion. Harmonic distortion is characterized by the presence of harmonics in the output not present in the excitation signal. Any nonlinearity in a transfer function – electrical or mechanical – can create harmonics. Typically, a tone at fundamental frequency F0 will generate harmonics distortion at nF0 with n=2, 3, 4, etc. Therefore, by observing the correlation of amplitude of harmonic in different fundamental frequency with pressure and current, we can describe the extent and relationship of nonlinear properties.

2.3 Measurement

2.3.1 Stimulus Signal – Chirp

Among different stimulus signals that can be used as measurement, a chirp signal was chosen, which was usually used to measure the frequency response and impedance of the loudspeaker. A linear chirp signal is a sinusoidal wave that increases in frequency linearly over time as shown in Figure 5 and Figure 6.

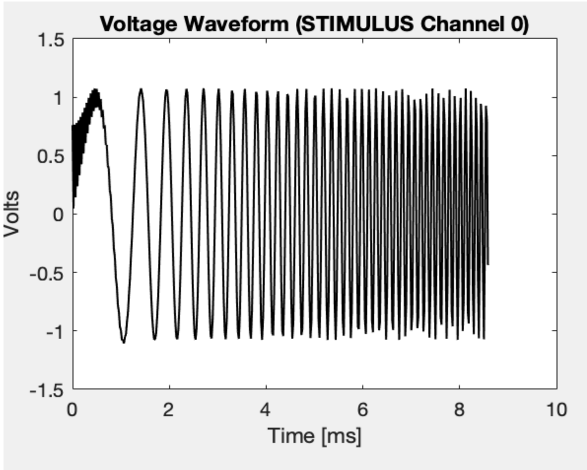


Figure 5: Waveform of stimulus signal: Chirp

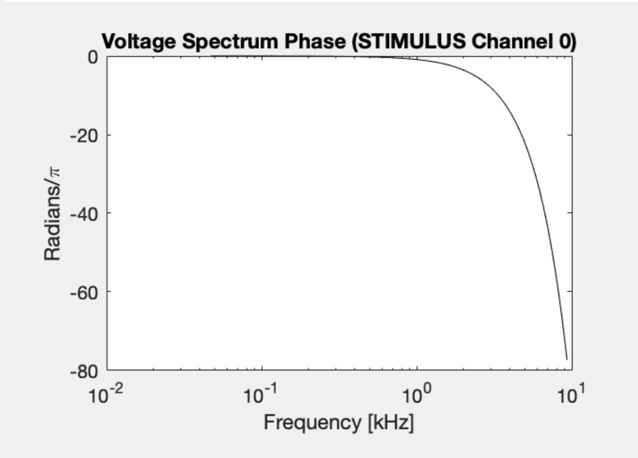


Figure 6: Phase of chirp signal.

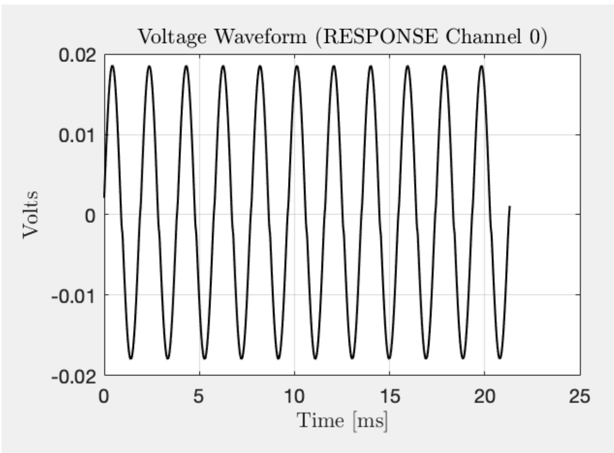
In the frequency domain:

$$C(\omega) = e^{-j\varphi(\omega)} \tag{5}$$

The reason for using the chirp signal rather than pure tones to measure the impedance of loudspeaker is that it covers continuous frequencies in increasing levels, and it has equal energy at all frequency. Therefore, it is more convenient to get data at all frequencies. However, to measure the impedance over a large frequency range using tones, it would take long time to generate different tones.

2.3.2 Stimulus Signal – Pure Tones

Definition of pure tone is the sine wave in time domain and impulse signal in frequency domain. In order to generate such tone, it was significant to use FFT frequencies, which is exactly periodic with length NFT. Sine wave in time domain can be obtained by function fsst() in Matlab.



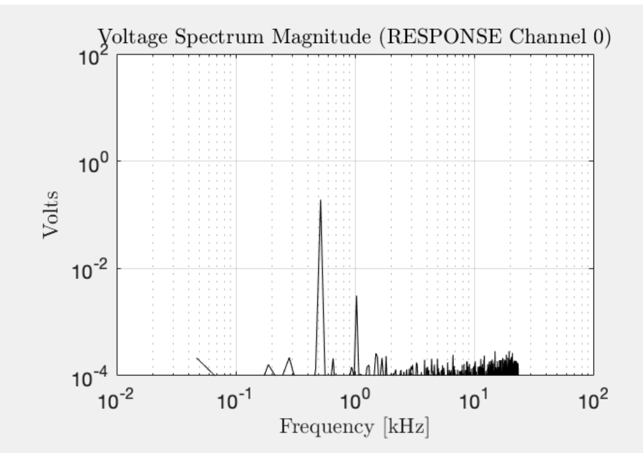


Figure 7: Voltage waveform of response is a sine wave at fundamental frequency F0 = 0.5kHz, while voltage spectrum magnitude is impulse function with harmonics in nF0, n = 1, 2, 3, 4, etc. High order harmonics drop fast with n, so that they typically disappear after n = 5.

2.3.3 Time-Averaging – Dechirp

When the output stimulus signal has been captured, the response blocks are simultaneously time-averaged block by block. Thus, the noise has been averaged out as well as the phase of the stimulus, called De-chirp. To do this, both the stimulus block and response block are Fourier transformed by FFT with NFT to be a power of 2, typically 1024. The MU-box will repeat the output stimulus several times and simultaneously record the response of two channels.

2.3.4 Pressure Ratio Measurement

The pressure response measurement can be achieved using a microphone. The microphone tubing was placed under the up-side-down bell-shaped speaker and then after each measurement, taking pressure in frequency band from output of microphone. Since this experiment needs three input channels to get pressure ratio with calibrated level.

2.3.5 Group Delay

The definition of group delay is the derivative of phase at one particular point, which is a linearization of the phase response at that point. Group delay represents how the phase response of the neighboring frequencies relate to the phase response at that point. In the equation (6), θ represents the phase of signal, while $\tau(\omega)$ represents the group delay. For the linear phase response, τ is the negative of the phase, known as the phase delay, while for nonlinear phase response, it specifies the delay experienced by a narrow band group of signals which have frequencies within a narrow frequency interval $\Delta\omega$.

$$\tau(\omega) = -\frac{d\tau(\omega)}{d\omega}\,[\mathrm{ms}] \tag{6}$$

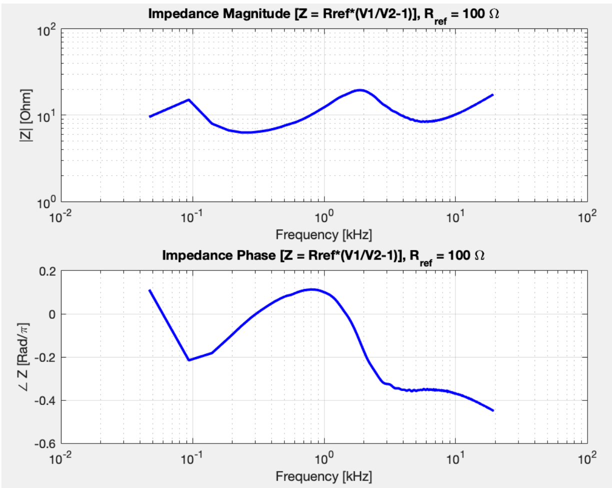


Figure 8: Magnitude and phase plot of the measured studio loudspeaker at a single gain of 0.5.

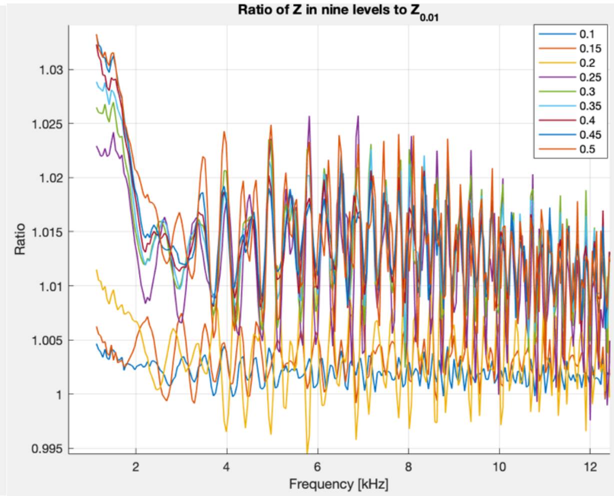


Figure 9: Magnitude ratio of impedance in different gain through voltage drive with range between 2 and 22 kHz.

3. Description of Research Results

Section 4.1 concludes the characteristics of the studio loudspeaker, while Section 4.2 and 4.3 include measurement analysis of a hearing aid.

3.1 Impedance Plot of Studio Loudspeaker

Figure 8 is the result of studio loudspeaker of experiment at a gain of 0.5. Obviously, both the magnitude and phase of impedance change with frequency, and at high frequency, there is a small amount noise shown as ripples. At roughly 2 kHz, the magnitude reaches its local maximum, while the phase is decreasing. From this plot of the pure level, it is hard to find out any information about nonlinearity. Thus, in order to visualize it, using a toning gain levels is efficient. Figure 9 depicts ratios of magnitude in nine different levels. The first gain 0.01 was used as the normalized level by dividing all the other magnitudes, which results in nine ratio functions with respect to frequency. All nine ratios maintain in the value of roughly 1, with periodic ripples at low frequency.

In the Figure 9, the ratios are all almost constant at one except with small ripples probably due to distortion of chirp stimulus. Therefore, it is better to use pure tones as stimulus signals. Overall conclusion draw for the studio loudspeaker is probably that when it was produced, manufacturer have already modified the linearity tones of times to keep improving the hearing quality.

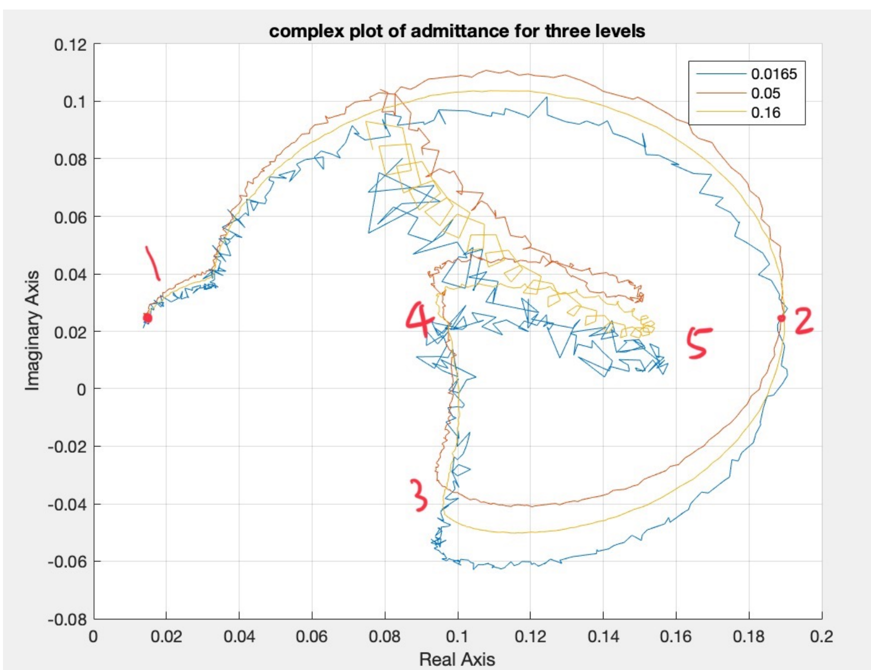


Figure 10 This plot shows the polar form of admittance of loudspeaker at three different levels. The change on real axis represents the change of magnitude show on upper portion of Fig. 11. The marked digits correspond with specific frequencies shown in Fig. 11.

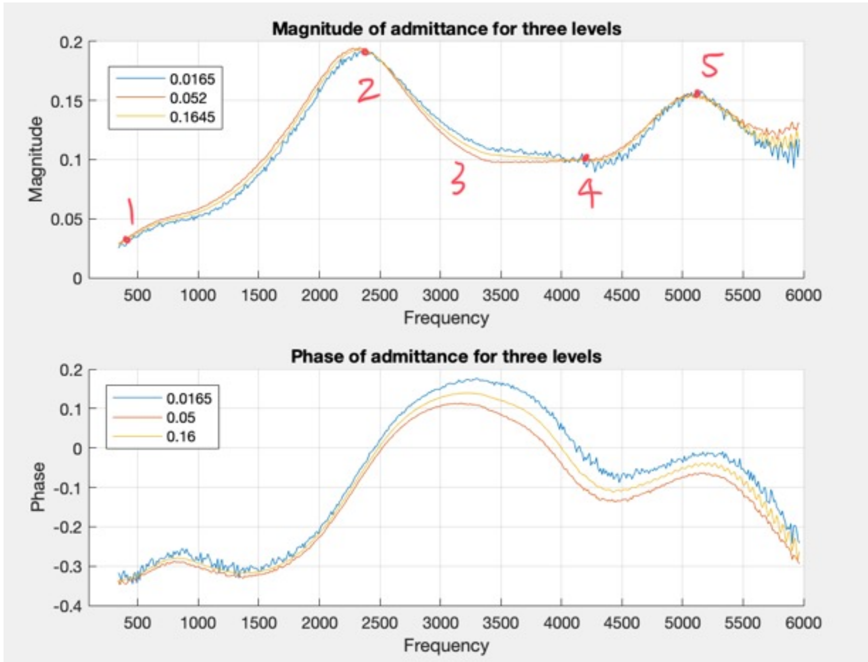


Figure 11: Magnitude and Phase of admittance in three levels. At frequency from 2500 Hz to 4000 Hz, both magnitude and phase plot show an obvious nonlinearity, especially in phase.

3.2 Admittance Plot of Hearing Aid

Because of failure to find out nonlinear characteristics by calibration in the studio speaker, the hearing aid was used as replacement. The datasets are collected in Mimosa Acoustic, with the same data type used previously, except the levels are fixed in three: 0.0165, 0.052 and 0.1645. Each level has its own pressure and impedance complex raw data, which should be pre-

processed before analysis. Admittance Y was taken to plot both polar plot and magnitude, phase plot, by taking the reciprocal of impedance Z. Figure 10 depicts the polar form of data, which is the truncated version taking frequency range from 250 Hz to 6 kHz, otherwise at extremely high and low frequency band, the wanted complex loop would be hidden behind noise at boundary.
In the Figure 10, three loops in the polar form are pointwise corresponding to the magnitude and phase in Figure 11. To be specific, starting from leftmost labeled point #1 in Figure 10, which is starting point from 250 Hz in Figure 11, point #1 move along real axis while magnitude raises until reaches point #2, the rightmost labeled point and also an intersection of three levels. After point #2, the real part of plot decreases to position #3, and then keep constant in real axis, so that in magnitude plot of Figure 11, three levels stay at 0.1 from 3500 to 4000 Hz. With the same theory, phase plot is representation of imaginary axis. Compared with magnitude plot in Figure 11, phase plot shows more separation between levels, from which the conclusion that nonlinear characteristics exist more in phasor can be drawn.

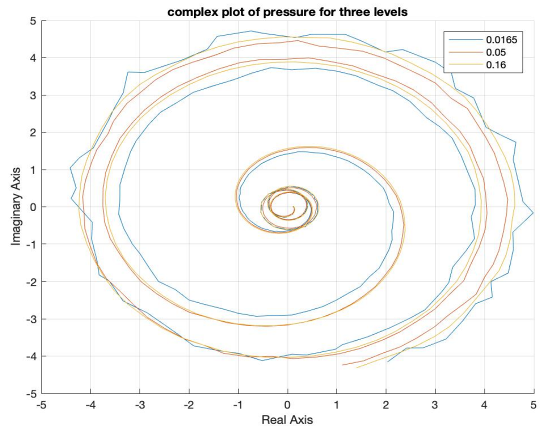


Figure 12: This polar plot shows pressure of speaker in terms of frequency at different three levels

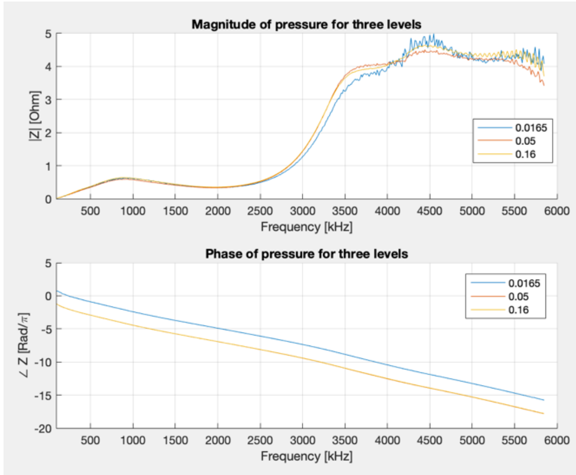


Figure 13: Magnitude and phase plot of pressure.

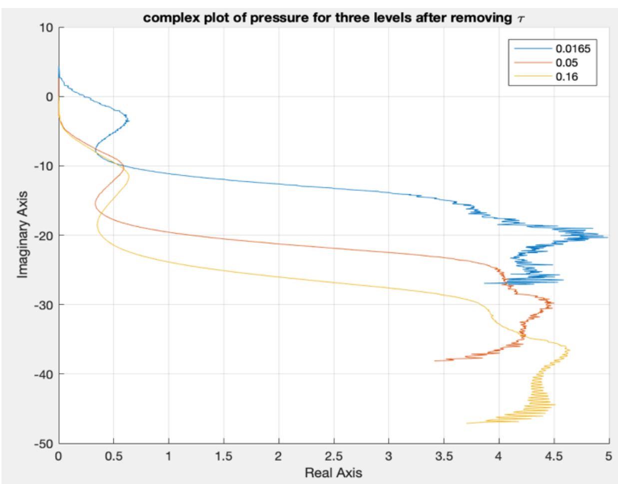


Figure 14: This complex plot is transformed from Figure 12 by subtracting the group delay.

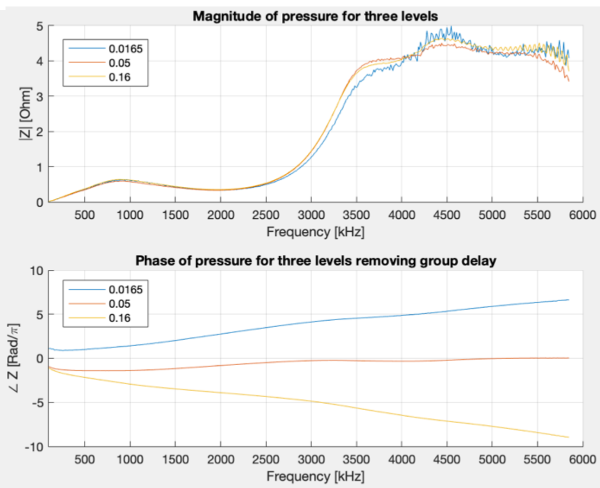


Figure 15: Magnitude and group delay of pressure, after removing the group delay.

3.3 Pressure Plot of Hearing Aid

Compared with the phase plot of admittance in lower part of Figure 11, the phase of pressure in lower part of Figure 13 depicts linear function of frequency with negative slope. From the plot of polar form, the swirling loop start from center of origin and extend around the origin, but for admittance, the origin locates in the lower left side of the loop instead of central. Due to group delay and phase delay, the complex plot in Figure 12 shows circular loops. To avoid it, group delay was subtracted from the unwrapped phase of original signals, shown in Figure 14 and Figure 15. Apparently, compared to the magnitude part, the phase plot is more useful to view the nonlinearity: with the increasing levels, the slopes of phase decreases.

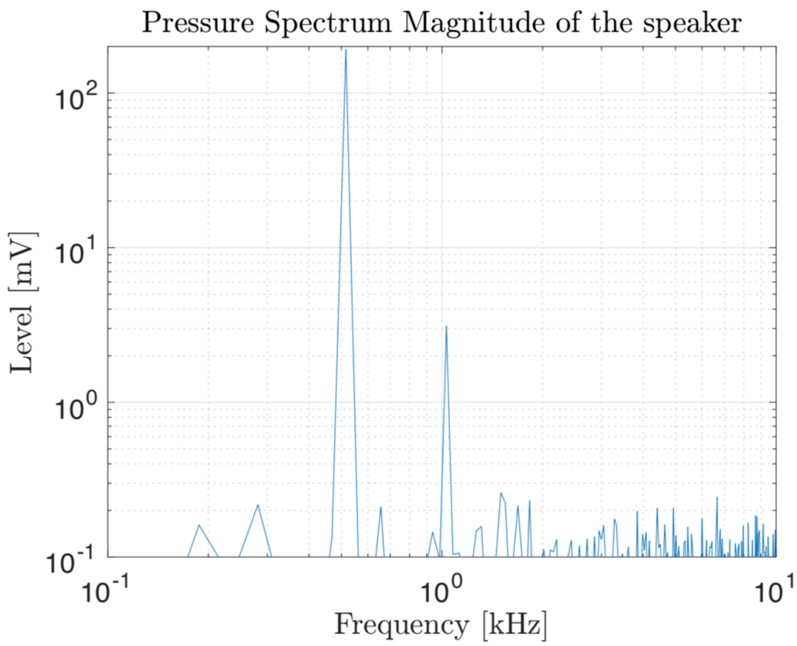


Figure 16: Pressure spectrum of speaker at 500Hz and a gain of 0.5. First impulse was the response of sine wave in fundamental frequency f0 = 500Hz, and the second impulse is the first harmonic sitting in 2*f0 = 1kHz. The magnitude of the first impulse is about 100 times the second impulse, which is the first harmonic.

3.4 Harmonic Distortion for Pressure and Current

Since there is no obvious nonlinearity reflect from impedance, the better approach is to visualize nonlinear property via harmonic distortion produced by pure tones stimulus, because any nonlinearity in a system can create harmonics. Figure 17 depicts the relationship of the current or pressure harmonic and voltage intensity in fundamental frequency from 0.5kHz to 3kHz respectively. Voltage intensity is toned set by a gain in the range of 0.05 and 0.5 in 0.5 interval. Current harmonics were calculated by dividing the voltage of CH1 by Rref, since speaker and reference resistor share the same current. (See Fig. 3) Pressure sound waves for each tone were collected through the microphone. From the plot, the slope of second harmonic is twice of first harmonic for both pressure and current, and the ratio is about a factor of 100.

In the Figure 18, the second harmonic for pressure, current and voltage of speaker has been pulled out and plotted as a function of fundamental frequency. Obviously, after the scaling of offset at 1kHz, the patterns of curve for pressure and current are exactly the same at low frequencies, which proves that there is a high correlation between both pressure, current with nonlinearity, harmonic distortion. In addition, the shapes of current and pressure are distinctive compared with that of voltage.

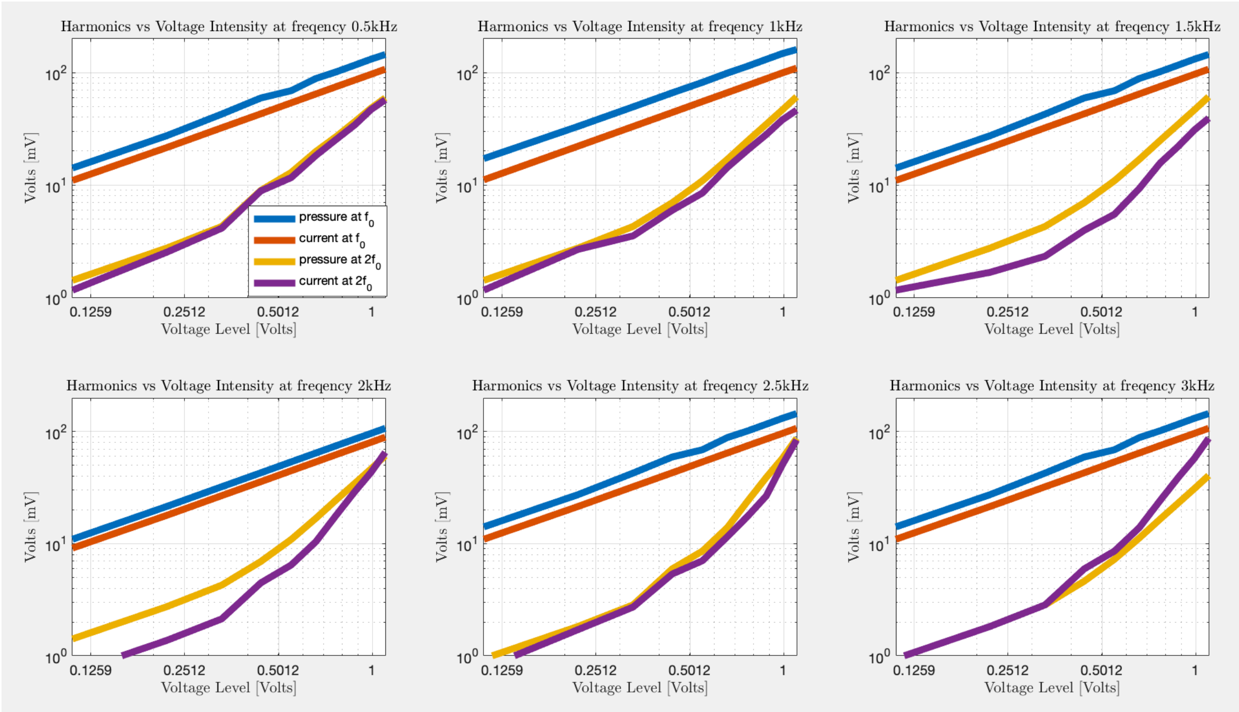


Figure 17: Trend of pressure and current at f0 and 2f0, first and second harmonic as a function of current intensity at 500Hz pure tone.

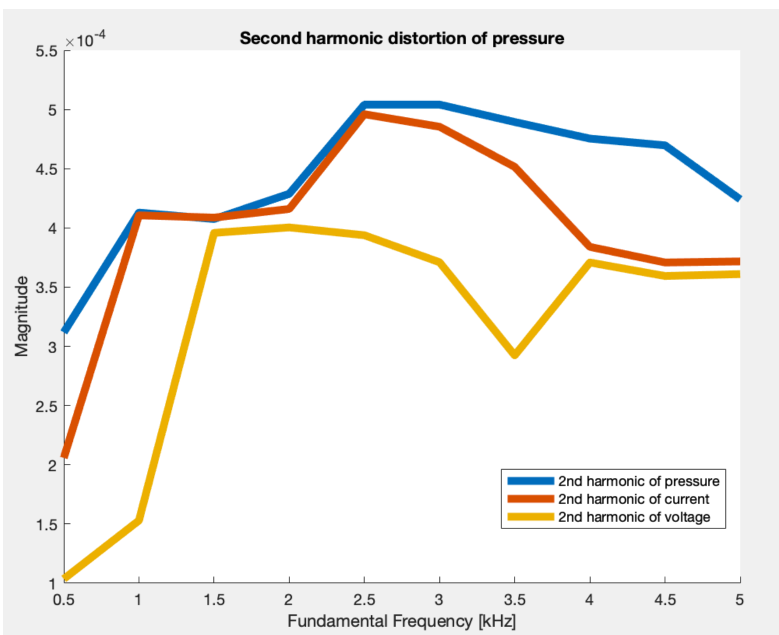


Figure 18: 2nd harmonic distortion of pressure, current, voltage

4. Conclusion

To investigate and model the nonlinear characteristics of loudspeaker, the hypotheses had been proposed, the nonlinear pattern of speaker pressure is proportional to that of current passing through, instead of voltage. The measurement system is built to calculate impedance of loudspeaker and to show the correlation of current and pressure harmonic distortion, the representation of nonlinear property. Supposedly, there are two approaches to achieve the goal: one is calibrating impedance in several levels using chirp signal as stimulus, the other is finding the relationship between pressure and current from nonlinearity shapes using pure tones as the stimulus. From the analysis for impedance of both studio speaker and hearing aid, it was hard to observe nonlinearity clearly, due to the distortion of a chirp. Thus, the experiment has been switched to the measurement of nonlinear property, via the second harmonic distortion of pure tones. Fortunately, the shape of second harmonic distortion for speaker pressure and current is the nearly the same at 0.5 to 3kHz, which matches the hypotheses. In addition, the harmonic trends for pressure and current in a function of voltage intensity are exactly the same no matter for different fundamental or different order of harmonic. This conclusion is therefore consistent with what stated in hypotheses that the pressure and current should be proportional at each harmonic. The result proved the case of fundamental frequency in the range from 0.5kHz to 3kHz in 0.5kHz interval. To further improve the correctness in the correlation of nonlinearity, more research should utilize the current source, which should give a more stable and precise result.

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