STRATHCLYDE

DISCUSSION PAPERS IN ECONOMICS

AN APPRAISAL OF FRIEDMAN'S POSITIVELY SLOPED PHILLIPS
CURVE CONJECTURE

BY

ROD CROSS

NO 18-14

DEPARTMENT OF ECONOMICS
UNIVERSITY OF STRATHCLYDE
GLASGOW

AN APPRAISAL OF FRIEDMAN'S POSITIVELY SLOPED

PHILLIPS CURVE CONJECTURE

ROD CROSS

OCTOBER 1985/
SEPTEMBER 2018

Summary: In this paper we ask what happens in the medium-term interregnum
between the domain of the short run Phillips curve and that of the long run
Phillips curve.   In this interregnum inflation is anticipated but has real
effects on the natural rate of unemployment, according to Friedman's conjecture.
We discuss the new classical and hysteresis alternatives to Friedman's conjecture,
and some of the methodological issues involved in appraising the conjecture.   A
simple model, and some data, are employed in this task.

In his Nobel Prize address Milton Friedman (1977) introduced a new dimension to Phillips curve analysis by postulating the existence of an interregnum between the domain of the short-run negatively sloped Phillips curve and that of the long-run vertical Phillips curve. In this interregnum people have come to anticipate correctly the mean value of the rate of inflation, so distinguishing the medium-run from the short-run domain in which this is not the case. Higher mean values of the rate of inflation, however, are postulated to be characterised by greater variability, or a higher variance, in the rate of inflation. Perceptions of relative prices in this medium-run domain are postulated to be blurred by the unaccustomed level of variability in the rate of inflation. The increased uncertainty regarding relative prices arising from this "blurring" effect is seen as being exacerbated by institutional and political obstacles to the free movement of prices, which in turn are taken to be greater in number and effect the higher is the mean value of the rate of inflation. Thus higher mean values of the rate of anticipated inflation are, according to this conjecture, accompanied by greater uncertainty regarding, and more inertia in, the movement of relative prices, so reducing the efficiency of market prices in the coordination of economic activities. This reduction in the efficiency of the price mechanism, ex hypothesi, increases the natural rate of unemployment. The implication is that money is not neutral in this medium-run domain, and that higher rates of anticipated inflation are accompanied by higher natural rates of unemployment, tracing out a positively sloped Phillips curve. In terms of calendar time the domain of the medium-run positively sloped Phillips curve is " -- a transitional period (which) may well extend over decades --" (Friedman 1977, p.465).

The obvious question that arises is whether what happens in this medium-term domain is consistent with the long-run neutrality of money, that is with convergence to a long-run vertical Phillips curve, the "super-natural" rate equilibrium postulated in Friedman's other major contribution to Phillips curve analysis (Friedman 1968). It is with question that this paper is

concerned.   Friedman's own answer to this question was yes:

"I am inclined to retain the long-long-run vertical Phillips curve ..
(A) high rate of inflation would have some real effects, by altering desired
cash balances, for example, but it need not alter the efficiency of labor
markets, or the length or terms of labor contracts, and hence it need not
change the natural rate of unemployment.   This analysis implicitly supposes,
first, that inflation is steady or at least no more variable at a high rate
than at a low, - otherwise, it is unlikely that inflation would be as fully
anticipated at high as at low rates of inflation;   second, that the inflation
is, or can be, open, with all prices free to adjust to the higher rate, so that
relative price adjustments are the same with a 20 per cent inflation as with a
zero inflation;   third, really a variant of the second point, that there are
no obstacles to indexing of contracts.   Ultimately, if inflation at an average
rate of 20 per cent per year were to prevail for many decades, these requirements
could come fairly close to being met .." (Friedman 1977, pp.464-465).

Friedman's conclusion here places the long-run "super-natural" rate of un-
employment equilibrium in a similar time frame to steady state growth paths in
neoclassical growth theory: estimates of the time required for adjustment to
steady states" .. may be considerably longer than that for which we have time
series .." (Atkinson 1969, p.144).

The answer given in the present paper to the question of whether convergence
to the long-run "super-natural" rate equilibrium is likely to occur is no.
Anticipating the argument to come, the contention is that the conjunction of a
short-run negatively sloped Phillips curve with a medium-run positively sloped
Phillips curve introduces an element of instability into models containing such
Phillips curves;   and that such instability can impart overall instability,
given feasible combinations of parameter values.   The instability in question
arises because any disturbance which in the short-run generates unanticipated
inflation and a decrease in the actual rate of unemployment will, in the medium-
run, stimulate an increase in the anticipated rate of inflation;   this will in

turn, according to the positively sloped Phillips curve conjecture, lead to an increase in the natural rate of unemployment, generating a gap between the actual and natural rates of unemployment and further unanticipated inflation; and so on. If this arg ent holds water, the long-run vertical Phillips curve could perhaps be more appropriately termed the dead-run case: the neutrality of money proposition ceases to hold.

The wider significance of the positively sloped Phillips conjecture lies in its ability to give formal expression to the view that inflation matters, and that there are real gains to be had from reducing the rate of inflation. Critics of the Weltanschaung held by many monetarists have argued that the costs of inflation highlighted by monetarists are likely to be relatively small in magnitude compared to the real output and employment costs of anti-inflation policies (see Tobin 1972 and Hahn 1984 for example). Monetarism is an easy target for such criticism if the only adjunct to the short-run negatively sloped Phillips curve is the long-run vertical line: the natural rate of un-employment is the same at high as at low mean rates of anticipated inflation, so why suffer the output and employment costs of unanticipated deflation if the old natural rate of unemployment will re-emerge once expectations have adjusted? If, however, the process of adjustment from the short-run to the long-run is characterised by a positively sloped Phillips curve then Phillips curve analysis contains an account of why it might be worthwhile to pursue anti-inflation policies. An alternative method of giving expression to the costs of inflation in Phillips curve analysis is, of course, the deterioration in the trade-off between unanticipated inflation and unemployment at high rates of inflation to be found in the new classical macroeconomics (see Lucas 1973 for example). Given the distinction between monetarism and the new classical macro-economics (see Laidler 1981 for discussion), particularly to be found in the issue of whether markets clear more or less instantaneously or not, it is worth-while to consider Friedman's positively sloped Phillips curve conjecture in its own right.

The rest of this paper is structured as follows. Section I outlines the three different members of the Phillips Curve genus identified by Friedman. In order to set Friedman's conjecture in perspective Section II takes a brief look at the competition to Friedman's conjecture in the form of the new classical and Keynesian approaches to the natural rate of unemployment. Section III discusses some considerations involved in appraising conjectures at a quasi-empirical level. Section IV appraises Friedman's conjecture in the context of a simple model which nests the positively sloped Phillips curve conjecture alongside auxiliarly hypotheses which are designed to provide a preferred rather than alien habitat for the conjecture. Section V concludes with a major qualification.

I THE PHILLIPS CURVE GENUS

Friedman distinguishes three stages in Phillips curve analysis. The first was the identification of the short-run curve by Fisher (1926) and A.W. Phillips (1958) as a stable relationship between a projected moving average of the rate of change of prices (Fisher), or the rate of change of money wage rates (Phillips) and the level of unemployment. During the second stage of Phillips curve analysis initiated by Friedman (1968) and Phelps (1967) the short-run curve was re-interpreted as a relationship between the unanticipated rate of inflation and the level of unemployment. Once inflation came to be anticipated the trade-off between inflation and unemployment would disappear, replacing the short-run negatively sloped Phillips curve with a vertical line corresponding to the "natural" rate of unemployment (Friedman) or U* (Phelps). The natural rate was postulated to depend on "real" factors ".... the structural characteristics of the labour and commodity markets ....." (Friedman 1968 p.8) and to be independent, or largely independent, of monetary factors. The inclusion of "... stochastic variability in demands and supplies..." (Friedman 1968, p.8) in the list of factors which determine the natural rate

can be seen as an anticipation of the positively sloped Phillips curve
formally unravelled by Friedman almost a decade later.   The rate of inflation
would accelerate if an economy were to experience rates of unemployment below
the natural rate, and decelerate at rates of unemployment above the natural
rate.   A distinction is sometimes drawn between the interpretation of the short-
run Phillips relationship which sees quantities as reacting to price changes -
Fisher (1926), Lucas (1972) and the later Friedman (1977) story; and the
interpretation which sees prices as reacting to quantity changes - Lipsey (1960),
Phelps (1967) and the earlier Friedman (1968) story.

Even at this juncture in Phillips curve analysis Friedman seemed to sense
something lacking in the story of inflation-unemployment interaction told in the
first two stages of Phillips curve analysis: "...a full adjustment to the new
rate of inflation takes about as long for employment as for interest rates,
say, a couple of decades" (Friedman 1968, p.11).   Given that even the slowest
human beings could be expected to adjust their expectations to a new, steady
rate of inflation within, say, a couple of years, the question is what happens
during the "couple of decades" to prevent "a full adjustment to the new rate
of inflation": that is, how is the interregnum between the short and long
run Phillips curves to be characterised?

The Medium Run Phillips Curve

This third stage in Phillips curve analysis was directed towards
accommodating the "... apparent empirical phenomenon ... (of) recent years ...
(that) ... higher inflation has often been accompanied by higher, not lower,
unemployment, especially for periods of several years in length ..." (Friedman
1977, pp.459-460).   The conditions for the vertical long-run Phillips curve
to hold are"... that inflation is steady or at least no more variable at a
high rate than at a low ... that the inflation is, or can be open, with all
prices free to adjust to a higher rate, so that relative price adjustments

are the same with a 20 per cent inflation as with a zero inflation ... (and)
that there are no obstacles to the indexing of contracts" (Friedman 1977,
pp.464-465). Such conditions, however, are not likely to be met unless "...
inflation at an average rate of 20 per cent per year were to prevail for many
decades ... when a country initially moves to higher rates of inflation,
these requirements will be systematically departed from ... and such a
transitional period may well extend over decades"(Friedman 1977, p.465).

The Volatility Hypothesis

In an attempt to explain what happens over the "decades" it may take to
adjust to the new rate of inflation Friedman conjectured the existence of a
positively sloped Phillips curve. Friedman outlines two sets of reasons for
expecting the slope of the medium term Phillips curve to be positive. First the
variability or volatility of inflation is postulated to be higher at higher rates
of inflation (see Moore 1984 for a review of some of the issues involved here).
An increase in the volatility of the rate of inflation is taken to raise the
natural rate of unemployment because the postulated greater variability in
relative prices renders"... market prices a less efficient system for coordinating
economic activity ... it seems plausible that the average level of unemployment
would be raised by the increased amount of noise in market signals, at least
during the period when institutional arrangements are not yet adapted to the
new situation". (Friedman 1977, p.467). This account can be seen as a new
Chicago view of the costs of anticipated inflation, contrasting with the stress
on the deadweight "triangle" losses associated with lower real money balance
holdings contained in the old Chicago view.

The investigation of the empirical relationship between the level and
variability of the rate of inflation has been an active area of empirical
research during the last decade. An extensive review of this literature
is available in Cukierman (1983), so here it is only necessary to outline
the main results which have emerged :   the variance of the

rate of inflation is found to be positively related to the level of inflation; and an increase in the level and variance of the rate of inflation is accompanied by "... increases in the · oss-sectional variabilities of relative prices, of inflationary expectations, and of the inflation forecast error" (Cukierman 1983, p.107). A word of caution is necessary here because most of the econometric results in this literature were gained using homoscedastic specifications of error terms in the relevant regression equations. Given that Friedman's conjecture is couched in terms of the variance of relative prices, and perhaps of inflation expectations and errors therein, being an increasing function of the mean rate of anticipated inflation, heteroscedastic error terms are likely to arise in regression equations which attempt to mimic Friedman's conjecture. When heteroscedastic regression models, such as Engle's ARCH (autoregressive conditional heteroscedastic) model, have been deployed the inflation level-variability relationship postulated by Friedman has been less apparent: "... a high rate of inflation does not necessarily imply a high variance of inflation ... in fact, the 1970s (in the U.S.) provide a clear counter-example" (Engle 1983, p.292).

Even if we accept that the positive level-variance inflation relationship holds empirically, the implication is not necessarily that higher mean values of the rate of inflation lead to greater variability in, and uncertainty regarding, relative prices: the line of "causality" may run in the opposite direction. Much of the debate in this literature has been concerned with assessing how much the level-variance relationship is due to changes in the variability of aggregate supply and demand on the one hand, and to changes in the variability of relative supplies and demands on the other. This issue of "causality" is particularly difficult to resolve. In econometric terms "causality" is usually defined as some form of link between exogenous and endogenous variables. The traditional Cowles Commission approach (see Hood and Koopmans ed., 1953, for example) is often criticised for requiring "incredible" identifying restrictions in order to estimate the relationships

involved; the "vector autoregression" approach of Granger and Sims (Sims 1980, for example) is more of a test for precedence than causality; the notions of "weak, strong and super exogeneity" suggested by Engle, Hendry and Richard (1983) are geared towards efficient estimation rather than identification of "causality"; and so on (see Leamer 1985 for further discussion). At a more general level it has been argued that the notion of "causality" appropriate to Economics is diachronic, and hence quite different to the notion appropriate to the natural sciences which has been adopted by many economists (see Hicks, 1979 for this thesis).

Political and Institutional Factors

The other set of reasons invoked by Friedman to explain why the medium run Phillips might be positively sloped are political and institutional in nature. The political effect arises because governments are postulated to be more prone to impose price controls at higher rates of inflation which, ex hypothesi are more volatile rates of inflation; and price controls are postulated to distort relative prices more the higher is the rate of inflation. "...Governments are themselves producers of services sold on the market ... other prices are regulated by government ... the social and political forces unleashed by volatile inflation rates will lead governments to try and repress inflation in still other areas ... by explicit price and wage control, or by pressuring private businesses or unions "voluntarily" to exercise "restraint", or by speculating in foreign exchange in order to alter the exchange rate ... the details will vary from time to time and country to country, but the general result is the same: reduction in the capacity of the price system to guide economic activity; distortions in relative prices because of the introduction of greater friction, as it were, in all markets; and, very likely, a higher recorded rate of unemployment" (Friedman 1977, pp.467-468). The argument deployed here is developed in greater detail in Sjaastad, 1974.

The behaviour of private sector institutions is also postulated to distort relative prices at higher rates of anticipated inflation: "...it takes time for actual practice to adjust ... in the meantime, prior arrangements introduce rigidities that reduce the effectiveness of markets ... in addition, indexing is, even at best, an imperfect substitute for stability of the inflation rate ...price indexes are imperfect ... they are available only with a lag and generally are applied to contract terms only with a further lag" (Friedman 1977, p.466). The aspect of this hypothesis which has attracted most attention is the way the practices adopted in capital markets fail to take account of the anticipated rate of inflation; in the U.K. for example, accounting for inflation is the exception rather than the norm. Many banks require a minimum 4 : 1 ratio of profits to nominal interest payments when issuing loans, despite the fact that this ratio will be a decreasing function of the rate of inflation; many analysts use historical cost profit figures when assessing share prices; corporate fixed debt is not usually index-linked; nominal rather than real interest payments are tax deductible; nominal rather than real discount rates are often applied to income streams; inflation induced gains on debt are not usually included in the definition of profits; and so on. Such factors provide the background to the Modigliani-Cohn theorem (1979) that the degree of undervaluation of shares is an increasing function of the rate of inflation. The evidence for the U.K. is that such effects are important enough to generate a positive relationship between inflation, bankruptcy rates and default premia (see Wadhwani 1984).

A Representation of Friedman's Conjecture

If Friedman's conjecture holds, the implication is that governments can achieve sustainable changes in unemployment by changing aggregate nominal demand over the "decades" to which the medium-run Phillips curve is relevant. Thus the slogan that "there is no trade-off between anticipated inflation and unemployment" relates only to the long-run. The slogan relevant to the medium-

run is that "higher anticipated inflation leads to higher unemployment". This slogan has been adopted in reverse by many governments since the late 1970s to rationalise anti-inflation policies: "inflation destroys jobs". The translation of such rhetoric into Phillips curve analysis becomes possible once the medium-term positively sloped Phillips curve is adjoined to the short and long-run curves.

Given that Friedman did not write down a formal representation of the positively sloped Phillips curve conjecture, the issue of how best to formulate this conjecture arises. In his discussion of the empirical phenomenon to be explained Friedman (1977 pp.459-464) the conjecture appears to relate the natural rate of unemployment in the interregnum to the level of the rate of inflation. This would suggest a medium-term Phillips curve of the form

$$U^* = U_R + \beta_1 \Delta P \qquad \text{Conjecture I}$$

where U^* is the natural rate, U_R represents the component of the natural rate determined by factors such as unemployment insurance arrangements which are taken to be exogenous to the system, and β_1 is the slope of the medium-run Phillips curve with regard to the rate of anticipated Inflation ΔP. This formulation can also be justified by reference to the postulate that the volatility of inflation, an increase in which is hypothesised to raise unemployment, is an increasing function of the level of the rate of inflation. Also, the very phrase "positively sloped Phillips curve" seems to suggest this formulation.

Elsewhere, however, Friedman emphasises that it is"...increasing volatility and increasing government intervention with the price system ... (which) ... are the major factors that seem likely to raise unemployment, not high volatility or a high level of "intervention" (Friedman 1977, p.468). This suggests an alternative formulation

$$U^* = U_R + \beta_2 \Delta^2 P \qquad \text{Conjecture II}$$

where $\Delta^2 P$ is the rate of change of the rate of inflation. If it is increasing volatility which raises unemployment, rather than the level of volatility in the system, it is more appropriate to represent the conjecture in terms of

the acceleration of the price level rather than its rate of change. This form of the conjecture i likely to lie more easily alongside the neutrality of money proposition, given that anticipated inflation will have no effect on the natural rate when $\Delta^2 P = 0$.

II TWO ALTERNATIVES

According to the neutrality of money proposition relative prices are determined by real factors, and are independent of the rate of inflation, the quantity of money and other nominal magnitudes (see Gale 1982, for discussion). The accumulation of evidence to the effect that the distribution of relative prices is not independent of the rate of inflation (see Cukierman 1983 for a review) has stimulated economists to modify their accounts of the neutrality of money proposition. In Friedman's account money is taken to be non-neutral over a medium run domain spanning "decades", but neutral in the long run "super-natural" domain. Two other reactions can be discerned: in the new classical macroeconomics the neutrality proposition is retained for the medium as well as long run domain - "...some were led to introduce informational and adjustment frictions while retaining the basic paradigm" (Cukierman 1983, p.103) and in the disequilibrium trading Keynesian case the proposition is dropped for all domains - "...others went further and abandoned the neutrality of money idea by introducing sticky prices and direct links between the relative prices of particular sectors and the general price level..." (Cukierman 1983, p.103). Friedman's account can be seen as occupying the middle ground between these two camps. If the argument of this paper holds water, however, the instability arising from Friedman's medium run Phillips curve serves to move Friedman into the Keynesian camp with the neutrality of money proposition not even to be seen on the horizon.

The New Classical Approach

This research programme is characterised by three main features:
equilibrium trading contingent on available information; rational expectations;
and a business cycle theory based on a "surprises" version of the short run
Phillips curve (see Sheffrin 1983, for a review). The features of the relationship
between the distribution of relative prices and inflation which generate a positively
sloped medium run Phillips curve in Friedman's conjecture are ruled out by this
approach: people are postulated to acquire the relevant information which it is
possible to acquire, and to exploit all advantageous trades. For example,
if indexed contracts were advantageous they would be used, and so no real
effects on the natural rates of output and unemployment could arise in the
medium term domain from non-indexation, contra Friedman. Instead the positive
relationship between the variability of relative prices and inflation enters
the theory in the short run domain, which is characterised by information
deficiency. The outcome is that the slope of the short run Phillips curve
depends on the variability of the rate of inflation.

The short run Phillips curve arises from unanticipated inflation.
Unanticipated inflation engenders deviations from the natural rate of unemployment
because of misperceptions regarding : real interest rates (Lucas and Rapping 1970);
capital gains (Lucas 1975); the intertemporal structure of real wage rates
(Ghez and Becker 1975); or the current distribution of real wage offers
(Alchian 1970) - see Lindbeck and Snower, 1985, for a survey. Perhaps the
best known presentation of the new classical short run Phillips curve is that
contained in Lucas (1973), and serves to illustrate the contrast between the
new classical treatment of relative price variability and Friedman's conjecture.

Producers have the problem of working out the extent to which variations
in their own price $P(i)$ reflect variation in the general price level P as
opposed to deviations of their own price from the general price level, i,
where $P(i) = P+i$. They are postulated to know the mean and variance of the

general price level, $\bar{P}$ and σ^2, and that z is distributed independently of P with zero mean and variance r^2. In this "island" paradigm producers are taken to have information on what has happened on other islands in the past, what is happening on their own island in the present, but not to know what is going on in other islands in the present. This implies a signal extraction problem for producers in estimating whether movements on their own price reflect price level or relative price changes. Confusion between the two provides the foundation for the Lucas supply curve:

$$Y(i)_t - Y^*(i)_t = a\left[P(i)_t - E(P_t|\phi(i)_t)\right] \tag{i}$$

where $Y(i)$ and $Y^*(i)$ are actual and equilibrium output supplies from island i, a is the slope of the supply curve, E is the expected value operator and $\phi(i)$ is the information available on island i. The information set $\phi(i)_t = \left[P(i)_t, \bar{P}_t\right]$, and $E(P_t|\phi(i)_t)$ is specified as $b\bar{P}_t + (1-b)P(i)_t$. Aggregating over individual markets or "islands" yields the familiar expression:

$$Y_t - Y_t^* = c\left[P_t - \bar{P}_t\right] \tag{ii}$$

where $c = ab$, $b = \dfrac{r^2}{\sigma^2+r^2}$ and Y and Y^* are the actual and natural output levels. Specifying Okun's Law as

$$U_t^* - U_t = d\left[Y_t - Y_t^*\right] \tag{iii}$$

where U and U^* are the actual and natural rates of unemployment, (ii) can be re-written as

$$U_t^* - U_t = cd\left[P_t - \bar{P}_t\right] \tag{iv}$$

Expression (iv) illustrates clearly the contrast between the new classical account of the real effects of the variability of inflation and the Friedman conjecture, which, as discussed in Section I of this paper, can be expressed as

$$U_t^* = U_{Rt} + \beta_1\Delta P_t + \beta_2\Delta^2 P_t \tag{v}$$

In the new classical case, expression (iv), the real effects of variability in the rate of inflation are confined to the short run domain of unanticipated inflation: the slope of the short run Phillips curve, cd, depends on the proportion of the variability in inflation which is due to relative price variance , r^2, rather than to variance in the general price level, σ^2. In the Friedman case, expression (v), the real effects of inflation variability are felt in the medium run domain of anticipated inflation: an increase in, or acceleration, in the rate of inflation leads to an increase in the natural rate of unemployment through the β_1 or β_2 effects.

The Persistence Problem

Time series for unemployment tend to be characterised by a quite high order autoregressive process (see Ashenfelter and Card 1983, for example): low un-employment rates tend to breed low unemployment, high unemployment rates high unemployment. Friedman's abandonment of the neutrality of money proposition in the medium term domain offers a means of explaining this "persistence" if it can be shown that high unemployment rates over this domain are associated with high, or rising rates of inflation, and low unemployment rates with low, or falling, rates of inflation: this is a big if, as we shall see in Section V of this paper. The new classical theory, by confining the real effects of inflation to the short run domain, does not have this means of explaining persistence. Four types of reaction to this problem can be discerned in the new classical theory. The first involves specifying a time series of relative price misper-ceptions in attempting to explain what appear to be persistent deviations from the natural rate of unemployment. The relative price misperception most commonly employed relates to real wages and involves the intertemporal substitution of leisure (see Lucas and Rapping 1970, and Hall 1980, for example). The weight of empirical evidence does not support this approach: "...the results of this essay and much of the other evidence cited raise serious doubts about the empirical

viability of the intemporal substitution - market clearing views of the labour market" (Altonji, 1982, pp.138-139). The second approach involves specifying exogenous factors which have moved the natural rate of unemployment in such a way as to be consistent with the observed persistence: see Minford 1983, for example. The evidence, of recent years in particular, does not lend support to this approach. In the U.K., for example, the exogeneous factors normally invoked to explain changes in the natural rate, such as unemployment insurance, trades union power and employee taxes have suggested a fall in the natural rate over the last five years at a time when unemployment has risen dramatically. The third approach has involved specifying why deviations from the natural rate of unemployment might persist for more than the one period information lag which gives rise to such deviations. The main modifications on such lines are the overtly ad hoc approach of Lucas (1973), "let there be autoregression"; the capital stock adjustment approach of Lucas (1975); the inventory stock adjustment approach of Blinder (1981); and the overlapping wage contract approach of, for example, Taylor (1980). These modifications are hard to justify on the basis of the "market clearing contingent on available information" heuristic central to the new classical theory: advantageous trades remain unexploited. In time this may prove to be the beginnings of a new, non-classical macroeconomics. Finally, a fourth approach specifies the natural rate of unemployment as an endogenous variable in the short run. In D.M. Lilien (1982), for example, the natural rate of unemployment varies with the degree of labour reallocation across sectors over the business cycle. This line of reasoning involves disequilibrium trading and contains no presumption that such false trading will disappear over time: a Keynesian stranger in the new classical camp.

The Keynesian Approach

Keynesian economists have traditionally taken the costs of anticipated inflation, variable or otherwise, to be relatively small in magnitude, certainly

compared to the output and employment costs of anti-inflation policies (see Tobin, 1972, and Hahn, 1984, for example). Experience of such policies, particularly over the last five years, has persuaded economists who would be known as monetarists in the U.K. to move towards this view: "...though some of us did-expect the implementation of a monetary strategy designed finally to bring the great inflation of the 1970's to an end to have significant adverse side effects on real income and employment, none of us expected the deep and prolonged depression that ensued" (Laidler, 1985b, p.35). At the centre of the Keynesian approach is the notion that markets, the labour market in particular, do not clear even when inflation is anticipated, and that dis-equilibrium trading is likely to persist beyond the short run domain. Instead of the natural rate of unemployment, Keynesians tend to use the concept of a non-accelerating inflation rate of unemployment (NAIRU) to provide a means of distinguishing the short run Phillips curve from what happens in the medium run domain. The medium run NAIRU concept is asymmetric in nature: inflation will accelerate at rates of unemployment below the NAIRU, but will not necessarily decelerate at rates above the NAIRU. This encapsulates the traditional argument for expecting Keynesian unemployment, relative output prices being too high, to be more likely than classical unemployment, relative wage rates being too high: "...if prices are more flexible upward than downward, a prima facie argument exists for suggesting that Keynesian unemployment is more likely than classical unemployment..." (Malinvaud 1984, p.38). A further difference lies in the lack of unique, one to one, relationship between the NAIRU and the rate of anticipated inflation. The NAIRU is partly determined by the exogenous factors invoked in the new classical and monetarist accounts of the natural rate, and could well be affected in some way by/real effects of variable inflation depicted in Friedman's conjecture. But stress is also placed on the recent history of disequilibrium trading as a determinant of the NAIRU, different histories being characterised by different levels of the NAIRU: "... a variety of types of evidence suggest that
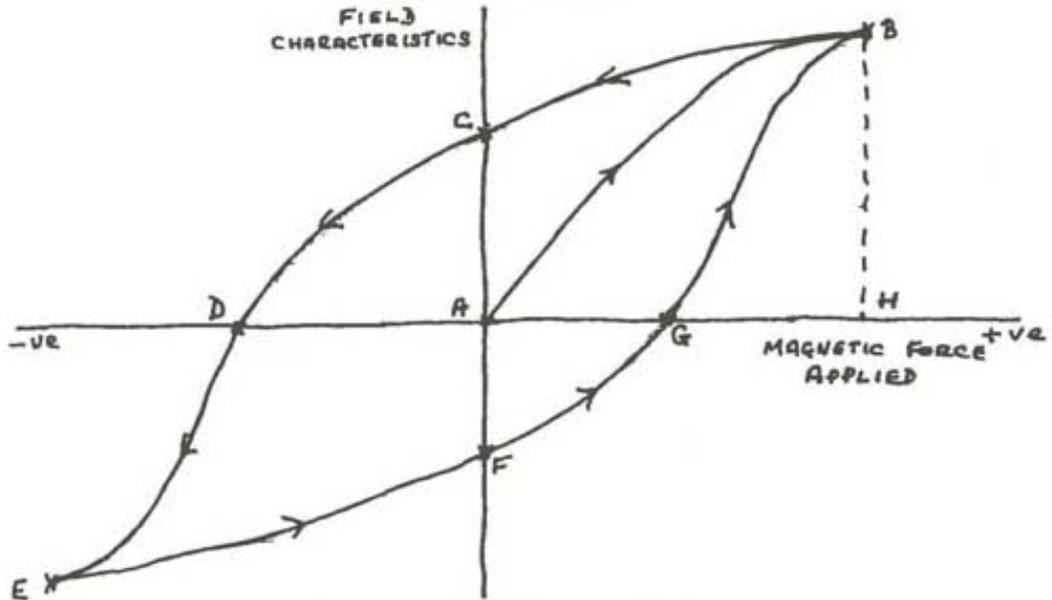
previous employment experience has an important effect on subsequent labour supply .. these results cast doubt on the medium run relevance of the natural rate hypothesis ... because policy affects the level of employment in the short run, it has a long run effect on the position of the labour supply schedule -- workers drawn into the labour force by cyclical upturns tend to remain even after the boom has ended -- the converse is true for shocks which reduce employment ... persistence elements are more important than timing elements in explaining fluctuations on either the number of persons employed or the number participating in the labour force .." (Clark and Summers, 1983, p.164).

Arguably this NAIRU version of the natural rate of unemployment represents the application of the notion of hysteresis to the natural rate concept. Hysteresis is a concept used in electrodynamics to modify Maxwell's field equations when accounting for the otherwise anomalous behaviour of magnetic fields in the presence of ferric metals (see Brailsford, 1966, pp.5-7, for example). The concept is illustrated in the accompanying Diagram I. The application of a positive magnetic force H to a field in which ferric metals are present changes the field characteristics from A to B; the removal of the force H, does not yield a return to the original field characteristics A, but rather involves movement to field characteristics C; to restore the system to its original state a negative force D has to be applied; and so on through E, F and G. The implication is that in order to explain field characteristics the history of the forces applied to the field has to be taken into account as well as the force being applied "currently".
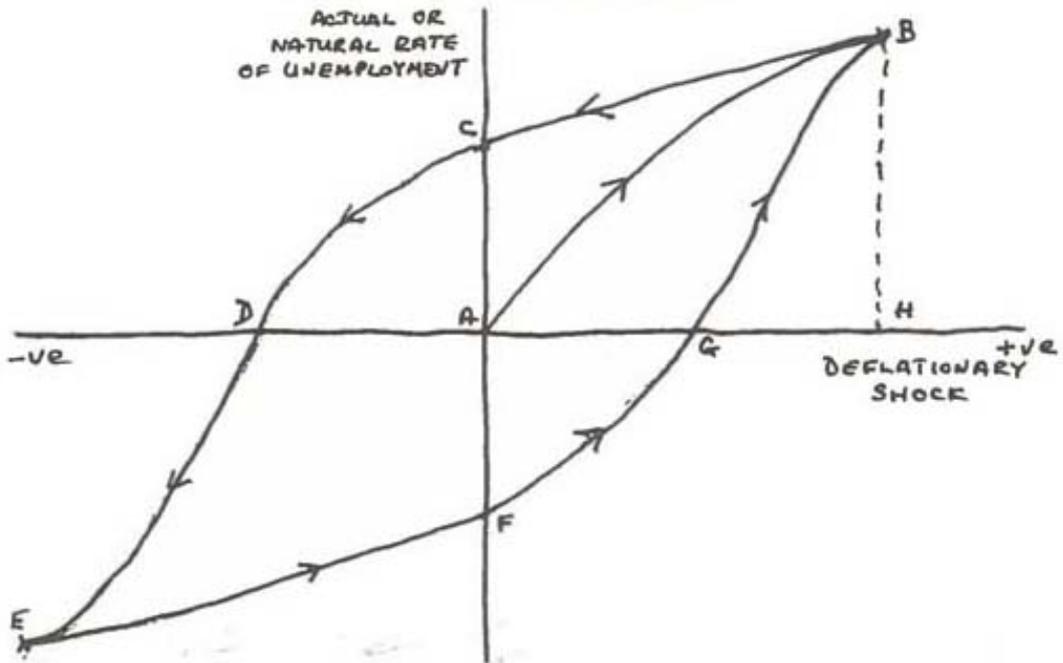
Phelps introduced this concept to the economics profession in an often neglected contribution to the theory of the natural rate of unemployment: "...the transition from one equilibrium to the other tends to have long-lingering effects on the labour force, and these effects may be discernible on the equilibrium rate of unemployment for a long time. The natural rate of unemployment at any future date will depend upon the course of history on the interim. Such a

I ELECTROMAGNETISM

FIELD
CHARACTERISTICS

B

C

D

A

G

H

MAGNETIC FORCE +ve
APPLIED

−ve

F

E

II   THE NATURAL RATE OF UNEMPLOYMENT

ACTUAL OR
NATURAL RATE
OF UNEMPLOYMENT

B

C

D

A

G

H

DEFLATIONARY +ve
SHOCK

−ve

F

E

property is often called hysteresis" (Phelps, 1972, p.xxiii).   An illustration
of the application of the hysteresis concept to the natural rate is given in
the accompanying Diagram I:.   A once-off deflationary shock H, associated with
anti-inflation policies for example, leads the actual rate of unemployment to
rise to B;   the removal of the shock H, however, does not yield a return to the
original natural rate A, but rather movement to a new natural rate C, which will
persist in the absence of further shocks;   to achieve a return to the original
natural rate A an expansionary shock D is required;   and so on through E, F and
G.   The implication is that to explain the natural rate or NAIRU it is necessary
to invoke the history of the way unemployment has reacted to the shocks impinging
on economic systems as well as current disturbances.   The hysteresis concept
can be seen as providing a unifying framework in which Keynesian theory,
Friedman's conjecture and new classical attempts to explain persistence can be
embedded.

   The contrast with Friedman's conjecture can be seen in the following simple
illustration of hysteresis effects.   The hysteresis-augmented version of the
natural rate of unemployment can be written as

$$U_t^{*} = U_{Rt} + e\Delta U_t + f \int_0^T (U_t^{*} - U_t).dt \qquad \text{(vi)}$$

where U^{*} and U are the natural and actual rates of unemployment, e and f are
hysteresis effect parameters e>0, f<0, and U_{Rt} reflects the vector of current
exogenous disturbances impinging on the natural rate (see Cross 1985 for further
discussion).   Differentiating (vi) with respect to time, and approximating,
yields:

$$\Delta U_t^{*} = \Delta U_{Rt} + e\Delta^2 U_t + f(U_t^{*} - U_t) \qquad \text{(vii)}$$

Employing the specification of Okun's Law given in (iii) above and substituting
this into (vii) yields

$$\Delta U^{*}_{t} = \Delta U_{Rt} + e\Delta^{2}U_{t} + df(Y_{t} - Y^{*}_{t}) \qquad \text{(viii)}$$

or, using (iii),

$$\Delta U_{t} = \Delta U_{Rt} + e\Delta^{2}U_{t} + df(Y_{t} - Y^{*}_{t}) - d(\Delta Y_{t} - \Delta Y^{*}_{t}) \qquad \text{(ix)}$$

which can be contrasted with Friedman's conjecture by rewriting (v) as

$$\Delta U^{*}_{t} = \Delta U_{Rt} + \beta_{1}\Delta^{2}P_{t} + \beta_{2}\Delta^{3}P_{t} \qquad \text{(x)}$$

or, using (iii),

$$\Delta U_{t} = \Delta U_{Rt} + \beta_{1}\Delta^{2}P_{t} + \beta_{2}\Delta^{3}P_{t} - d(\Delta Y_{t} - \Delta Y^{*}_{t}) \qquad \text{(xi)}$$

The fundamental difference is that Friedman's conjecture explains persistence in the natural rate by various differences of the price level which are taken to mimic the variability of relative prices; whereas the hysteresis approach explains persistence by reference to the history of the actual and natural rates of unemployment and output. In other words Friedman's conjecture explains the natural rate in terms of relative prices, the hysteresis approach by reference to the history of unemployment and output. Given that we know that there is substantial autoregression in unemployment time series the hysteresis concept is a promising one: hysteresis implies persistence, whereas the Friedman conjecture and new classical approach do not necessarily imply persistence.

So far we have not explained why hysteresis effects such as those contained in expression (vii) should arise. The following is a by no means exhaustive list of conjectures regarding economic behaviour which can imply hysteresis effects: labour force participation being endogenous to economic activity (see Clark and Summers 1983); the efficiency wage hypothesis (see Stiglitz, 1984); increasing returns to scale (see Weitzman, 1982); implicit contracts (see Grossman and Hart, 1981); social aspects of exchange relationships (see Akerlof, 1984); changes in the proportions of workers in dual labour markets (see Solow, 1985); institutional aspects of wage bargaining (see McDonald and Solow, 1981);

the probability of gaining employment being a function of the duration of
unemployment, after allowing for heterogeneity (see Nickell, 1979); asymmetry
between the ease of bankruptcy and starting up firms (see Hahn, 1984); and
prolonged capital and inventory stock adjustment (see Blinder, 1981). So
far little has been done to elaborate the hysteresis implications of such
aspects of economic behaviour and few empirical tests have been conducted:
Phelps (1972), Hargreaves Heap (1980), Clark and Summers (1983), Cross (1983)
and Blanchard et al (1985) provide examples of the application of hysteresis
reasoning to the explanation of unemployment.

III CONJECTURE APPRAISAL

In the recent literature there has been much discussion of the "best"
methods of appraising the empirical content of conjectures in Economics: see
Hendry, 1983, 1985 for discussion from an econometrician's perspective; and
the contributions to Caldwell, ed., 1985, for the issues viewed from a more
explicitly methodological perspective. Less space has been devoted to
appraisal of the "quasi-empirical" content associated with conjectures. The
discussion in this section focusses on the latter quasi-empirical aspect of
the appraisal problem and employs the Lakatos appraisal criterion of content
to organise the discussion: see Lakatos, 1978, for the rationale for using
content as an appraisal criterion, and Cross, 1982, for an application of this
criterion in Economics.

Appraisal of the quasi-empirical aspects of conjectures has at least two
dimensions. In one dimension this activity is concerned with phenomena which
start life as mental constructs, an octahedron, or the notion of convergence
from the medium run Phillips curve to a "super-natural" rate of unemployment
equilibrium, for example. In some sense this quasi-empirical content can be
distinguished from that relating to phenomena which started life external to
the proponent of the conjecture, the Earth's moon, or the U.K. unemployment

rate since 1855, for example. The distinction is by no means watertight: mental constructs play a significant role in the interpretation of the external world, and the external world is often shaped by the use of mental constructs in human action. Despite this the distinction could well prove to be a useful one to deploy in certain contexts, such as the problem with which this paper deals. The terminology is due to Lakatos who used the term quasi-empirical to refer to the content of the mathematical conjectures appraised in his early work on the philosophy of mathematics: the aim of this work was "... to elaborate the point that informal, quasi-empirical, mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and reputations" (Lakatos 1976, p.5). Lakatos used the example of Euler's conjecture regarding the relationship between vertices, V, edges, E, and faces, F, in polyhedra to illustrate this method of appraisal. This conjecture, $V + F = E + 2$, holds, inter alia, for cubes, triangular and pentagonal prisms, squares, triangular and pentagonal pyramids and octahedra but not for cylinders and certain transformations of the polyhedra mentioned earlier. The problem, given that the notion of mathematics yielding "indubitably true" theorems has been abandoned, is one of how to assess the amendments and reformulations of Euler's conjecture by mathematicians such as Cauchy (see Lakatos 1976). Lakatos employed the criterion of content in this task to criticise amendments which barred counterexamples by redefinition, "a cylinder is not a polyhedron" for example; or which excluded counterexamples by limiting the domain of the conjecture, "the conjecture only applies when polyhedra have no cavities" for example. The application of this method of appraisal to amendments of models in mathematical economics might, by distinguishing between content increasing and content reducing amendments, help produce a more informative body of mathematical models in Economics (see Morishima 1984 for discussion of the uses and abuses of mathematical economics).

In a second dimension the quasi-empirical appraisal of conjectures is concerned with assessing the empirical content of conjectures before they have actually been tested. In Lakatos an amendment to a theory, or research programme, is progressive if it increases the content of the theory. Amendments to theories can be progressive in a theoretical sense or an empirical sense: amendments to a theory generate "... a series of theories $T_1, T_2, T_3 \cdots$ where each subsequent theory results from adding auxiliary clauses to (or from semantical reinterpretations of) the previous theory in order to accommodate some anomaly, each theory having at least as much content as the unrefuted content of its predecessor ... let us say that a series of theories is theoretically progressive ... if each new theory has some excess empirical content over its predecessor, that is, predicts some novel, hitherto unexpected fact. Let us say that a theoretically progressive series of theories is also empirically progressive ... if some of this excess empirical content is also corroborated, that is, if each new theory leads us to the actual discovery of some new fact ..." (Lakatos 1978, pp.33-35). Given that there is often a substantial time lag between the proposal and thorough empirical examination of conjectures, as in Einstein's general theory of relativity or Phelps' hysteresis-augmented theory of the natural rate of unemployment, there is a prima facie case that some means of assessing progression or degeneration in content during this phase might be of use.

The Quasi-Empirical Content Arising From Friedman's Conjecture

The appraisal problem with which this paper deals arises from Friedman's positively sloped Phillips curve conjecture. The Duhem-Quine thesis, or Duhem's thesis, implies that, because conjectures are invariably conjoined together with other hypotheses, axioms, lemmas, proofs, initial conditions, boundary conditions and so on, the appropriate level at which to appraise such conjectures is the "theoretical whole". In this case the "theoretical whole" is monetarism. The question asked here is whether the mental construct

of a positively sloped Phillips curve introduced into monetarism by Friedman's conjecture increases or reduces the content of monetarism when assessed at a quasi-empirical level. This question has several aspects.

Could the conjecture be, at least in principle, refuted? This is an elementary requirement elaborated by Popper, 1959: "... one can say that there are no abominable snowmen, for this could be falsified by finding them; but one cannot say that there are abominable snowmen, for this could not be falsified; the fact that one had failed to find any would not prove conclusively that none existed ..." (A.J. Ayer, cited in Caldwell 1982, p.21). In the case of Friedman's conjecture a negative association between the mean rate of anticipated inflation and the natural rate of unemployment is excluded, so the conjecture is in principle refutable.

Is the content arising from the conjecture consistent with the content arising from the rest of the theory in which the conjecture is embedded? Monetarism contains the proposition that in the long run money is neutral. The conjecture introduces non-neutrality into the medium-run domain. The question then is whether what happens in the medium term is consistent with convergence to a long run equilibrium in which money is neutral. Friedman's answer is yes: "I am inclined to retain the long-long-run vertical Phillips curve ..." (Friedman 1977, p.464). If the analysis in Section IV of this paper holds water the answer to the question could well be no, that convergence to a "super-natural" rate equilibrium does not occur. If this is the case the content introduced by the conjecture has the effect of removing the content of monetarism dealing with the long run domain, and the theory no longer implies long run monetary neutrality.

Is the conjecture ad hoc? An ad hoc amendment to a theory can be defined as one that serves to explain away otherwise anomalous evidence without generating content other than with regard to the anomaly which is to be explained: in other words an ad hoc amendment makes a theory wise after the event at the cost of reducing the content of the theory, that is at the cost of immunising the

theory from criticism (see Cross 1984 for discussion of this notion of ad hocness).   On the surface Friedman's conjecture may appear ad hoc in this respect, given that it was directed at "... accommodating (an) apparent empirical phenomenon (of) recent years ..." (Friedman 1977, pp.459-460).   If Friedman's conjecture were to be confined to explaining what happened in France, West Germany, Italy, Japan, Sweden, the U.K. and the U.S. from 1956 to 1975 (see Friedman, 1977, Table 1) this would be the case.   Friedman, however, does not say anything to indicate that the range of application of the conjecture is limited to these instances: if this is so, the conjecture is not ad hoc (see though the qualification discussed in Section V of this paper).

Does the conjecture predict novel facts?   Here there is a problem as to what is meant by novel facts: does novelty imply "not used in the formulation of the theory", "not known to the proponent of the theory", "not directly relevant to the initial problem faced", or what?   (see Gardner 1982 for discussion).   According to Friedman it was "... the course of events ..." which gave rise to the conjecture, and the fact that "... in recent years higher unemployment has often been accompanied by higher, not lower, unemployment, especially for periods of several years in length ..." was obviously known to him (Friedman 1977, p.45).   Thus the conjecture has not predicted novel facts in the senses referred to above.

Does the conjecture ease or restrict the boundary conditions which delineate the domain over which the theory can be applied?   Before the introduction of the conjecture monetarism could be applied to a short run domain of unanticipated inflation - let us assume that this domain occupies two years of real time, the time taken for people to come to anticipate correctly the mean rate of inflation; and to the long run domain of the vertical Phillips curve - "... a full adjustment to the new rate of inflation takes about as long for employment as for interest rates, say, a couple of decades" (Friedman 1968, p.11).   Assuming a real time domain which always starts at the beginning of the short run, the

implication is that the boundary conditions employed in monetarism before the introduction of Friedman's conjecture left a gap of a period of at least eighteen years inside which the theory was, by injunction, not applicable. Thus the introduction of the conjecture substantially eased the boundary conditions associated with monetarism, yielding a substantial increase in content, and concomitantly a substantial increase in the range of criticism which could be brought to bear on the theory.

The above discussion of how Friedman's conjecture might be appraised is by no means exhaustive: we have not asked whether the conjecture holds surprising implications, whether severe tests can be applied to the conjecture, whether the tests are easily replicable, whether the conjecture imports a simpler structure to the theory, and most importantly, we have not assessed the comparative development of content in new classical and Keynesian theories at this time. Nevertheless the discussion serves to highlight how a conjecture such as Friedman's can be appraised at a quasi-empirical level.

IV   A SIMPLE MODEL

The following model is designed to investigate the implications of introducing a positively sloped Phillips curve into monetarist analysis.   In particular the exercise is designed to ask whether the presence of such a Phillips curve in the medium-run is consistent with convergence to a long-run natural rate equilibrium where money is neutral (for a review of other factors which might violate the long-run neutrality proposition, see, for example, Gale (1982) and Sheffrin (1983)).   The model attempts to nest Friedman's conjecture in a preferred, monetarist, habitat.

$$\Delta P_t = \Delta P_t^e + \alpha(U_t^* - U_t) \qquad\qquad \alpha > 0 \quad (1)$$

$$U_t^* = U_R + \beta_1 \Delta P_t + \beta_2 \Delta^2 P_t \qquad\qquad \beta_1, \beta_2 > 0 \quad (2)$$

$$U_t = \bar{U} - \gamma Y_t \qquad\qquad \gamma > 0 \quad (3)$$

$$Y_t = M_t + \delta i_t - P_t \qquad\qquad \delta > 0 \quad (4)$$

$$M_t = M^* + \epsilon_t \qquad\qquad (5)$$

$$i_t = \bar{r} + \Delta P_t^e \qquad\qquad (6)$$

All variables except unemployment are measured in logs; Δ and Δ^2 are the first
and second difference operators; P_t is the general price level; ΔP_t^e is the
expected rate of inflation, $P_t^e - P_{t-1}$; U_t^* is the natural rate of unemployment,
U_t the actual rate of employment; U_R represents the component of the natural
rate of unemployment which is determined by "real" factors, $\bar{U}$ the component of
actual unemployment which is independent of output; Y_t is the level of real
output; M_t is the actual money stock, M^* is the target for the money stock set
by the authorities and ϵ_t is the deviation of the actual from target value of
the money stock; i_t is the nominal rate of interest, and $\bar{r}$ is the fixed real
rate of interest.

Equation (1) represents the short-run Phillips curve, with inflation rising
above the expected rate when the actual rate of unemployment is less than the
natural rate - according to the Lipsey-early Friedman-Phelps interpretation;
or, alternatively, with unemployment falling below the natural rate when the
actual is greater than the expected rate of inflation - according to the
Fisher-Lucas-later Friedman interpretation. Equation (2) nests the two alter-
native formulations of Friedman's medium-run Phillips curve conjecture discussed
in the last section: setting $\beta_2=0$ yields conjecture I; setting $\beta_1=0$ yields con-
jecture II. Equation (3) is the inverted form of a primitive production function
similar to that involved in Okun's Law (see Gordon 1982 for a study using this
relationship). Equation (4) is the equilibrium condition for a money market
characterised by an exogenous money stock, a transactions demand for money of

unit elasticity and an asset demand for money. Equation (5) describes the determination of the actual money stock. And equation (6) captures Fisher's theory of the determination of the nominal interest rate.

Solving the model for P yields

$$\{1 - \alpha\beta_1 - \alpha\beta_2 + \alpha\gamma\}P_t + \{\alpha\beta_1 + 2\alpha\beta_2 + \alpha\gamma\delta\}P_{t-1} - \alpha\beta_2 P_{t-2} =$$
$$\{1 + \alpha\gamma\delta\}P_t^e + \alpha\{U_R - \bar{U} + \gamma(M^* + \epsilon + \delta\bar{r})\} \tag{7}$$

After some initial scepticism about rational expectations (see the comment by Samuelson 1983, for example), Friedman seems now to have incorporated rational expectations into his view of how economies work (see Friedman and Schwartz 1982, for example). In line with our intention of setting the medium-run Phillips curve conjecture in its preferred habitat then, the appropriate way to close (7) is to take the expected values of equations (1) - (6) and set $P_t^e = EP_t$. Taking the expected values of the endogenous variables in (1) - (6) on the basis of the information available one period ago yields

$$EU_t = EU_t^* \tag{E1}$$
$$EU_t^* = U_R + (\beta_1 + \beta_2)EP_t - (\beta_1 + 2\beta_2)P_{t-1} + \beta_2 P_{t-2} \tag{E2}$$
$$EU_t = \bar{U} - \gamma EY_t \tag{E3}$$
$$EY_t = EM_t + \delta Ei_t - EP_t \tag{E4}$$
$$EM_t = M^* \tag{E5}$$
$$Ei_t = \bar{r} + EP_t - P_{t-1} \tag{E6}$$

Solving for EP yields

$$E\left[P_t | \phi_{t-1}\right] = \left[\frac{\beta_1 + 2\beta_2 + \gamma\delta}{\beta_1 + \beta_2 + \gamma\delta - \gamma}\right] P_{t-1} - \left[\frac{\beta_2}{\beta_1 + \beta_2 + \gamma\delta - \gamma}\right] P_{t-2}$$
$$+ \frac{\bar{U} - U_R - \gamma(M^* + \delta\bar{r})}{\beta_1 + \beta_2 + \gamma\delta - \gamma} \tag{8}$$

where ϕ_{t-1} is the information available one period ago, and E is the expected value operator.

Substituting (8) for P_t^e in (7) yields:

$$P_t - \left[\frac{\beta_1 + 2\beta_2 + \gamma\delta}{\beta_1 + \beta_2 + \gamma(\delta-1)}\right]P_{t-1} + \left[\frac{\beta_2}{\beta_1 + \beta_2 + \gamma(\delta-1)}\right]P_{t-2}$$

$$= \left[\frac{\bar{U} - U_R - \gamma(M^* + \gamma\bar{r})}{\beta_1 + \beta_2 + \gamma(\delta-1)}\right]\left[1 - \frac{2\alpha\gamma\delta}{1 - \alpha\beta_1 - \alpha\beta_2 + \alpha\gamma}\right]$$

$$+ \frac{\alpha\gamma\epsilon_t}{1 - \alpha\beta_1 - \alpha\beta_2 + \alpha\gamma} \tag{9}$$

Given that the point of this modelling exercise is to investigate the effects of introducing a medium run Phillips curve into a monetarist model, it makes sense to see first how the model behaves in the absence of this curve, i.e. without β_1 or β_2 effects. If the model were to display unstable behaviour without β_1 or β_2 effects, a demonstration that such effects generate unstable behaviour would not have much force.

Case I: No Medium Run Phillips Curve

In this case the model contains short and long run Phillips curves, but no medium run curve. Setting β_1, $\beta_2 = 0$ in (9) yields

$$P_t + \frac{\delta}{(1-\delta)}P_{t-1} = K_1 + K_2\epsilon_t \tag{10}$$

where K_1 and K_2 are constants. The stability condition is $\left|\frac{\delta}{1-\delta}\right| < 1$, or $\delta < 1/2$. If this condition holds $\frac{\delta}{1-\delta} > 0$, so the price level will display convergent alternations in response to a disturbance arising from $U_R, \bar{U}, M^*, \bar{r}$ or ϵ_t. This property is not unattractive given that combining the deterministic structure of the model with stochastic disturbances will yield something approximating the more or less regular cycles in economic activity observed. The $\delta < 1$ condition is consistent with the heuristic underlying monetarism, given that high values of δ would move the model towards the liquidity trap case; and is easily satisfied by empirical estimates of the interest elasticity of the demand for money gained using a wide array of econometric methods (see Laidler 1985a for a survey and Hendry 1985 for interest elasticity estimates which are substantially higher than those reported elsewhere in the literature).

Case II: The Medium Run Phillips Curve, $\beta_1 > 0$

Here the natural rate increases in tandem with the rate of inflation, which by hypothesis bears a one to one relationship with the level of variability in relative prices. This immediately introduces an element of instability into the model: an expansionary shock, such as an overshoot of the monetary target $\varepsilon_t > 0$, pushes the actual above the expected rate of inflation and the actual below the natural rate of unemployment; the expected rate of inflation adjusts upwards with a one period lag, closing the gap between the actual and natural rates; but the natural rate has moved upwards in the previous period in response to the increase in the variability of relative prices hypothesised to arise from the higher rate of inflation, widening the gap between the actual and natural rate; thus the higher value of β_1, the more likely it is that the model will be unstable. In the money market the increase in the expected rate of inflation increases the nominal interest rate, reducing the asset demand for money; this leads to an increase in the level of real output consistent with the money stock target, reducing the actual rate of unemployment by way of Okun's Law; thus the higher the δ value, and the lower the γ value, the more likely is instability. The question then is whether the fact that people are taken to use all the non-current information contained in the model when forming expectations allows the gap between the actual and natural rates of unemployment to be closed, and equilibrium to be restored.

Setting $\beta_2 = 0$ in (9) yields

$$P_t - \frac{(\beta_1 + \gamma\delta)}{\beta_1 - \gamma(\delta - 1)} P_{t-1} = K_3 + K_4 \varepsilon_t \qquad (11)$$

where K_2 is a constant. The stability condition is that $\left| \dfrac{\beta_1 + \gamma\delta}{\beta_1 + \gamma(\delta-1)} \right| < 1$, or $\delta + \beta_1 / \gamma < \frac{1}{2}$, which will be violated for sufficiently high δ and β_1 values, and

for sufficiently low γ values, as indicated in the discussion above. For example, if we take $\gamma = \frac{1}{3}$ according to tradition (see Gordon 1982), and $\delta = \frac{1}{10}$ (see Laidler 1985a), the model will be unstable when $\beta_2 > \frac{2}{15}$. If Friedman's conjecture is taken seriously, values of β_2 in this range are quite plausible. At the very least the introduction of the medium run Phillips curve introduces an element of instability into the model, making the stability condition more stringent, $\delta + \beta_1/\gamma < \frac{1}{2}$ rather than $\delta < \frac{1}{2}$. In this stable case money is non-neutral, and the "super-natural" vertical Phillips curve will only be regained, and neutrality restored, if $\beta_1 \to 0$ as $t \to$ "decades". In the unstable case convergence to the "super-natural" rate equilibrium will not occur even when $\beta_1 \to 0$: the characterisation of what happens in the interregnum between the domains of the short and long run Phillips curves is inconsistent with convergence to this equilibrium, and inconsistent with the neutrality of money proposition, even in its long run form.

Case III: The Medium Run Phillips Curve, $\beta_2 > 0$
_____

Here the natural rate increases in tandem with the acceleration rather than rate of change of the price level, the former acting as a proxy for the increasing volatility of relative prices which is postulated to occur at higher mean rates of anticipated inflation. In this form the natural rate will take the value implied by the long run Phillips curve only when the rate of inflation is steady: that is $U^*_t = U_R$ only if $\Delta^2 P_t = 0$. A major difference between this Conjecture II, $\beta_2 > 0$, $\beta_1 = 0$ and the Conjecture I, $\beta_1 > 0$, $\beta_2 = 0$ just discussed is that $\beta_2 \to 0$ as $t \to$ "decades" is no longer necessary for convergence to a "super-natural" rate equilibrium and monetary neutrality.

Setting $\beta_1 = 0$ in (9) yields:

$$P_t - \frac{(2\beta_2 + \gamma\delta)}{\beta_2 + \gamma(\delta - 1)} P_{t-1} + \frac{\beta_2}{\beta_2 + \gamma(\delta - 1)} P_{t-2} = K_5 + K_6 c_t \qquad (12)$$

The condition for real roots in (12) is that

$$\frac{(2\beta_2 + \gamma\delta)^2}{\left[\beta_2 + \gamma(\delta - 1)\right]^2} - \frac{4\beta_2}{\beta_2 + \gamma(\delta - 1)} = \frac{\gamma^2\delta^2 + 4\beta_2\gamma}{\left[\beta_2 + \gamma(\delta - 1)\right]^2} > 0 \qquad (13)$$

which clearly holds. The stability condition for (12) is that

$$\frac{2\beta_2 + \gamma\delta}{\beta_2 + \gamma(\delta - 1)} \pm \frac{\sqrt{\gamma^2\delta^2 + 4\beta_2\gamma}}{\beta_2 + \gamma(\delta - 1)} < 2 \qquad (14)$$

A sufficient condition for the violation of (14) and hence instability is that $\beta_2 + \gamma(\delta-1) > 0$: the dominant root is not that involving $+\sqrt{\quad}$ in (14) and stability requires

$$\sqrt{\gamma^2\delta^2 + 4\beta_2\gamma} < \gamma(\delta - 2) \qquad (15)$$

which is clearly violated. Following a similar line of reasoning for the case of $\beta_2 + \gamma(\delta - 1) < 0$, the dominant root now involves $-\sqrt{\quad}$ in (14), and a sufficient condition for stability is $4\beta_2 + 3\gamma\delta - 2\gamma > 0$. These are sufficient conditions for instability and stability respectively because there is a third possible conjunction of parameter values, $\beta_2 + \gamma(\delta - 1) < 0$ and $4\beta_2 + 3\gamma\delta - 2\gamma < 0$ which is difficult to resolve.

The behaviour of the model under Conjecture II is similar, qualitatively, to the behaviour under Conjecture I: the stronger the β_2 effect arising from the medium run Phillips curve the more likely is instability overall; similarly the stronger the δ effect arising from the asset demand for money, and the weaker the γ effect arising from Okun's Law, the more likely is instability; the model is unstable if $\beta_2 > \gamma(1-\delta)$. Taking the $\gamma = \frac{1}{3}$, $\delta = \frac{1}{10}$ illustration used for Conjecture I, the requirement for instability using Conjecture II is now more stringent, $\beta_2 > \frac{9}{30}$. Money is non-neutral in the medium-term only when $\Delta^2 p_t \neq 0$, and convergence to a "super-natural" rate equilibrium now requires only $\frac{\gamma(\delta-2)}{4} < \beta_2 + \gamma(\delta-1) < 0$ rather than $\beta_2 \to 0$ as $t \to$ "decades" in addition. Nevertheless there is still the clear possibility that what happens in the medium term makes the long run vertical Phillips case redundant.

Implications

The instability arising from the medium-run Phillips curve is somewhat similar to that arising from the Wicksell process in other models of monetary equilibrium (see Gale 1982, pp.136-144, for example). The irony here is that Friedman, in borrowing Wicksell's natural rate of interest concept for application to unemployment, has also taken on board the type of instability arising from Wicksell's cumulative process. Overall instability has only been demonstrated for the range of parameter values $\delta + \beta_1/\gamma > \frac{1}{2}$ (Conjecture I) and $\beta_2 > \gamma(1-\delta)$ (Conjecture II), but we would argue that such parameter values are quite feasible and consistent with the analytical system employed in monetarism.

If the instability results hold water this means that the second stage in Phillips curve analysis, the introduction of the vertical long-run Phillips curve may prove to have been a stimulating but misleading diversion in the history of Phillips curve analysis. The instability arising from the real effects of perturbations in the rate of inflation can be such as to ensure that economic systems will not return to a long-run natural rate of unemployment, $U^* = U_R$. This suggests that the medium-run concept of the natural rate of unemployment is the more interesting one to pursue. Such a re-direction of attention to the medium-run domain would also allow Phillips curve analysis to give an account of the real effects of anticipated inflation, such effects being absent from the system of Phillips curves which contains only a simple dichotomy between short and long-run curves. The real effects of anticipated inflation are widely discussed elsewhere in the academic literature (see Phelps 1972 and Cukierman 1983 for example), and motivate major innovations in economic policy, such as the anti-inflation policies pursued by many governments since the late 1970s. Without an account of the way such effects impinge on the natural rate of unemployment monetarism contains a major gap in content.

V   CONCLUSION

So far in this paper we have refrained from explicit reference to the question of whether Friedman's conjecture is consistent with the medium run associations between unemployment and inflation actually observed: the appraisal method employed has been concerned with the quasi-empirical aspects of the conjecture.   The appraisal problem at the empirical level can be framed in terms of whether the content of monetarism, as augmented by Friedman's conjecture, can encompass the content of the two main rival theories, namely the hysteresis augmented version of Keynesian theory, and the new classical theory (see Section II of this paper).   Hendry, 1983, argues persuasively that the model encompassing principle in econometrics embodies many of the consider-ations involved in the Lakatos criterion of comparative empirical content. Whilst this is not the place for an econometric evaluation of Friedman's conjecture along such lines, some summary empirical statistics regarding inflation - unemployment associations are of interest here because they follow Friedman's procedure of presenting evidence in this form in support of his conjecture - see Friedman 1977, Table I and Figures 3 and 4.

A raw characterisation of the way the three competing theories depict the natural rate in the medium term is as follows (see Section II of this paper, equations (ix) and (xi)).

Friedman: $\Delta U_t = \Delta U_{Rt} + \beta_1 \Delta^2 P_t - d(\Delta Y_t - \Delta Y^*)$——FM

Hysteresis: $\Delta U_t = \Delta U_{Rt} + e\Delta^2 U_t + df(Y_t - Y_t^*) - d(\Delta Y_t - \Delta Y_t^*)$——HK

New
Classical : $\Delta U_t = \Delta U_{Rt} - d(\Delta Y_t - \Delta Y_t^*)$——NCM

where $\beta_1 > 0$, $d > 0$, $e > 0$, $f < 0$, $df < 0$.

As a preliminary to econometric investigation it is useful to know the problem which is to be addressed.   Tables I and II assemble summary statistics of the first order correlation coefficients between ΔU and $\Delta^2 U_t$, $\Delta^2 P_t$, $(Y_t - Y_t^*)$ and $(\Delta Y_t - \Delta Y_t^*)$ for the U.K. 1860-1984, and for various OECD countries 1953-1983.

FIRST ORDER CORRELATION COEFFICIENTS

TABLE I : U.K. 1860 - 1984

	ΔU_t	$\Delta^2 U_t$	$\Delta^2 P_t$	$Y_t - Y_t^*$	$\Delta Y_t - \Delta Y_t^*$
1.	1860-1984	0.67	-0.37	-0.25	-0.80
2.	1860-1913	0.61	-0.01	-0.17	-0.80
3.	1920-1938	0.54	-0.27	-0.29	-0.88
4.	1946-1984	0.52	-0.47	-0.61	-0.65

Sources of Data

U - Unemployment % : C.H. Feinstein, National Income, Expenditure and Output of the U.K. 1855-1965, Cambridge U.P., 1972, and Employment Gazette, various issues.

P - Log of Retail Price Index: Feinstein, op. cit., and Economic Trends, June 1985.

Y - Log of Real G.D.P. : Feinstein, op. cit., and Economic Trends, May 1985.

Y^* - Trend of Real G.D.P. : gained by fitting a linear time trend $Y_t = a + bt + \varepsilon_t.$

FIRST ORDER CORRELATION COEFFICIENTS

TABLE II : OECD COUNTRIES 1953-1983

		$\Delta^2 U_t$	$\Delta^2 P_t$	$Y_t - Y_t^*$	$\Delta Y_t - \Delta Y_t^*$
	ΔU_t				
1.	U.K.	0.55	-0.22	-0.34	-0.20
2.	U.S.	0.73	-0.17	-0.45	-0.64
3.	W. Germany	0.50	-0.10	-0.34	-0.60
4.	France	0.49	-0.03	0.12	-0.40
5.	Italy	0.67	-0.20	0.10	-0.19
6.	Sweden	0.69	-0.41	-0.08	-0.53
7.	Netherlands	0.56	-0.22	-0.51	-0.57
8.	Japan	0.66	-0.51	0.14	-0.33

Sources of Data

U – Unemployment % : B.R. Mitchell, European Historical Statistics, 2nd Rev. Ed., Macmillan 1980; B.R. Mitchell, International Historical Statistics: the Americas and Australasia, Macmillan 1983; and International Labour Statistics various issues. For France U is measured as the number of people registered as unemployed.

P – Log of Retail Price Index: International Financial Statistics, various issues.

Y – Log of Real G.D.P. : International Financial Statistics, various issues.

Y^* – Trend of Real G.D.P. : gained by fitting a non-linear time trend
$$Y_t = a + b_1 t + b_2 t^2 + \varepsilon_t$$

In relation to Friedman's conjecture the striking feature to emerge from Tables I and II is that $(\Delta U_t, \Delta^2 P_t)$ correlation coefficients are all negative, not positive as might be expected if the conjecture holds. In relation to the hysteresis theory, the $(\Delta U_t, \Delta^2 U_t)$ correlation coefficients all have the expected sign and appear to be remarkably stable across countries, and over time for the U.K. The $(\Delta U_t, Y_t - Y_t^*)$ and $(\Delta U_t, \Delta Y_t - Y_t^*)$ correlations all have the signs expected, with the exception of France and Japan for $(\Delta U_t, Y_t - Y_t^*)$.

The suggestion is, and the stress is that in the absence of an econometric examination of the data and encompassing tests this is only a suggestion, that Friedman's conjecture is likely to encounter severe problems in accounting for the data in question: the raw data suggest a negative rather than positive association between unemployment and inflation over "decades". This negative correlation between unemployment and inflation is, however, consistent with the hysteresis theory, as is the positive correlation between unemployment and its own history strikingly evident in Tables I and II.

Friedman's reaction to the possibility of a negative correlation between inflation and unemployment over "decades" was to suggest that the positively sloped Phillips curve might be relevant only to periods characterised by an upward shift in the mean rate of inflation, a negatively sloped Phillips curve being relevant to periods in which the mean rate of inflation moves downwards. In what is virtually a footnote to his conjecture Friedman says: "...I have stressed the effect(s) ... produced by a transition from a monetary system in which there was a "normal" price level to a monetary system consistent with long periods of high ... inflation ... it should be noted that once ... economic agents have adjusted their practices ... a reversal to the earlier monetary framework or even the adoption in the new monetary framework of a successful policy of low inflation would in turn require new adjustments, and these might have many of the same adverse transitional effects on the level of employment ... there would appear to be an intermediate run negatively sloped Phillips curve instead of the positively sloped one I have tried to rationalise" (Friedman 1977, p.468).

Apart from hinting that Friedman is perhaps thinking of a fourth stage in Phillips curve analysis in which he is resurrected as a Keynesian, this modification to the positively sloped Phillips curve conjecture serves to destroy the foundations of the conjecture: the variability of relative prices has to be specified as either higher, lower or the same at higher mean rates of anticipated inflation, or the medium run Phillips curve has no refutable content which can be appraised.

If this paper has served its purpose it will have helped clarify some of the issues involved in explaining unemployment in the medium term. The main conclusion is that what happens in the medium term makes convergence to the long run vertical Phillips curve unlikely, and that money is unlikely to be neutral, even in the long run. There is also a prima facie case that hysteresis augmented theories, rather than Friedman's conjecture or the new classical theory, will encounter fewer difficulties, and be led to introduce fewer content reducing modifications, in order to account for the unemployment-inflation associations observed in the medium term domain.

BIBLIOGRAPHY

Akerlof, G.A. (1984), An Economic Theorist's Book of Tales, Cambridge, U.P., Cambridge.

Alchian, A.A. (1970), "Information Costs, Pricing and Resource Unemployment" in E.S. Phelps et al, eds., The Microeconomic Foundations of Employment and Inflation Theory, 27-52, W.W. Norton & Co., New York.

Altonji, J.G. (1983), "The Intertemporal Substitution Model of Labour Market Fluctuations: an Empirical Analysis", 105-146 in C.A. Greenhalgh et al,eds. op. cit.

Ashenfelter, O. and Card, D. (1983), "Time Series Representations of Economic Variables and Alternative Models of Labour Markets", 83-103, in C.A. Greenhalgh et al, eds., op. cit.

Atkinson, A.B. (1969), "The Timescale of Economic Models: How Long is the Long Run?", Review of Economic Studies, 36, 137-152.

Blanchard, O. et al (1985), "Employment and Growth in Europe: a Two Handed Approach", Centre for European Policy Studies, June, Brussels.

Blinder, A.S. (1981),"Inventories and the Structure of Macro Models", American Economic Review, 71.2, 11-16.

Brailsford, P. (1966), Physical Principles of Magnetism, D. Van Nostrand Company, London.

Caldwell, B. (1982), Beyond Positivism, Allen and Unwin, London.

Caldwell, B., ed. (1984), Appraisal and Criticism in Economics, Allen and Unwin, London.

Clark, K. & Summers, L. (1983), "Labour Force Participation: Timing and Persistence", in C.A. Greenhalgh et al, eds., op. cit.

Cross, R. (1982), "The Duhem-Quine Thesis, Lakatos and the Appraisal of Theories in Macroeconomics", Economic Journal, June, 92.366, 320-340.

Cross, R. (1983), "Long Term Unemployment, Hysteresis and the Natural Rate of Unemployment", Business Economist, Summer, 4-15.

Cross, R. (1984), "Monetarism and Duhem's Thesis", 78-99 and 111-120 in P. Wiles and G. Routh, eds., op. cit.

Cross, R. (1985), "On the Long Run Phillips Curve Genus", mimeo, University of St. Andrews.

Cukierman, A. (1983), "Relative Price Variability and Inflation: a Survey and Further Results" in Carnegie-Rochester Conference Series on Public Policy, 19, 103-157.

Engle, R.F. (1983), "Estimates of the Variance of U.S. Inflation Based Upon the Arch Model", Journal of Money, Credit and Banking, 15.3, 286-301.

Engle, R.F., Hendry, D.F. & Richard, J-F. (1983), "Exogeneity", Econometrica, 51, 277-304.

Fisher, I. (1926), 'A Statistical Relation between Unemployment and Price Changes', International Labour Review, June 1926, 13.6, 785-792.

Friedman, M. (1968), 'The Role of Monetary Policy', American Economic Review, March 1968, 58.1, 1-17.

Friedman, M. (1977), 'Nobel Lecture: Inflation and Unemployment', Journal of Political Economy, June 1977, 85.3, 451-472.

Friedman, M. & Schwartz, A. (1982), Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices and Interest Rates, University of Chicago Press, for the National Bureau, Chicago.

Gale, D. (1982), Money: In Equilibrium, Cambridge U.P.

Gardner, M. (1982), "Predicting Novel Facts", British Journal for the Philosophy of Science.

Gordon, R.J. (1982), "Inflation, Flexible Exchange Rates and the Natural Rate of Unemployment", in Baily, M.N., ed., Workers Jobs and Inflation, 89-158, The Brookings Institution, Washington D.C.

Greenhalgh, C.A., Layard, P.R.G. & Oswald, A.J., eds (1983), The Causes of Unemployment, Oxford U.P., 1983.

Grossman, S.J. and Hart, O.D. (1981), "Implicit Contracts, Moral Hazard and Unemployment", American Economic Review, May, 71.2, 301-307.

Hahn, E. (1984), Equilibrium and Macroeconomics, Blackwell, Oxford.

Hall, R.E. (1980), "Labour Supply and Aggregate Fluctuations", Carnegie-Rochester Series on Public Policy, 12, 7-33.

Hargreaves Heap, S.P., (1980), "Choosing the Wrong Natural Rate: Accelerating Inflation or Decelerating Employment and Growth", Economic Journal, September, 90.359, 611-620.

Hendry, D.F. (1983), "Econometric Modelling: the "Consumption Function" in Retrospect", Scottish Journal of Political Economy, November, 30.3, 193-220.

Hendry, D.F. (1985), "Monetary Econometric Myth and Econometric Reality", Oxford Review of Economic Policy, Spring, 1.1, 72-84.

Hicks, J. (1979), Causality in Economics, Basic Books, New York.

Hood, W.C. & Koopmans, T.C. (1953), eds., Studies in Econometric Method, Yale U.P., New Haven.

Laidler, D.E.W. (1981), 'Monetarism: an Interpretation and Assessment', Economic Journal, March 1981, 91.1, 1-28.

Laidler, D.E.W. (1985a), The Demand for Money, 3rd ed., Harper and Row, New York.

Laidler, D.E.W. (1985b), "Monetary Policy in the U.K. - Successes and Shortcomings", Oxford Review of Economic Policy, 1.1, Spring, 35-43.

Lakatos, I. (1976), Proofs and Refutations, eds. J. Worrall & E. Zahar, Cambridge U.P.

Lakatos, I. (1978), Philosophical Papers, Vol.1: The Methodology of Scientific Research Programmes, Vol. 2, Mathematics, Science and Epistemology, J. Worrall and G. Currie, eds., Cambridge U.P.

Leamer, E.E. (1985), "Vector Autoregressions for Causal Inference?", in Carnegie-Rochester Conference Series on Public Policy, 22, 255-303.

Lilien, D.M. (1982), "Sectoral Shifts and Cyclical Unemployment", Journal of Political Economy, 10, 777-793.

Lindbeck, A. & Snower, D.J. (1985), "Explanations of Unemployment", Oxford Review of Economic Policy, Summer, 1.2, 34-59.

Lipsey, R.G. (1960), 'The Relationship between Unemployment and the Rate of
  Change of Money Wage Rates in the U.K. 1862-1957: A Further Analysis'
  Economica, February 1960, XXVII.105, 1-31.

Lucas, R.E. Jr (1972), 'Expectations and the Neutrality of Money,' Journal of
  Economic Theory, April 1972, 4.2, 103-124.

Lucas, R.E. Jr (1973), 'Some International Evidence on Output-Inflation Trade-
  Offs', American Economic Review, June, 63.3, 326-334.

Lucas, R.E. Jr (1975), "An Equilibrium Model of the Business Cycle", Journal
  of Political Economy, 83, 1113-1144.

Lucas, R.E. Jr and Rapping, L.A. (1970), "Real Wages, Employment and Inflation",
  in E.S. Phelps et al, eds., Microeconomic Foundations of Employment and
  Inflation Theory, W.W. Norton & Co., New York.

Malinvaud, E. (1984), Mass Unemployment, Blackwell, Oxford.

McDonald, I.M. & Solow, R.M. (1981), "Wage Bargaining and Unemployment",
  American Economic Review, December, 71.5, 896-908.

Minford, P. (1983), "Labour Market Equilibrium in an Open Economy", 207-244 in
  C.A. Greenhalgh et al, eds., op. cit.

Modigliani, F. & Cohn, R. (1979), "Inflation, Rational Valuation and the Market",
  Financial Analysts Journal, March/April, 24-44.

Moore, J. (1984), 'The Costs of Inflation: Some Theoretical Issues' Centre for
  Economic Policy Research Discussion Paper No. 19, May 1984, 6, Duke of York
  St., London SW1Y 6LA.

Morishima, M. (1984), "The Good and Bad Uses of Mathematics", 51-77 in P. Wiles
  and G. Routh, eds., Economics in Disarray, Blackwell.

Nickell, S. (1979), "Estimating the Probability of Leaving Unemployment",
  Econometrica, September, 47.5, 1249-1266.

Phelps, E.S. (1967), 'Phillips Curves, Expectations of Inflation and Optimal
  Unemployment over Time', Economica, August 1967, 34, 254-281.

Phelps, E.S. (1972), Inflation Policy and Unemployment Theory, Macmillan, London.

Phillips, A.W. (1985), 'The Relation Between Unemployment and the Rate of
    Change of Money Wages Rates in the U.K. 1861-1957', Economica, November
    1985, 25.100, 283-299.

Samuelson, P.A. (1983), 'Comment' on 'What Would Keynes Have Thought of
    Rational Expectations', in Worswick, David and Trevithick, James, eds.,
    Keynes and the Modern World, Cambridge U.P. 1983.

Sheffrin, S.M. (1983), Rational Expectations, Cambridge U.P.

Sims, C. (1980), "Macroeconomics and Reality", Econometrica, 48, 1-48.

Sjaastad, L.A. (1974), "Monetary Policy and Suppressed Inflation in Latin
    America", in R.Z. Aliber, ed., National Monetary Policies and the
    International Financial System, Chicago U.P.

Solow, R.M. (1985), "Insiders and Outsiders in Wage Determination", Scandinavian
    Journal of Economics, 87.2, 411-428.

Stiglitz, J.E. (1984), "Price Rigidities and Market Structure", American
    Economic Review, May, 74-2, 350-355.

Taylor, J.B. (1980), "Aggregate Dynamics and Staggered Contracts", Journal of
    Political Economy, 88.1, 1-23.

Tobin, J. (1972), 'Inflation and Unemployment', American Economic Review,
    March , 62.1, 1-18.

Wadhwani, S.W. (1984), "Inflation, Bankruptcy, Default Premia and the Stock
    Market", Centre for Labour Economics Discussion Paper No. 194, March 1984,
    L.S.E.

Weitzman, M.L. (1982), "Increasing Returns and Unemployment Theory", Economic
    Journal, December, 92.368, 787-804.

Wicksell, K. (1898), Geldzins and Guterpreise, Gustav Fischer, Jena, translated
    by R.F. Kahn as Interest and Prices, Macmillan, London, 1936.

Wiles, P. & Routh, G., eds. (1984), Economics in Disarray, Blackwell, Oxford.