SLA-Formulation for Squeezed Protection in Elastic Optical Networks Considering the Modulation Format

K. D. R. Assis, R. C. Almeida Jr, H. Waldman, A. F. Santos, M. S. Alencar, M. J. Reed, A. Hammad and D. Simeonidou

Abstract—In Spectrum-Sliced Elastic Optical Path Networks (SLICE), the lightpath bandwidth is variable and the virtual topology overlay on a physical topology shall be designed to optimize the spectrum utilization. Under static traffic, SLICE networks are typically designed through a Mixed Integer Linear Programming (MILP) with the aim of minimizing the spectrum utilization. In this paper, a new MILP formulation for protection in SLICE networks is proposed, which uses the concept of bandwidth squeezing and grooming to guarantee a minimum agreed bandwidth for each source-destination pair in the surviving bandwidth. The route for each demand on the physical topology is determined by balance equations together with physical layer constraints in the formulation, so that no pre-calculated routes are required and the modulation format of each established lightpath may be chosen with enough quality of transmission and save network spectrum. Therefore, the proposed formulation jointly solves the virtual topology design and physical topology design problems. The first results evaluate the effectiveness of the MILP formulation for two small networks when connections are under different Service Level Agreement (SLA) requirements and are provisioned by an appropriate protection scheme and different modulation formats. Due to the NP-hard nature of the proposed MILP formulation, a heuristic algorithm for moderately large networks is also proposed. Case studies are carried out in order to analyze the basic properties of the formulation and the performance of the proposed heuristic. With the proposed formulation, it is possible to identify the configurations that ensure minimum spectrum occupation with different kinds of protection for each lightpath. Different kinds of modulation formats are considered and contrasted to the benchmark case of a single modulation format and using the same kind of protection for all lightpaths.

Index Terms—Elastic Optical Networks, Optimization, Routing, Survivability, Modulation Format, Virtualization

I. Introduction

AS the traffic load over optical networks grows rapidly, how to properly utilize the optical network capacity has become a major interest in both academia and industry in recent

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years. Wavelength-Division Multiplexing (WDM) has been used to increase the spectrum efficiency of optical networks. However, when the wavelength channels with different line rates are multiplexed into one fiber, the gap between the neighboring wavebands is large when the effective bandwidth of a channel is short. On the other hand, high bit-rate demands have to be fit into the fixed bandwidth of a channel, requiring the use of a modulation format with high spectral efficiency, which impacts its reach. Consequently, a difficult tradeoff occurs in WDM optical networks under heterogeneous traffic demands. To overcome this limitation, an elastic spectrum allocation scheme capable of using the spectrum in a manner closer to a grid-less system has been adopted to increase the spectrum utilization [1], [2]. In this type of network, usually referred to as SLICE, Elastic or Flexible optical networks, the bandwidth of a waveband on the spectrum is varied to fit the traffic demand from the upper layer, instead of being limited to rigid wavelength channels, as in WDM.

In either WDM or Elastic optical networks, proper recovery schemes are needed to guarantee that the associated client demands continue being served even in the case of failures [3], [4],[5]. Recovery can be provided by either protection, in which the failed working path is substituted by a pre-assigned backup path, or restoration, in which no resource is pre-assigned and the working path is rerouted on the fly. Backup paths can use the resources that are dedicated to protect a single working path, or that are shared to provide protection to multiple working paths. The former scheme is known as Dedicated Path Protection (DPP) and the latter as Shared Path Protection (SPP) [3].

As mentioned before, in SLICE networks, paths can use a variable number of slots, depending on the requested bitrate, modulation format and slot width. As a consequence, new recovery schemes and Service Level Agreements (SLA) can be devised exploiting the variation in the amount of resources assigned to each path. For instance, the SLA between network operators and clients can include the possibility that all or only part of the requested bit rate is restored in case of failure [3], [6], [7]. A new protection mechanism with traffic partitioning, squeezed bandwidth after a failure and traffic grooming was proposed in [8], in which the same SLA is used for every demand. The mechanism not only provided the possibility of assuring protection for a fraction of the traffic, but also allowed careful distribution of the traffic throughout the network to considerably save spectrum resource.

This work investigates the problem of how to efficiently

design SLICE networks with different SLAs for individual demands. The corresponding MILP formulation is derived, and a comparison between the results of the new proposal and the design with single SLA for all demands, [8], is presented. Furthermore, differently from the previous work [8], in this current MILP formulation the grooming, routing and spectrum allocation (RSA) and the modulation format are integrated during the optimization process. The MILP provides a route from the source to the destination node of each demand, either through a direct lightpath or by performing grooming on existing lightpaths. An adequate allocation of channels in the optical spectrum (i.e. indices of frequency slots) is provided, so that the same set of contiguous spectrum (contiguity constraint) is maintained in all links along the chosen route (continuity constraint). Also in this proposed MILP formulation, the route for each demand (over virtual and physical topology) is determined by flow balance equations in the formulation. Therefore, pre-calculated routes are not required. The authors believe that the proposed MILP formulation, by efficiently provisioning each connection in the network, may help a network operator to avoid high resource consumption; this becomes particularly important if either conventional DPP or a single modulation format is employed.

The remainder of this paper is structured as follows: The related works are discussed in Section II. The problem and the methodology that are used in the paper are presented in Section III. The MILP formulation is described in Section IV. A heuristic approach for larger networks is presented in Section V. In Section VI, the performance of the proposed method is compared with traditional approaches, and the obtained results are discussed. Section VII concludes the paper.

II. RELATED WORKS

In a previous work [9], a model for the offline routing and spectrum allocation problem is proposed with a set of predetermined routes for each demand. Although the authors in [9] have shown that their version of the RSA problem can reach near-optimal solutions, this model does not guarantee a global optimum as it excludes many feasible paths that can generate optimal solutions. Apart from that, the formulations do not consider the integrated problem of virtual topology design (VTD) and grooming[10],[11] on the virtual topology, as well as routing and spectrum assignment, and do not take into account modulation format constraints.

The virtual and physical topology design problems have been extensively studied in both optical "Grid" and "Gridless" networks with or without survivability aspects. The authors in [12],[13] studied the mapping problem in optical networks, taking into consideration some optical layer constraints, such as the transmission reach constraint and the spectral continuity/conflict constraints. The more recent work [14], focused on investigating how to provision topology mapping with survivability criteria against link failures. The authors in [12] proposed an efficient link protection scheme that relies on constructing an enhanced topology with survivability in the virtual layer. Another proposed restoration scheme is termed *squeezed restoration* [6]. It is a type of recovery scheme where

the backup path is established with a bandwidth reduction in relation to the working path's bandwidth and may reach a required minimum amount considering the client requirement; this is known as *bandwidth squeezing*. This generates cost-effective restoration in terms of spectral resource utilization, which increases the number of surviving paths for the mission-critical data when there are insufficient backup resources in a disastrous failure situation. The authors in [8] developed a scheme similar to [6], but aimed at protection, which is referred to as *partial protection*: after any single link failure, the flow can drop to the partial protection requirement, where a fraction of the demand is guaranteed to remain available between the source and destination after any failure. However, these works have not addressed the problem of providing different protection characteristics for each connection. That is important because different connections can demand different protection requirements.

Notice that the terms *squeezed restoration* and *partial protection* were originally used, respectively, for restoration and protection mechanisms.

III. METHODOLOGIES

In optical networks, a connection is routed through many nodes in the network between its source and destination, and there are many elements along its path that can fail. The only practical way of obtaining good availability is to make the network survivable, that is, able to continue providing service in the presence of failures. Protection switching is the key technique used to ensure survivability whereby redundant capacity is provided within the network to allow automatic rerouting of the affected traffic around a failure using the redundant capacity. The following explains some basic concepts as well as describing alternative forms of protection mechanisms that can be efficiently used in Elastic Optical Networks (EONs).

A. Survivability Design in Optical Networks with Traffic Squeezing

Recently, there have appeared some studies about squeezing restoration/protection in EONs [3], [6] and [15]. To illustrate the squeezing protection concept, Fig. 1a shows a simple network topology with an active lightpath between nodes 1 and 3. Let us assume that the lightpath is transporting 100 Gbit/s of traffic. To protect such a lightpath against a link failure, it is possible to find another lightpath, including route and available spectrum, for the same 100 Gbit/s of capacity, as shown in Fig. 1b. In the event of a failure at link 1-3, the disrupted lightpath is obviously restored using the backup path 1-2-3. This is the DPP scheme that has been traditionally used in optical networking.

However, in EONs, *traffic squeezing* can be applied as a new feature during service recovery, in addition to the conventional DPP. By applying traffic squeezing with to the protection capability, the traffic of disrupted lightpaths at failure time may be reduced in a manner commensurate to the previously running working traffic. This case is named in this paper as DPP with squeezing capability (DPP+S) and is illustrated in Fig.1c. Note that if, under a link failure, the original 100 Gbit/s of traffic

may be squeezed to 50% of its normal operation bitrate, just an extra of 50 Gbit/s has to be reserved for protection purpose, requiring from the network 150 Gbit/s, i.e., much less capacity than with DPP.
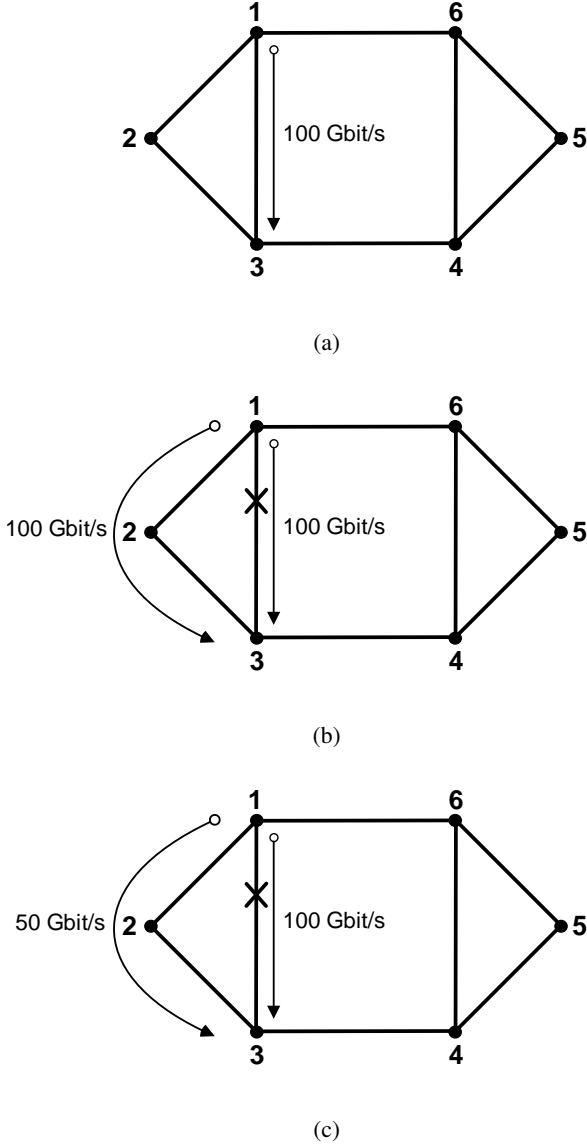

(a)

(b)

(c)

Fig. 1: Topology 1. Protection mechanism examples for a source-destination traffic transmission: (a) Normal condition, (b) DPP, (c) DPP+S

B. SLA Mechanism for several cases: The PDDP and (PDPP+S) approaches.

The main driving factor in the adoption of elastic networking is the need to save spectrum to accommodate an ever-increasing traffic demand that is leading the bandwidth mining progress to exhaust the capacity of the fixed grid WDM network. Bandwidth mining was predicated on the assumption of spectral resource overprovisioning in which a bandwidth-hungry, inflexible protection scheme such as DPP was welcome without any major objection. In the framework of elastic networking, though, in which spectrum is treated as a scarce resource, a spectrally more efficient substitute for DPP is now needed.

The core idea of the protection mechanism proposed in [8] is then to reduce the amount of extra bandwidth used for protection whenever possible; to allow the network to distribute the total bandwith (working plus redundant) in a flexible way among as many disjoint paths between source and destination as needed within the connectivity constraints of the physical topology; to coordinate this traffic distribution with the optimization of a network-wide objective function; and to guarantee in the SLA that, in the event of a single failure of one of the participating paths, the user will still receive at least an agreed specified fraction of the committed traffic.

Let $\alpha(s,d)$ be the ratio between the extra bandwidth reserved for protection of the s-d demand and the committed bandwidth Λ^{sd} under normal operation. In conventional DPP, $\alpha(s,d) = 1$, since no bandwidth sqeezing is tolerated and only two link-disjoint paths are engaged in the process, even if the physical topology provides more connectivity. Assume that $0 < \alpha(s,d) \leq 1$, so that the total amount of reserved bandwidth for both working and protection traffic on connection s-d is $[1 + \alpha(s,d)]\Lambda^{sd} \leq 2\Lambda^{sd}$, i.e., possibly lower than if conventional DPP were used.

On the other hand, the user may negotiate in the SLA a specified maximal loss of bandwidth that he/she is willing to tolerate during the repair time in the event of a single failure in one of the link-disjoint paths that carry his/her traffic. If $\beta(s,d)$ is the maximal fraction of the committed traffic Λ^{sd} the user is willing to lose during this time, then the SLA will allow the s-d traffic to be squeezed to at least a guaranteed bandwidth of $[1 - \beta(s,d)]\Lambda^{sd}$.

In order to provide this guarantee, it is necessary that each of the participating disjoint paths that carry the $s - d$ traffic be allowed to carry at most a bandwidth of $[\alpha(s,d) + \beta(s,d)]\Lambda^{sd}$, so that, in the event of its failure, the network may still provide a total bandwidth of at least $[1 + \alpha(s,d)]\Lambda^{sd} - [\alpha(s,d) + \beta(s,d)]\Lambda^{sd} = [1 - \beta(s,d)]\Lambda^{sd}$, as agreed in the SLA.

In order to comply with both the network and the user requirements, the choice of $\alpha(s,d)$ and $\beta(s,d)$ cannot be arbitrary. Since each of the $g(s,d)$ participating link-disjoint paths can carry a bandwidth of at most $[\alpha(s,d) + \beta(s,d)]\Lambda^{sd}$, a total bandwidth of $g(s,d)[\alpha(s,d) + \beta(s,d)]\Lambda^{sd}$ may be made available between s to d by the network. Since this amount is upper-bounded by the network requirement to $[1 + \alpha(s,d)]\Lambda^{sd}$, the following inequality must hold:

$$g(s,d)\left[\alpha(s,d) + \beta(s,d)\right]\Lambda^{sd} \leq [1 + \alpha(s,d)]\Lambda^{sd}, \quad (1)$$

with equality holding for most efficient use of the spectrum. Therefore, the best compromise between $\alpha(s,d)$ and $\beta(s,d)$ is given by

$$\alpha(s,d) = [1 - \beta(s,d).g(s,d)] / [g(s,d) - 1]. \quad (2)$$

Moreover, since $\alpha(s,d) \geq 0$ so as to keep the committed bandwidth fully provided under normal operation, then

$\beta(s,d) \leq 1/g(s,d)$, with $g(s,d) \geq 2$.

If $\beta(s,d) = 0$, the strategy may still be referred to as DPP, since the extra traffic is still fully dedicated to the connection. In the presence of traffic partitioning, DPP is hereby referred to as Partitioning Dedicated Path Protection (PDPP). The benefits of PDPP can be inferred by analyzing the example in Fig.2a, where $\alpha(s,d)$ has been assumed as 0.5 (50%). Notice that the working traffic may be partitioned between two lightpaths (1-3 and 1-2-3), which consumes the same 100 Gbit/s of working traffic as under DPP, but just 50% of extra traffic is reserved on lightpath 1-6-4-3. Therefore, less bandwidth is required (150 Gbit/s) when compared to conventional DPP (200 Gbit/s).

The network objective is to minimize the spectrum slot utilization by all connections while complying with all SLAs. Each user objective is to keep his/her value of $\beta(s,d)$ as small as possible. The two objectives are compatible with (1), but they are contingent on the connectivity of the network. However, if all participating paths make full utilization (for both work and protection) of the maximum rate $[\alpha(s,d) + \beta(s,d)]\Lambda^{sd}$ under normal operation, then (2) must hold. Under this condition, there is a clear compromise between $\alpha(s,d)$ and $\beta(s,d)$, as shown on Figs. 2a and 2b. Whereas the PDPP solution of Fig. 2a does not allow any squeezing by keeping $\beta(s,d) = 0$, leading to $\alpha(s,d) = 0.5$ for $g(s,d) = 3$ in (2), the solution of Fig. 2b, by allowing squeezing with $\beta(s,d) = 0.2$ in PDPP with squeezing (PDPP+S), makes feasible the reduction of $\alpha(s,d)$ from 0.5 to 0.2 in (2), thus relieving the stress on the network spectral resources. Note that in multipath routing, differential delay is a problem that must be dealt with for the correct operation of the network. References [16] and [17] address this problem for networks for enabling its practical applications.

In the next Section, a MILP formulation is presented in which $\beta(s,d)$ is specified for each (s,d) pair according with a given SLA, and $\alpha(s,d)$ is a variable to be obtained in the optimization of a network-wide objective function. This strategy is different from [8] and [14] where all (s,d) pairs use the same SLA. In this way, one can obtain differentiated protection for different users by specifying $\beta(s,d) = 0$ for PDPP; $\beta(s,d) = 0$ with $\alpha(s,d) = 1$ for DPP; or $\beta(s,d) \geq 0$ according with each SLA for PDPP+S.
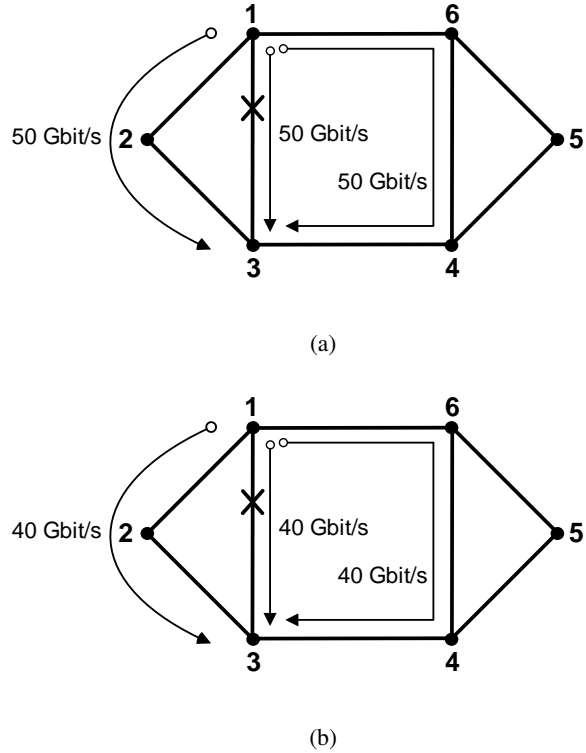

(a)

(b)

Fig. 2: Protection mechanism examples for a source-destination traffic transmission: (a) PDPP with $\alpha(s,d) = 0.5$; (b) PDPP+S with $\alpha(s,d) = 0.2$ and $\beta(s,d) = 0.2$.

IV. MILP FORMULATION

Most approaches [9],[18] divide the compound client and optical design into two separate problems: virtual topology design, in which best connections among nodes are derived from traffic demand; and the RSA, in which physical paths are accommodated in the physical topology to support the requested connections. Other works in the literature [19], [18] assume that the virtual topology is known beforehand. Differently from the previous works [8], [14], in this paper, a novel Mixed Integer Linear Programming (MILP) formulation is proposed to solve both problems (VTD and RSA) jointly, taking into account grooming, modulation format selection, without pre-calculated routes and the possibility of different SLA for each connection.

After defining the nomenclature IV-A, input parameters IV-B and variables IV-C used in the formulation, the complete MILP is presented in IV-D.

A. Notation

- s and d denote the source and destination nodes of the traffic demands in the network, respectively.
- i and j denote originating and terminating nodes of a variable bandwidth lightpath, respectively.
- m and n denote endpoints of a physical link in the network.
- z denotes the kind of mudulation format from a set of M available modulation formats.

B. Given

- $G = (N, E)$: A graph with a node set N where each node is associated with a node of the physical network, and an edge set E where each edge is associated to a physical link of the network.
- Δ_i: virtual degree of node i.
- Λ^{sd}: Traffic matrix element, used to denote the traffic intensity (in Gbps) from source node s to destination node d.
- Maximum squeezed bandwidth ratio: $\beta(s, d)$, where [1-$\beta(s, d)$] is the minimum admitted bandwidth fraction after a link failure, as agreed in the SLA on pair sd.
- Minimum physical degree: $g(s, d)$ is the minimum physical degree of a node along the path between sd. For example, if $s=1$ and $d = 5$ in Fig.2; the minimum physical degree will be from node 5 with degree 2. Therefore, $g(1, 5)=2$.
- F: Filter Guard Band (FGB), which represents the minimum spectrum width between wavebands.
- slot width: Ω.
- d_{mn}: Distance between the nodes m and n on physical topology.
- Spectral efficiency of the modulation format z: η_z, where $z \in 1, \ldots, M$.
- Maximum reach for a lightpath using modulation format z: d_z, where $z \in 1, \ldots, M$.
- χ: A large number.

C. Variables

- Lightpath bandwidth V_{ij}: bandwidth of an elastic lightpath from node i to node j in the virtual topology (Gbps).
- Lightpath indicator b_{ij}: A binary variable that indicates whether an elastic lightpath from node i to node j exists in the virtual topology.
- λ_{ij}^{sd}: Amount of traffic flow (in Gbps) from source s to destination d that is routed on lightpath from node i to node j.
- $\alpha(s, d)$: The relation between the reserved bandwidth for protection on demand s–d and its working bandwidth $\alpha(s, d).\Lambda^{sd}$. It is an input when the strategy used is DPP.
- B_{ij}^{sd}: A binary variable to indicate whether a fraction of traffic from node s to node d is routed through a lightpath from node i to node j. B_{ij}^{sd} equals to 1 if $\lambda_{ij}^{sd} > 0$; equals to 0 if $\lambda_{ij}^{sd} = 0$.
- p_{ij}: An integer variable that quantifies the bandwidth (in terms of number of slots) of an elastic lightpath from node i to node j in the physical topology.
- P_{mn}^{ij}: Amount of bandwidth that a lightpath from node i to node j uses in a fiber link m-n (in terms of number of slots).
- A_{mn}^{ij}: A binary variable to indicate whether the lightpath from node i to node j passes through a link m-n. A_{mn}^{ij} equals to 1 if $P_{mn}^{ij} > 0$; equals to 0 if $P_{mn}^{ij} = 0$.
- Modulation format indicator e_{ij}^z: A binary variable that indicates if an elastic lightpath from node i to node j employs the modulation format z.

- S_{ij}: An integer variable that denotes the starting frequency for lightpath i-j.
- $W_{ij,kt}$: A binary variable that equals 1 if the starting frequency of lightpath i-j is smaller than the starting frequency of lightpath k-t. (i.e, $S_{ij} < S_{kt}$) and 0 otherwise.
- C: Maximum utilized spectrum slot index.

The objective function is to minimize the maximum utilized spectrum slot index, C.

D. Proposed MILP formulations

The formulations proposed in this paper are based on the new concepts described in Section II, whereas different strategies of lightpath protection are used and the optimum VTD and RSA solution under grooming and different modulation formats is found.

- *Objective Function:*

$$Minimize : C \tag{3}$$

- *Grooming and SLA Constraints:*

$$\sum_j \lambda_{ij}^{sd} - \sum_j \lambda_{ji}^{sd} = \begin{cases} ([1 + \alpha(s, d)]).\Lambda^{sd} & if \quad i = s \\ -([1 + \alpha(s, d)]).\Lambda^{sd} & if \quad i = d \\ 0 & if \quad i \neq d \end{cases} \tag{4}$$

$$\lambda_{ij}^{sd} \leq ([\alpha(s, d) + \beta(s, d)]).\Lambda^{sd} \quad \forall s, d, i, j \tag{5}$$

$$\alpha(s, d) \geq [1 - \beta(s, d) * g(s, d)] / [g(s, d) - 1] \quad \forall s, d \tag{6}$$

$$B_{ij}^{sd} \geq \lambda_{ij}^{sd}/\chi \quad \forall s, d, i, j \tag{7}$$

$$\sum_{ij} B_{ij}^{sd}.A_{mn}^{ij} \leq 1 \quad \forall s, d, m, n \tag{8}$$

$$\sum_{sd} \lambda_{ij}^{sd} = V_{ij} \quad \forall ij \tag{9}$$

$$V_{ij}/\chi \leq b_{ij} \quad \forall ij \tag{10}$$

$$\sum_j b_{ij} \leq \Delta_i \tag{11}$$

$$\sum_j b_{ji} \leq \Delta_i \tag{12}$$

- *Modulation Formats' Constraints:*

$$p_{ij} \geq \left(\frac{V_{ij}}{\Omega \cdot \eta_z} \right) - (1 - e_{ij}^z) \cdot \chi \quad for \ z = 1, 2, \ldots, M \tag{13}$$

$$p_{ij} \leq \left(\frac{V_{ij}}{\Omega \cdot \eta_z} \right) + 1 + (1 - e_{ij}^z).\chi \quad for \ z = 1, 2, \ldots, M \tag{14}$$

$$p_{ij} \leq \chi \cdot \sum_z e_{ij}^z \tag{15}$$

$$\sum_z e_{ij}^z \leq 1 \tag{16}$$

$$\sum_z e_{ij}^z \le \chi \cdot V_{ij} \qquad (17)$$

$$\sum_z e_{ij}^z \ge \frac{V_{ij}}{\chi} \qquad (18)$$

$$\sum_{mn} A_{mn}^{ij} \cdot d_{mn} \le \sum_z d_z \cdot e_{ij}^z \qquad (19)$$

- *Routing on Physical Topology Constraints:*

$$\sum_n P_{mn}^{ij} - \sum_n P_{nm}^{ij} = \begin{cases} p_{ij} & m = i \\ -p_{ij} & m = j \\ 0 & m \ne i, j \end{cases} \quad \forall ij, m \qquad (20)$$

$$\sum_{ij} \left(P_{mn}^{ij} + F.A_{mn}^{ij} \right) - F \le C \quad \forall mn \qquad (21)$$

$$A_{mn}^{ij} \ge \frac{P_{mn}^{ij}}{\chi} \quad \forall ij, mn \qquad (22)$$

$$A_{mn}^{ij} + A_{ml}^{ij} \le 1 \quad \forall i, j, m; \quad n \ne l \qquad (23)$$

-*Spectrum Continuity and Consecutive Sub-Carrier Constraints:*

$$p_{ij} + S_{ij} \le C \quad \forall ij \qquad (24)$$

$$S_{ij} \ge 0, p_{ij} \ge 0 \quad \forall ij \qquad (25)$$

$$W_{ij,kt} + W_{kt,ij} \le 1 \quad \forall ij, kt : ij \ne kt \qquad (26)$$

$$W_{ij,kt} + W_{kt,ij} \ge \left[\left(A_{mn}^{ij} + A_{mn}^{kt} \right) - 1 \right] \\ \forall ij, kt : ij \ne kt \qquad (27)$$

$$p_{ij} + S_{ij} + F \le S_{kt} + \chi * [1 - W_{ij,kt}] \\ \forall ij, kt : ij \ne kt \qquad (28)$$

$$p_{kt} + S_{kt} + F \le S_{ij} + \chi * [1 - W_{kt,ij}] \\ \forall ij, kt : ij \ne kt \qquad (29)$$

Equation (3) denotes the objective function, which aims to minimize the highest slot index used by any link in the network. Equation (4) is the conservation constraints of flows on the virtual topology (grooming layer). Equations (5) – (6) are the bandwidth partitioning and squeezing constraints. Equation (7) is used to indicate the virtual hops used by the source-destination node traffic. Equation (8) denotes that multiple lightpaths used to route the traffic from a source-destination node pair must use different physical links in order to enable the proposed traffic partitioning mechanism. Equation (9) denotes that low-speed traffic flows are groomed into bandwidth-variable lightpaths. Equation (10) indicates whether there exists an elastic lightpath between nodes i and j. Equations (11) and (12) limit the number of transceivers (virtual degree) on each node. Equations (13) and (14) provide the number of slots for the lightpath given an assigned modulation format. A lightpath is assigned to a modulation format that satisfies the constraints from (15) until (19), which implies that a single modulation format is employed to a lightpath, and its distance limitation must be satisfied. Equation (20) is the

flow conservation constraints of routing at the optical layer. Equation (21) denotes that the utilized bandwidth (including FGB) should not exceed the spectrum capacity of the fiber. Equation (22) is used to evaluate the FGB overhead and Equation (23) guarantees that the traffic of a lightpath can not be partitioned in the physical topology.

The constraint (24) along with the objective function (3) and constraint (20) is used to minimize the maximum number of spectrum slots used among all demands. Equation (25) is trivial, since a connection starting frequency and its number of slots must both be positives. Equations (26) and (27) guarantee that, if lightpaths $i-j$ and $k-t$ share any fiber in the network, ex. m-n, then $A_{mn}^{ij} = A_{mn}^{kt} = 1$ and, therefore, $W_{ij,kt} + W_{kt,ij} = 1$, which states that either $W_{ij,kt}$ or $W_{kt,ij}$ is equal to one and the other is equal to zero. Therefore, together with equations (28) and (29), one can guarantee that their spectrums do not overlap. On the other hand, if lightpaths $i-j$ and $k-t$ do not share any fiber in the network, then A_{mn}^{ij} and A_{mn}^{kt} can never be both equal to one, which, by (26) and (27), allows that $W_{ij,kt}$ and $W_{kt,ij}$ can be both equal to zero. Therefore, there is no spectrum overlapping constraint for these lightpaths, as there should be.

Unfortunately, the constraint (8) is non-linear, but it is formed by the multiplication of two binary variables. Therefore, variable w can be defined as $B_{ij}^{sd}.A_{mn}^{ij} = Z_{sdmn}^{ij}$, (28) may be replaced by equations (30) to (34) that follow, and thus convert the MILP formulation into a linear problem.

$$\sum_{ij} Z_{sdmn}^{ij} \le 1 \qquad \forall s, d, m, n \qquad (30)$$

$$Z_{sdmn}^{ij} \le B_{ij}^{sd} \qquad \forall s, d, m, n, i \qquad (31)$$

$$Z_{sdmn}^{ij} \le A_{mn}^{ij} \qquad \forall s, d, m, n, i, j \qquad (32)$$

$$Z_{sdmn}^{ij} \ge 0 \qquad \forall s, d, m, n, i, j \qquad (33)$$

$$Z_{sdmn}^{ij} \ge B_{ij}^{sd} + A_{mn}^{ij} - 1 \qquad \forall s, d, m, n, i, j \qquad (34)$$

V. HEURISTIC ALGORITHM

Due to the complexity of the problem for large networks, the complete strategy presented in the proposed MILP formulation may be very time consuming. For instance, running the MILP formulation for the network shown in Fig. 1 (6-node topology) in an Intel i3 2.27 GHz 2 GB machine took about 2 h for all values tested for $\beta(s,d)$, except when $\beta(s,d) = 1$ (no-protection) when it took about 40 min. If two additional nodes are added, one connected to nodes 1 and the other to node 6, the required simulation time increases considerably, which emphasizes that a heuristic model is necessary for moderate or large networks.

The complexity of the complete MILP formulation may be reduced by decomposing the problem into two sub-problems (phases): virtual topology design of the lightpaths and then the protection strategy. In the first phase the lightpaths of the virtual topology (b_{ij}'s) are defined by virtual topology design with some constraints imposed by the virtual topology equations studied in this paper (for example, virtual degree set to a specific number). The virtual topology can also be

created from a heuristic algorithm [20]. Therefore, some b_{ij}'s will be set to 1 dealing with the virtual degree limit and they are going to be parameter for the MILP formulation.

Subsequently, in the second phase, a portion of the complete MILP formulation is used to find the protection strategy of lightpaths, as well as their routes, set of contiguous and continuous slots in the physical topology and modulation format. Since the b_{ij}'s are pre-determined in phase 1, a solution as good as the one provided by the fully integrated problem, where b_{ij}'s is variable, may not be found. However, the processing time is substantially reduced and a good solution may still be acquired. The problem is still complex but the modulation format constraints tend to influence the choice of the shortest routes. This happens as the modulation formats with high spectral efficiency (they get less slots) have shorter reaches than others, so they becoming predominant due to the objective function. A good solution depends on the use of efficient strategies. The strategy used in this paper has been named as *two-step heuristic* or, for simplicity, we refer to it as HEUR.

VI. DISCUSSION AND RESULTS

A. Small Networks

For evaluating the effectiveness of the proposed optimization, two different 6-node network topologies were analyzed (Fig.1, topology 1 and Fig. 3, topology 2). IBM ILOG CPLEX v.11.0 [21] was used on an Intel i7 3.6 GHz 32GB machine to solve the formulation. An upper limit of 2h was specified as the maximum allowed computation time for solving the MILP formulation in small networks. The simulations using the complete formulation needed less time (around 40 minutes) than the upper limit assigned for solving the problem in small networks.

1) Performance Gain Due To Traffic Partitioning and Squeezing: The performance of DPP, PDPP and PDPP+S were first compared with the complete MILP formulation for the topology 2, Fig.3. We chose this network because it has more physical links and therefore more possibilities of traffic partitioning. This first simulation is important because it shows the advantages of the PDPP strategy over DPP, as discussed in Fig.2. For the simulations it was assumed a pair of unidirectional fiber on each link. The traffic demand is uniform for each source-destination pair and it was assumed in the range from $20\,\mathrm{Gbps}$ to $100\,\mathrm{Gbps}$.

The slot width, Ω, is set as 12.5 GHz and the filter guard band between wavebands is set to one slot. We assumed 3 available modulation formats ($M = 3$). The spectral efficiency of each modulation format, η_z, is set as $\eta_1 = 2$, $\eta_2 = 4$ and $\eta_3 = 8$ bit/s/Hz. The maximum reach of a lightpath under each modulation format z is $d_1 = 4$ hops, $d_2 = 2$ hops and $d_3 = 1$ hop.

Firstly, the given squeezed bandwidth ratio for each demand, $\beta(s,d)$, was randomly obtained to generate four different SLA scenarios, as shown in Table I. They are called MTS (Mixed Traffic Squeezing) and are used for the PDPP+S strategy. $\alpha(s,d)$ variable is calculated according to Equation 2 given the squeezing bandwidth ratio ($\beta(s,d)$) provided for each source-destination pair $s - d$.

The PDPP strategy (100% protection with $\alpha(s,d)$ variable and no-squeezing for the pair s-d given by $\beta(s,d) = 0$) and DPP protection (100% protection with $\alpha(s,d) = 1$ and no-squeezing for the pair s-d given by $\beta(s,d) = 0$) are compared. The comparison of the maximum utilized spectrum slot number of the different analyzed approaches is shown in Fig.4. The demonstrates that PDPP+S, which provides different SLAs for each node pair $s - d$, can save a large amount of the total spectrum when compared to PDPP and DPP. This occurs because DPP and PDPP do not benefit from traffic squeezing, as occurs with PDPP+S. The PDPP strategy also saves resources when compared to DPP strategy under high traffic condition, generating almost 20% of reduction under $100\,\mathrm{Gbps}$ of traffic for any source-destination pair.

TABLE I: Squeezed bandwidth ratio, $\beta(s,d)$, for each pair $s - d$ of 6-node network (4 cases)

CASE I (MTS 1)						CASE II (MTS 2)					
-	0.5	0.9	0.1	0.2	0.3	-	0.2	0.8	0.4	0.9	0.6
0.5	-	0.9	0.3	0.8	0.2	0.7	-	0.9	0.1	0.9	0.4
0.2	0.5	-	0.1	0.9	0.2	0.2	0.4	-	0.2	0.9	0.5
0.1	0.5	0.2	-	0.3	0.3	0.6	0.1	0.6	-	0.3	0.8
0.5	0.6	0	0.4	-	0.3	0.9	0.5	0.6	0.4	-	0.5
0.6	0.2	0.1	0.7	0.8	-	0.6	0.9	0.2	0.9	0.6	-

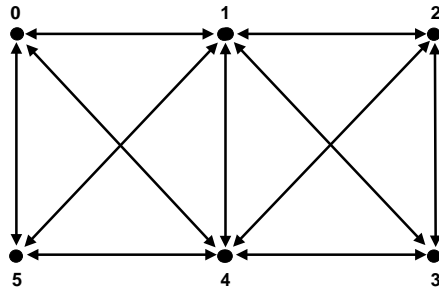| CASE III (MTS 3)						CASE IV (MTS 4)					
-	0.2	0.1	0.5	0.6	0.6	-	0.9	0.9	0.8	0.5	0.1
0.9	-	0.9	0.7	0.6	0.1	0.5	-	0.9	0.4	0.2	0.5
0.3	0.2	-	0.3	0.2	0.3	0.3	0.1	-	0.4	0.8	0.1
0.5	0.2	0.8	-	0.5	0.1	0.1	0.7	0.7	-	0.7	0.8
0.6	0.9	0.7	0.4	-	0.7	0.3	0.5	0.7	0.2	-	0.1
0.3	0	0.6	0.9	0.6	-	0.9	0.3	0.5	0.5	0.4	-

Fig. 3: Topology 2. Illustrative example of six-node network physical topology

2) Performance Gain Due To Modulation Format Assignment: For a more comprehensive comparison, it is interesting to investigate the performance of the network when the three assumed modulation formats (z_1, z_2 and z_3) compared to when only a single modulation format (z_1) is used. This is shown in Fig. 5 using matrix MTS 1 as traffic-squeezing values. The reason to choose z_1 as the only available modulation format is because it can be used to establish all connections in the network, since 2 hops is enough to connect any pair of nodes in the topology 2 under study. The format z_2 also could be used. The advantage of using multiple modulation formats is evident, since all requests could be established with a considerably lower number of maximum number of slots in the network when compared to using a single modulation format.
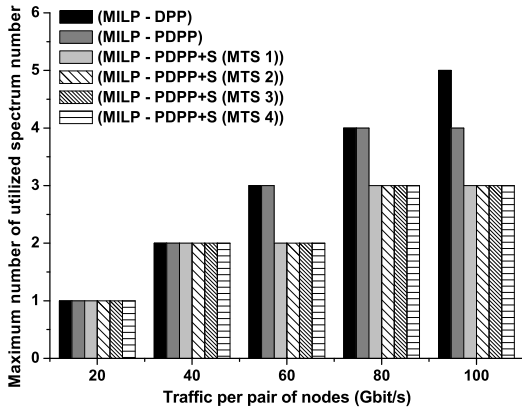
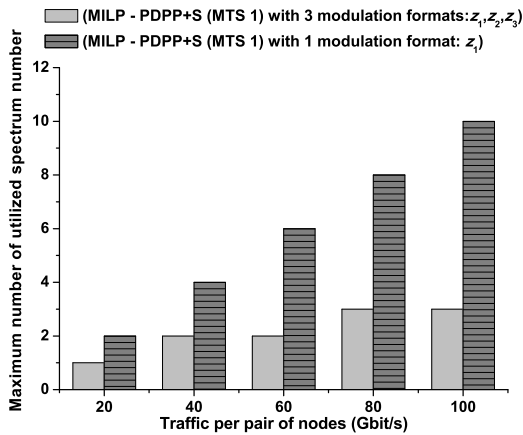Fig. 4: Maximum number of utilized spectrum number for the 6-node network

Fig. 6: Expansion traffic factor $\alpha(s, d)$ for each node pair $s - d$ of traffic demand

Fig. 5: Maximum number of utilized spectrum index for the 6-node network, topology 2

3) The Extra-Traffic Saving: Fig. 6 analyzes the extra bandwidth, $\alpha(s, d)$, required for protecting the traffic of each node pair considering DPP, PDPP and PDPP+S with the squeezing values described in Case I of Table I when $100\,\mathrm{Gbps}$ of traffic is assumed for each source-destination pair over the topology 2. As observed, traditional DPP has the worst result. This occurs because, under this kind of protection, each demand needs to be assigned with the double of the required capacity. On the other hand, PDPP is able to reduce the amount of traffic required by traditional DPP for the majority of source-destination node pairs, and PDPP+S outperforms PDPP in practically every node pairs with a large difference. Such techniques provide a more efficient use of resources, since extra bandwidth resource for protection can be reduced with traffic partitioning, as $\alpha(s, d) < 1$, and even more reduced when bandwidth squeezing is allowed.

4) The Heuristic Performance: The heuristic has also been investigated on the topology 1 and compared to performance when using the exact MILP formulation.

We have assumed again a traffic demand for each source-destination node pair from $20\,\mathrm{Gbit/s}$ to $100\,\mathrm{Gbit/s}$ in steps of $20\,\mathrm{Gbit/s}$. For the separated VTD, we have assumed a pre-determined random virtual topology with average virtual
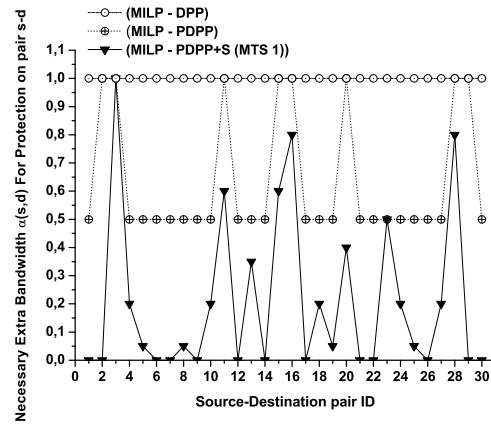
degree set to three for all transceivers.

Table II shows the maximum number of required slots (C) in the network when the values of traffic squeezing is the same per node and assumes the values $\beta(s, d) = 0, 0.2, \ldots, 0.8$. A number of SLA levels have been assumed as shown in Table II for $\beta(s, d)$. It can be observed that the *two-step heuristic* (HEUR) produces results similar to those from the optimal MILP solution for all cases. Therefore, that suggests the efficiency of the proposed heuristic strategy. Table II also depicts the number of times a modulation formats is used for the transmission of the traffic requests along the different lightpaths. Modulatoin format z_1, corresponding to a maximum lightpath length of four hops, is rarely used, since more spectrally efficient modulation formats are feasible and preferred.

B. Larger Network (NSFNET)

Due to the complexity of solving the MILP formulation for large networks, analysis were performed with the algorithm-based heuristic for a moderately large network (NSFNET, [22]) with 14 nodes and 21 bidirectional links. For comparison purposes, DPP, PDPP and PDDP+S with the MTS squeezing matrix described in Table III were used.

It may be impractical to use the proposed heuristic in larger networks (around 20 or more nodes). The simulation for the NSFNET was limited to 24 h either with DPP, PDPP of PDPP+S schemes. Although the VTD burden has been alleviated with the lightpaths given as input to the MILP, the RSA problem, with protection and modulation constraints, is still hard to be solved. However, since the numbers of nodes and links in backbone optical networks are usually not very large and a more powerfull computer may be used, it is expected that the proposed heuristic may be applied in to a number of practical SLICE networks, and consequently be a benchmark for other heuristics.

The slot width, Ω, is set as 12.5 GHz and the filter guard band between wavebands of one slot is 12.5 GHz. We assumed 3 available modulation formats ($M = 3$). The data rate-to-bandwidth ratio, η_z, is set as $\eta_1 = 2$, $\eta_2 = 4$ and $\eta_3 = 8$ bit/s/Hz for each z modulation format. Now the maximum reach

TABLE II: Maximum Number of Slots and Number of kinds Modulation Format fot the Connections

	$\beta(s,d)$	0	0.2	0.4	0.6	0.8	Random (MTS 1)
20 Gbit/s	MILP $C :=$	2	2	2	1	1	2
	z_1	0	0	0	0	0	0
	z_2	0	0	3	3	3	0
	z_3	16	16	13	13	12	16
	HEUR $C :=$	2	2	2	1	1	2
	z_1	0	0	0	0	1	0
	z_2	0	0	3	3	3	0
	z_3	16	16	13	13	12	16
40 Gbit/s	MILP $C :=$	4	3	3	2	2	3
	z_1	0	0	0	0	0	0
	z_2	0	0	0	0	0	0
	z_3	16	16	16	16	16	16
	HEUR $C :=$	4	3	3	2	2	3
	z_1	0	0	0	0	1	0
	z_2	0	0	0	1	2	1
	z_3	16	16	16	15	13	15
60 Gbit/s	MILP $C :=$	6	5	4	3	3	4
	z_1	0	0	0	0	0	0
	z_2	0	0	0	0	0	16
	z_3	16	16	16	16	16	16
	HEUR $C :=$	6	5	4	3	3	4
	z_1	0	0	0	0	1	0
	z_2	0	0	0	1	3	0
	z_3	16	16	16	15	12	16
80 Gbit/s	MILP $C :=$	8	6	5	4	4	5
	z_1	0	0	0	0	0	0
	z_2	0	0	0	0	0	0
	z_3	16	16	16	16	16	16
	HEUR $C :=$	8	6	5	4	4	5
	z_1	0	0	0	0	2	0
	z_2	0	0	0	0	1	1
	z_3	16	16	16	16	13	15
100 Gbit/s	MILP $C :=$	9	8	6	5	5	6
	z_1	0	0	0	0	0	0
	z_2	0	0	0	0	0	0
	z_3	16	16	16	16	16	16
	HEUR $C :=$	9	8	6	5	5	6
	z_1	0	0	0	0	1	0
	z_2	0	0	0	0	3	1
	z_3	16	16	16	16	12	15

TABLE III: MTS Matrix. Squeezed bandwidth ratio, $\beta(s,d)$, for the traffic of each node pair $s-d$ of the NSFNet network

-	0.3	0.8	0.9	0.6	0.9	0.5	0.5	0.0	0.7	0.8	0.9	0.9	0.4
0.2	-	0.2	0.1	0.9	0.8	0.1	0.6	0.6	0.2	0.4	0.4	0.8	0.9
0.6	0.5	-	0.5	0.2	0.2	0.1	0.1	0.3	0.8	0.8	0.9	0.6	0.9
0.1	0.6	0.1	-	0.7	0.9	0.5	0.1	0.3	0.0	0.8	0.9	0.3	0.0
0.2	0.6	0.4	0.5	-	0.9	0.9	0.3	0.0	0.3	0.0	0.4	0.2	0.1
0.4	0.2	0.7	0.6	0.3	-	0.6	0.5	0.1	0.6	0.7	0.8	0.5	0.2
0.6	0.6	0.6	0.7	0.7	0.5	-	0.5	0.8	0.9	0.2	0.6	0.6	0.4
0.3	0.2	0.6	0.8	0.8	0.7	0.3	-	0.7	0.5	0.9	0.8	0.7	0.4
0.6	0.1	0.4	0.3	0.4	0.4	0.5	0.4	-	0.4	0.7	0.5	0.7	0.8
0.5	0.5	0.8	0.2	0.3	1.0	0.2	0.1	0.1	-	0.0	0.6	0.3	0.1
0.2	0.5	0.3	0.3	0.2	0.4	0.3	0.5	0.2	0.0	-	0.1	0.7	0.5
0.8	0.4	0.5	0.7	0.5	0.9	0.1	0.5	0.9	0.2	0.1	-	0.4	0.5
0.3	0.7	0.2	0.5	0.8	0.2	0.9	0.5	0.8	0.5	0.7	0.4	-	0.3
0.2	0.2	0.9	0.0	0.3	0.8	0.9	0.1	0.3	0.9	0.7	0.3	0.2	-
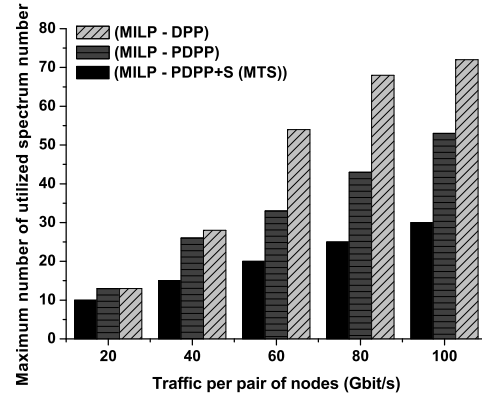
Fig. 8: Maximum number of utilized spectrum number for the NSFNET network
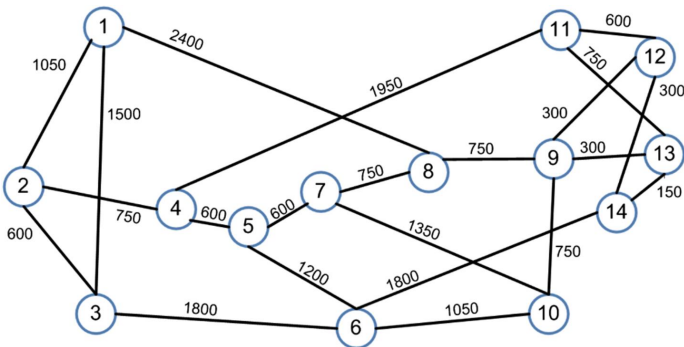

Fig. 7: NSFNET Network, distance in km

of a lightpath under each modulation format z is $d_1 = 4000$ km, $d_2 = 2000\ km$ and $d_3 = 1000\ km$. The squeezing matrix is given by TableIII and we have assumed a pre-determined virtual topology with average virtual degree setup to three ($\Delta_i = 3, \forall i \in N$).

Fig.8 shows the maximum number of slots used, C, in any fiber in the network for the *two-step heuristic* when DPP, PDPP and PDPP+S schemes are employed. When comparing

the three protection schemes, it can be seen that, for all values of traffic per node, PDPP used less slots than DPP. For instance, from 60 Gbit/s until 100 Gbit/s a reduction of about 25% was achieved with the use of PDPP instead of DPP. In addition, the PDPP+S strategy gets good results with the given MTS matrix compared to PDPP with 100% protection ($\beta(s,d) = 0$), even for low values of traffic. On the other hand, for low traffic, DPP and PDPP provide equal or similar performance. The results clearly demonstrate that applying traffic partitioning with squeezing is much more advantageous for resource savings compared to the other techniques, as also observed for the small networks previously analyzed.

VII. CONCLUSIONS

In this paper, a novel and unified MILP formulation for DPP and two new protection schemes in EON networks with multiple SLAs, grooming and RSA was proposed. The proposed formulation enables different survivability levels for the network traffic demands subject to committed service profiles, including bandwidth squeezing for each source-destination pair, which can increase the number of surviving paths in the network at the price of reducing the traffic bandwidth under a link failure. Using extensive simulation experiments, it has been demonstrated the effectiveness of the complete MILP formulation. The performance obtained in terms of objective value and protection

is very good. A heuristic for large networks was proposed. It has been noticed that RSA, by using an MILP formulation with protection and modulation format constraints in step 2, still takes a long time to find its objective function. This processing time burden can be alleviated with new heuristics that are under study.

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REFERENCES

[1] M. Jinno, H. Takara, B. Kozicki, Y. Tsukishima, Y. Sone, and S. Matsuoka, "Spectrum-efficient and scalable elastic optical path network: architecture, benefits, and enabling technologies," IEEE Communications Magazine, vol. 47, no. 11, pp. 66–73, November 2009.

[2] B. Kozicki, H. Takara, T. Yoshimatsu, K. Yonenaga, and M. Jinno, "Filtering characteristics of highly-spectrum efficient spectrum-sliced elastic optical path (slice) network," in Conference on Optical Fiber Communication, March 2009.

[3] R. Goscien, K. Walkowiak, M. Klinkowski, and J. Rak, "Protection in elastic optical networks," IEEE Network, vol. 29, no. 6, pp. 88–96, Nov 2015.

[4] C. Ma, J. Zhang, Y. Zhao, M. F. Habib, S. S. Savas, and B. Mukherjee, "Traveling repairman problem for optical network recovery to restore virtual networks after a disaster [invited]," IEEE/OSA Journal of Optical Communications and Networking, vol. 7, no. 11, pp. B81–B92, November 2015.

[5] D. S. Yadav, S. Babu, and B. Manoj, "Quasi path restoration: A post-failure recovery scheme over pre-allocated backup resource for elastic optical networks," Optical Fiber Technology, vol. 41, pp. 139 – 154, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1068520017302158

[6] Y. Sone, A. Watanabe, W. Imajuku, Y. Tsukishima, B. Kozicki, H. Takara, and M. Jinno, "Bandwidth squeezed restoration in spectrum-sliced elastic optical path networks (slice)," IEEE/OSA Journal of Optical Communications and Networking, vol. 3, no. 3, pp. 223–233, Nov 2011.

[7] G. Shen, H. Guo, and S. K. Bose, "Survivable elastic optical networks: Survey and perspective (invited)," Photonic Netw. Commun., vol. 31, no. 1, pp. 71–87, Feb. 2016. [Online]. Available: http://dx.doi.org/10.1007/s11107-015-0532-0

[8] K. D. R. Assis, R. C. Almeida, and H. Waldman, "Milp formulation for squeezed protection in spectrum-sliced elastic optical path networks," in IEEE International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS), March 2012, pp. 1–7.

[9] L. Velasco, M. Klinkowski, M. Ruiz, and J. Comellas, "Modeling the routing and spectrum allocation problem for flexgrid optical networks," Photonic Network Communications, vol. 24, no. 3, pp. 177–186, April 2012.

[10] H. Wang and G. N. Rouskas, "Hierarchical traffic grooming: A tutorial," Computer Networks, vol. 69, pp. 147 – 156, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1389128614001820

[11] H. Wang and G. Rouskas, "Traffic grooming in optical networks: Decomposition and partial linear programming (lp) relaxation," IEEE/OSA Journal of Optical Communications and Networking, vol. 5, pp. 825–535, 2013.

[12] Z. Ye, A. N. Patel, P. N. Ji, and C. Qiao, "Survivable virtual infrastructure mapping with dedicated protection in transport software-defined networks," IEEE/OSA Journal of Optical Communications and Networking, vol. 7, no. 2, pp. A183–A189, February 2015.

[13] A. Pages, S. J.Perello, J. A.Garcia-Espin, J. F. Riera, and S. Figuerola, "Optimal allocation of virtual optical networks for the future internet," in IEEE International Conference on Optical Network Design and Modeling (ONDM), April 2012, pp. 1–6.

[14] K. D. R. Assis, S. Peng, R. C. Almeida, H. Waldman, A. Hammad, A. F. Santos, and D. Simeonidou, "Network virtualization over elastic optical networks with different protection schemes," IEEE/OSA Journal of Optical Communications and Networking, vol. 8, no. 4, pp. 272–281, April 2016.

[15] F. Paolucci, A. Castro, F. Cugini, L. Velasco, and P. Castoldi, "Multipath restoration and bitrate squeezing in sdn-based elastic optical networks," Photonic network communications, vol. 28, no. 1, pp. 45–57, August 2014.

[16] R. Alvizu, G. Maier, M. Tornatore, and M. PiÃro, "Differential delay constrained multipath routing for sdn and optical networks," Electronic Notes in Discrete Mathematics, vol. 52, pp. 277 – 284, 2016, iNOC 2015 â" 7th International Network Optimization Conference. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1571065316300427

[17] K. Yedugundla, S. Ferlin, T. Dreibholz, Özgü Alay, N. Kuhn, P. Hurtig, and A. Brunstrom, "Is multi-path transport suitable for latency sensitive traffic?" Computer Networks, vol. 105, pp. 1 – 21, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1389128616301396

[18] K. Christodoulopoulos, I. Tomkos, and E. A. Varvarigos, "Routing and spectrum allocation in ofdm-based optical networks with elastic bandwidth allocation," in IEEE Global Telecommunications Conference (GLOBECOM), December 2010, pp. 1–6.

[19] Y. Wang, X. Cao, and Y. Pan, "A study of the routing and spectrum allocation in spectrum-sliced elastic optical path networks," in IEEE International Conference on Computer Communications (INFOCOM), April 2011, pp. 1503–1511.

[20] R. Ramaswami and K. N. Sivarajan, "Design of logical topologies for wavelength-routed optical networks," IEEE J.Sel. A. Commun., vol. 14, no. 5, pp. 840–851, Jun. 1996. [Online]. Available: http://dx.doi.org/10.1109/49.510907

[21] ILOG, Inc, "ILOG CPLEX: High-performance software for mathematical programming and optimization," 2018, see https://www.ibm.com/analytics/cplex-optimizer.

[22] P. S. Khodashenas, J. Comellas, S. Spadaro, J. Perello, and G. Junyent, "Using spectrum fragmentation to better allocate time-varying connections in elastic optical networks," IEEE/OSA Journal of Optical Communications and Networking, vol. 6, no. 5, pp. 433–440, May 2014.