Philosophia Mathematica (III) **00** (2010), 1–8.

Critical Studies/Book Reviews

C.S. JENKINS. ***Grounding Concepts: An Empirical Basis for Arithmetical Knowledge***. Oxford: Oxford University Press, 2008. ISBN 978-0-19-923157-7. Pp. xiv + 290.

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This book is written so as to be 'accessible to philosophers without a mathematical background'. The reviewer can assure the reader that this aim is achieved, even if only by focusing throughout on just one example of an arithmetical truth, namely '$7 + 5 = 12$'. This example's familiarity will be reassuring; but its loneliness in this regard will not. Quantified propositions — even propositions of Goldbach type — are below the author's radar.

The author offers 'a new kind of arithmetical epistemology', one which 'respects certain important intuitions' (p. 1)[1]: apriorism, realism, and empiricism. The book contains some clarification of these 'isms', and some thoughtful critiques of major positions regarding them, as espoused by such representative figures as Boghossian, Bealer, Peacocke, Field, Bostock, Maddy, Locke, Kant, C.I. Lewis, Ayer, Quine, Fodor, and McDowell. The philosophical reader will find some interest and value in these wider-ranging discussions. Our concern in this review, however, is to examine closely the original positive proposal on offer.

Arithmetical truths, the author maintains, are *conceptual* truths. Knowing truths like $7 + 5 = 12$ involves no 'epistemic reliance on any empirical evidence'; but that, she says, is not to claim 'epistemic independence of the senses altogether'. She wants to show that

> experience grounds our concepts . . . and then mere conceptual examination enables us to learn arithmetical truths (p. 4).

Concepts that are 'appropriately sensitive' to 'the nature of [an independent] reality' she calls *grounded*. Because of the role of grounded concepts, 'arithmetical truths explain our arithmetical beliefs in the right sort of way for those beliefs to count as knowledge' (p. 9).

In the context of her concentration on the special nature of arithmetical knowledge, the author offers (in Chapter 3) what could strike some bystanders as an unnecessarily over-ambitious account of knowledge *tout court*. Knowledge, for the author, is

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[1] Page numbers in parentheses refer to the book under review.

> true belief which can be well explained, to someone not acquainted with the details of the subject's situation (an 'outsider'), just by citing the proposition believed. (p. 74)

Clearly this proposal places an enormous burden on the notion of explanation. Space does not permit the reviewer to develop in detail any of the obvious counterexamples (both empirical and mathematical) to this account of knowledge. Those conversant with post-Gettier epistemology will readily think up such counterexamples for themselves.

The heart of the work is Chapter 4, 'A theory of arithmetical knowledge'. The author believes (p. 109) that 'arithmetical propositions can sometimes be good explanations of why we have the arithmetical beliefs we do'. But surely this is not good enough. She ought to be asserting that *every one* of our arithmetical beliefs can be explained by citing the proposition believed, if her general account of knowledge is to be faithfully applied. (Indeed, as we shall see below, she does eventually make the more general assertion; but subsequently retracts it.)

Setting aside this problem of scope, let us examine the author's account of how it is that an arithmetical fact p can supposedly explain a subject's belief that p.

> I shall hypothesize that this kind of explanatory link holds in virtue of three sub-links, one between the arithmetical facts and our sensory input, one between our sensory input and our arithmetical concepts, and one between our arithmetical concepts and our arithmetical beliefs. (p. 116)

The links are never explained.

> When we attempt to conceive of something's being the case, what we are doing is investigating how our conceptual representations fit together. ... When we attempt to conceive of $7 + 5$ not being equal to 12, we investigate whether our concepts of 7, 5 and 12 are concepts as of things that stand in the relation denoted by our concept $\cdots + \cdots = \ldots$ [.] We find that they are, and hence report that we cannot conceive of $7 + 5$ not being equal to 12 (or, sometimes, just that $7 + 5 = 12$, and/or that necessarily $7 + 5 = 12$). (p. 123)

The method, then, appears to be to try to conceive of not-p's being the case; finding (upon investigation or examination) one cannot conceive this; and thereupon concluding that p. Once again, the author's restrictive focus on simple computational statements, all of which are decidable, results in her not being aware of an obvious counterexample to her suggested epistemic path to a grasp of necessary arithmetical truth. Can one conceive the negation of the consistency statement for Peano arithmetic? Yes, one can, and one can do so consistently with Peano arithmetic, provided only that Peano arithmetic is itself consistent. This contradicts her view; for her method as stated above does not give the right result (namely, that the consistency statement for Peano arithmetic is true).

As the title of the book suggests, the notion of grounding is central to the author's account. It is introduced via a sequence of definitions, the most important ones of which we shall now quote. It will be shown that her exercise in framing definitions is to no productive effect.

> (6) I shall say that a concept *refers* iff it is a representation of some real feature of the world. ... a property may be a real aspect of the world even when nothing has that property, and even when *necessarily* nothing has it. (p. 126)

From this it presumably follows that the concept 'unicorn' refers, in the sense of 'refers' here defined, as does the concept 'non-self-identical'.

> (9) *Fitting* concepts are concepts which either refer themselves or else are correct compounds of referring concepts ... (p. 127)

Next:

> (10) ... Given some purported a priori knowable proposition p, we can say that a concept C is *relevantly accurate* (or, sometimes, just *accurate*) iff C is fitting and neither C nor any concept from which C is composed *misrepresents its referent* in any respect relevant to our purported *a priori* way of knowing that p. (Note that which concepts count as 'relevantly accurate' depends on which item(s) of a priori knowledge we are interested in.) (pp. 127–128; third emphasis added)

The author admits that she is 'not entirely clear on what it would be for a concept to misrepresent its referent'. So she allows that for

> those who do not think that a concept can misrepresent its referent, all fitting concepts will automatically count as [relevantly] accurate ... and all ... inaccuracy will amount to unfittingness. That is fine by me. (p. 128)

Let us take the author's concession seriously, then, and set aside the specter of conceptual misrepresentation, in this more limited context of a discussion of arithmetical concepts.

The question now arises: what are we to make of the mention of the purported a priori knowable proposition p in definition (10)? The second conjunct on the right-hand side becomes trivially true upon assuming that no concepts misrepresent their referents. So that conjunct contributes nothing to the definition of relevantly accurate concepts. Hence it can be suppressed. Definition (10) then becomes

> (10*) . . . Given some purported *a priori* knowable proposition p, we can say that a concept C is *relevantly accurate* (or, sometimes, just *accurate*) iff C is fitting.

One is then left wondering what connection is being assumed between p and C, and why the clause 'Given . . . p' remains in place at all. The author's parenthetical note that

> which concepts count as 'relevantly accurate' depends on which item(s) of a priori knowledge we are interested in

also becomes otiose, if not nonsensical, in this new context where the possibility of conceptual misrepresentation is ruled out. So, (10*) boils down to

> (10**) A concept C is *relevantly accurate* (or, sometimes, just *accurate*) iff C is fitting.

The author's next definition is as follows.

> (11) A concept is *grounded* just in case it is relevantly accurate and there is nothing lucky or accidental about its being so. (p. 128)

By this stage one is also wondering what the adverb 'relevantly' could be contributing, since there does not seem to be any *a priori* knowable, parametric, proposition p left in the context (*vide* (10**)), with respect to which the concept in question could be at all relevant.

From (10**) it follows by (11), (9), and (6) that a concept is grounded just in case it represents some real feature of the world or is a correct compound of concepts that represent some real feature of the world and there is nothing lucky or accidental about its doing so. This does not strike one as saying much. The author does not oblige the reader with any paradigm examples, or foils, of the notions being deployed (fittingness, grounding, *etc*.). Indeed, on p. 180 one learns that 'examples of ungrounded concepts are hard to find'. This is perhaps not surprising, on reflection, since the definition of groundedness, as unwound above, seems to impose hardly any significant constraints on concepts in general.

Concept grounding is important, the author tells us, because

> examining *grounded* concepts can help us acquire knowledge of [the world]. (p. 131)

This promissory note is subsequently expanded:

> If our concepts are sensitive to sensory input and hence the world, in such a way that they (or their ultimate constituents) are likely to represent real aspects of the world *in an accurate way*, then an examination of those concepts is no longer just an examination of ourselves, but becomes an examination of a reliable on-board conceptual map of the world. And if this is the case, we have reason to think that propositions believed solely on the basis of such an examination can supply us with knowledge of the independent world. ... concepts can give us access to some truths about the world, ... most importantly for our current purposes, *the truths of arithmetic*. (pp. 134–135; emphases added)

One hastens to ask: *Which* truths of arithmetic? *All* of them? Or just the axioms of Peano? Or just the quantifier-free truths? Or just the true sentences with no unbounded quantifiers? By this stage (the midpoint of the work) the reader is owed at least one detailed account of how a conceptual 'examination' can be carried out, so as to arrive at knowledge of an exemplary arithmetical truth. The reader is owed an account of exactly which concepts, for the purpose of such an exercise, are taken to be grounded; and how they might deliver up arithmetical knowledge upon the right kind of examination. The reader wishes to know whether quantifier-free computational statements (the only sorts of examples considered in this book) are the only arithmetical truths that will succumb to such an 'aprioristic yet empiricist' treatment; or whether the author has the wherewithal to deal also with the Gödel phenomena, once quantification over all the natural numbers is admitted.

The reader, however, is left in the dark concerning such questions. The second half of the book proceeds without any concern for what by this stage will be the most importantly unanswered questions in the reader's mind. From this point on, also, there are some textural tones that tend to belie the author's confident architectonic impulses. With notable frequency one encounters such hedges as 'I am not entirely clear on what it would be for ... ' (p. 128), 'If I were forced to guess, I would say it was likely that ... ' (p. 157), 'I have not, of course, said enough here to count as having given a serious argument ... ' (p. 177).

The reader looking for answers to these questions will be disappointed on reaching the author's eventual admission that

> I have not said anything substantial about the way in which examining the concepts leads to possession of the belief. I have not said how we go about conducting an 'examination' of our concepts, nor how and why the examination process leads us to adopt certain beliefs and not others. I have not said how it is that we can tell by examining our concepts of *7*, +, *5*, =, and *12* that they stand in a certain relation, or

> [how] our noticing this leads to our believing that $7 + 5 = 12$. ... I am not offering a worked-out account of this process. (pp. 246–247)

She explains her failure, or principled reluctance, to pursue such matters as follows:

> Attempting to describe the processing stages of concept examination in detail would be akin to attempting to describe the processing stages involved in visual perception: not immediately relevant to the epistemological issues we're focussing on, and not the sort of thing that should be done *a priori* by a philosopher without input from empirical psychology. (p. 248)

Mathematicians arrive at their known truths of mathematics by *a priori* means, namely deductive proofs based on intuitively evident axioms. Nevertheless, it would appear that the author is telling us that the real reason why mathematicians can be regarded as knowing these truths is that a certain kind of empirical investigation could be undertaken, an investigation that must involve the participation of empirical psychologists.

It would be better if this claim were confined only to what the mathematician calls 'first principles', *i.e.*, axioms; but nowhere does the author limit her claim about provenance to the axioms. Indeed, in the usual Peano axiomatization, '$7 + 5 = 12$' is not an axiom, but a theorem. So, if the author is correct in her view, the average mathematician will be left staring at any convincingly rigorous proof of a theorem p, wondering how on earth he could be so wrong (concerning his grounds for knowing p). For the author is telling him that it is not his understanding of the proof that secures him his knowledge that p, but rather some sort of examination of his concepts, concepts which are somehow grounded in his sensory input. Yet for all that, the author concedes

> I have not attempted to give a full account of grounding for concepts, or even for arithmetical concepts. I have not even given a full account of what a concept is, or what arithmetical concepts are like. And I have not given anything like a full account of conceptual examination. ... I have not asked what sorts of pre-conceptual sensory input could ultimately ground our arithmetical concepts, and I have not attempted to give any details of the relationship between this input and our arithmetical concepts in virtue of which the latter count as grounded in the former. (p. 260)

Perhaps most disappointing is the author's failure to appreciate the scope and profundity of the extant tradition of both logicism and neo-logicism. She pays scant regard to the intuitions behind logicism. Nowhere does she anticipate or address an imagined anti-empiricist opponent (in her sense of ‘empiricist’) who appeals to the logical possibility of a disembodied Cartesian soul in a universe containing no material objects. It is easy to provide a convincing thought-experimental account of how such a soul could attain to at least the axioms of Peano arithmetic (indeed: even at second order). The strategy of course is Fregean in spirit (if not in the details of its logical execution), and it is *a priori* in the strict traditional sense. Moreover it is independent, not only of empirical evidence, but also of any concepts that might be empirical in their acquisition or grounding.

So, the unaddressed question arises for the author’s brand of aprioristic *empiricism*: why have you ignored this time-honored form of logicist reflection on the logically possible *purely logical* provenance of our concept of natural number? Is it not *this* that makes arithmetical knowledge *a priori*? — rather than some (obscure and unexplained) form of ‘grounding’ of arithmetical concepts in ‘sensory input’? In light of the logicist tradition, and all that it has accomplished, is it not over-hastily dismissive to write that

> … there is no reason to suppose that [the concepts *7*, +, *5*, =, and *12*] could be grounded *without* reliance on sensory input (p. 150)?

We adverted above to an appearance of slight strain between two assertions in the work. The first of these is confidently sweeping:

> … arithmetical truths are conceptual truths; or, at least, enough arithmetical truths are conceptual truths to enable us to account for *all* of our *a priori* arithmetical knowledge once we add in knowledge secured by inference from other truths known in this way. (p. 123; emphasis added)

But, thirty pages later the author retracts:

> To account for all of our knowledge of arithmetic is a tough call, even when we allow that much can be achieved by deduction from previously known arithmetical facts. Gödel’s incompleteness results are a measure of how tough a call this is. (p. 153)

One is left with the misgiving that this was a work conceived and executed with an eye only to Kant’s famous example (‘$7 + 5 = 12$’), which subsequently had to be hedged and qualified as soon as an expert critic drew attention to Gödelian incompleteness (for, surely, any epistemology for arithmetic must be able to deal with quantified sentences).

Readers of this journal are unlikely to be persuaded by this brand of empiricist apriorism, interesting though the prospects for some account of that kind might be. The details of execution are neither adequate nor convincing. The author observes that ‘The approach I am advocating does not seem to have been considered before' (p. 199). If her account, as here executed, turns out to be the only possible one of this kind, then we have an explanation, perhaps, as to *why* it has not been considered before — despite the author's anticipation of extensions of her account to 'other kinds of *a priori* knowledge like knowledge of other parts of mathematics, and of logic' and 'even … ethics' (p. 265).

doi:10.1093/phimat/nkq019