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Key Points:
- Equatorial magnetosonic waves have a discrete spectrum at harmonics of the proton gyrofrequency
- The Chirikov resonance criterion is applied to investigate the validity of applying quasi-linear theory with the assumption of a continuous Gaussian spectrum
- Cluster observations of equatorial magnetosonic waves

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Equatorial Magnetosonic Waves Observed by Cluster Satellites: The Chirikov Resonance Overlap Criterion

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Abstract Magnetosonic waves play an important role on the overall dynamics of relativistic radiation belt electrons. Numerical codes modeling the evolution of the radiation belts often account for wave-particle interaction with magnetosonic waves. The diffusion coefficients incorporated in these codes are generally estimated based on the results of statistical surveys of the occurrence and amplitude of these waves. These statistical models assume that the spectrum of the magnetosonic waves can be considered as continuous in frequency space. This assumption can only be valid if the discrete nature of the waves satisfy the Chirikov overlap criterion. Otherwise, the assumption of a continuous frequency spectrum could produce erroneous results in wave models and hence estimates of the electron diffusion coefficients used in numerical models of the inner magnetosphere. Recently, it was demonstrated, through a case study conducted on a single short (10 s) period snapshot within a longer wave event, that the discrete nature of the equatorial magnetosonic waves do satisfy the Chirikov overlap criterion and so the assumption of a continuous frequency spectrum is valid for the calculation of diffusion coefficients. This paper expands this study to a broader range of time with many magnetosonic wave events to determine whether the discrete nature of the waves always satisfy the Chirikov overlap criterion. The results show that most, but not all, discrete magnetosonic emissions satisfy the Chirikov overlap criterion. Therefore, the use of the continuous spectrum, employed in quasi-linear theory, may not always be justified.

1. Introduction

Magnetosonic waves, also known as electromagnetic equatorial noise, were first observed by OGO 3 almost 50 years ago as an oscillation in the magnetic field (Russell et al., 1970). They are highly oblique whistler mode waves that occur as a series of narrow tones close to harmonics of the proton gyrofrequency and the lower hybrid resonance frequency (Balikhin et al., 2015; Gurnett, 1976; Horne et al., 2000; Laakso et al., 1990; Němec et al., 2005; Perraut et al., 1982). They are abundant in the near Earth plasma environment, predominantly confined close to the magnetic equator of the terrestrial magnetosphere at radial distances between 2 and 8 R_E, both inside and outside the plasmapause primarily in the afternoon and premidnight (Cornilleau-Wehrlin et al., 2003; Meredith et al., 2008; Němec et al., 2005; Perraut et al., 1982; Russell et al., 1970; Santolík et al., 2002; Shprits et al., 2013). Magnetosonic wave emissions have been observed in conjunction with ring-like proton distributions (Chen, 2011; Meredith et al., 2008; Perraut et al., 1982; Santolík et al., 2002) where resonance interactions provide the free energy for their growth, generating a spectrum of discrete emissions at harmonics of the proton gyrofrequency (Balikhin et al., 2015). Magnetosonic waves consist of intense electromagnetic emissions that propagate nearly perpendicular to the ambient magnetic field. They are believed to be generated during active times due to a ring distribution of injected ring current ions (Balikhin et al., 2015; Boardsen et al., 1992; Chen, 2011; Horne et al., 2000; Ma et al., 2014; Perraut et al., 1982). In fact, it has been shown that the variations in wave amplitude are strongly correlated with variations in geomagnetic indices and solar wind parameters (Kim & Shprits, 2017; Ma et al., 2013), in a similar fashion to the amplitudes of chorus and plasmaspheric hiss waves (Aryan et al., 2014, 2016; Meredith et al., 2012).

Magnetosonic waves are responsible for the scattering and acceleration of radiation belt electrons from 10 keV to relativistic energies via Landau resonant interactions on a time scale of approximately 1 day (Gurnett, 1976; Horne, 2007; Shprits et al., 2009). According to recent observations, magnetosonic waves could have significant impact on pitch angle diffusion and acceleration of the relativistic radiation belt

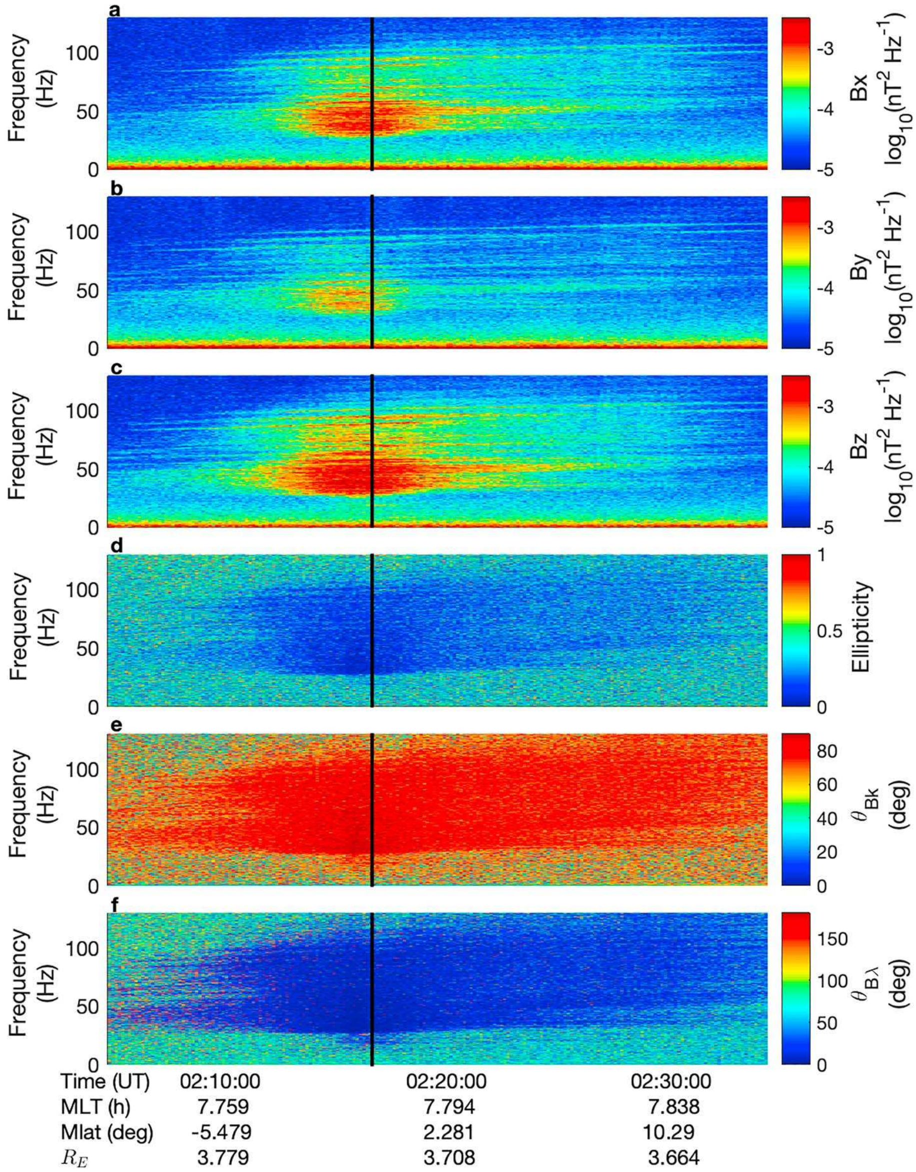

Figure 1. Observation of a magnetosonic wave event measured by Cluster 2 on 16 November 2006 at around 02:08 to 02:33 UT. (a–c) The dynamic wave spectrogram (*Bx*, *By*, and *Bz*, respectively) measured by Spatio-Temporal Analysis of Field Fluctuations search coil magnetometer. (d) The ellipticity of the waves, (e) the angle between the propagation direction and the external magnetic field (θ_{Bk}), and (f) the angle between the maximum variance direction and the external magnetic field ($\theta_{B\lambda}$). The vertical black line indicates the equator crossing. The foot of the figure shows the time (UT), magnetic local time (MLT), magnetic latitude (Mlat), and radial distance (R_E).

electrons that dictates the overall dynamics of the radiation belts (Albert et al., 2016; Horne, 2007; Ma et al., 2013, 2016; Meredith et al., 2008; Mourenas et al., 2013). It is known that magnetosonic waves can act as intermediaries in an energy transfer between high-energy ions and relativistic electrons (Meredith et al., 2008; Zhou et al., 2014).

The variability of relativistic electrons within the radiation belts is often modeled using numerical codes, such as the Pitch Angle and Energy Diffusion of Ions and Electrons (PADIE) (Glauert & Horne, 2005), Versatile Electron Radiation Belt (VERB) (Shprits et al., 2009), and Comprehensive Inner-Magnetosphere Ionosphere (CIMI) (Fok et al., 2014). These codes account for particle interaction with various wave modes such as chorus, plasmaspheric hiss, electromagnetic ion cyclotron, and magnetosonic waves. The effects of the waves on the particle population are characterized by tensors of diffusion coefficients. The magnetosonic wave diffusion coefficients are generally estimated based on the use of quasi-linear theory with the

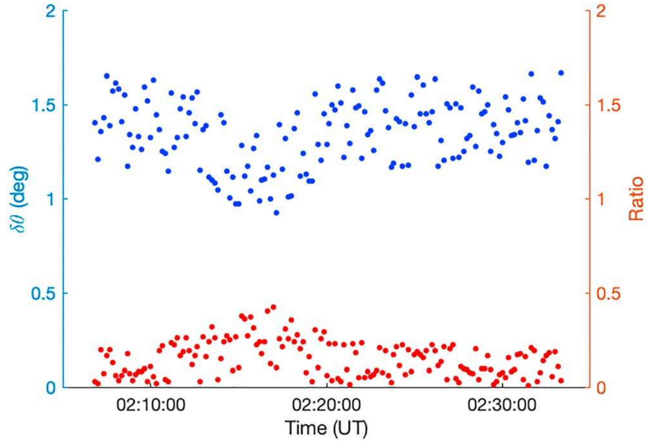

Figure 2. The analysis of the Chirikov resonance overlap criterion for the magnetosonic wave event observed by Cluster 2 on 16 November 2006 as presented in Figure 1. The blue and red dots represent 10 s averaged values of $\delta\theta$ and the ratio ($\frac{vl/\tan\theta_m}{1-\omega^2/v\Omega_{ce}^2}$) on the right-hand side of equation (2), respectively.

assumption of a continuous gaussian spectrum (Mourenas et al., 2013; Shprits et al., 2013). However, it is known that magnetosonic waves exhibit a discrete spectrum consisting of emissions close to a number of harmonics of the proton gyrofrequency. Therefore, the assumption of a continuous spectrum may only be justified if the harmonic elements satisfy the Chirikov resonance overlap criterion (Artemyev et al., 2015). Otherwise, the contribution of each harmonic to the diffusion coefficient should be evaluated separately.

Using the observations of equatorial magnetosonic waves made during the Cluster Inner Magnetospheric Campaign, Walker et al. (2015) demonstrated that at least in one case study the discrete nature of the waves do satisfy the Chirikov resonance overlap criterion and so the use of quasi-linear theory with the assumption of a continuous frequency spectrum is valid for the calculation of diffusion coefficients. The study by Walker et al. (2015) was conducted on a single short (10 s) period snapshot within a longer wave event. This paper expands this study to a broader range of time with many magnetosonic wave events to determine whether the discrete nature of the waves at harmonics of the proton gyrofrequency always satisfy the Chirikov resonance overlap criterion. In addition, it is widely accepted that magnetosonic waves are predominantly confined very close (≤3°) to the magnetic equator of the terrestrial magnetosphere (Russell et al., 1970). This paper also aims to validate this assumption using observations of equatorial magnetosonic waves by Cluster satellites.

2. Data

This study uses data collected by the Cluster satellites from the start of the mission up to February 2017. The Cluster mission consist of four identical satellites that were launched in 2000 in a polar orbit with an apogee of approximately 20 R_E, initial perigee of approximately 4 R_E, and a period of 57 hr. This initial orbit has evolved over time to allow Cluster to sample plasma and wave activity at the magnetic equator over a range of different radial distances. This study uses burst science mode observations collected by the FGM (fluxgate magnetometer) (Balogh et al., 1997), the STAFF (Spatio-Temporal Analysis of Field Fluctuations) search coil magnetometer (Cornilleau-Wehrlin et al., 1997), and the EFW (Electric Fields and Waves) (Gustafsson et al., 1997) instruments (Escoubet et al., 1997). During burst science mode operation fluxgate magnetometer and STAFF collect magnetic field waveform measurements with sampling rates of 67 and 450 Hz, respectively. This study investigates 112 visually selected discrete magnetosonic wave events that were at least partially observed within 10° of the magnetic equator and $7R_E$ radial distance, to determine whether the discrete nature of the waves always satisfy the Chirikov resonance overlap criterion.

3. Chirikov's Resonance Overlap Criterion

The diffusion coefficients required for the numerical codes are generally estimated based on the use of quasi-linear theory with the assumption of a continuous spectrum. However, it is known that magnetosonic waves exhibit a discrete spectrum close to a number of harmonics of the proton gyrofrequency (Boardsen et al., 1992; Chen, 2011; Horne, 2007; Mourenas et al., 2013; Němec et al., 2005; Russell et al., 1970). Nevertheless, the assumption of a continuous spectrum may be justified providing that the harmonic elements satisfy the Chirikov resonance overlap parameter for stochastic motion.

The Chirikov resonance overlap criterion states that, in a Hamiltonian system, a deterministic trajectory will begin to move between two resonances in a chaotic and unpredictable manner as soon as these unperturbed resonances overlap (Chirikov, 1960). This implies that when the resonance widths of two adjacent harmonic emission bands are large enough in comparison with the fundamental frequency as formulated in equation (1), where Δw_r is the frequency half width of the unperturbed resonance and Ω_d is the frequency distance between two unperturbed resonances, the particles may move between different resonance frequencies in

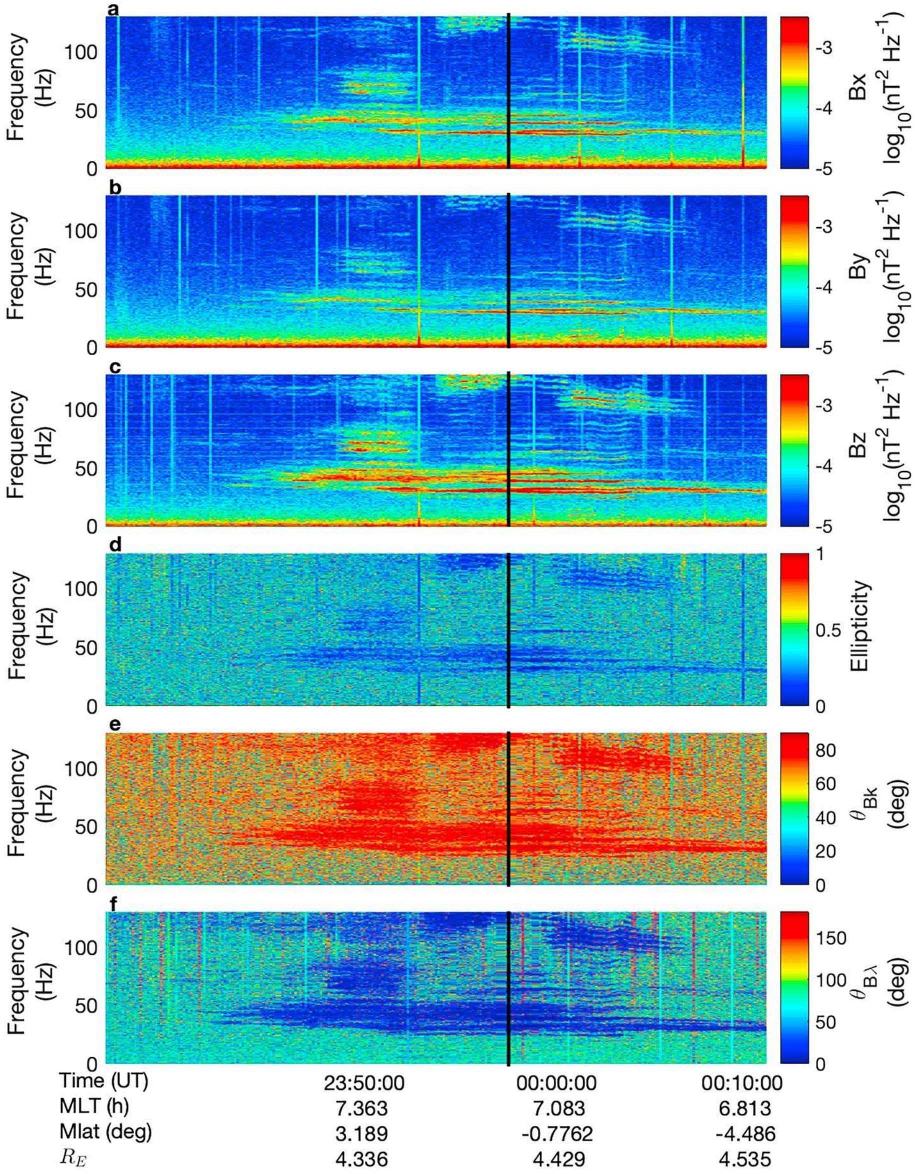

Figure 3. Observation of a magnetosonic wave event measured by Cluster 1 on 4 November 2014 at around 23:42 to 00:12 UT. Caption of Figure 1 applies.

a chaotic manner and may not be associated with one particular resonance (Walker et al., 2015).

$$S^2 = \left(\frac{\Delta w_r}{\Omega_d}\right)^2 > 1 \tag{1}$$

Overlap of neighboring harmonic emissions may occur for two adjacent resonances when magnetosonic waves interact with electrons in Landau resonance as formulated in equation (2) where θ_m is the mean angle between the propagation direction and the external magnetic field, $\delta\theta$ is the standard deviation of the wave propagation angles θ_m, l is the harmonic number, $\nu = m_e/m_p$ is the electron to proton mass ratio, and Ω_{ce} is the electron gyrofrequency (Artemyev et al., 2015). It is assumed that the more general Chirikov resonance overlap criterion is only fulfilled if equation (2) is satisfied. Otherwise, the contribution of each harmonic to the diffusion coefficient should be evaluated separately to avoid potentially erroneous results in wave models.

$$\delta\theta > \frac{\nu l/\tan\theta_m}{1 - \omega^2/\nu\Omega_{ce}^2} \tag{2}$$

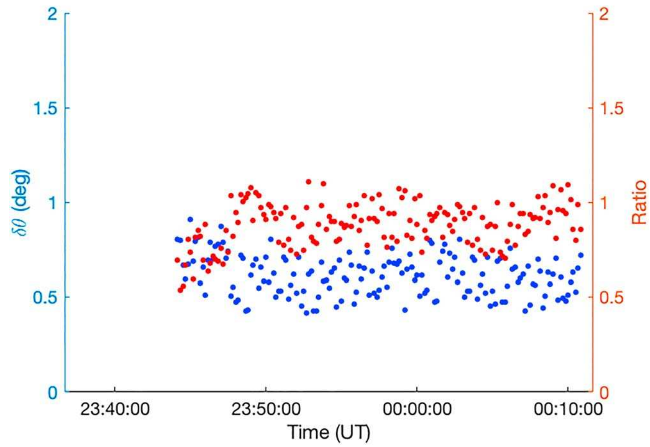

Figure 4. The analysis of the Chirikov resonance overlap criterion for the magnetosonic wave event observed by Cluster 1 on 4 November 2014 as presented in Figure 3. Caption of Figure 2 applies.

In this study, the Morlet wavelet transform is applied to transform the components of the STAFF magnetic waveform signal into the frequency domain. The evaluation of the overlap criterion is based on the mean and distribution of the wave normal angles at each of the discrete emission frequencies corresponding to peaks in the wavelet spectra. The wavelet filtered waveform are divided into intervals of 0.25 s to determine the wave normal angles. A minimum variance analysis is then performed on data with wave power amplitudes greater than a threshold value of 0.2 to determine the direction of minimum variance, which corresponds to the propagation direction of the wave. The resulting sets of propagation angles obtained from minimum variance analysis with respect to the external magnetic field (θ_{Bk}) are analyzed to determine the mean direction of propagation (θ_m) and its standard deviation ($\delta\theta$) as a measure of its variance, where all units are converted to degrees for the analysis of the Chirikov resonance overlap criterion outlined in equation (2) (Artemyev et al., 2015). A more detailed description of the methodology is provided in Walker et al. (2015).

4. Observation

In this study, we analyzed 112 discrete magnetosonic wave events observed by the Cluster satellites. For each wave event the Chirikov resonance overlap criterion is investigated in relation to the harmonic nature of the wave emissions based on the Chirikov resonance overlap parameter formulated in Artemyev et al. (2015) as discussed in previous section. Figure 1 shows an observation of a magnetosonic wave event measured by Cluster 2 on 16 November 2006 at around 02:08 to 02:33 UT. Figures 1a–1c show the dynamic wave spectrogram (Bx, By, and Bz) geocentric solar equatorial components measured by STAFF search coil magnetometer, respectively. The bottom three panels show (Figure 1d) the ellipticity of the waves, (Figure 1e) the angle between the propagation direction and the external magnetic field (θ_{Bk}), and (Figure 1f) the angle between the maximum variance direction and the external magnetic field ($\theta_{B\lambda}$). The vertical black line indicates the equator crossing. The foot of the figure shows the time (UT), magnetic local time (MLT), magnetic latitude (Mlat) and radial distance (R_E). Cluster 2 begins observing discrete banded magnetosonic wave signatures at around 02:08 UT as it travels toward the equator. The banded nature of the emissions can be clearly seen, and their amplitudes are not constant but vary independently. The emissions intensify near the equator with maximum wave intensities observed when the spacecraft crosses the equator consistent with the findings of Russell et al. (1970). As the spacecraft moves away from the equator the emissions slowly fade away. The emissions are observed approximately in the frequency range 20 to 110 Hz with the most intense emissions observed at around 50 Hz near the equator corresponding to approximately the twentieth harmonics of the local proton gyrofrequency. As is evident from Figure 1d, the emissions are highly elliptical (the values of ellipticity of magnetic field fluctuations, which is defined as a ratio of the minor to the major polarization axes, may range from 0 to 1. The values of ellipticity equal to 0 correspond to a linear polarization. The values of ellipticity equal to 1 correspond to a circular polarization). Figure 1e shows that the waves propagate perpendicularly to the external magnetic field with θ_{Bk} close to 90°. In addition, Figure 1f shows that the oscillations of the wave magnetic field occur in the direction parallel to the external magnetic field. This is all consistent with the properties of magnetosonic waves. The Chirikov resonance overlap criterion was investigated in relation to the harmonic nature of the wave emissions observed in this event. The results are presented in Figure 2,

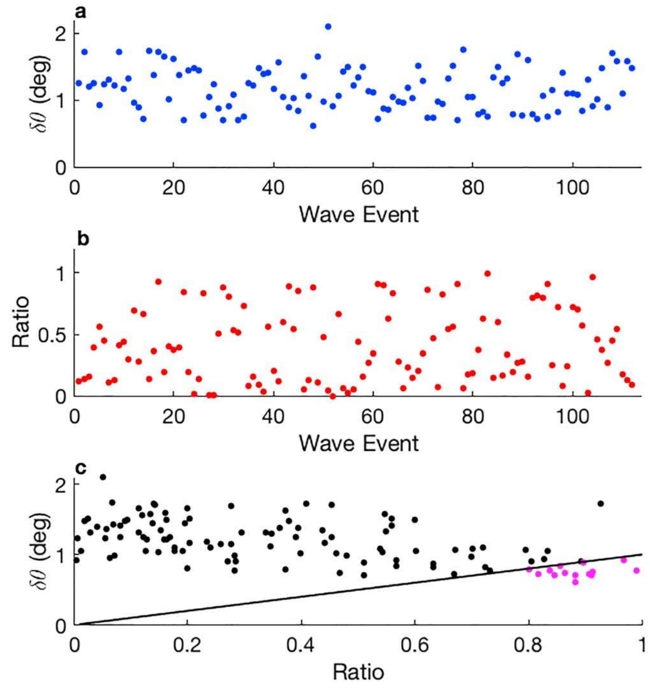

Figure 5. The analysis of the Chirikov resonance overlap criterion for 112 magnetosonic wave events studied in this paper. From top to bottom the panels show the average values of (a) $\delta\theta$, (b) the ratio of the right-hand side of equation (2), and (c) the variation of $\delta\theta$ versus the ratio for each magnetosonic wave event. The black and magenta dots represent wave events that satisfy and did not satisfy the Chirikov resonance overlap criterion, respectively. The black line corresponds to $\delta\theta$ = ratio.

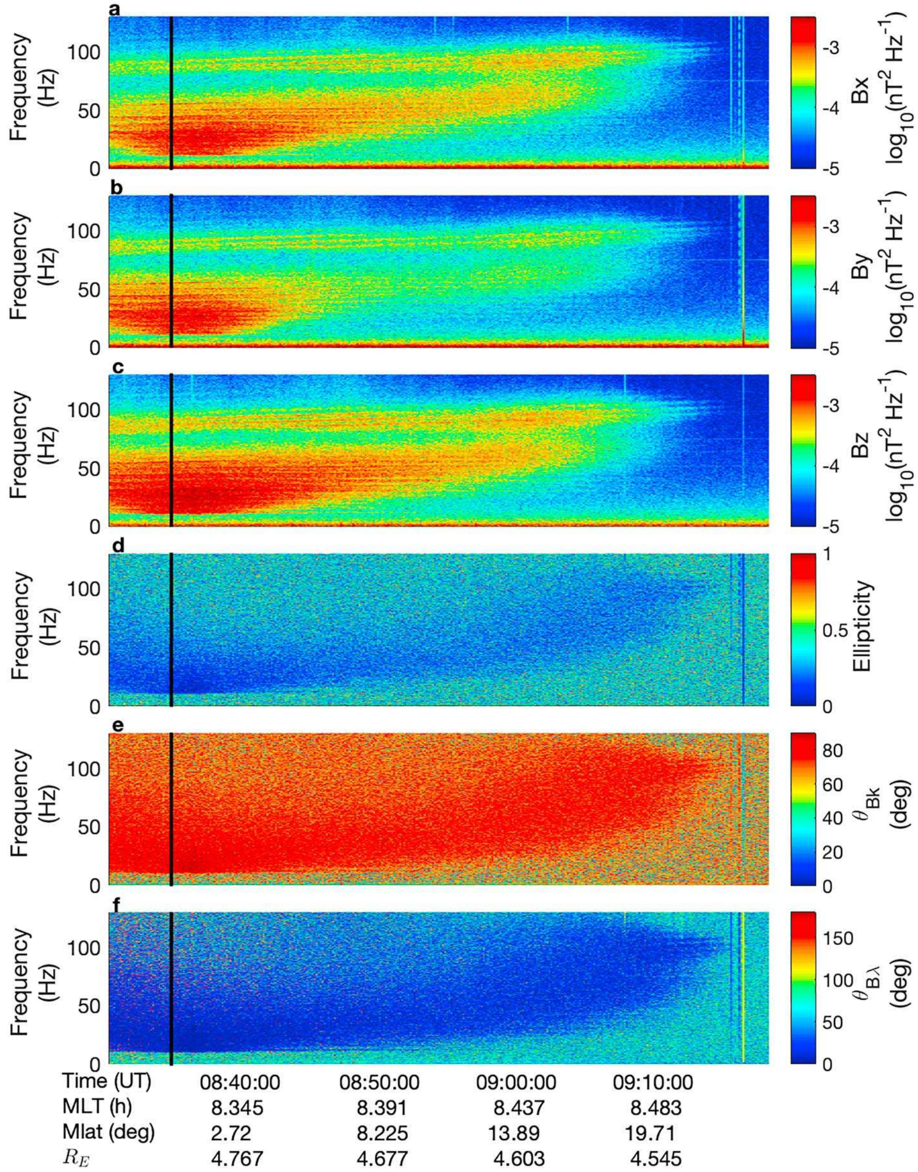

Figure 6. Observation of a magnetosonic wave event measured by Cluster 4 on 12 November 2005 at around 8:35 UT. Caption of Figure 1 applies.

where the blue and red dots represent 10 s averaged values of $\delta\theta$ and the ratio ($\frac{\nu l/\tan\theta_m}{1-\omega^2/\nu\Omega_{ce}^2}$) on the right-hand side of equation (2), respectively. Generally, the $\delta\theta$ values are around 1.4 ° and the ratio values are around 0.1. However, near the equator as Cluster 2 observes more intense and broad wave emissions the values of $\delta\theta$ decrease, while the values of the ratio increase. Overall, the average $\delta\theta$ value, 1.35°, is much larger than the average ratio value, 0.15, for the whole event. This means that the discrete nature of the magnetosonic wave observed in this particular event do satisfy the Chirikov resonance overlap criterion and so the use of quasi-linear theory with the assumption of a continuous frequency spectrum is valid for the calculation of diffusion coefficients.

However, some of the magnetosonic wave events investigated in this study did not satisfy the Chirikov resonance overlap criterion. Figure 3 shows the observation of a magnetosonic wave event measured by Cluster 1 on 4 November 2014 at around 23:42 to 00:12 UT, using the same format as Figure 1. Cluster 1 begins observing discrete magnetosonic wave signatures at around 23:42 UT as it travels toward the equator. Evidently, the emissions observed in this case have properties that are consistent with the properties of magnetosonic waves; there are a number of banded emissions that intensify as the spacecraft passes the equator, the ampli-

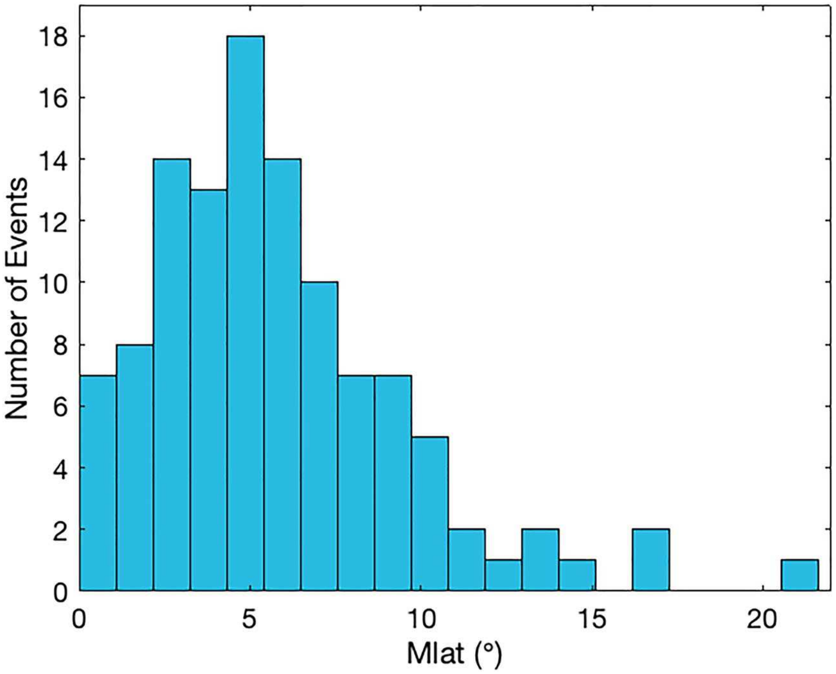

Figure 7. The distribution of magnetosonic wave events across magnetic latitude (Mlat).

tudes of the emissions are not constant but vary independently, and the banded emissions propagate almost perpendicular to the external magnetic field since they are highly elliptical in nature and the wave magnetic field oscillates parallel to the external field. The Chirikov resonance overlap criterion was investigated in relation to the harmonic nature of the wave emissions observed in this event. The results are presented in Figure 4 in the same format as Figure 2. In this event the difference between the values of $\delta\theta$ and ratio are much less. Crucially, the $\delta\theta$ values are generally smaller than the ratio values. This means that the discrete nature of the magnetosonic wave observed in this particular event does not satisfy the Chirikov resonance overlap criterion and so the use of quasi-linear theory with the assumption of a continuous frequency spectrum is not valid for the calculation of diffusion coefficients and may lead to erroneous results in wave models.

Overall, we found that most, but not all, discrete magnetosonic emissions satisfy the Chirikov resonance overlap criterion. In fact, out of 112 selected wave events studied in this paper, 98 events satisfied the Chirikov resonance overlap criterion, but 14 wave events did not satisfy the Chirikov resonance overlap criterion. Figure 5 shows the result of the analysis of the Chirikov resonance overlap criterion for 112 magnetosonic wave events. From top to bottom the panels show the average values of (Figure 5a) $\delta\theta$, (Figure 5b) the ratio of the right-hand side of equation (2), and (Figure 5c) the variation of $\delta\theta$ versus the ratio for each magnetosonic wave event. The black and magenta dots represent wave events that satisfy and did not satisfy the Chirikov resonance overlap criterion, respectively. The black line corresponds to $\delta\theta =$ ratio. Evidently, for the majority of the wave events the $\delta\theta$ values are large and the ratio values are much smaller, as shown in Figure 5c. For these cases the Chirikov resonance overlap criterion is satisfied. However, for the wave events where the Chirikov resonance overlap criterion is not satisfied the $\delta\theta$ values are relatively small while the ratio values are marginally larger. This suggests that $\delta\theta$ could be the controlling parameter in defining the validity to use quasi-linear theory with the assumption of a continuous frequency spectrum.

In addition, it is widely accepted that magnetosonic waves are predominantly confined to approximately 3° of the magnetic equator (Cornilleau-Wehrlin et al., 2003; Němec et al., 2005; Russell et al., 1970; Santolík et al., 2002). While in most wave events studied in this paper, the peak intensities of the waves are observed close to the equator, the emissions may still be observed at much higher latitudes. For example, Figure 6 demonstrates a discrete magnetosonic wave event measured by Cluster 4 on 12 November 2005. The panels are as described in Figure 1. The banded emissions are consistent with magnetosonic waves as they propagate almost perpendicular to the eternal magnetic field and the wave magnetic field oscillates parallel to the external field. In this case, Cluster 4 observes strong discrete magnetosonic wave emissions near the equator at around 08:36 UT. The satellite continues to observe these emissions as it moves away from the equator up to approximately 21° away from the equator, consistent with some past studies that suggested the existence of low-amplitude magnetosonic waves at high latitudes (Tsurutani et al., 2014; Zhima et al., 2015). In fact, our analysis shows that the emissions of approximately 75% of the magnetosonic wave events studied in this paper were observable beyond 3° of the magnetic equator. Figure 7 shows the distribution of all 112 magnetosonic wave events studied in this paper across magnetic latitude. Evidently, the majority of the wave events are observed close to the equator, but the emissions can be observed at much higher latitudes consistent with past studies (Tsurutani et al., 2014; Zhima et al., 2015). The results presented in this study are based on magnetosonic wave events that were at least partially observed within 10° of the magnetic equator and within 7 R_E radial distance. Therefore, there could be even more magnetosonic wave emissions at even higher latitudes as suggested by past studies (Tsurutani et al., 2014; Zhima et al., 2015).

5. Conclusion

In this study 112 discrete magnetosonic wave events observed by Cluster satellites were selected and analyzed. For each wave event we evaluated the overlap criterion based on the same methodology described in Walker et al. (2015) to determine whether the discrete nature of the identified magnetosonic wave events satisfy the Chirikov resonance overlap criterion formulated in Artemyev et al. (2015). The results show that the majority of the magnetosonic wave emissions do satisfy the Chirikov resonance overlap criterion. In fact, out of 112 selected wave events, 98 events satisfied the Chirikov resonance overlap criterion. This means that the use of quasi-linear theory with the assumption of a continuous frequency spectrum is valid for the calculation of diffusion coefficients for the majority of wave events. However, there are magnetosonic wave events that do not satisfy the Chirikov resonance overlap criterion. In this study 14 magnetosonic wave events did not satisfy the Chirikov resonance overlap criterion. In those cases the use of quasi-linear theory with the assumption of a continuous frequency spectrum is not valid for the calculation of diffusion coefficients and may lead to erroneous results in wave models and hence estimates of the electron diffusion coefficients used in numerical models of the inner magnetosphere.

In addition, this study also found that not all magnetosonic wave events are confined very close to the magnetic equator as it is widely assumed. In fact, the results show that there are approximately 75% of wave events that are observed outside 3° and some were observed at much higher latitudes, approximately 21° away from the magnetic equator, consistent with some past studies that suggested the existence of low-amplitude magnetosonic waves at high latitudes (Tsurutani et al., 2014; Zhima et al., 2015).

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