DEVELOPMENT OF A SEGMENTED FRACTIONAL PLAN (SFP)TO AID THE IMPLEMENTATION OF SIX SIGMA IN SMEs

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ABSTRACT

Six sigma is a statistically based, project-oriented process improvement strategy with documented successes of its usage in both large and small to medium sized enterprises. Although six sigma has been successful used by some small and medium sized enterprises (SMEs), a number of SMEs have cited the lack of resources as a factor impeding the use of six sigma. This research aims to develop an experimental plan to aid manufacturing SMEs implement six sigma at low cost.

A statistical technique used with six sigma which can be resource intensive is the factorial technique used in the design of experiments. As the number of factors in a full factorial experiment increases, so does the number of experimental runs needed to conduct the experiment. This can lead to a considerably high amount of experimental runs when the factors to be studied are each represented in three levels (3-level experiments).

To aid the implementation of six sigma in SMEs, this research developed an experimental plan referred to as a Segmented Fractional Plan (SFP) for fractionating 3-level full factorial experiments when 3 and 4 factors are to be studied (3^3 and 3^4 full factorial experiments). The SFP was tested using published data on designed experiments and its performance was compared to Orthogonal Arrays (OAs) using the aforementioned data on designed experiments and a laboratory experiment. The findings from the comparisons show that to identify the process setting that produces the desired product quality, with a reduced number of experimental runs, the SFP can perform as well as or better than some OAs thus, providing an option for economic experimentation when fractionating 3^3 and 3^4 full factorial experiments.

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List of abbreviations and symbols

ANOVA	Analysis of variance
DFSS	Design for Six Sigma
DMAIC	Define, Measure, Analyse, Improve and Control
DMADV	Define, Measure, Analyse, Design, Verify
DOE	Design of Experiments
DPMO	Defects per million opportunities
EC	European Commission
FMEA	Failure mode and effects analysis
Gage R&R	Gage repeatability and reproducibility studies
JIT	Just-in-time-manufacturing
KN	Kilo Newton
LCL	Lower Control Limit
LSL	Lower Specification Limit
LSS	Lean Six Sigma
M-C plot	Main effects and centre point plot
NVA	Non-Value Added activities
OA	Orthogonal Arrays
OFAT	One-factor-at-a-time
PDSA	Plan Do Study Act

psi	Pounds per square inch
RNVA	Required Non-Value Added activities
RSM	Response Surface Methodology
SF	Shear Factor
SFP	Segmented Fractional Plan
SMED	Single Minute Exchange of Die
SMEs	Small and Medium sized Enterprises
TQM	Total Quality Management
UCL	Upper Control Limit
UK	United Kingdom
USA	United States of America
USL	Upper Specification Limit
VA	Value Added activities
μ	Mean
σ	Standard deviation

Dedication

I will like to dedicate this research to my father '*Engr. Okechukwu Ozoemena*' for financing it, and my entire family: my father, mother, brothers and sister for the love they showed me throughout the course of this study.

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Firstly, I would like to express my gratitude to my principal supervisor Dr Phillip Hackney for his continuous support and motivation in the course of this study. Your support and motivation enabled me to keep a positive attitude in the course of this research.

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Declaration

I declare that the work contained in this thesis has not been submitted for any other award and that it is all my own work. I also confirm that this work fully acknowledges opinions, ideas and contributions from the work of others.

Name: Chukwunonso Emmanuel Ozoemena

Signature: 

Date: 25/08/2016

Chapter 1

Introduction

1.1. Quality improvement initiatives in the manufacturing industry

Process quality improvement has become the focus of many organisations due to the competitive advantage a process with high quality can provide [1]. To improve the quality of manufacturing processes, various quality improvement initiatives have been proposed [1-3]. Three of these initiatives are described herein due to the popularity of their use and the documented success stories of their usage [4-6]. These are: the lean manufacturing system, the total quality management system and the six sigma system.

Introduced in Japan in the 1940s, the lean manufacturing system focuses on eliminating seven types of process waste using a collection of tools and techniques. These wastes include: overproduction, unnecessary waiting times, unnecessary transportation, excessive inventory, over processing, unnecessary motion and defects. By eliminating these wastes, a lean organisation removes all processes that add no value from the customer's perspective. The tools and techniques associated with lean manufacturing require no statistical knowledge and have proved to be effective in minimising these seven wastes [6, 7].

Secondly, total quality management (TQM) focuses on reducing the variation in work processes. It uses a collection of statistical and non-statistical tools and techniques to improve the quality of a process. This system recommends the implementation of quality improvement projects on an organisation wide basis and is usually deployed using quality councils, workforce-level teams and cross-functional teams. The quality council addresses strategic initiatives, the workforce-level teams attend to quality issues associated with routine production activities and the cross-functional teams address quality concerns that require personnel from different departments within the organisation. Despite the popularity of the TQM system, its success rate can be classified as moderate. Reasons put forward for the moderate success of the TQM system include; a lack of high-level management commitment and involvement, insufficient recognition of variability reduction as the primary objective and the inadequate use of statistical tools and techniques [1, 4].

Finally, six sigma, like TQM, focuses on minimising process variation to improve the quality of a process. It employs a collection of statistical and non-statistical tools and techniques implemented using a sequence of steps known as DMAIC to achieve the desired goal [5]. DMAIC is an abbreviation for the steps: define, measure, analyse, improve and control. These steps provide a roadmap for process improvement when conducting quality improvement projects [8]. Developed by Motorola in the 1980s, six sigma has generally been more successful than TQM [1].

1.2. Research motivation

The globalisation of markets and the growing inter-dependence of economies are shaping national and international competitive environments. To compete in an increasingly global market, organisations need to establish mechanisms that enable them to commence and sustain business improvement efforts when needed [9]. One way to attain competitive advantage in competitive markets is by improving the quality of goods and services being produced [9-11]. An organisation that can delight its customers by improving and controlling the quality of its goods and services has the potential to dominate its competitors [1].

Large organisations competing in an increasingly global market tend to rely on subcontracting certain jobs to other organisations, most of which will be small to medium sized enterprises (SMEs). As such, SMEs act as subcontractors to large organisations. As the success of the quality programmes of large organisations is dependent on the quality of goods and services from their suppliers, it is logical for large organisations to encourage the application of quality improvement initiatives such as six sigma among their suppliers to be assured of obtaining high quality goods and services. [9, 12]. In addition, to be competitive, the increasing demand on quality by large organisations puts pressure on SMEs to consider using quality improvement initiatives [9].

An organisation can improve its process quality by using quality initiatives such as lean manufacturing, TQM and six sigma. This research focuses on six sigma implementation in SMEs due to two reasons. These are:

1. ***Its focus on the financial impact of a project:*** The focus on the financial impact of projects draws strong management leadership and support to project activities. This minimises the risk of project failure due to insufficient allocation of resources by top management [13].

2. ***The infrastructure of quality personnel it creates to lead and deploy six sigma:*** The infrastructure of quality personnel employed by six sigma in descending order of hierarchy are: Champions, Master Black Belts, Black Belts, Green Belts, Yellow Belts and White Belts [14]. Champions provide the business focus for projects while the Master Black Belts, Black Belts, Green Belts, Yellow Belts and White Belts are individuals specially trained to deploy six sigma. These specially trained individuals receive differentiated training tailored to their ranks to improve their project management and problem solving skills. The hierarchical structure of quality personnel employed by six sigma helps to control and coordinate work across organisational levels to ensure that project activities match the overall business aim [15].

Despite the successful use of six sigma by large organisations and SMEs for process improvement, some SMEs have not benefited from the use of six sigma. One of the reasons cited by SMEs for not using six sigma is the lack of resources to do so [16-18]. Fundamental to the six sigma approach is the use of statistical tools and techniques [19, 20]. SMEs may lack resources in the form of time and personnel to use statistical tools and techniques for quality improvement [21]. A statistical technique used with six sigma which can be resource intensive is the factorial technique used in the design of experiments. This technique helps identify and positively adjust important variables that affect process performance [22-24].

Factorial experimentation is an experimental strategy in which all variables (factors) that are believed to have an influence on the performance of a process are varied together instead of one at a time. Factorial designs are widely used to study the joint effect of several factors on the output (response) of a process. In factorial designs, all possible combinations of the settings of the factors to be studied are tested [23, 25].

As the number of factors in a factorial experiment increases, so does the number of experimental runs needed for experimentation. An experimental run is a specific combination of the settings of several factors. This increase in experimental runs can lead to situations where the number of experimental runs outgrows the resources available for experimentation. Thus, when resources are limited, it may not be feasible to conduct some factorial experiments [23, 25].

Orthogonal arrays and fractional factorial designs (also orthogonal arrays) are commonly used to fractionate factorial experiments when resources are limited.

Orthogonal arrays minimise the experimental runs needed for experimentation compared to factorial designs by assuming the effects of certain interactions between factors are negligible [23]. To distinguish factorial designs from fractional factorial designs, factorial designs are referred to as full factorial designs [23, 25]. This research focuses on developing an experimental plan for fractionating full factorial experiments when 3 and 4 factors are to be studied at three levels (3^3 and 3^4 full factorial experiments). 3-level experiments are useful in situations where the process settings are represented in three levels or the experimenter wishes to analyse the process performance based on three factor levels.

1.3. Research aim

This research aims to develop an experimental plan for fractionating 3^3 and 3^4 full factorial experiments in manufacturing SMEs. The experimental plan is developed with the aim of reducing the number of experimental runs compared to orthogonal arrays thus, minimising the resources needed for experimentation.

1.4. Research objectives

The objectives of this study are stated as follows:

1. Develop an experimental plan for fractionating 3^3 and 3^4 full factorial experiments which will require a lower number of experimental runs compared to orthogonal arrays.
2. Test the performance of the developed experimental plan using data from literature on designed experiments.

3. Compare the performance as well as the number of experimental runs required by the developed experimental plan to orthogonal arrays using the data from literature on designed experiments and a laboratory experiment.
4. Based on the comparisons develop a model for carrying out 3-level experiments in SMEs.

1.5. Outline of thesis chapters

Chapter 1 briefly introduces various quality improvement initiatives, discusses the research motivation, outlines the aim and objectives of the research and, presents an outline of the thesis chapters.

Chapter 2 discusses the lean manufacturing, TQM and six sigma quality improvement initiatives and reviews the literature on six sigma implementation in SMEs.

Chapter 3 introduces the subject of experimental design and reviews orthogonal arrays used to fractionate 3^3 and 3^4 full factorial experiments.

Chapter 4 presents the development of an experimental plan for fractionating 3^3 and 3^4 full factorial experiments in SMEs and compares the performance of the developed experimental plan to orthogonal arrays using data from literature and a laboratory experiment.

Chapter 5 concludes the research work by outlining the original contribution to knowledge (contribution to research and practise) provided by the research.

Chapter 2

Quality improvement initiatives – An Overview

2.1. Introduction

This chapter presents an overview of the lean manufacturing system, the total quality management system and the six sigma system. Following from the overview of these quality improvement initiatives, this chapter outlines the reasons why this research focuses on six sigma implementation in SMEs, reviews the literature on six sigma implementation in SMEs and outlines the rationale behind developing an experimental plan to aid the implementation of six sigma in SMEs.

2.2. Lean manufacturing

2.2.1. Fundamentals of lean manufacturing

Lean production started within Toyota in Japan in the 1940s and was pioneered by Taiichi Ohno and Shigeo Shingo [26-28]. Lean thinking can be conceptualised as a holistic paradigm that focuses on delivering value to the customer while eliminating waste from all activities involved in rendering the service [28, 29]. As a result of its successes in manufacturing, lean thinking has been applied to many other areas such as supply chain management, accounting, administration, health care and government. All of which it has been successfully applied to with documented benefits [29].

Lean manufacturing revolves around five concepts [6, 26, 29]. These are presented in figure 2.1 and are briefly discussed.

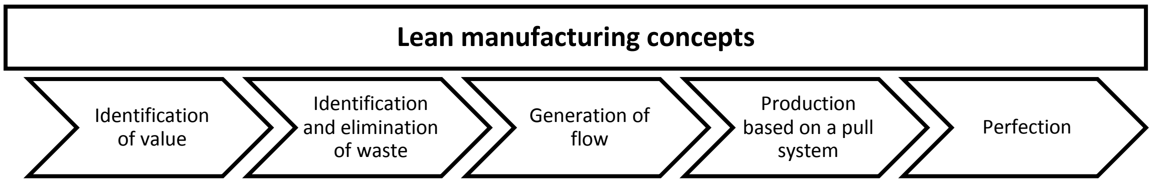


Figure 2.1. Concepts of lean manufacturing

1. **Identification of value:** The identification of value starts with the definition of value propositions for certain customers. These value propositions range from customer to customer and it is the challenge of the manufacturer to develop a product portfolio based on these value propositions. Value can be defined as what the customer says it is, considers important and is willing to pay for. When defining what is of value to the customer, the definition must be clear, unambiguous, complete, representing the need of the customer during the product life cycle and also allow for value clarification without giving rise to an escalation of requirements. For value creation to be successful, it is of high importance that everyone involved in the process channel their energy on capturing the final value proposition with the best of competence, experience, wisdom and consensus [6, 26, 29].

2. **Identification and elimination of waste:** Waste can be defined as any activity in a process which does not add value to the customer [6, 29, 30]. Lean thinking classifies all work activities into three categories. These are [29]:

Value Added activities (VA): These are activities which create value in the production process by transforming information or material towards the completion of a product and also reduce uncertainty in the process. These are activities which the customer must be willing to pay for. Also crucial to these activities is that it is done right the first time.

Required Non-Value Added activities (RNVA): These activities do not add value to the process but cannot be eliminated due to certain issues such as requirement by law, contract, company mandate, current technology, etc. Sometimes this waste is a necessary part of the process and cannot be eliminated e.g. financial controls.

Non-Value Added activities (NVA): These do not add any value to the process and also consume resources.

Wastes in lean manufacturing can be classified into seven main types [7, 26]. These are:

Over Production: This type of waste is brought about by the over production of goods. The products are made for no specific customer; hence are of no value.

Waiting: This type of waste occurs as a result of waiting times. It is brought about when time is used ineffectively. As people, equipment, or products wait to be processed, no value is added to the customer. An ideal state would be devoid of waiting times in which goods flow in a consistent and orderly manner.

Transport: When goods are moved from one place to another, some damage can be done unknowingly. Minimising transportation can reduce the amount of damage done as a result of moving goods from one place to another.

Inventory: This is waste brought about as a result of excessive inventory. Excessive inventory can lead to long lead times, space consumption, increase in storage costs and also prevent quick identification of problems.

Over processing: This occurs when complex/inflexible machines or procedures are used to manufacture goods when simpler/flexible machines or procedures can be used instead. This causes workers to sometimes over produce goods to compensate for the large investments in the complex machines. These large complex machines can also take up space leading to poor layout, excessive transportation and poor communication.

Motion: Unnecessary motion could affect the work place ergonomics as stretching, bending and unnecessary pick-ups that can be avoided could lead to operator fatigue which in turn leads to poor productivity.

Defects: Defects are the most visible type of waste which also translates to direct costs. These are errors on products which require rework or are designated as scrap. The Toyota Production System views defects as opportunities for continuous improvement rather than a problem associated with bad management. This way the system is improved as continuous improvement on the seven types of waste are undertaken.

3. Generation of flow: Understanding flow is probably the most challenging task in lean manufacturing. It is the lack of flow in manufacturing processes that leads to the storage of huge inventories in businesses which consume working capital. Flow revolves around one piece manufacturing as opposed to batch-and-queue processes. Flow can be summarised as the linkage of activities spanning across processes, people and culture which delivers value to the customer [6, 26, 29].

The theory of constraints introduced by Goldratt and Cox [31] aligns with lean thinking in the sense that it depicts an organisation as a system of resources connected by processes which makes a product to be sold, recognises the value stream of a system and identifies major barriers (constraints) to the lack of flow in a system. To guide the operation of a production plant, Goldratt and Cox [31] suggested monitoring three major process indicators. These are:

***Throughput*:** This is the rate at which a system generates revenue through sales.

***Inventory*:** This takes into account all the money invested by the system in purchasing things it intends to sell.

***Operational Expense*:** This accounts for all expenditure put in by the system in order to turn inventory into throughput.

4. Production based on a pull system: In a lean system, customer demand pulls finished goods through the system to prevent the build up of inventories [32].

5. Perfection: Strive for perfection by continuously eliminating non-value added activities [32].

Lean thinking aims to improve the performance of the whole supply chain and not just individual production processes. Hence, the improvement efforts must target the entirety of the supply chain [26, 29, 33].

2.2.2. Tools and techniques of lean manufacturing

Lean manufacturing is implemented using a variety of tools and techniques. Some of the common tools and techniques used to implement lean manufacturing are briefly described in table 2.1 [7, 26]:

Table 2.1. Common lean manufacturing tools and techniques

Tools and techniques	Uses
5S	Eliminates waste resulting from a poorly organised work area by describing proper methods of house keeping
Gemba (The real place)	Promotes a thorough understanding of manufacturing processes by first-hand observation and communicating with employees
Heijunka (level scheduling)	Reduces lead time by mixing product variants during production and reduces inventory by manufacturing in smaller batches
Kaizen (Continuous improvement)	A strategy which combines the collective talents of employees to improve the performance of a manufacturing process and eliminate waste
Kanban (Pull system)	Eliminates waste from inventory and over production by regulating the flow of goods with signal cards
Poka-yoke (Error proofing)	Detects and prevents errors during production based on simple devices designed to detect and prevent errors
Single minute exchange of die (SMED)	Reduces change over time to less than 10 mins based on a four step changeover process
Value stream map	Used to visually map the flow of production so as to expose waste in the current processes and provide a roadmap for improvement via a desired future state
Just-in-Time manufacturing (JIT)	A strategy used to pull parts through production based on customer demand instead of projected demand. It is implemented by combining other lean manufacturing tools and techniques

2.2.3. Benefits of lean manufacturing

Application of lean thinking to manufacturing has been able to improve performance standards as compared to non-lean environments. In a nut shell, lean thinking improves performance across the whole supply chain which translates to improved business performance. Benefits obtained as a result of the use of lean manufacturing tools and techniques are: Increased supply chain speed, inventory savings as a result of a reduction in inventory, reduction in floor space, release of working capital, shorter lead times, higher product quality and increased learning [26, 27, 34]. A technology transfer project in the North East of England by North East Productivity Alliance (NEPA) showed that the application of lean manufacturing management practices and knowledge in 15 local companies was able to bring about returns eight times greater than the total cost of implementation [27].

2.2.4. Limitations of lean manufacturing

Despite the benefits that can be obtained from implementing lean manufacturing, the lean manufacturing system is not without limitations. Limitations of lean manufacturing are:

1. Vulnerable to unplanned disruptions in the supply chain: A lean system releases working capital, reduces inventories, reduces lead times and improves material flow. However, a lean organisation can become susceptible to disruptions as the leanness in itself makes the organisation vulnerable to events such as equipment breakdown, labour absenteeism, disruptions to material flow, disruptions to transportation routes and supplier failures [35].
2. JIT deliveries can cause congestion in the supply chain. This can lead to delays, pollution, etc. [36].

3. The leanness of an organisation means that highly dynamic conditions cannot be dealt with as the focus on perfection which is a function of particular market conditions at certain periods of time reduces the ability to react to new conditions [32, 37].

2.3. Total quality management

Total quality management (TQM) draws on the teachings of three primary authorities in quality management, namely: Edward Deming, Joseph Juran and Kaoru Ishikawa [4]. TQM defines quality as conformance to customer requirements and quality at an affordable price [38, 39]. The origins of TQM can be traced back to the late 1940s in Japan. Following the destruction of Japan during the Second World War, the Japanese with the help of quality experts from the United States of America (USA) embarked on a process of rebuilding Japanese industries [2, 38].

As of the 1950s and 1960s, goods made in Japan were regarded as junk and unfit for purpose while goods manufactured in the USA where seen as the best. However, this changed during the 1970s and 1980s as Japan became the benchmark for the production of automobiles, consumer electronics, machine tools, heavy machinery, etc. This lead to the USA paying more attention to product quality and striving to develop quality management frameworks. TQM in the USA was born out of the need to catch up with the pace of Japanese manufacture in the 1970s [38]. Table 2.2 shows the ideological differences between Japanese and American businesses in quality management after the Second World War [38].

Table 2.2. Quality attitudes of American and Japanese businesses

USA Perspective	Japanese Perspective
Product quality is close enough	Get it right the first time
We are better than everyone	Always continue to improve as good enough is never good enough
We are the experts so we will dictate to the customers what they want	Understand what the customer wants and give them more

2.3.1. Assumptions and principles of TQM

TQM is governed by certain assumptions and process improvement principles. These are described briefly as follows [4]:

Assumptions

The cost of poor quality (inspection, rework, lost customers, etc.) is greater than the cost of developing processes that produce high quality products and services.

Employees will take initiatives to improve the quality of their work as long as they are provided with the tools and training that are needed for quality improvement and management pays attention to their ideas.

Organisations are systems of highly interdependent parts and the problems central to the organisation cross traditional functional lines.

Quality is the responsibility of the top management.

Process improvement principles

Focus on work processes**:** The quality of products and services is dependent on the quality of the processes in which they are designed and produced. Thus, employees must be trained to assess, analyse and improve work processes.

Analysis of variability**:** Uncontrolled variance in work processes is the primary cause of quality problems. To improve product quality, these must be analysed and controlled by persons directly involved with the daily operations of the work processes.

Management by fact**:** Use systematically collected data at every point during problem solving.

Learning and continuous improvement**:** Commit to continuous improvement as opportunities to develop better methods for carrying out work always exist.

Explicitly identify and measure customer requirements**:** This entails assessing what the customer wants and providing products and services that meet those requirements. The knowledge of customer requirements provides a test for evaluating and considering process changes.

Create supplier partnerships on the basis of quality**:** Create supplier partnerships on the basis of quality rather than solely on price. Also it is recommended organisations work directly with the suppliers of raw materials to ensure that the materials supplied are of the highest quality possible.

Use cross functional teams to identify and solve quality problems**:** The use of cross-functional teams ensures all relevant information and expertise are available when making decisions on system wide problems.

Use scientific methods to monitor process performance and to identify areas for quality improvement**:** These include statistical tools such as control charts, pareto charts, cost-of-quality analysis, etc. Using these tools to collect and analyse data provides improvement teams with fact-based and trust worthy data to use in their decision making.

***Use process improvement techniques that enhance the effectiveness of quality teams*:** These include techniques such as flowcharts, brainstorming, fishbone diagram, etc. The use of these techniques improves the quality of the decision-making process.

The assumptions and process improvement principles briefly described define the core of the TQM system.

2.3.2. Implementing TQM

The TQM system recommends the implementation of quality improvement projects on an organisation wide basis and is usually deployed using quality councils, workforce-level teams and cross-functional teams. The quality council addresses strategic initiatives, the workforce-level teams attend to quality issues associated with routine production activities and the cross-functional teams address quality concerns that require personnel from different departments within the organisation [1, 4].

Unlike the six sigma DMAIC approach to problem solving, there is no such approach specific to TQM. However, the Plan-Do-Study-Act (PDSA) process improvement methodology is associated with the implementation of TQM projects [32]. Like DMAIC, the PDSA methodology provides a roadmap for carrying out improvement projects in a structured manner [40]. The activities conducted at the various steps of the PDSA methodology are outlined in figure 2.2 [40]:

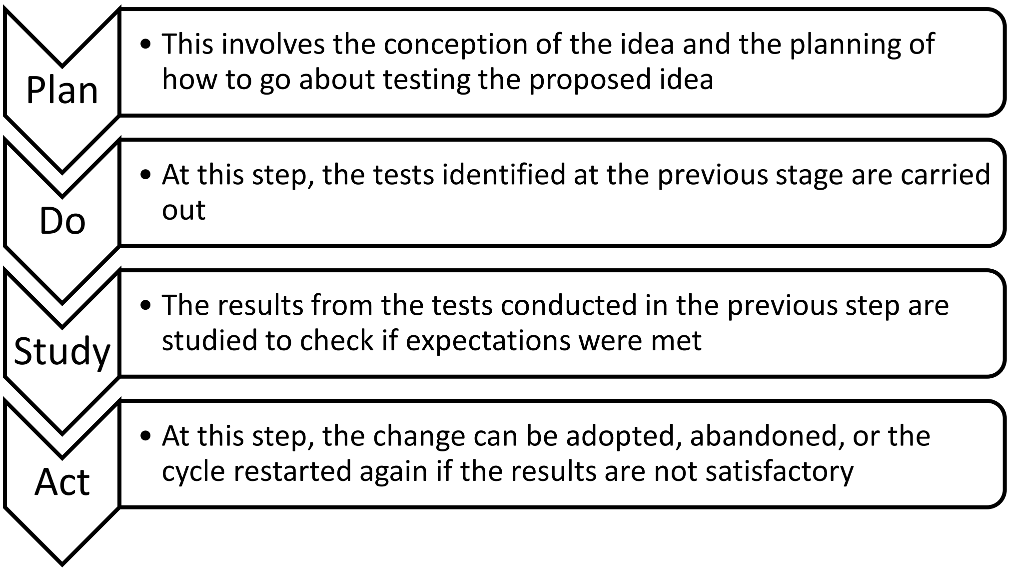


Figure 2.2. Activities conducted at the various steps of the PDSA methodology

The tools and techniques used with TQM are also used with six sigma [4, 5] hence, they are presented with the six sigma system in further sections of this thesis.

2.3.3. Benefits of TQM

Organisations that have successfully embarked on TQM projects have reported on the benefits of such projects. Documented benefits of successful TQM projects include: reduction in scrap and rework costs, reduction in cycle times, reduction of defect rates, reduction in warranty costs, increased throughput, improved customer satisfaction, etc. [9, 38].

2.3.4. Limitations of TQM

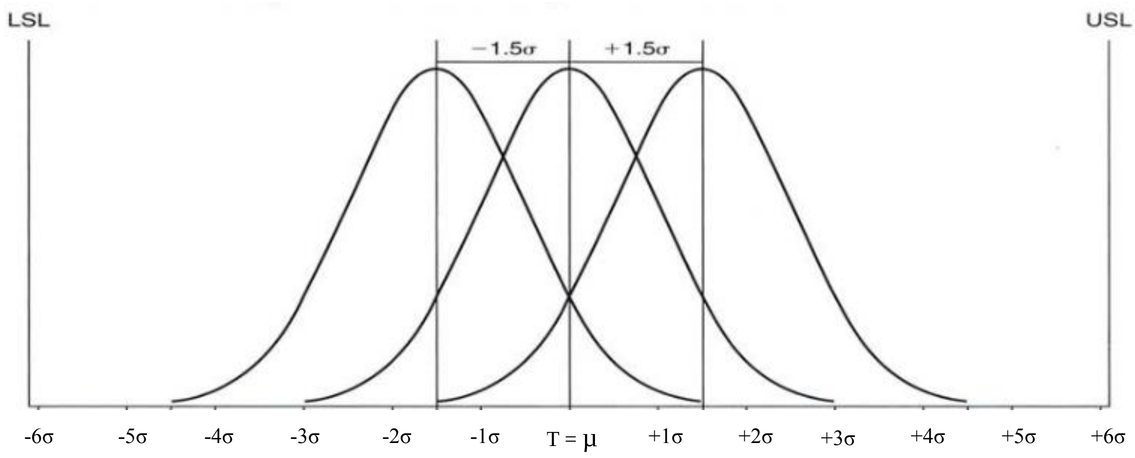
Despite the successful use of TQM by some organisations for process improvement, the system is not without limitations. A major limitation of the TQM system is that the business objectives of TQM projects are general as opposed to specific in the sense that there is no defined metric for justifying the selection of projects. This can affect project selection as well as project support from management and staff [1, 32].

2.4. Six sigma

Six sigma is a statistically based, project oriented, process improvement strategy which aims to improve the quality of a process by minimising the process variability [5, 22, 41]. Six sigma enables companies to use proven statistical methods for achieving and sustaining business excellence [42-44]. Developed in 1986 by Motorola, it has been successfully used by a number of organisations to improve the quality of their business processes [41, 42, 45].

2.4.1. Fundamentals of six sigma

Six sigma centres on reducing variability around specified target values in key process and product quality characteristics to a level in which defects are extremely unlikely. To check if a process is performing up to six sigma standards, the upper specification limit (USL) and lower specification limit (LSL) must be at least six standard deviations from the target mean. This level of quality results in two parts per billion not conforming to specifications. When a process operates at six sigma quality, the six sigma concept allows for a shift in process mean by as much as 1.5 standard deviations on either side of the specification limit to accommodate for process disturbances as no system is truly stable. With this shift in process mean, a six sigma process would produce 3.4 parts per million (3.4 ppm) not conforming to specification. The concept of the shifting mean has been a source of controversy as critics have argued that the concept allows only for accurate process predictions when the drift in the process mean is within 1.5 standard deviations. Though this is true, the six sigma concept is just a way to model the shift in process stability via an approximation of the mean shift [1, 5, 43]. Figure 2.3 graphically presents the six sigma concept [1].



Sigma (σ) level	Percentage inside sigma level	ppm Defective
±1	30.23	697700
±2	69.13	608700
±3	93.32	66810
±4	99.3790	6210
±5	99.97670	233
±6	99.999660	3.4

Figure 2.3. The six sigma concept (Normal distribution with the mean (μ) shifted by 1.5 standard deviations (1.5*σ*) from the target mean (T))

2.4.2. Implementing six sigma

Six sigma belt system

Six sigma is implemented by persons trained in its workings [1, 5]. Following from this training, these personnel receive Belt titles such as [1, 5]:

Green Belt: Green belts receive between 1-2 weeks training. They receive thorough training in core statistical tools and either assists in major projects lead by black belts or lead teams engaged in smaller improvement projects.

Black Belt: Black belts usually have a minimum of four weeks specialised training which could be sometimes spread over a four month period and combined with concurrent work on a six sigma project. They are trained in all six sigma tools (advanced, intermediate and beginner) and are well grounded in the underlying statistical theory in which the tools operate. The main function of a black belt is to lead teams in carrying out improvement projects for the organisation. In most organisations, black belts train green belts and could lead up to three projects at a time.

Master Black Belt: Master black belts are recruited from the ranks of an organisation's black belts and are often involved in training black belts and other master black belts. In most cases, they usually have advanced technical degrees and extensive black belt experience. Master black belts often write and develop training manuals, are heavily involved in the project selection process, and they work closely with the six sigma champion (sponsor). Six sigma champions are normally part of the organisation's executive board. They provide the resources for embarking on the project and ensure that the right projects are selected.

In addition to the green, black and master black belts, six sigma training providers offer yellow and white belt courses to enable organisation that do not have the resources to train green and black belts and also use these personnel for full-time projects as well as cross-functional projects, implement six sigma at a lower cost [46-49].

DMAIC methodology

Six sigma uses a sequential five step approach to improve the performance of an existing process. These are: Define, Measure, Analyse, Improve and Control (DMAIC). The DMAIC roadmap provides a structured approach for executing quality improvement projects. The DMAIC process is described thus [1, 5, 13, 42]:

Define: The define step entails identifying the project opportunity and verifying that it presents a legitimate potential for breakthrough improvement. Pareto charts (graphs which display the frequency of problems in a prioritised order) can be used to identify the improvement opportunity and a project charter can be used to organise the improvement efforts.

A project charter details a description of the project, its scope, start date and anticipated completion date, provides an initial description of primary and secondary project measurement metrics and how those metrics align with operational objectives and corporate goals, critical to quality characteristics that are impacted by the project, potential benefits to the customer, potential financial benefits to the company, team members and their roles, project milestones, and any other additional information of importance to the project completion. Improvement projects might fail when different people have different understandings of what the project is supposed to accomplish. The project charter helps to avoid this by clearly defining the project in a language everybody understands.

Measure: The measure step aims to evaluate and understand the current state of the process. In the measure step, data is collected on the current state of process performance. This provides a base line which can be used to judge how well the process improves from its current state. Such data could include measures of quality, cost, throughput, cycle time, etc. Fundamental to process measurement is determining how much data needs to be collected to allow for a thorough analysis and understanding of current process performance. Data may be collected via historical records or in real-time. Historical records may not always be reliable as they may be incomplete, distorted, or the methods of record keeping may have changed over time. It is advisable and often necessary to collect current data empirically. It is important that a sufficient amount of data is collected to allow for proper process measurement and also any assumptions made during the data collection process must be documented.

Also during the measurement step, the capability of the measurement system is evaluated to ensure that the improvement team are not wasting their efforts trying to solve a non-existent problem in the process when the problem may actually stem from a faulty measurement system. A scientific and quantitative way of performing measurement system analysis is the use of the gage repeatability and reproducibility study (Gage R&R). This entails the use of designed experiments to quantify the accuracy and variation of the measurement system.

Analyse: The analyse step closely examines the data gathered during the measure step to understand the cause and effect relationship of the problem and also to identify any special causes of variability in the system. Under normal working conditions, such variations should not be present. Some examples of special-cause variations might be tool failure, change in operating personnel, etc. The analyse step aims to determine the potential causes of the quality problems which can then be worked on for improvement in the improve step.

Improve: In this step, ways to improve the identified quality problems are set out. The improve step aims to develop a solution to the identified problem and test the solution for its validity. This test is a form of confirmation experiment to evaluate the solution, document the solution, and confirm that the solution attains the desired project goals. Such testing may lead to further refinement of the solution via an iterative activity in which the proposed solution is revised and improved several times to arrive at the most desirable outcome. Before implementing the solution, the risk associated with doing so must be considered and appropriate risk management plans put in place.

Control: In the control step, measures are set in place to control the improved process. Control charts are an effective and important statistical tool used to monitor processes for deviation from the acceptable. When possible, control actions must include control charts of key process metrics to enable monitoring of such metrics and taking action when the process is out of control. When changing to a new process, it is not unusual to find that something has gone wrong. Hence, a transition plan must accommodate for a validation check several months after the project completion to ensure that the gains realised from the improved process are still in place.

Figure 2.4 presents the objectives of the different steps of the six sigma DMAIC process [1, 5].

DMAIC steps	**Objectives**
Define	• Identify the critical-to-quality issue • Develop a project charter • Document the process
Measure	• Validate measurement systems. • Determine the measurement characteristic of critical-to-quality issue • Determine process baseline performance (process sigma level and process capability analysis)
Analyse	• Identify and analyse the sources of variation • Identify and verify potential root causes
Improve	• Identify potential solutions. • Implement the chosen solution
Control	• Develop and implement process control plans

Figure 2.4. Objectives of the different steps of the six sigma DMAIC process

2.4.3. Six sigma tools and techniques

In implementing six sigma, a variety of tools and techniques are used at the different steps of the six sigma DMAIC methodology. Some of the common tools and techniques are [1, 5, 50, 51]:

Pareto chart: A pareto chart is a vertical bar graph which displays the frequency of problems in a prioritised order so as to enable identification of the most significant problem.

Flow chart: A flow chart is a graphical tool used to document the flow of a process. It is used to symbolically represent the sequence of operations in a process.

Gage Repeatability and Reproducibility study: Repeatability can be defined as the variations in measurement from one measurement instrument when used several times by one appraiser to measure the identical characteristic of the same part. A measurement system is repeatable if its variability is consistent. Reproducibility is the variation in the average measurements taken by different appraisers, using the same measuring instrument to measure the identical characteristic of the same part. A measurement system is said to be reproducible when different appraisers produce consistent results.

Gage repeatability and reproducibility studies (Gage R&R) are used to determine the capability of a measurement system by determining the amount of variability in the collected data that can be attributed to the measurement system. If the variation of the measurement system is small compared to the process variation, then the measurement system is considered capable.

Process capability analysis: A process capability index is a metric used to indicate the performance of a process relative to requirements. A commonly used measure of the process capability is the capability index C_p. C_p measures the potential capability of a process and is calculated using the following formula:

$$C_p = \frac{USL - LSL}{6\sigma}$$

USL and LSL represent the upper and lower specification limit and σ represents the standard deviation.

Another capability index is the C_{pk} index. This measures the realised process capability relative to the actual operation and is given by the formula:

$$C_{pk} = \text{minimum}\ (\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma})$$

μ represents the process mean. When $C_{pk} > 1$, the process is deemed capable; and when $C_{pk} < 1$, the process is deemed incapable. C_{pk} is regarded as a more practical measure of process capability than C_p.

Another metric used to assess the performance of a process is the DPMO (defect per million opportunities). DPMO represents the proportion of the process output outside the specification limits multiplied by one million. For example, if 2.5% of the process output is outside the specification limits, then the DPMO of the process is calculated as:

$$DPMO = \frac{1{,}000{,}000 * 2.5}{100} = 25{,}000$$

Using the DPMO, the performance of a process in terms of its sigma level can be obtained.

Cause and effect diagrams: The cause and effect diagram graphically presents the potential causes of a given effect. It is also known as the Ishikawa diagram or a fishbone diagram. This diagram assists in brain storming and enables an improvement team to identify and graphically display the root causes of a problem.

Design of experiments (DOE): DOE is used to understand how several factors affect the output of a process and to determine the optimal combination of factor settings. They can be applied to a physical process or to a computer simulation of a process.

Statistical process control charts: A statistical process control chart is a graphical tool used to determine if a process is stable. By comparing the process output against an upper control limit (UCL) and a lower control limit (LCL), it can be determined if the process is in statistical process control or not. Points above or below the UCL and LCL indicate the process is not in control due to the presence of special causes of variation. When this is not the case, it is assumed that only common causes of variation are present and the process is under control.

Table 2.3 presents some of the common tools and techniques used at the different steps of the DMAIC methodology [1, 5, 22, 52].

Table 2.1. Common tools and techniques used with the six sigma DMAIC methodology

Step	Tools and techniques
Define	Project charter, Process maps, Flow charts, Benchmarking, Pareto charts
Measure	Process capability analysis, Statistical process control charts, Process control plans, Gage R&R study
Analyse	Hypothesis tests, Brainstorming, Regression analysis, Failure mode and effects analysis (FMEA), Process maps, DOE, Simulations, Statistical process control charts
Improve	DOE, FMEA, Force field diagrams, Simulations
Control	Statistical process control and Process control plans

2.4.4. Other elements of six sigma

DFSS: DFSS is an acronym for Design For Six Sigma. It is used to design processes, products and services that are six sigma capable. DFSS is implemented using a variation of DMAIC called DMADV, an acronym for Define, Measure, Analyse, Design and Verify. Many of the statistical tools used with DMAIC are also used with DFSS. DFSS is focused on improving business results via an increase in sales revenue generated from new products and services and finding opportunities to apply existing products [1, 20].

2.4.5. Benefits of six sigma

Six sigma has been successfully used by many organisations to improve the performance of their business processes. Some documented benefits of six sigma implementation include [13]: Reduction and elimination of defects, improved productivity, increased profit, improved customer satisfaction, etc. Rodin and Beruvides [53] analysed six sigma projects conducted over a period of seven years at a government contractor in the USA and their analysis showed that six sigma projects where cost effective, achieving a benefit to cost ratio of 2.66.

2.4.6. Limitations of six sigma

A limitation of the six sigma methodology is the assumption of a shift in process mean by at most 1.5 sigma. This is so as a shift of more than 1.5 sigma may result in the calculation of erroneous defect rates [1, 54].

2.4.7. A focus on six sigma

Table 2.4 summarises the characteristics of the lean manufacturing, TQM and six sigma quality initiatives. Table 2.4 is based on the overview of lean manufacturing, TQM and six sigma provided by this research and the work of Andersson et al [32].

Table 2.4. Characteristics of lean manufacturing, TQM and six sigma

	Lean manufacturing	**TQM**	**Six sigma**
Characteristics			
Principle	Remove waste (NVA)	Reduce process variation	Reduce process variation
Process objective	Improve process flow	Minimise defects	Minimise defects
Business objective	Reduce lead time	Improve customer satisfaction	Save money
Approach	Project management	Project management	Project management
Problem solving methodology	N/A	PDSA	DMAIC or DMADV
Tools and techniques	Non-statistical tools and techniques	Statistical and non-statistical tools and techniques	Statistical and non-statistical tools and techniques
Documented benefits	Increased supply chain speed, reduced lead times, release of working capital	Reduction of defect rates, reduction in cycle times, increased productivity, improved customer satisfaction	Reduction and elimination of defects, increased productivity, increased profit, improved customer satisfaction
Limitations	Vulnerable to unplanned disruptions in the supply chain. Congestion caused by JIT deliveries can lead to pollution, delays, etc. Reduces ability to react to change in highly dynamic conditions	Business objectives are broad as opposed to specific	The 1.5 sigma shift in process mean for all processes is unrealistic and may lead to the calculation of erroneous defect rates

The overview of lean manufacturing, TQM and six sigma provided by this research (summarised in table 2.4) has shown that while TQM and six sigma improve process quality via variation reduction and lean manufacturing improves process quality via waste elimination, each of these quality initiatives have been successfully used to attain their respective process and business objectives. However, this research focuses on six sigma implementation in SMEs due to two reasons. These are:

1. ***Its focus on the financial impact of the project****:* With six sigma, there is a strong focus on projects that positively impact the financial performance of a business. The focus on the financial impact of projects draws strong management leadership and support to project activities. This minimises the risk of project failure due to insufficient allocation of resources by top management [13].

2. ***The infrastructure of quality personnel it creates to lead and deploy six sigma****:* The hierarchical structure of quality personnel (Champions, Master Black Belts, Black Belts, Green Belts, Yellow Belts and White Belts) employed by six sigma helps to control and coordinate work across organisational levels. This ensures that project activities match the overall business aim [14, 15].

Following the introduction of six sigma in the 1980s, the lean manufacturing and six sigma initiatives have been combined into a single initiative known as lean six sigma (LSS) to enable organisations benefit from the strengths of both initiatives [55]. LSS is deployed using the six sigma belt system and is implemented with lean manufacturing and six sigma tools and techniques using the six sigma DMAIC methodology [56]. Though the term six sigma is generally used in this thesis, the discussions on six sigma in SMEs in this thesis are not particular to six sigma but also LSS in SMEs.

2.5. Six sigma in SMEs

2.5.1. Definition of SMEs

The definition of SMEs varies in different countries [57, 58]. In defining SMEs, this research adopts the European Commission (EC) definition of SMEs [59]. Table 2.5 shows the EC definition of SMEs. The definition applies to firms which are not part of a larger group.

Table 2.5. EC definition of SMEs

Company category	Staff headcount	Turnover	Balance sheet total
Medium-sized	<250	≤ € 50 million	≤ € 43 million
Small-sized	<50	≤ € 10 million	≤ € 10 million
Micro-sized	<10	≤ € 2 million	≤ € 2 million

Summarily, the EC defines SMEs as businesses that have a workforce of less than 250 persons and have an annual turnover of at most €50million. SMEs cover a wide range of activities and products and range from high-tech firms to small retail shops [60].

2.5.2. Why process improvement in SMEs?

A study by Sawang and Unsworth [61] on the adoption of innovation initiatives in SMEs and large organisations in Australia provides an insight into factors that may affect the adoption of process improvement initiatives such as six sigma in these organisations. They define product innovations as new products or services introduced to meet customer needs and, define process innovations as new task specifications, new elements, new work and information flow mechanisms, new equipment and new materials used to render a service or produce a product. Innovation initiatives investigated in their research included six sigma. By reviewing the literature on innovation adoption in firms and conducting preliminary case studies, they state that because the challenges of adopting innovation initiatives (financial requirements and effort required) may be too great for SMEs, SMEs tend to display a lack of willingness to adopt innovation except it becomes nearly imperative due to external pressure or there is an availability of in-house skill base to execute the project (non-financial readiness). External pressure refers to opinions expressed by external stake holders such as customers, competitors, suppliers etc. and how much this opinion affects the way the company runs its business.

On the other hand, they state that large organisations were less likely to be affected by scarce financial resources, non-financial readiness and external pressures in adopting innovation initiatives compared to SMEs. Instead environmental factors such as market environment dynamism, competitiveness and the opportunity to innovate were more defining factors. Market environment dynamism refers to how quickly advances in technological developments spring up in such markets and the opportunity to innovate refers to the rate at which opportunities to innovate arise in the industry.

Based on the literature review and preliminary case studies, they proposed and tested a number of hypothesis using case studies of a larger number of companies. The hypotheses are as follows:

1. That pressure would positively affect innovation adoption in SMEs but not large organisations.
2. That scarce financial resources and non-financial readiness would affect innovation adoption in SMEs to a greater extent as compared to large organisations.

3. That the market environment (dynamism, competitiveness and opportunity to innovate) would affect innovation adoption in large organisations but not SMEs.

The findings from the case studies showed that SMEs were not significantly affected by a lack of financial resources in adopting innovation initiatives. This can also be interpreted as SMEs not being willing to spend money adopting innovation initiatives especially when they do not see a compelling need to do so and are not sure of the benefit. However, they would spend money to adopt such initiatives if there is a compelling need to do so (external pressure). On the other hand, when SMEs possess the in-house ability (non-financial readiness) to pursue innovation initiatives, even if they are not sure of the rewards or don't have a compelling need to do so, they tend to adopt these initiatives. In addition to scarce financial resources, SMEs where not significantly affected by the market environment.

As for large organisations, it was discovered that the adoption of innovation initiatives was not significantly affected by external pressure, scarce financial resources, non-financial readiness and competition. What mattered to these organisations were the opportunity to innovate and the dynamism of the market environment. Hence Sawang and Unsworth concluded that large organisations were more likely to pull innovation while SMEs were most likely to have innovation pushed unto them. These findings will or may vary in other geographical regions however, the research identifies factors that may influence the adoption of quality initiatives such as six sigma in SMEs.

The globalisation of markets and the growing inter-dependence of economies are shaping national and international competitive environments. To compete in an increasingly global market, organisations need to establish mechanisms that enable them to commence and sustain business improvement efforts when needed. One way to attain competitive advantage is by improving the quality of goods and services being produced [9, 62]. An organisation that can delight its customers by improving and controlling the quality of its goods and services has the potential to dominate its competitors [1].

Large organisations competing in an increasingly global market tend to rely on subcontracting certain jobs to other organisations, most of which will be SMEs. As such, SMEs act as subcontractors to large organisations. As the success of the quality programmes of large organisations is dependent on the quality of goods and services provided by their suppliers, it is logical for large organisations to encourage the use of quality improvement initiatives amongst their suppliers to ensure the supply of high quality products and services [9, 12]. Besides, competition means that for SMEs wanting to become suppliers to large organisations, the increasing demand on quality by large organisations puts pressure on SMEs to consider using proven quality improvement initiatives [9].

2.5.3. Challenges facing the use of six sigma in SMEs

The use of formal quality improvement methods like six sigma in SMEs has been a subject of debate due to resource constraints in SMEs [62, 63]. In assessing the impact of a six sigma programme in a Finnish SME, Silen [64] cited the lack of resources as one of the reasons commonly put forward by SMEs for not using six sigma. Other reasons cited by Silen include: suitable only for large organisations, inapplicable to SME processes and businesses, complex statistics, too expensive to implement, lack of time, complex accompanying theory, presence of an existing quality system and offers nothing new.

Thomas and Barton [16] in developing a six sigma strategy for SMEs cited the lack of resources as well as the complexity of the experimental design methods used with six sigma as factors preventing SMEs from taking the initiative to implement six sigma. Reporting on a study carried out to investigate the use of six sigma within U.K manufacturing SMEs, Antony et al [17] also concluded that SMEs did not have the resources to implement six sigma projects. Analysing the implementation of structured quality systems within Welsh SMEs, Thomas and Webb [63] reported the lack of human and financial resources within SMEs as factors impeding the use of these systems. Lee et al [65] in developing a six sigma readiness model for Chinese enterprises cited intellectual and financial limitations as some of the factors impeding six sigma adoption.

Timans et al [66] investigated the use of Lean Six Sigma (LSS) in Dutch manufacturing SMEs and identified: internal resistance, availability of resources, changing business focus and a lack of leadership as the most common factors impeding the use of LSS for process improvement. Furthermore, Deleryd et al [21] in investigating the use of statistical methods for process improvement in SMEs reported that SMEs lack resources in the form of time and personnel to use these methods and further elaborated on this by stating that as SMEs tend to have a lean organisation, they find it difficult to appoint a co-ordinator or facilitator for quality improvement activities.

In SMEs, employees are occupied with daily business activities and have limited time for extra activities. The reduced workforce means that employees are responsible for different functions with little backup. In addition, as implementing quality improvement initiatives requires the management and staff to dedicate a significant amount of time to the task, this can cause problems in SMEs as it lacks the economies of scale enjoyed by large organisations [9].

Despite these concerns, there is a growing recognition that six sigma is applicable to companies of all sizes [67]. A study conducted by Antony [62] on the view point of some of the leading academics and practitioners of six sigma showed that many of them believed six sigma can be used by SMEs and some of them drawing from previous experiences stated that the results would be quicker compared to large organisations. In SMEs, the reduced management layers and reduced staff hierarchy are likely to increase the speed of improvement activities compared to large organisations. The reduced layers of staff and management means that the power of decision making does not depend on extensive staff and management hierarchies. These along with informal operating procedures bring about faster communication lines and a short decision making chain thus, increasing the speed of improvement activities. Furthermore, the reduced human resources in SMEs may lead to a low resistance to change thus, enhancing improvement activities [9, 68].

In implementing six sigma, SMEs can minimise costs by investing in yellow and white belt training as opposed to green and black belt training and, use reasonably priced six sigma training providers [47]. Six sigma implementation strategies can range from across the whole business to using an approach in which specific problems are targeted thus, it does not require massive investments in companywide training before significant benefits can be achieved [67].

Literature exists on how the six sigma DMAIC methodology has been successfully used to improve the process quality of some SMEs [16, 64, 69-71]. Silen [64] reported on how six sigma was used within a plastics manufacturing SME in Finland to improve process yield and reduce defects across different processes. In addition to the improved product yield and reduced defect rate, the projects resulted in improved company profits. Thomas and Barton [16] presented a case where six sigma was used to resolve a critical-to-quality issue faced by a company which designs and manufactures office seating and furniture. The project resulted in gains of £60,000 compared to the project cost of £5,000. The successful use of statistical tools and techniques in the course of the project enabled the company to accept the use of these tools for process improvement and also become more technical in their approach to problem solving. Kaushik et al [69] presented a case where six sigma was used to improve the process quality of a bicycle chain manufacturing unit. Application of six sigma to the process improved the process performance from 1.40 sigma to 5.46 sigma, and resulted in a process saving of Rs 0.288 million per annum. Desai [70] reported on the successful use of six sigma to improve customer delivery time and turnover of a manufacturing firm involved in the production of a range of products. The project enabled an improved understanding of the problem both qualitatively and quantitatively, and enabled a structured approach to improvement through effective analysis of the root causes of the problem. Furthermore, Reddy and Reddy [71] presented a case where six sigma was used to improve the process performance of a bearing manufacturing facility. The project reduced the production of defective bearing rings from 2.7% to 0.65%, reduced the production of defective bore diameters from 65% to 35% and, increased the process sigma level from 4.04 to 4.44. These examples demonstrate how six sigma has been used to improve the process performance of SMEs and also show the benefits that can be obtained from the use of the DMAIC approach to problem solving.

2.5.4. Critical success factors of six sigma implementation

The success of six sigma projects is dependent on various factors [72]. Coronado and Antony [72] provided a well defined list of factors key to the successful implementation of six sigma. Some of these factors are discussed as follows [72]:

Management commitment and involvement: The success of business improvement activities is dependent on the commitment and involvement of top management. Although commitment is necessary, it must be followed by active involvement. Top management should be actively involved in quality improvement activities by devoting considerable personal energy, participating in improvement projects and, participating in the creation and management of process management systems. When this is not the case, the importance of the quality initiative may be undermined and employees may lack the motivation to conduct improvement projects.

Communication: Communicating project information is important as this will help employees understand the need for the project. A communication plan should describe what should be communicated, by whom and how often it is communicated. Also, it is advisable to publish successful and unsuccessful project results. The successful project results will help to motivate employees while the unsuccessful project results will help other projects learn from mistakes.

Training: Training enables people to better understand the fundamentals, tools and techniques of six sigma. The belt system used by six sigma provides a training structure for employees and identifies the key roles of the persons directly involved in a six sigma project. Though the training associated with the six sigma belt system provides the trained personnel with the knowledge to improve process quality, it is important that persons trained in six sigma constantly seek knowledge from outside to reinforce what they already know. Persons trained to a certain belt level in six sigma should share their knowledge with untrained persons to improve the skill base of the company. Training improves the comfort level of employees.

Organisational infrastructure: The infrastructure of an organisation can aid in the successful completion of six sigma projects. Such infrastructure can include a long term strategy and the availability of resources to embark on six sigma. By having a long term strategy, projects which the gains can be realised fast can be conducted first. The successful results from those projects can be used to improve the morale of employees before embarking on more difficult projects.

It is important that organisations have the resources needed to conduct six sigma projects. Organisations that have succeeded in implementing six sigma have made a significant amount of investment. Some of these may include: consultancy costs, training costs, cost of implementing a proposed improvement plan, etc. Thus, in conducting six sigma projects, organisations need to make sure that the cost of not improving the process will be greater than the cost of improving the process.

Furthermore, the organisational infrastructure should provide a system which enables proper project selection and definition, provides excellence and consistency in the training regime and integrates financial systems with project activity to aid in proper financial evaluation of completed projects.

The complete list of factors presented by Coronado and Antony [72] is presented in table 2.6.

Table 2.6. Critical success factors of six sigma implementation

	Critical success factors
1.	Management involvement and commitment
2.	Communication
3.	Training
4.	Organisational infrastructure
5.	Culture change
6.	Linking six sigma to business strategy
7.	Linking six sigma to customers
8.	Linking six sigma to human resources
9.	Linking six sigma to suppliers
10.	Project management skills
11.	Understanding six sigma tools and techniques
12.	Project prioritisation and selection

In analysing six sigma implementation in SMEs, Kumar [73], Antony et al [17], Brun [74] and, Timans et al [66] investigated the importance of six sigma critical success factors in SMEs. The critical success factors investigated across the four studies included those identified by Coronado and Antony [72]. Kumar's work [73] was based on a single SME in the UK, the work of Antony et al [17] was based on multiple SMEs in the UK, Brun's work [74] was based on the opinion of six sigma experts in Italy and, the work of Timans et al [66] was based on multiple SMEs in the Netherlands. The results of their findings are presented in table 2.7.

Table 2.7. Importance of six sigma critical success factors in SMEs

Rank	**Study by Kumar [73]**	**Study by Antony et al [17]**	**Study by Brun [74]**	**Study by Timans et al [66]**
1	Management involvement	Management involvement and participation	Management involvement and commitment	Linking LSS to customers
2	Linking six sigma to customers	Linking six sigma to business strategy	Linking six sigma to business strategy, cultural change	Vision and plan statement
3	Communication, Cultural change, Education and training, Vision and plan statement	Linking six sigma to customers	Linking six sigma to customers, communication	Communication, Management involvement and participation, Linking to business strategy

Table 2.7. (Continued)

Rank	Study by Kumar [73]	Study by Antony et al [17]	Study by Brun [74]	Study by Timans et al [66]
4	Project prioritisation and selection, Understanding of six sigma methodology, Linking six sigma to suppliers, Linking six sigma to business strategy	Organisational infrastructure	Project management skills, Understanding tools and techniques, Project prioritisation and selection	Understanding of LSS Project management skills
5	Organisational infrastructure	Understanding of six sigma methodology	Education and training	Organisational infrastructure
6	Linking six sigma to employees, Project management skills	Project prioritisation and selection	Organisational infrastructure and culture, Linking six sigma to human resources	Project prioritisation and selection
7		Training	Linking six sigma to suppliers	Cultural change, Education and training, Linking six sigma to suppliers
8		Project management skills		
9		Cultural change		
10		Linking six sigma to suppliers		
11		Linking six sigma to employees		

Table 2.7 shows that management involvement was considered the most important factor across the studies of Kumar [73], Antony et al [17] and Brun [74] while linking LSS to customers was considered the most important factor in the study conducted by Timans et al [66]. In addition to management involvement, linking six sigma to business strategy and linking six sigma to customers were common among the top four factors across all studies. On the other hand, linking six sigma to employees and linking six sigma to suppliers were common among the three least important factors across all the studies. The studies by Kumar [73], Antony et al [17], Brun [74] and Timans e al [66] show the importance of six sigma critical success factors and can serve as a guide for SMEs about to start six sigma or in the process of using six sigma.

2.6. A focus on the Design of Experiments (DOE)

The challenges of six sigma implementation in SMEs brought about by resource constraints have led various researchers to develop models/frameworks to aid SMEs implement six sigma with minimal cost. Kumar et al [75] developed a framework to enable SMEs run six sigma programmes at a low cost. The framework developed by Kumar et al [75] provides SMEs with a detailed roadmap on how to successfully deploy six sigma across the organisation while minimising costs at the same time. Thomas and Barton [16] developed a six sigma model to aid SMEs implement six sigma with lower costs by modifying existing tools and techniques. Thomas and Barton’s six sigma model [16] provides SMEs with a step by step approach to resolve quality problems using certain tools and techniques in an economic manner. As an extension to the six sigma model developed by Thomas and Barton [16], Thomas et al [76] developed an LSS model to aid SMEs minimise resources when implementing LSS. This model provides SMEs with a step by step approach to resolving quality problems using lean manufacturing and six sigma tools and techniques while minimising costs at the same time.

In addressing the challenges of six sigma implementation in SMEs brought about by resource constraints, this study will focus on developing an experimental plan (a DOE technique) to aid SMEs minimise resource usage when carrying out designed experiments. The studies of Kumar [73], Antony et al [17] and Timans et al [66] (discussed previously in section 2.5.4) investigated the use of DOE and other six sigma tools and techniques in SMEs. Table 2.8 presents the ratings of tools and techniques from these studies based on usage and usefulness.

Table 2.8. Ratings on the usage and usefulness of six sigma tools and techniques in SMEs

	Study by Kumar [73]		**Study by Antony et al [17]**		**Study by Timans et al [66]**	
Tools and techniques	**Usage**	**Usefulness**	**Usage**	**Usefulness**	**Usage**	**Usefulness**
Process mapping	3.8	3.7	4.438	4.600	3.36	3.57
Project charter	n/a	n/a	3.857	3.500	3.36	3.59
Cause and effect diagram	3.6	3.8	4.188	4.333	3.40	3.69
Histogram	3.5	3.0	4.125	4.357	3.56	3.70
Scatter plot	3.4	3.2	2.333	2.462	2.53	2.97
Run charts	4.3	3.8	3.111	4.200	3.13	3.41
Control charts	4.6	4.3	3.267	4.154	3.03	3.70
ANOVA	3.4	3.6	3.429	3.538	2.79	3.27
Regression analysis	2.2	3.4	3.4	3.167	2.48	3.03

Table 2.8. (Continued)

	Study by Kumar [73]		Study by Antony et al [17]		Study by Timans et al [66]	
Tools and techniques	**Usage**	**Usefulness**	**Usage**	**Usefulness**	**Usage**	**Usefulness**
DOE	3.2	3.8	3.071	3.230	2.44	3.03
Taguchi methods	3.0	3.2	2.846	3.100	2.09	2.50
Measurement system analysis	3.7	3.7	2.700	3.500	2.76	3.13
Non-parametric tests	n/a	n/a	2.000	2.333	n/a	n/a
Hypothesis tests	3.2	3.0	1.867	3.571	2.28	2.67
Quality function deployment	n/a	n/a	3.273	3.889	2.29	2.59
FMEA	n/a	n/a	3.938	4.200	3.31	3.53
Process capability analysis	3.2	3.9	3.188	4.231	2.07	3.21
Affinity diagram	n/a	n/a	2.400	2.333	2.21	2.80
Benchmarking	n/a	n/a	3.067	3.714	2.83	3.07
Quality costing analysis	n/a	n/a	3.000	3.667	3.21	3.65
SIPOC diagram	n/a	n/a	3.286	3.167	3.45	3.59

The usage and usefulness of the tools and techniques were rated from 1 to 5 on a Likert scale. For usage, the scale was in ascending order from 1 (indicating ‘never been used’) to 5 (indicating ‘used continuously’). For usefulness, the scale was in ascending order from 1 (indicating ‘not useful’) to 5 (indicating ‘extremely useful’). The studies by Kumar [73], Antony et al [17] and Timans et al [66] show that the less complex tools and techniques were used more frequently than more complex tools and techniques. Tools and techniques such as histograms, cause and effect diagrams, process mapping, control charts had an average usage value greater than 3.3 while more complex tools and techniques such as DOE and Taguchi methods had an average usage value less than 3.3. With regards to the usage and usefulness of DOE, it can be concluded that these methods are not frequently used but are considered to be important in terms of their usefulness to resolve complex quality problems. Examples of the use of DOE for process improvement in SMEs are:

Identification of process settings that minimise the premature loosening of chair arms as well as reduce the variability of the joint strength of the chair arms [16].

Identification of process settings that minimise bush diameter variation in a bicycle chain manufacturing process [69].

The use of statistical methods for process improvement is not limited to SMEs or large organisations as these methods are needed when they are the only means to an objective analysis of data. This means that a deeper understanding of the concept of variation, identification of the sources of variation and the minimisation of variation are crucial to improve business performance in SMEs [21]. Thus statistical methods used with six sigma such as process control theory, experimental design techniques, etc., have a major part to play in improving the process quality of SMEs [16]. By focusing on developing an experimental plan to aid SMEs minimise resource usage when using designed experiments, this research will contribute to addressing the challenges of six sigma implementation in SMEs brought about by resource constraints.

2.7. Summary

This chapter presented an overview of three commonly used quality improvement initiatives in the manufacturing industry, put forward reasons why this study focused on six sigma implementation in SMEs, identified resource constraints as a factor impeding the use of six sigma in SMEs and outlined the rationale behind developing an experimental plan to aid SMEs minimise resource usage when conducting designed experiments.

The review on the use of statistical methods for process improvement in SMEs revealed that SMEs lack resources in the form of time and personnel to use these methods due to their reduced workforce. The reduced workforce in SMEs means that employees are responsible for different functions with little backup hence, have limited time for extra activities [9]. As using statistical methods require staff to dedicate a significant amount of time to the task [21], this can cause problems in SMEs as it lacks the economies of scale enjoyed by large organisations [9].

In developing an experimental plan to aid SMEs conduct designed experiments with minimal costs, this research will focus on experiments in which the factors to be studied are represented in three levels (3-level experiments). These experiments are useful in situations where the process settings are represented in three levels or the experimenter wishes to analyse the process performance based on three factor levels. Thus, they have a role to play in improving process quality.

The next chapter introduces the subject of DOE and reviews experimental plans commonly used to conduct 3-level experiments.

Chapter 3

The Design of Experiments

3.1. Introduction

This chapter introduces the subject of DOE by reviewing the role of designed experiments, history of designed experiments, principles of designed experiments and, experimental design strategies. Following from this, experimental plans commonly used for 3-level experimentation are reviewed in further sections of this chapter.

3.2. The role of designed experiments

An experiment can be defined as a test in which the factors believed to have an influence on the output of a process are identified and manipulated according to a predetermined plan so as to study their effect on the process output. The output of a system can be influenced by several factors some of which are controllable and some uncontrollable. Thus, when possible, an experimental plan should include provisions for dealing with uncontrollable factors [5, 23, 25].

Experiments are conducted for various reasons. These may include:

1. To determine which factors have the most influence on the process output [77],
2. To determine the settings of the influential factors that optimise the process output [78],
3. To determine the settings of the influential factors that minimises the variability in the process output [79, 80],

4. To determine the settings of the influential factors that minimise the effect of uncontrollable factors on the process output [79, 80].

The statistical design of experiments refers to the process of planning an experiment to enable appropriate data to be collected and analysed by statistical methods thus, resulting in valid and objective conclusions. In cases where the problem involves data that are subject to experimental errors, statistical methods are the only objective approach to analysis. All experiments are designed experiments however, some are better designed than others [25]. Examples of the use of statistically designed experiments for process improvement include:

Reduction of wire induced waviness in a grinding process for wire sawn silicon wafers [81],

Optimisation of accelerated solvent extraction of cocaine and benzoylecgonine from coca leaves [82],

Optimisation of biodiesel production [83],

Improving the quality of a wire bonding process [84].

3.3. DOE terminology

To enhance the understanding of subsequent material in this thesis, DOE terminology used in this study are defined as follows [23, 25, 85, 86]:

Factor: A variable believed to have an influence on the output of a process.

Response: The output from the process under investigation.

System: A process under investigation.

Factor setting or level: The specific value which a factor is set at.

Main effect: The individual effect of each factor in an experiment.

Interaction effect: The combined effect of two or more factors in an experiment.

Main effect plot: Main effects can be presented graphically using main effect plots which show the average of all the responses at each level of a factor connected by a line.

Interaction effect plot: Interaction effect plots display the average response at each of the combinations of the factors. The optimal factor settings of a process can be determined using main and interaction effects plot by identifying the factor settings with the most desired average response

Aliasing: The combination of two or more factor effects into a single effect. Such effects are said to be aliased.

Treatment combination or Experimental run or Run: A combination of the settings of several factors.

Experimental plan or experimental design: A given arrangement of experimental runs.

Design resolution: A characteristic which identifies the degree to which factor effects in an experimental design are aliased. The most prevalent resolution types are defined as follows:

> *Resolution III designs*: Main effects are aliased with two-factor interactions.
>
> *Resolution IV designs*: No main effects are aliased with two-factor interactions, but two-factor interactions are aliased with each other.
>
> *Resolution V designs*: Main effects and two-factor interactions are not aliased with other main effects or two factor interaction but two-factor interactions are aliased with three factor interactions.

Orthogonality: Two factors are orthogonal if the number of runs of all their level combinations is the same. An experimental plan is orthogonal if all pairs of its factors are orthogonal.

Degree of Freedom: The degrees of freedom in a set of data are the amount of information needed to estimate the values of unknown parameters and estimate the variability of these parameter estimates. The number of degrees of freedom associated with a factor and an experimental plan is equal to one less the number of factor levels and experimental runs respectively.

Experimental error: Unexplained variation in replicated experimental runs.

Screening design: An experimental design that identifies the most important factors from a large number of potential factors in a system.

Analysis of variance (ANOVA): A mathematical process for separating the variability of experimental effects into assignable causes and setting up significance tests.

Sum of squares: A measure of variation from the mean. In ANOVA, the sum of squares of the experimental error and other factor effects (main, interaction and curvature effects) represent the variation in the system response that can be attributed to these effects.

3.4. Brief history of experimental design

The history of experimental design can be traced back to Ronald A. Fisher's work on agricultural experiments in the 1920s. Fisher's initial experiments (randomised block designs) were concerned with determining the effect of various fertilisers on the yield of various plants. Fisher introduced statistical thinking and principles into designing experiments. His work constituted the basis of all subsequent developments in experimental design and led to the birth of the concept of factorial analysis. Fisher's experiments were a scientific breakthrough from an older method of varying one factor at a time [79].

To facilitate industrial experimentation, Box and Wilson introduced the concept of Response Surface Methodology (RSM). RSM utilises sequential experiments to determine the optimum operating conditions of a process or to determine the region of the factor space in which operating requirements are satisfied [87]. Examples of the use of RSM include:

Describing the performance of coated carbide tools when turning AISI 1045 steel [88]

Optimisation of the critical medium components for Pullulan production by Aureobasidium pullulans FB-1 [89]

Early applications of experimental design focused on optimising the average value of a product characteristic. Taguchi went a step further by focusing on minimising the variation around a target mean of a product characteristic. Taguchi developed on-line quality improvement techniques to improve product quality during production and introduced off-line quality improvement techniques to improve product quality at the design stage. These techniques are collectively known as Taguchi methods [79, 90]. Though the statistical concept behind Taguchi designs have been controversial [91], Taguchi designs have been used successfully to improve the performance of manufacturing processes [92]. Examples of the use of Taguchi designs for process improvement include:

Design optimisation of cutting parameters for turning operations [93]

Optimisation of a die casting process [94]

Design optimisation of aluminium recycling process [95]

3.5. Principles of experimental design

In conducting statistically designed experiments, it is recommended that when possible, three basic principles be adhered to. These are described as follows [23, 25, 96]:

Blocking: Blocking is used to improve the precision of comparisons among factors of interest. This design technique is often used to reduce or eliminate the variability transmitted from factors that may influence the experiment but are of no interest to the experimenter. A block generally consists of a set of relatively homogeneous experimental conditions.

Randomisation: Randomisation involves randomly determining the allocation of experimental material and the order in which the experimental runs are to be performed. In using statistical methods, it is required that the observations (or errors) be independently distributed random variables. This assumption is usually made valid by randomisation. Properly randomising an experiment assists in averaging out the effects of extraneous factors that may be present.

Replication: Replication means an independent repeat of each experimental run. There are two important properties of replication. The first allows the experimenter to obtain an estimate of the experimental error which is used as a basic unit of measurement for determining whether observed differences in the data are really statistically different. Secondly, replication allows the experimenter to obtain a more precise estimate of the true mean response for one of the factor levels in the experiment if the sample mean is used to estimate this parameter.

3.6. Strategies of experimentation

The best-guess approach: The best-guess approach is frequently used in practice by engineers and scientists. This strategy works reasonably well when the experimenter has a great deal of technical or theoretical knowledge of the system being studied. However, there are some disadvantages associated with this approach. Firstly, if the initial best-guess does not produce the desired result, another guess has to be taken to determine the correct combination of factor settings. This may continue for a prolonged period of time without any guarantee of success. Secondly, suppose the initial best-guess produces a good result, there is a temptation for the experimenter to stop testing despite the fact that there is no guarantee that the optimal solution has been found [25, 97].

One-factor-at-a-time approach (OFAT): OFAT is another strategy of experimentation used extensively in practice. It consists of selecting a starting point (base level) for the levels of the factors being studied and then successively varying each factor over its range while other factors are held constant at their base level. By analysing how the response variable was affected when each factor was varied with all other factors held constant, the optimal process setting can be determined. The major disadvantage of the OFAT strategy is that it does not consider possible interactions between the factors [25, 98].

Factorial Experimentation: The correct approach to experimenting with several factors is the factorial experimental strategy [25, 98]. Factorial experimentation is an experimental strategy in which all factors are varied together instead of one at a time. Compared to the best guess and one factor-at-a-time approach, factorial designs provide greater precision when estimating main effects and also, they allow for the exploration of interaction effects [25, 77]. This study focuses on factorial experimental plans used in 3-level experimentation.

3.7. The 3^k factorial design

For a 3^k factorial design, k represents the number of factors and 3 represents the number of factor levels. The 3^2 factorial design is the simplest design in the 3^k class of factorial designs. This design has two factors at three levels each. In this design, the nine treatment combinations result in eight degrees of freedom between these treatment combinations. The main effects of factors A and B each have two degrees of freedom and the AB interaction effect has four degrees of freedom. If the experiment is replicated, there will be $n3^2 - 1$ total degrees of freedom and the error term will have $3^2 (n - 1)$ degrees of freedom [25].

Interaction effects in 3-level factorial designs can be parameterised using two systems, namely: the linear quadratic system and the orthogonal components system. In the linear quadratic system, the A and B main effects can be decomposed into linear and quadratic components. This method of parameterisation requires that the factor settings be quantitative (continuous factor settings). Let y_0, y_1 and y_2 represent the observations at the factor levels 0 (low), 1 (medium), and 2 (high), the linear effect is defined as $y_2 - y_0$, and the quadratic effect is defined as $(y_2 - y_1) - (y_1 - y_0)$. The quadratic effect can also be re-expressed as $(y_2 + y_0) - 2y_1$. The four degrees of freedom in the AB interaction are decomposed into $(AB)_{ll}$, $(AB)_{lq}$, $(AB)_{ql}$, $(AB)_{qq}$. These are called linear-by-linear, linear-by-quadratic, quadratic-by-linear and quadratic-by-quadratic interaction effects. The sum of squares of the AB interaction is the sum of the sum of squares of these four interaction components [23, 25].

In the second method of parameterisation, the orthogonal components system, the AB interaction has two components: the AB and AB^2 components of interaction. These have two degrees of freedom each. Unlike the linear quadratic system, the components of the AB interaction in the orthogonal components system have no actual meaning and are usually not displayed in the analysis of variance table. This method of parameterisation is usually associated with the case where all factor settings are qualitative (discrete factor settings) [23, 25]. This study focuses on the case where the factor settings are analysed in a qualitative manner.

3.8. The 3^{k-p} fractional factorial design

Fractional factorial designs are used to fractionate factorial designs when resource constraints prevent the use of factorial experiments for process improvement [23, 25]. To distinguish factorial designs from fractional factorial designs, factorial designs are referred to as full factorial designs [23]. Fractional factorial designs are constructed based on the effect sparsity principle which states that the number of relatively important effects in a full factorial experiment is small. Based on the assumption that certain interaction effects are negligible, main effects are aliased with interaction effects in fractional factorial designs [23]. The 3^{k-p} design is the fractional factorial design with a (3^{-p})th fraction of the 3^k full factorial design. The general mechanism for generating 3-level fractional factorial designs is similar to fractional factorial designs in which the factors are represented in two levels (2-level fractional factorial designs). By starting with a full factorial design of some of the factors to be studied, the interactions in a full factorial design (involving the complete list of factors) can be used to construct new factors (or blocks) by making their factor levels identical to those for the respective interaction terms. For example, for a 2^{3-1} fractional factorial design, the design is constructed by starting with a 2^2 full factorial design, listing factors A and B in the first two columns and constructing factor C from the AB interaction [23, 99]. This is shown in table 3.1.

Table 3.1. Construction of a 2^{3-1} fractional factorial design

Standard run	**A**	**B**	**C = AB**
1	-	-	+
2	+	-	-
3	-	+	-
4	+	+	+

Factor C can be expressed mathematically as C = AB and this is known as the design generator of the 2^{3-1} fractional factorial design. For 2-level fractional factorial designs, the design generators can be + or -.

For 3-level fractional factorial designs, the values for factor C are computed as follows:

$C = 3 - \mathrm{mod}_3\,(A+B)$

$\mathrm{Mod}_3\,(x)$ stands for a modulo-3 operator which will find a number y that is less than or equal to x and is evenly divisible by 3. It then computes the difference (remainder) between y and x. For example, $\mathrm{mod}_3\,(3) = 0$, $\mathrm{mod}_3\,(5) = 2$. If this is applied to the sum of columns A and B, column C can be obtained. This aliasing of interactions with new main effects can be summarised in the expression:

$0 = \mathrm{mod}_3\,(A+B+C)$

Looking at a 3^{3-1} fractional factorial design presented in table 3.2, it can be observed that addition of the numbers in the three columns will sum to 0, 3 or 6. This means that the values are evenly divisible by 3, hence the expression $\mathrm{mod}_3\,(A+B+C) = 0$. This expression is known as its design generator or fundamental identity. Some other designs will have a fundamental identity that contain the number 2 as a multiplier. E.g. $0 = \text{mod}_3\,(B + C*2 + D + E*2 + F)$ [23, 99, 100].

Table 3.2. Construction of a 3^{3-1} fractional factorial design

Standard run	A	B	C
1	0	0	0
2	0	1	2
3	0	2	1
4	1	0	2
5	1	1	1
6	1	2	0
7	2	0	1
8	2	1	0
9	2	2	2

Aliasing patterns for 3-level fractional factorial designs may be computed as follows:

Consider the experimental run from a 3^{4-1} fractional factorial design with factors A, B, C and D; where the levels for A, B and C are 1, 0, 2.

The design generator for this design is D = ABC. I.e. the column associated with factor D is derived from the addition of columns A, B, C. Mathematically, this is represented as: $1 + 0 + 2 = 0$ (*mod 3*). Thus the level of factor D is 0 as in modulus 3 calculus, any multiple of 3 equals zero.

If x_1, x_2, x_3 and x_4 are used to represent factors A, B, C and D, then,

$$x_4 = x_1 + x_2 + x_3 \ (mod\ 3) \quad (3.1)$$

or equivalently,

$$x_1 + x_2 + x_3 + 2x_4 = 0 \ (mod\ 3) \quad (3.2)$$

Equation (3.2) can be referred to as $I = ABCD^2$. This is known as the defining relation. The squared power of D corresponds to the coefficient of x_4.

Aliasing patterns can be deduced from the defining relation. For example by adding $2x_1$ to both sides of equation (3.2), we have

$$2x_1 = 3x_1 + x_2 + x_3 + 2x_4 = x_2 + x_3 + 2x_4 \ (mod\ 3) \quad (3.3)$$

This means that the three groups (factors) defined by $2x_1$ or $x_1 = 0, 1, 2$ (mod 3) are identical to the three groups defined by $x_2 + x_3 + 2x_4$ (mod 3). The contrasts (factor levels of each of factors B, C and D summed to zero in the orthogonal array) among the three groups in $2x_1$ or $x_1 = 0, 1, 2$ (*mod 3*) define the main effect A and the contrasts among the three groups in $x_2 + x_3 + 2x_4$ (*mod 3*) define the interaction BCD^2. Hence, A and BCD^2 are aliased. Similarly, if x_1 is added to both sides of equation 3.2, A and AB^2C^2D become aliased. Therefore, A has two aliases given as $A = BCD^2 = AB^2C^2D$. Following this derivation, it can be shown that the following effects in a 3^{4-1} fractional factorial design are aliased [23]:

$$A = BCD^2 = AB^2C^2D$$

$$B = ACD^2 = AB^2CD^2$$

$$C = ABD^2 = ABC^2D^2$$

$$D = ABC = ABCD$$

$$AB = CD^2 = ABC^2D$$

$$AB^2 = AC^2D = BC^2D$$

$$AC = BD^2 = AB^2CD$$

$$AC^2 = AB^2D = BC^2D^2$$

$$AD = AB^2C^2 = BCD$$

$$AD^2 = BC = AB^2C^2D^2$$

$$BC^2 = AB^2D^2 = AC^2D^2$$

$$BD = AB^2C = ACD$$

$$CD = ABC^2 = ABD$$

As the number of factors in a full factorial design increases, so does the number of experimental runs needed to conduct the experiment. For instance, a 3^2 full factorial experiment will require 9 experimental runs, a 3^3 full factorial experiment will require 27 experimental runs, a 3^4 full factorial experiment will require 81 experimental runs, etc. Thus in situations where experiments are affected by the availability of resources, it may not be feasible to conduct some 3-level full factorial experiments [23, 25]. This research focuses on the fractionation of 3^3 and 3^4 full factorial experiments.

3.9. Experimental plans for fractionating 3^3 and 3^4 full factorial designs

Orthogonal arrays (OAs) and fractional factorial designs (also orthogonal arrays) are commonly used to fractionate 3-level full factorial designs [23, 101, 102]. A 9-run fractional factorial design (3^{3-1} fractional factorial design) is used to fractionate a 3^3 full factorial design while a 9-run fractional factorial design (3^{4-2} fractional factorial design) and a 27-run fractional factorial design (3^{4-1} fractional factorial design) are used to fractionate a 3^4 full factorial design [23, 25]. In this thesis, factor effects that are free from aliasing with any two-factor interaction components but are aliased with components of three-factor interactions are regarded as clear [23]. The aliasing patterns of the 3^{3-1}, 3^{4-2} and 3^{4-1} fractional factorial designs are presented in table 3.3 [23].

Table 3.3. Aliasing patterns of the 3^{3-1}, 3^{4-2} and 3^{4-1} fractional factorial designs

Number of factors	Design	Resolution	Design generators	Clear effects
3	3^{3-1}	III	C = AB	None
4	3^{4-2}	III	C = AB, D = AB^2	None
4	3^{4-1}	IV	D = ABC	A, B, C, D, AB^2, AC^2, AD, BC^2, BD, CD

From table 3.3, it can be seen that the 3^{3-1} and 3^{4-2} fractional factorial designs have no clear effects and in the 3^{4-1} fractional factorial design, the main effects are clear but some two-factor interaction components are aliased with other two-factor interaction components. 3^{3-1} and 3^{4-2} fractional factorial designs are analysed by assuming that two-factor interaction effects and higher are negligible thus, only main effects can be analysed using these designs. On the other hand, the 3^{4-1} fractional factorial design is analysed by assuming that three-factor interaction effects and higher are negligible thus, main and two-factor interaction effects can be analysed using this design [23].

Based on the assumption that three-factor interaction effects and higher are negligible, Xu et al [103] developed non-regular 3-level 18-run OAs for screening important factors from a large number of potential factors and also detecting interactions among a subset of active factors when 3 to 7 factors are to be studied. Non-regular experimental designs are designs in which any two factorial effects cannot be estimated independently of each other (orthogonal) or are not fully aliased while regular experimental designs are designs in which any pair of factorial effects are either orthogonal or fully aliased. The 3^{k-p} fractional factorial designs are regular experimental designs. An example of the aliasing pattern of a non-regular experimental design will be: $A = A - \frac{1}{3}BC + \frac{1}{3}BD$, compared to those of regular experimental designs in the form of: $A = A - BC + BD$, where A, B, C and D represent factor effects [23]. To simplify discussions on the 18-run OAs of Xu et al and the 3^{3-1}, 3^{4-2} and 3^{4-1} fractional factorial designs, the $3^{3-1}$and 3^{4-2} fractional factorial designs are referred to as 9-run OAs, the 3^{4-1} fractional factorial design is referred to as a 27-run OA and the 18-run OAs of Xu et al are referred to as 18-run OAs.

3.10 Summary

This chapter presented an overview of DOE, identified full factorial experiments as the correct approach to experimentation when dealing with several factors, narrowed down the focus of this research to experiments in which 3 and 4 factors are to be studied at three levels and reviewed experimental plans used to fractionate 3^3 and 3^4 full factorial experiments. In reviewing experimental plans used to fractionate 3-level full factorial experiments, OAs were identified as experimental plans commonly used to fractionate these experiments. OAs are constructed by assuming certain interaction effects are negligible (effect sparsity principle). Based on this assumption, OAs minimise the number of runs needed for experimentation compared to full factorial experiments by aliasing the negligible interaction effects with main effects and other interaction effects [23].

This study compares the performance of the 9-run and 18-run OAs (for 3-factor experiments) and the 18-run and 27-run OAs (for 4-factor experiments) to a developed experimental plan referred to as a Segmented Fractional Plan (SFP). Although the 18-run OA was designed to be used to test for interactions when the factor settings are quantitative in form, the results of the 18-run OA are based on main effects analysis as the factor settings are analysed in a qualitative manner in this study. For 4-factor experiments, the 18-run OA was analysed instead of the 9-run OA as the additional number of experimental runs in the 18-run OA provides more experimental data and as such will increase the chances of correctly estimating the main effects.

The next chapter presents the development of the SFP as well as the comparison of its performance to the 9-run and 18-run OAs (for 3-factor experiments) and the 18-run and 27-run OAs (for 4-factor experiments). The OAs and the SFP are analysed based on their ability to use their respective experimental runs to identify the optimal setting of a process.

Chapter 4

The Segmented Fractional Plan

4.1. Introduction

This chapter presents the development of the SFP and compares its performance to OAs using data from literature and a laboratory experiment. The SFP is developed with the aim of reducing the experimental runs needed for experimentation compared to OAs thus, minimising the resources needed for experimentation. Following the development of the SFP, the performance of the SFP relative to the OAs as well as the advantages and disadvantages of these experimental plans are discussed in further sections of this chapter.

4.2. Development of the Segmented Fractional Plan

In the design of experiments, experiments with more than one replicate are recommended. However, situations may arise where due to a lack of resources, a minimal number of experimental runs is sought to improve the process [23, 25]. The SFP is developed for cases where a single replicate of the experiment is preferred due to the minimal availability of resources and the magnitude of the factor effects across three levels are not to be saved for archival purposes.

The SFP uses a full factorial experiment (2^3 or 2^4 full factorial experiment) at the high and low factor settings to identify the most important factor in the system and to also determine the optimal process setting when curvature resulting from the medium settings of the factors is not detected. The importance of the factors is determined by the magnitude (sum of squares) of their main effects.

By adding a centre point run (all factors set at their medium setting) to the full factorial design, a test for curvature is conducted using a statistical test and, a main effects and centre point plot (m-c plot). Curvature as referred to herein signifies that the medium setting is the best setting of one or more factors. The statistical test uses centre point runs to check for the possibility of a curvilinear relationship between the factors being studied and the response of interest [87, 104, 105] by measuring the difference between the average response at the factorial points and the average response at the centre point (shown in figure 4.1). If this difference is small, then the average response at the centre point lies on or close to the response plane passing through the factorial points; signifying curvature is not present. On the contrary, if the difference is large, then curvature exists [25]. As a curvilinear relationship may mean that the medium setting of a factor produces a better result than its high and low settings, the statistical test is employed to test for curvature.

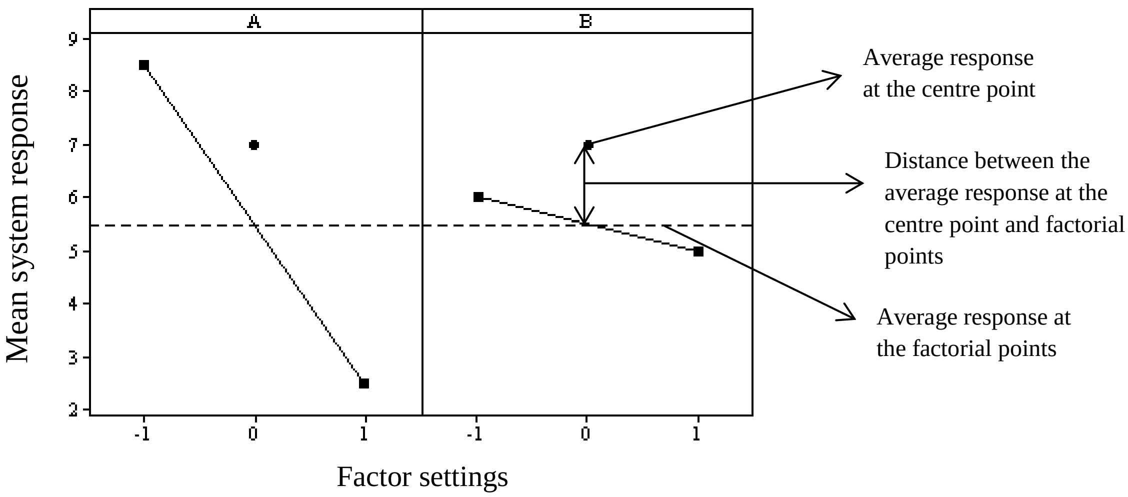


Figure 4.1. Main effect plot of a 2^2 full factorial design with centre point, ■ represents the runs at the factorial points, ● represents the centre point runs, -1 = low factor setting, 0 = medium factor setting, 1 = high factor setting

The procedure for determining the significance of curvature using the statistical test is as follows [25]:

$$SS_{CV} = \frac{n_F * n_C\ (\bar{y}_F - \bar{y}_C)^2}{n_F + n_C}$$

SS_{CV} = Sum of squares due to curvature, n_F = number of factorial points, n_C = number of centre points, $\bar{y}_F$ = average response from factorial point runs, $\bar{y}_C$ = average response from centre point runs,

$$MS_{CV} = \frac{SS_{CV}}{DF_{CV}}$$

MS_{CV} = Mean square due to curvature, DF_{CV} = Degrees of freedom due to curvature

$$\text{F-value}_{CV} = \frac{MS_{CV}}{MS_E}$$

F-value_{CV} = F-value due to curvature, MS_E = Mean square of error

$$MS_E = \frac{SS_E}{DF_E}$$

SS_E = Sum of squares due to error, DF_E = Degrees of freedom for error

A P-value (P-value due to curvature) is a statistic used to determine the significance of curvature. On obtaining the F-value due to curvature, the P-value due to curvature can be determined from the F-value. The smaller the P-value and the larger the F-value, the more important curvature is or other experimental effects (main effects, interaction effects, experimental error) [25].

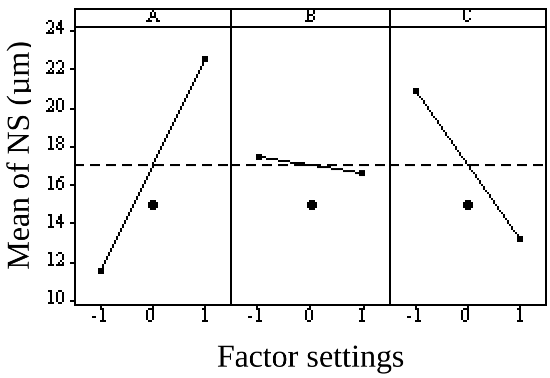
In a single replicate experiment, the statistical test can be used by removing the least significant interaction effect from the response prediction model and allocating the degrees of freedom as well as the sum of squares of the removed interaction term to error. However, this may compromise the goodness of the statistical test when the interaction effect is not small enough as the larger the sum of squares of the removed interaction term, the smaller the F-value due to curvature and the larger the P-value due to curvature [25].

To minimise the problem associated with the statistical test due to lack of replicates, the SFP uses the statistical test in conjunction with an m-c plot to improve the detection of curvature. Using both tests, if the response at the centre point is worse than the mean response from the full factorial experiment, it is assumed curvature is unlikely.

The m-c plot shows the position of the response of the centre point run relative to the mean response from the full factorial experiment and the mean response of the high and low setting of each factor. Across the 3-level full factorial design space, if the mean response at the high and low setting of a factor or a combination of factors is worse than the mean response of its medium setting, it may be reflected in the response of the centre point run as the response resulting from the main effects of these factors at their medium setting as well as their interactions with the medium settings of other factors may better the response associated with their optimum settings (best average response between the factor settings) across the full factorial design at their high and low settings. In using the m-c plot, when the centre point run produces a better result than the mean response of the optimal setting of at least one factor, it is assumed that curvature may be present. When this is not the case, it is assumed curvature is unlikely.

Figure 4.2 which is based on a 3^3 full factorial experiment [106] used to investigate the influence of polymer concentration (factor A), amount of nanoparticles (factor B) and stirring speed (factor C) on the size of nanoparticles produced (NS), is used to demonstrate how the m-c plot is used. Figure 4.2a is an m-c plot from the experiment and figure 4.2b is a main effects plot from the 3^3 full factorial of the same experiment. The low, medium and high factor settings are represented by the numbers -1, 0 and 1 respectively. This is the same for all other experiments described in this thesis. For a minimal size of nanoparticles, the m-c plot showed that the response of the centre point run (15 µm) was better than the response associated with the optimal setting of factor B (16.65 µm) across the 2^3 full factorial design space. Based on the interpretation of the m-c plot, this may suggest the presence of curvature. Comparing the m-c plot to the main effects plot from the 3^3 full factorial experiment, it can be seen that curvature exists as the optimal setting of factor B is its medium setting.

(a)



(b)

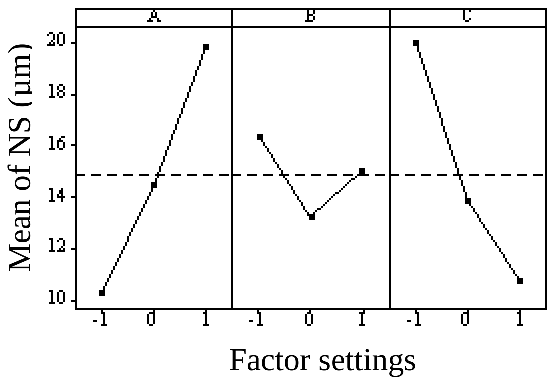


Figure 4.2 (a) M-C plot for the NS experiment, ■ represents the runs at the factorial points, ● represents the centre point run. (b) Main effects plot for the NS experiment

If either the statistical test or the m-c plot, or both, suggest the possibility of curvature, the medium setting of the factors should be explored. If both tests suggest curvature is unlikely, the full factorial experiment can be analysed to identify the optimal process setting. To identify the optimal process setting when the tests for curvature suggest curvature may be present, an experimental run is conducted by changing the most important factor to its medium setting while keeping other factors at their best setting from the full factorial experiment. Retaining the setting of the most important factor that produced the best response, a second 2-level full factorial experiment of the less important factors at their best setting from the first full factorial experiment and their medium setting is performed. The optimal factor setting corresponds to the factor settings that produce the best response across all experiments conducted. In explaining why the most important factor was changed to its medium setting and why a full factorial of the less important factors was used, a synergistic and anti-synergistic interaction are defined as follows:

A synergistic interaction is an interaction which provides an additional improvement to the system response when main effects are positively exploited compared to a model of main effects only. On the other hand, an anti-synergistic interaction worsens the system response when main effects are positively exploited compared to a model of main effects only. Positive exploitation of main effects mean that the main effects are set at levels that improve the system response while negative exploitation of main effects mean that the main effects are set at levels that worsen the system response [107]. The following example from a 2^3 full factorial experiment investigating the effects of temperature (factor A), initial pH of solution (factor B) and the ionic strength of dispersion (factor C) on the maximum adsorption of an anionic dye

(Brilliant Yellow) onto sepiolite [108] is used to illustrate how synergistic and anti-synergistic interactions work.

The regression equation based on the maximum dye adsorption (Qe) was:

$$Qe = 1.7458 - 0.1433A - 0.3400B + 0.0808C + 0.0675AB - 0.0450AC + 0.2117BC + 0.0125ABC \quad (4.1)$$

Positively exploiting the main effects, the following statements hold:

1. For a main effects model only, Qe = 2.3099 mg/g
2. For a model of main effects with AB, AC and ABC synergistic interactions, Qe = 2.4349 mg/g. The synergistic interaction improved the response of the main effects model.
3. For a model of main effects with BC anti-synergistic interaction, Qe = 2.0982 mg/g. The anti-synergistic interaction worsened the response of the main effects model.

In using the SFP, the most important factor is selected to be changed to its medium setting instead of other factors due to the following reasons: Firstly, by varying the setting of the most important factor first in a 2-level full factorial experiment, there is a reduced chance that the interaction effects which act opposite to the direction of exploitation of its main effect will overcome its main effect as well as the interaction effects acting in the direction of exploitation of its main effects compared to when other factors are changed first [107]. This is demonstrated using an experiment [109] conducted to investigate the influence of adsorbent type (factor A), pH of solution (factor B) and temperature (factor C) on the adsorption of boron from aqueous solution (Y).

The regression equation from the designed experiment is as follows:

$$Y = 0.4840 + 0.0790A - 0.0206B - 0.0666C + 0.0071AB + 0.0421AC - 0.0161BC + 0.0146ABC \quad (4.2)$$

From the regression equation, it can be seen that factor A is the most important followed by factors C and B respectively.

When the aim of the experiment is to increase the amount of boron adsorbed, the following statements hold:

1. The optimal process settings are A = +, B = - and C = - .
2. Positively exploiting the main effects of factor A, B and C, the main and interaction effect model produced a response of Y = 0.5995 mgL^{-1}. This response corresponds to the optimal process setting.
3. Positively exploiting the effect of factor B (B = -) while keeping factor A at its less optimal setting from its main effect analysis (A = -) and keeping factor C at its optimal setting from its main effect analysis (C = -), the main and interaction effects model produced a response of Y = 0.5107 mgL^{-1}.
4. Negatively exploiting the effect of factor B (B = +) while keeping factors A and C at the same settings from the previous step (A = -, C = -), the main and interaction effects model produced a response of Y = 0.5167 mgL^{-1}. The main effect of factor B is not reflected as the response has improved compared to when factor B was positively exploited. Even though the main effect of factor B and the ABC synergistic interaction had a larger value than the AB, AC and BC anti-synergistic interactions at the optimal process setting (A = +, B = -, C = -), which is a representation of the positive exploitation of all the factors, at the process setting (A = -, B = +, C = -), the

interaction effects which acted opposite to the direction of exploitation of the main effect of factor B (AC, BC, ABC) overcame the main effect of factor B as well as the interaction effects which acted in the direction of exploitation of the main effect of factor B (AB).

5. However, positively exploiting the effect of factor A across all combinations of factor settings of factors B and C improves the system response compared to when factor A is negatively exploited. This is shown in table 4.1.

Table 4.1. Boron adsorption at A = - and A = +

B	C	Boron adsorption (mgL^{-1}) at A = -	Boron adsorption (mgL^{-1}) at A = +
-	-	0.5107	0.5995
+	-	0.5167	0.5755
-	+	0.3547	0.5535
+	+	0.2379	0.5235

The second reason the most important factor was chosen to be changed first to its medium setting compared to other factors was due to the hierarchical ordering principle for factorial effects which states that lower order effects are more likely to be important than higher order effects. In other words, main effects are more likely to be important than two factor interaction effects, two factor interaction effects are more likely to be important than three factor interaction effects, etc. Focusing on the main effect of the most important factor as it is the most likely to obey the principle, this signifies that the main effect of the most important factor is more likely to be larger than any interaction effect [23]. Thus, by changing first, the most important factor in a process to its medium setting, there is a reduced chance that the interaction effects which act opposite to the direction of exploitation of its main effect will overcome its main effect as well as the interaction effects acting in the direction of exploitation of its main effects compared to when other factors are changed first. When the most important factor across the 3-level full factorial experiment is different from that obtained from the full factorial involving the medium settings of the factors and their best setting from the full factorial experiment at their high and low settings, the likelihood of the SFP identifying the optimal process setting is reduced.

Responses due to anti-synergistic interactions can only be confirmed by using a full factorial design as it explores all possible combinations of factor settings. Where the optimal process setting obtained from the main effect analysis of a full factorial experiment does not correspond to the optimal process setting across the full factorial design matrix, it is as a result of anti-synergistic interactions present in the system [107]. The first full factorial employed by the SFP will identify the optimal process setting resulting from anti-synergistic interactions within the design space of the full factorial experiment at the high and low factor settings and the second full factorial of the less important factors at the best setting of the most important factor will identify an optimal process setting which results from anti-synergistic interactions at the best setting of the most important factor and the settings of the less important factors being studied. .

For 3^3 experiments, the SFP will require 9, 12 or 13 experimental runs, and for 3^4 experiments, 17, 24 or 25 experimental runs. This is dependent on the presence of curvature in the system. A flow chart of the SFP is shown in figure 4.3.

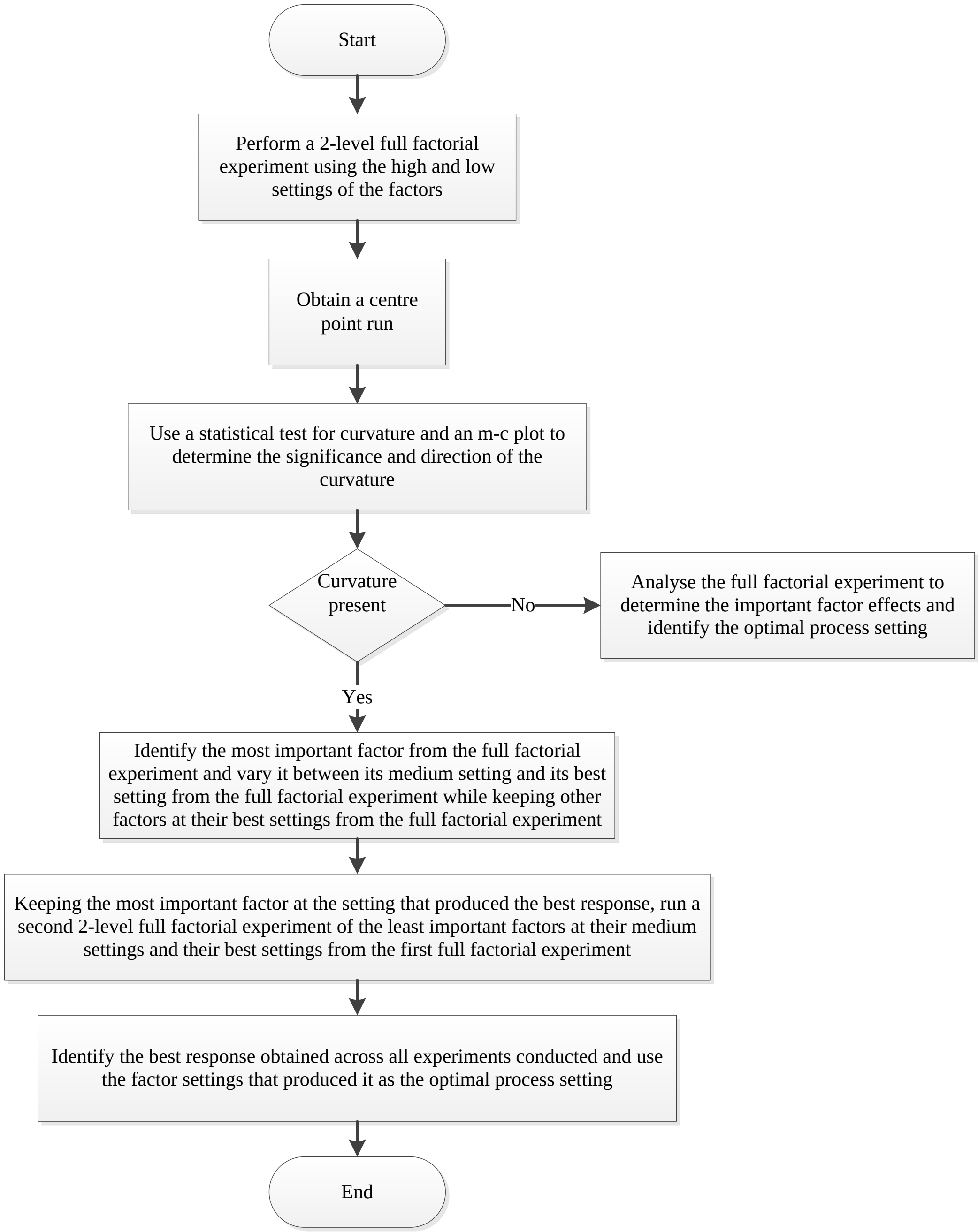


Figure 4.3. A flow chart of the Segmented Fractional Plan

4.3. Experimental data set

Six full factorial experiments [106, 110-114] identified from literature on designed experiments and a full factorial experiment conducted in a laboratory were used to compare the performance of the OAs and the SFP. Full factorial experiments were chosen to compare the performance of the OAs and the SFP as they contain the responses from the treatment combinations in the OAs and the SFP as well as the responses from all possible combinations of the settings of the factors. For the experiments obtained from literature, the responses investigated and the factors studied are given in table 4.2. Experiments 1, 2 and 3 are 3^3 full factorial experiments and experiments 4, 5 and 6 are 3^4 full factorial experiments.

Table 4.2. Experimental data set (experiments from literature)

Experiment number	**Response, units**	**Factors**
Experiment 1	Xylanese production, U/ml	Xylan (A), pH (B) and cultivation time (C)
Experiment 2a (maximum response)	Size of nanoparticles-in-microsphere, µm	Polymer concentration (A), amount of nanoparticles (B) and stirring speed (C)
Experiment 2b (minimum response)	Size of nanoparticles-in-microsphere, µm	Polymer concentration (A), amount of nanoparticles (B) and stirring speed (C)
Experiment 3	Coating bond strength of micro friction surfacing process, N	Rotational speed (A), traverse rate of the substrate (B) and feed rate of the mechtrode (C)

Table 4.2 (continued)

Experiment number	**Response, units**	**Factors**
Experiment 4	Surface roughness of medium carbon steel, µm	Speed (A), feed (B), radial rake angle (C) and nose radius (D)
Experiment 5	Damage factor in the end milling of glass fibre reinforced plastic composites, mm	Number of flutes (A), cutting speed (B), depth of cut (C) and feed rate (D)
Experiment 6a	Surface roughness values of Inconel 718 superalloy across the feed, µm	Cutting speed (A), feed (B), axial depth of cut (C) and radial depth of cut (D)
Experiment 6b	Surface roughness values of Inconel 718 superalloy transverse to the feed, µm	Cutting speed (A), feed (B), axial depth of cut (C) and radial depth of cut (D)

The response in experiment 2 was analysed based on its maximum and minimum response values as both responses were desirable, depending on the aim of the experiment. Also, two responses were analysed in experiment 6 namely: the surface roughness values across the feed and the surface roughness values transverse to the feed. Thus, eight responses from the six full factorial experiments were analysed. The responses from the experiments are coded herein as follows; Experiment 1 (XA), Experiment 2 (NS), Experiment 3 (ST), Experiment 4 (SR), Experiment 5 (DF), Experiment 6a (SAF), and Experiment 6b (STF).

The 3^3 and 3^4 full factorial experiments obtained from literature were selected to represent the following:

1. Interactions of varying strengths
2. Cases where the optimal process setting was influenced by synergistic and anti-synergistic interactions
3. Experiments with and without curvature

The methodology used for classifying the strength of interactions in this thesis was adopted from Frey et al [115]. It is based on the contribution of the interactions to the total sum of squares of the system. The interaction strength is calculated by dividing the sum of squares due to interaction effects by the sum of squares due to the factor effects (main and interaction effects). Based on this ratio, the interaction strengths are classified. The calculation of the interaction strength is shown mathematically in equation 4.3.

$$\text{Interaction strength} = \frac{\mathrm{SS_{INT}}}{\mathrm{SS_{FE}}} = \frac{\mathrm{SS_{FE}} - \mathrm{SS_{ME}}}{\mathrm{SS_{FE}}} \tag{4.3}$$

Where SS_{INT} = sum of squares due to interaction effects, SS_{ME} = sum of squares due to main effects, SS_{FE} = sum of squares due to factor effects.

The calculation of SS_T (a measure of the total variability in the data), SS_{ME}, SS_{INT} and SS_E are demonstrated using a 2-factor experiment as follows [25]:

$$SS_{ME}\text{ (factor A)} = \frac{1}{bn}\sum_{i=1}^{a} y_i^2 - \frac{y^2}{abn}\ ,\ SS_{ME}\text{ (factor B)} = \frac{1}{an}\sum_{j=1}^{b} y_j^2 - \frac{y^2}{abn}$$

$$SS_{INT} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y^2}{abn} - SS_{ME}$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y^2}{abn}$$

$SS_E = SS_T - SS_{FE}$

a = number of levels for factor A, b = number of levels for factor B, i = specific level of factor A, j = specific level of factor B, y_i = response at i level of factor A at all levels of factor B, y_j = response at j level of factor B at all levels of factor A, y_{ij} = response at i level of factor A and j level of factor B, k = specific replicate, n = total number of replicates, y_{ijk} = response at the i level of factor A and j level of factor B for the kth replicate, y = summation of responses from experimental plan.

To facilitate the grouping of the experiments used in this thesis based on their interaction strengths, three classes of interactions are used. These are given in table 4.3 as follows:

Table 4.3. Classification of interaction strength

Class of interaction	Strength of interaction
Mild	0 to 0.1
Moderate	0.1 to 0.25
Strong	Above 0.25

In identifying experiments from literature, experiments with various interaction strengths were chosen to increase the chances of interactions negatively affecting the ability of the OAs and the SFP to identify the optimal process setting and, to increase the chances of interactions compromising the goodness of the statistical test. The 3^3 and 3^4 full factorial experiments identified from literature are each representative of the three classes of interactions.

In analysing experimental data in this thesis, it was assumed that the errors associated with experimental runs were at their lowest. Thus, it is assumed the main and interaction effects represent their best possible estimate. All experiments in this thesis were analysed using Minitab statistical software. The results from the experiments obtained from literature are subsequently discussed while those from the laboratory experiment are discussed in later sections of this thesis.

4.4. Results from the 3^3 full factorial experiments

4.4.1. Testing for curvature using the SFP

Table 4.4. Magnitude of main effects from 2^3 and 3^3 full factorial experiment

	Experiment 1 [XA]		**Experiment 2 [NS]**		**Experiment 3 [ST]**	
	2^3 full factorial	3^3 full factorial	2^3 full factorial	3^3 full factorial	2^3 full factorial	3^3 full factorial
A	3.96 [2]	9.76 [2]	240.90 [1]	417.01 [1]	224115 [2]	383905 [2]
B	2.25 [3]	6.24 [3]	1.36 [3]	44.68 [3]	49770 [3]	308896 [3]
C	242.11 [1]	796.66 [1]	117.81 [2]	403.21 [2]	624403 [1]	1268752 [1]

Table 4.4 ranks and compares the magnitudes of the main effects for experiments 1, 2 and 3 based on a 2^3 full factorial experiment at the high and low factor settings, and a 3^3 full factorial experiment. The ranks are indicated in brackets. Across all experiments, the ranks indicate that the magnitudes of the main effects at two and three levels are the same. However, this may not always be the case as will be seen with the 3^4 full factorial experiments.

On identifying the most important factor, the smallest interaction effect can be removed from the response prediction model so as to perform a statistical test for curvature. Across experiments 1 to 6, the statistical test was interpreted based on a 95% confidence interval.

Experiment 1 (XA): In this experiment, the system response with the largest value was desired. Removing the interaction effect of xylan (factor A) and pH (factor B) from the response prediction model as it was the least significant interaction effect, the statistical test for curvature generated a p-value of 0.045. This indicated that curvature was likely. Checking for curvature with the m-c plot as shown in figure 4.4, it showed that the response of the centre point run was better than the response associated with the optimal setting of all three factors across the 2^3 full factorial design space. This also gives an indication that exploring the design space associated with the medium settings of the three factors might yield improved results compared to what has already been obtained from the 2-level full factorial design.

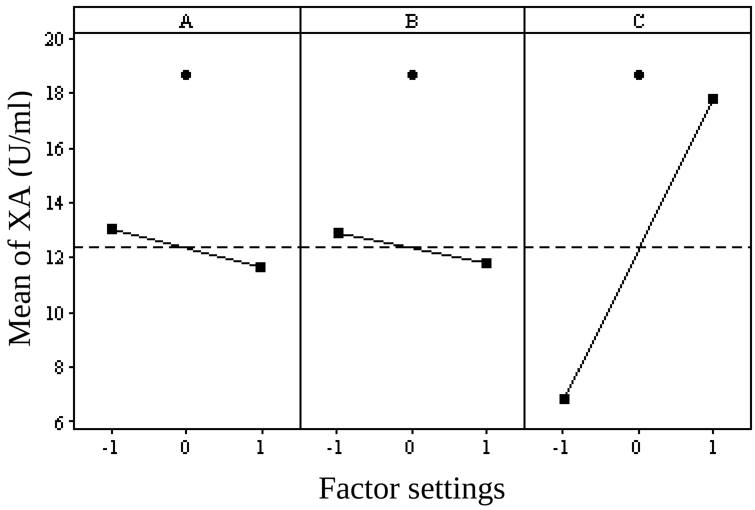


Figure 4.4. M-C plot for the XA experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 2a (NS): In this experiment, the m-c plot in figure 4.5 showed that the response of the centre point run was worse than the mean response from the full factorial experiment. Hence it is assumed that curvature is unlikely. In this case the experimenter need not explore the design space associated with the medium settings of the factors, but instead, analyse the 2-level full factorial experiment and use the information gained to improve the process.

Experiment 2b (NS): Removing the effect of the interaction between polymer concentration (factor A), amount of nanoparticles (factor B) and stirring speed (factor C) from the response prediction model, the statistical test for curvature generated a p-value of 0.361 which implied that curvature was unlikely. The m-c plot in figure 4.5 showed that the response at the centre point was better than the response associated with the optimal setting of factor B (amount of nanoparticles) across the 2^3 full factorial design space. In this case, additional experiments should be conducted to identify the optimal factor settings. As this was the same experiment described in figure 4.2, it can be seen from figure 4.2 that a main effect analysis of the 3^3 full factorial experiment confirmed the presence of curvature as the optimal setting of factor B (amount of nanoparticles) was its medium setting. This shows the advantage of combining the statistical test for curvature with the m-c plot. Even though the statistical test suggested curvature was unlikely, the use of the m-c plot improved the chance of identifying curvature associated with the medium settings of the factors.

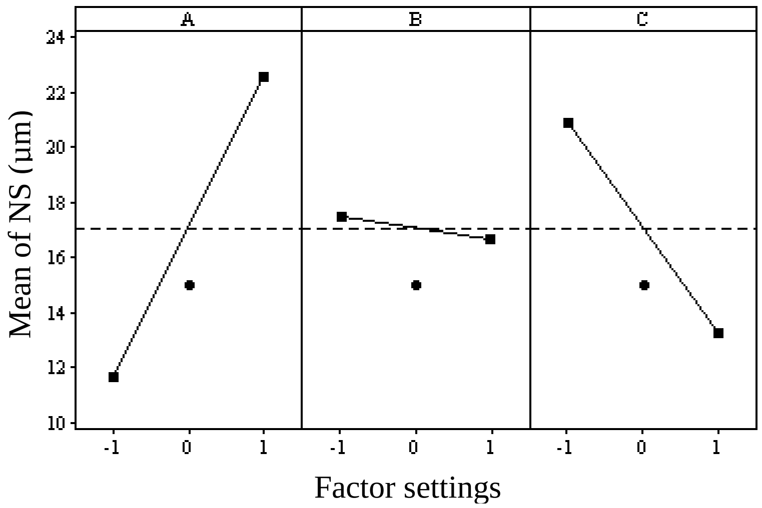


Figure 4.5. M-C plot for the NS experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 3 (ST): In this experiment, the system response with the largest value was desired. Removing the effect of the interaction between the traverse rate of the substrate (factor B) and feed rate of the mechtrode (factor C) from the response prediction model, the statistical test for curvature generated a p-value of 0.214 which implied that curvature was unlikely. Checking for curvature with the m-c plot as shown in figure 4.6, it showed that the response at the centre point was better than the response associated with the optimal setting of all three factors across the 2^3 full factorial design space, hence a suggestion for the experimenter to explore the design space associated with the medium settings of all the factors. A main effect analysis of the 3^3 full factorial experiment showed that the optimal setting of factor A was its medium setting as the optimal process setting was A = medium, B = low, C = high. This again, shows the advantage of combining the statistical test for curvature with the m-c plot.

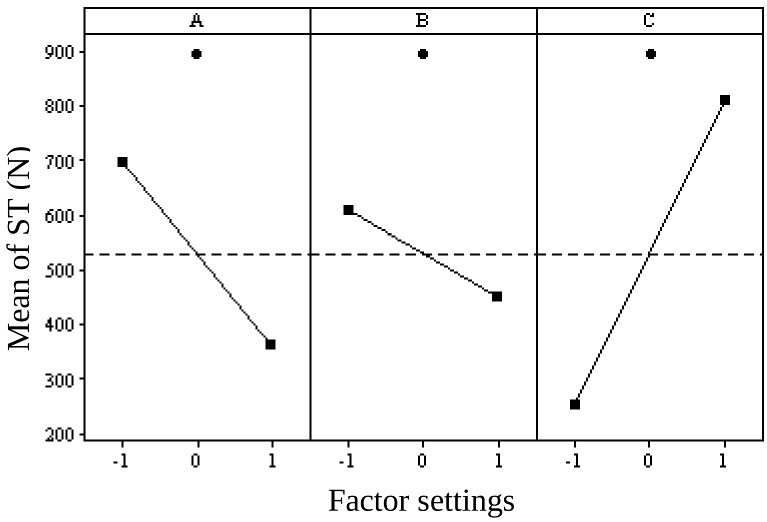


Figure 4.6. M-C plot for the ST experiment, ■ represents the runs at the factorial points, ● represents the centre point run

4.4.2. Comparing the performance of the OAs and the SFP

In experiment 1, the interaction strength is classified as mild. The 9-run OA produced a result of 15.81 U/ml, while the 18-run OA and the SFP produced a result of

22.45 U/ml. The main effect analysis of the 3^3 full factorial experiment showed that the mean response of the best two settings (medium and high settings) of the most important factor were practically identical. Thus, the optimal process settings produced by the main effect analysis of the 3^3 full factorial design was regarded to be the same as those of the 18-run OA and the SFP. In this experiment, the 18-run OA and the SFP performed better than the 9-run OA.

In experiment 2a, the strength of the interactions are classified as moderate. For the case were the system response with the largest value is desired, the 9 and 18-run OA produced a result of 27 µm. On the other hand, the SFP produced a result 31.60 µm. This corresponds to the result obtained from the main effect analysis of the 3^3 full factorial experiment and is also the optimal process setting identified across the 3^3 full factorial design matrix. Using 9 experimental runs, the SFP identified the optimal process setting as curvature was not detected in the system.

In experiment 2b, the 9 and 18-run OA produced a result of 7.51 µm, while the SFP produced a result of 6.80 µm. The results of the 9 and 18-run OA correspond to the results obtained from the main effect analysis of the 3^3 full factorial experiment. However, due to anti-synergistic interactions, the best response across the 3^3 full factorial design space was 6.80 µm. The SFP was able to identify this setting using 13 experimental runs.

In experiment 3, the interactions are classified as strong. The 9-run OA produced a result of 1026 N, the 18-run OA produced a result of 882 N and the SFP produced a result of 1249 N. From the point of view of array efficiency, the 18-run OA produced a better result as its result was the same as that obtained from the main effect analysis of the 3^3 full factorial experiment. However, due to anti-synergistic interactions, the optimal process setting across the 3^3 full factorial design was 1249 N. Though curvature was present, the anti-synergistic interactions associated with the high and low factor settings resulted in the optimal process setting of 1249 N. As a result, the SFP was able to identify the optimal process setting.

4.5. Results from the 3^4 full factorial experiments

4.5.1. Testing for curvature using the SFP

Table 4.5. Magnitude of main effects for 2^4 and 3^4 full factorial experiments (SR and DF experiments)

	Experiment 4 (SR)		**Experiment 5 (DF)**	
	2^4 full factorial	3^4 full factorial	2^4 full factorial	3^4 full factorial
A	0.1881 [2]	0.4971 [4]	0.1423 [2]	0.4319 [2]
B	0.2614 [1]	0.7928 [1]	0.0427 [3]	0.1159 [4]
C	0.1305 [3]	0.6967 [3]	0.0424 [4]	0.1760 [3]
D	0.1216 [4]	0.7677 [2]	0.2657 [1]	0.7904 [1]

Table 4.6. Magnitude of main effects for 2^4 and 3^4 full factorial experiment (SAF and STF experiments)

	Experiment 6a (SAF)		**Experiment 6b (STF)**	
	2^4 full factorial	3^4 full factorial	2^4 full factorial	3^4 full factorial
A	0.0912 [4]	0.0170 [4]	0.0938 [4]	0.0603 [4]
B	0.2012 [3]	2.1769 [2]	0.2418 [2]	1.4532 [3]
C	0.4001 [2]	0.0816 [3]	0.1598 [3]	2.0410 [2]
D	1.0201 [1]	4.4140 [1]	1.5744 [1]	4.3535 [1]

Tables 4.5 and 4.6 rank and compare the magnitude of the main effects for experiments 4, 5 6a and 6b based on a 2^4 full factorial experiment at the high and low factor settings and a 3^4 full factorial experiment. The ranks are indicated in brackets. From tables 4.5 and 4.6, it can be seen that in these experiments, the ranks of some of the factors vary between the 2^4 and 3^4 full factorial experiments however, the ranks of the most important factors are the same. If the most important factor in the 2-level design differs from that in the 3-level design, the performance of the SFP may be affected.

In experiments 4, 5, 6a and 6b, the response with the smallest value was desired.

Experiment 4 (SR): Removing the interaction between speed (factor A) and nose radius (factor D) from the response prediction model, the statistical test for curvature generated a p-value of 0.003 while the m-c plot in figure 4.7 showed that the response at the centre point was better than the response associated with the optimal setting of all four factors across the 2^4 full factorial design space. Both tests suggest curvature may be present in the system. Thus, further experiments need to be conducted to investigate the performance of the medium settings of the factors.

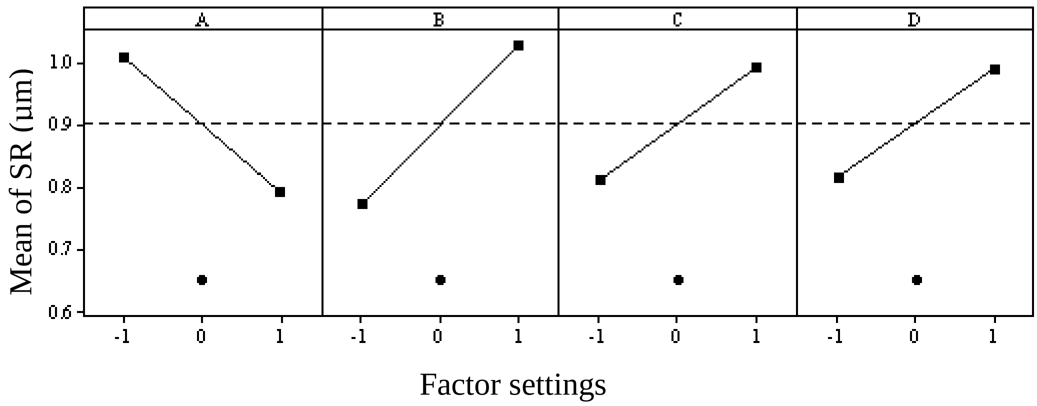


Figure 4.7. M-C plot for the SR experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 5 (DF): In this experiment, the statistical test for curvature generated a p-value of 0.216 when the interaction effect between speed (factor B), depth of cut (factor C) and feed rate (factor D) was removed from the response prediction model and the m-c plot in figure 4.8 showed that the response at the centre point was worse than the response associated with the optimal setting of all four factors across the 2^4 full factorial design space. In this case, both tests indicated that curvature was unlikely. A main effect analysis of the 3^4 full factorial experiment showed that the optimal process setting due to main effects did not contain the medium setting of any of the four factors. On this occasion, the tests for curvature successfully detected the absence of curvature in the system.

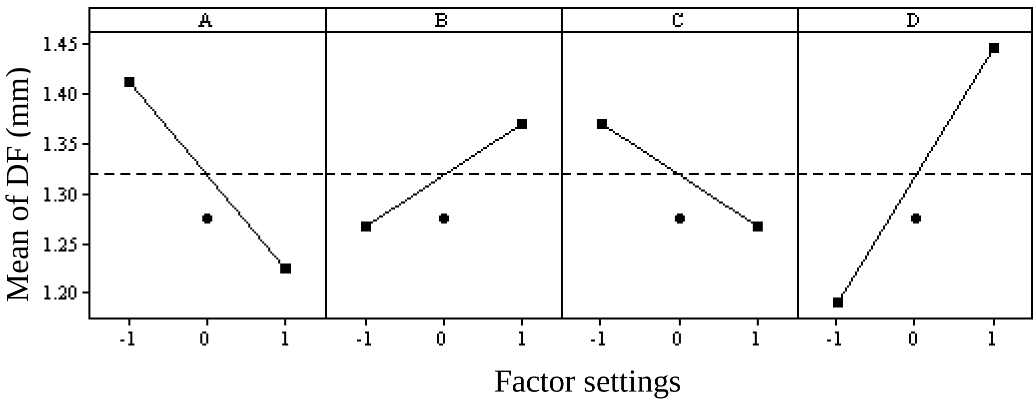


Figure 4.8. M-C plot for DF experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 6a (SAF): In this case the m-c plot in figure 4.9 showed that the response of the centre point run was worse than the mean response from the 2^4 full factorial experiment. Hence, the performance of this system should be analysed and improved using the 2^4 full factorial experiment.

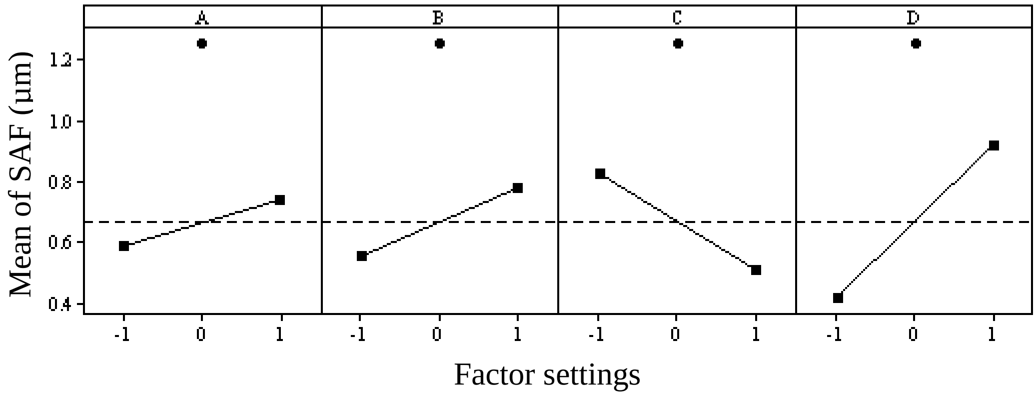


Figure 4.9. M-C plot for SAF experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 6b (STF): The statistical test for curvature generated a p-value of 0.106 when the interaction effect between cutting speed (factor A), feed (factor B), axial depth of cut (factor C) and radial depth of cut (factor D) was removed from the response prediction model. This indicated curvature was unlikely. On the other hand, the m-c plot in figure 4.10 showed that the response at the centre point was better than the response associated with the optimal setting of all four factors across the 2^4 full factorial design space. Thus, indicating that curvature may be present. A main effect analysis of the 3^4 full factorial experiment revealed that the optimal process setting included the medium setting of the cutting speed (factor A) and the axial depth of cut (factor C) which signified the presence of curvature. On this occasion, the m-c plot proved to be useful as it was able to identify the curvature associated with the medium settings of the factors.

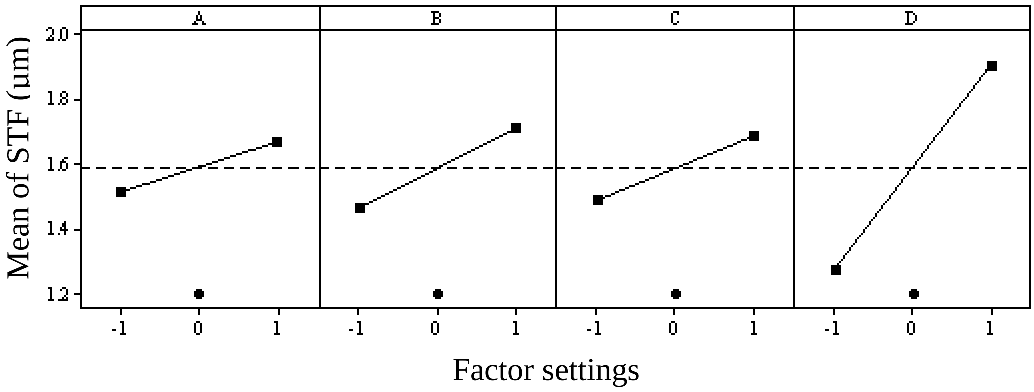


Figure 4.10. M-C plot for STF experiment, ■ represents the runs at the factorial points, ● represents the centre point run

4.5.2. Comparing the performance of the OAs and the SFP

In experiment 4, the interaction strength is classified as mild. Anti-synergistic interactions did not influence the optimal process setting in this experiment. The 18-run OA, 27-run OA and the SFP produced a result of 0.460 µm which was the same as that obtained from the main effect analysis of the 3^4 full factorial experiment. The SFP required 25 experimental runs to identify the optimal process setting compared to the 18 and 27 experimental runs used by the 18 and 27-run OAs respectively. In this case, all the experimental plans produced the optimal result as the optimal process setting across the 3^4 full factorial design matrix was 0.460 µm.

In experiment 5, where the strength of the interactions are moderate, the 18 and 27-run OA produced a result of 1.1401 mm which corresponded to the result produced by the main effect analysis of the 3^4 full factorial experiment. However, due to anti-synergistic interactions, the best response across the 3^4 full factorial design space was 1.1383 mm. The SFP was able to identify this response using 17 runs from a 2^4 full factorial experiment with a centre point run as no curvature was detected.

In experiment 6a, the interaction strength is classified as strong. The main effect analysis of the 3^4 full factorial experiment produced a result of 0.280 µm and the best response across the 3^4 full factorial design space was 0.245 µm. The 18-run OA produced a result of 0.315 µm while the 27-run OA produced a result of 0.245 µm. In this case, the result of 0.245 µm produced by the 27-run OA is by chance as it was not obtained from an analysis of the interaction effects. Using 17 experimental runs, the SFP produced a result of 0.270 µm. This was due to an anti-synergistic interaction from the 2^4 full factorial experiment as the medium settings of the factors were not explored.

In experiment 6b, the best response across the 3^4 full factorial design space was 0.480 µm. In this experiment, the strength of the interactions is classified as strong. The 18-run OA and the SFP produced a result of 0.520 µm which corresponded to the result obtained from the main effect analysis of the 3^4 full factorial experiment. On the other hand, the 27-run-OA produced a result of 1.083 µm which was worse than that of the 18-run OA and SFP. In this experiment the SFP used 25 experimental runs compared to the 18 and 27 experimental runs of the 18 and 27-run OAs respectively.

4.6. The laboratory experiment

The laboratory experiment was carried out on a spot welding machine used for resistance welding. Welding is the process of joining two pieces of metals or non-metals by heating them to their melting point. Resistance welding is the process of joining two metals together by the heat produced due to the resistance of the flow of electric current by these metals [116]. The materials to be welded were mild steel strips of 1.5 mm thickness cut to length and breadth dimensions of 90 mm by 25 mm respectively.

The response of interest was the force needed to shear the weld and this was recorded in Kilo Newton (KN). The optimum weld was determined to be the weld with the highest shear force. In measuring the shear force (SF) of the weld, an Instron 3382 machine was used to shear the weld at a speed of 2 mm/min.

By studying the mode of operation of the spot welding machine, five factors which affect the quality of the weld produced were identified. These are [117, 118]:

1. **Squeeze time**: This is the time interval between the initial application of the electrode force on the work surface and the first application of current. It is measured in cycles, with 1 cycle = 1/60 of a second.
2. **Weld time**: The weld time starts after the squeeze time and it is the time allocated for electrical current to pass through the materials that are to be welded. It is measured in cycles, with 1 cycle = 1/60 of a second.
3. **Hold time**: The hold time occurs after the weld time. It is the time that the metals remain under force from the electrode with no electrical current passing through them. Like the squeeze time and weld time, it is measured in cycles with 1 cycle = 1/60 of a second.
4. **Current**: The current supplied during the welding process is measured as a percentage.
5. **Pressure**: The pressure supplied by the electrode during the welding process is measured in pounds per square inch (psi).

In comparing the performance of the SFP to OAs using the laboratory experiment, the author decided to use three factors instead of four to minimise the resources needed for experimentation. To narrow down the list of factors to three, a resolution V 2^{5-1} fractional factorial experiment replicated twice was conducted on the spot welder machine. The factor levels used in the experiment are presented in table 4.7.

Based on the authors experience in using the spot welding machine, the factors were set at levels sufficient to reflect the magnitude of their effect on the weld quality.

Table 4.7. Factor levels for 2^{5-1} fractional factorial design

Factors	Symbol	Levels	
		-1	1
Pressure (psi)	A	20	60
Current (%)	B	30	50
Hold time (cycles)	C	10	40
Squeeze time (cycles)	D	30	90
Weld time (cycles)	E	30	90

The experimental design matrix of the resolution V 2^{5-1} fractional factorial experiment as well as the observations from the experimental runs are presented in table 4.8 and table 4.9 shows the magnitude of the main effect of the five factors of study.

Table 4.8. Results for the 2^{5-1} fractional factorial experiment

Replicate 1

Run	A	B	C	D	E	Shear Force (KN)
1	-	-	-	-	+	17.01
2	+	-	-	-	-	13.50
3	-	+	-	-	-	12.05
4	+	+	-	-	+	12.40
5	-	-	+	-	-	15.25
6	+	-	+	-	+	16.39
7	-	+	+	-	+	13.89
8	+	+	+	-	-	11.95
9	-	-	-	+	-	16.50
10	+	-	-	+	+	15.10
11	-	+	-	+	+	11.99
12	+	+	-	+	-	9.50
13	-	-	+	+	+	17.45
14	+	-	+	+	-	15.40
15	-	+	+	+	-	11.99
16	+	+	+	+	+	12.92

Replicate 2

Run	A	B	C	D	E	Shear Force (KN)
17	-	-	-	-	+	17.15
18	+	-	-	-	-	14.02
19	-	+	-	-	-	12.09
20	+	+	-	-	+	12.20
21	-	-	+	-	-	16.07
22	+	-	+	-	+	16.52
23	-	+	+	-	+	13.60
24	+	+	+	-	-	10.30
25	-	-	-	+	-	16.30
26	+	-	-	+	+	14.80
27	-	+	-	+	+	13.95
28	+	+	-	+	-	8.99
29	-	-	+	+	+	18.10
30	+	-	+	+	-	15.75
31	-	+	+	+	-	12.95
32	+	+	+	+	+	13.75

Table 4.9. Magnitude of main effects for 2^{5-1} fractional factorial design

Factors	Sum of squares
Pressure (psi)	16.316
Current (%)	115.482
Hold time (cycles)	6.780
Squeeze time (cycles)	0.034
Weld time (cycles)	18.927

The results in table 4.9 reveal that the current, weld time and pressure were the most important factors affecting the quality of the weld. Thus these three factors were used to conduct a 3^3 full factorial experiment to be used to compare the performance of the SFP, the 18-run OA and the 9-run OA. In conducting this experiment, the hold time and squeeze time were set at their best levels from the 2^{5-1} fractional factorial experiment.

The factor levels used in the 3^3 full factorial experiment are shown in table 4.10 and the experimental design matrix of the 3^3 full factorial experiment as well as the observations from the experimental runs are shown in table 4.11. The factor levels were chosen arbitrarily.

Table 4.10. Factor levels for the 3^3 full factorial design

Factors	Factor levels		
Pressure (A)	20 (-1)	40 (0)	60 (1)
Current (B)	20 (-1)	40 (0)	60 (1)

Factors	Factor levels		
Weld time (C)	53 (-1)	63 (0)	73 (1)

Table 4.11. Results for the 3^3 full factorial experiment

Run	A	B	C	Average Shear Force (KN)
1	-1	-1	-1	17.48
2	-1	-1	0	17.19
3	-1	-1	1	17.19
4	-1	0	-1	15.83
5	-1	0	0	16.16
6	-1	0	1	17.03
7	-1	1	-1	6.08
8	-1	1	0	6.97
9	-1	1	1	5.63
10	0	-1	-1	17.24
11	0	-1	0	18.75
12	0	-1	1	18.95
13	0	0	-1	15.73
14	0	0	0	13.98

Run	A	B	C	Average Shear Force (KN)
15	0	0	1	16.12
16	0	1	-1	4.49
17	0	1	0	4.57
18	0	1	1	4.85
19	1	-1	-1	16.98
20	1	-1	0	19.55
21	1	-1	1	18.59
22	1	0	-1	14.46
23	1	0	0	15.13
24	1	0	1	14.30
25	1	1	-1	3.41
26	1	1	0	4.37
27	1	1	1	4.22

The 3^3 full factorial experiment was replicated twice and the average was used to analyse the performance of the SFP and the OAs.

4.6.1. Results from the spot welder experiment

4.6.1.1. Testing for curvature using the SFP

Table 4.12 shows the magnitude of the main effect of factor A (pressure), factor B (current) and factor C (weld time) for the 2^3 and 3^3 full factorial experiments.

Table 4.12. Magnitude of main effects for 2^3 and 3^3 full factorial experiment

Factors	**2^3 full factorial design**	**3^3 full factorial design**
A	1.264	4.088
B	323.851	858.069
C	0.353	1.910

From table 4.12, it can be seen that the most important factor in the 2^3 full factorial experiment (current) was also the most important factor in the 3^3 full factorial experiment.

Removing the interaction effect between pressure (factor A), current (factor B) and weld time (factor C), the statistical test for curvature generated a p-value of 0.055 (at 95% confidence interval). The m-c plot as presented in figure 4.11 showed that the response at the centre point was better than the response associated with the optimal setting of pressure (factor A) and weld time (factor C) across the 2^3 full factorial design space. Thus, suggesting that curvature may be present. However, it was determined that there was no curvature resulting from the medium settings of the factors as a main effects analysis of the 3^3 full factorial experiment revealed that the optimal process setting of factor A was its low setting, the optimal process setting of factor B was its low setting and the optimal process setting of factor C was its medium or high setting as the results of both settings were practically identical.

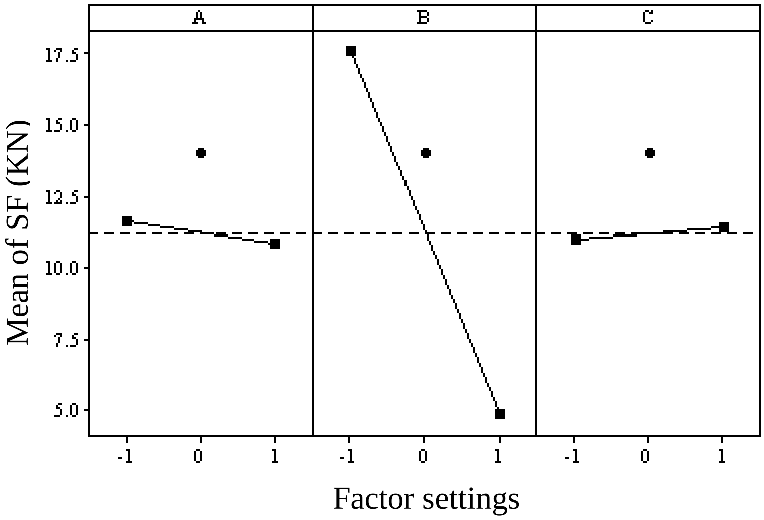


Figure 4.11. M-C plot for the spot welder experiment

4.6.1.2. Comparing the performance of the OAs and the SFP

In this system, the interactions are classified as mild in strength and the best response across the full factorial design space was 19.55 KN. The 18-run OA produced the same result as the main effect analysis of the 3^3 full factorial experiment which was an optimal process setting of factor A = low, factor B = low and factor C = high. This produced a response of 17.19 KN. Analysing the performance of the 9-run OA, the mean of the low and medium setting of factor A was practically identical thus, it was regarded that the 9-run OA produced the same result as the main effect analysis of the 3^3 full factorial experiment. Due to anti-synergistic interaction in the system, the optimal process setting of factor A was its high setting. Hence, the optimal process setting of the system was factor A = high, factor B = low and factor C = medium. This produced a response of 19.55 KN and was identified by the SFP.

4.7. Discussions on the performance of the OAs and the SFP

Table 4.13 and 4.14 compares the results and the number of experimental runs used by the OAs, the SFP and the 3-level full factorial design for the experiments obtained from literature and the laboratory experiment respectively. In table 4.13, L/B (larger-the-better) signifies that a larger response value was desired and S/B (smaller-the-better) signifies that a smaller response value was desired.

Table 4.13. Summary of the results from the OAs, the SFP and the 3-level full factorial design (XA, NS, ST, SR, DF, SAF and STF experiments)

Exp. 1 (XA). Exp. Plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	**15.81**	**22.45**	**22.45**	**22.45**
Run size	**9**	**18**	**13**	**27**
Exp. 2a (NS). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in µm) (L/B)	**27**	**27**	**31.60**	**31.60**
Run size	**9**	**18**	**9**	**27**
Exp. 2b (NS). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in µm) (S/B)	**7.51**	**7.51**	**6.80**	**6.80**
Run size	**9**	**18**	**13**	**27**
Exp. 3 (ST). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in N) (L/B)	**1026**	**882**	**1249**	**1249**
Run size	**9**	**18**	**13**	**27**
Exp. 4 (SR). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in µm) (S/B)	**0.460**	**0.460**	**0.460**	**0.460**
Run size	**18**	**27**	**25**	**81**
Exp. 5 (DF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in mm) (S/B)	**1.1401**	**1.1401**	**1.1383**	**1.1383**
Run size	**18**	**27**	**17**	**81**
Exp. 6a (SAF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in µm) (S/B)	**0.315**	**0.245**	**0.270**	**0.245**
Run size	**18**	**27**	**17**	**81**
Exp. 6b (STF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in µm) (S/B)	**0.520**	**1.083**	**0.520**	**0.480**
Run size	**18**	**27**	**25**	**81**

Table 4.14. Results of the OAs and the SFP for the spot welder experiment

Experimental plans	Run size	Response (KN)
9-run OA	9	17.19
18-run OA	18	17.19
SFP	13	19.55
3^3 full factorial	27	19.55

From table 4.13, with the exception of experiment 6a, the SFP performed as well as or better than the OAs. In experiment 1, the SFP and the 18-run OA identified the optimal process setting however the SFP reduced the number of experimental runs used by the 18-run OA by 28%. In experiment 2 (L/B), the SFP reduced the number of experimental runs used by the 18-run OA by 50% and outperformed it by identifying the optimal process setting. In experiment 2 (S/B) and experiment 3, the SFP reduced the number of experimental runs used by the 18-run OA by 28% and outperformed it by identifying the optimal process setting.

In experiment 4, the SFP performed as well as the OAs and reduced the number of experimental runs used by the 27-run OA by 7%. In experiment 5, the SFP outperformed the 18 and 27-run OAs by identifying the optimal process setting. In addition to outperforming the 18 and 27-run OAs, the SFP reduced the number of experimental runs used by these OAs by 6% and 37% respectively. In experiment 6a, the SFP produced a better system response than the 18-run OA and reduced the experimental runs used by it by 6%. In experiment 6b, the SFP reduced the number of experimental runs used by the 27-run OA by 7 % and outperformed it by producing a better system response.

For the laboratory experiment, table 4.14 showed that the SFP reduced the number of experimental runs used by the 18-run OA by 28% and outperformed it by identifying the optimal process setting. The four features of the SFP are:

1. Conducting a full factorial experiment at the high and low settings of the factors to identify the most important factor and to determine the optimal process setting when curvature is unlikely. The full factorial experiment at the high and low settings of the factors will identify an optimal factor setting due to anti-synergistic interactions within its design space.

2. Combining the m-c plot with the statistical test for curvature to test for curvature. To avoid exploring design spaces that may be of little significance, the SFP utilises the tests for curvature to check for the likelihood of the medium settings of the factors being the optimal setting. By combining the m-c plot with the statistical test for curvature, any deficiency of the statistical test for curvature due to the lack of replicates is minimised.

3. Varying the most important factor between its best setting from the first full factorial experiment and its medium setting to determine the best setting of the most important factor. Varying the setting of the most important factor while other factors are held constant increases the likelihood of identifying the optimal process setting resulting from the main and interaction effects of all the factors compared to when a less important factor is changed first.

4. Conducting a 2-level full factorial experiment of the least important factors at their best setting from the first full factorial experiment and their medium setting, while keeping the most important factor at its best setting to identify the optimal process setting. The use of a second full factorial experiment by the SFP to

identify the optimal process setting when curvature may be present ensures that all the combinations of the settings of the least important factors are tested at the best level of the most important factor. By doing so, an optimal process setting which results from anti-synergistic interactions at the best setting of the most important factor and the settings of the less important factors being studied will be identified.

Compared to the OAs, an advantage of the full factorial experiment at the high and low factor settings used by the SFP is the identification of the optimal factor setting due to anti-synergistic interactions within this design space. For instance, in the damage factor experiment (experiment 5), using 17 experimental runs, the SFP identified the optimal process setting resulting from anti-synergistic interactions which produced a result of 1.1383 µm. On the other hand, both the 18 and 27-run OA produced the same result of 1.1401 µm with 18 and 27 experimental runs respectively. Because the OAs are not full factorial designs, the experimenter cannot identify for certain responses due to anti-synergistic interactions. Also, the interaction tests of the 27-run OA may or may not identify them. Cases may exist when the optimal process setting produced by the main and interaction effect analysis of the OAs is present in their design matrix. In such a case, a comparison can be made between the optimal process settings identified from the main and interaction effect analysis to the optimal process setting across the OA design matrix. The better response can then be selected based on the comparison.

A second advantage of the SFP over the 18 and 27-run OA is that in minimising the chances of exploring insignificant design spaces by means of the tests for curvature, the process performance can be improved by identifying anti-synergistic interactions with nine less experimental runs than the 18-run OA (for 3^3 full factorial experiments), one less experimental run than the 18-run OA (for 3^4 full factorial experiments) and ten less experimental runs than the 27-run OA (for 3^4 full factorial experiments). Table 4.15 presents the advantages of the SFP relative to the 9, 18 and 27-run OAs.

Table 4.15. Advantages of the SFP over the OA

SFP	**9-run OA**	**18 and 27-run OA**
Minimises resource usage by using the tests for curvature to reduce the risk of exploring insignificant design spaces	Requires an equal or reduced number of runs than the SFP hence, the SFP provides no advantage over it in terms of minimising the risk of exploring insignificant design spaces	Resources might be wasted by exploring design spaces of little significance
Anti-synergistic interactions associated with the design space of the high and low factor settings can be identified for certain	Anti-synergistic interactions cannot be identified for certain	Anti-synergistic interactions cannot be identified for certain

In using the SFP, the most important factor across the first 2-level full factorial experiment and the full factorial involving the medium settings of the factors and their best setting from the first full factorial experiment may not be the same. In such a case, changing the most important factor to its medium setting at the best settings of other factors from the first full factorial experiment may produce sub optimal results as the response at the medium setting of the most important factor may be affected by anti-synergistic interactions. To minimise this, the factor settings should be evenly spaced out when possible. This can reduce the chances of choosing factor settings that do not reflect the true importance of the factors.

A disadvantage of the SFP compared to the OAs is that the optimal factor setting due to main effects in the full factorial experiment at the high and low factor settings may differ from those obtained from a 3-level full factorial experiment due to interactions. This is more likely to affect small main effects. In such situations, when curvature is present and anti-synergistic interactions do not determine the optimal process setting, the OAs may outperform the SFP.

For 3^3 full factorial experiments, the SFP provides a reduction in the number of experimental runs compared to the 18-run OA as it requires a maximum of 13 experimental runs, and for 3^4 full factorial experiments, the SFP provides a reduction in the number of experimental runs compared to the 27-run OA as it requires a maximum of 25 experimental runs. When curvature is not detected, for 3^3 full factorial experiments, the SFP will require 9 experimental runs which is the same for the 9-run OA and for 3^4 full factorial experiments, the SFP will require 17 experimental runs which is one run less than the 18-run OA.

In large organisations where resources are more readily available and the magnitude of factor effects across three levels are not to be saved for archival purposes, for 3-factor experiments, the author recommends running a full factorial at the high and low factor settings and then running a second full factorial using the best settings of the initial full factorial experiment and the medium settings of the factors. This experimental plan is recommended as it is able to identify an optimal process setting resulting from anti-synergistic interactions within the design space of the two full factorial experiments it employs. Furthermore, for 4-factor experiments, the author recommends the experimental procedure outlined in the SFP with the exception of the test for curvature to minimise the chances of not identifying better system responses when the tests for curvature do not detect curvature. Also, this experimental plan is recommended due to its ability to identify an optimal process setting due to anti-synergistic interactions within the design space of the full factorial of the factors at their high and low settings.

4.8. Summary

This chapter documented the development of the SFP as well as a comparison of its performance to 9-run and 18-run OAs (for 3-factor experiments) and 18-run and 27-run OAs (for 4-factor experiments). The comparisons showed that with a reduced number of experimental runs, the SFP can perform as well as or better than the 18-run OA (for 3-factor experiments) and the 27-run OA (for 4-factor experiments) thus, providing an option for economic experimentation.

Although the tests for curvature employed by the SFP are not guaranteed to detect curvature nor guard against improved process settings brought about by continuing with further experiments in the SFP, these tests provide SMEs the option of minimising resource usage by minimising the risk of exploring insignificant design spaces. In addition to the tests for curvature, the second full factorial experiment at the best setting of the most important factor reduces the runs needed for experimentation compared to the 18-run OA (for 3-factor experiments), the 27-run OA (for 4-factor experiments) and, a full factorial experiment involving the medium settings of the factors and their best setting from a full factorial experiment at their high and low settings.

The analyses of the OAs and the SFP have been based on the fact that the experiments are not replicated. A disadvantage of conducting non-replicated experiments with the OAs and the SFP is that the observations from the experimental runs are prone to experimental error. Thus, when the experimental error is large, the results generated from these experimental plans may not be optimal. The next chapter concludes this thesis by presenting the original contribution to knowledge (contributions to research and practice) provided by this study.

Chapter 5

Conclusion

This chapter concludes this thesis by presenting the original contribution to knowledge (contribution to research and practice) provided by this study and identifying areas worthwhile for future research.

5.1. Original contribution to research

This research has investigated ways to aid SMEs minimise resource usage when implementing six sigma. In doing so, the SFP was developed to minimise the number of experimental runs needed to fractionate 3^3 and 3^4 full factorial experiments compared to OAs. In investigating ways to aid SMEs implement six sigma with minimal resources, this study has provided the following original contributions to research:

1. The SFP was developed for fractionating 3^3 and 3^4 full factorial experiments when a single replicate of an experiment is preferred due to the minimal availability of resources and the magnitude of the factor effects across three levels are not to be saved for archival purposes. To the best of the author's knowledge, the author is not aware of an experimental plan like the SFP that has been developed.
2. The SFP was tested using 3^3 and 3^4 full factorial experiments each representative of:
   i. Interactions of mild, moderate and strong strengths.
   ii. Cases in which the optimal process setting were affected by synergistic and anti-synergistic interactions.

iii. Experiments with and without curvature.

3. Using the 3^3 and 3^4 full factorial experiments aforementioned, the performance of the SFP was compared to 9-run and 18-run OAs (for 3-factor experiments) and, 18-run and 27-run OAs (for 4-factor experiments). Based on the comparisons, it was shown that with a reduced number of runs, the SFP can perform as well as or better than the 18-run OA (for 3-factor experiments) and the 27-run OA (for 4-factor experiments).

5.2. Original contribution to practice

Based on the SFP and the OAs, a model for conducting 3-level experiments in SMEs is developed to help managers, engineers and other personnel involved in quality improvement select economic experimental plans when conducting 3-level experiments. The model is developed for two to seven factors represented at three levels each. In developing the model, the choice of which design to employ among the 9-run OA, the 18-run OA, the 27-run OA and the SFP was made based on the likelihood of interactions in the system, the ability of the designs to identify an optimal process setting due to synergistic and anti-synergistic interactions, and the number of experimental runs needed. Based on the number of factors to be studied, the experimental plans are selected as shown in figure 5.1.

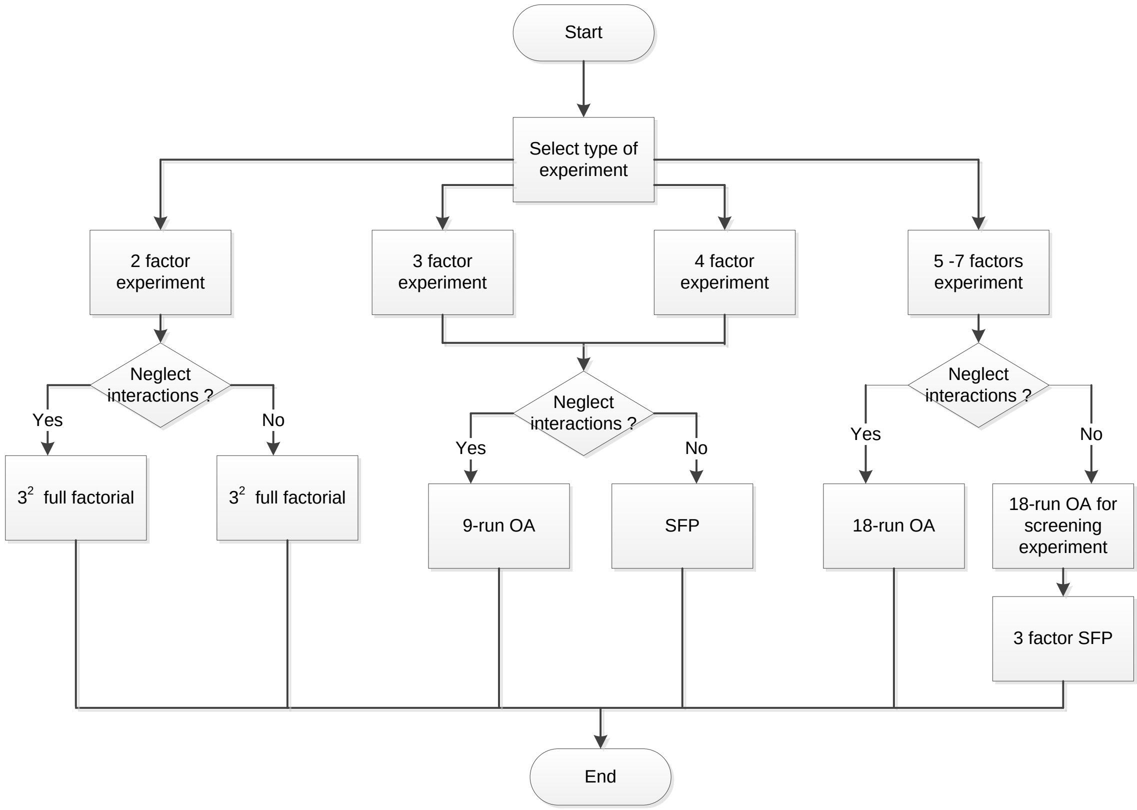


Figure 5.1. A model for conducting 3-level experiments in SMEs

For two factors of study, whether interactions are anticipated or not, the 3^2 full factorial experiment is advised as it requires 9 experimental runs to fully understand the system and identify the optimal process setting.

For three and four factors of study, when interactions are to be neglected, the 9-run OA is recommended as it requires a reduced number of runs to explore the low, medium and high settings of the factors compared to the 18-run OA, 27-run OA and the SFP. When interactions are not to be neglected, the SFP is advised due to its ability to identify the optimal process setting resulting from anti-synergistic interactions associated with the design space of the high and low factor settings and its reduced number of runs compared to the 18-run OA (for three factors) and 27-run OA (for four factors).

For five to seven factors, when interactions are to be ignored, the 18-run OA is recommended as it can identify the optimal setting resulting from main effects with a reduced number of runs compared to the 27-run OA. When interactions are not to be neglected, the 18-run OA is advised to screen out important factors from the potential factors compared to the 27-run OA due to its reduced number of runs. Following the screening exercise, the 3-factor SFP is recommended for determining the optimal settings of the most important factors due to its ability to identify the optimal process setting resulting from anti-synergistic interactions associated with the design space of the high and low factor settings and its reduced number of runs compared to the 18-run OA (for three factors) and 27-run OA (for four factors).

5.3. Recommendations for future research

This research was focused on developing an experimental plan for fractionating 3^3 and 3^4 full factorial experiments when the factors are qualitative. In developing experimental plans to address the issue of resource constraints, future work can focus on the development of experimental plans for modelling the performance of a process when the factors are quantitative by focusing on minimising the number of experimental runs needed for experimentation compared to existing strategies. The performance of the developed experimental plans should then be compared to existing strategies so as to measure its effectiveness.

In addition, future research can focus on using additional case studies to validate the performance of the SFP relative to the OAs. The testing of the SFP and the OA using real industrial experiments may not be feasible due to the number of experimental runs required to conduct 3^3 and 3^4 full factorial experiments and also, tailoring the experiments to represent the scenarios tested in this study will be difficult as it is not possible for the experimenter to select the scenarios to be tested before experimentation.

Hence it is suggested that less expensive laboratory experiments be used to identify factors and factor levels which represent the experimental scenarios investigated in this study.

Finally, further research is needed to identify ways to better quantify the performance of the SFP. One way to do this is by characterising the relative probabilities of interactions which act opposite to and in the direction of main effects when the main effects are positively and negatively exploited. This way, the performance of the SFP can be quantified when the most important factor in the 3-level full factorial experiment and the full factorial involving the medium settings of the factors and their best setting from the first full factorial experiment are the same or otherwise.

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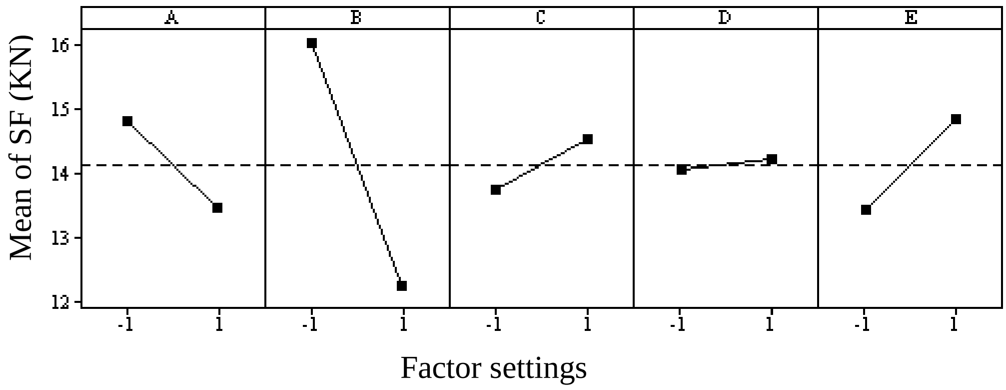
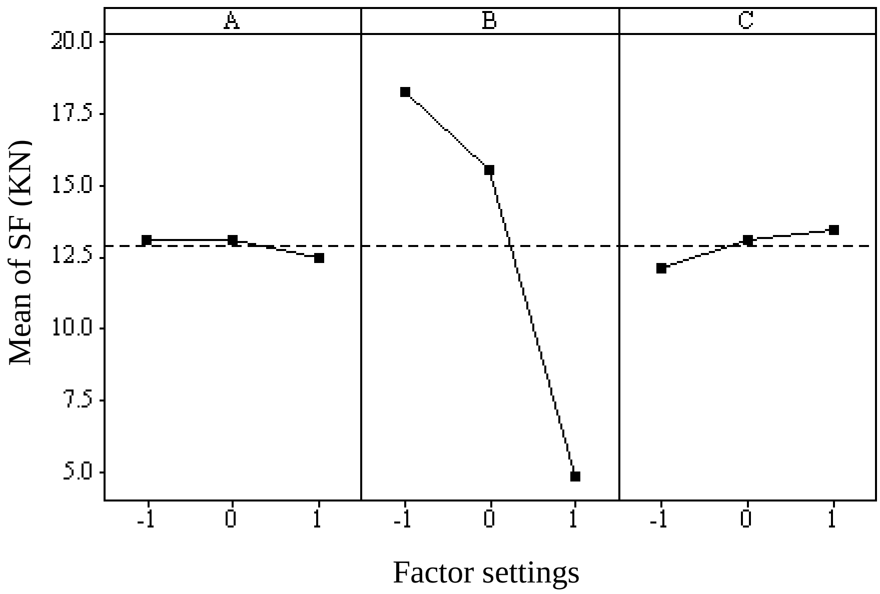
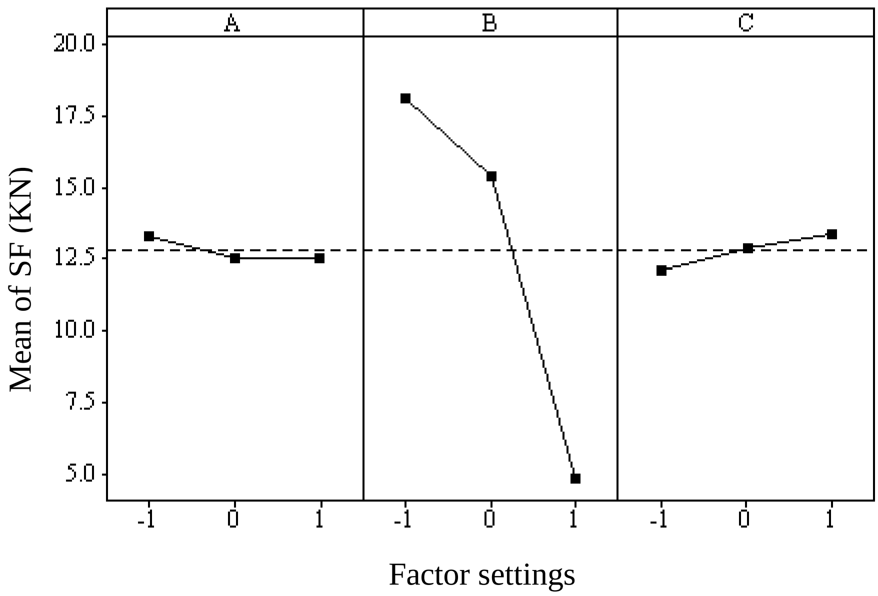
APPENDIX A Factor effects plot for experimental data set



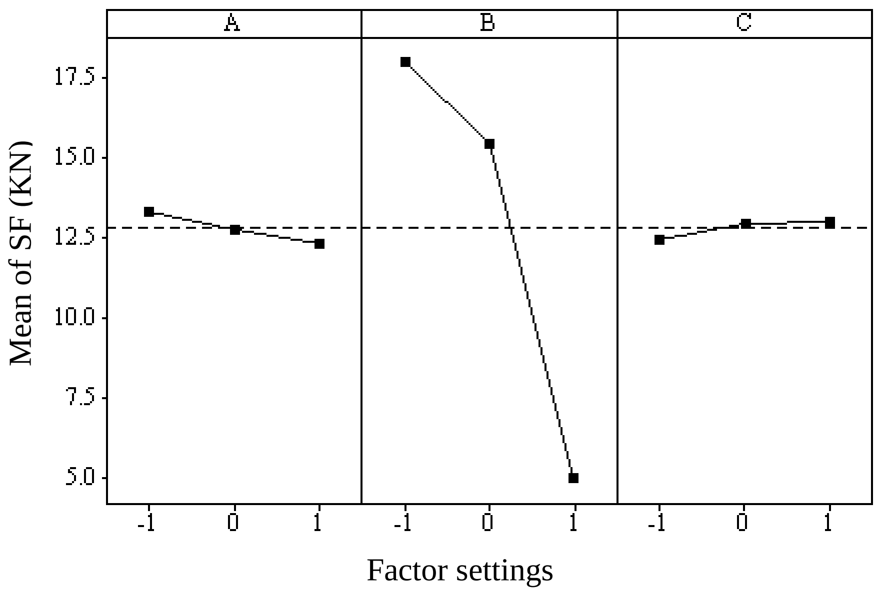
Main effect plot for the 2^{5-1} fractional factorial spot welder experiment



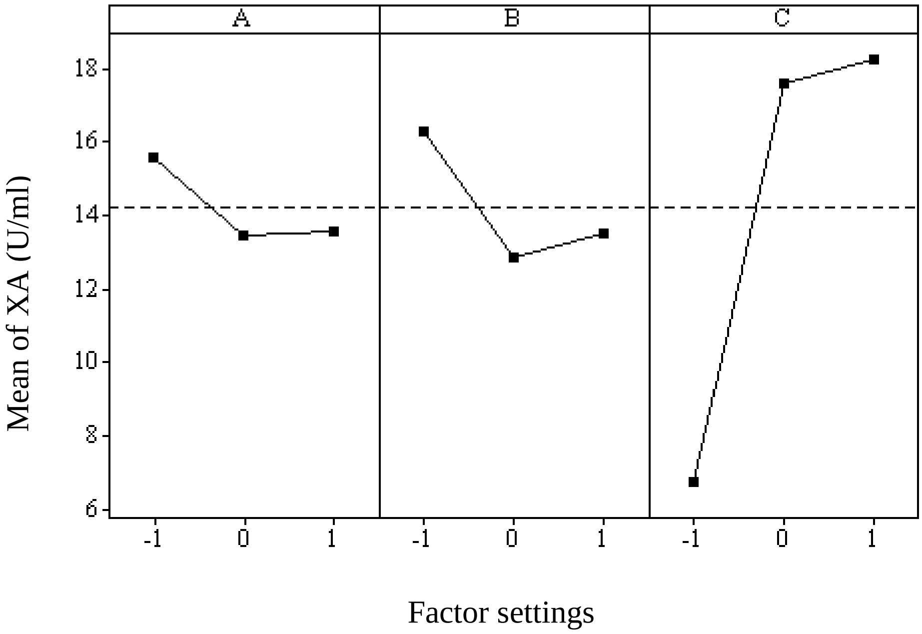
Main effect plot for the 9-run OA (spot welder experiment)



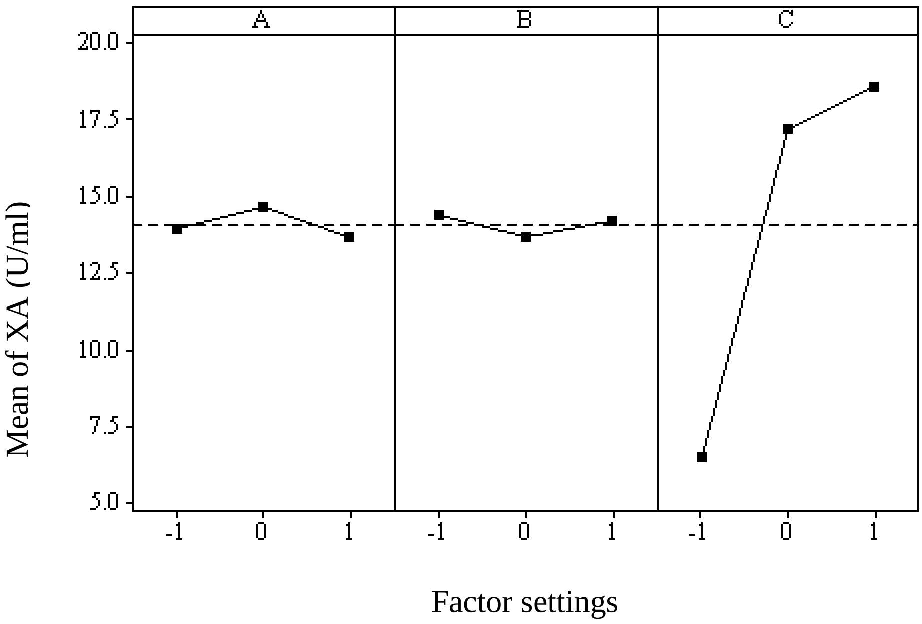
Main effect plot for the 18-run OA (spot welder experiment)



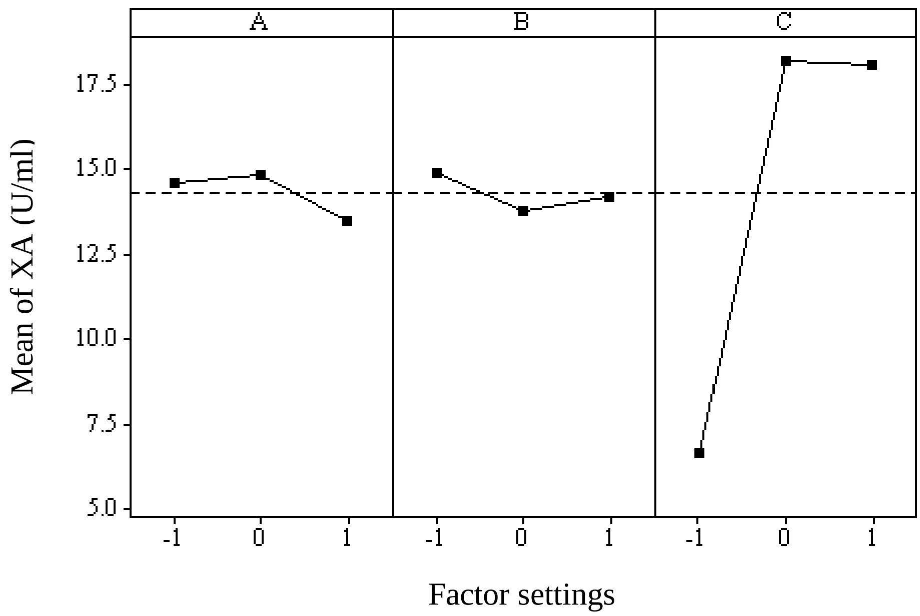
Main effect plot for the 3^3 full factorial (spot welder experiment)



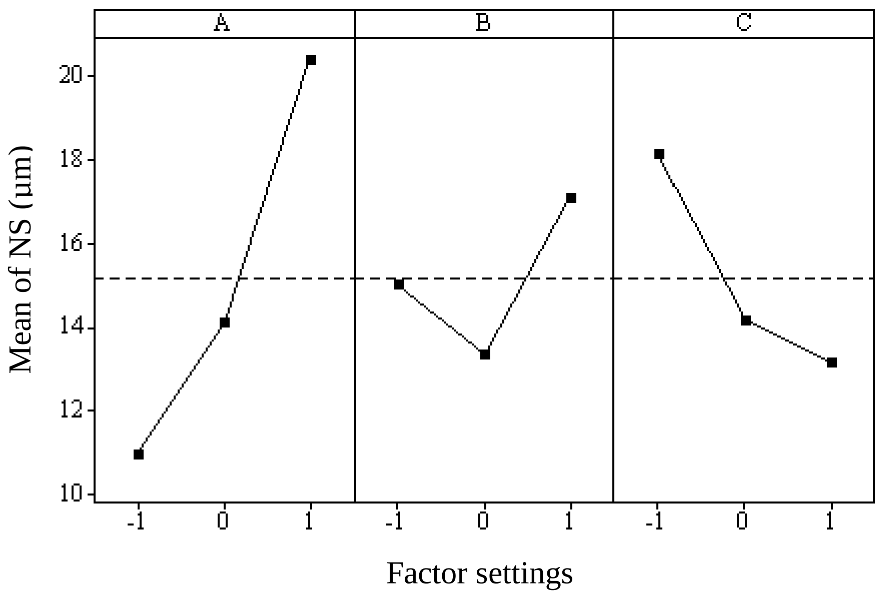
Main effect plot for the 9-run OA (XA experiment)



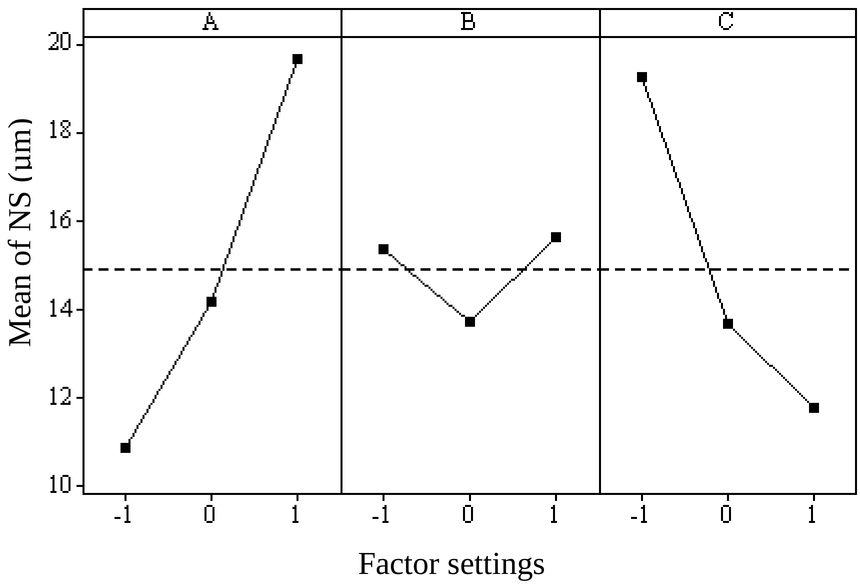
Main effect plot for the 18-run OA (XA experiment)



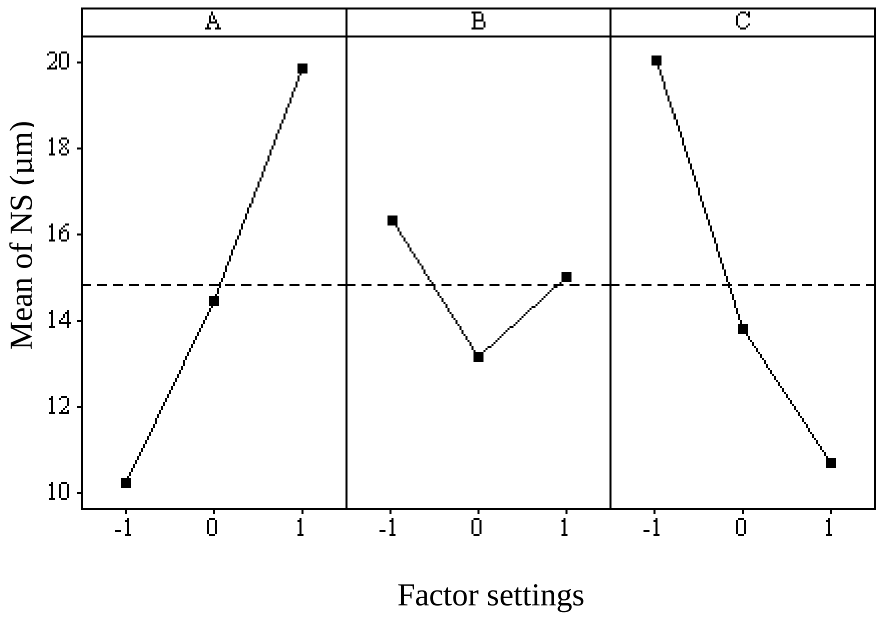
Main effect plot for the 3^3 full factorial (XA experiment)



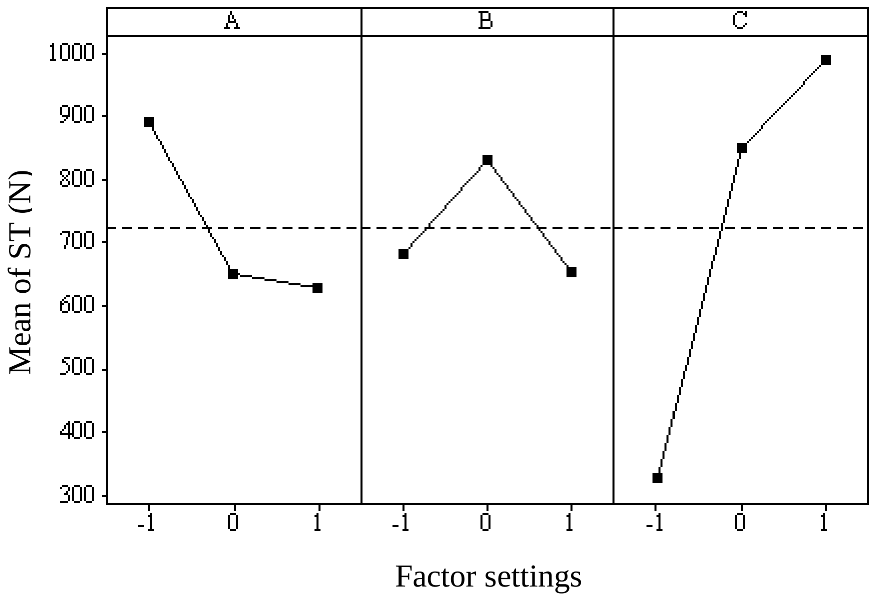
Main effect plot for the 9-run OA (NS experiment)



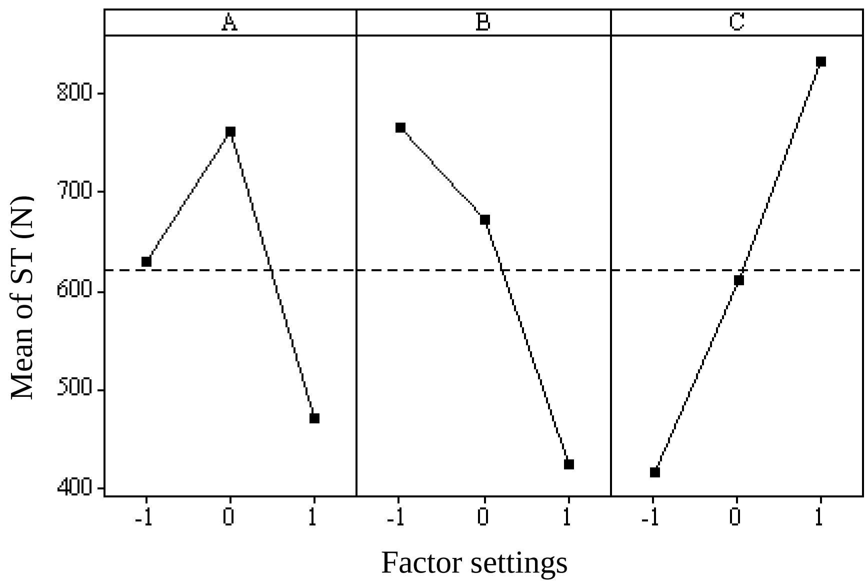
Main effect plot for the 18-run OA (NS experiment)



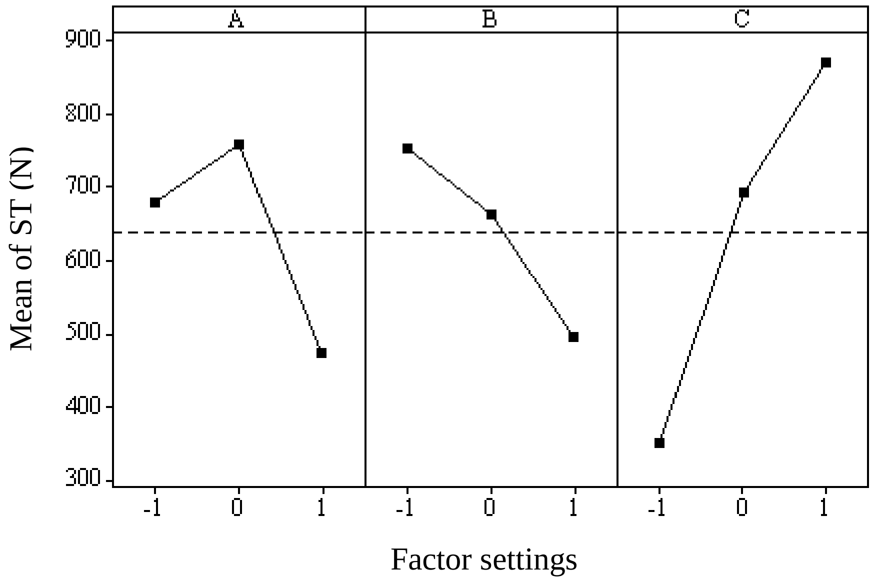
Main effect plot for the 3^3 full factorial (NS experiment)



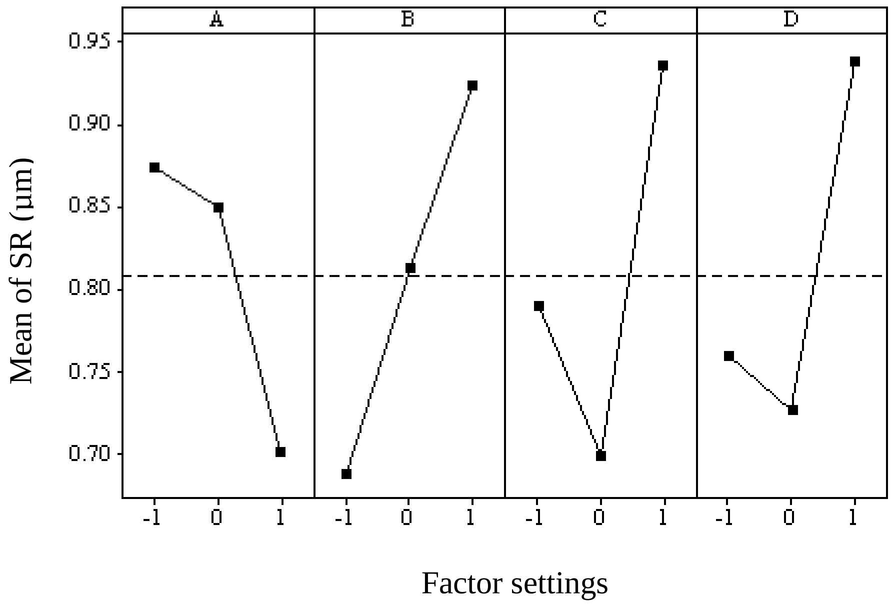
Main effect plot for the 9-run OA (ST experiment)



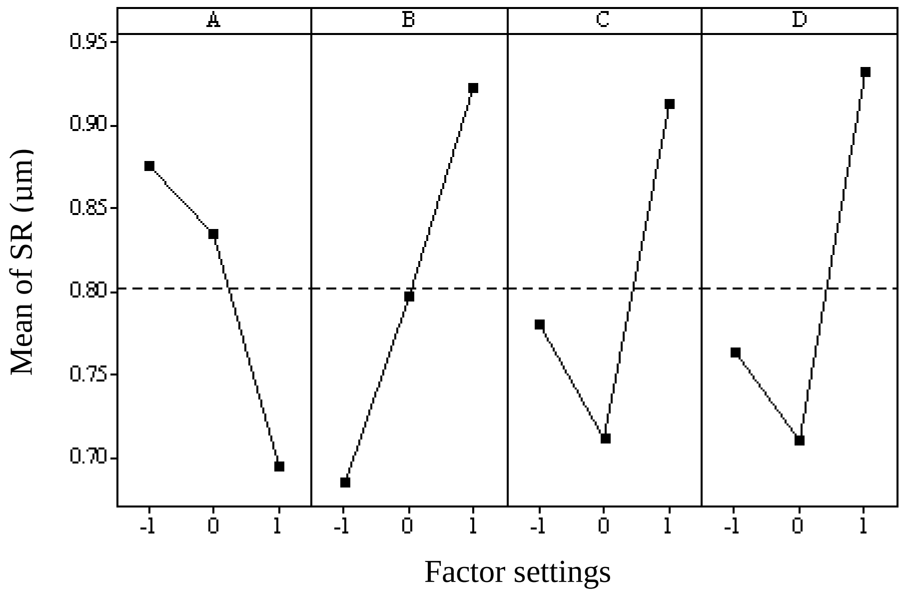
Main effect plot for the 18-run OA (ST experiment)



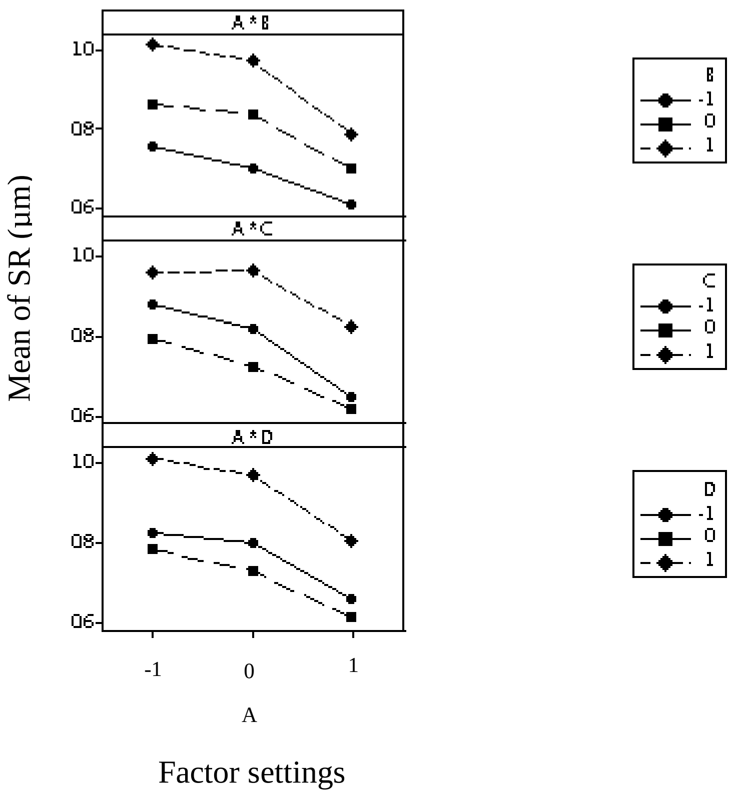
Main effect plot for the 3^3 full factorial (ST experiment)



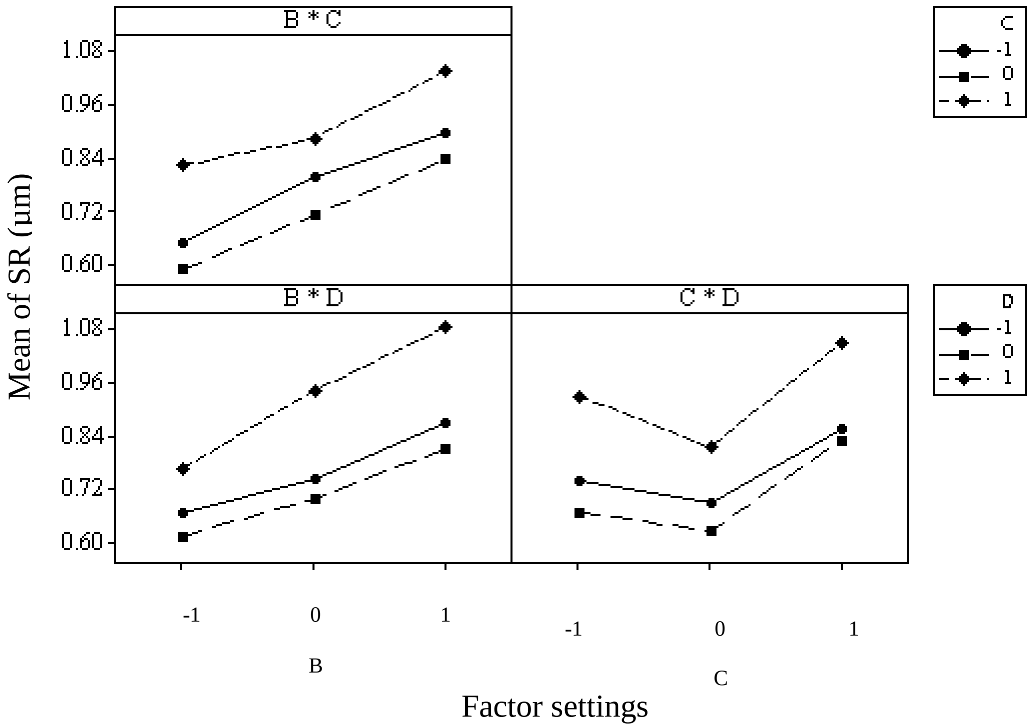
Main effect plot for the 18-run OA (SR experiment)



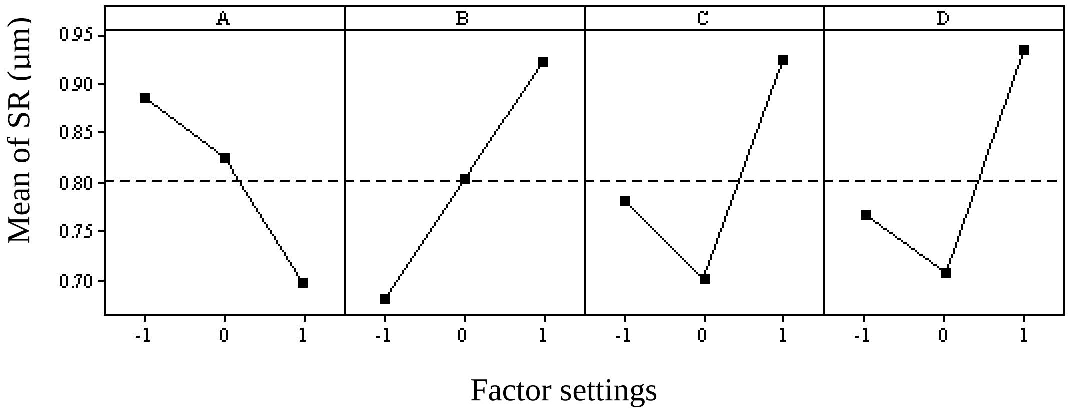
Main effect plot for the 27-run OA (SR experiment)



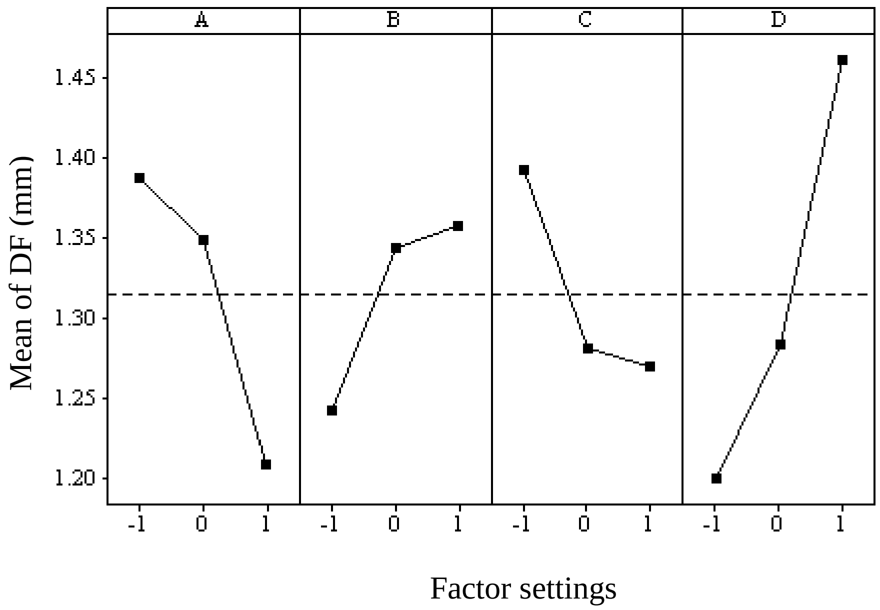
Interaction effect plot for the 27-run OA (SR experiment)



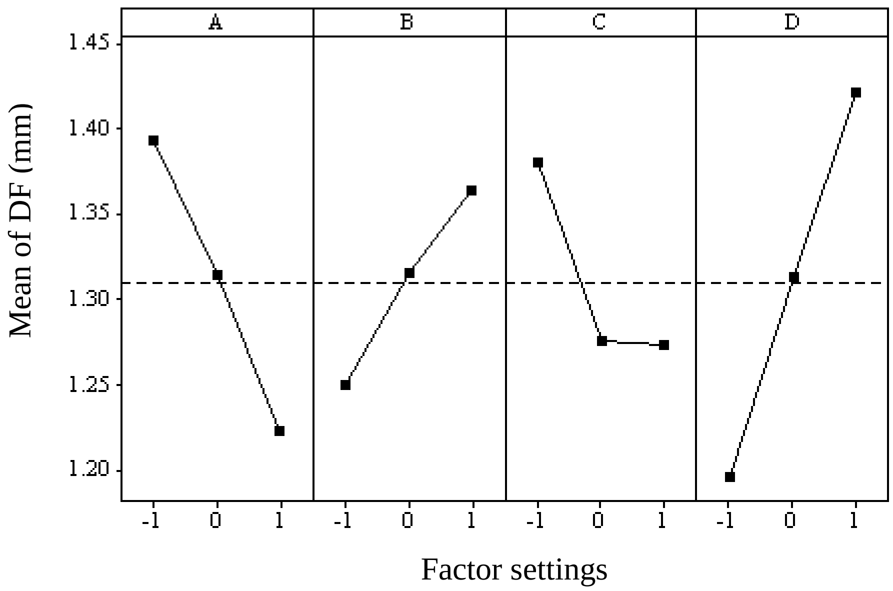
Interaction effect plot for the 27-run OA (SR experiment)



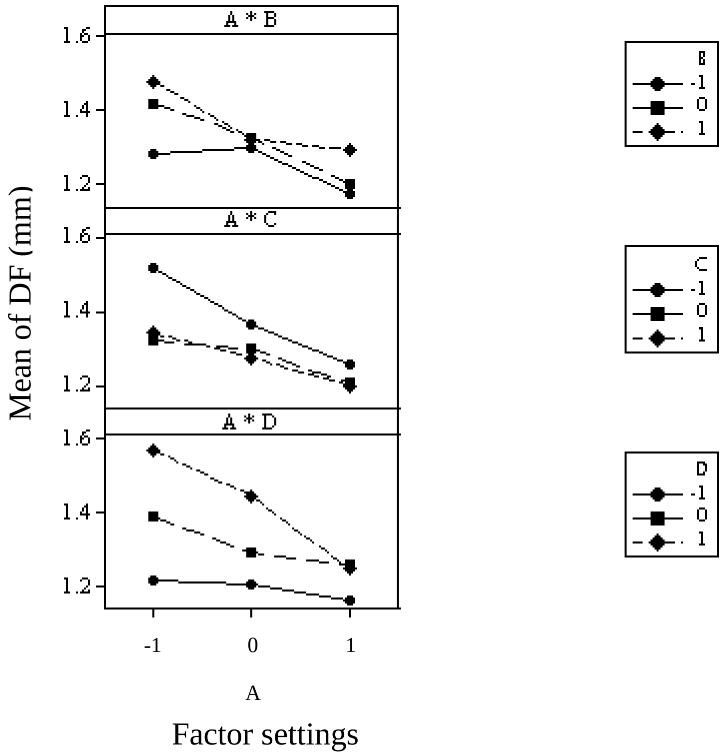
Main effect plot for the 3^4 full factorial (SR experiment)



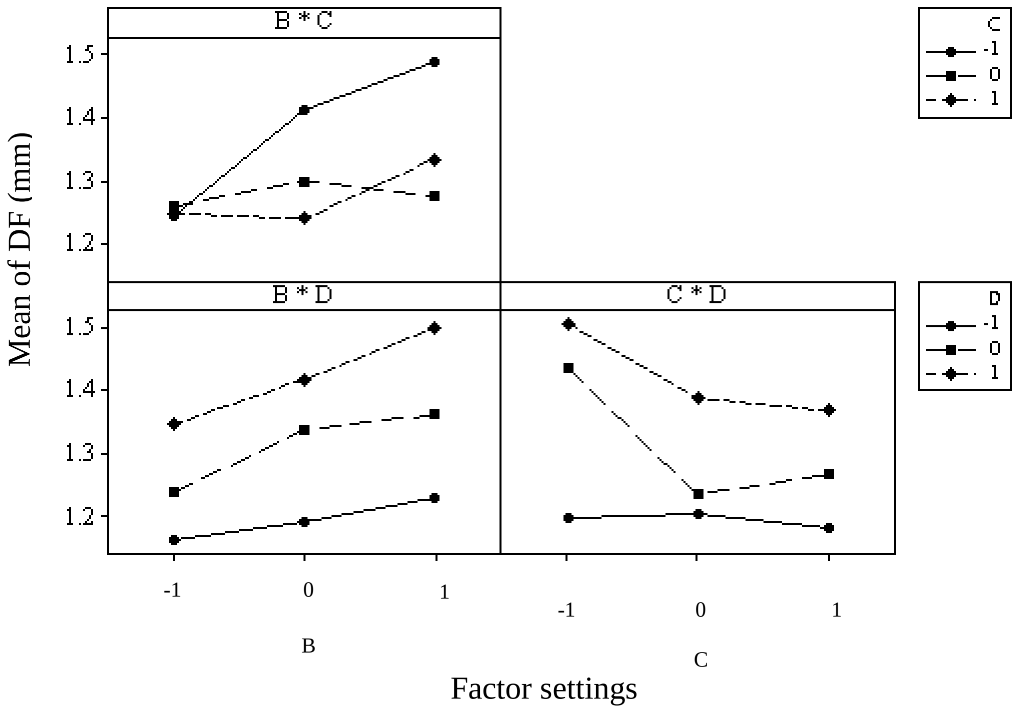
Main effects plot for 18-run OA (DF experiment)



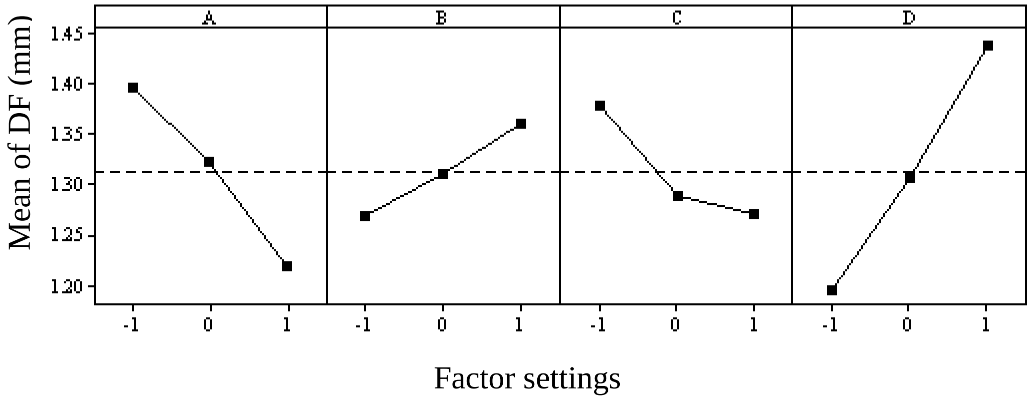
Main effect plot for 27-run OA (DF experiment)



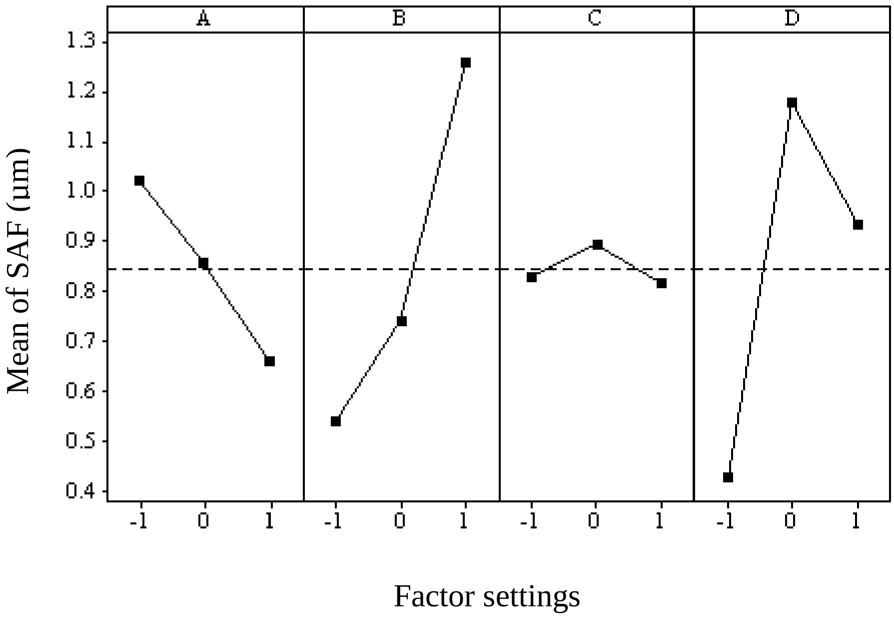
Interaction effect plot for 27-run OA (DF experiment)



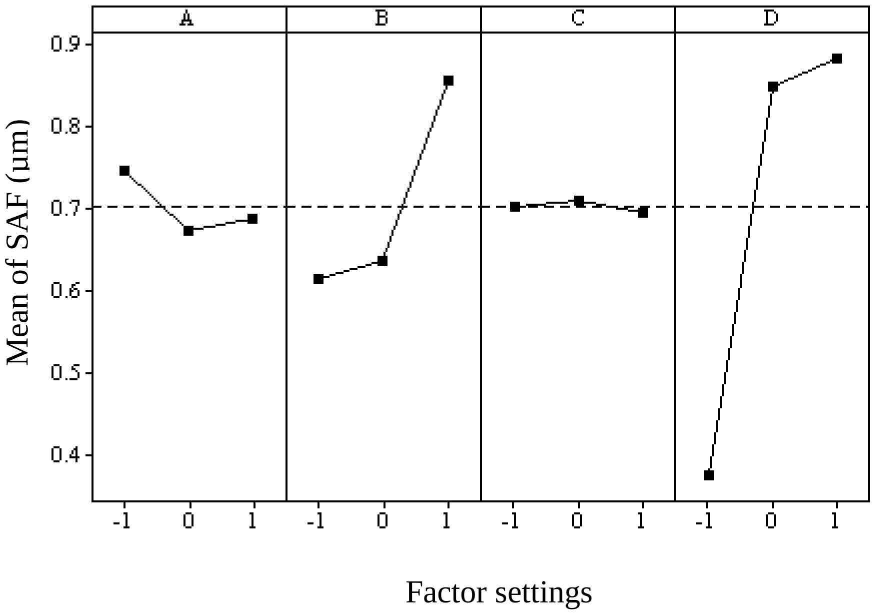
Interaction effect plot for 27-run OA (DF experiment)



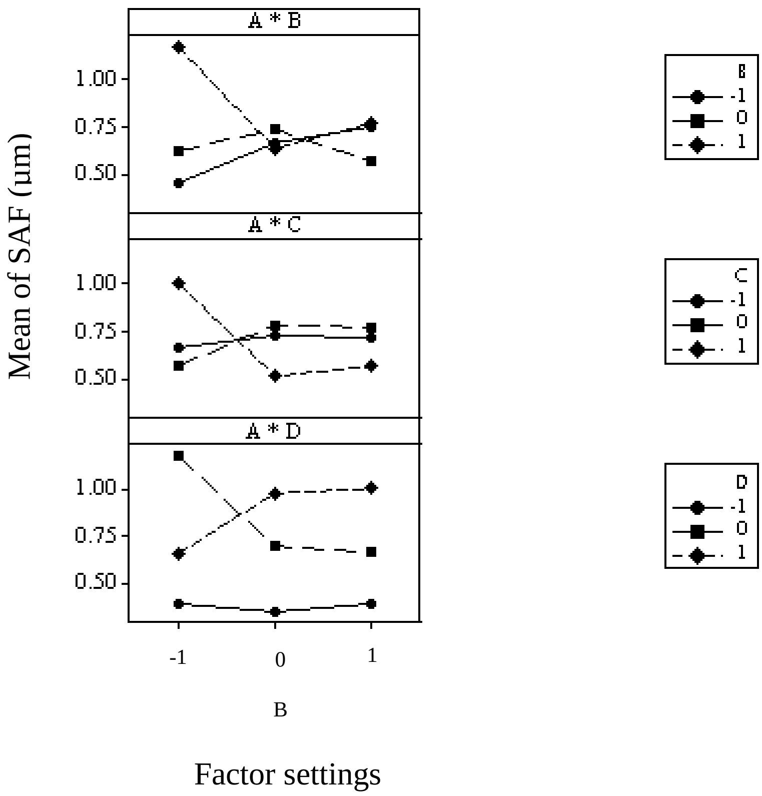
Main effect plot for 3^4 full factorial (DF experiment)



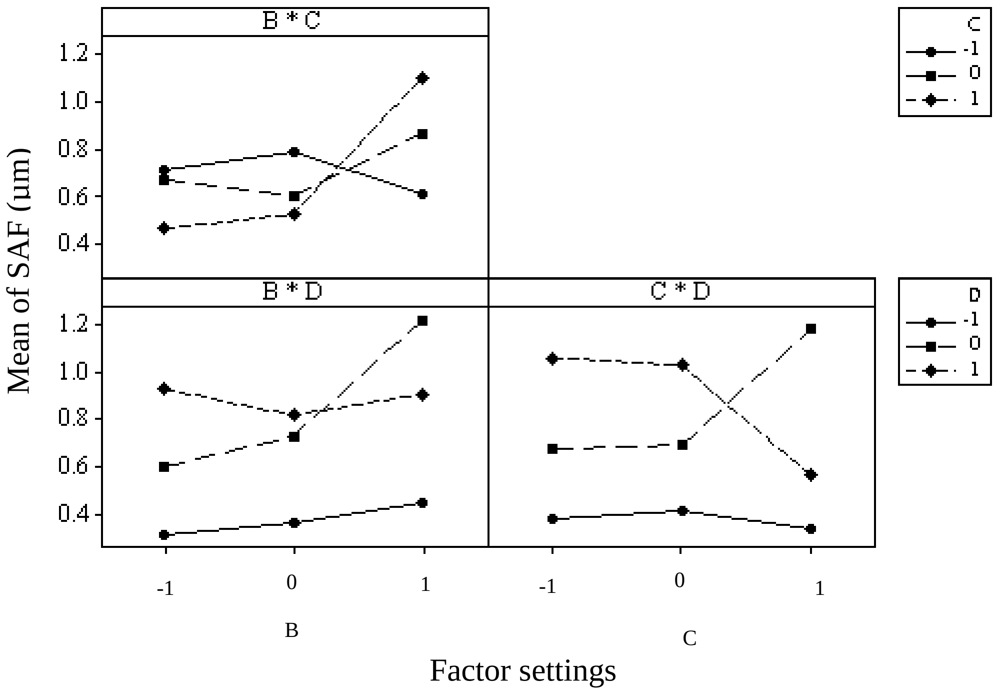
Main effect plot for 18-run OA (SAF experiment)



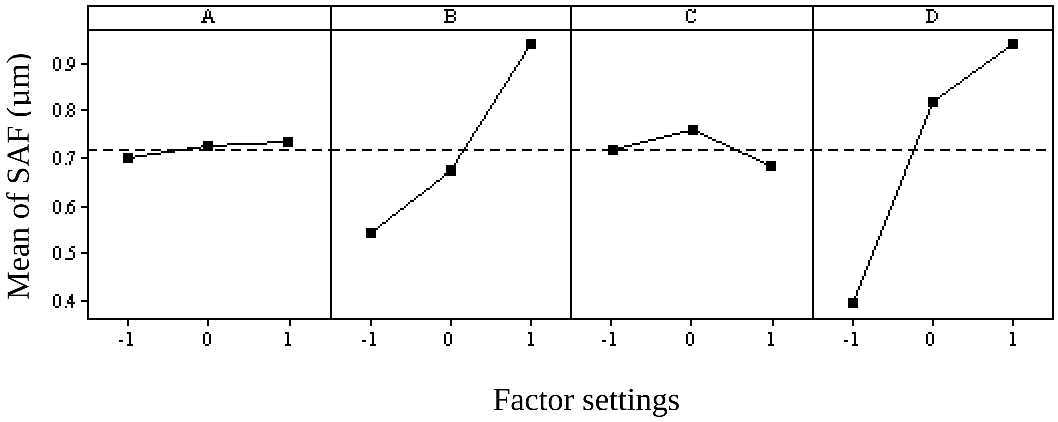
Main effect plot for 27-run OA (SAF experiment)



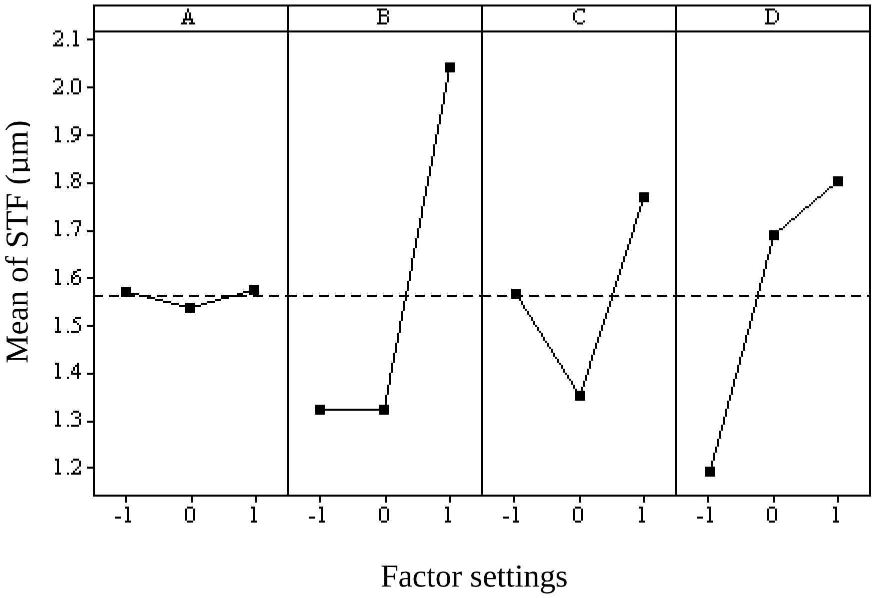
Interaction effect plot for 27-run OA (SAF experiment)



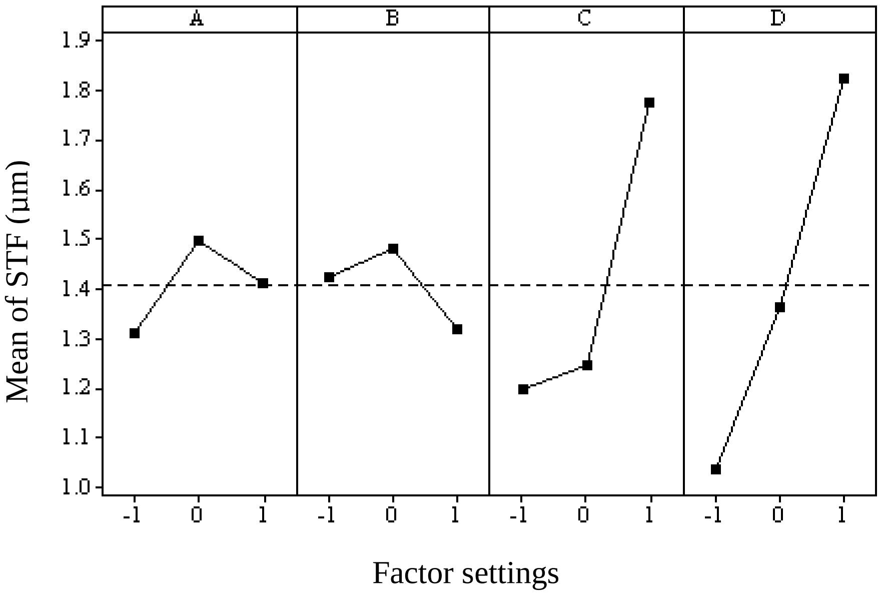
Interaction effect plot for 27-run OA (SAF experiment)



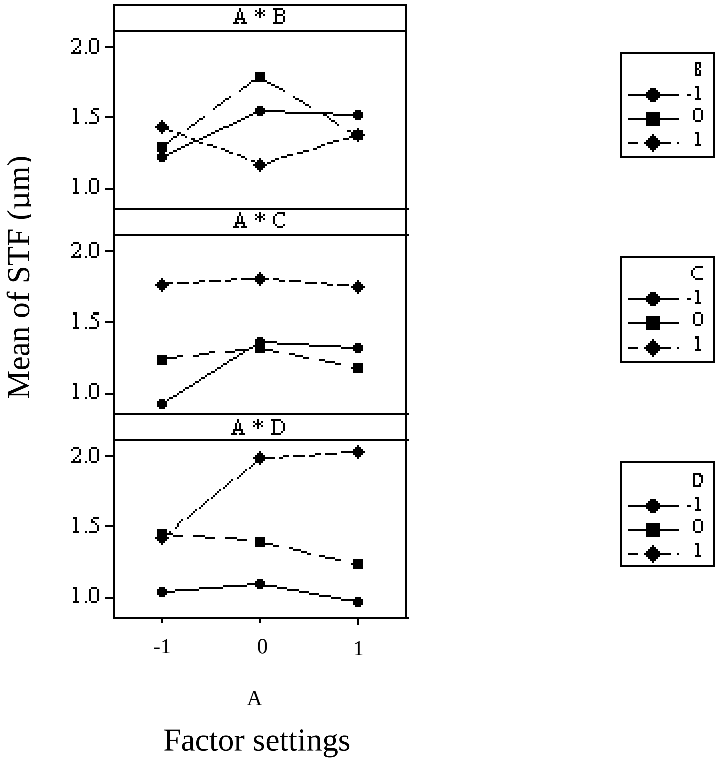
Main effects plot for 3^4 full factorial (SAF experiment)



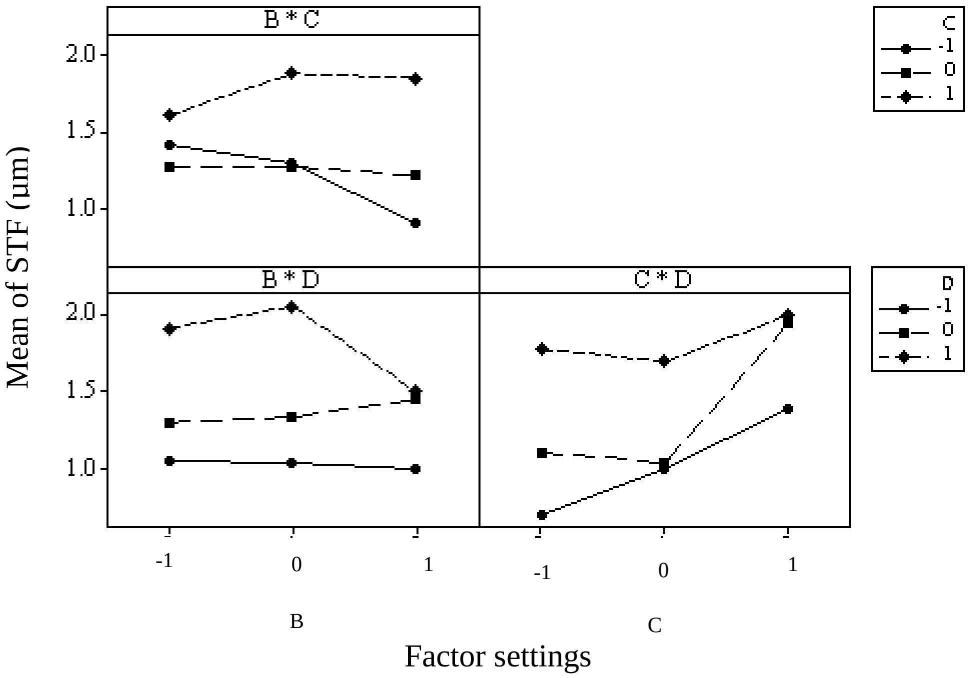
Main effect plot for 18-run OA (STF experiment)



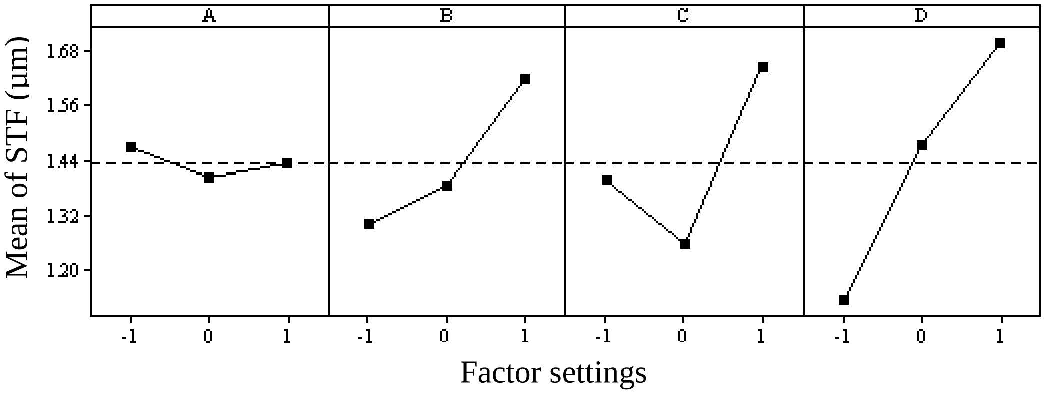
Main effect plot for 27-run OA (STF experiment)



Interaction effect plot for 27-run OA (STF experiment)



Interaction effect plot for 27-run OA (STF experiment)



Main effect plot for 3^4 full factorial (STF experiment)

APPENDIX B Orthogonal array tables

9-run OA for 3 factors

	Factor columns		
Run	**1**	**2**	**3**
1	0	0	0
2	0	1	1
3	0	2	2
4	1	0	1
5	1	1	2
6	1	2	0
7	2	0	2
8	2	1	0
9	2	2	1

18-run OA for 3 factors

	Factor columns		
Run	**1**	**2**	**3**
1	0	0	0
2	1	1	1
3	2	2	2
4	0	0	1
5	1	1	2
6	2	2	0
7	0	1	0
8	1	2	1
9	2	0	2
10	0	2	2
11	1	0	0
12	2	1	1
13	0	1	2
14	1	2	0
15	2	0	1
16	0	2	1
17	1	0	2
18	2	1	0

18-run OA for 4 factors

	Factor columns			
Run	**1**	**2**	**3**	**4**
1	0	0	0	0
2	1	1	1	1
3	2	2	2	2
4	0	0	1	1
5	1	1	2	2
6	2	2	0	0
7	0	1	0	2
8	1	2	1	0
9	2	0	2	1
10	0	2	2	1
11	1	0	0	2
12	2	1	1	0
13	0	1	2	0
14	1	2	0	1
15	2	0	1	2
16	0	2	1	2
17	1	0	2	0
18	2	1	0	1

27-run OA for 4 factors

	Factor columns			
Run	**1**	**2**	**3**	**4**
1	0	0	0	0
2	0	0	1	1
3	0	0	2	2
4	0	1	0	1
5	0	1	1	2
6	0	1	2	0
7	0	2	0	2
8	0	2	1	0
9	0	2	2	1
10	1	0	0	1
11	1	0	1	2
12	1	0	2	0
13	1	1	0	2
14	1	1	1	0
15	1	1	2	1
16	1	2	0	0
17	1	2	1	1
18	1	2	2	2
19	2	0	0	2
20	2	0	1	0
21	2	0	2	1
22	2	1	0	0
23	2	1	1	1
24	2	1	2	2
25	2	2	0	1
26	2	2	1	2
27	2	2	2	0