How periodic driving heats a disordered quantum spin chain

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We study the energy absorption in real time of a disordered quantum spin chain subjected to coherent monochromatic periodic driving. We determine characteristic fingerprints of the well-known ergodic (Floquet-ETH for slow driving/weak disorder) and many-body localized (Floquet-MBL for fast driving/strong disorder) phases. In addition, we identify an intermediate regime, where the energy density of the system – unlike the entanglement entropy a local and bounded observable – grows *logarithmically* slowly over a very large time window.

Introduction. Periodically-driven, or Floquet, many-body quantum systems are a current focus of out-of-equilibrium physics. Generically, an external forcing pushes the system away from equilibrium and heats up the system, as is natural for a non-adiabatic perturbation. Ergodic many-body systems in particular heat up to reach a fully-mixed state (also known as "infinite-temperature" or Floquet-ETH state) [1–3]. Interestingly, such a heat death of the correlations can be avoided in the presence of constraints frustrating the entropy increase. One possibility is in a Floquet-integrable system where there exist quantities conserved even in the presence of driving. This leads to a synchronised state maximizing entropy, but now subject to constraints imposed by those conserved quantities [4]; this is known as periodic Gibbs ensemble, in analogy to the generalised Gibbs ensemble of static systems [5].

An alternative, more robust way to prevent full heating – not requiring the fine-tuning needed for integrable or dynamically localized [6–8] behavior – is provided by many-body localization (MBL) [9–11]. Here, the addition of sufficiently strong disorder to an interacting, ergodic system leads to vanishing energy, particle and spin transport so that ergodicity is broken in the static (time-independent) limit. When switching on periodic driving, a region in the driving frequency-amplitude plane exists in which the system approaches a state that is not fully mixed, and in particular has finite energy with respect to, e.g., the static, or average, Hamiltonian [12, 13]. In this regime the effective Hamiltonian governing the stroboscopic dynamics may exhibit sharply distinct phases, characterized by order parameters, including ones with no equilibrium counterparts [14].

What this classification leaves open entirely is how the process of synchronization takes place, i.e. how the steady state is approached in real time. This question is not only of intrinsic fundamental importance, but it also occurs in the context of practical applications, such as in Floquet-engineering cold atomic systems [16, 17], where one is interested to what extent generating interesting effective Hamiltonians unavoidably goes along with heating [18].

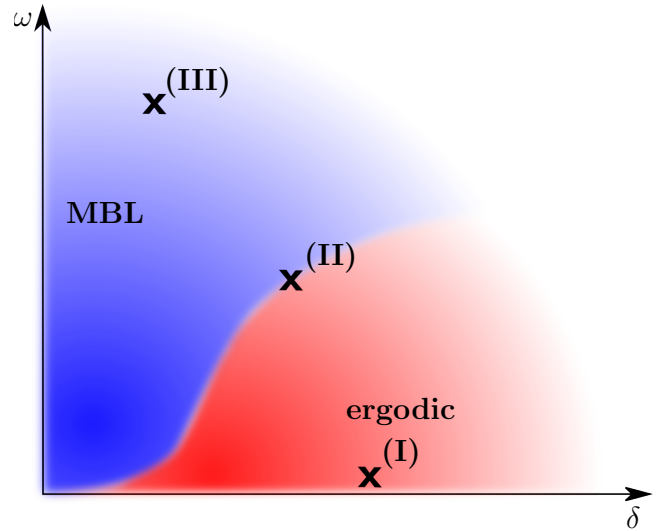In this article, we analyze this process in a setting which combines disorder, interactions, and driving. In

FIG. 1. (color online) Proposed phase diagram for the long time state of a driven strongly, disordered system as a function of the driving frequency ω and strength δ. Red (I) and blue (III) indicate Floquet-ETH and Floquet-MBL behavior, where the system approaches a fully-mixed, "infinite-temperature" state or remains localized, respectively. At (II), heating leads to energy growing logarithmically slowly with time (Fig. 2(II)) over a broad time window. Our numerics cannot access small ω and δ near the origin where Ref. 15 suggests the indicated behavior.

particular, we study how energy is absorbed in real time. We consider a disordered spin chain initially in the ground state of a static Hamiltonian, and monitor stroboscopically its energy density with respect to this Hamiltonian upon switching on the periodic drive.

In the strongly disordered case, with parameters chosen such that the static model is MBL, we confirm the existence of the localized and ergodic regimes under driving, and describe characteristic properties of their heating process. In the ergodic regime, we find that the energy indeed saturates at the maximum-entropy, "infinite-temperature" value (as it does when starting from the weakly disordered ergodic regime); the rate at which this

is reached scales quadratically with the driving amplitude, δ, consistent with a Fermi Golden rule-type picture.

In the localized regime driving results in a non-zero excess energy density, ϵ_∞, which is reached relatively swiftly. For a driving frequency, ω, well above the otherwise dominant local disorder strength, η, we find an asymptotic dependence $\epsilon_\infty \propto \delta^2/\eta^2$, varying parameters beyond which leads to deviations presaging the delocalization transition. This can be understood via the behaviour of driven two-level systems.

Most remarkably, at the crossover between the two we find a *logarithmically slow* heating process, with energy entering the system over a window extending over several decades in time. This is superficially reminiscent of the characteristic logarithmic growth of the entanglement entropy in a static MBL system [19–23]. However, the internal energy is a quantity which is a local observable, for which no such result is known. Also, given the boundedness of the internal energy in the model we study, the time window of the logarithmic growth is necessarily limited; however, this phenomenon is comfortably visible in our numerics already for chains of length $L = 20$; an extrapolation of our results suggests it may extend over a window extending over more than eight decades in time.

Model. We study the spin-1/2 XXZ chain in a disordered longitudinal field subject to a monochromatically driven staggered field with period $T = 2\pi/\omega$:

$$H(t) = H_0 + H_D(t) \tag{1}$$

$$H_0 = J_\perp \sum_{i=0}^{L}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_z \sum_{i=0}^{L} S_i^z S_{i+1}^z \tag{2}$$

$$+ \sum_{i=0}^{L} h_i^z S_i^z, \tag{3}$$

where $h_i^z \in [-\eta, \eta]$, $J_\perp, J_z \geq 0$, and with driving

$$H_D(t) = -\delta \cos \omega t \sum_{i=0}^{L}(-1)^i S_i^z. \tag{4}$$

The static part H_0 is known to be MBL for $J_z \neq 0$ and sufficiently strong disorder $\eta > \eta_c$ [11, 24]. Driving such MBL systems is expected to lead to delocalization [12, 13] for low enough frequencies at fixed δ, yielding the phase diagram schematically shown in Fig. 1, with a high-frequency regime Floquet-MBL and a low-frequency Floquet-ETH regime where a fully-mixed state is approached [15].

Here we concentrate on real-time dynamics, in particular the (stroboscopic) time evolution of the energy in the system, expressed via the rescaled excess energy density:

$$\epsilon(nT) = \frac{\langle \psi | H(nT) | \psi \rangle - E_{\min}}{\overline{E} - E_{\min}}, \tag{5}$$

with $\overline{E} = D_H^{-1}\text{tr}[H(0)]$ and D_H being the Hilbert space dimension, so that $\epsilon = 0$ in the ground state of $H(0)$, while $\epsilon = 1$ for the fully mixed ensemble.

In what follows we discuss in detail different parameter regimes and characterize their dynamical properties. In the case of strong disorder, where the static system is in the MBL regime, we study three points on the phase diagram in Fig. 1, corresponding to the above mentioned behaviors (I), (II) and (III), in detail numerically by finite-size simulations.

For all our numerical studies we initialize the system in the ground state of the static Hamiltonian at $t = 0$, $H(0)$, so that $\epsilon(0) = 0$ and fix the interaction strength to $J_\perp = 1$ and $J_z = 0.5$. The ground state is obtained from the sparse matrix representation of the static Hamiltonian using the Lanczos method and for the time evolution of the driven system we use an iterative Krylov space based algorithm [25]. The harmonic driving is discretized using sufficiently small time steps of $\delta t = 0.0250 J_\perp^{-1}$ ($\delta t = 0.0025 J_\perp^{-1}$ for large frequencies). We average the results over $N_{\text{disorder}} = 300\text{-}500$ disorder realizations. For all simulations we utilise the conservation of S_z^{total} and consider only the $S_z^{\text{total}} = 0$ sector, allowing for a considerable speedup.

Strong disorder. We begin by setting the disorder strength to $\eta = 5$, which puts the static Hamiltonian comfortably in its MBL phase, and choose a driving amplitude $\delta = 2$. For a driving frequency of $\omega = 0.314$, corresponding to (I) in Fig. 1, we find that the system is in the ergodic regime, approaching the "infinite temperature" state, with energy $\overline{E}$ corresponding to $\epsilon_\infty = 1$, in the long time limit (Fig. 2, leftmost panel). While finite-size effects are visible at the sizes displayed, there is convergence to $\epsilon_\infty = 1$ with increasing system size.

Next we increase the frequency to $\omega = 8.38$ and choose $\delta = 0.6, 1.8, 3.0$, finding that all these points lie in the Floquet-MBL phase corresponding to the neighborhood of point (III) in Fig. 1. Here the system at first absorbs energy comparatively quickly but stops well short of the "infinite-temperature" point (Fig. 2, rightmost panel). The saturated value of ϵ_∞ depends on the system parameters, and increases with δ. This is the Floquet-MBL phase and it is stable for a range of δ. Notice also that there is very little system-size dependence in the results, indicating that for the Floquet-MBL regime finite-size effects are much weaker than for the ergodic regime of panel (I), as expected for a system with a finite localization length. This is also consistent with the results of Ref. [12], where level statistics results also lie in-between the localized (Poisson) and ergodic (circular unitary ensemble) results. In this regime ergodicity is broken, there exist local integrals of motion and therefore the final state is not fully-mixed. We explore the dependence of the absorbed energy on δ further down.

Finally we select $\omega = 4.19$ and $\delta = 1.8$, corresponding to point (II) in Fig. 1 which lies in between the well-localized and comfortably ergodic regimes. While the energy density grows to its "infinite-temperature" value as in the ergodic case (I), its logarithmically slow growth
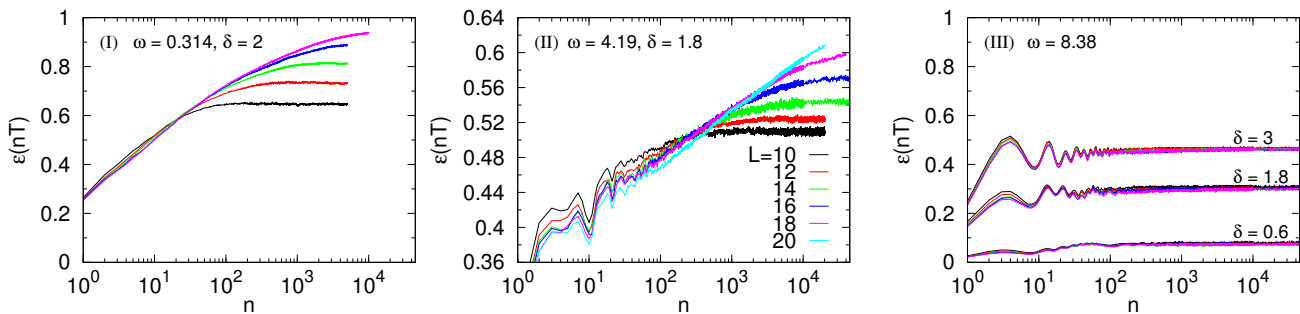
FIG. 2. (color online). Energy absorption in a strongly disordered system with $\eta = 5$ and $J_z = 0.5$, corresponding to different locations in the phase diagram, Fig. 1. At (I), an initially MBL system delocalizes and heats up to a fully-mixed state. In the intermediate regime, (II), the system heats up to the fully-mixed state, but logarithmically slowly. This slow growth persists for longer times as we increase L. For Floquet-MBL, (III), driving does not delocalize the system, leading instead to a localized long-time state which has partially heated up to some intermediate energy. Fig. 3 explores the dependence of the final energy on the driving amplitude δ.

extends over several decades (Fig. 2, middle panel)! This logarithmic growth strikingly visible in the crossover regime has not been observed before and is the central result of our paper. We emphasize that this phenomenon is only superficially reminiscent of the logarithmic growth of the entanglement entropy described in Ref. [19, 20] and later on also observed in Floquet-MBL systems [13]. The entanglement growth is by now well-understood in terms of the structure of the MBL Hamiltonian in terms of local "l-bits" [26–29]. Most fundamentally, the quantity ϵ_∞ we consider is a fully local observable. Further, it is bounded, so that the logarithmic growth eventually has to terminate. However, this leaves a very large scope for the logarithmic growth—extrapolating the growth in the middle panel of Fig. 2 all the way until the maximum $\epsilon_\infty = 1$ is reached allows for a time window covering 8 or 9 decades.

The slow logarithmic growth in regime (II) is astonishingly pronounced. By contrast, the curves for (I) and particularly (III) plateau out much earlier, so that it is not possible to determine whether the rise is sensibly described by a quickly-terminated logarithmic growth akin to the slowly-terminated window in (II), or rather by a different functional form. We do note that the curves for all values of L agree for short times in Fig. 2(I), defining a limiting curve in the thermodynamic limit from which finite-size systems peel off at a time which grows with L. The shape of the limiting curve is not inconsistent with logarithmic growth, albeit over an inconclusively limited timespan.

Let us now turn to the behavior of ϵ_∞ as a function of δ, plotted in Fig. 3 for different system sizes L (upper panel). In the ergodic regime (upper set of points) and for sufficiently large δ, the energy density ϵ_∞ increases with increasing system size L, indicating that it will approach the fully-mixed value $\epsilon_\infty = 1$ for large enough L.

For smaller δ, ϵ_∞ still increases with L but more slowly. This is consistent with the phase diagram in Fig. 1, since

a smaller δ means the system is closer to the ergodic regime. The finite-size numerics reported here cannot determine whether or not there eventually is a critical δ below which MBL survives for any value of driving frequency ω.

In the localized regime (lower set of points) the saturation value grows approximately linearly for an intermediate range of δ, sandwiched between ergodic saturation for large δ, and a quadratic regime for small δ. Here, the saturation value does not depend on system size, indicating the presence of an appropriately defined localization length beyond which an increase in L no longer changes ϵ_∞, so that even in the thermodynamic limit the system absorbs only a finite amount of energy per unit length but stops short of heating up to completely. This behavior is consistent with the presence of local integrals of motion [26–30] in the effective Hamiltonian. These can then play the role of conserved quantities for the driven problem, restricting in turn the growth of entropy and thus making the fully-mixed state with $\epsilon_\infty = 1$ inaccessible for the given initial condition. The accessible region of Fock space grows as the transition point is approached, leading to the observed increase of ϵ_∞.

The lower panel of Fig. 3 shows the final value of the energy density for large values of ω focusing on the region of asymptotically small driving amplitudes. For frequencies large enough (but still below the many-body bandwidth of our finite-size systems), data corresponding to different disorder strengths η collapse and scale with δ^2/η^2 (lower set of points). This dependence is in agreement with that exhibited by a set of independent, driven two-level systems, again indicating that the local integrals of motion do indeed survive in the Floquet-MBL system, as described in the appendix. Once this behavior sets in, transitions to higher energy states are suppressed, and the system absorbs almost no energy.

Weak disorder. We finally turn to the case where the static Hamiltonian itself is still disordered but not in the
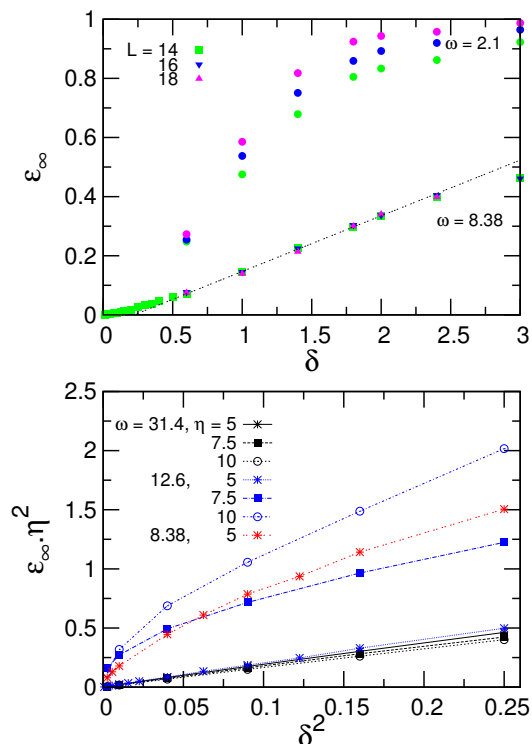
FIG. 3. (color online). Upper panel: Final excess energy density ϵ_∞ vs. driving amplitude δ for two values of the frequency, interaction strength $J_z = 0.5$ and disorder strength $\eta = 5$ corresponding to MBL in the static case. For the upper set of data, corresponding to the Floquet-ETH regime, increasing system size leads to increasing ϵ_∞, so that it approaches 1 in the thermodynamic limit, the fully-mixed result. For the lower group of curves, corresponding to the localized regime, the energy saturates to a finite value $\epsilon_\infty < 1$ depending on δ, implying a limited absorption of energy and the presence of a finite localization length. Lower panel: Same quantity for large values of ω, in a regime of small driving amplitudes. For frequency sufficiently large compared to η (lower set of points) scaling collapse of data corresponding to different disorder amplitudes η is possible ($L = 14$). In this panel, points of the same color have the same ω while points of the same shapes have the same η.

MBL phase, Fig. 4. In this regime the system fully heats up as in the clean case [12]. The inset in Fig. 4 shows the bare data while the main plot has the time rescaled by a factor of δ^2. The heating thus depends on the amplitude δ and time n (measured in units of the stroboscopic step) via the combination $\delta^2 n$. This is consistent with the expectation from leading-order perturbation theory (as in Fermi's Golden Rule), which gives the observed dependence on the square of the driving amplitude δ. This is in contrast to the Floquet-MBL case (Fig. 2) where no such collapse occurs, demonstrating the breakdown of linear response for the long-time behavior of the system.

Conclusions We have studied the energy absorption in real time of a disordered quantum spin chain subjected to coherent monochromatic periodic driving in different
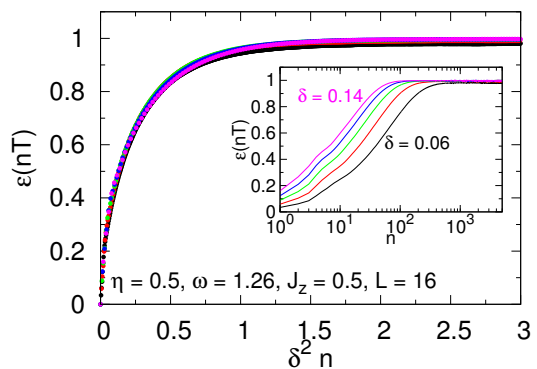

FIG. 4. (color online). Energy absorption for different driving amplitudes in a weakly disordered system with $\eta = 0.5$ and $J_z = 0.5$. The inset shows the bare data while the main plot shows the collapsed data after rescaling the time coordinate by δ^2.

parameter regimes. For strongly disordered systems, in which the static Hamiltonian is in the many-body localized phase, we have identified three regimes: An ergodic regime in which the system heats up to "infinite temperatures"; a well-localized regime in which the system quickly plateaus at some finite energy density; and an intermediate regime in which the system slowly heats up with a logarithmic increase of energy over several decades. This logarithmic growth is very distinct from the characteristic logarithmic growth of entanglement entropy in that the energy density is a locally observable quantity. For weakly disordered systems, where the static Hamiltonian is in the extended phase, we observe that energy is quickly absorbed until reaching a fully mixed state, with the heating curves collapsing upon rescaling time with a factor δ^2. Also, in the strongly disordered case, driving at high frequencies yields to behaviour which can be understood in terms of driven two-level systems.

Our results provide the first detailed study of the energy absorption over time in a Floquet-MBL system. It ties in with the broader interest in how Floquet systems reach their steady states, where an increasingly rich phenomenology is being uncovered. This also comprises the case of clean systems and fast driving, where it has been argued that approaching the fully mixed state can be extremely slow [31, 32]. It is an open question whether the pronounced logarithmic growth we have uncovered might be related to the glassy behavior seen in Ref. [33] for a clean system, where the authors argue that the appearance of rare resonances which eventually proliferate are what causes the heating. The extent to which this set of phenomenologies generalizes to higher dimension remains a tantalizing open question, with our capacity to find an answer limited by the usual difficulties in treating systems combining interactions and disorder.

More broadly, our work advances our understanding of

the phenomenon of MBL as well as of the properties of the newly discovered Floquet ensembles, both of which continue to constitute intensely studied and rapidly advancing subfields of out-of-equilibrium many-body quantum dynamics.

Note added. As we were writing up this work, a preprint [34] appeared which studies the question of energy absorption in an MBL system driven by a strong field. At the same time as this work, a preprint is being prepared which focuses on heating in the Floquet-MBL phase [35].

Appendix. Here, we provide details of the computation of $\epsilon_\infty \sim \delta^2/\eta^2$ for a driven two-level system to model the strongly disordered regime at high frequency. The driven two-level system is described by

$$H(t) = \frac{1}{2}\eta S^z + \delta\cos(\omega t)S^x.$$

(note that η here is related, but not identical, to the disorder amplitude in the main text). In the high-frequency regime, $\omega \gg \eta \gg \delta$, we calculate the time-averaged energy of this system with the ground state of the $t = 0$ Hamiltonian as the initial state. To this end we find the Floquet Hamiltonian H_F to leading order in the Magnus expansion [36, 37],

$$H_F = h_z S^z + h_x S^x.$$

with

$$h_z = \frac{1}{2}\eta + \frac{1}{8}\frac{\delta^2\eta}{\omega^2} \qquad (6)$$

$$h_x = \frac{\delta\eta^2}{2\omega^2}. \qquad (7)$$

This may be diagonalised as follows: a rotation $U = \exp(iS^y\theta)$ with $\theta = \arctan(h_x/h_z)$ aligns H_F with the z-spin axis, whence the eigenvectors and eigenvalues may be read off. Rotating back, the eigenvectors and eigenvalues are $|e_{\uparrow/\downarrow}\rangle = U^\dagger|\uparrow/\downarrow\rangle$, with $|\uparrow/\downarrow\rangle$ the eigenvectors of σ^z, and $\pm\sqrt{h_x^2 + h_z^2}$, respectively.sf

The initial state is the ground state of $H(0) = \frac{1}{2}\eta S^z + \delta S^x$, which can be written as $|\psi_0\rangle = V^\dagger|\downarrow\rangle$ with $V = \exp(iS^y\arctan(2\delta/\eta))$. The time-averaged energy starting from this initial state is given by

$$E_\infty = \sum_{\alpha=\uparrow/\downarrow}\langle e_\alpha|H(0)|e_\alpha\rangle\,|\langle\psi_0|e_\alpha\rangle|^2$$

Rescaling this as in the main text and expanding to leading order in the parameters δ/ω and δ/η we obtain

$$\varepsilon_\infty = 2\frac{\delta^2}{\eta^2}$$

which has the same form as the high-frequency (i.e., the lowest set of) curves of the lower panel of Fig. 3 of the main text.

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[1] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 012110 (2014).

[2] L. D'Alessio and M. Rigol, Phys. Rev. X **4**, 041048 (2014).

[3] P. Ponte, A. Chandran, Z. Papić, and D. A. Abanin, Annals of Physics **353**, 196 (2015).

[4] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

[5] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys. Rev. Lett. **98**, 050405 (2007).

[6] A. Das, Phys. Rev. B **82**, 172402 (2010).

[7] A. Eckardt, M. Holthaus, H. Lignier, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Phys. Rev. A **79**, 013611 (2009).

[8] M. Grifoni and P. Hänggi, Physics Reports **304**, 229 (1998).

[9] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. **321**, 1126 (2006).

[10] I. Gornyi, A. Mirlin, and D. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005).

[11] V. Oganesyan and D. A. Huse, Phys. Rev. B **75**, 155111 (2007).

[12] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **115**, 030402 (2015).

[13] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Phys. Rev. Lett. **114**, 140401 (2015).

[14] V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, (2015), arXiv:1508.03344 [cond-mat.dis-nn].

[15] D. A. Abanin, W. de Roeck, and F. Huveneers, (2014), 1412.4752.

[16] P. Hauke, O. Tieleman, A. Celi, C. Ölschläger, J. Simonet, J. Struck, M. Weinberg, P. Windpassinger, K. Sengstock, M. Lewenstein, and A. Eckardt, Phys. Rev. Lett. **109**, 145301 (2012).

[17] M. Holthaus, J. Phys. B: At. Mol. Opt. Phys. **49**, 013001 (2016).

[18] M. Genske and A. Rosch, Phys. Rev. A **92**, 062108 (2015).

[19] M. Žnidarič, T. Prosen, and P. Prelovšek, Phys. Rev. B **77**, 064426 (2008).

[20] J. H. Bardarson, F. Pollmann, and J. E. Moore, Phys. Rev. Lett. **109**, 017202 (2012).

[21] M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **110**, 260601 (2013).

[22] R. Vosk and E. Altman, Phys. Rev. Lett. **110**, 067204 (2013).

[23] A. Nanduri, H. Kim, and D. A. Huse, Phys. Rev. B **90**, 064201 (2014).

[24] A. Pal and D. A. Huse, Phys. Rev. B **82**, 174411 (2010).

[25] Y. Saad, SIAM Journal on Numerical Analysis **29**, 209 (1992).

[26] D. A. Huse, R. Nandkishore, and V. Oganesyan, Phys. Rev. B **90**, 174202 (2014).

[27] M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **111**, 127201 (2013).

[28] A. Chandran, I. H. Kim, G. Vidal, and D. A. Abanin,

Phys. Rev. B **91**, 085425 (2015).

[29] V. Ros, M. Müller, and A. Scardicchio, Nuclear Physics B **891**, 420 (2015).

[30] J. Z. Imbrie, (2014), arxiv:1403.7837 [math-ph].

[31] D. A. Abanin, W. de Roeck, and F. Huveneers, Phys. Rev. Lett. **115**, 256803 (2015).

[32] T. Mori, T. Kuwahara, and K. Saito, (2015), arXiv:1509.03968 [cond-mat.stat-mech].

[33] M. Bukov, M. Heyl, D. A. Huse, and A. Polkovnikov, (2016), arxiv:1512.02119 [cond-mat.quant-gas].

[34] M. Kozarzewski, P. Prelovsek, and M. Mierzejewski, (2016), arXiv:1602.06055 [cond-mat.str-el].

[35] S. Gopalakrishnan, M. Knap, and E. Demler, (2016).

[36] A. Eckardt and E. Anisimovas, New J. Phys. **17**, 093039 (2015).

[37] M. Bukov, L. D'Alessio, and A. Polkovnikov, (2014), arXiv:1407.4803v6 [cond-mat.quant-gas].