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Technical Report No. 72

The use of optical flow and splay angle in steering a central path

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October 1999

This Technical Report will be published in:
Gale A.G. et al. (Eds.): *Vision in Vehicles VIII.* (to appear early 2000).

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The use of optical flow and splay angle in steering a central path

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Abstract. In the present experiment we investigated the importance of velocity information during a lane-centering task between the walls of a simulated tunnel. We varied both simulated velocity and the spatial frequency content of the walls' surfaces, in order to address the influence of each parameter on steering performance. Further, this performance was compared to the effectiveness of lateral control using lane border information. We found that drivers used both velocity and spatial frequency information to maintain a centered position on a path, and that the presence of lane borders improved accuracy. The results suggest that multiple sources of visual information, rather than mere demarcating lines, are used for lateral control on a straight path.

1 Introduction

For animal locomotion, steering a path through an environment requires both the avoidance of obstacles and the maintenance of an appropriate distance to objects. Insects can easily fly through a window or corridor, keeping at an appropriate distance from both sides. Similarly, humans can steer down a roadway keeping a secure middle position between the lane borders, successfully compensating for lateral deviation. Despite the similarity of both behaviors, different visual cues have been emphasized to control these two types of visually guided locomotion.

When bees fly through a corridor or window they tend to fly through the middle of the opening. To fulfil this task, they appear to balance the retinal image speeds of both sides of the tunnel, a behavior termed 'the centering response' by Srinivasan and colleagues (1991). The authors have shown that the centering response is independent of the spatial frequency content of the environment, since the bees' flying path remains unaffected when the vertical oriented stripes on the left and the right side of the corridor differ in their spatial period. On the other hand, when one corridor wall is moving opposite to the bees' flying direction, the bees deviate from the center and fly on a path closer to the stationary wall in order to equalize the apparent velocity of both sides.

In driving research, experimenters have focused mainly on the utility of lane borders for steering control (Donges, 1978; Godthelp et al., 1984, Land and Lee, 1994, Beall and Loomis, 1996). Lane borders and also in a more general sense, surface textures which are marked by lines parallel to the direction of motion, give rise to an optical variable called splay angle. That is, the angle in the optical projection between a straight line and the line perpendicular to the horizon. During lateral displacement an optical rotation of this angle will occur. One important characteristic of the splay angle of a road edge is that it is independent of forward speed. Beall and Loomis (1996) have shown in a simulator study the superiority of straight road driving control based on splay angle information in contrast to control guided by pure motion parallax information. On the other hand, on curved roads the localization and tracking of the tangent point of a bend seems to be important (Land and Lee, 1994; Land, 1998). Motion information, which is derived from forward translation, seems not to be a likely candidate for lane-keeping. Indeed, Riemersma (1981) tested the detectability thresholds for lateral deviation in simulated driving condition in two road environments. When the road environment was characterized by continuous edge lines, the detectability was a function of the lateral speed and with a precision well suitable for course control. When only random-dots defined the surface, performance deteriorated to chance level, rejecting optical flow as a cue for lateral control. Recently, the visual cues for steering were summarized as follows: "The locations of the edges of the road in the field of view appear to provide the principal visual cues for steering. They are necessary and sufficient, and the rest of the flow-field does not seem to be involved in any very direct and essential way." (Land, 1998, p.178).

However, when the lateral control in driving has to be carried out in a non-flat environment, which is also defined by vertical surfaces, velocity cues may become important. For instance, when driving through a tunnel, the control dimension is perpendicular to the tunnel walls, generating a strong optical flow. Since the optical speed of an environmental point depends on its visual angle from the heading direction and the shortest line between observer and environmental point (Nakayama and Loomis, 1974), velocity information in the optical flow field may well be used to control the distance. The velocity of a moving pattern in the frequency domain is defined as the ratio of temporal and spatial frequency (TF/SF). A long debate exists as to whether humans can compute velocity independently from temporal frequency of the stimulus[1] (e.g. McKee et al., 1986; Chen et al., 1998; Smith and Edgar, 1991). We shall consider this debate in our present approach.

The following experiment was designed to address two main questions: First, is velocity information, used in a lane-centering task and is it independent of the temporal frequency of the stimulus? Second, how is the performance based on velocity cues, related to the performance based on splay angle information?

To address these questions we designed an experiment quite similar to the experimental set-up of Srinivasan et al. (1991). The drivers' ability to maintain a central position in a simulated tunnel was tested by using an active-control task. The visual information in the tunnel was varied, so that we could separate the effect of temporal, spatial, and lane-border information upon the lane-centering task.

2 Methods

2.1 Participants

Nine participants, four male and five female, at the age of 19 to 31 (mean: 23.6 years) were paid 15 DM/hour for their participation in the experiment. All participants had normal or corrected-to-normal vision, and were all licensed drivers.

2.2 Apparatus

A Silicon Graphics ONYX2 InfiniteReality computer with three graphic pipes was used to generate the real-time display. These three 1280 × 1024 pixel displays were projected onto a curved projection screen (vertically oriented half-cylinder of 7 m diameter and 3.15 m height) using three Electrohome Marquee 8000 projectors. For an observer seated in the center of the cylindrical projection screen, the image subtended 180° of visual angle horizontally and 50° vertically. Real-time simulation was programmed using the Silicon Graphics Performer programming library. The refresh-rate of the rendering was 72 Hz. Time lag between steering control and the visual update was within one refresh period (14 ms).

2.3 Task

Subjects controlled their lateral position in the tunnel by using a computer mouse with kinematics analogous to a steering-wheel. Participants had second-order (acceleration)

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[1] With the expression 'velocity is independent from temporal frequency' is meant that velocity is *not exclusively dependent* on temporal frequency, but rather *dependent on both* temporal *and* spatial frequency of the stimulus. This inconsistent use in the literature may lead to confusion.

control over the lateral position: i.e. a leftward change in lateral position required a sinusoidal deflection (center-left-center-right-center) of the mouse. Viewing direction was held constant at the middle of tunnel. Accordingly, the participants controlled only the lateral position in the tunnel, without changes in heading direction. This design of control was intended to keep the task sufficiently difficult without the need to introduce random deviations (such as simulated gust winds) during the trial, and also, so as to prevent any rotatorary optical flow information being available to the observers.

2.4 Visual stimuli

The display simulated the view of a wide tunnel with parallel, vertical side walls of 10 m height (i.e. 8 eye heights wide[2]; given a simulated eye height of 1.25 m). The walls were covered with a black and white square-wave grating. Ceiling and floor borders of the tunnel were not visible. Beginning at a depth of 120 m the walls gradually disappeared into black fog until they were completely covered at 180 m. Height of the walls were adjusted so that they were outside the visible regions. The stimulus covered the whole field of view of 180° in horizontal direction, except for a central region of 3.6° around the vertical midline of the projection, where black fog covered the far end of the tunnel.

The wavelength of the pattern was either 10 or 20 m, which corresponds to a spatial frequency of 0.1 and 0.05 cycles(c)/m, respectively. Forward velocity was simulated, by moving the wall pattern horizontally with a speed of 10 m/s (slower forward speed, 36 km/h) or 20 m/s (faster forward speed, 72 km/h) towards the observer. The corresponding maximum angular velocity at a viewing direction of 90° from the center of the projection, and with a lateral distance to the wall of 5 m was 114.6°/s in the slower and 229.2°/s in the faster case. The combination of the two spatial periods and the two speeds generated a temporal modulation of 0.5, 1.0 or 2.0 c/s. In some trials no forward motion was simulated.

Splay angle information was provided by a set of blue lines, which indicated the left and right border of the tunnel. They ran in parallel to the direction of self-motion, located at the same height 2 m beneath the drivers' eye point. At the beginning of the trial the two lines formed a splay angle of 56.3° and 68.2°.

2.5 Design and procedure

The experimental session was comprised of a velocity discrimination-task and the tunnel-centering task. The velocity discrimination task was performed at the beginning of the experimental session. In this psychophysical experiment, thresholds for velocity discrimination were obtained using the constant stimuli method. The same view of the tunnel was used, but adapted to a 2AFC paradigm. One wall represented the constant stimulus with one of the four combinations of spatial frequency and velocity. The velocity of the second wall was varied, moving 1, 1.05, 1.2, 1.5, or 2 times faster or slower than the constant stimulus. After a presentation duration of 4 seconds, participants indicated which side moved faster by pressing a button.

In the subsequent tunnel-centering task, participants first had the opportunity to accustom to the experimental set-up, running 8 to 10 practice trials. In the practice trial, visual feedback about the lateral position was given. Participants completed four blocks of trials (A-D) of the tunnel-centering task, where each block lasted approx. 15 minutes and each trial 20 s. In the experimental trials no feedback about the position was given. The order of presentation of the four experimental blocks was randomized.

Block A and C contained 48 trials, while block B and D contained 32 trials. The number of trials in block A and C resulted from the crossing of two spatial frequencies (10 or 20 c/m) with three speeds (0, 10 or 20 m/s)

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[2] It has become common practice to refer dimensions in simulated environments in eye height units, since the scale of the environment is not otherwise specified. However, it is easier to form a picture when dimensions are given in familiar units. For that reason, we shall henceforth only refer to distances in meters.

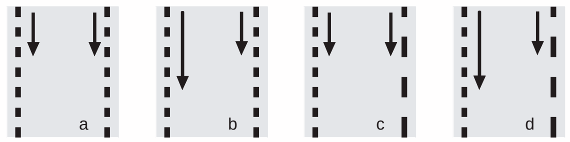

Figure 1: Schematic representation (top-down view) of the pairing of spatial frequency and velocity of the tunnel wall used in the experiment (block A - D). Magnitude of vectors represents the velocity for each side.

two visibility conditions of splay angle (absent or present), where each combination was presented four times. Blocks B and D consisted of a factorial combination of two spatial frequencies, two speeds (10 or 20 m/s), and two visibility conditions, in which each combination was repeated four times.

In Block A, the non-conflict condition, the spatial frequency and the velocity of the stripes of the left and the right wall were equal (sSFsVEL) - see also figure 1. Block B, C and D were conflict conditions. In block B, both walls carried the same spatial frequency pattern, but the apparent velocities were inconsistent (sSFdVEL). Walls in block C differed in their spatial frequency (dSFsVEL), but moved with the same speed. In block D spatial frequency as well as velocity differed between the left and right wall (dSFdVEL).

2.6 Instructions

Each trial started with a displacement of 2 m, randomly assigned to the left or to the right from the centerline. The participants were instructed to achieve a center position in the tunnel and to maintain it throughout the trial. They were also required to respond as quickly and accurately as possible as soon as a deviation was noticed. They were informed that they would control their lateral position without any change in heading direction. They were also informed that the spatial frequency of the stripes of the walls could differ, but not about the diversity of the velocity of the simulated forward motion. Participants were notified that no unpredictable changes in their position would occur (no lateral disturbances).

2.7 Data analysis

The first 10 seconds of the trial were excluded from the analysis, since during this period the participants had to achieve and maintain a centered position in the tunnel. The mean of the lateral position was calculated using data from the second half of the trial (10 - 20 s after start). For each block a separate analysis of variance was conducted for the four factors (i) 'subject', (ii) 'availability of splay information', (iii) 'spatial frequency' and (iv) 'velocity'[3]. Root-mean-square-error terms were calculated from the same data to indicate the stability of the drivers' performance.

3 Results

The purpose of the velocity discrimination task, was to test the ability of the participants to reliably discriminate the velocities of the wall patterns when they differed by a factor of two. We calculated the Weber fractions for the four constant stimuli by fitting a psychometric function and determining the 75%-level. The performance of the participants showed substantial differences among the participants with Weber fractions ranging from 0.07 to 0.47 (mean: 0.21). To compare, findings in literature report Weber fractions of 0.05-0.10 over a wide range of velocities (McKee et al., 1986). However, in the light of our experiment this performance sufficed, since all participants could clearly discriminated velocity when it differed by a factor of two.

3.1 Control condition

The mean lateral position during block A yielded no significant difference in any experimental factor. The drivers' performance in terms of lateral position in all conditions was

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[3] In block A and C the factor spatial frequency indicated the variation of the two spatial frequencies between the trials. In block B and D the factor spatial frequency indicated the variation of the location of the higher spatial frequency, to the left or to the right, within the trial. Accordingly, the factor velocity indicated the location of the higher velocity within the trials of block C and D. Since velocities were equal for both sides in block A and B, it referred here to the variation of absolute velocity between the trials.

comparable. The overall mean of all trials was -0.36 m, which significantly deviates from the center position to the left (z = 5.34, $p << 0.001$). However, to anticipate results which will be discussed later, this systematic deviation to the left occurred in all conditions tested. One reason for this tendency may be that the participants behaved as they were driving in a real car. Clearly, if their aim was to place the car's axis in the middle of the lane, the driver's position must be shifted (to the left, when used to driving on continental roads).

When the stability of the path was taken into account in terms of root-mean-square error (RMSE), we found that simulated velocity was a significant factor, F(2,16) = 25.76, $p < 0.001$, as was the availability of splay information, F(1,8) = 17.87, $p < 0.01$ (figure 1). RMSE was significantly lowered at zero speed, that is when subjects controlled their position only in a side-slipping fashion. When lines indicated the lateral tunnel borders, providing splay angle information, RMSE also decreased. Furthermore, we found a significant interaction between the factors velocity and availability of splay information, F(2,16) = 10.58, $p < 0.001$. The lateral control at zero speed was equally accurate for both types of visual cues. The variation of the spatial frequency of the wall pattern had no effect on RMSE data.

These results confirm previous findings in altitude control, showing a deterioration with increasing forward speed when the surface was textured with lines perpendicular to the direction of motion and an interaction between the availability of splay angle information and forward velocity[4] (Flach et al., 1997). Lateral control during translation on a straight path without splay angle information is not as efficient as with splay angle information.

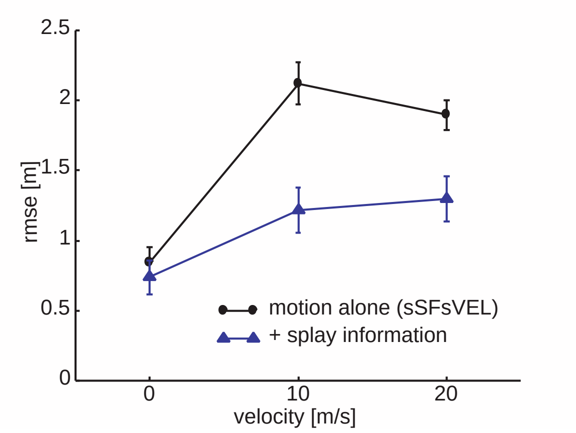

Figure 2: Root-mean-square error as a function of velocity in the control condition (no conflict - same spatial frequency and same velocity on both sides of the tunnel). Error bars represent one standard error of the mean.

3.2 The effect of wall velocity

As expected we found a difference in the lateral position when only motion information was available and the simulated speed of the walls differed by a factor of two (block B). When the apparent velocity of the right walls was higher, the participants steered away from that wall to the left and vice versa (see figure 3). An analysis of variance confirmed this difference. The only significant factor in this block was the location of the higher velocity, F(1,8) = 8.41, $p = 0.020$. The relative deviation from the overall mean of -0.74 m (to the left; higher velocity on the right side) and 0.68 m (to the right; higher velocity on the left side) is of comparable size. However, the observed deviation differs from the theoretically expected position of 1.67 m, where the optical velocity of both sides would have been equalized. Reasons for this will be discussed later.

The variation of the spatial frequency of the wall pattern (both walls with 0.1 or 0.05 c/m) did not produce any significant difference in the resulting lateral position. When splay angle information was also available, drivers maintained a position nominally closer to the center of the tunnel (figure 3, light bars).

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[4] Differing from our results, during altitude control the performance at zero speed without splay angle information is still inferior than with splay angle information (Flach et al., 1992; 1997). It is worth noting, that the centering in the tunnel can be performed by a comparison of the visual cues on both wall sides, whereas in altitude control a change in the relevant variable has to be perceived, since only one surface defines the ground.

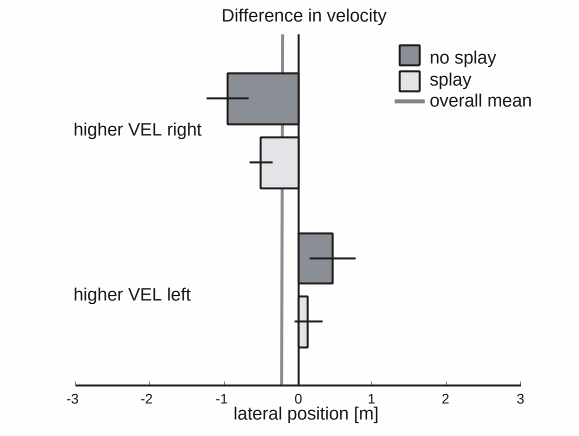

Figure 3: Lateral position in the tunnel depends on which side of the wall the higher velocity (20 m/s vs. 10 m/s) of the wall pattern appeared. Error bars represent +/-1 standard error of the mean. Overall mean is -0.21 m (dashed line).

However, this difference failed to reach statistical significance, which is shown by the absence of an interaction between the location of the higher velocity and the availability of splay information, F(1,8) = 2.44, ns. The overall mean in this block also deviates significantly from the center of the tunnel (-0.21 m).

These results suggest that the drivers used a strategy of comparing the velocity information from both hemifields to maintain a central position in the tunnel. A displacement caused by unequal velocity, although reduced in size, persisted even when border lines providing splay angle information were visible. Similar findings are reported in recent experiments showing that people shift their lateral position while walking on a treadmill when velocity of simulated corridor walls were unequal, and that implicit splay angle information provided by the ground line of the corridor walls attenuated this effect (Duchon and Warren, 1998; Warren, 1998). However, in these experiments the spatial frequency of the stimulus was not varied systematically.

3.3 The effect of difference in spatial frequency

Similar to the previous results, we found a left offset bias in the overall mean of the lateral position across all conditions of -0.27 m. The lateral position in the condition of different spatial frequency of the wall pattern is illustrated in figure 4. The results show that when splay information was absent the position deviated systematically towards that side, which carried the higher spatial frequency pattern, F(1,8) = 14.59, $p < 0.01$, (figure 4, dark bars). When the left wall carried the pattern with the higher spatial frequency, the participants steer to the left and vice versa, with an almost equal offset of 0.79 m (higher spatial frequency right) and -0.80 m (higher spatial frequency left). This rather surprising result is at odds with the flying path of bees, where no such effect was found (Srinivasan et al., 1991). Moreover, in the present experiment the effect of different spatial frequency has a size comparable to the effect of different velocity information (see 3.1). When splay angle information was available, drivers maintained a position closer to the middle of the tunnel (figure 4, light bars). This result is confirmed by the significant interaction between the location of higher spatial frequency and the availability of splay information, F(1,8) = 8.83, p = 0.018. With the aid of splay angle information drivers can clearly compensate for the drift from the centerline, caused by divergent spatial frequency information. No effect was found for the factor of forward velocity.

Since the effect of different spatial frequency information is of comparable size during both simulated motion and zero motion, we must consider a strategy different from pure velocity comparison. A likely source of information is the angular extent between grating borders or the number of stripes in a given optical angle[5]. The angular extent

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[5] A similar concept is 'optical density' (Warren, 1982), which is defined as OD = z / g, where z is altitude (here: distance to wall) and g is the extent of texture elements (distance of grating borders). For a given spatial period, reducing the distance will reduce

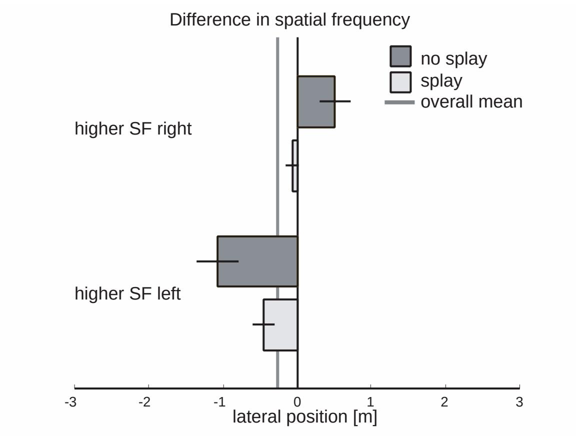

Figure 4: Position in the tunnel depends on which side of the wall the higher spatial frequency of the wall pattern appeared. Error bars represent 1 standard error of the mean. Overall mean is -0.27 m (dashed line).

between grating borders decreases with decreasing distance to the wall. Therefore, the systematic deviation towards the side of the higher spatial frequency can be explained as an attempt to equalize the angular separation of the grating borders in corresponding areas of both sides of the visual field, erroneously assuming equal spatial frequency. Then, theoretically, a displacement of 1.67 m would result. Finally, the effect of different spatial frequency cannot be explained by the perceived velocity. Findings in velocity discrimination tasks suggest that for two gratings moving at the same physical velocity but having different spatial frequency, the high spatial frequency stimulus appears to move faster (Chen et al., 1998). Thus, this would predict a displacement *away* from, and not *towards*, the wall with the higher spatial frequency pattern.

3.4 The combined effect of both difference in velocity and difference in spatial frequency

The results of the mean lateral position in block D are presented in figure 5. We found a significant first-order interaction between the location of the higher spatial frequency pattern and location of the higher velocity, F(1,8) = 10.02, p = 0.013. When splay angle information was absent (see 5a), lateral position was shifted towards the wall with the higher spatial frequency pattern, but only when the velocity of this side was lower (figure 5a, inner bars). When the higher spatial frequency pattern moved with a higher velocity, no such displacement was found. The lateral position tends to lie around the overall mean of -0.27 m (figure 5a, top and bottom bar), suggesting that the effects of different spatial frequency and different velocity were at least partly independent and cancelled each other. Finally, as before, splay angle information forced the driver to steer more towards the centerline of the tunnel, confirmed by a second-order interaction of location of higher spatial frequency × location of higher velocity × availability of splay angle information, F(1,8) = 10.41, p = 0.012.

4 Discussion

The main result of the present experiment is that velocity cues can be used to control the lateral position on a straight path, given an environment that allows a strategy of balancing the apparent speed on both sides of the visual field. When the velocity between the walls differs, a displacement can be measured even when border lines mark the edges of the path. These findings are in line with previous experiments investigating the influence of velocity information on the control of walking on a treadmill (Duchon and Warren, 1998; Warren, 1998). Situations in which velocity cues can become important are not hard to find. Demarcating lines may be badly visible, forcing the driver to consider other cues. On the other hand, when overtaking a large vehicle, large changes will result in areas of the optical flow field which should not motivate a large avoidance steering movement.

That it was velocity, and not the temporal frequency, which affected the displacement, can be seen by comparing the results in 3.2. and 3.3. The displacement in 3.2. could be

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the optical density. When two gratings differ in their optical densities by a factor of 2, they could either be perceived, as equidistant to the observer but with different spatial frequency, or as having the same spatial frequency but located at different distances.

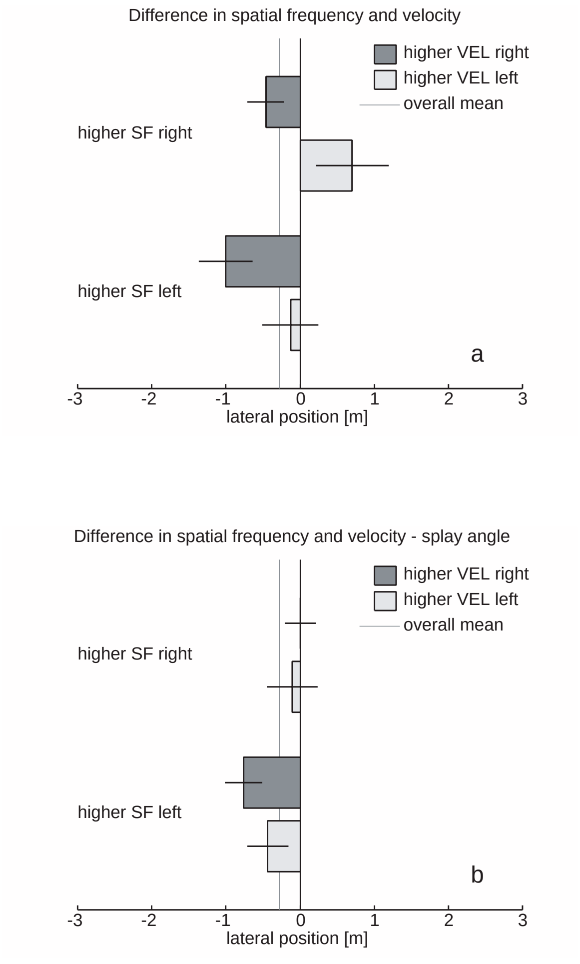

Figure 5: Mean lateral position in the tunnel, when the walls differed in both spatial frequency of the pattern and simulated forward velocity: (a) velocity information alone and (b) velocity information and splay angle inform-ation. The two inner bars in each subplot represent data from the pairing of low SF / high VEL with high SF / low VEL, so that the temporal frequency of the grating on both sides was equal (1.0 c/s). The top and bottom bars represent the condition when temporal frequency differred (0.5 vs. 2.0 c/s and 2.0 vs. 0.5 c/s). Here the effect of different spatial frequency and different velocity are opposed and counteract. Overall mean is -0.27 m.

explained also by the differing temporal frequency of both wall patterns. Thus, a correction away from the higher temporal frequency could have been initiated, although temporal frequency will not change with distance. But since temporal frequency differed in 3.3., by increasing the spatial frequency of one side, participants corrected towards, and not away from the side with the higher temporal frequency. So, there is at least some evidence that velocity was taken into account independently of temporal frequency. This interpretation is further supported by results suggesting that experienced velocity in a rotating drum is also dependent on physical velocity, not temporal frequency of the pattern (de Graaf et al., 1990).

Furthermore, the drivers also relied on the spatial frequency of the wall pattern (3.3), contrary to the findings with bees. Here, we suggest that the different spatial frequency between the wall patterns elicited a process different from that merely specifying perceived velocity. It seems that the drivers used a strategy of comparing the spatial frequency or angular extent of the pattern borders of both walls in itself. The conclusion from this is twofold: First, it suggests that the drivers produced an expectation about the spatial layout, assuming equal spatial frequency at both sides in a conflict condition. Second, when deviating from the centerline towards the wall with the higher spatial frequency pattern, an occurring difference in velocity between both sides was tolerated. Moreover, it seems that both spatial and velocity information were considered equally, since the observed deviation was about the half of the theoretically expected value (results in 3.2 and 3.3). In addition, the effect of different spatial frequency was able to neutralize the effect of different velocity in direct comparison (results in 3.4). Nevertheless, when the environment also consisted of lane borders, providing splay angle information, the performance was mostly enhanced, but not under all condition (i. e. at zero speed). These results suggest that lateral control during driving does not depend solely on the perception and processing of lane borders, at least when motion information is available over a large area of the visual field.

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