

Application of Compressed Sensing to

Quantum Chemistry

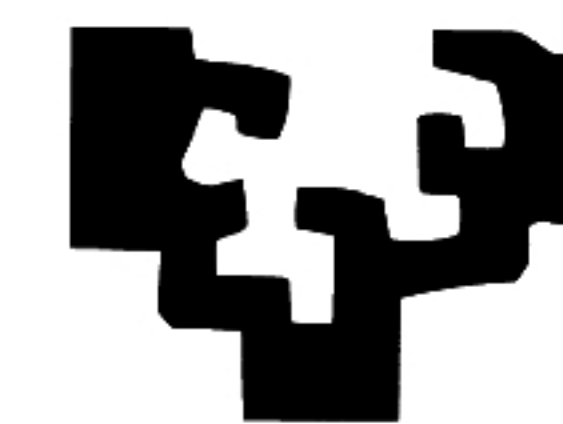
Jacob N. Sanders*, Alejandro Pérez Paz*, Alán Aspuru-Guzik*, Angel Rubio*◇

* Department of Chemistry and Chemical Biology, Harvard University, Cambridge, MA, USA.

* Nano-Bio Spectroscopy Group, University of the Basque Country UPV/EHU, San Sebastián, Spain.

◇ Max Planck Institute for the Structure and Dynamics of Matter, Hamburg, Germany.

eman ta zabal zazu



Universidad del País Vasco

Euskal Herriko Unibertsitatea

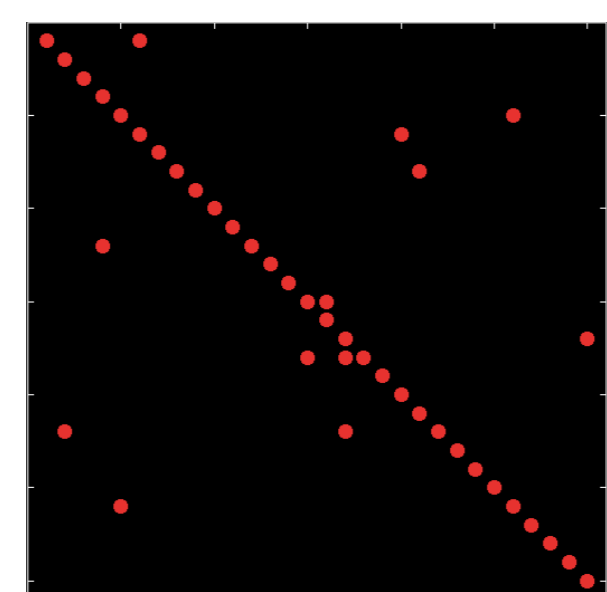
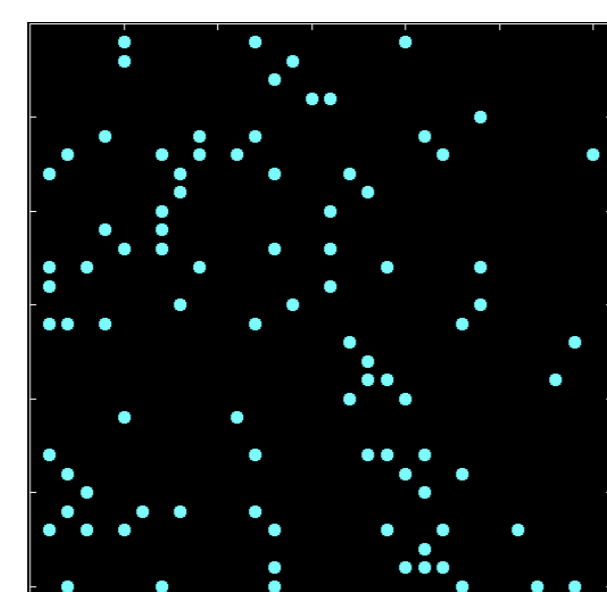
(A) Research Objective:

Use compressed sensing to calculate quantum chemistry matrices (Hamiltonian, Fockian, etc) cheaply without having to compute each matrix element.

(B) Recovering a Matrix with Compressed Sensing

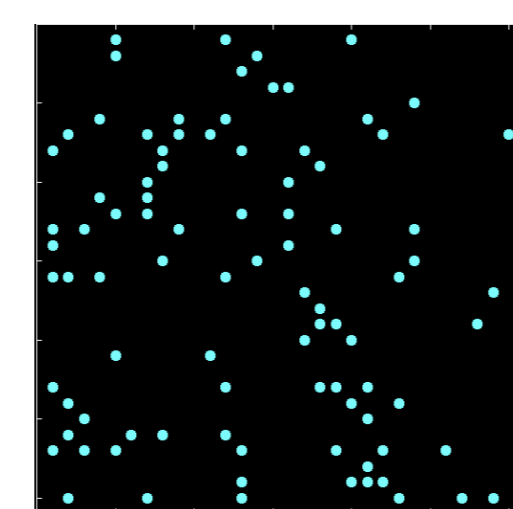
- **Step 1:** Cheaply locate a basis in which the matrix is **sparse**.

Example: cheaply approximate the **eigenvectors**.

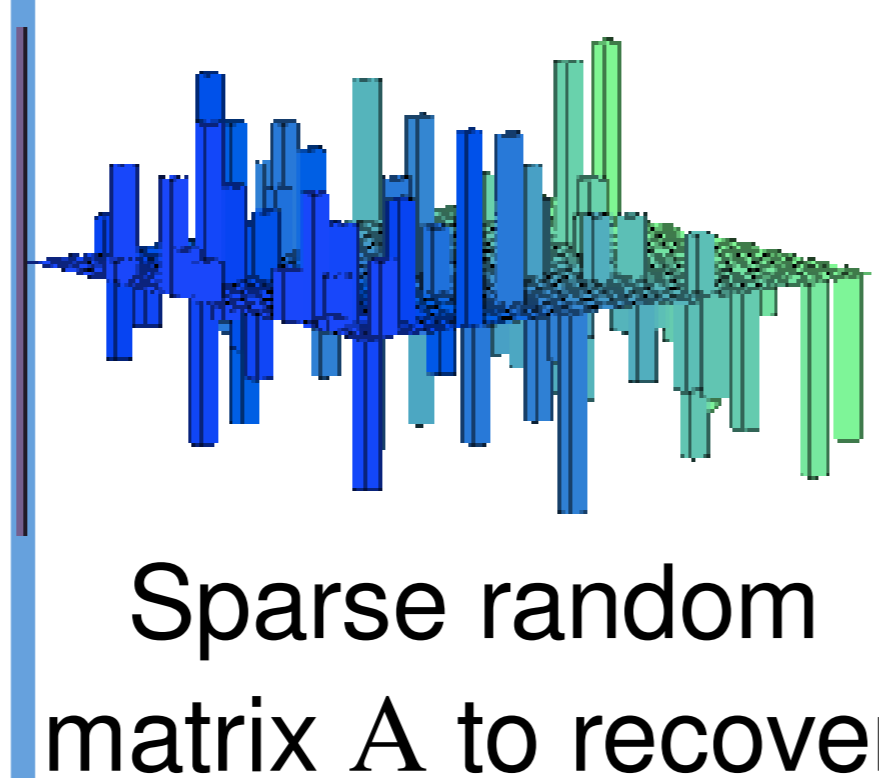
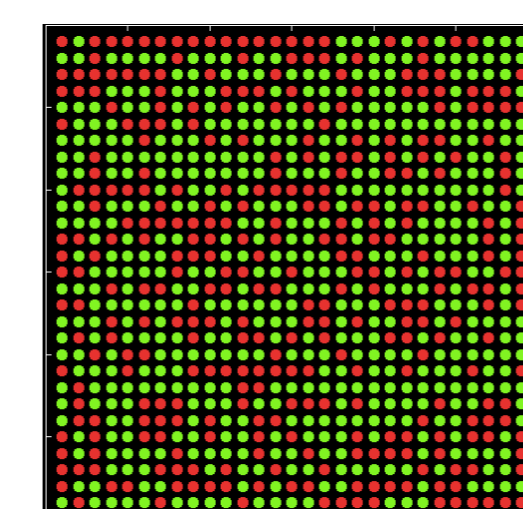


- **Step 2:** Cheaply recover this sparse matrix using compressed sensing.

(C) Compressed Sensing for Sparse Matrices

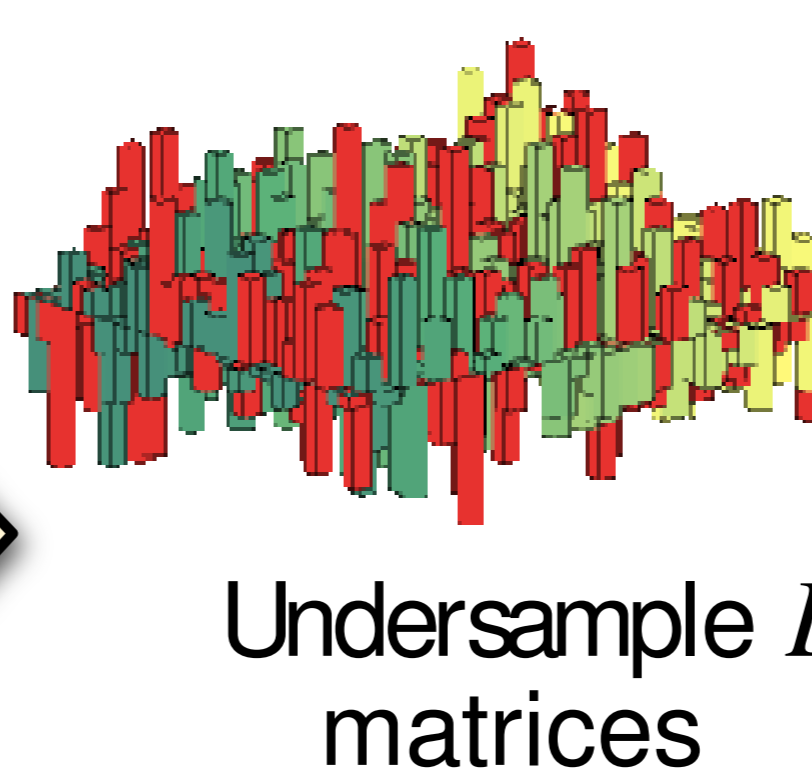


Apply Incoherent Change-of-Basis Matrix P (e.g. discrete cosine transform)



Sparse random matrix A to recover

$$B = PAP^T$$



Undersample B matrices

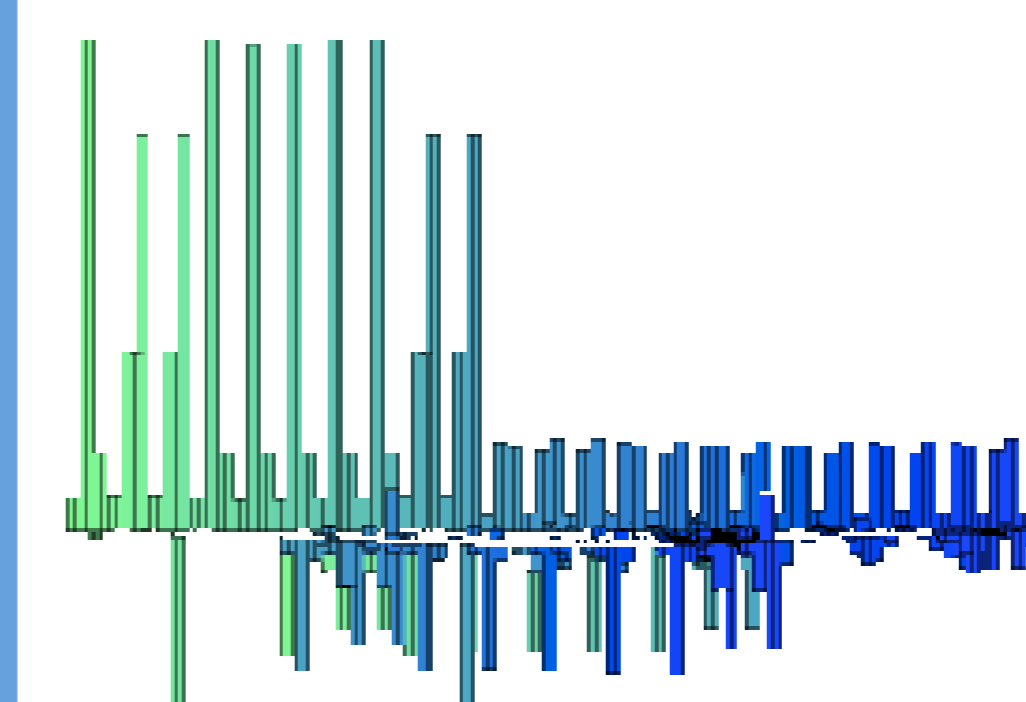
$$\min_A \|A\|_1 \text{ s.t. } \|PAP^T - B\|_2 < \eta$$

Recover A via compressed sensing

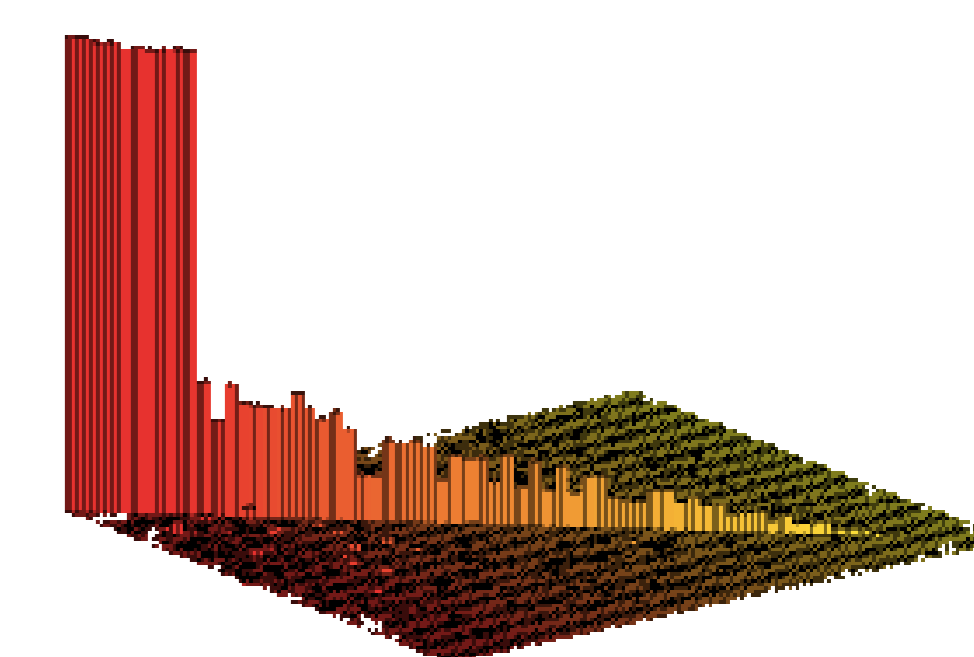
(D) Approach for Electronic Structure Calculations

$H(X)$ in PW/AO basis

$A=H(X)$ in basis of eigenvectors of $H(G)$



$H(\text{hybrid})$ in PW/AO basis



$A=H(\text{hybrid})$ in basis of cheap KS orbitals

A is a sparse matrix that can be recovered with our procedure.

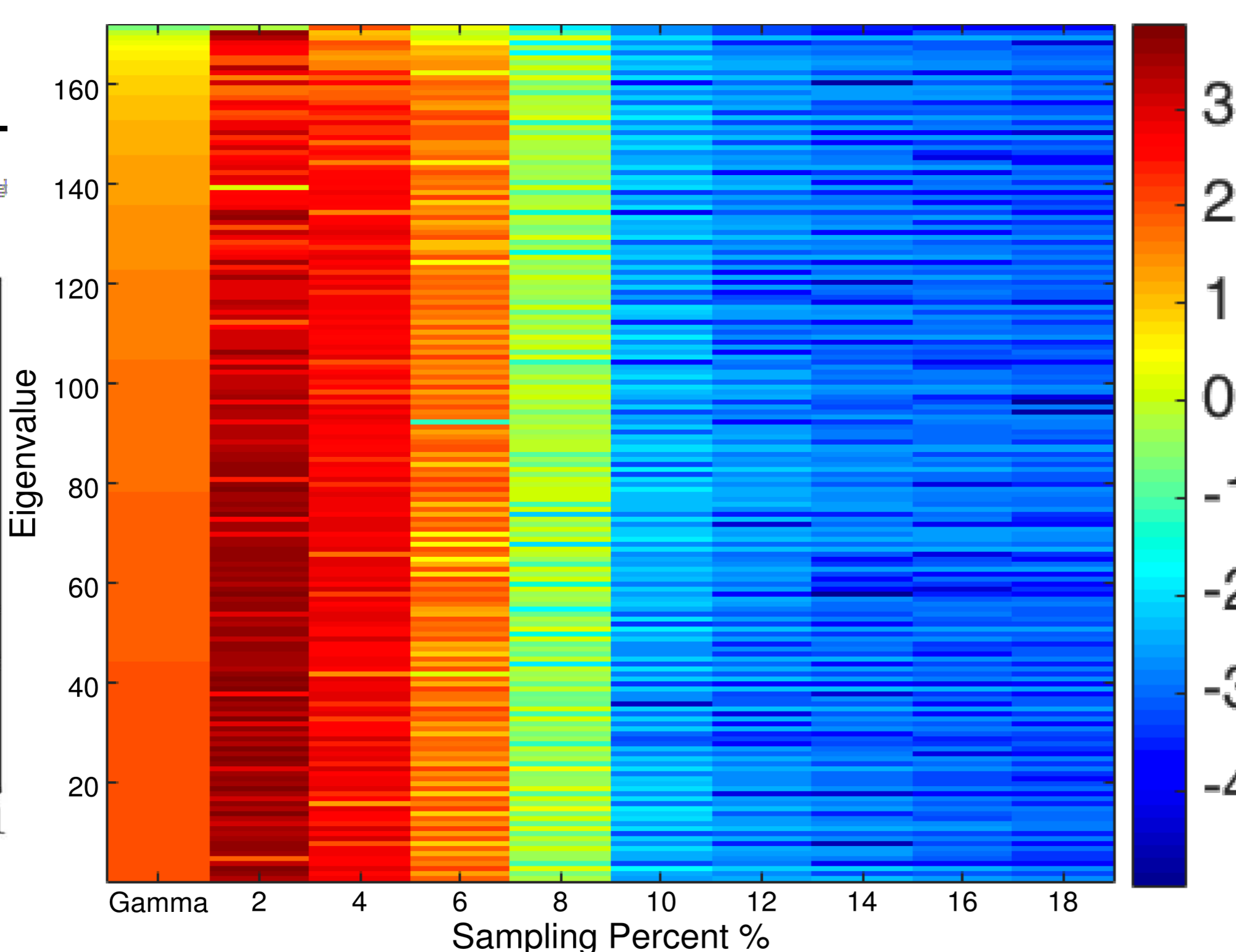
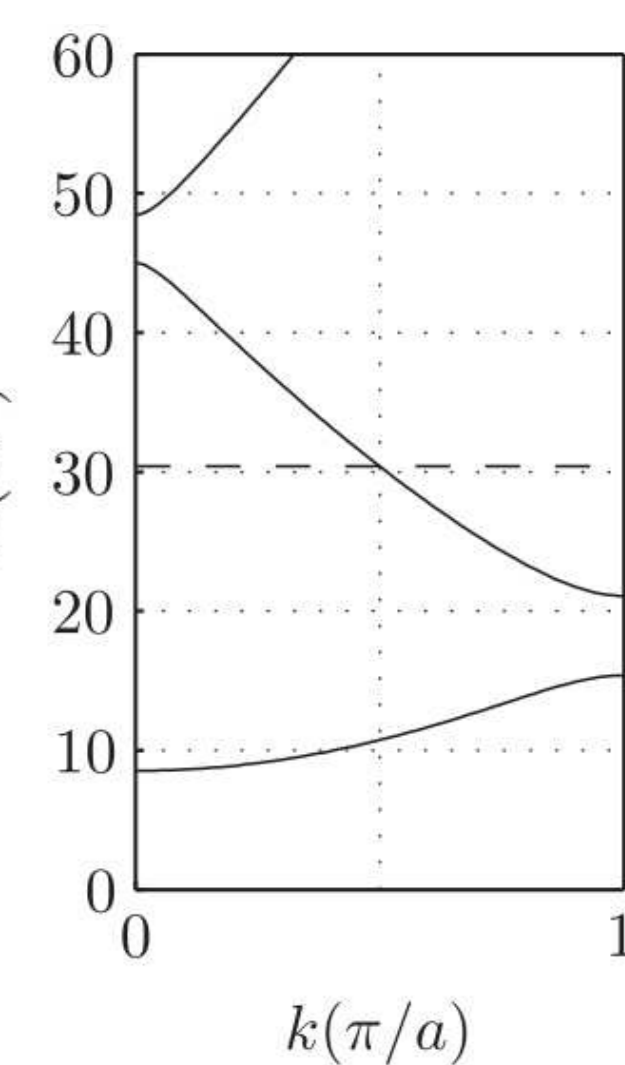
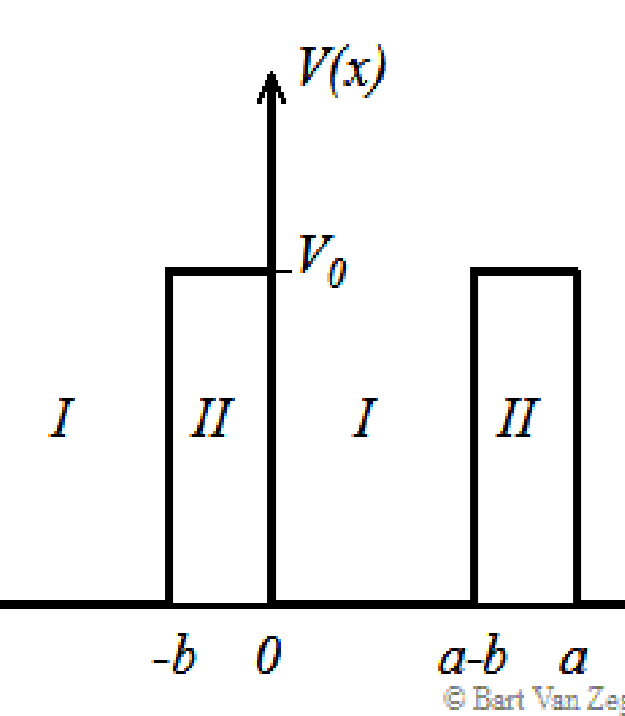
(E) The Kronig-Penney model

Physica B **406**, 4373-4380 (2011)

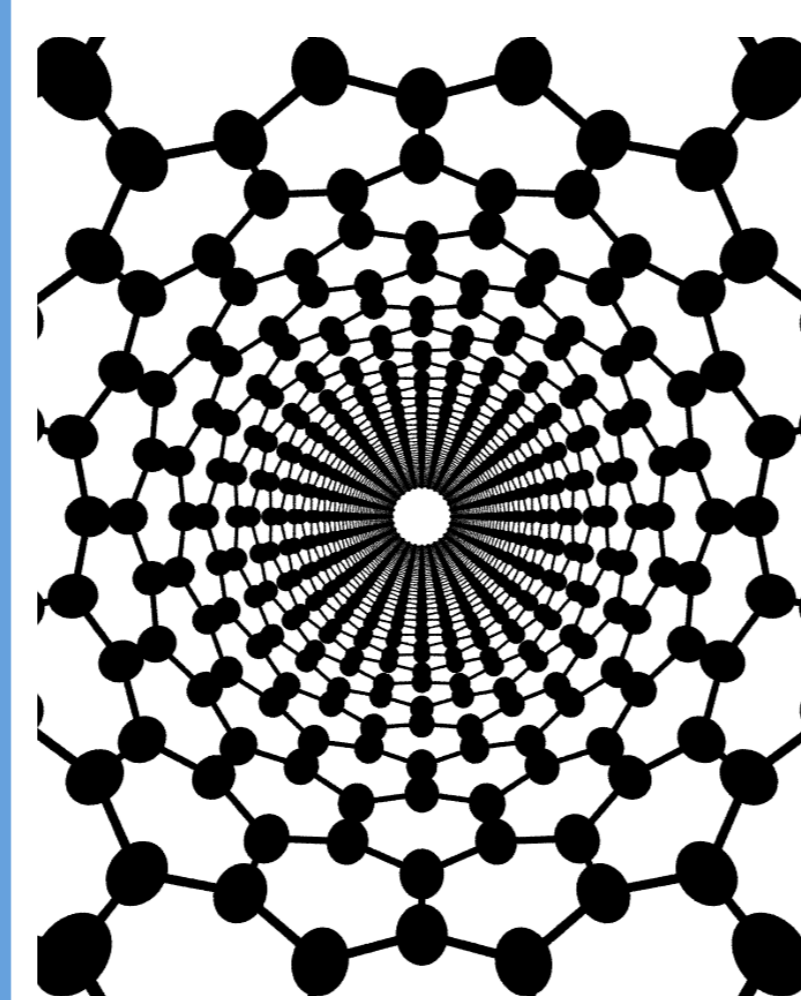
$$V(x) = V_0 \text{ for } a - L < x < a; 0 \text{ for } 0 < x < a - L.$$

$$V_0 = 0.459, a = 3.779, L = 2.834 \text{ a.u.}$$

$H(\Gamma)$ vs. $H(X)$ errors in a.u.

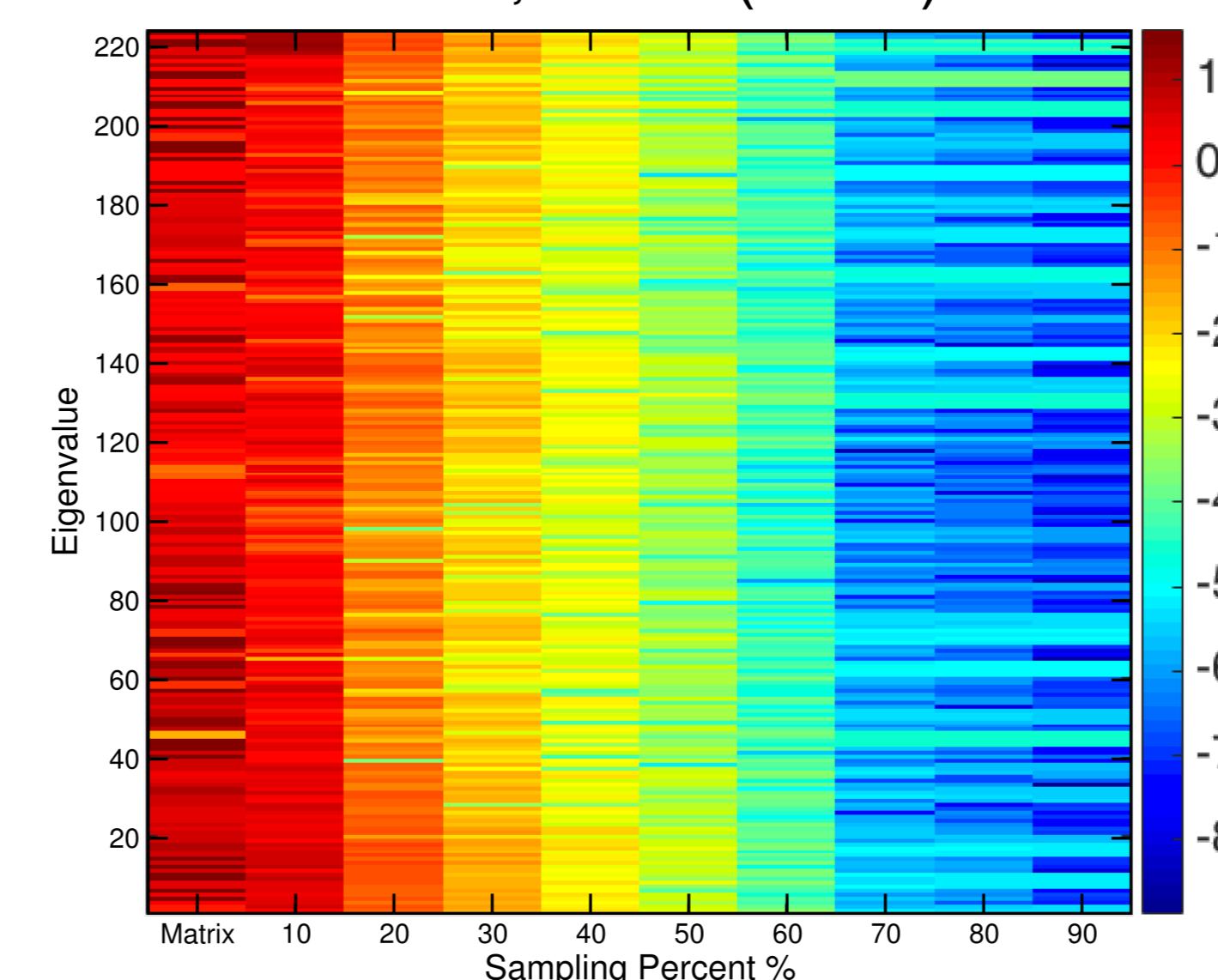


(F) (14,0) carbon nanotube: tight-binding



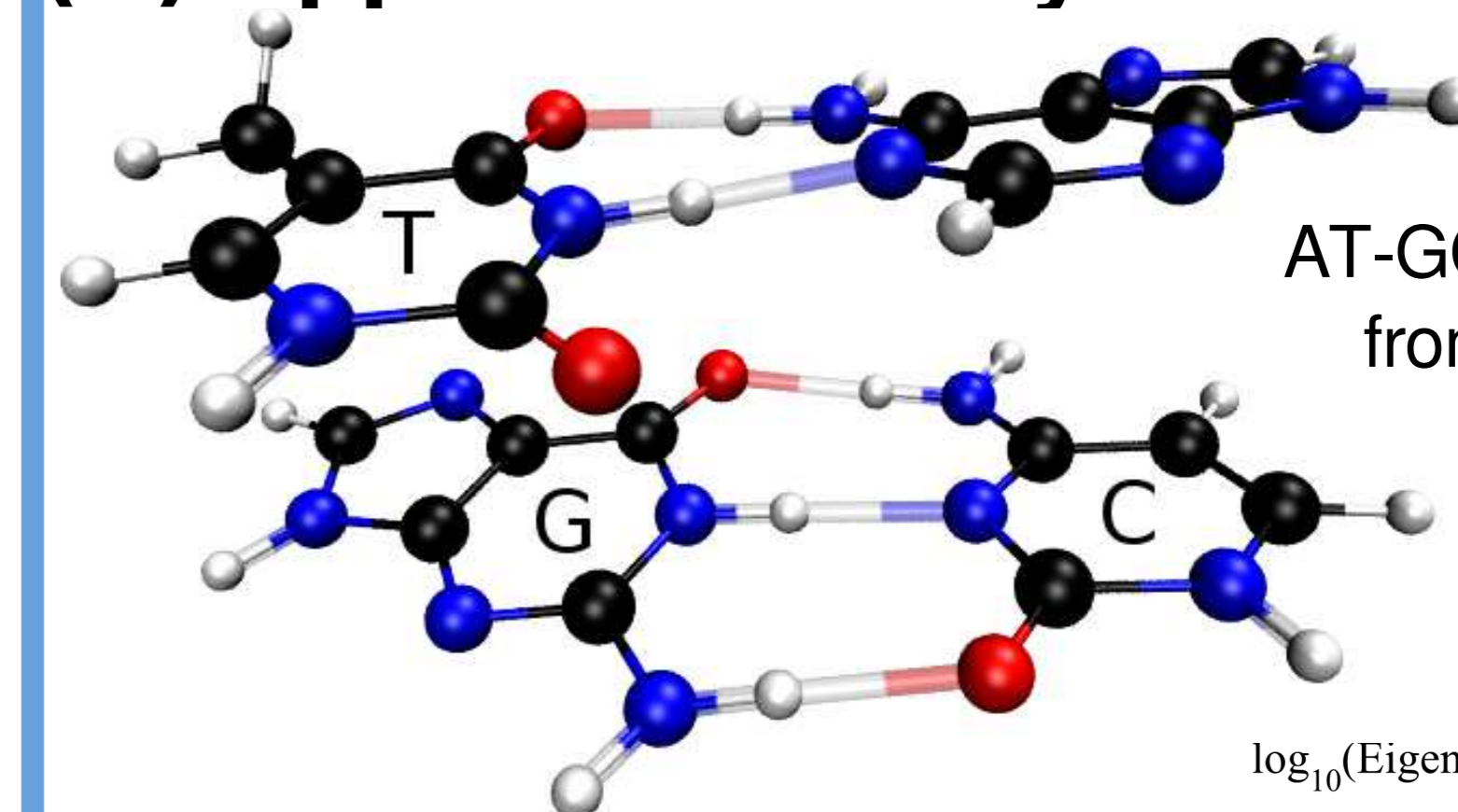
Errors in eV

56 C atoms/unit cell, diameter 1.1065 nm.
DFTB+ code: PRB **58**, 7260 (1998).



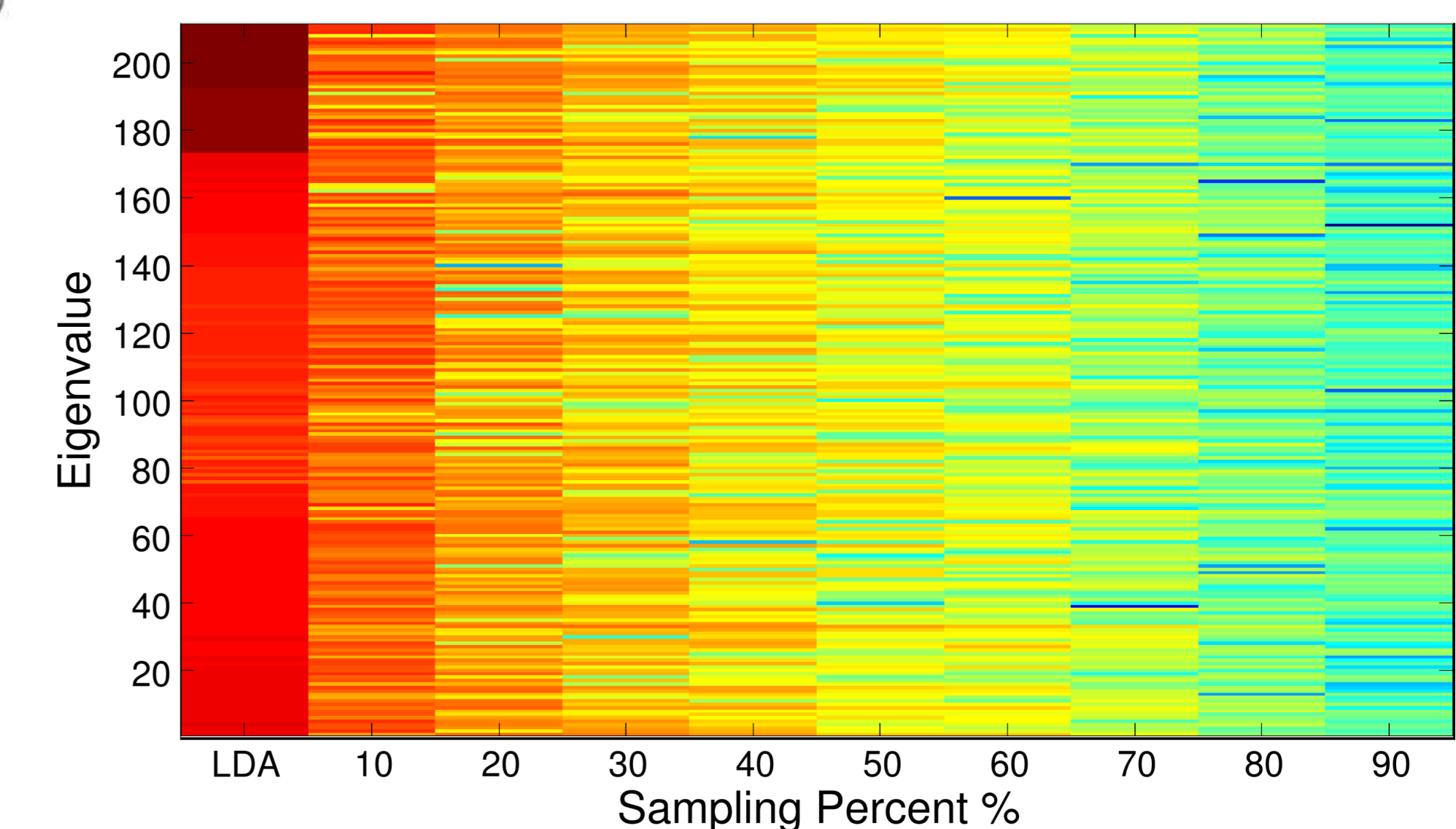
Compressed Sensing recovers sparse matrices regardless where non-zeros are.

(G) Application to hybrid PBE0/B3LYP Fockians



AT-GC Watson-Crick stacked base pairs from JPCB **114**, 1191-1203 (2010).
Errors in a.u.

$\log_{10}(\text{Eigenvalue Error})$ vs. Undersampling



We accurately recover all matrix elements with **as low as 40 % of undersampling**.

(H) Future Research Plan

- Investigate electron-phonon coupling matrix elements.
- Solving +Dyson Equation, Hubbard Hamiltonian?
- Application to other matrix problems in physics/chemistry.

Publications:

• X. Andrade, J. N. Sanders, & A. Aspuru-Guzik. Application of compressed sensing to the simulation of atomic systems. PNAS **109**, 35 (2012). arXiv: 1205.6485.

• J. N. Sanders, X. Andrade, & A. Aspuru-Guzik. Compressed Sensing for the Fast Computation of Matrices: Application to Molecular Vibrations. ACS Cent. Sci. **1** (1) 24-32 (2015).

Acknowledgments:

A.P.P. thanks the support from "Ayuda para la Especialización de Personal Investigador del Vicerrectorado de Investigación UPV/EHU-2013" & Duncan J. Mowbray for his help with Inkscape