MAX-PLANCK-INSTITUT FÜR WISSENSCHAFTSGESCHICHTE
Max Planck Institute for the History of Science

2017

PREPRINT 488

Jens Høyrup

Otto Neugebauer and the Exploration of Ancient Near Eastern Mathematics

Abstract

The exploration of Mesopotamian mathematics took its beginning together with the decipherment of the cuneiform script around 1850. Until the 1920s, "mathematics in use" (number systems, metrology, tables and some practical calculations of areas) was the object of study – only very few texts dealing with more advanced matters were approached before 1929, and with quite limited results.

That this situation changed was due to Otto Neugebauer – but even *his* first steps in 1927–28 were in the prevailing style of the epoch, so to speak "pre-Neugebauer". They can be seen, however, to have pushed him toward the three initiatives which opened the "Neugebauer era" in 1929: The launching of *Quellen und Studien*, the organization of a seminar for the study of Babylonian mathematics, and the start of the work on the *Mathematische Keilschrift-Texte*. After a couple of years François Thureau-Dangin (since the late 1890s the leading figure in the exploration of basic mathematics) joined in. At first Thureau-Dangin supposed Neugebauer to take care of mathematical substance, and he himself to cover the philology of the matter. Very soon, however, both were engaged in substance as well as philology, working in competitive parallel until both stopped this work in 1937–38. Neugebauer then turned to astronomy, while Thureau-Dangin, apart from continuing with other Assyriological matters, undertook to draw the consequences of what was now known about Babylonian mathematics for the history of mathematics in general.

Contents

Prolegomenon

This is the third time in six years that I have been invited to look at Otto Neugebauer's role in the exploration of Mesopotamian mathematics.[1] I have tried to approach the matter from different angles, and therefore, in Leopold von Ranke's words ([1885: vi], first published in 1824), had to offer "nur Geschichten, nicht die Geschichte" – yet still trying on all three occasions to set out "wie es eigentlich gewesen" (once more with von Ranke, now p. vii).

Before Neugebauer

Today – or at least some decades ago – Otto Neugebauer and François Thureau-Dangin are/were often regarded as those who had first and almost on their own discovered and deciphered "Babylonian mathematics". To some extent the veracity of this understanding depends on what we mean by "mathematics", but to make it true we need to apply a much narrower understanding than any of the two would have done – more or less that understanding of mathematics which was produced in order to fit the Cold War story of a specific "Western rationality", reaching "from Plato to NATO" and then, by miracle, foreboded in ancient Mesopotamia (again, by miracle, part of "the West" until the times of Solon or Alexander the Great but then no more).

Even if unfamiliar with the topic, Orientalists may be sceptical. It may, however, be useful to recall to them what went before the two heroes, (mis)interpreting for a short while the "and" of my title as if it meant that Neugebauer and exploration are not meant to be correlated.

Apart from speculations based on the Old Testament and ancient Greek sources, the study of Ancient Mesopotamian mathematics (even if understood as broadly as Neugebauer and Thureau-Dangin would have done) could only begin when the cuneiform script was deciphered. But it did not wait one moment longer. In a paper read in 1847 (published as [Hincks 1848]), Edward Hincks believed to have identified 76 syllabic values; only 18 turned out eventually to be correct or almost correct, 46 had the consonant right but a wrong vowel, and 12 were wholly wrong [Fossey 1904: 185]. In the same paper, however, he described the "non-scholarly" number system correctly.[2] The use of the place value notation in an astronomical text was described by Hincks

[1] See [Høyrup 2016, presented in 2010; and [Høyrup 2016c], presented in 2013. The present paper was my contribution to the Colloquium "Aus der Vergangenheit lernen: Altorientalische Forschungen in Münster im Kontext der internationalen Fachgeschichte", Institut für Altorientalische Philologie und Vorderasiatische Altertumskunde der Westfälischen Wilhelms-Universität Münster, 30. November bis 2. Dezember 2016.

[2] The system which is sexagesimal but not positional until 100, and combines with word-signs for 100 and 1000 for higher values.

in [1854: 407], with a note added in print referring to Henry Rawlinson's similar observation on a table of squares [1855: 218*f*] (which already suggests the use of sexagesimal *fractions*).

Within the same five-page footnote in the same paper (pp. 217–221), Rawlinson points out that the values ascribed by Berossos [ed. Cory 1832: 32] to σάρος (*šār*), νῆρος (*nēru*) and σώσσος (*šūši*), respectively 3600, 600 and 60 years, are "abundantly proved by the monuments" (p. 217). After presenting the square table, the note goes on as follows:

> while I am now discussing the notation of the Babylonians, I may as well give the phonetic reading of the numbers, as they are found in the Assyrian vocabularies

– and after that a fanciful explanation of the origins of the *šār*, *nēru šūši* proposed by Jules Oppert is presented and rejected on p. 220.

During the decades following upon the stabilization of Assyriology as a particular field (which for the sake of anecdote we may date with Léon Heuzey [1906: 7] to the appearance of [Oppert 1859]), the approach initiated by Hincks and Rawlinson was continued, and the domain of numbers and metrologies was considered a necessary part of the decipherment of the cuneiform documents – how else, indeed, would it have been possible to understand what these had to tell about chronologies, economy, etc.? Numbers and metrology, so to speak, constitute the mathematical analogue of Ignace Gelb's onionology [1967: 8], and they are the *sine qua non* of onionology proper.

Some approaches to the topic, building on comparison with Biblical and other non-Mesopotamian material, were from slightly to outright fanciful. Even Oppert, well aware [1886: 90] that there were "en Assyrie et en Chaldée, comme partout ailleurs, des variations continuelles dans les mesures", believed in [1872] in a shared "ancient" metrology (merely with slightly varying values). Without the slightest textual support (whence "sous toute réserve") on p. 427 he even postulated the existence of a parallel to a particular Hebrew measure.[3] Others, not least George Smith [1872], insisted on using primarily autochthonous Mesopotamian material – stone weights, lexical lists and other written material, as was to be the standard after [Weißbach 1907].

A new perspective was introduced by Josef Epping's and Franz Xaver Kugler's work on the Late Babylonian astronomical texts. They firmly established the use of sexagesimal place value *fractions* at least in astronomy; already in [1866: 18], Johannes Brandis had suggested that the corresponding Greek system had to be of "Chaldean" origin.

In [1906], Hermann Hilprecht failed to take this astronomical insight into account in his otherwise path-breaking publication of a large number of arithmetical and metrological tables. Failing understanding of the floating-point character of the place-value notation and a conviction that everything in Babylonian thought was determined

---

[3] In [1859], Hebrew had indeed not only been his chief parallel language to Akkadian but also a tool for transcription alongside the Latin translation.

by an over-arching Gnostic-Hermetic worldview made him miss the potentialities of his work.

Already before that, one of the two protagonists of the eventual understanding of Mesopotamian "higher" mathematics had arrived on the scene. In [1897], Thureau-Dangin published an intricate analysis of a Neosumerian field plan, inaugurating thus his life-long interest in matters mathematical. Over the next decades, François-Marie Allotte de la Fuÿe also worked along these lines, mostly on third-millennium material [1906; 1909; 1915a; 1915b]. Vincent Scheil [1915; 1916] reached a good understanding of the place-value system and the function of the appurtenant tables (although not everybody understood what Scheil told).

All of this concerned what we may speak of as "mathematics in [extra-mathematical] use" – even Hilprecht's tables were, after all, so intimately connected to scribal real-work practice (though how was only understood to the full by Thureau-Dangin in [1932]) that they can be characterized in this way. The "higher" level of Mesopotamian mathematics (the mathematical analogue of Gelb's "Tammuz") remained a closed book. So much so, indeed, that when Thureau-Dangin published hand copies of a Seleucid text of this kind (AO 6484) in [1922: pl. LXI–LXII], he only saw that it contained "arithmetical operations".

That he could see that much presupposed the initial insights in the mathematical terminology which had been produced by Ernst Weidner [1916], Heinrich Zimmern [1916] and Arthur Ungnad [1916] (the latter two were commentaries and philological complements/corrections to Weidner's pioneering publication). The young Weidner, having the good luck to make his military service in Berlin, analyzed two problems from the tablet VAT 6598, seeing from a diagram drawn on it that it probably dealt with the determination of the diagonal of a rectangle.

Weidner's understanding of the terminology was not always correct from a philological point of view (in part, Zimmern and Ungnad corrected that[4]) – in part, he had had to guess from context, that is, from what the operations *did* on the numbers involved, and from how he expected a mathematical text to be organized; none the less, it was technically adequate (so much so that Neugebauer took over one of the mistakes without much damage to his understanding[5]).

---

[4] In particular, Ungnad pointed out that igi.*n*.gál stands for 1 divided by *n* (that is, the reciprocal of *n*, but he does not use that word). This allows him to put to rest Hilprecht's Gnostic fancies, in a paragraph so short that it was probably never really noticed. Unfortunately Ungnad still believed in [1917: 41*f*] that this expression was an ellipsis for the division of any number by *n*, and he did not link it to Zimmern's understanding of *paṭārum* as "splitting"; he therefore translated Assurbanipal's claim (Prism L) to be able to "find reciprocals and make difficult multiplications" as if he was speaking about performing *divisions*. This was taken over by Neugebauer in an editorial note in [Schuster 1930: 196] and still repeated by Adam Falkenstein as the "solution of division-problems" in [1953: 126]. Recently Jeanette Fincke [2003: 111] has transformed it into the solution of "mathematical problems".

[5] Weidner's translation of *en-nam* as "berechne" is repeated in [Neugebauer 1929: 88]. Thureau-

In [1922], John Gadd analyzed the main fragment of BM 15285 (a text about the subdivision of squares). Since it contains only problem statements and no calculations, it did not cast new light on the terminology for operations, but it helped with names for geometrical objects. The last contribution from the pre-Neugebauer period to the investigation of advanced mathematics was Carl Frank's *Straßburger Keilschrifttexte* from [1928]. Six of these texts dealt with mathematical problems, and Frank did not understand much of them – not least, as Neugebauer [1929: 67] was to point out, because Frank translated the sexagesimal floating-point place-value numbers into fixed-point decimal numbers, regularly choosing incoherent orders of magnitude: thereby, in Frank's own words, getting "phantastically high numbers" (p. 20) – as once Hilprecht.

Neugebauer "before Neugebauer"

Though his work was soon to be continued in polite competition with Thureau-Dangin, it was Neugebauer who inaugurated the new era – but only after some preparations. Born in Graz in 1899, he entered the army and the Great War in 1917 so as to receive a *Not-Abitur* and not need to pass the scary examination with its Greek and Latin [Pyenson 1995: 264; Swerdlow 1998: 4]. After sharing a pencil with Ludwig Wittgenstein in a post-war Italian prisoners' camp he studied, first in Graz (1919–1921, electrical engineering and physics) and then in Munich (physics). In 1922, he relocated and reoriented his interests, to Göttingen and mathematics, studying with Richard Courant (just back in Göttingen after three years in Münster), Emmy Noether and Edmund Landau; he soon became Courant's assistant and responsible for the Institute library.

We may link Neugebauer's library work with his life-long engagement in bibliographic projects, first as a founder of *Zentralblatt für Mathematik* and then, as the *Zentralblatt* came under Nazi control, of *Mathematical Reviews*. What concerns us here, however, was a consequence of yet another change of interest. Firstly, in 1926, he edited the first volume of Felix Klein's *Vorlesungen über die Geschichte der Mathematik im 19. Jahrhundert* [Klein 1926] together with Richard Courant – yet since Courant's collaborator for the second volume was St. Cohn-Vossen and Courant had been involved in Klein's lectures as they were held [Rowe 2016a: 32], Neugebauer's role was probably really that of an assistant. Secondly, and decisively, he made the necessary studies of ancient Egyptian for his dissertation *Die Grundlagen der ägyptischen Bruchrechnung* [Neugebauer 1926]. For this, according to [Swerdlow 1998: 4], he had studied ancient Egyptian under

Dangin [1931: 195*f*], when trying to correct, did not do much better – only Albert Schott would identify it as a pseudo-Sumerogram for *mīnûm*, see [Neugebauer 1932b: 8 n. 18].

Hermann Kees and Kurt Sethe; in any case they were examiners for his doctoral examination [Rowe 2016a: 31].[6]

The introduction to this work runs:

> [...] auch die moderne Wissenschaft hat erst allmählich lernen müssen, "vorurteilslos" an die Dinge heranzutreten und sie so zu verstehen, wie sie *geworden* sind. Neben die Forderung, nicht alle Phasen eines Prozesses wie Gleichzeitiges und für unser Verständnis Gleichwertiges zu betrachten, tritt die andere, sich soweit als irgend möglich davor zu hüten, uns geläufige moderne Begriffe und Anschauungen auf antike Verhältnisse kritiklos zu übertragen. So selbstverständlich diese beiden Forderungen zu sein scheinen, so schwierig hatten und haben sie es, sich durchzusetzen. Auch in Untersuchungen über die Mathematik der Ägypter ist oft genug gegen sie verstoßen worden [...]. Kritik und Sorgfalt der Historiker der Mathematik haben es in diesem Punkte nicht vermocht, mit der gleichzeitigen philologischen Arbeit Schritt zu halten.
>
> Auch die Mathematik der letzten Jahrhunderte hat eine große Wandlung erfahren; ihre "Arithmetisierung" hat große Fortschritte gemacht und die Untersuchungen über ihre logischen Grundlagen sind in ein entscheidendes Stadium getreten. Beide Richtungen haben den Blick dafür geschärft, den begrifflichen Kern mathematischer Sätze und Operationen herauszuschälen. Es ist klar, daß auch die Geschichte gerade der *Anfänge* der Mathematik danach streben muß, das Verhältnis zu erkennen, in dem die Begriffe, die in der gegebenen geschichtlichen Entwicklung die ursprünglichen sind, zu jenen Begriffen stehen, die nach modernen Anschauungen diesen Platz in rein logischer Hinsicht einnehmen müßten. [...].
>
> Es war mein Bestreben, beide Tendenzen, sowohl die der historischen wie der mathematischen Wissenschaften, soweit es in meinen Kräften stand, hier zur Geltung zu bringen.

As motto above the introduction we find a quotation from Herman Hankel,

> Es ist eben Mathematik auch eine Wissenschaft, die von *Menschen* betrieben wird, und jede Zeit, sowie jedes Volk hat nur *Einen* Geist.

Together, these formulations show the tensions inherent in Neugebauer's first step as a historian.

Firstly, being brought up in an environment dominated by such figures as David Hilbert, Landau and Noether, he was familiar with the formalist view of mathematics, according to which the search for logical foundations and concepts constitute its essence. Secondly, as expressed in Hankel's formulation of the *Volks-* and *Zeitgeist*-idea, the logical and conceptual structure of mathematics is not the same in all epochs and locations.[7] Thirdly, to trace out these differences and leaps in the development asks

[6] Since Noel Swerdlow's obituary is based in part on oral information from witnesses who themselves are likely to have built upon what Neugebauer had told them concerning his early years, it is likely to be less reliable than David Rowe's archive-based data. Neugebauer himself indeed only refers to Kees's and Sethe's interest in the work and their "valuable remarks and corrections" in his *Vorwort*.

[7] In [2016b: 124*f*], David Rowe summarizes a characterization of Neugebauer's historiographic approach given by his collaborators in later decades thus:

for historical, meticulous work, not lazy reliance on fables or export of present-day ideas. It seems almost certain that the phrase “wie sie *geworden* sind”, with its emphasis, is to be understood as a deliberate echo of Leopold von Ranke’s famous “wie es eigentlich gewesen”, formulated when Ranke was 29 years old and even he making his first step (Neugebauer in 1926 was still younger).

Neugebauer was already preparing to take his second step as a historian. In [1927] he published an article “Zur Entstehung des Sexagesimalsystems” (as long as the dissertation, and indeed his *Habilitationsschrift* [Rowe 2016a: 149]). He was inspired by a desire to use the Sumerian parallel in order to elucidate the character of Egyptian mathematics, but in the article this is only a minor concern (and mostly it has to do with the grammar of numerals and fractions). As told on p. 5, it was made possible by a grant from the Gesellschaft der Wissenschaften zu Göttingen (recommended by Courant), which allowed Neugebauer to pay a visit of several weeks to Anton Deimel in Rome, where the latter “kindly put at disposition his rich familiarity with the Sumerian language and culture”; publications by Thureau-Dangin and Deimel also provide the fundament – in particular Thureau-Dangin’s conviction that *metrology* is the starting point, which Neugebauer sharpened more than Thureau-Dangin would accept. Paradoxically as it may seem, the work belongs squarely in the “pre-Neugebauer period” – and given the basis available (not least the total lack of information about the proto-literate period, but not only that), the outcome is rather fanciful as a *Entstehungsgeschichte;* however, many acute observations on pre-theoretical numeration are made here and in the sequel [Neugebauer 1928b] that might today be counted under the heading of ethnomathematics.

Even Neugebauer’s next “Babylonian” publication, from [1928a], is “pre-Neugebauer”, although it points to what was soon to come. Neugebauer uses Weidner’s translation of a calculation of the diagonal of a rectangular “door”; as already Weidner had seen [1916: 261], the calculation follows the formula $c = a + \frac{b^2}{2a}$, c being the diagonal, a the height and b the base of the door. Since Weidner had not understood the floating-point character of the sexagesimal notation (or, rather, had not expected his readers to know it), he had been forced do insert a number 60 in the translation that is not

> Neugebauer firmly believed in the immutable character of mathematical knowledge, which meant that his field of historical inquiry, the exact sciences, differed from all other forms of human endeavor in one fundamental respect: in this realm there was no room for historical contingency.

I shall not exclude that Neugebauer’s attitudes changed with time; nor, however, that pupils and collaborators imputed their own attitudes on the master (in one case at least, not cited by Rowe, this was clearly the case – I shall omit identification, this not being the adequate occasion for facile polemics), or that the characterization is unduly generalized from the particular case of mathematical astronomy. In any case it is clear that it does not fit Neugebauer’s historiographical starting point.

native to the text; Neugebauer, however, explains it and so gets rid of it. He sees that this formula is an approximation to the "Pythagorean" formula $c = \sqrt{a^2 + b^2} = a\cdot\sqrt{1 + \frac{b^2}{a^2}}$, and he therefore concludes that the "Pythagorean theorem" must have been known in Old Babylonian times. He argues for the possibility of a transmission from India, where the theorem is used in the Āpastamba-*Śulba-sūtra*, or from a common ancestor. So, at this point Neugebauer is still a mathematician-historian with generic knowledge of the field but relying on the translations established by the philologists belonging to the single fields.

A concluding remark states that the identification of the Old Babylonian calculation as an approximate instance of the "Pythagorean rule"

> erst dann zur Basis weiterer Schlüsse gemacht werden kann, wenn man sie in ein weit vollständigeres Bild der babylonischen und indischen Mathematik einzuordnen vermag, als wir es heute besitzen. Es ist wieder einmal ersichtlich, wie dringend nötig für alle geschichtlichen Fragen die Publikation der in unseren Museen vorhandenen Texte wäre. Was trotzdem aus den bereits heute zugänglichen Quellen erschlossen werden kann, hoffe ich an anderer Stelle darlegen zu können.

This announced the "Neugebauer era" – a shift described by Wolfram von Soden [1937] in these words:

> Wenn die babylonische Mathematik Jahrzehnte hindurch nicht die Beachtung in der Wissenschaft gefunden hat, die ihr ihrer Bedeutung nach zukommt, so lag das nicht zum wenigsten daran, daß lange Jahre nur geringe Teile der uns erhaltenen mathematischen Literatur veröffentlicht waren. Wohl besitzen faßt alle großen Tontafelsammlungen meist schon seit längerer Zeit auch mathematische Tafeln in größerer oder geringerer Anzahl; herausgegeben wurden von diesen im allgemeinen aber nur die leicht zu lesenden Rechentabellen, während die viel schwierigeren Aufgabentexte mit wenigen Ausnahmen immer wieder zurückgestellt wurden. Es ist eines der großen Verdienste O. NEUGEBAUER's, daß er bei Inangriffnahme der umfassenden Erforschung der babylonischen Mathematik diesen Mangel nicht nur erkannte, sondern auch alsbald an seine Abstellung ging.

The "Neugebauer era"

The beginning of this *Inangriffnahme* and this era is marked by three initiatives: the *Quellen und Studien*, Neugebauer's seminar, and the planning of the *Mathematische Keilschrift-Texte*.

In 1929, Neugebauer launched the *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* together with Julius Stenzel and Otto Toeplitz. The first fascicle of its *Abteilung B*, appearing in March 1929, is introduced in these words [Neugebauer, Stenzel & Toeplitz 1929:1–2]:

> [...]
>
> Durch den Titel "Quellen und Studien" wollen wir zum Ausdruck bringen, daß wir in der steten Bezugnahme auf die Originalquellen die notwendige Bedingung aller

> ernst zu nehmenden historischen Forschung erblicken. Es wird daher unser erstes Ziel sein, *Quellen* zu erschließen, d. h. sie nach Möglichkeit in einer Form darzubieten, die sowohl den Anforderungen der modernen Philologie genügen kann, als auch durch Übersetzung und Kommentar den Nichtphilologen in den Stand setzt, sich selbst in jedem Augenblick von dem Wortlaut des Originales zu überzeugen. Den berechtigten Ansprüchen beider Gruppen, Philologen und Mathematikern, nach wirklicher Sachkenntnis Genüge zu leisten, wird nur möglich sein, wenn es gelingt, eine enge Zusammenarbeit zwischen ihnen herzustellen. Diese anzubahnen soll eine der wichtigsten Aufgaben unseres Unternehmens sein.
>
> Die technische Durchführung dieses Programmes denken wir uns so, daß in zwangloser Folge zwei Publikationsreihen erscheinen. Die eine, A, "Quellen", soll die eigentlichen Editionen größeren Umfanges umfassen, enthaltend den Text in der Sprache des Originales, philologischen Apparat und Kommentar und eine möglichst getreue Übersetzung, die auch dem Nichtphilologen den Inhalt des Textes so bequem als irgend tunlich zugänglich macht. Jedes Heft dieser "Quellen" wird ein für sich geschlossenes Ganzes bilden - Die Hefte der Abteilung B, "Studien", sollen jeweils eine Reihe von Abhandlungen zusammenfassen, die in engerem oder weiterem Zusammenhang mit dem aus den Quellen gewonnenen Material stehen können.

The co-editors were 17 respectively 19 years older than Neugebauer; moreover, their interest was not genuine history but the pseudo-historical "genetic method" in teaching.[8] We may be confident, also because of the closeness to Neugebauer's programme from 1928 as quoted above, that Neugebauer was the driving force behind the endeavour.

Next, "Neugebauer's Göttingen seminar". In 1985 Kurt Vogel told me about the immense astonishment when one morning Hans-Siegmund Schuster related at Neugebauer's seminar that he had discovered solutions of second-degree equations in a cuneiform text. As any piece of oral history told at 56 years' distance (and committed to writing by the historian another three decades later), this might be suspected to mistake some particulars; however, the central details, the existence of a seminar and the key role played by Schuster (at most 19 years old, and not yet officially enrolled in university!) are confirmed by Neugebauer in [1929: 80], where the "Mitarbeit von Herrn H. S. Schuster an einem Seminar über babylonische

---

[8] In Toeplitz's own words [1927: 94]:

> Nichts liegt mir ferner als eine Geschichte der Infinitesimalrechnung zu lesen; ich selbst bin als Student aus einer ähnlichen Vorlesung weggelaufen. Nicht um die Geschichte handelt es sich, sondern um die Genesis der Probleme, der Tatsachen und Beweise, um die entscheidenden Wendepunkte in dieser Genesis.

He was neither interested, as we see, in *wie es eigentlich gewesen*, nor in *wie es* [*eigentlich*] *geworden*. His historiography was close to that from which von Ranke had once distanced himself, that which sees as its task "die Vergangenheit zu richten, die Mitwelt zum Nutzen zukünftiger Jahre zu belehren" ([von Ranke 1885: vii], first published in 1824). Cf. also [Folkerts, Scriba & Wußing 2002: 134].

Mathematik" is acknowledged.[9] Vogel did not date the event, but it must have taken place in early 1929.

Thirdly, the introduction to [MKT, I, v] begins in these slightly ironical words:

> Das Ziel dieser Arbeit war von Anfang an die Schaffung einer vollständigen Sammlung aller mathematischen Keilschrift-Texte. [...]
>
> Wenn ich sage, daß es von Anfang an meine Absicht gewesen wäre, eine Edition aller erfaßbaren mathematischen Keilschrift-Texte zu veranstalten, so soll das heißen, daß diese Arbeit zwar nie ihre Grundtendenz verändert hat, um so mehr aber ihren Umfang.

In 1929 Neugebauer had thus

– created a publication channel for the programme he had outlined in 1928;
– organized collaborative work on Babylonian mathematics;
– planned his own publication of the available corpus in agreement with the outline from 1928 and the *Geleitwort* from the *Quellen und Studien*.

In 1929 we also see the earliest *results*, published in *Quellen und Studien* B. In the first fascicle, Neugebauer has two articles, the second of which in collaboration with V. V. Struve. His alone (except for the collaboration of Schuster mentioned above) is "Zur Geschichte der babylonischen Mathematik" [Neugebauer 1929]. This unspecific title covers a reinterpretation of two of the mathematical texts which Frank had published the year before (#10 and #8). As the basis for his revision, he keeps the sexagesimal place-value numbers instead of trying to translate them into decimal notation. In Neugebauer's own words (p. 67):

> Obwohl Nichtassyriologe, sehe ich mich doch gezwungen, im folgenden die Frankschen Übersetzungen nicht einfach abzudrucken; diese sind nämlich in ihren Zahlenangaben so gründlich an dem babylonischen Sexagesimalsystem gescheitert, daß oft gerade die entscheidenden Stellen nicht zu verwenden sind. Die dezimale Umschreibung der Zahlen des Originales erweist sich wieder einmal als eines der größten Hindernisse in der Erschließung eines Textes, solange dieser nicht bis in alle Einzelheiten hinein verstanden ist. - Schließlich lassen sich, nachdem einmal der sachliche Inhalt klargestellt ist, auch die Lesungen selbst nicht unerheblich verbessern - kleine philologische Ungenauigkeiten bitte ich mir nicht zu schwer anzurechnen.

[Høyrup 2016: 175–178] confronts Frank's transliteration and translation of #10 with Neugebauer's new translation – Neugebauer gave no complete transliteration, only pointwise philological corrections.[10] These were few, but just enough to make mathematical sense of the text. As pointed out above, one innocuous philological

---

[9] [Schuster 1930: 194 n. 1], beyond Göttingen, gives the probably more precise "Seminar über Fragen aus der Geschichte der antiken Mathematik" as his institutional abode. The archives of Göttingen University may contain more detailed information, but for the present purpose this has no importance.

[10] He was to offer one in [MKT I, 259 *f*], now based on the values given in Thureau-Dangin's *Syllabaire*, which Frank had not used.

mistake is taken over from Weidner by Frank as well as by Neugebauer without damage – see note 5.

The new philological insights and the correct identification of the numerical parameters (and the courage to ascribe unexpected insights to the Babylonians) allowed Neugebauer to grasp both what is meant by the far from trivial statement of the problem (kindly supported by a drawing though not at all to scale) and to explain the calculations by which it is solved. As spin-off he discovered, and explained, that the Babylonians solved the division by *n* through a multiplication by the reciprocal.[11]

In this connection he also introduced his new transcription of sexagesimal place-value numbers. In [1911: 32], Louis Delaporte (the first to do so?) had used the mark ´ for the order of "minutes" ($\frac{1}{60}$) and ˝ for "seconds" ($\frac{1}{60^2}$). Scheil [1916: 139], immediately followed by Ungnad [1916: 366], had used ° as a mark for "order zero". In [1932: 12], Thureau-Dangin was to introduce `, ``, ```, etc. for the ascending orders. In 1929, Neugebauer established instead the system which has become the standard in all discussions of Babylonian mathematical astronomy, Thureau-Dangin's 1`5°2´20˝ corresponding to Neugebauer's 1,5;2,20. Neugebauer must have known the traditional Assyriologists' notation from [Ungnad 1916], which he cites repeatedly, but he prefers to ignore its existence – unless his words (p. 68 n. 3)

> Die Einführung besonderer Namen ("Minuten, Sekunden" usw.) bedeutet nur eine überflüssige Belastung des Rechnens,

are meant to polemicize against it.[12]

Frank's #8 is heavily damaged, and contains only a sequence of related statements, no solutions. However, even here Neugebauer's method allowed him to understand; as he concluded,

> Ganz abgesehen von der Verwendung von Dreiecks- und Trapezformel sehen wir, daß komplizierte lineare Gleichungssysteme aufgestellt und gelöst werden, daß man ganz systematisch Aufgaben quadratischen Charakters stellt und zweifellos auch zu lösen verstand – und all dies mit einer Rechentechnik, die der unseren völlig äquivalent ist.

A "Zusatz nach Abschluß der Korrektur" then adds that a problem actually *solving* a mixed quadratic problem has been found, and acknowledges the role of Schuster, cf. above.

---

[11] He refers to Zimmern [1916: 24], who explains *igi* n *dù-a* (Zimmern's transliteration) to mean "Teil n spalten". In an editorial note to [Schuster 1930: 196] he none the less accepts as an alternative Ungnad's more general reading from [1917] as "division" (above, note 4). Neugebauer apparently did not yet feel confident enough to correct renowned Assyriologists. He also did not provide a new transliteration but only corrected that of Frank pointwise.

[12] In [1932a: 231], Neugebauer was to present a valid argument against the °-´-˝ notation (including Thureau-Dangin's extension, which had been introduced in the meantime) – but valid only for astronomical work, where conflicts might arise with the writing of angular measures.

With all the immaturity that is by necessity inherent in a *first*, this earliest article by Neugebauer in *Quellen und Studien* B shows not only that a watershed had been passed but also points to what was to characterize Neugebauer's future work on Babylonian mathematics.

One further aspect of both – article and future work – should be highlighted. Neugebauer makes use of modern algebraic symbolism in his analysis; this, however, must not be read as a claim that this mirrors the Babylonian method. The words *völlig Äquivalent* in the conclusion are chosen with care. One particular passage in the solution, seen by Neugebauer (p. 73) to be *Vom besonderen Interesse*, makes this very clear. In Neugebauer's view the procedure here is *umwegig*; moreover, as he observes,

> Der Sinn eines solchen Umweges wird erst zu klären sein, wenn man die Technik des babylonischen Zahlenrechnens, d. h. die Verwendungsregeln ihrer Tabellen besser kennt, als es heute der Fall ist.

That is, firstly: the Babylonian method is not, or at least not always, isomorphic with what follows from the use of modern formulas; but secondly, the difference is supposed to depend on the Babylonian use of arithmetical tables.

As suggested to me by Jöran Friberg, the explanation is rather the use of a geometric stratagem – see [Høyrup 2002: 241*f*]. Since Weidner, however, the chief key to the terminology of the mathematical texts had been what the operations effected on the numbers; most of the apparently geometric references of the vocabulary were supposed to be dead metaphors (as our "square" of the number 3, which certainly does not have four sides). This was to remain the standard way, not only of Neugebauer but of all discussions of Babylonian mathematics for more than 50 years.

Obviously, not *all* geometric references were explained away. The two texts dealt with in the article treat of subdivided trapezia, that was not doubted. The following article in the fascicle, [Neugebauer & Struwe 1929], also deals with geometry, namely that of the circle (thus the title, actually also that of the truncated cone). It enriched the understanding of the mathematical terminology; ascertained that the circular diameter was supposed to be $^1/_3$ of the perimeter, and that the area was found correspondingly (in modern terms, with π approximated as 3); and identified one of the Babylonian ways to find (that is, approximate) the volume of a truncated cone.

Three articles in the second fascicle also deal with Babylonian mathematics, two by Neugebauer and one by Schuster. [Neugebauer 1930a], "Beiträge zur babylonischen Arithmetik", continues the revision of Frank's Old Babylonian Straßburger texts, still without a new full transliteration but now going more into detail with the philological corrections and including attempts to trace some of the ideas that underlie the formulations. As to mathematical substance, the texts are shown to deal (correctly) with distribution of silver in arithmetical progression and with mixed second-degree problems (about the sides of a rectangle and of two squares). Neugebauer does not speak of "algebra"; his title shows that at this point he regards the problems – even the problems about rectangles and squares – as instances of arithmetical thinking.[13]

[Neugebauer 1930b], "Sexagesimalsystem und babylonische Bruchrechnung I", is a continuation of [Neugebauer 1928b] and in debate with [Thureau-Dangin 1928], and as such it may still be classified as "pre-Neugebauer".

[Schuster 1930], "Quadratische Gleichungen der Seleukidenzeit aus Uruk", on the other hand, belongs firmly in the Neugebauer era. It analyzes precisely that text in which Thureau-Dangin [1922: pl. LXI–LXII] had only seen "arithmetical operations". Beyond the mathematical interpretation, Schuster here makes important observations about the mathematical terminology of the Seleucid epoch (obviously not yet knowing that part of what he finds is specifically Seleucid – only familiarity with a fuller corpus would allow that discovery).

In [1932], Thureau-Dangin published his *Esquisse d'une histoire du système sexagésimal*. This important booklet can be regarded as the culmination of his work from the preceding decades – indeed, as the crown upon work in which Assyriologists had been engaged since Hincks and Rawlinson. It is in critical dialogue with [Neugebauer 1928b] – but not with the new discoveries made around the "Neugebauer seminar".

Dialogue of the latter kind had begun (somewhat reluctantly) the year before, in [Thureau-Dangin 1931]. This article begins as follows:

> Notre connaissance de la mathématique babylonienne a pour principales sources des textes remontant au temps de la première dynastie : [...] La terminologie des textes mathématiques babyloniens est déjà en grande partie élucidée, grâce notamment à Zimmern et Ungnad. D'autre part, les études d'O. Neugebauer, qui ont pour objet plutôt le fond que la forme de ces textes, apportent au philologue d'utiles données.

That Ungnad and Zimmern (forgetting Gadd) had elucidated most of the terminology was a rash judgment, given how much of even the few texts listed by Thureau-dangin was still in the dark at the moment. That Neugebauer (and his group of collaborators, of whom Schuster and Struve were so far visible in print) was rather going for mathematical substance (*fond*), and that the philology was left for Assyriologist was also a problematic opinion. On the second account, it is true, many of Thureau-Dangin's Assyriological notes over the next seven years deal with terminology and grammar – which shows how overly optimistic he has been on the first account. But over the same years, as it turned out, Thureau-Dangin did not keep away from the mathematical substance of the texts, nor did Neugebauer and his group[14] abstain from philology.

[13] Neugebauer did use this designation in [1932b] and [1936], the first and third in a series of three "Studien zur Geschichte der antiken Algebra" (the second deals with Apollonios). The particular conceptualization of "algebra" in these articles is discussed in [Høyrup 2016b].

[14] Beyond Schuster and Struve, Neugebauer refers to the collaboration of Heinz Waschow and Albert Schott. Whereas Schuster seems not to have worked on mathematics after 1930, and Struve became head of the Cuneiform collection of the Leningrad Ermitage (and intensely engaged in the social history of Ur III), Waschow and Schott continued their collaboration – the treatment

Over these following six to seven years, a tense, hardly friendly but always polite race between the two took place. I shall not follow it in detail, but only mention the two chief accomplishments: Neugebauer's *Mathematische Keilschrift-Texte* [MKT], published in three tomes in *Quellen und Studien* A in 1935–1937, and Thureau-Dangin's *Textes mathématiques babyloniens,* TMB.

The introduction to the latter work ends in these words [TMB, xl]:

> Le présent volume ne comprend aucun texte qui n'ait été édité ailleurs dans sa forme originale. Le principal objet que je me suis proposé en le rédigeant a été de mettre des documents à la disposition des historiens de la pensée mathématique. Puisse-t-il leur être de quelque utilité et les aider à préciser ce que la science mathématique, l'algèbre notamment, doit aux scribes de Sumer et d'Accad.

In his review, von Soden [1939: 144] elaborates and adds:

> Dieses neue Werk hat nicht die Aufgabe, Neugebauer's MKT zu ersetzen; werden doch weder die Lichtdrucke und Autographien der Texte wiederholt noch auch alle Texte neubearbeitet. Th.-D.'s Ziel war es vielmehr, unter vollständiger Beiseitelassung der Rechentabellen [...] diejenigen Aufgabentexte, deren Erhaltungszustand ein wenigstens im großen und ganzen befriedigendes Verständnis ermöglicht, in einer wohlfeileren Ausgabe möglichst vielen Forschern zugänglich zu machen, da der leider so hohe Preis der MKT ihrer weiteren Verbreitung im Wege steht.

Von Soden goes on with a praise of the philological contributions of the TMB (corrections to Neugebauer's readings as well as glossaries). None the less, MKT stands out, also philologically, as the basis on which one has to build; TMB, instead, is aimed at readers who are interested in the history of mathematics. The 42 pages long introduction to the volume confirms this: it deals with mathematical substance, not with matters that would interest Assyriologists alone. Moreover, while Neugebauer's calculations by means of symbolic algebra serve verification purposes and are not claimed to represent Babylonian thinking, Thureau-Dangin's symbolic translations really seem to be meant as *translations* of Babylonian thought.

This agrees with the paths taken by the two after 1937/38. In 1937, Neugebauer seems to have considered his work on Babylonian *mathematics* as on the whole completed. Only in [1945] would he publish *Mathematical Cuneiform Texts* in collaboration with Abraham Sachs, containing texts to which he had not had access when preparing the MKT; his popularization *The Exact Sciences in Antiquity* from [1951] contains a chapter on Babylonian mathematics (no new insights, of course). Apart from that, he only published two or three small items on the topic. Instead, from 1937 onward he concentrated on astronomy (Babylonian and otherwise). Thureau-Dangin still

---

of BM 34568 in [MKT III, 14–22] is said to be ("bis auf einige Kleinigkeiten) due to Waschow, while Schott, though mostly contributing philological advice for the MKT, was an intended chief collaborator in the astronomical project that eventually gave rise to [Neugebauer 1955] (in which Schott, deceased in 1945 or earlier and obviously isolated from Neugebauer during the War, could not participate) – see [Neugebauer 1937: 30].

published a few items in 1938 about Babylonian mathematics [1938b, 1938c], and of course TMB. After that his interest *within Assyriology* turned to other matters, as can be seen from the notes he published in *Revue d'Assyriologie*. But he did not give up the history of mathematics.

One article from [1940a], it is true, carries the title "Notes sur la mathématique babylonienne"; these notes, however, are a discussion of topics that would interest historians of mathematics: the treatment of irreducible third-degree problems; the question whether the Babylonians had a notion of negative numbers; and the relation between Babylonian "algebra" and Euclid's *Elements* II. These discussions are in explicit dialogue with Solomon Gandz, Ettore Bortolotti, Vogel, and others; the connection with *Elements* II had been the topic of [Neugebauer 1936], but in that case the dialogue is not explicit. Thureau-Dangin also has the dubious honour to misread (as quite a few historians of mathematics were to do in subsequent decades) Neugebauer's symbolic formulas as translations and thus to attribute to him a claim that the Babylonians made use of negative numbers.[15] [Thureau-Dangin 1938a], "Fausse position et origine de l'algèbre", and [Thureau-Dangin 1940b], "L'Origine de l'algèbre", on their part, are clear contributions to the history of mathematics, using Babylonian mathematics as a resource only, and one resource among others.

After 1937 / 38, Assyriologists, when dealing with matters numerate (as they had to when discussing many administrative matters), returned to mathematical onionology, leaving texts containing too many numbers in place-value notation as a "matter for Neugebauer" (not being aware that Neugebauer had left the field since long);[16]

---

[15] Thureau-Dangin offers two references, [MKT I, 455*f*] and [Neugebauer 1936: 256]. In the latter, Neugebauer confronts two expressions

(a) (b) (c) dirig
(b) (a) (c) ba-lal

where a, b and c are the same numbers in the two lines. Neugebauer explains that d i r i g means "exceed" and l a l "subtract". He translates the latter into $b - a = c < 0$, and that is what he refers to as having a negative right-hand side. In [MKT I, 456] he also speaks of the "negative right-hand side" of the formula

$$2{,}30(x - y) - (x + y)^2 = -16{,}40$$

and explains that this corresponds to the "ba-lal «abgezogen»" of the text.

A look at [Neugebauer 1936: 255] is informative. Here it is said that the "series texts" expand "den babylonischen Zahlbereich zum vollen Bereich der Rationalzahlen". What Neugebauer speaks of here points back to the task formulated in [Neugebauer 1926: 1], *den begrifflichen Kern mathematischer Sätze und Operationen herauszuschälen*. Thureau-Dangin did not set himself such tasks, and therefore, we may say, supposed Neugebauer to use mathematical terms and algebraic formulas naively, as he did himself.

[16] The exceptions until 1970 are rare enough to be listed: four publications of new texts by Taha Baqir [1950a; 1950b; 1951; 1962]; one by Albrecht Goetze [1951]; some discussions around Baqir's publications [von Soden 1952], [Bruins 1953], and [Gundlach & von Soden 1963]; the belated and problematic edition of the mathematical Susa texts [Bruins & Rutten 1961], with the indispensable review [von Soden 1964]; and two suggested interpretations of the "Pythagorean

historians of mathematics, on their part, accepted Neugebauer's translations (or, more often, his analysis in symbols) as the final work on the matter.

So, the "Neugebauer era", heroic but shortlived, ended in 1937/38.

References

Allotte de la Fuÿe, François-Maurice, 1906. "La mesure des volumes dans les textes archaïques de la Chaldée". *Revue d'Assyriologie et d'Archéologie Orientale* **6** (1904–07), 75–78.

Allotte de la Fuÿe, François-Maurice, 1909. "Le gour saggal et ses subdivisions d'après les documents présargoniques de Lagaš". *Revue d'Assyriologie et d'Archéologie Orientale* **7** (1909–10), 33–47.

Allotte de la Fuÿe, François-Maurice, 1915a. "Un cadastre de Djokha". *Revue d'Assyriologie et d'Archéologie Orientale* **12**, 47–54.

Allotte de la Fuÿe, François-Maurice, 1915b. "Mesures agraires et formules d'arpentage à l'époque présargonique". *Revue d'Assyriologie* **12**, 117–146.

Baqir, Taha, 1950a. "An Important Mathematical Problem Text from Tell Harmal". *Sumer* **6**, 39–54.

Baqir, Taha, 1950b. "Another Important Mathematical Text from Tell Harmal". *Sumer* **6**, 130–148.

Baqir, Taha, 1951. "Some More Mathematical Texts from Tell Harmal". *Sumer* **7**, 28–45.

Baqir, Taha, 1962. "Tell Dhiba'i: New Mathematical Texts". *Sumer* **18**, 11–14, pl. 1–3.

Brandis, Johannes, 1866. *Das Münz-, Mass- und Gewichtswesen in Vorderasien bis auf Alexander den Großen*. Berlin: Wilhelm Herz.

Bruins, Evert M., 1953. "Revision of the Mathematical Texts from Tell Harmal". *Sumer* **9**, 241–253.

Bruins, Evert M., 1957. "Pythagorean Triads in Babylonian Mathematics". *The Mathematical Gazette* **41**, 25–28.

Bruins, Evert M., & Marguerite Rutten, 1961. *Textes mathématiques de Suse*. (Mémoires de la Mission Archéologique en Iran, XXXIV). Paris: Paul Geuthner.

Cory, Isaac Preston, 1832. *Ancient Fragments of the Phoenician, Chaldæan, Egyptian, Tyrian, Carthaginian, Indian, Persian, and Other Writers.* 2London: William Pickering.

Delaporte, Louis, 1911. "Document mathématique de l'époque des rois d'Our". *Revue d'Assyriologie et d'Archéologie Orientale* **8**, 131–133.

Falkenstein, Adam, 1953. "Die babylonische Schule". *Saeculum* **4**, 125–137.

Fincke, Jeanette C., 2003. "The Babylonian Texts of Nineveh: Report on the British Museum's *Ashurbanipal Library Project*". *Archiv für Orientforschung* **50** (2003–2004), 111–149.

Folkerts, Menso, Christoph J. Scriba & Hans Wussing. "Germany", pp. 109–150 *in* Joseph W. Dauben & Christoph J. Scriba (eds), *Writing the History of Mathematics: Its Historical Development*. Basel etc.: Birkhäuser.

Fossey, Charles, 1904. *Manuel d'assyriologie*. Tôme premier. *Explorations et fouilles, déchiffrement des cunéiformes, origine et histoire de l'écriture*. Paris: Leroux.

Frank, Carl, 1928. *Straßburger Keilschrifttexte in sumerischer und babylonischer Sprache*. (Schriften der Straßburger Wissenschaftlichen Gesellschaft in Heidelberg, Neue Folge, Heft 9). Berlin & Leipzig: Walter de Gruyter.

triples" of the tablet Plimpton 322 [Bruins 1957], [Price 1964]. And finally Aisik A. Vajman's book-length treatment of "Sumero-Babylonian mathematics" from [1961], in which several new texts were presented and analyzed.

Gadd, Cyril John, 1922. “Forms and Colours”. *Revue d’Assyriologie et d’Archéologie Orientale* **19**, 149–159.

Gelb, Ignace, 1967. “Approaches to the Study of Ancient Society”. *Journal of the American Oriental Society* **87**, 1–8.

Goetze, Albrecht, 1951. “A Mathematical Compendium from Tell Harmal”. *Sumer* **7**, 126–155.

Gundlach, Karl-Bernhard, & Wolfram von Soden, 1963. “Einige altbabylonische Texte zur Lösung »quadratischer Gleichungen«”. *Abhandlungen aus dem mathematischen Seminar der Universität Hamburg* **26**, 248–263.

Heuzey, Léon, 1906. “À la mémoire de Jules Oppert”. *Revue d’Assyriologie et d’Archéologie Orientale* **6** (1904–07), 73–74.

Hilprecht, Hermann V., 1906. *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur*. (The Babylonian Expedition of the University of Pennsylvania. A: Cuneiform Texts, XX,1). Philadelphia: Department of Archaeology, University of Pennsylvania.

Hincks, Edward, 1848. “On the Third Persepolitan Writing, and on the Mode of Expressing Numerals in Cuneatic Characters”. *Transactions of the Royal Irish Academy. Polite literature* **21**, 249–256.

Hincks, Edward, 1854. “On the Assyrian Mythology”. *Transactions of the Royal Irish Academy* **22**:6, 405–422.

Høyrup, Jens, 2002. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. New York: Springer.

Høyrup, Jens, 2016a. “As the Outsider Walked In: The Historiography of Mesopotamian Mathematics Until Neugebauer”, pp. 165–195 *in* Alexander Jones, Christine Proust & John M. Steele (eds), *A Mathematician’s Journey: Otto Neugebauer and Modern Transformations of Ancient Science*. Cham etc.: Springer.

Høyrup, Jens, 2016b. “What is ‘geometric algebra’, and what has it been in historiography?”. *Preprint*, Roskilde University, 15.1.2016.

Høyrup, Jens, 2016c. “Mesopotamian Mathematics, Seen ‘from the Inside’ (by Assyriologists) and ‘from the Outside’ (by Historians of Mathematics)”, pp. 53–78 *in* Volker R. Remmert, Martina Schneider & Henrik Kragh Sørensen (eds), *Historiography of Mathematics in the 19th and 20th Centuries*. Basel: Birkhäuser.

Klein, Felix, 1926. *Vorlesungen über die Geschichte der Mathematik im 19. Jahrhundert*. 2 vols. Berlin: Julius Springer, 1926–27.

MKT: O. Neugebauer, *Mathematische Keilschrift-Texte*. 3 vols. (Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung A: Quellen. 3. Band, erster-dritter Teil). Berlin: Julius Springer, 1935, 1935, 1937.

Neugebauer, Otto, 1926. *Die Grundlagen der ägyptischen Bruchrechnung*. Berlin: Julius Springer.

Neugebauer, Otto, 1927. “Zur Entstehung des Sexagesimalsystems”. *Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse*, Neue Folge **13**:1.

Neugebauer, Otto, 1928a. “Zur Geschichte des pythagoräischen Lehrsatzes”. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse*, 45–48.

Neugebauer, Otto, 1928b. “Ueber die Entstehung des Sexagesimalsystems”. *Archeion* **9**, 209–216.

Neugebauer, Otto, 1929. “Zur Geschichte der babylonischen Mathematik”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 67–80.

Neugebauer, Otto, 1930a. “Beiträge zur Geschichte der Babylonischen Arithmetik”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 120–130.

Neugebauer, Otto, 1930b. “Sexagesimalsystem und babylonische Bruchrechnung”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31).

Neugebauer, Otto, 1932a. “Zur Transkription mathematischer und astronomischer Keilschrift-texte”. *Archiv für Orientforschung* **8** (1932–33), 221–223.

Neugebauer, Otto, 1932b. “Studien zur Geschichte der antiken Algebra I”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **2** (1932–33), 1–27.

Neugebauer, Otto, 1936. “Zur geometrischen Algebra (Studien zur Geschichte der antiken Algebra III)”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **3** (1934–36), 245–259.

Neugebauer, Otto, 1937. “Untersuchungen zur antiken Astronomie I”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik.* Abteilung B: *Studien* **4** (1937–38), 29–33.

Neugebauer, Otto, & Abraham Sachs, 1945. *Mathematical Cuneiform Texts.* (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society.

Neugebauer, Otto, 1951. *The Exact Sciences in Antiquity*. (Acta historica scientiarum naturalium et medicinalium, 9). København: Munksgaard.

Neugebauer, Otto, 1955. *Astronomical Cuneiform Texts: Babylonian Ephemerides of the Seleucid Period for the Motion of the Sun, the Moon, and the Planets*. London: Lund Humphries, 1955.

Neugebauer, Otto, Julius Stenzel & Otto Toeplitz, 1929. Geleitwort”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 1–2.

Neugebauer, Otto, & W. Struve, 1929. “Über die Geometrie des Kreises in Babylonien”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 81–92.

Oppert, Jules, 1859. *Expédition scientifique en Mésopotamie exécutée par ordre du Gouvernement de 1851 à 1854*. II. *Déchiffrement des inscriptions cunéiformes*. Paris: Imprimerie Impériale.

Oppert, Jules, 1872. “L’étalon des mesures assyriennes fixé par les textes cunéiformes”. *Journal asiatique*, sixième série **20** (1872), 157–177; septième série **4**, 417–486.

Oppert, Jules, 1886. “La notation des mesures de capacité dans les documents juridiques cunéiformes”. *Zeitschrift für Assyriologie und verwandte Gebiete* **1**, 87–90.

Price, Derek J. de Solla, 1964. “The Babylonian ‘Pythagorean Triangle’ Tablet”. *Centaurus* **10**, 219–231.

Pyenson, Lewis, 1995. “Inventory as a Route to Understanding: Sarton, Neugebauer, and Sources” *History of Science* **33**, 253–282.

Rawlinson, Henry, 1855. “Notes on the Early History of Babylonia”. *Journal of the Royal Asiatic Society of Great Britain and Ireland* **15**, 215–259.

Rowe, David E., 2016a. “From Graz to Göttingen: Neugebauer’s Early Intellectual Journey”, pp. 1–59 *in* Alexander Jones, Christine Proust & John M. Steele (eds), *A Mathematician’s Journey: Otto Neugebauer and Modern Transformations of Ancient Science*. Cham etc.: Springer.

Rowe, David E., 2016b. “Otto Neugebauer’s Vision for Rewriting the History of Ancient Mathematics”, pp. 123–141 *in* Volker R. Remmert, Martina Schneider & Henrik Kragh Sørensen (eds), *Historiography of Mathematics in the 19th and 20th Centuries*. Basel: Birkhäuser.

Scheil, Vincent, 1915. “*Les tables* | *igi x gal-bi*, etc.”. *Revue d’Assyriologie et d’Archéologie Orientale* **12**, 195–198.

Scheil, Vincent, 1916. “Notules. XX.–Le texte mathématique 10201 du Musée de Philadelphie”. *Revue d’Assyriologie et d’Archéologie Orientale* **13**, 138–142.

Schuster, H. S., 1930. “Quadratische Gleichungen der Seleukidenzeit aus Uruk”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 194–200.

Smith, George, 1872. “On Assyrian Weights and Measures”. *Zeitschrift für Ägyptische Sprache und Altertumskunde* **10**, 109–112.

Swerdlow, Noel M., 1998. “Otto E. Neugebauer, May 26, 1899–February 19, 1990”. *The National Academies Press* **75**.
http://www.nap.edu/readingroom/books/biomems/oneugebauer.pdf (accessed 4.12.2005).

Thureau-Dangin, François, 1897. “Un cadastre chaldéen”. *Revue d’Assyriologie* **4** (1897–98), 13–27.

Thureau-Dangin, François, 1922. *Tablettes d’Uruk à l’usage des prêtres du Temple d’Anu au temps des Séleucides.* (Musée de Louvre – Département des Antiquités Orientales. Textes cunéiformes, 6). Paris: Paul Geuthner.

Thureau-Dangin, François, 1928. “L’origine dy système sexagésimal”. *Revue d’Assyriologie et d’Archéologie Orientale* **25**, 115–118.

Thureau-Dangin, François, 1931. “Notes sur la terminologie des textes mathématiques”. *Revue d’Assyriologie et d’Archéologie Orientale* **28**, 195–198.

Thureau-Dangin, F., 1932. *Esquisse d’une histoire du système sexagésimal.* Paris: Geuthner.

Thureau-Dangin, F., 1938a. “La méthode de fausse position et l’origine de l’algèbre”. *Revue d’Assyriologie* **35**, 71–77.

Thureau-Dangin, F., 1938b. “Notes assyriologiques. LXXVI. – Histoire d’un problème babylonien. LXXXVII. – « Multiplier par » dans les textes cunéiformes du temps des Séleucides”. *Revue d’Assyriologie et d’Archéologie Orientale* **35**, 104–106.

Thureau-Dangin, F., 1938c. “Le « grain », mesure de surface”. *Revue d’Assyriologie et d’Archéologie Orientale* **35**, 156–157.

Thureau-Dangin, F., 1940a. “Notes sur la mathématique babylonienne”. *Revue d’Assyriologie* **37** (1940–41), 1–10.

Thureau-Dangin, F., 1940b. “L’Origine de l’algèbre”. *Académie des Belles-Lettres. Comptes Rendus*, 84e année, N. 4,, 292–319.

TMB: François Thureau-Dangin, *Textes mathématiques babyloniens*. Leiden: Brill, 1938.

Toeplitz, Otto, 1927. “Das Problem der Universitätsvorlesungen über Infinitesimalrechnung und ihrer Abgrenzung gegenüber der Infinitesimalrechnung an den höheren Schulen”. *Jahresbericht der deutschen Mathematiker-Vereinigung* **36**, 88–100.

Ungnad, Arthur, 1916. “Zur babylonischen Mathematik”. *Orientalistische Literaturzeitung* **19**, 363–368.

Ungnad, Arthur, 1917. “Lexikalisches”. *Zeitschrift für Assyriologie und verwandte Gebiete* **31** (1917–18), 38–57.

Vajman, A. A., 1961. *Šumero-vavilonskaja matematika. III-I Tysjačeletija do n. e.* Moskva: Izdatel’stvo Vostočnoj Literatury.

von Ranke, Leopold, 1885. *Geschichten der romanischen und germanischen Völker von 1494 bis 1514.* 3Leipzig: Duncker & Humblott.

von Soden, Wolfram, 1937. [Review of MKT]. *Zeitschrift der Deutschen Morgenländischen Gesellschaft* **91**, 185–203.

von Soden, Wolfram, 1939. [Review of TMB]. *Zeitschrift der Deutschen Morgenländischen Gesellschaft*

93, 143–152.
von Soden, Wolfram, 1952. “Zu den mathematischen Aufgabentexten vom Tell Harmal”. *Sumer* **8**, 49–56.
von Soden, Wolfram, 1964. [Review of TMS]. *Bibliotheca Orientalis* **21**, 44–50.
Weidner, Ernst F., 1916. “Die Berechnung rechtwinkliger Dreiecke bei den Akkadern um 2000 v. Chr.” *Orientalistische Literaturzeitung* **19**, 257–263.
Weißbach, Franz Heinrich, 1907. “Über die babylonischen, assyrischen und altpersischen Gewichte”. *Zeitschrift der Deutschen Morgenländischen Gesellschaft* **61**, 379–402, 948–950.
Zimmern, Heinrich, 1916. “Zu den altakkadischen geometrischen Berechnungsaufgaben”. *Orientalistische Literaturzeitung* **19**, 321–325.

MAX-PLANCK-INSTITUT FÜR WISSENSCHAFTSGESCHICHTE

Max Planck Institute for the History of Science

Preprints since 2013 (a full list can be found at our website)

442 William G. Boltz and Matthias Schemmel **The Language of 'Knowledge' and 'Space' in the Later Mohist Canon** (TOPOI fi Towards a Historical Epistemology of Space)

443 Stefano Bordoni **Looking for a Rational Thermodynamics in the late XIX century**

444 Sonja Brentjes and Jürgen Renn **The Arabic Transmission of Knowledge on the Balance**

445 Horst Nowacki **Archimedes and Ship Design**

446 Matthias Schemmel **Elements of a Historical Epistemology of Space** (TOPOI fi Towards a Historical Epistemology of Space)

447 Martin Thiering and Wulf Schiefenhövel **Spatial Concepts in Non-Literate Societies: Language and Practice in Eipo and Dene Chipewyan** (TOPOI fi Towards a Historical Epistemology of Space)

448 Jürgen Renn **Einstein as a Missionary of Science**

449 Hubert Laitko **Der Ambivalenzbegriff in Carl Friedrich von Weizsäckers Starnberger Institutskonzept**

450 Stefano Bordoni **When Historiography met Epistemology.** Duhem„s early philosophy of science in context

451 Renate Wahsner **Tausch – Allgemeines – Ontologie** oder **Das Auseinanderlegen des Konkreten und seine Aufhebung**

452 Jens HÃyrup **Algebra in Cuneiform.** Introduction to an Old Babylonian Geometrical Technique

453 Horst Nowacki **Zur Vorgeschichte des Schiffbauversuchswesens**

454 Klaus Geus and Mark Geller (eds.) **Esoteric Knowledge in Antiquity** (TOPOI fi Dahlem Seminar for the History of Ancient Sciences Vol. II)

455 Carola Sachse **Grundlagenforschung. Zur Historisierung eines wissenschaftspolitischen Ordnungsprinzips am Beispiel der Max-Planck-Gesellschaft (1945–1970)**

456 David E. Rowe and Robert Schulmann **General Relativity in the Context of Weimar Culture**

457 F. Jamil Ragep **From Tūn to Turun: The Twists and Turns of the Ṭūsī-Couple**

458 Pietro Daniel Omodeo **Efemeridi e critica all'astrologia tra filosofia naturale ed etica: La contesa tra Benedetti e Altavilla nel tardo Rinascimento torinese**

459 Simone Mammola **Il problema della grandezza della terra e dell'acqua negli scritti di Alessandro Piccolomini, Antonio Berga e G. B. Benedetti e la progressiva dissoluzione della cosmologia delle sfere elementari nel secondo '500**

460 Stefano Bordoni **Unexpected Convergence between Science and Philosophy: A debate on determinism in France around 1880**

461 Angelo Baracca ***Subalternity vs. Hegemony* – Cuba's Unique Way of Overcoming Subalternity through the Development of Science**

462 Eric Hounshell & Daniel Midena **"Historicizing Big Data" Conference, MPIWG, October 31 – November 2, 2013** Report

463 Dieter Suisky **Emilie Du Châtelet und Leonhard Euler über die Rolle von Hypothesen.** Zur nach-Newtonschen Entwicklung der Methodologie

464 Irina Tupikova **Ptolemy's Circumference of the Earth** (TOPOI fi Towards a Historical Epistemology of Space)

465 Irina Tupikova, Matthias Schemmel, Klaus Geus **Travelling along the Silk Road: A new interpretation of Ptolemy's coordinates**

466 Fernando Vidal and Nélia Dias **The Endangerment Sensibility**

467 Carl H. Meyer & Günter Schwarz **The Theory of Nuclear Explosives That Heisenberg Did not Present to the German Military**

468 William G. Boltz and Matthias Schemmel **Theoretical Reflections on Elementary Actions and Instrumental Practices: The Example of the *Mohist Canon*** (TOPOI fi Towards a Historical Epistemology of Space)

469 Dominic Olariu **The Misfortune of Philippus de Lignamine's Herbal or New Research Perspectives in Herbal Illustrations From an Iconological Point of View**

470 Fidel Castro Díaz-Balart **On the Development of Nuclear Physics in Cuba**

471 Manfred D. Laubichler and Jürgen Renn **Extended Evolution**

472 John R. R. Christie **Chemistry through the 'Two Revolutions': Chemical Glasgow and its Chemical Entrepreneurs, 1760-1860**

473 Christoph Lehner, Helge Wendt **Mechanik in der Querelle des Anciens et des Modernes**

474 N. Bulatovic, B. Saquet, M. Schlender, D. Wintergrün, F. Sander **Digital Scrapbook – can we enable interlinked and recursive knowledge equilibrium?**

475 Dirk Wintergrün, Jürgen Renn, Roberto Lalli, Manfred Laubichler, Matteo Valleriani **Netzwerke als Wissensspeicher**

476 Wolfgang Lefèvre **„Das Ende der Naturgeschichte" neu verhandelt**

477 Martin Fechner **Kommunikation von Wissenschaft in der Neuzeit: Vom Labor in die Öffentlichkeit**

478 Alexander Blum, Jürgen Renn, Matthias Schemmel **Experience and Representation in Modern Physics: The Reshaping of Space** (TOPOI fi Towards a Historical Epistemology of Space)

479 Carola Sachse **Die Max-Planck-Gesellschaft und die Pugwash Conferences on Science and World Affairs (1955–1984)**

480 Yvonne Fourès-Bruhat **Existence theorem for certain systems of nonlinear partial differential equations**

481 Thomas Morel, Giuditta Parolini, Cesare Pastorino (eds.) **The Making of Useful Knowledge**

482 Wolfgang Gebhardt **Erich Kretschmann. The Life of a Theoretical Physicist in Difficult Times**

483 Elena Serrano **Spreading the Revolution: Guyton's Fumigating Machine in Spain. Politics, Technology, and Material Culture (1796–1808)**

484 Jenny Bangham, Judith Kaplan (eds.) **Invisibility and Labour in the Human Sciences**

485 Dieter Hoffman, Ingo Peschel (eds.) **Man möchte ja zu seinem Fach etwas beitragen**

486 Elisabeth Hsu, Chee Han Lim **Enskilment into the Environment: the *Yijin jing* Worlds of *Jin* and *Qi***

487 Jens Høyrup **Archimedes: Knowledge and Lore from Latin Antiquity to the Outgoing European Renaissance**

488 Jens Høyrup **Otto Neugebauer and the Exploration of Ancient Near Eastern Mathematics**