

Supporting information for:
Combination of MD Simulations with
Two-State Kinetic Rate Modeling Elucidates
the Chain Melting Transition of Phospholipid
Bilayers for Different Hydration Levels

Bartosz Kowalik,[†] Thomas Schubert,[‡] Hirofumi Wada,[¶] Motomu Tanaka,^{‡,§}

Roland R. Netz,[†] and Emanuel Schneck^{*,||}

[†]*Fachbereich Physik, Freie Universität Berlin, D-14195, Berlin, Germany*

[‡]*Institute of Physical Chemistry, Heidelberg University, D-69120 Heidelberg, Germany*

[¶]*Department of Physics, Ritsumeikan University, Kusatsu, 525-8577 Shiga, Japan*

[§]*Institute for Integrated Cell-Material Sciences (WPI iCeMS), Kyoto University, 606-8501
Kyoto, Japan*

^{||}*Biomaterials Department, Max Planck Institute of Colloids and Interfaces, D-14476
Potsdam, Germany*

E-mail: schneck@mpikg.mpg.de

Supporting Material

Derivation of Eq. 12 of the Main Text

According to Eq. 11

$$f'_\beta := \frac{df_\beta}{dt} = -f_\beta k_{\beta\alpha}. \quad (\text{S1})$$

Thus,

$$f''_\beta := \frac{d^2 f_\beta}{dt^2} = -f'_\beta k_{\beta\alpha} - f_\beta k'_{\beta\alpha} = f_\beta (k_{\beta\alpha}^2 - k'_{\beta\alpha}), \quad (\text{S2})$$

where $k'_{\beta\alpha} := dk_{\beta\alpha}/dt$. According to Eq. 8

$$f''_\beta(t_m) = 0, \quad (\text{S3})$$

so that

$$k_{\beta\alpha}^2(t_m) - k'_{\beta\alpha}(t_m) = 0. \quad (\text{S4})$$

Using Eqs. 3, 5, 7, and 9, $k'_{\beta\alpha}(t_m)$ is obtained as

$$k'_{\beta\alpha}(t_m) = \frac{r\Delta H_\beta^*}{k_B T_m^{app2}} k_{\beta\alpha}(t_m), \quad (\text{S5})$$

so that Eq. S4 can be rewritten as

$$\frac{r\Delta H_\beta^*}{k_B T_m^{app2}} k_{\beta\alpha}(t_m) - k_{\beta\alpha}^2(t_m) = 0, \quad (\text{S6})$$

which further simplifies to

$$\frac{r\Delta H_\beta^*}{k_B T_m^{app2}} - k_{\beta\alpha}(t_m) = 0. \quad (\text{S7})$$

Using Eq. 5 again, we obtain

$$\frac{r\Delta H_\beta^*}{k_B T_m^{app2}} = k_0 e^{\Delta S_\beta^*/k_B} e^{-\Delta H_\beta^*/k_B T_m^{app}} \quad (\text{S8})$$

and multiplication with $\Delta H_\beta^*/4k_B$ yields

$$\frac{r\Delta H_\beta^{*2}}{4k_B^2 T_m^{app2}} = \frac{k_0 H_\beta^*}{4k_B} e^{\Delta S_\beta^*/k_B} e^{-\Delta H_\beta^*/k_B T_m^{app}}, \quad (\text{S9})$$

which can be rewritten as

$$\left(\frac{\Delta H_\beta^*}{2k_B T_m^{app}} e^{\Delta H_\beta^*/2k_B T_m^{app}} \right)^2 = \frac{k_0 \Delta H_\beta^*}{4k_B r} e^{\Delta S_\beta^*/k_B}. \quad (\text{S10})$$

By introducing

$$\theta = \Delta H_\beta^*/k_B \quad (\text{S11})$$

and

$$r^* = \frac{k_0 \Delta H_\beta^*}{4k_B} e^{\Delta S_\beta^*/k_B} \quad (\text{S12})$$

we obtain

$$\left(\frac{\theta}{2T_m^{app}} e^{\theta/2T_m^{app}} \right)^2 = r^*/r, \quad (\text{S13})$$

which can be solved for T_m^{app} using the principal branch of the LambertW function, $W(x)$, by exploiting its property $W(xe^x) = x$:

$$T_m^{app} = \theta \cdot \left[2W \left(\sqrt{r^*/r} \right) \right]^{-1}. \quad (\text{S14})$$

Derivation of Eqs. 16 and 19 of the Main Text

In the following we expand the right hand side of Eq. 12 in powers of the natural logarithm around an arbitrary heating rate r_0 . We start with the first derivative of T_m^{app} with respect to $\ln r$:

$$\frac{dT_m^{app}}{d(\ln r)} = r \frac{dT_m^{app}}{dr} = \theta \cdot \left[4W \left(\sqrt{r^*/r} \right) \left(1 + W \left(\sqrt{r^*/r} \right) \right) \right]^{-1}, \quad (\text{S15})$$

which follows from the property $W'(x) = W(x)/[x(1 + W(x))]$ of the LambertW function. Around r_0 , T_m^{app} can then be approximated as

$$T_m^{app}(r) = T_0 + C \ln(r/r_0) + O([\ln(r/r_0)]^2), \quad (\text{S16})$$

with

$$T_0 = T_m^{app}(r_0) = \theta \cdot \left[2W\left(\sqrt{r^*/r_0}\right) \right]^{-1} \quad (\text{S17})$$

and

$$C = \left(\frac{dT_m^{app}}{d(\ln r)} \right)_{r_0} = \theta \cdot \left[4W\left(\sqrt{r^*/r_0}\right) \left(1 + W\left(\sqrt{r^*/r_0}\right) \right) \right]^{-1}. \quad (\text{S18})$$

For the particular choice $r_0 = r_{eq}$ we obtain $T_0 = T_m^{eq}$ and C assumes the form

$$C = \frac{T_m^{eq2}}{\theta + 2T_m^e} \approx \frac{T_m^{eq2}}{\theta}, \quad (\text{S19})$$

where we used $W\left(\sqrt{r^*/r_{eq}}\right) = \theta/(2T_m^{eq})$ and the last approximation is valid for $\theta \gg T_m^{eq}$.

Heating-Rate-Dependence of the Transition Path Time

For a harmonic, large-enough barrier height ΔG^* , the transition path time can be expressed as

$$\tau_{tp}(T) \propto \frac{\ln[2e^\gamma \Delta G^*(T)/(k_B T)]}{(\omega^*)^2 D(T)/(k_B T)} \propto \ln[2e^\gamma \Delta G^*(T)/(k_B T)], \quad (\text{S20})$$

where $\gamma \approx 0.577$ denotes the Euler constant, $(\omega^*)^2$ is the curvature of the barrier, and $D(T) \propto T$ is a diffusion coefficient^{S1}. According to Eq. S20 and with $\Delta G_\beta^* = \Delta H_\beta^* - T\Delta S_\beta^*$, τ_{tp} is almost constant in the narrow temperature range 310 K $< T <$ 330 K, corresponding to the heating rates realized in this study. This is illustrated in Fig. S1, where τ_{tp} for $\Delta H_\beta^* = 318$ kJ/mol and $\Delta S_\beta^* = 0.77$ kJ/(mol K) (compare with Table II, parameters for $n_w = 31$ und $N_l = 288$) is plotted as a function of the temperature.

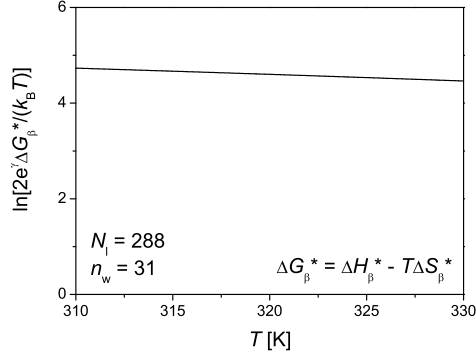


Figure S1: Transition path time τ_{tp} for $\Delta H_{\beta}^* = 318$ kJ/mol and $\Delta S_{\beta}^* = 0.77$ kJ/(mol K) according to Eq. S20 as a function of the temperature.

Determination of θ and r^* via the Kissinger method

Eq. S8 can be rewritten in the Kissinger form^{S2},

$$\ln\left(\frac{r}{T_m^{app2}}\right) = \frac{A}{T_m^{app}} + B, \quad (\text{S21})$$

with

$$A = -\theta \quad (\text{S22})$$

and

$$B = \ln\left(\frac{k_0}{\theta}\right) + \frac{\Delta S_{\beta}^*}{k_B} \equiv \ln r^* + 2 \ln\left(\frac{2}{\theta}\right). \quad (\text{S23})$$

A linear fit in the Kissinger plot ($\ln(r/T_m^{app2})$ versus $1/T_m^{app}$) then yields A and B . In Fig. S2 this is presented exemplarily for the large system ($N_l = 288$) at excess hydration ($n_w = 31$), where we obtain $\theta = -A = (38.5 \pm 3.0) \times 10^3$ K and $\ln r^* \equiv B - 2 \ln 2/\theta = 129 \pm 11$, both consistent with Table II in the main text.

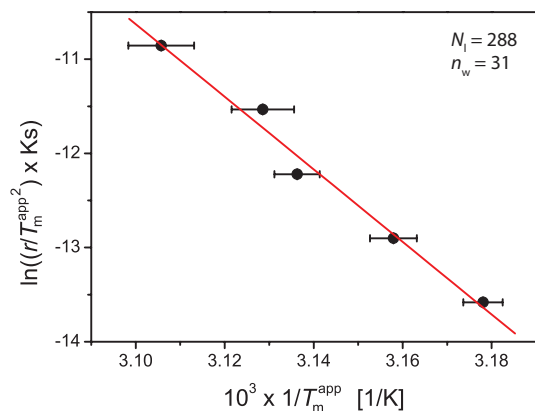


Figure S2: Kissinger plot representation of r -dependent apparent melting temperatures T_m^{app} for the large system at excess hydration. The solid straight line is a linear fit to the data points.

Evolution of membrane area and number of gauche bonds for several heating runs

Fig. S3 shows the numbers of gauche bonds in both monolayers and the membrane area for three independent representative heating runs across the chain melting transition. Data are from large systems ($N_l = 288$) at excess hydration ($N_l = 288$) and heating rate $r = 0.125$ K/ns. It is seen that the number of gauche bonds increases roughly simultaneously, while the membrane area always responds with a delay of several nanoseconds.

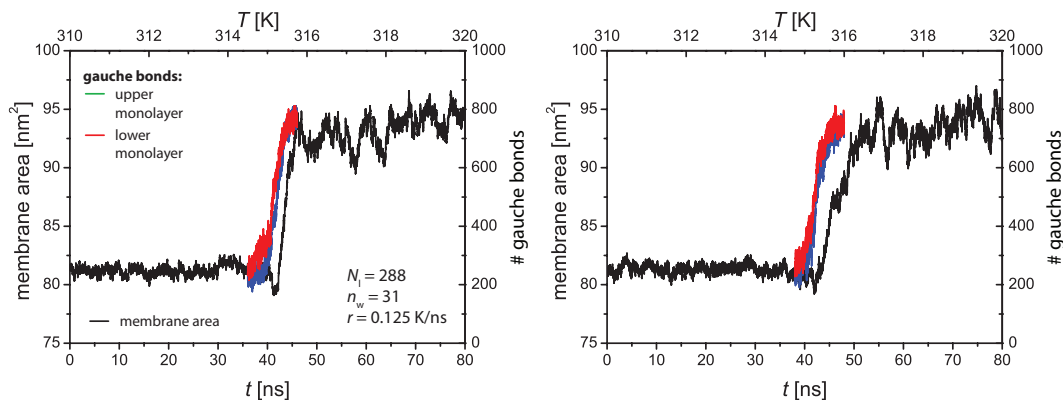


Figure S3: Numbers of gauche bonds in both monolayers and membrane area for two independent representative heating runs across the chain melting transition.

References

- (S1) Chung, H. S.; Louis, J. M.; Eaton, W. A. Experimental determination of upper bound for transition path times in protein folding from single-molecule photon-by-photon trajectories. *Proceedings of the National Academy of Sciences* **2009**, *106*, 11837–11844.
- (S2) Farjas, J.; Roura, P. Modification of the Kolmogorov–Johnson–Mehl–Avrami rate equation for non-isothermal experiments and its analytical solution. *Acta Materialia* **2006**, *54*, 5573–5579.