36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.33E, O-2.005 (2009)

Global nonlinear particle-in-cell gyrokinetic simulations in tokamak geometry

A. Bottino[1], B.D. Scott[1], R. Hatzky[2], S. Jolliet[3], B.F McMillan[3],
A.G. Peeters[4], T.M. Tran[3], T. Vernay[3] and L. Villard[3]

[1] *Max Planck Institut für Plasmaphysik, IPP-EURATOM Association, Garching, Germany*

[2] *Computer Center of the Max-Planck-Gesellschaft, D-85748 Garching, Germany*

[3] *CRPP, Association Euratom - Confédération Suisse, EPFL, Lausanne, Switzerland*

[4] *CFSA, University of Warwick, CV4 7AL, Coventry UK*

Particle-in-cell (PIC) methods have been widely used for solving the gyrokinetic equations and simulating turbulence in tokamak and stellarator. Most of the existing PIC codes are based on the δf method [1]. The distribution function f of each plasma species can be split into a time independent background distribution function f_0 and a time dependent perturbation δf, $f = f_0 + \delta f$. In the δf method, the perturbed part only (δf) is discretised using numerical particles, also called markers. As long as the perturbation δf keeps small as compared to f_0, the δf method reduces the statistical noise. The δf method can be interpreted as a "control variates" algorithm [2, 3], a variance reduction technique widely used in Monte Carlo methods.

Many linear and nonlinear global gyrokinetic δf PIC codes exist and are routinely used for simulating electrostatic perturbations. However, the electrostatic approximation is expected to break down in the core of high β_e ($\beta_e \equiv n_e T_e / B^2$) plasmas or in any region where pressure gradients are large. For a finite value of β_e, magnetic fluctuations modify the evolution of the electrostatic instabilities and eventually introduce new electromagnetic modes [4]. Electromagnetic simulations using a conventional δf method are much more demanding in respect of numerical resources than electrostatic simulations. In particular, the parallel electron dynamics imposes a strong constraint on the size of the time step. In addition to this, electromagnetic simulations require a much larger number of numerical particles in order to correctly describe the evolution of the nonadiabatic part of the electron distribution function. Indeed, the physically relevant nonadiabatic part of the electron distribution function is overwhelmed by the adiabatic response to the magnetic potential $A_{||}$ leading to a severe accuracy problem, known in the literature as "cancellation problem" (see [3] and references therein). An accurate enough description of this small signal requires a very low statistical noise or, in other words, a huge number of numerical particles.

The code used in this work is the global δf PIC code ORB5 [5]. ORB5 solves the set of gyrokinetic equations in the whole plasma core down to the magnetic axis. The use of MHD equilibria leads to a totally consistent inclusion of geometrical parameters such as the Shafranov shift and allows for simulating most of the existing tokamak experiments and future reactor

size machines. A field-aligned filtering procedure and sophisticated noise-control and heating operators allow for accurate simulations with smaller numbers of markers than standard δf PIC simulations [6]. The code ORB5 has been proved to scale up to 32k cores on a BlueGene-P architecture. The strong scaling for the standard ITM CYCLONE base-case, described in Ref. [4], is shown in Fig. 1.

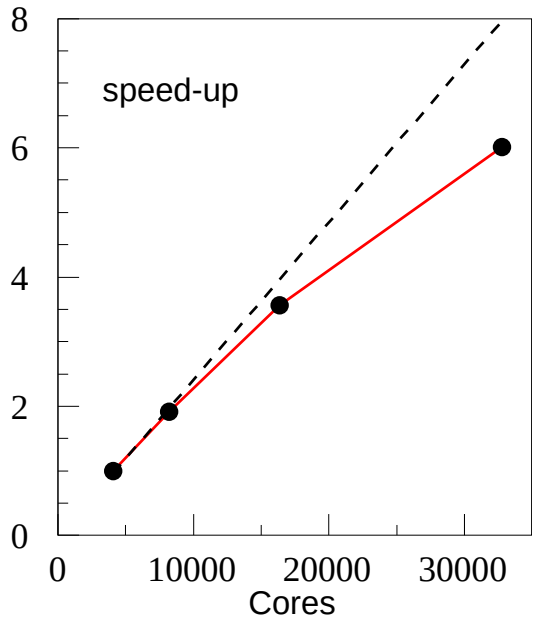

Figure 1: *Electrostatic ORB5: strong scaling. Relative speed-up from 4k to 32k cores for the Cyclone base case [4]; grid size:* $(128,512,256)$, $3x10^9$ *markers. Simulations performed on BlueGene/P, in collaboration with RZG Garching.*

The code ORB5 has been extended to include magnetic perturbations in $A_\parallel$. Ampère's law, as well as the Poisson equation, are discretised with finite elements (B-splines). The cancellation problem of the unphysical adiabatic currents is solved using an adjustable control variates method. The control variate, in this case, correspond to the part of the distribution function of the electrons responding adiabatically to the magnetic potential $A_\parallel$. This scheme is described in detail in Section 8.2 of Ref. [3]. Note that the same scheme has been successfully applied in linear PIC simulations in tokamak geometry [7]. The gyrokinetic model implemented in ORB5 is derived from the Vlasov-Poisson-Ampére model of Ref. [8], in the $p_\parallel$ formulation. The code ORB5 solves the following Ampère's law:

$$C_A\left(\frac{\beta_i}{\rho_i^2}+\frac{\beta_e}{\rho_e^2}\right)A_\parallel-\nabla_\perp^2 A_\parallel=\mu_0\left(j_{\parallel,i}+j_{\parallel,e}\right) \tag{1}$$

where $j_{\parallel,s}$ is the gyrocenter current, ρ_s is the thermal gyroradius and $\beta_s\equiv\mu_0 n_0 T_s/B_0^2$ of the species s. The first two terms $\propto\beta_s/\rho_s^2$ on the left-hand side of Eq. (1) are the ion and electron skin terms which exactly cancel the adiabatic part of currents on the right-hand side. The C_A factor in front of the skin terms is due to the finite extent of the velocity-space domain in the simulations. The value of C_A is close to unity and varies with the radius. The inclusion of this factor is crucial for the correct solution of the cancellation problem as it was shown in Ref. [9]. The electromagnetic version of ORB5 has been tested in simpler geometry and benchmarked against the linear electromagnetic code GYGLES (see Ref. [7] and referenced therein) in tokamak geometry. For this benchmark we have considered a circular equilibrium with major radius $R_0=2.0$ m, minor radius $a=0.5$ m and $\rho^*=1/110$ at mid-radius. The value of the density on the axis has been varied in the simulations in order to perform a scan in

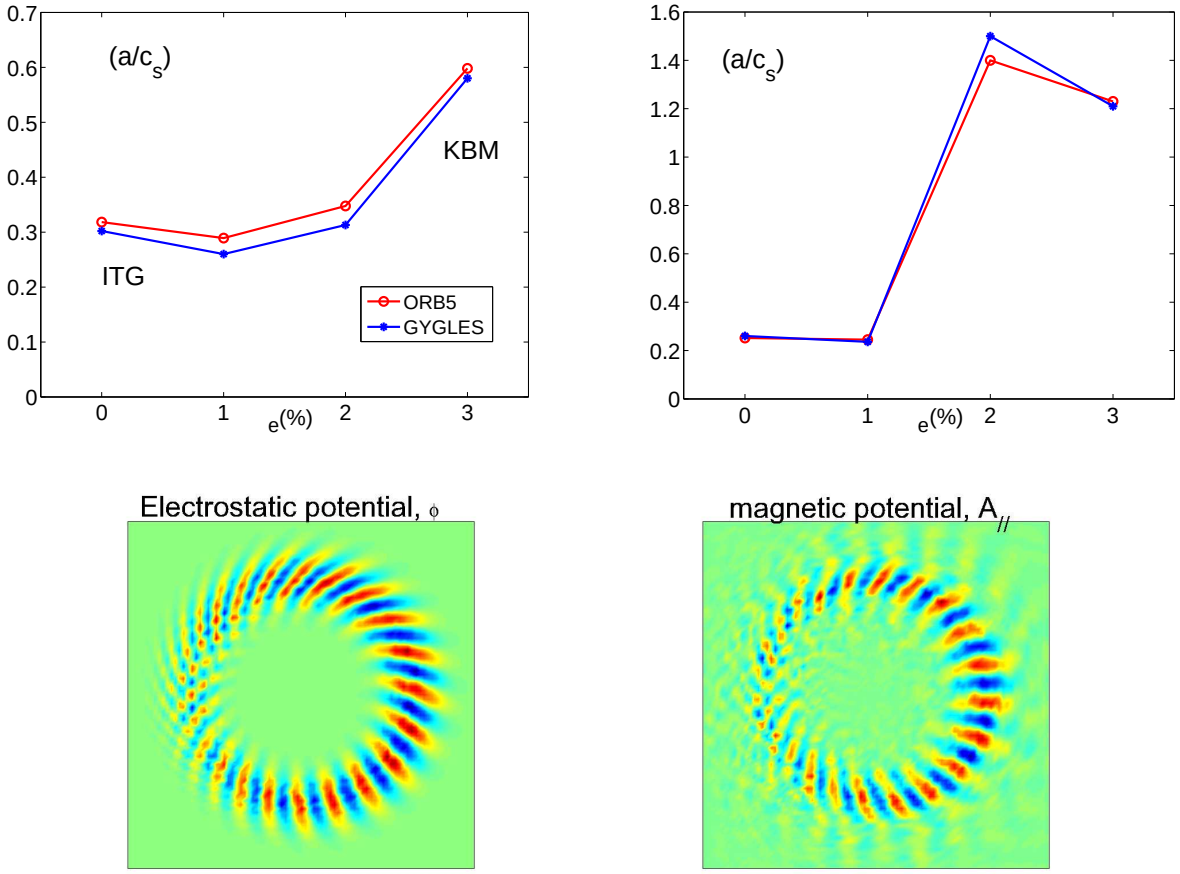

Figure 2: *Top: linear benchmark between ORB5 (red open symbols) and GYGLES (blue). Left: growth rate as a function of* β_e. *Right: real frequency as a function of* β_e. *Bottom: linear ORB5 simulations, poloidal cross sections. Left: electrostatic potential,* ϕ. *Right: parallel magnetic potential,* $A_{\|}$.

β_e. Details about the equilibrium profiles can be found in Ref. [7]. The two codes are in good agreement in both growth rates and real frequencies. For the case $\beta_e = 1\%$ the dominant mode is still an ion temperature gradient driven (ITG) mode partially stabilized by finite β_e effects. For $\beta_e > 2\%$ the most unstable mode is clearly an electromagnetic kinetic ballooning mode (KBM) (see Fig. 2). Figure 2 (bottom) shows the poloidal cross section of the potentials for the $\beta_e = 2\%$ case. All the ORB5 linear simulations were performed with 128 m ion markers and 256 m electron markers.

The nonlinear simulations of Fig. 3 are based on parameters and profiles of the ITM Cyclone base case described in Ref. [4]. The mass ratio is $m_i/m_e = 1000$ and the value of the central density has been adjusted to have $\beta_e = 0.3\%$. Note that in these simulations (as well as in Ref. [4]) no heat sources are applied, the initial temperature gradient ($R/L_T \simeq 9$) relaxes during the time evolution toward the critical gradient value. The EM simulation was performed using 512 million numerical particles per species and with a time step 20 times smaller as compared to the electrostatic case ($\Delta t = 1\ \Omega_{c,i}$ where $\Omega_{c,i}$ is the ion cyclotron frequency). The ion thermal

diffusivity is clearly larger in the $\beta_e = 0.3\%$ case as compared to the electrostatic case due to the trapped electron contribution to the ITG instability. This effect is obviously not present in all the simulations of Ref. [4] were adiabatic electrons were used.

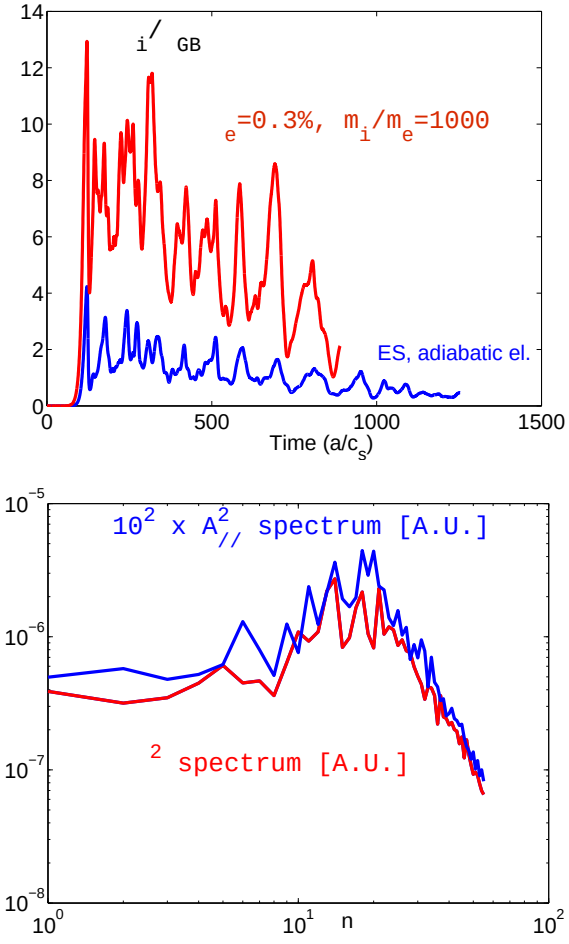

Figure 3: *Top: time evolution of the ion thermal diffusivity for an electromagnetic $\beta_e = 0.3\%$ EM simulation (red). Bottom: spectrum of the potential fields at $t \simeq 800\ [a/c_s]$; n, toroidal mode number.*

A more detailed analysis of the heat fluxes shows that the magnetic flutter terms are negligible and do not affect ions, in agreement with existing flux-tube results [10]. During the simulation the signal/noise diagnostics [11] show a noise/signal ratio ranging between 5% and 10%.

The inclusion of Ampère's law does not degrade the scaling properties of ORB5, since the field solver time is a small fraction of the total computational time. The first nonlinear simulations show that a high radial resolution is required for describing the nonadiabatic electron dynamics in the vicinity of resonant surfaces. When the radial resolution is too poor spurious modes appear in electromagnetic simulations [12]. In general, achievement of converged global nonlinear electromagnetic simulations requires a large amount of numerical resources due to the constraints imposed by Alfvén dynamics and the kinetic electrons. Indeed, electromagnetic simulations of the Cyclone base case for $\beta_e >$ 0.3% seem to require even higher number of markers than the case presented in this work.

References

[1] S.E. Parker, W.W. Lee, J. Comput. Phys., 107 (1993) 309.
[2] A.Y. Aydemir, Phys. Plasmas, 1 (1994) 5480.
[3] R. Hatzky, A. Könies, A. Mishchenko, J. Comput. Phys., 225 (2007) 568.
[4] G.L. Falchetto et al., Plasma Phys. Contr. Fusion, 50 (2008) 124015.
[5] S. Jolliet, A. Bottino, P. Angelino et al., Comput. Phys. Comm, 177 (2007) 409.
[6] B.F. McMillan, S. Jolliet, T.M. Tran, A. Bottino et al., Phys. Plasmas, 16 (2009) 022310.
[7] A. Mishchenko, R. Hatzky, A. Köenis, Phys. Plasmas, 15 (2008) 112106.
[8] T.S. Hahm, W.W. Lee, A. Brizard, Phys. Plasmas, 31 (1988) 1940.
[9] A. Mishchenko, R. Hatzky, A. Köenis, Phys. Plasmas, 11 (2004) 5480.
[10] J. Candy, Phys. Plasmas, 12 (2005) 072307.
[11] A. Bottino at al., Phys. Plasmas, 14 (2007) 010701.
[12] B. Scott, Plasma Phys. Contr. Fusion, 48 (2006) B277.