FIXED- AND FREE-BOUNDARY $n \geq 1$ MODES IN TOROIDAL $l = 2$ STELLARATORS

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1. Introduction.

The magnetohydrodynamic equilibrium and stability properties of toroidal $l = 2$ stellarator configurations of large aspect ratio A ($A = R_T/a = 10$ to 20 as for the WENDELSTEIN VII-A device, R_T is the major torus radius and a the mean minor plasma radius) are accessible to the asymptotic stellarator expansion procedure STEP [1,2,3]. So this is an adequate tool to study some characteristic equilibrium and stability properties of stellarator configurations without longitudinal net-current, which also apply to Advanced Stellarators [4]. Results of a mode analysis of Heliotron-E configurations using the three-dimensional BETA- code are given in Ref.5, and in Ref.6 ideal and resistive $n = 1$ modes are studied for that configurations based on the average MHD equations. In the present paper a classification of unstable free-boundary modes occurring in toroidal $l = 2$ stellarators is given. The normalized eigenvalues of fixed- and free-boundary modes with mode numbers $n = 1, 2, 3, ..$ in toroidal direction and various radial node numbers of the eigenfunctions are given as functions of the position of an electrically conducting wall the mean minor radius of which is denoted by b.

2. Asymptotic Equilibria.

The $l = 2$ equilibrium configurations consist of $M = 5$ field periods of period length L_P/a where the aspect ratio A is related to the period length by $A = ML_P/2\pi a = M/ha$. The value of the twist (angle of rotational transform divided by 2π) at the boundary is kept below 0.5 for the finite-beta magnetic field configurations. For these equilibria, the lowest-order resonance condition $n/m = 1/2 = \iota$ is not satisfied inside the plasma region (m is the poloidal mode number of the dominant Fourier mode).

The vacuum magnetic field is given by $\vec{B} = B_0[\vec{e}_z + (\delta/h)\nabla I_2(hr)\ sin(2\theta - hz)]$ in the pseudo-cylindrical coordinate system (r, θ, z), where the Bessel function $I_2(hr)$ appears in the solution of the Laplace equation for the straight system (Bessel model); δ describes the amplitude of the helical $l = 2$ field. The asymptotic value of the twist on magnetic axis is given by $\iota = M\delta^2/16$ for the vacuum field. The ι-profile of the vacuum field as function of the mean minor radius r of the magnetic surfaces is approximately given by $\iota(r) \approx \iota_0[1 + (hr)^2/2 + 7(hr)^4/96]$. At finite β, the ι-profile is changed as shown in Fig.1. The local shear $r\iota'/\iota$ ($\approx\ (hr)^2$ in leading order) of these configurations is rather small compared to Heliotron-E configurations because of the long period length. The pressure profile $p = p_0(1 - \psi) \approx p_0(1 - (r/a)^2)$ is approximately a parabolic function in r (ψ is the normalized poloidal flux). The aspect ratio is $A \approx\ 7.7$.

Figure 1 shows the twist ι and the normalized specific volume $(V' - V_0')/V_0'$ as functions of r/a and $(r/a)^2$, respectively, for two different β-values and vanishing longitudinal net current. The corresponding vacuum fields have a large average magnetic hill meaning that $(V' - V_0')/V_0'\ > 0$. By superimposing $l = 3$ fields with the same period length

L_P or half of that, vacuum fields with a magnetic well can be obtained having more favorable stability properties [7].

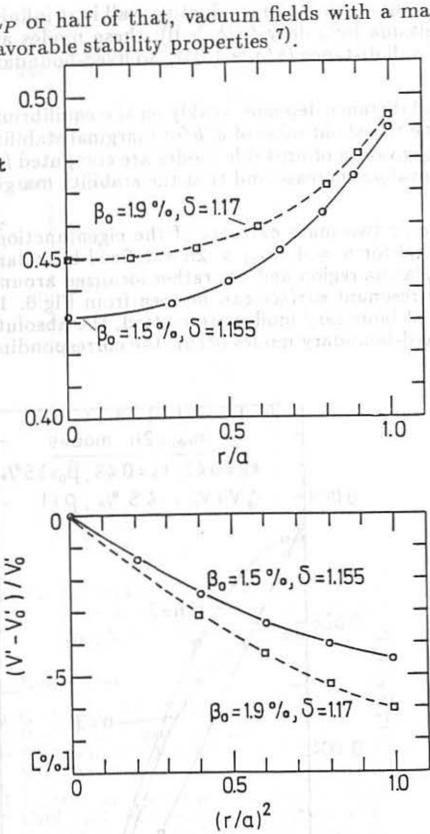

Fig.1. Twist and normalized specific volume $(V' - V_0')/V_0'$ as functions of r/a and $(r/a)^2$, respectively ($A \approx 7.7$).

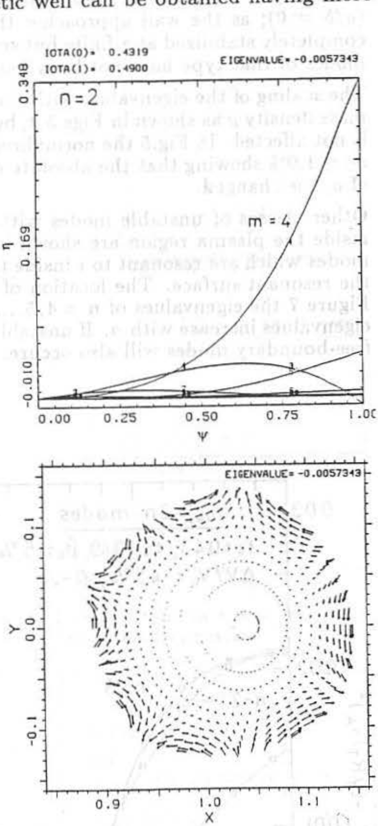

Fig.2. Eigenfunction and displacement vector of a free-boundary $n = 2$ mode with one maximum ($\iota_0 = 0.43, \iota_b = 0.49$).

3. Stability Results.

For all stability computations the equilibrium mass density ρ is assumed to be $\rho \sim \sqrt{p}$ except for the results in Fig.4. The eigenfunction η of a free-boundary $n = 2, m_{res} = 4$ mode is shown in Fig.2 as function of ψ together with the corresponding displacement vector. The eigenfunction assumes its maximum at the boundary. The equilibrium configurations have been chosen so that the $n = 1, 2, 3$ and $m_{res} = 2n$ ($n/m_{res} = 1/2$) modes are not resonant to ι inside the plasma region. As shown in Fig.3, the free-boundary $n = 1, 2, 3$ ($m_{res} = 2n$) modes are unstable if the conducting wall is at infinity ($a/b = 0$); as the wall approaches the plasma boundary ($a/b > 0$), these modes are completely stabilized at a finite but small wall distance ($b/a \approx 1.05$). So fixed-boundary modes of that type have not been found.

The scaling of the eigenvalues with the wall distance depends weakly on the equilibrium mass density ρ as shown in Figs.3,4, but the threshold value of a/b for marginal stability is not affected. In Fig.5 the normalized eigenvalues of unstable modes are computed for $\beta_0 = 1.9\%$ showing that the absolute eigenvalues increase and that the stability margin of a/b is changed.

Other classes of unstable modes with one or two main extrema of the eigenfunctions inside the plasma region are shown in Fig.6 for $n = 4, m_{res} = 2n + 1$ fixed-boundary modes which are resonant to ι inside the plasma region and are rather localized around the resonant surface. The location of the resonant surface can be seen from Fig.6. In Figure 7 the eigenvalues of $n = 4, 5, \ldots$ fixed boundary modes are plotted, the absolute eigenvalues increase with n. If unstable fixed-boundary modes occur, the corresponding free-boundary modes will also occure.

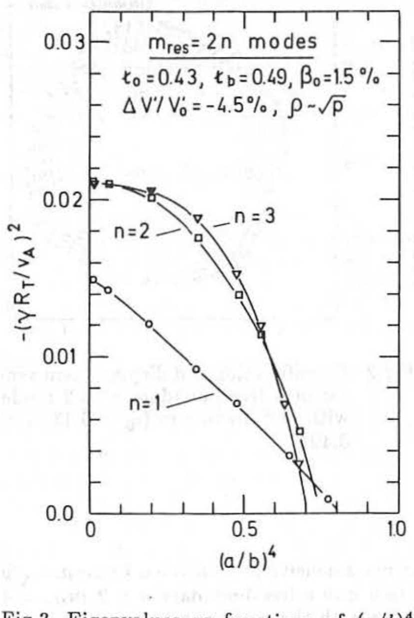

Fig.3. Eigenvalues as functions of $(a/b)^4$ for $n = 1, 2, 3$ free-boundary modes.

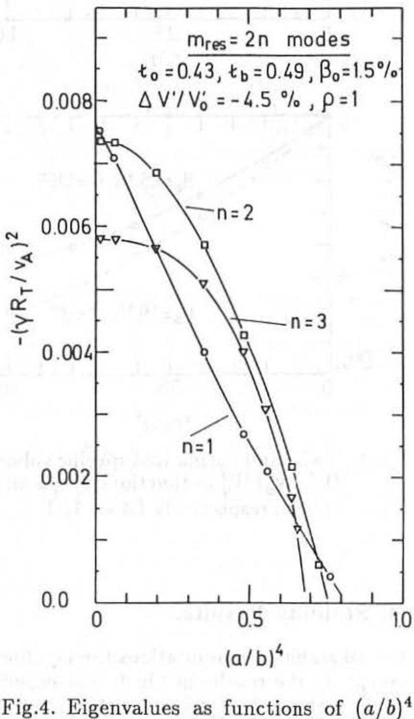

Fig.4. Eigenvalues as functions of $(a/b)^4$ for $n = 1, 2, 3$ free-boundary modes ($\rho = 1$).

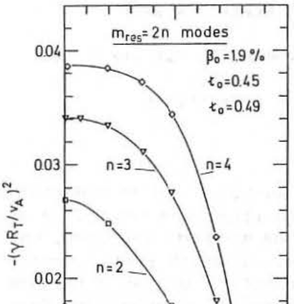

Fig.5. Eigenvalues as functions of $(a/b)^4$ for $n = 1,2,3,4$ modes with $m_{res} = 2n$ ($\beta_0 = 1.9\%$).

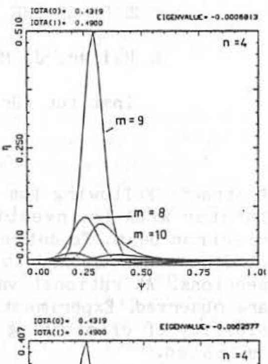

Fig.6. Eigenfunctions of $n = 4$, $m_{res} = 9$ fixed-boundary modes

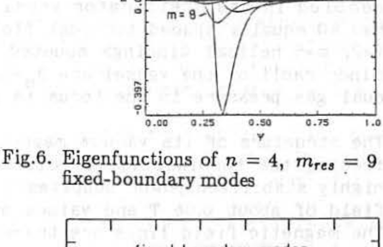

Fig.7. Eigenvalues as functions of n for fixed-boundary modes with $m_{res} = 2n+1$ and $m_{res} = 2n+2$.

4. References

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