Characterization of Edge Plasma Fluctuations in ASDEX

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Abstract:

Nonlinear dynamical characterizations of the edge plasma fluctuations measured by both H_{α}-light diagnostic and Langmuir probes in ASDEX are presented. The edge plasma fluctuations are stochastic rather than chaotic, they have a higher-dimensional structure in phase space. In time, the edge turbulence is found to have memory properties, the time required to lose the memory is different in the different cases.

1. Introduction

Edge plasma fluctuations are experimentally investigated by both H_α-light diagnostic and Langmuir probes in ASDEX. Much work has been done on the parameter scaling and the statistical characterizations of these fluctuations, such as correlation, power spectra, coherence and so on[1]. The edge plasma fluctuations are proven to be turbulent, having broad spectra, and the fluctuation-induced transport estimated agrees with the anomalous transport observed. In this paper, we would like to present some nonlinear dynamical characterizations of edge plasma fluctuations, which, combined with the previous work[1], would be helpful to our understanding of the nature of the edge turbulence.

The trajectory of a dynamical system in phase space can be described by the correlation dimension which gives the minimum number of degrees of freedom, or yields the lower bound to the number of dominant modes in turbulence. In some sense, the correlation dimension can be used as a geometrical measure (in phase space) for the fluctuation systems.

Loss of memory, on the other hand, can serve as a temporal measure for the fluctuation systems. Here, loss of memory means that if the evolution of a system depends on the initial conditions (i.e. has the memory of initial conditions), the system would gradually lose the memory of the initial conditions with time. The time required to lose the memory completely depends on the Liapunov exponents[2] that the system has.

For the edge plasma fluctuations observed in ASDEX, it is found that their correlation dimensions are very high. This means that the edge plasma fluctuations appear to be stochastic (many degrees of freedom) rather than chaotic (few degrees of freedom). In other words, the edge turbulence has a complicated structure, there exist many dominant modes in the edge turbulence and between these modes are strong nonlinear interactions.

It is also found that the edge turbulence has memory properties in time. The time required to lose completely the memory of the initial conditions is different for different fluctuation systems. This may mean that there are some different processes of instabilities occurring in systems.

2. Methods

Deducing the dynamics of a system from only a single observable is based on the fact that a system's dynamics can be reproduced from a single degree of freedom[3,4]. Consider an observable $x(t)$ as a time series $\{x_i\}$, $i=1,2,3,\ldots$, with the time interval between consecutive points x_i and x_{i+1} equals the sampling time. A set of n-dimensional vectors $\mathbf{r}_i$ can be constructed from this time series $\{x_i\}$ by the time-delay method:

$$\mathbf{r}_i = \{x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(n-1)\tau}\}, \quad i=1,2,3,\ldots \qquad x_i\,(1)$$

where τ is called the delay time, n is the embedding dimension. The miraculous thing is that the trajectory of $\mathbf{r}_i$ in its n-dimensional space is simply related to the trajectory of the initial system in its full phase space, if n is large enough and the appropriate value of τ is chosen. Almost any value chosen for τ will be acceptable, in practice one should find a better choice of τ. n may have to be up to $2d+1$ if the original system has d degrees of freedom.

One simple procedure to chose the delay time τ is to find the autocorrelation time t_a at which the autocorrelation function first passes through zero. Another procedure to chose the delay time τ is to evaluate the mutual information by means of information theory[5]. The mutual information measures the amount of information that a measurement of $x(t)$ predicts about a measurement of $x(t+\tau)$. It will typically decay as τ is increased, and will finally reach a base value due to external noise. The value of τ at which the mutual information first reaches its asymptotic value is the best choice for the delay time. Generally speaking, the mutual information measures the general dependence of two variables, while the autocorrelation function measures only the linear dependence. We will use the second procedure to chose the delay time in the following.

To calculate the mutual information, we divide the value range of the variable x into N intervals of equal length l. Let P_i be the probability that the variable $x(t)$ lies in the ith interval, and P_j is the probability that the variable $x(t+\tau)$ lies in the jth interval. The mutual information I_m is given by:

$$I_m(\tau)=\sum_{i,j=1}^{N} P_{i,j} \log\left(\frac{P_{i,j}}{P_i P_j}\right) \tag{2}$$

where $P_{i,j}$ is the joint probability that the variable $x(t)$ lies in the ith interval and the variable $x(t+\tau)$ lies in the jth interval.

There are several different types of dimension, however the correlation dimension is the only one that can be obtained efficiently and easily from the experimental signals. The algorithm of calculating the correlation dimension from the experimental signals is as follows[6]: with the set of n-dimensional vectors $\boldsymbol{r}_i$, one can evaluate the correlation sum $C(r)$ defined by:

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \theta(r - |\boldsymbol{r}_i - \boldsymbol{r}_j|) \tag{3}$$

where $\theta(x)$ is the Heaviside function, and N is the number of vectors $\boldsymbol{r}_i$. For an intermediate range of parameter r, this correlation sum $C(r)$ will scale like:

$$C(r) \propto r^{\upsilon} \tag{4}$$

and the exponent υ is the correlation dimension. The value of correlation dimension calculated in this procedure will increase as the embedding dimension n is increased for small n values, then approach a saturation value as n is increased to be large enough, this saturation value is the real dimension of the system. For a linear physical system, the correlation dimension is always an integer. But for some complicated nonlinear systems (e.g., the chaotic and the turbulent systems), the correlation dimensions are non-integer values, and moreover a chaotic system possesses a lower dimension while the dimension of a turbulent system is very high.

Restricted by the computation time, the total number of data N used in Eq.(3) could not be too large (N=16k-32k, 1k=1024, usually used in this paper). For the longer time series of signal, one can also use the algorithm of "average" pointwise dimension[7], which usually yields almost the same value as the correlation dimension and provides a great saving over the computation time. In this approximate algorithm, the correlation sum $C(r)$ is calculated approximately by:

$$C(r) \approx \frac{1}{M}\sum_{i}^{M} \frac{1}{N}\sum_{j=1}^{N} \theta\,(r - |r_i - r_j|) \qquad (5)$$

where M is the number of reference points chosen randomly. Usually M=100 is enough for obtaining a good estimate of the correlation dimension, and N can be increased up to 10^6 points.

The method to characterize the loss of memory for dynamical systems from a time series of signal was proposed recently by Gade et al.[8]. They introduced the time-dependent generalized dimension $D_{q,l}(t)$ defined by:

$$D_{q,l}(t) = \frac{\ln\left(\sum' p_{i,j}^{q}(t)\right)}{(q-1)\ln l}, \qquad -\infty < q < \infty \qquad (6)$$

The prime in the summation means that the sum is only over those i and j subscripts for which the value of $P_{i,j}(t)$ is not zero. q is a parameter varying from $-\infty$ to ∞. When q is taken equal to 0, 1 and 2, the dimension given by Eq.(6), under the limit $l \to 0$, is the fractal dimension, information dimension and correlation dimension, respectively[7].

There are two limiting cases for the time-dependent generalized dimension $D_{q,l}(t)$ versus t. In the first case $t=0$, we have $P_{i,j}(0)=P_i\delta_{ij}$ and $D_{q,l}(0) \approx D_q$, where D_q is the time-independent generalized dimension[7]. In the other case for long times, the system has completely lost the memory of the initial conditions, for two isolated events we have $P_{i,j}(t) = P_i P_j$, and we get $D_{q,l}(t) = 2D_q$. Here we see that the function of $D_{q,l}(t)$ is equal to D_q at $t=0$, then increases gradually as t is increased, and finally reaches the asymptotic value equal to $2D_q$. The value of t at which $D_{q,l}(t)$ reaches its asymptotic value is the value of time required to lose the memory completely. Note that this method is specially effective for the lower-dimensional systems (one- or two-dimensional systems), since the joint probability $P_{i,j}(t)$ is easy to calculate. For the higher-dimensional systems, the difficulty is to calculate the joint probability $P_{i,j}(\mathbf{r},t)$, for the vectors $\mathbf{r}_i$ replace the scalars x_i.

Another useful relationship between the time-dependent fractal dimension $D_{0,l}(t)$ and the Liapunov exponent λ for the given interval length l is as follows[8]:

$$D_{0,l}(t) = D_0 - \lambda t / \ln l \tag{7}$$

This relation will hold over the straight-line region at the beginning of the curve $D_{0,l}(t)$ versus t, and can be used directly to estimate the Liapunov exponent for the experimental system, if the system is a low-dimensional system.

3. Data and Results

In this paper both fluctuation signals measured by H_α-light diagnostic and Langmuir probes are analyzed. H_α emission observations measure electron density fluctuations, while Langmuir probes can measure density fluctuations and potential fluctuations as well. Each kind of diagnostic has totally 16 channels covering a distance of 10-20cm in poloidal direction near the midplane of ASDEX, and the signals are all digitized at the sampling rate of at least 1MHz. A total number of 10^6 points for H_α signals and of 2×10^4 points for probe signals are recorded per channel and shot.

Typical fluctuation signals analyzed are shown in Fig.1a-Fig.1f, which are three sets: (1) H_α signals measured in ordinary discharges (indicated by H_α in Fig.1a, the number in brackets is the shot-number of ASDEX discharge); (2) Langmuir probe signals for both density fluctuations $\tilde{n}_e$ and potential fluctuations $\tilde{\phi}$ in ordinary discharges (shown in Fig.1b and Fig.1c); (3) H_α signals in the transient phases of Ohmic, L- and H-mode (indicated by OH, L and H in Fig.1d-1f). Their corresponding power spectra are shown in Fig.2a-Fig.2f, respectively, and more other detailed work on statistical analysis of these fluctuations can be found in Ref.1.

In order to calculate the dimension for plasma fluctuations, we should first choose the delay time τ to construct the vectors $\boldsymbol{r}_i$ by using Eq.(1). Fig.3 shows an example of the mutual information I_m versus τ for H_α fluctuation signals. The number of data used in calculating I_m is usually 10^4 points, and each interval has 100 points on the average. It is seen that the base value of this mutual information is about 0.4 (a.u.), and at $\tau \approx 20\mu s$ the mutual information I_m has reached the asymptotic value. Thus $\tau = 20\mu s$ should be chosen as the delay time used in constructing the vectors $\boldsymbol{r}_i$ for this H_α fluctuation signals. For other sets of fluctuation signals we use the same procedure to choose the delay time.

Both algorithms defined by Eq.(3) and Eq.(5) are used in calculating the dimension for the density fluctuations presented by H_α signals. In the case of calculating the correlation dimension, the total number of 16k-32k data points are used. The value of correlation dimension obtained with 32k points is larger only by 2% than that obtained with 16k points; while in the case of calculating the "average" pointwise dimension, the number of data is increased up to as many as 5×10^5 points, and the dimension value obtained with $N = 5 \times 10^5$ and $M = 100$ is larger by 10% than that obtained with N=32k and M=100. One example of the correlation sum $C(r)$ obtained with $N = 5 \times 10^5$ and $M = 100$ in Eq.(5) is shown in Fig.4 as a log-log plot versus r for the embedding dimension n =3, 4, ..., 10. It is seen that each curve has a well-defined linear region over which the scaling law $C(r) \propto r^{\nu}$ holds. For simplicity, we will only show the values of correlation dimension obtained with N=16k in the following.

All the dimension results calculated from the fluctuation signals are summarized in Fig.5 as functions of the embedding dimension *n,* where the diagonal line corresponds to the dimension of a noise system. It is seen that the dimension obtained from the signal H_{α}(28089) keeps a lower value under the given embedding dimension, but is still very high (at least 6), the result obtained from the probe signal $\tilde{n}_e$(31146) agrees very well with this dimension. All the channels of signals (16 channels of H_{α} and 8 channels of Langmuir probes measuring the ion saturation current) give the same value of dimension. It is also seen that a little higher dimension is obtained from the potential fluctuation $\tilde{\phi}$(31146), and all the 8 channels of potential signals give the same dimension as well. These results suggest that the edge plasma fluctuations in ASDEX display a high-dimensional structure, and the structure is homogeneous over the investigated region.

For the density fluctuations in the transient phases of Ohmic, L- and H-mode, the dimension values obtained are much higher (at least 8). Owing to such high dimension, it is difficult to distinguish differences between them. The available results show that the structure of edge turbulence could become more complicated (i.e., increase in dimension) under the certain conditions, and the number of plasma quantities involved in edge turbulence is always large, regardless of during Ohmic, L and H phase.

Note that all the correlation dimensions shown in Fig.5 have not yet displayed the saturated behavior with increasing *n* up to n=10. This means that the real dimensions for the edge plasma fluctuations are still higher, higher than that we have obtained here. However, we are not interested to find finally the real dimensions by increasing the number of data and the embedding dimension, because the dimension of 10, 20 or 30 here does not mean any new things except the same fact that the edge plasma fluctuations appear to be stochastic (many degrees of freedom) rather than chaotic (few degrees of freedom).

In order to find the memory properties of the edge turbulence, we have calculated the time-dependent fractal dimension $D_{0,l}(x,t)$ [9] from the fluctuation signals. The number of data used in this calculation is 10^4 points at least, and the interval length l is taken to be 1% of the value-range of signals (i.e., $l=0.01$ if the data are mapped into the interval of [0,1]). The results obtained from different fluctuation signals and from a normally distributed noise are shown in Fig.6 (the function $D_{0,l}(t)$ for the signal $\tilde{n}_e$(31146) or for both signals L(32838) and H(32838) is quite similar to that for H_α(28089) or for OH(32838), respectively). Since the noise is independent of the initial conditions, its function $D_{0,l}(t)$ should immediately go to the asymptotic value after $t>0$. This behavior is shown clearly in Fig.6 (the jump from $t=0$ to $t=1$ is an artifact, due to using equal length scales rather than the natural length scales of the system), but the asymptotic value of $D_{0,l}(t)$ is observed to be ~$1.8D_0$ rather than $2D_0$, this is because with a finite number of data, we cannot get the relation $P_{i,j}(t)=P_i P_j$ when the system has completely lost the memory of the initial conditions.

In Fig.6, it is seen that all the functions of $D_{0,l}(t)$ for the edge plasma fluctuations increase gradually to their asymptotic values with time t is increasing. This kind of behavior of $D_{0,l}(t)$ means that the edge plasma fluctuations lose their memory of initial conditions in time. The time required to lose the memory completely is observed to be different for the different fluctuation signals: ~20μs for both density fluctuation signals H_α and $\tilde{n}_e$; ~15μs for the potential fluctuation signal $\tilde{\phi}$; ~10μs for the density fluctuation signals OH(32838), L(32838) and H(32838), respectively. It seems that the higher the dimension of fluctuations is, the shorter this time scale tends to be.

Although the values of memory time shown here are obtained from the functions of $D_{0,l}(x,t)$, not from the functions of $D_{0,l}(\mathbf{r},t)$, we believe that the different values of this time scale represent the different processes of instabilities occurring in the system.

It is difficult to get the ordinary Liapunov exponents[2,7] for these fluctuations, because their dimension is too high to evaluate $D_{0,l}(\mathbf{r},t)$. As a rough estimation, here we evaluate the exponents from the functions of $D_{0,l}(x,t)$. Because there are no straight-line regions on the curves (the jump from t=0 to t=1 is the useless data), we can only take a few points on the curve to get an averaged exponent. The first three points of the curve are taken to estimate the averaged exponent by using Eq.(7), and the exponent obtained will be indicated by λ^* which is approximately equal to the mean value of Liapunov exponents that the system has.

The results of exponent λ^* obtained are shown in the following table. It is seen that the density fluctuations of H_α and $\tilde{n}_e$ (first two columns in the table) have the same value of $\lambda^* \approx 0.3$ (μs^{-1}). This is consistent with their dimension results. In addition, it is very interesting to find that the density fluctuations during H-mode have the same value of λ^* as during Ohmic phase, while the value of λ^* during L-mode increases. Since Liapunov exponents are equivalent to the exponential growth rates of instabilities, the larger the exponent λ^*, the stronger on an average the activity of instabilities in plasma.

Table (Unit: μs^{-1})

	H_α(28089)	$\tilde{n}_e$(31146)	$\tilde{\phi}$(31146)	OH(32838)	L(32838)	H(32838)
λ^*	0.31	0.29	0.15	0.09	0.14	0.11

4. Conclusions

In summary, some nonlinear dynamical characterizations of the edge plasma fluctuations observed in ASDEX were performed. The edge turbulence displays the higher-dimensional structures in phase space. It is impossible to describe this edge turbulence by the theoretical models with a few degrees of freedom. In particular, two kinds of diagnostic (H_α and Langmuir probe) give the consistent result for the density fluctuations; potential fluctuations have a different dimension from density fluctuations; and it is difficult to distinguish between the density fluctuations during Ohmic phase, L-mode and H-mode in terms of dimension.

On the other hand, the edge turbulence possesses the memory property in time. The time for loss of memory is 10-20μs in the different cases. The time scale of memory is determined by all the Liapunov exponents that the system has. For the edge plasma fluctuations, we obtained the averaged exponent λ^* which is approximately equal to the mean value of Liapunov exponents of system. It is found that the value of λ^* increases during L-mode, as compared with that during Ohmic phase and H-mode, this indicate that the activity of instabilities during L-mode becomes strong on an average.

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[9] for the edge fluctuations, it is impossible to calculate $D_{0,l}(\mathbf{r},t)$ if they have a very high dimension.

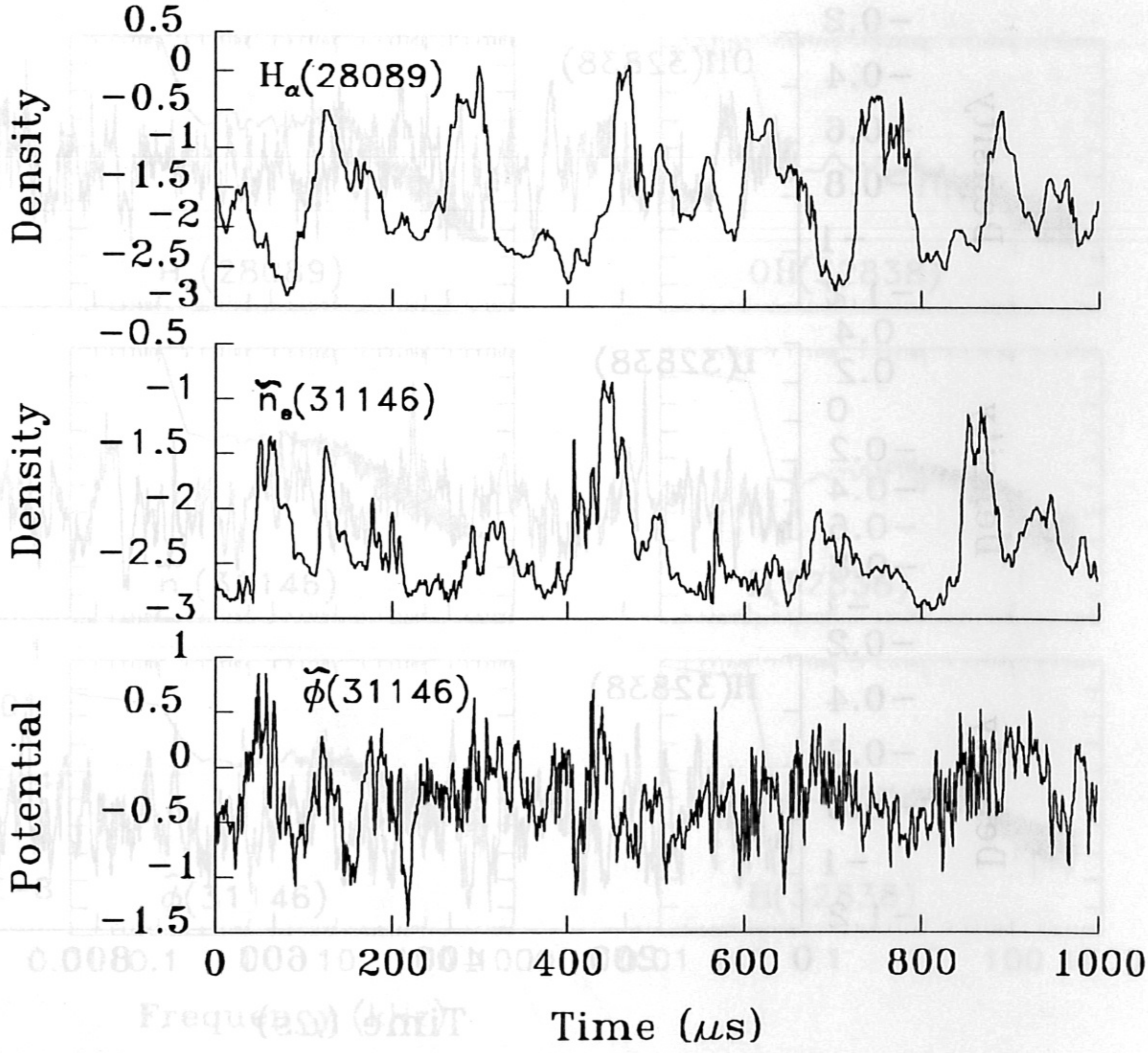

Fig.1a-1c. Typical plasma fluctuation signals measured by H_α diagnostic (shot: 28089) and Langmuir probes (shot: 31146) in ASDEX.

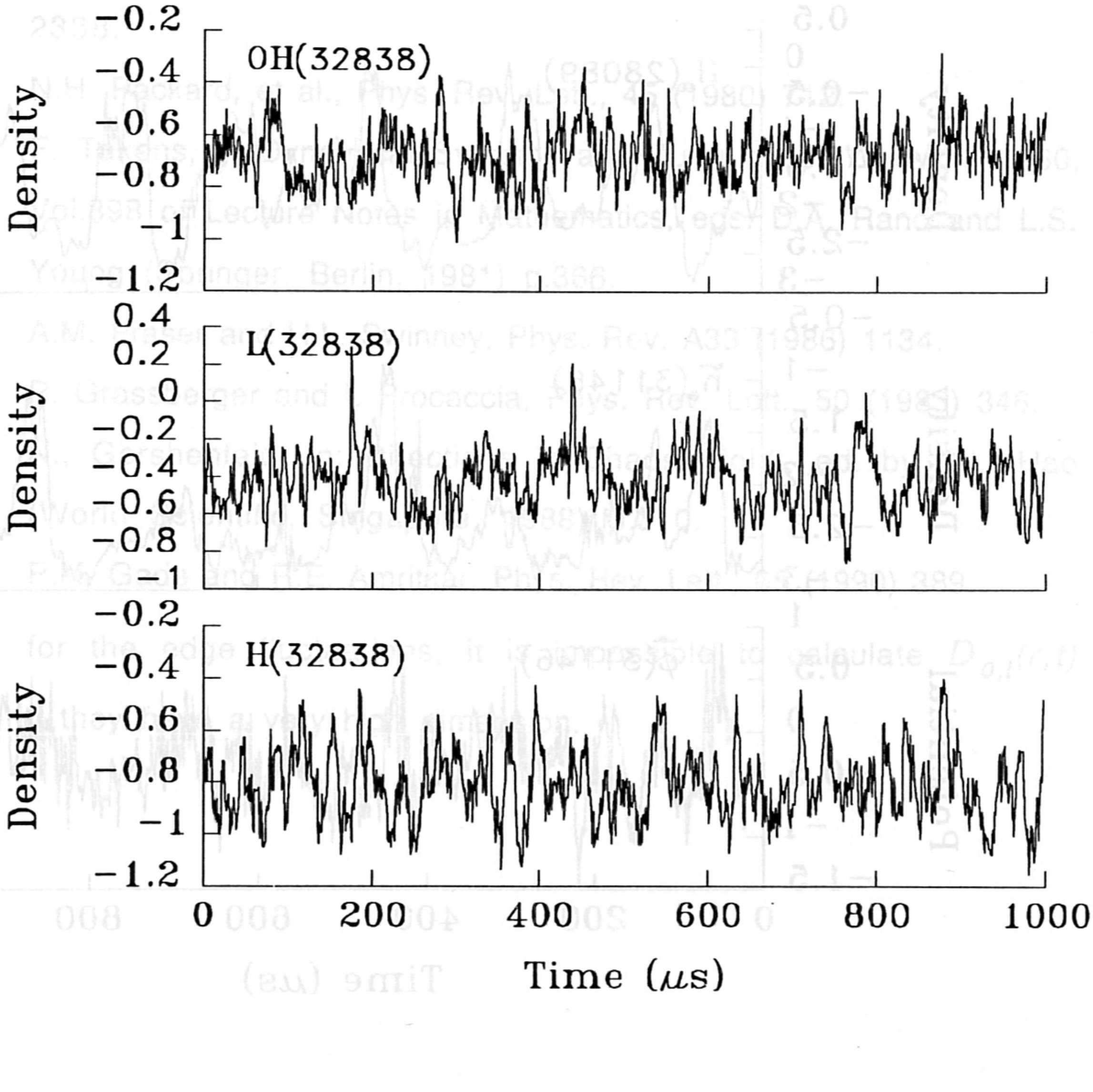

Fig.1d-1f. Typical density fluctuation signals measured by H_α diagnostic in Ohmic, L- and H-phase (shot: 32838).

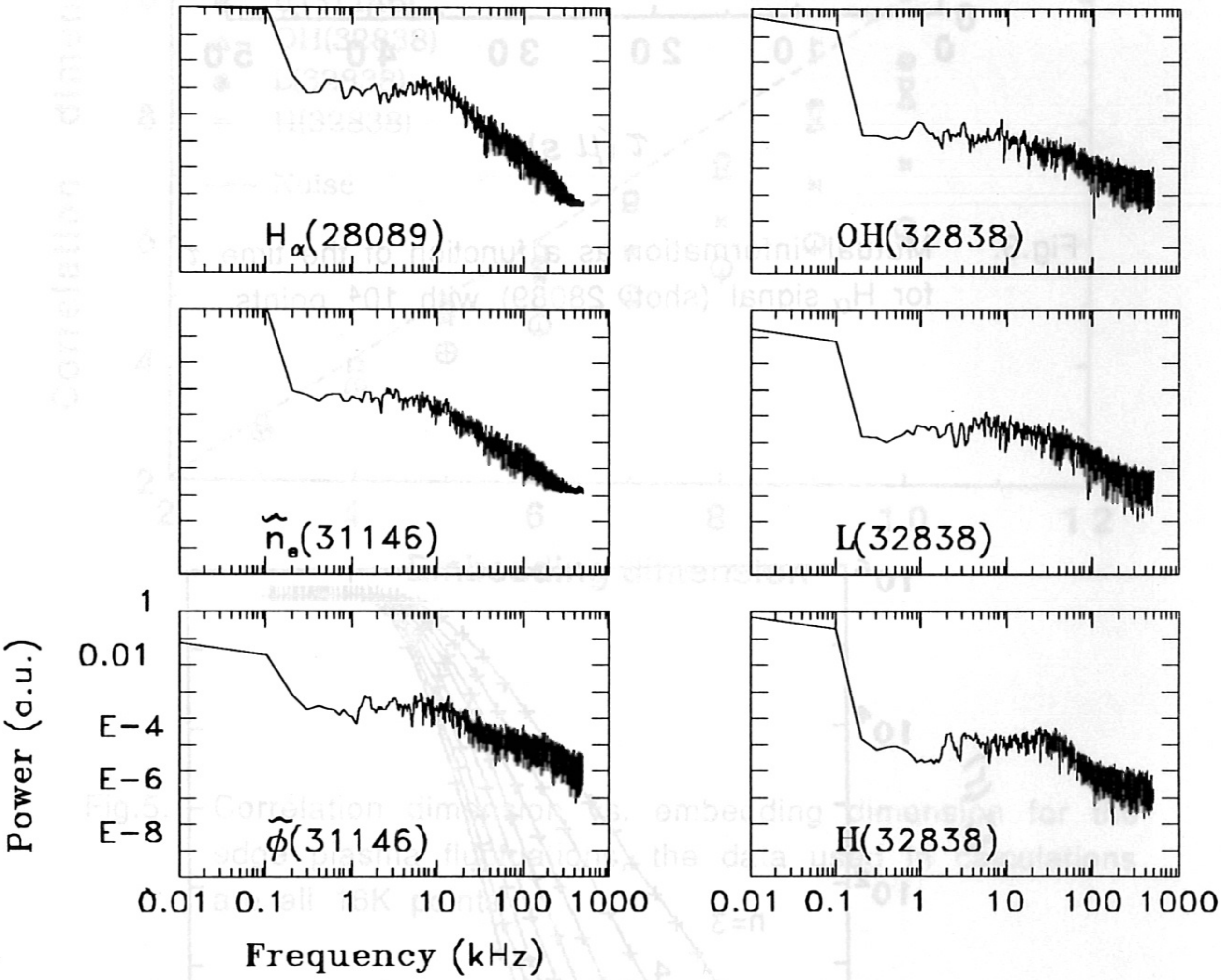

Fig.2. The corresponding power spectra of plasma fluctuation signals shown in Fig.1a-Fig.1f.

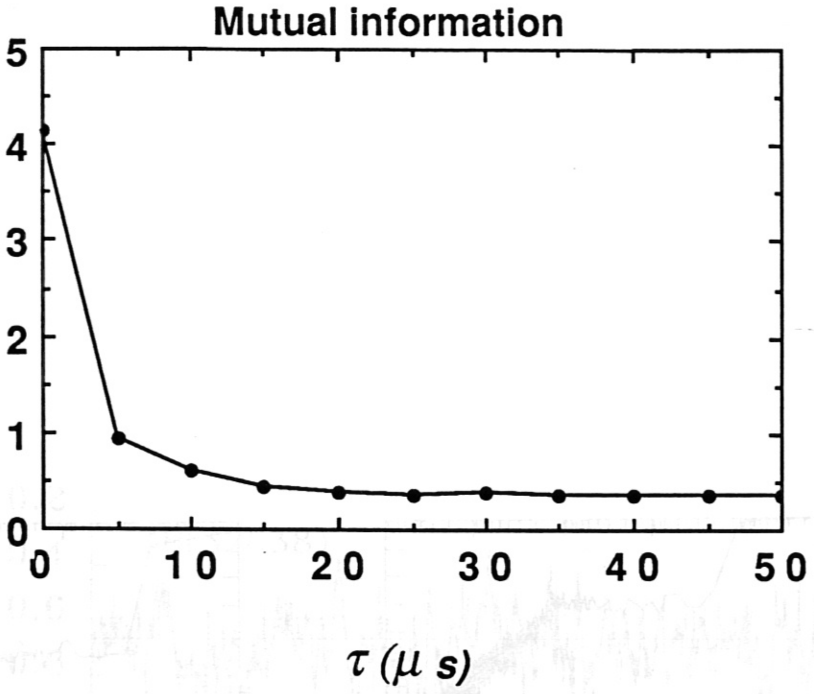

Fig.3. Mutual information as a function of the time τ for H_α signal (shot: 28089) with 10^4 points.

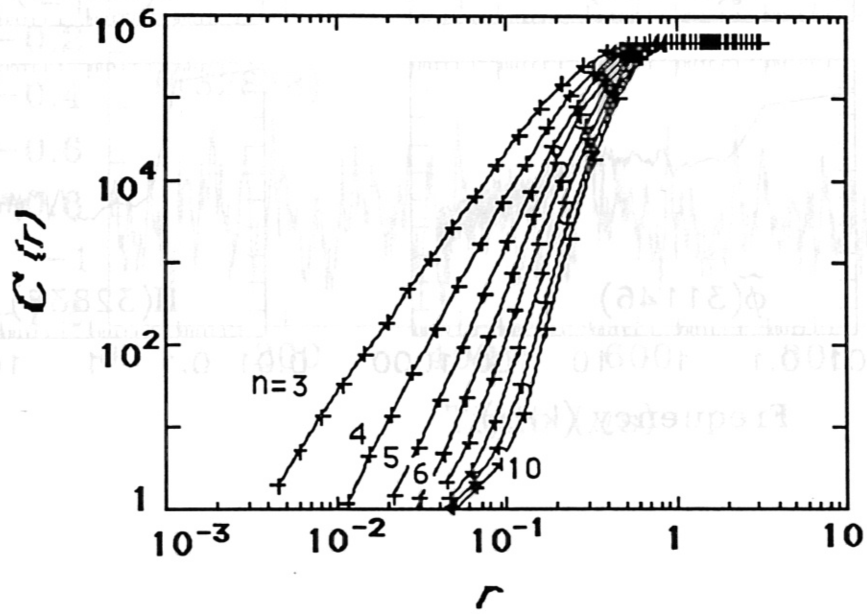

Fig.4. Log-log plots of $C(r)$ versus r for n =3, 4, ..., 10 with N=5x10^5 and M=100 for H_α signal (shot: 28089).

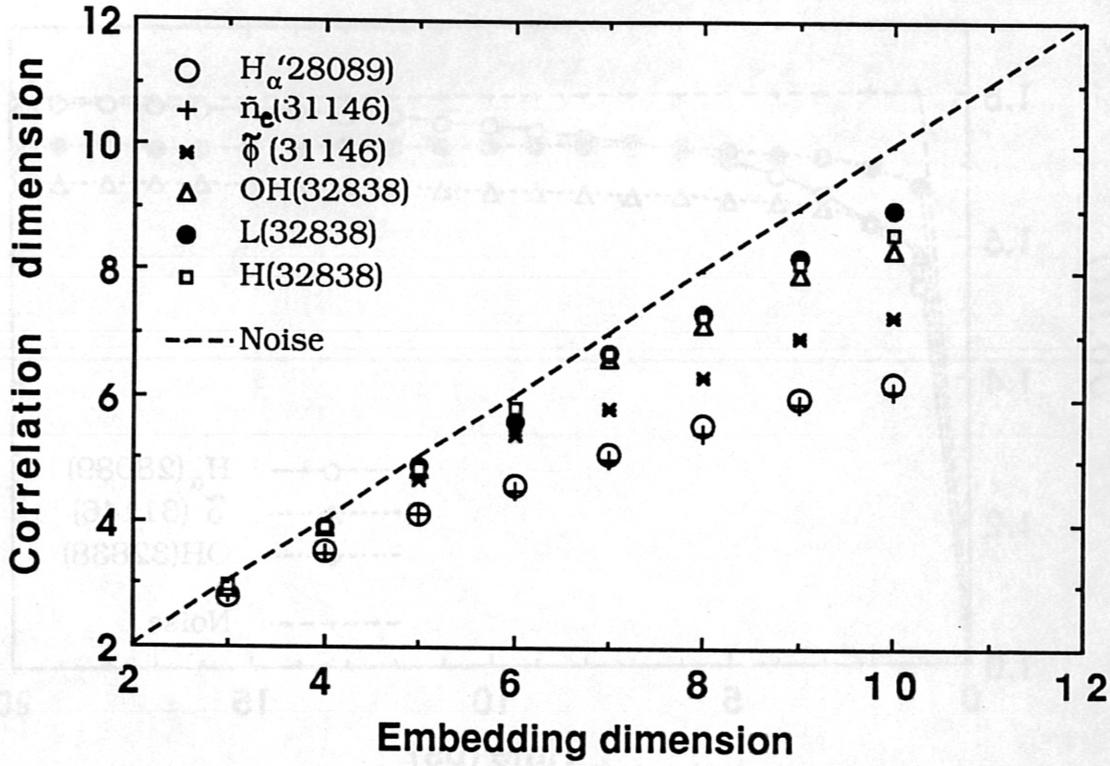

Fig.5. Correlation dimension vs. embedding dimension for the edge plasma fluctuations, the data used in calculations are all 16K points.

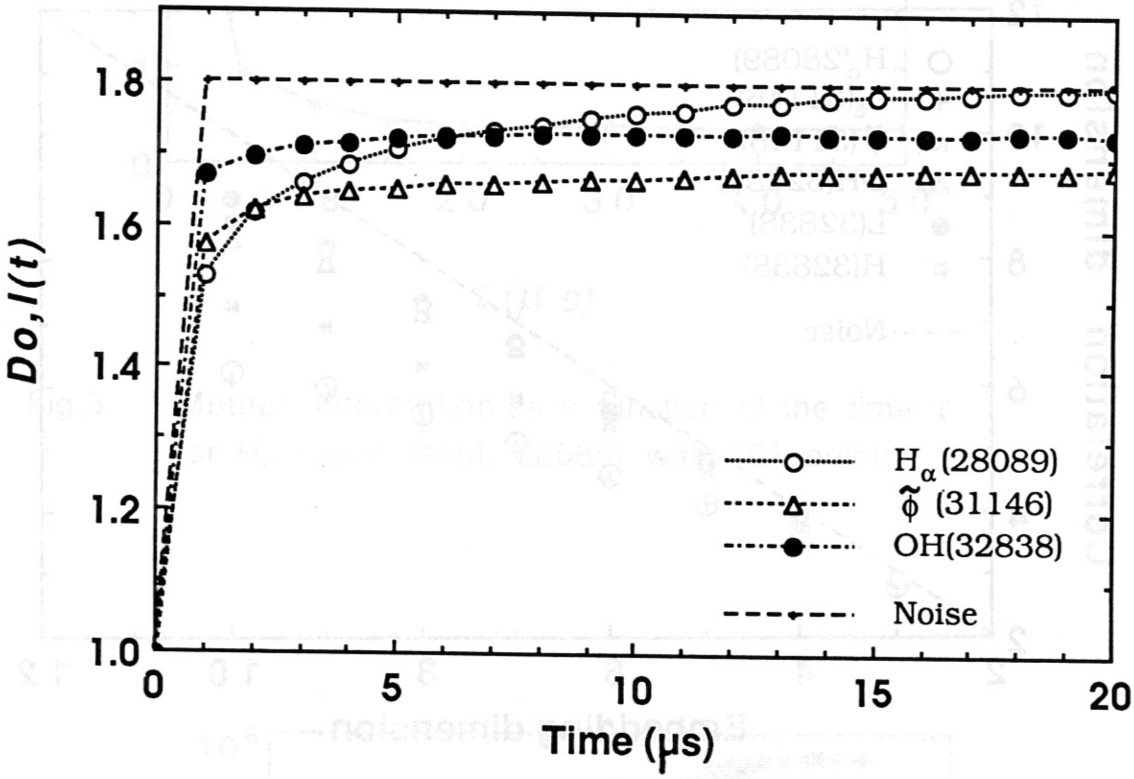

Fig.6. Fractal dimension $D_{0,l}(t)$ versus the time t for the edge plasma turbulence ($N=10^4$, $l=0.01$).