Dynamo and Alfvén effect in MHD turbulence

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This extended abstract reports a spectral relation between residual and total energy, $E_k^{\mathrm{R}} = |E_k^{\mathrm{M}} - E_k^{\mathrm{K}}|$ and $E_k = E_k^{\mathrm{K}} + E_k^{\mathrm{M}}$ respectively, as well as the influence of an imposed mean magnetic field on the spectra. The proposed physical picture, which is confirmed by accompanying direct numerical simulations, embraces two-dimensional MHD turbulence, globally isotropic three-dimensional systems as well as turbulence permeated by a strong mean magnetic field. The results have direct implications on the current understanding of the energy cascade in MHD turbulence.

In the following reference is made to two high-resolution pseudospectral direct numerical simulations of incompressible MHD turbulence which we regard as paradigms for isotropic (I) and anisotropic (II) MHD turbulence. The dimensionless MHD equations

$$\partial_t \boldsymbol{\omega} = \nabla \times [\mathbf{v} \times \boldsymbol{\omega} - \mathbf{b} \times (\nabla \times \mathbf{b})] + \mu \Delta \boldsymbol{\omega} \quad (1)$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \eta \Delta \mathbf{b} \quad (2)$$

$$\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{b} = 0 \,. \quad (3)$$

are solved in a 2π-periodic cube with spherical mode truncation to reduce numerical aliasing errors [1]. The equations include the flow vorticity, $\boldsymbol{\omega} = \nabla \times \mathbf{v}$, the magnetic field expressed in Alfvén speed units, $\mathbf{b}$, as well as dimensionless viscosity, μ, and resistivity, η.

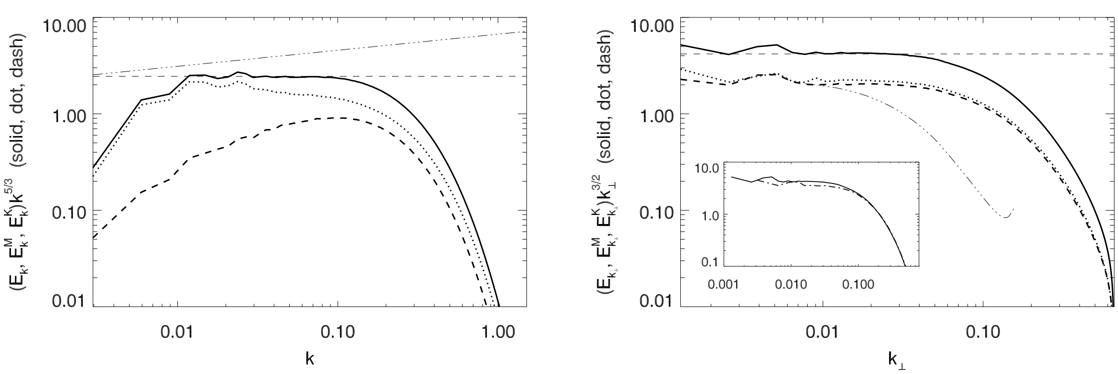

Figure 1: Total (solid), kinetic (dashed), and magnetic (dotted) energy in a 1024^3 simulation of decaying isotropic MHD turbulence (left) and in a $1024^2 \times 256$ simulation of anisotropic turbulence permeated by a strong mean magnetic field, $b_0 = 5$ (right, spectra are based on field perpendicular fluctuations). The dash-dotted line in the graph on the left illustrates a $k^{-3/2}$ power-law while the dashed horizontals indicate $k^{-5/3}$-behavior (left) and $k^{-3/2}$-scaling (right). The dash-dotted curve on the right shows the high-k part of the field-parallel total energy spectrum. The inset displays the difference in the perpendicular total energy spectrum when switching resolution from 512^2 (dash-dotted) to 1024^2 (solid).

Simulation I evolves globally isotropic freely decaying turbulence represented by 1024^3 Fourier modes. Total kinetic and magnetic energy are initially equal with $E^{\mathrm{K}} = E^{\mathrm{M}} = 0.5$. The dissipation

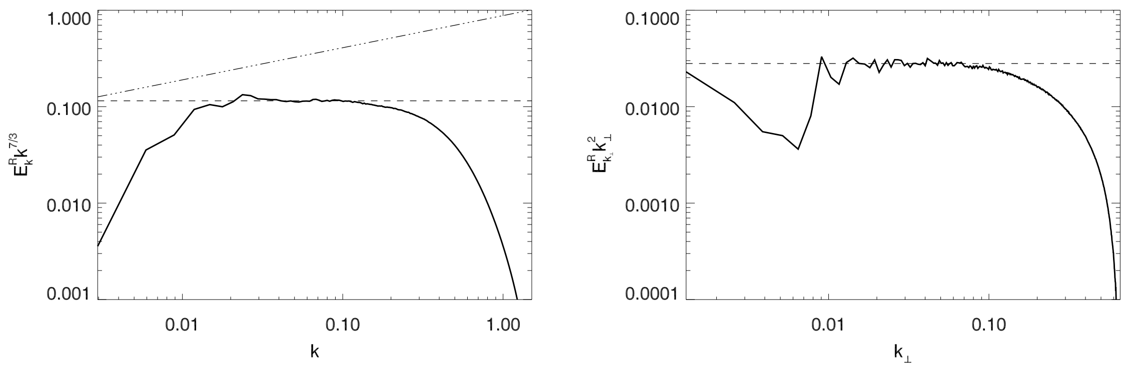

Figure 2: Compensated and space-angle-integrated residual energy spectrum, $E_k^{\rm R} = \left|E_k^{\rm M} - E_k^{\rm K}\right|$, for the same systems as in Fig. 1 (isotropic:left, mean magnetic field: right). The dash-dotted line depicts scaling expected for a total energy spectrum following Iroshnikov-Kraichnan scaling.

parameters are set to $\mu = \eta = 1 \times 10^{-4}$. Case II is a $1024^2 \times 256$ forced turbulence simulation with an imposed constant mean magnetic field of strength $b_0 = 5$ in units of the large-scale rms magnetic field $\simeq 1$ with $\mu = \eta = 9 \times 10^{-5}$.

Fourier-space-angle integrated spectra of total, magnetic, and kinetic energy for case I are shown in Fig. 1 (left). To neutralize secular changes as a consequence of turbulence decay, amplitude normalization assuming a Kolmogorov total energy spectrum, $E_k \to E_k/(\varepsilon\mu^5)$, $\varepsilon = -\partial_t E$, with wavenumbers given in inverse multiples of the associated dissipation length, $\ell_{\rm D} \sim (\mu^3/\varepsilon)^{1/4}$. Clearly, Kolmogorov scaling applies for the total energy in the well-developed inertial range, $0.01 \lesssim k \lesssim 0.1$.

In case II, pictured in Fig. 1 (right), strong anisotropy is generated due to turbulence depletion along the mean magnetic field, $\mathbf{b}_0$. This is visible when comparing the normalized and time-averaged field-perpendicular one-dimensional spectrum, $E_{k_\perp} = \int\int {\rm d}k_1\ {\rm d}k_2 E(k_\perp, k_1, k_2)$ (solid line) with the field-parallel spectrum, defined correspondingly and adumbrated by the dash-dotted line in Fig. 1 (right).

While there is no discernible inertial range in the parallel spectrum, its perpendicular counterpart exhibits an interval with Iroshnikov-Kraichnan scaling, $E_{k_\perp} \sim k_\perp^{-3/2}$ [2, 3]. This is in contradiction with the anisotropic cascade phenomenology of Goldreich and Sridhar for strong turbulence predicting $E_{k_\perp} \sim k_\perp^{-5/3}$ [4].

The observation that field-parallel fluctuations are restricted to large scales while the perpendicular spectrum extends more than half a decade further suggests that the strong $\mathbf{b_0}$ constrains turbulence to quasi-two-dimensional field-perpendicular planes as is well known and has been shown for this particular system [5].

Another intriguing feature of system II is that $E_k^{\rm K} \simeq E_k^{\rm M}$ with only slight dominance of $E^{\rm M}$ (cf. Fig. 1, right) in contrast to the growing excess of spectral magnetic energy with increasing spatial scale for case I. Both states presumably represent equilibria between two competing nonlinear processes: field-line deformation by turbulent motions on the spectrally local time scale $\tau_{\rm NL} \sim \ell/v_\ell \sim \left(k^3 E_k^{\rm K}\right)^{-1/2}$ leading to magnetic field amplification (turbulent small-scale dynamo) and energy equipartition by shear Alfvén waves with the characteristic time $\tau_{\rm A} \sim \ell/b_0 \sim (kb_0)^{-1}$ (Alfvén effect).

By using the spectral EDQNM equation for the residual energy in spectrally local and non-local approximations [6] and by assuming that the residual energy is a result of a dynamic equilibrium between turbulent dynamo and Alfvén effect, one obtains for stationary conditions and in the inertial range,

$$E_k^{\mathrm{R}} \sim k E_k^2 \sim \left(\frac{\tau_{\mathrm{A}}}{\tau_{\mathrm{NL}}}\right)^2 E_k \,. \tag{4}$$

with $\tau_{\mathrm{A}} \sim (kb_0)^{-1}$, where b_0 is the mean magnetic field carried by the largest eddies, $b_0 \sim (E^{\mathrm{M}})^{1/2}$, and by re-defining $\tau_{\mathrm{NL}} \sim \ell/(v_\ell^2 + b_\ell^2)^{1/2} \sim (k^3 E_k)^{-1/2}$. The modification of τ_{NL} is motivated by the fact that turbulent magnetic fields are generally not force-free so that magnetic pressure and tension contribute to eddy deformation as well.

Apart from giving a prediction which allows to verify the proposed model of nonlinear interplay between kinetic and magnetic energy, relation (4) also has some practical utility. It is a straightforward consequence of (4) that the difference between possible spectral scaling exponents, which is typically small and hard to measure reliably, is enlarged by a factor of two in E_k^{R}. Even with the limited Reynolds numbers in today's simulations such a magnified difference is clearly observable (e.g. dash-dotted lines in Figs. 1 and 2).

In summary, based on the structure of the EDQNM closure equations for incompressible MHD a model of the nonlinear spectral interplay between kinetic and magnetic energy is formulated. The quasi-equilibrium of turbulent small-scale dynamo and Alfvén effect leads to a relation linking total and residual energy spectra, in particular $E_k^{\mathrm{R}} \sim k^{-7/3}$ for $E_k \sim k^{-5/3}$ and $E_k^{\mathrm{R}} \sim k^{-2}$ for $E_k \sim k^{-3/2}$. Both predictions are confirmed by high-resolution direct numerical simulations of isotropic turbulence exhibiting Kolmogorov scaling and forced anisotropic turbulence displaying Iroshnikov-Kraichnan scaling perpendicular to the mean field direction. The findings limit the possible validity of the Goldreich-Sridhar phenomenology to MHD turbulence with weak mean magnetic fields and emphasize the important role of the Iroshnikov-Kraichnan picture for a large class of turbulent MHD systems.

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