Review of 3-D equilibrium calculations and reconstructions for W7-AS

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Abstract

Knowledge of the 3-dimensional structure of the plasma equilibrium is a prerequisite for experiments in stellarators and for the interpretation of the results. Therefore, since the calculation of equilibria consistent with the experimental data is an important task, we review the calculations done for W7-AS with applications to high β and large toroidal currents for stellarator-tokamak hybrid operation. We also present a novel method for fast equilibrium reconstruction for stellarators based on function parameterization.

Introduction

Although the geometry of the flux surfaces of a stellarators is largely determined by the external coil system, the plasma current densities induced by finite β, internal or external current drive (bootstrap-, Okhawa-, ohmic- and ECCD currents) may lead to considerable changes in the equilibrium fields.

To calculate 3-D MHD equilibria with free boundary, we use the NEMEC-code [1] assuming nested flux surfaces. The input consists of profiles for pressure and toroidal current, an estimate of the magnetic axis position and the plasma boundary, and the vacuum magnetic field. Based on an energy principle the equilibrium is determined iteratively using a steepest gradient method. The equilibrium quantities like flux surface geometry and magnetic field are given in Fourier series with respect to the cylindrical toroidal angle and a poloidal angle coordinate on a radially discretized grid.

An equilibrium reconstruction of a discharge at a given time point clearly implies an iterative process of adjusting the input parameters such that the resulting equilibrium data best match the experimental ones.

Equilibrium Calculations(1) : β effects

Usually, the pressure induced current densities dominate the changes in the magnetic configuration in net toroidal current free discharges. Although the reduced average toroidal curvature of W7-AS leads to smaller Pfirsch-Schlüter (PS) currents (a factor $\sqrt{2}$ compared to a conventional stellarator) they may give rise to appreciable Shafranov shifts, changes in the rotational transform (ι) profile and a displacement of the plasma as a whole in the accessible ι-range of W7-AS ($0.26 < \iota_{vac} < 0.56$). Very good agreement of the NEMEC calculations with experimental data was shown in Refs 2 and 3 for the high β cases. For W7-X the higher reduction of the average toroidal curvature as well as the higher operational range in ι ($\iota_{vac} \approx 1$) leads to much more stable magnetic configurations in the sense of finite-β equilibrium changes.

Equilibrium Calculations(2) : Toroidal current effects

Internally or externally driven net toroidal currents also affect the ι-profile and the geometry of the equilibrium flux surfaces, depending on their magnitude and distribution. At W7-AS, the usual operation is net current free, which means that the toroidal plasma current is kept at zero by inductive compensation. However, the toroidal current (I_{pl})

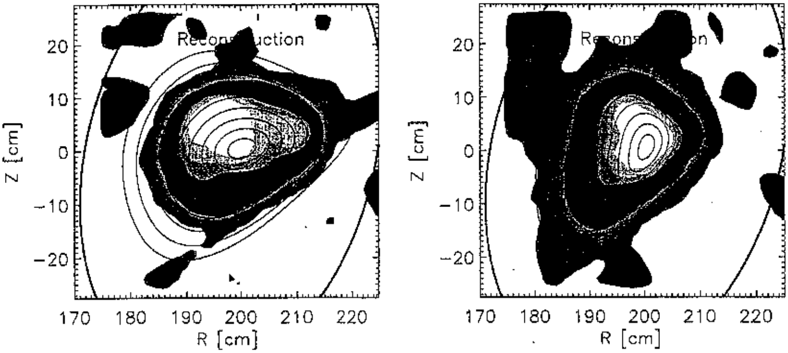


Figure 1: Comparison of Soft-Xray emissivity with NEMEC flux surfaces for $I_{\text{pl}} = 30\text{kA}$ (right) and -30kA (left) shows good agreement in the gradient regions. A very flat central emission profile dominates the central contours for $I_{\text{pl}} = 30\text{kA}$.

within each flux surface does not vanish and thus alters the ι-profile. A large aspect ratio estimate for the change in ι is given by $\Delta\iota(r_{\text{eff}}) = I_{\text{pl}}(r_{\text{eff}})R/(2\Phi(r_{\text{eff}}))$ with r_{eff} and R the minor and major radii, respectively, and Φ the toroidal magnetic flux. Nevertheless, for vanishing net current there is generally no influence on the flux surface geometry. Therefore, to evaluate ι-profiles in net-current free discharges, it is sufficient to know the finite β changes on ι and then correct the profile using the toroidal current profile deduced from neoclassical theory and deposition profiles.

Introducing a net toroidal current also changes the flux surface geometry depending on the current's magnitude. Calculations show that the excursion of the flux surfaces in R increases with positive net current (the shape tends to be more oblate) and decreases with negative net current (higher vertical elongation). Good agreement of the calculated flux surfaces in 2 cases with large toroidal currents ($\pm 30\text{kA}$ at $B=2.5\text{T}$, $\iota_{\text{tot}} \approx 0.4$) with the tomographically reconstructed soft X-ray emissivity measured by the new MiniSoX camera system [4] is shown in Fig. 1.

Discussion of NEMEC calculations

Up to high β and rather high currents, the equilibrium calculations for W7-AS with NEMEC are in good agreement with the experiment. However, the computational effort is orders of magnitude higher compared to the calculation of tokamak equilibria. For fast reconstructions accompanying the experiment on a shot to shot basis or even online, NEMEC is not suitable with present-day computing power. Therefore, other approaches like function parameterization, which is discussed below, have to be explored.

Despite the good agreement with the experiment, there are limitations in the NEMEC applications. The assumption of nested flux surfaces excludes the detection of ergodic regions or islands. For this, more advanced equilibrium solvers have to be applied, like HINT or PIES. However, their computational requirements are orders of magnitude higher again than that of NEMEC. Furthermore, the Fourier representation of the equilibrium quantities together with the energy minimization method applied limits the resolution of boundary structures which require high Fourier harmonics and contain comparably small energies. Such 5/m resonant structures ($m \geq 12$) have been seen by video observations of visible light in high β discharges. Nevertheless, NEMEC shows a smooth plasma boundary without the actual indentation.

A hybrid FP/interpretive method for equilibrium recovery

Interpretive methods for determining plasma equilibria are widely used in tokamak analysis. Input parameters to an equilibrium code are iteratively adjusted such that simulated diagnostic signals from the resulting equilibrium best match experimental data. These methods are generally unsuitable for stellarators since each iteration involves the full solution of a 3D equilibrium code, a task requiring roughly one hour of CPU time on the Cray J-90 at IPP for standard W7-AS equilibrium calculations using NEMEC.

The application of function parameterization[5] (FP) to W7-AS is under development. FP seeks simple functional relationships between plasma parameters and diagnostic measurements over a database of simulated equilibria. This facilitates rapid equilibrium reconstruction, here in terms of magnetic data and a prescribed pressure profile. We have developed a novel interpretive method for equilibrium identification based on FP reconstructions that can be performed in the order of a few tens of seconds on a workstation.

FP database

Here, the database consists of *circa* 400 NEMEC equilibria with zero net toroidal current. They are chosen by randomly varying 8 input parameters over ranges appropriate to W7-AS, namely 3 ratios of the 4 field coil currents, a limiter position and a 4-parameter pressure profile chosen from the following family (s is normalized toroidal flux):

$$p(s) = p_0\,(1-s)^2\,\exp(as + bs^2 + cs^3)$$

Interpretive scheme

The Thomson scattering diagnostic on W7-AS gives electron temperature and density (and thus the electron pressure p_e) on ≤ 20 channels along a horizontal line-of-sight through the magnetic axis in a symmetry plane (ϕ=0) at a single timepoint during a discharge, i.e. $p_e = p_e(R_i)$, R_i being the major radii of the Thomson channels. A smoothing polynomial in R is fitted to the p_e data, allowing evaluation between channels. The iterative procedure attempts to reproduce the spatial to flux transformation $s(R)$ in $p_e(R)$ since p_e is constant on flux surfaces. Starting from an initial guess, the equilibrium pressure profile $p_{\rm eq}(s)$ is varied to minimize the quantity

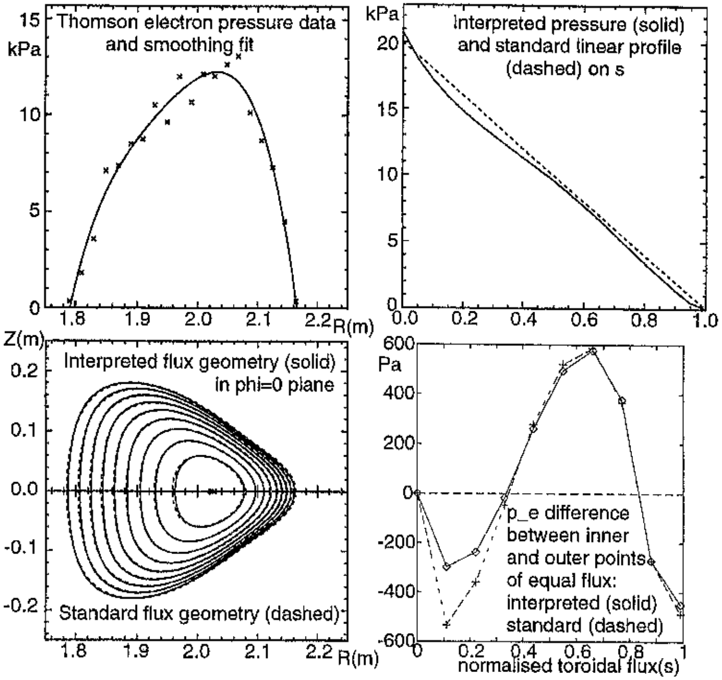
$$\int \left(p_e[R_{\rm in}(s)] - p_e[R_{\rm out}(s)]\right)^2\, ds$$

subject to the constraints that $p_{\rm eq}(s)$ is non-hollow and the boundary flux surface coincides with the zeroes of $p_e(r)$. An additional restriction that the kinetic energy content match the diamagnetic energy from the experiment can be optionally enforced.

The interpreted fit thus depends only on the topology of the $p_e(R)$ profile and not on its magnitude. In fact, it could be done separately for the Thomson temperature and density profile. Clearly, mismatches between the theoretical and physical magnetic configurations could falsify both the present scheme and the standard calculations. Consistency checks with additional spatially resolved or global diagnostic data would reveal such discrepencies, if present.

Comparison with standard calculations

Results from the interpretive method and the standard NEMEC simulation for shot 31909 are illustrated. The upper plots show the Thomson $p_e(R)$ with smoothing polynomial and the interpreted $p_{\rm eq}(s)$ together with the standard NEMEC profile. The lower plots show the equilibrium flux surfaces in the Thomson ($\phi = 0$) plane and the final asymmetry error in $p_e(s)$.



Though the two fits are of comparable quality, the interpreted fit is better than the standard calculation in the inner half of the plasma. This is not unexpected since the standard calculation uses the same optimization criterion with manual intervention required between iterations, rather than an automated least-squares fit. The p_e symmetry can be further improved by varying additional pressure parameters during iterations. Only three of a possible six were varied here to prevent overfitting. The interpretation for this case took roughly 20 seconds on a workstation, whereas the standard simulation required several NEMEC calculations.

Work in progress includes investigation of the effects of signal noise on the recovery. Moreover, although scalar equilibrium parameters and flux geometry are well recovered, a model that reproduces the ι-profile well has so far proved elusive. Once this is achieved, the procedure will be applied to equilibria with finite net toroidal current.

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