Comparison of different sawtooth crash models for transport analysis

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Introduction

In magnetically confined fusion plasmas, a variety of magnetohydrodynamic (MHD) instabilities can occur, driven by gradients of kinetic pressure or current density. The sawtooth oscillation is one of the fundamental instabilities in tokamaks. It is associated with abrupt changes in central plasma confinement due to growth of an (m,n)=(1,1) mode. This growing mode leads to rapid crashes of the central electron temperature and proper modelling of such crashes is necessary for an accurate prediction of the plasma confinement.

Transport codes for analysis and prediction of the plasma profiles evolution use different modelling for the sawtooth instability. The typical sawtooth crash models have several contradictions with experiments [1-4]. In this paper we discuss these contradictions and present a new model for the sawtooth crash which is motivated by stochastic model of the crash [5,6]. The model is compared with other typical transport models for the sawtooth crash and with Motional Stark Effect measurements (MSE).

Definitions of sawtooth models for transport calculations

In this paper, the transport code ASTRA [7] is used to simulate two different discharges in ASDEX Upgrade assuming three different crash models in each case. The models implemented in ASTRA are the following:

(i) Kadomtsev's full reconnection model [8].
(ii) Porcelli partial reconnection model [9].
(iii) A new model based on stochastic sawtooth crash assumption.

The first Kadomtsev model assumes that during the crash phase the O-point of the island becomes a new plasma centre. This leads unavoidably to the elevation of the safety factor profile in the plasma core during the crash as shown in figure 1a. Thus, *the position q=1 is shifted inside the plasma*. This result is in clear contradiction with experimental measurements with post-cursors activity which shows that the mode position remains the same after the crash [4]. The model also removes the island structure which contradicts the experimental observations as well.

The second Porcelli model is similar to Kadomtsev but the reconnection is stopped at a particular radius (island width radius). The main feature of this model is a constant flux between the original q=1 position and the mixing radius which creates a ring with a constant safety factor (q=1). The inner core in this ring is Taylor relaxed[1]. It is clear that the island width is an additional new parameter. In the following, two cases are considered. Porcelli model one reefers to the case of a very small island (about 1% of the q=1 radius, see figure 1b). Porcelli model two represents the experimentally relevant situation with about 50% of the flux mixing (see figure 1c). The inconsistencies to the experimental observations are the following:

- absence of the (1,1) mode after the crash (only q=1 ring is present),

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[1] In a later paper [10], Porcelli formulates a Hamiltonian which includes the conservation ansatz from Kadomtsev's model and at the same time provides the mode at the right position (around the original q=1 surface). But the flux surfaces inside the inner core are still not affected during the crash.

- physically unclear mechanism of temperature mixing between the Taylor relaxed core region and the outer region during the crash phase.

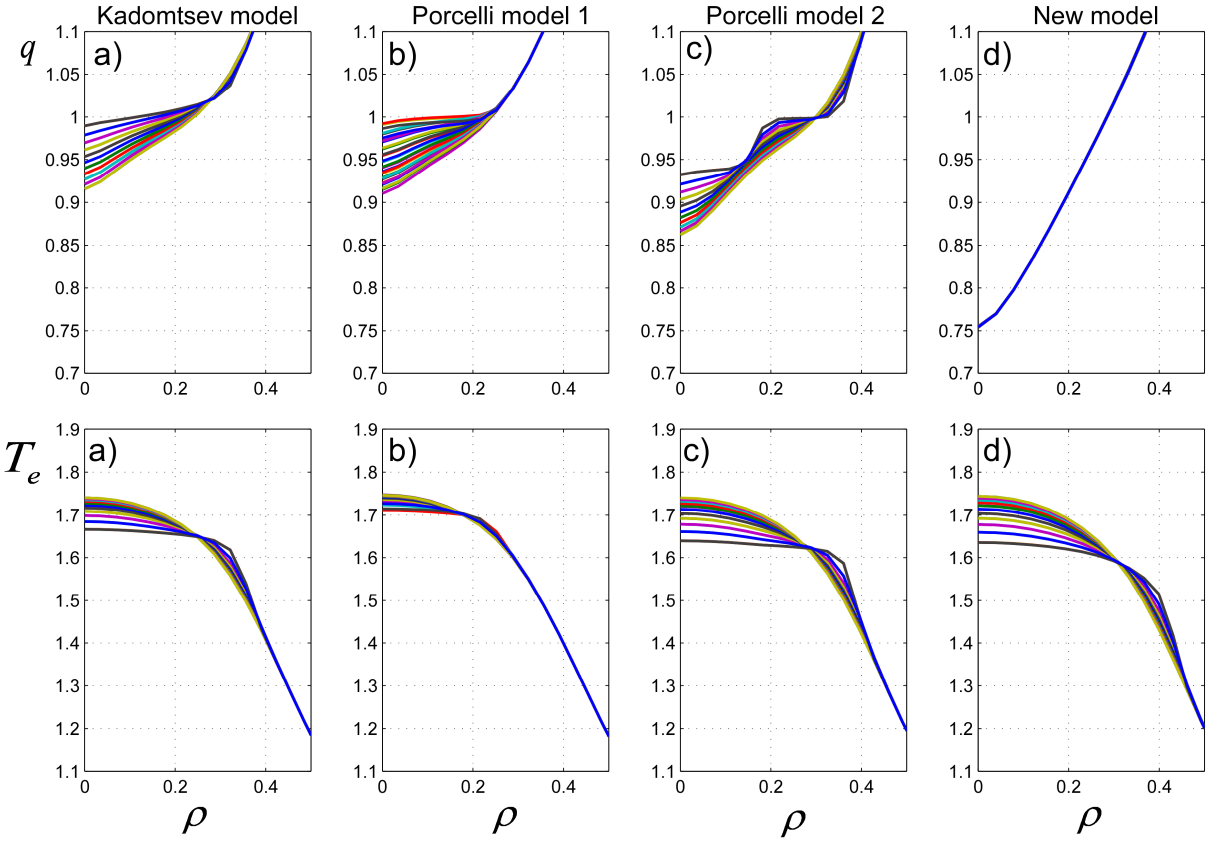

Figure 1. Comparison of changes in safety factor profile and temperature profile during the sawtooth cycle for different models. The profiles were calculated in transport code ASTRA for discharge 22401 in ASDEX Upgrade. (a) Kadomtsev model ; (b) Porcelli model with small island size; (c) Porcelli model with big island size; (d) new model for the crash. The new model (d) produces negligible changes of the safety factor profile in comparison to other cases (a-c).

The third “New model” is motivated by the stochastic hypothesis for the sawtooth crash. It was shown recently, that poloidally average safety factor after the crash remains almost the same even in case of strong stochastisity [4]. Since, this influence is small one can neglect changes in the safety factor during the crash phase. At the same time, stochastisity during the crash removes the heat from the plasma core which leads to strong temperature redistribution as it is found in the experiments. Thus, one can formulate the following assumptions for the new crash model:

- safety factor is not affected and remains the same after the crash,
- temperature profile is relaxed in the same way as in the other models.

These two constrains are sufficient to keep the mode at its original position and avoid any further complications with (1,1) island. (It was shown in previous paper [4] that the poloidally average safety factor profile with (1,1) mode is almost identical to the original one without the mode.) Starting from other sawtooth models only a small modification is necessary to implement this model into the transport code. One has to modify the Kadomtsev model such that safety factor changes will be removed and the temperature changes are kept during the crash. The result profiles are shown in figure 1(d). The main physical difference between the new model and Kadomtsev/Porcelli models is absence of the flux changes during the crash. This fact strongly suppresses the evolution of the safety factor profile. There are still small changes of the safety factor profile due to changes in plasma temperature (and thus in plasma conductivity), but these changes are much smaller and much slower compare to the previous models.

All sawtooth models require a trigger conditions to initiate the crash. The trigger conditions for the sawtooth crash are the same in all calculations and based on the Porcelli assumption for linear stability thresholds of resistive and ideal (1,1) mode. The threshold values are adjusted such that the sawtooth period is the same as in the experiment for all calculations.

It is useful to compare changes of the safety factor profile close to the plasma centre for these models since this is the place where the maximal changes of the safety factor would appear for all models (see figure 1). The evolution of the safety factor values are shown for a point close to the plasma centre where one of the central MSE channels is located, $\rho_{pol} = 0.1$ (figure 2).

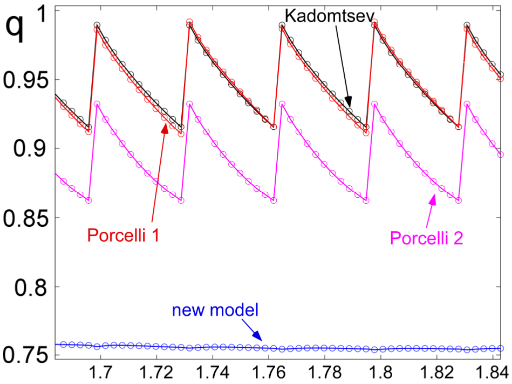

Figure 2. Changes of the safety factor profiles during the time evolution for the same simulations as in figure 1.

Comparison of the sawtooth models with measured MSE angles

Porcelli and Kadomtsev models produce jumps in the safety factor value during the crash (see figure 2). This jump is not present in the new model which has only small variation of the safety factor compare to the other models. Presence of these jumps gives us an opportunity to compare between the conventional models and the new model. One has to compare MSE measurements of the safety factor profile before and after the sawtooth crash. In order to make this comparison as direct as possible, measured MSE angles were calculated directly in the transport code ASTRA for all models. These calculations show sudden increase of the MSE angle during the crash for Kadomtsev/Porcelli models and almost constant values for the new model. The predicted increase of MSE values for central channels are approximately the same for Kadomtsev and Porcelli models and is about 0.1 degree. Unfortunately, the direct comparison of the MSE angles is not possible because of the noise in the MSE signals. Thus, a statistical analysis has to be applied for comparison of MSE angles.

In this paper we have chosen two discharges with stable and long sawtooth phases. The first discharge 13691 contains 33 identical sawteeth. The second discharge 22401 has 14 identical sawteeth. This allows statistical analysis of the MSE data with assumption that all sawteeth for a given discharge provide identical changes of the safety factor profile. This is a good approximation, because in the chosen intervals the amplitudes and the periods of the sawteeth are the same. For each of these discharges two data sets were created. The first one contains MSE values before the sawtooth crashes. The second includes MSE measurements after the crashes. All data sets are checked on the "normal distribution" assumption. The normally distributed data have to be distributed linearly on a logarithmic scale as it is shown in figure 3. One has to note that not all MSE channels provide the data with normal distribution. Some channels show strong deviation from the normal distribution assumption before and after the sawtooth crash and can not be included in the analysis. In spite of such difficulties, at least two channels with good distributed data close to the plasma core are found for each of the analysed discharges. The result distributions show no changes of the MSE angles during the crash which is in agreement with new model of the sawtooth crash. Calculations with ASTRA show that changes of the safety factor profile just before and just after the crash are negligible compare to the changes during the crash. Thus, it is possible to use several MSE measurements before each crash in the first data set (and do the same for the second data set after the crash). This improves the statistic but does not change the results. We have varied the number of data points from (1;1)=(number of points before the crash ; number after the crash) to (5;5). All analysis gives the same result. It is interesting that some channels show shift in the direction opposite to the predicted by Kaomtsev/Porcelli (see channel 10 in figure 3). One has to note also that width of the distribution function is relatively large (about 0.6 deg.) compare to the expected changes of the angle in Kadomtsev/Porcelli model (about 0.1 deg.). Thus, application of the statistical analysis is necessary. It will be also advantageous to repeat the presented analysis on other tokamaks.

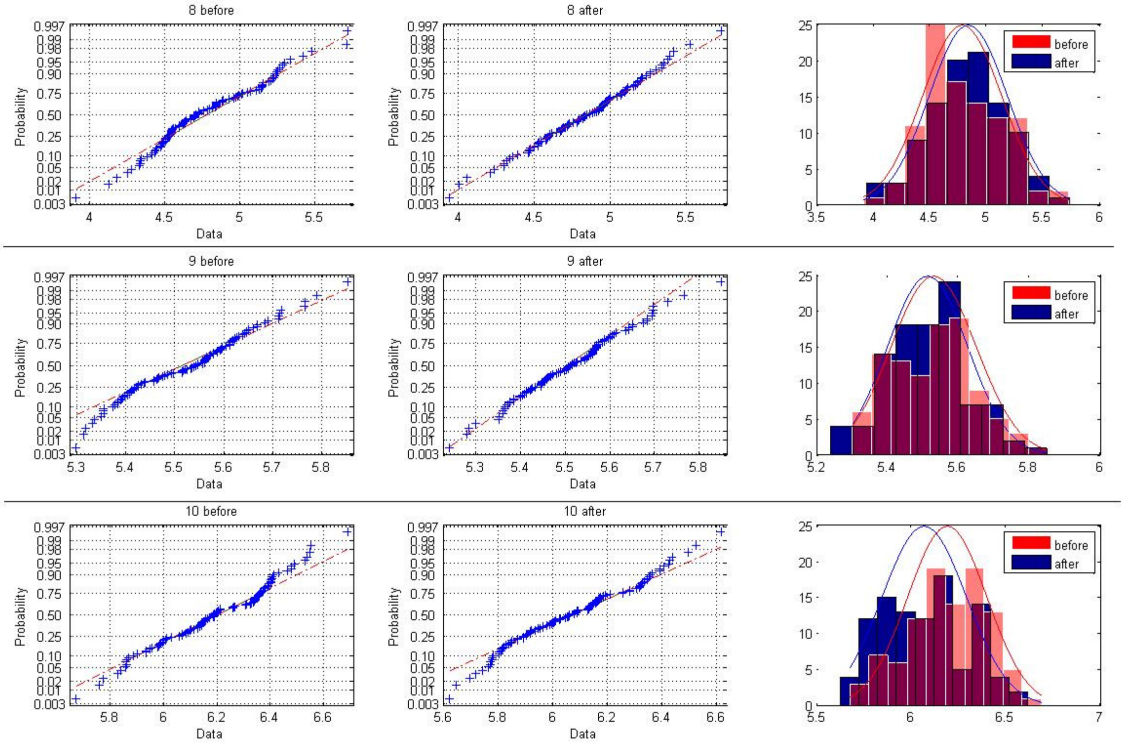

Figure 3. MSE angles for three most central channels (channel 10 $\rho_{pol} = 0.1$, channel 9 $\rho_{pol} = 0.09$, channel 8 $\rho_{pol} = 0.2$) are shown for discharge 13691 (sawteeth from 2 to 3.6s). For each channel three figures are shown: (a) test for "normal distribution" for points before crashes; (b) test for "normal distribution" for points after the crashes; (c) comparison of the two sets of data before and after the crash.

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