Theory of external geodesic acoustic mode excitation

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Introduction

From the cornucopia of literature on flow shear suppression of turbulence it appears the fusion problem could be facilitated greatly if only the global flows could be made strong enough to suppress the convective turbulence, preferably by external means and not in reliance on the turbulence itself. It can be shown that injection of momentum by neutral particle beams or plasma waves tends to be rather inefficient as the ratio of the applied power to the momentum transfer rate is given by the rather large phase- or particle velocity, respectively. In contrast, the resonant generation of geodesic acoustic modes – essentially oscillating zonal flows – by magnetic perturbations by external coils is a potentially far more efficient method [1]. This could already be done presently, e.g., in the DIII-D tokamak with the in-vessel ELM stabilization coils [I-coils]. In contrast, inducing GAM plasma pressure oscillations by pulsed external heating has very low efficiency independent of the spatial distribution of the heating, as the power deposited in the GAM motion is a factor of order $O(\delta T/T)$ (δT being the temperature oscillation associated with the GAM and T the background temperature) smaller than the heating power used, as this is the *free energy* deposited by the heating [2].

An elegant way to compute the action of external coil currents on the interior flux surfaces has been discovered, which allows analytical estimates and the coupling of a novel dynamic equilibrium code with a turbulence code to study the resonance layer in detail.

Since GAMs are axisymmetric modes they can only interact with axisymmetric external currents. Of these only toroidal currents need be considered, since any poloidal "ring coil" current (not encircling the plasma column) would just produce a field internal to the coil and not affect the plasma. Heuristically, an external quadrupole ($m = 2$) toroidal current (fig. 1a) would attract/repulse the toroidal currents within an elliptic plasma column in such a way as to cause a rotation (tilting force) of the column, i.e., act as a source for the GAMs. The current distribution should be up-down and left-right *anti-symmetric*, since otherwise the plasma column is just deformed, not tilted. The corresponding perturbative *magnetic field* has left-right and up-down *symmetry*, since it is a pseudo vector. (A perturbation with $m = 1$ corresponds to a pure shift of the plasma column, assuming high aspect ratio, and is ineffective.)
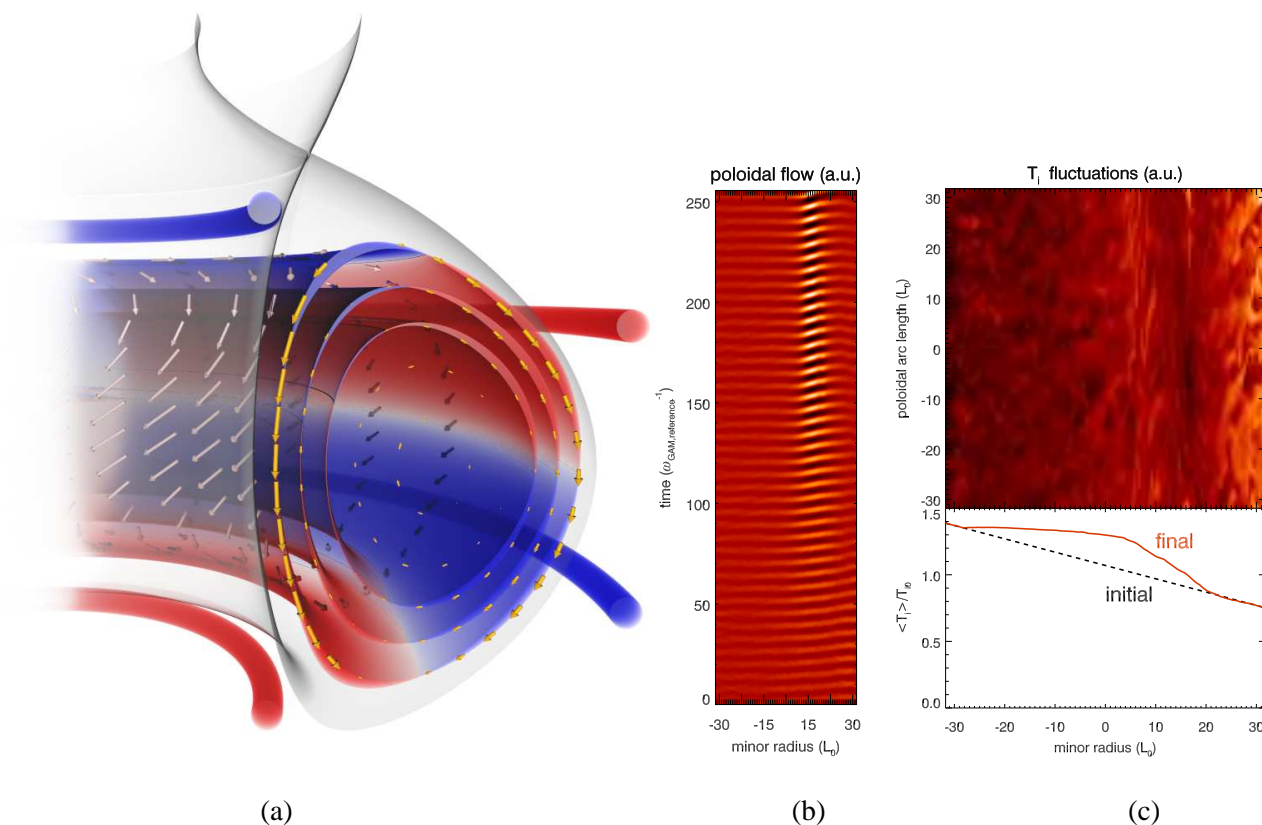
| (a) | (b) | (c) |

Figure 1: (a) Linear displacement and flux surface perturbation visualized for hypothetical quadrupole perturbation current $I = 90kA$ for DIII-D plasma configuration from discharge #119527, including the effect of screening currents in the limiter structures (not shown). Gray: unperturbed separatrix. Red/blue toroidal loops: position of positive/negative perturbation current relative to the plasma current. Red/blue shells: volume traversed by positive/negative radial displacement of flux surfaces. Yellow arrows in cross section: poloidal displacement component. White arrows: total displacement tangential to flux surfaces. GAM excitation in a turbulent nonlocal resistive ballooning scenario in a thin shell at the edge of a circular tokamak discharge; (b) flux surface averaged poloidal flow velocity with resonance, (c) shearing of turbulence fluctuations, top: cut from outboard midplane, bottom: steepening of ion temperature profile.

Dynamic equilibrium perturbation

The action of external coil currents on the interior flux surfaces of the plasma has been studied by means of a novel dynamic equilibrium code as well as analytically, deriving a surprisingly simple transfer function. Discounting very low aspect ratios R/a, the radial and poloidal displacement amplitude of the quasistatic equilibrium perturbation, augmented by the resonance quality factor, can serve as an estimate for the GAM amplitude. As shown exemplary for the code output for a DIII-D configuration in fig. 1a, the flux surfaces are indeed repulsed/attracted by negative/positive currents.

At this point the potentially greatly facilitating effects of the presence of a depression or a null (X-point, saddle point of ψ) in the magnetic field become apparent. First, the radial displacement amplitude increases inversely proportional to the poloidal field $B_{pol} = |\nabla \psi|/R$, as visible close to the "X" on top in the displacement field shown in fig. 1a, since a given flux perturbation ($\delta \psi$) causes a larger shift at a smaller background flux gradient.

Furthermore, the poloidal displacement resulting from the incompressibility of the toroidal magnetic field is amplified by the nozzle effect of close flux surface spacing at the midplane. Specifically, for the outermost shown flux surface displacement in the figure $\delta B_{pol} \sim 25mT$, $|\xi_{rad,max}| \sim 4.8cm\ |\xi_{pol}| \sim 13cm$, and the nozzle effect amounts to $\max(|\nabla \psi|)/\min(|\nabla \psi|) \sim 2.9$ – the displacement diverges at the separatrix.

In addition it becomes clear that the plasma is not screening (as one might expect) but rather amplifying the perturbation current. This is due to the attraction/repulsion of the toroidal plasma currents by the external current in the same/opposite direction.

Interaction with turbulence

To describe the induced resonance layer within a turbulence simulation it is essential to retain the radial variation of the GAM frequency throughout the computational domain, i.e. use a "nonlocal" code and not rely on the flux tube/local approximation.

Fig. 1b,c show the possible characteristic features in an electrostatic turbulence run with the NLET code [3] for the following reference parameters at the middle of domain ($r = 0$): $\alpha = 0.1, \varepsilon_n = 0.05, \eta_i = 1, \tau = 1, q = 3, s = 1$. The externally induced poloidal displacement was $L_0/2$, (L_0 is the ballooning scale length; typical ballooning scale lengths in a tokamak edge are $\sim 0.3cm$). The resonance layer is obvious in the time series of the flow profiles (fig. 1b) from the amplitude peaking and the characteristic radial phase variation due to the radial variation of the local oscillator frequency. The quality factor at the resonance is found to be $Q \sim 25$, i.e., the externally induced displacement is amplified by a factor Q at the resonance. The peaking and phase variation lead to a strong flow shear at the resonance, and a modulation and partial suppression of the turbulence by the GAMs. For high enough external forcing, this leads to something akin to a transport barrier (fig. 1c).

Discussion

According to the definition of the resonance quality factor Q the power lost by GAM damping is $P_{GAM} = \omega_{GAM}E_{GAM}/Q$. The energy stored in the GAM oscillation is $E_{GAM} = m\omega_{GAM}^2 d^2/2$, where $d \sim \xi_{pol}Q$ is the resonance-amplified displacement amplitude, $m = 4\pi^2 aRwnm_i$ the oscillating plasma mass, w the width of the GAM resonance layer, n the ion density and m_i the

ion mass. For circular slender plasma, the GAM frequency is $\omega_{GAM} = \sqrt{2\gamma}c_s/R$, $c_s = \sqrt{2T/m_i}$. Inserting typical tokamak edge values $T \sim 200\text{eV}$ $n \sim 10^{19}\text{m}^{-3}$, $R \sim 1.5\text{m}$, $a \sim 0.5\text{m}$, and assuming the width and displacement of the GAMs to be relevant to the turbulence, $w, d \sim 2\text{cm}$, and deuterium as medium results in $\omega_{GAM} \sim 175\text{krad/s}$, $E_{GAM} = md^2\omega_{GAM}^2/2 \sim 0.09\text{J}$ and $P_{GAM} = E_{GAM}\omega_{GAM}/Q \sim 0.7\text{kW}$ for $Q \sim 20$. For realistic tokamak experiments the power requirements are about an order of magnitude smaller, since the observed ω_{GAM} is reduced compared to the value for a circular plasma by about a factor of two [4, 5].

To achieve GAM amplitudes sufficient to significantly affect the turbulent diffusivity certainly requires a dedicated setup, which to our knowledge presently is nowhere installed. However for a first demonstration or even diagnostic purposes (to measure plasma acoustic resonance spectra) in many machines existing positioning coils or perturbation coils for edge instability control (e.g. resistive wall, edge localized or quasi coherent modes) could be used, provided they can be driven at the GAM frequency. For example in DIII-D the ELM suppression coils (I-coils) can produce a perturbation field of 0.02mT at 7kHz yielding a displacement of $\sim 6\text{mm}$ under good conditions with the above estimates, while a displacement amplitude of $\sim 2\text{mm}$ magnitude is routinely detected by spectroscopic imaging and Doppler measurements [6, 7]. The parameters could even be chosen more favorable to the GAM excitation, e.g. by lowering the equilibrium magnetic field and the GAM frequency. Moreover, even while not sufficient to suppress the turbulence completely, raising the GAM amplitude somewhat may lower the LH transition threshold [4].

The optimal conditions for GAM generation can be found by computing the induced displacement for varying plasma shapes, coil and passive conductor positions. In addition, the perturbation itself may also be maximized. For example elongating the plasma column increases the sensitivity to changes of the vertical magnetic forces, up to the well known vertical displacement instability. Higher order shaping, such as indentation, can convert the thus amplified vertical shift into the desired poloidal displacement.

References

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