Avalanche Dynamics of Collapse and Non-local Model of Transport

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I. Introduction

The phenomena of the collapses and the transient responses in toroidal plasmas are the challenging issues in the physics of high temperature plasmas. In this article we study the nature of the energy transport of plasma from two points of view. At first we study the heat transport with the hysteresis characteristics of the heat conductivity and obtain the avalanche at the energy transport. Secondly we consider the non-local effect in the heat flux and obtain the hysteresis in gradient-flux relation.

II. Avalanche Dynamics

In magnetically confined plasmas, the rapid crash of the temperature profile has been observed in many experiments. The examples are the giant-ELM, the sawtooth crash, the internal collapse, and so on. Recently the model of the transport bifurcation between the H-mode and magnetic braiding mode (M-mode) was proposed to explain the giant-ELM[1].

We analyze the 1-dimensional dynamics of the heat transport of the edge plasma inside the separatrix with the hysteresis characteristics of the heat conductivity to study the energy burst of the giant-ELM. We introduce the hysteresis characteristics of the heat conductivities to the energy transport equations and obtain the avalanche at the energy flux and the rapid collapse of the temperature profile.

II-1. Model of Avalanche

We investigate the energy burst of the giant-ELM as the bifurcation of the transport coefficients with the hysteresis characteristics, which are mentioned as follows.

It is predicted that the heat conductivities bifurcate between the H-mode and the M-mode[1]. We assume, for the simplicity, that the heat conductivity at the H-mode, χ^H, is constant and independent of other parameters. The electron and ion heat conductivities in the M-mode are obtained in [1] as, $\chi_i^M = \sqrt{m_i / m_e} M \chi^H$ and $\chi_e^M = \sqrt{m_i / m_e} M \chi_i^M$, where M is constant parameter and j denote the particle species ($j = i,e$). The threshold value of the bifurcation are defined that when the pressure gradient α exceeds α_c, the transition from the H-mode to the M-mode occurs, and when the value of α reduces to α_1, the back-transition from the M-mode to the H-mode occurs, where α is defined as $\alpha \equiv -q^2 R d\beta/dr$ and q, R and β are the safety factor, the major radius and the beta value, respectively. The threshold values, α_c and α_1, are the functions of magnetic well, R/a and s, where s is the shear parameter. Here, we assume that the threshold values, α_c and α_1, are given to be constant.

The hysteresis characteristics give rise to the collapse of the temperature profile accompanied with the avalanche process. The mechanism of the avalanche is as follows. When the pressure gradient, α, at any point exceeds the threshold value, α_c, the transition from the H-mode to the M-mode occurs and the heat flux in the M-mode region is enhanced due to the large heat conductivities, therefore α decreases. At the region adjacent to the M-mode region, α is forced to increase düe to the enhanced heat flux from the M-mode region and if α exceeds the threshold value, α_c, a successive transition from the H-mode to the M-mode occurs. Therefore, this crash of the temperature profile propagates and the M-mode region spreads rapidly from the first transition point. It is the avalanche. α at the region which is subject to the M-mode is decreased due to the rapid outflow of enhanced heat flux. When the value of α is reduced to α_1, the back-transition from the M-mode to the H-mode occurs and the value of α starts to increase again.

II-2. Calculation of the 1-D Transport code

We include the hysteresis model of the heat conductivities among the energy transport equation. We solve the development of the profiles of plasma temperatures, T_e and T_i. The density profile and the magnetic field are fixed in time. The energy transport equation is ,

$$\frac{3}{2}\frac{\partial}{\partial t}\left(n(r)\,T_j(r,t)\right)=\frac{\partial}{\partial r}\left\{\chi_j\frac{\partial}{\partial r}\left(n(r)\,T_j(r,t)\right)\right\} \quad , \tag{1}$$

where n, T_j and χ_j are the density, the temperature and the heat conductivities, respectively, r is taken along the minor radius. χ_j obeys the hysteresis characteristics. The slab region near the plasma edge is the region of interest, $-L<r<0$. The boundary conditions are as follows. At the edge ($r=0$) we impose the constraint of the constant temperatures of the ion and the electron. The heat flux from the core plasma ($r=-L$) is constant as $Q=Q_{in}$. The background heat conductivity is assumed as $\chi^L=1.0\,m/s^2$. We choose the standard tokamak parameters as follows, the major and the minor radii are $R=1.3$ m, $a=0.35$ m, the toroidal magnetic field is $B_T=1.3$ T. We assume that $L=0.1$ m, the density is the linear function of r and $n(-L)=2.0\times10^{19}$ and $n(0)=5.0\times10^{18}$, the safety factor is also the linear function of r and $q(-L)=1.2$ and $q(0)=2$, the temperatures at the edge are $T_{e\,s}$=$T_{i\,s}$=$50eV$. The input heat flux Q_{in} is chosen so that Q_{in} preserves the temperature as $T_{e\,0}$=$0.34keV$, $T_{i\,0}$=$0.30keV$ at r=$-L$ under the background heat conductivity, that is, Q_{in}=$1.97\times10^4Jm^{-2}s^{-1}$. The H-mode is imposed at the region from $r_1=-0.03m$ to $r_0=0m$. And χ^H is assumed as the parabolic function of r, whose minimum value is $\chi^H_{min}=0.1m^2/s$ at $r=r_1/2$. The threshold values is assumed as $\alpha_c=1.2$ and $\alpha_1=0.15$ and the numerical factor in the M-mode heat conductivities, M, is assumed as $M=0.5$. Each variable is normalized by a, $T_{e\,0}$ and τ_{Ap}, where τ_{Ap} is the poloidal Alfven time ($\tau_{Ap}=a\sqrt{\mu_0 n_i m_i}\,/\,B_p$).

At the H-mode region, if α exceeds the threshold value α_c, the transition from the

H-mode to the M-mode occurs and the avalanche process begins. The temperature is decreased by the enhanced heat transport. In Fig. 1, the time slice of the profile of the electron temperature, $\hat{T}_e$, is drawn, where the hat denotes the normalized value. The curve labeled (a) shows the profile of $\hat{T}_e$ just before the start of the avalanche process and the curves labeled (b) and (c) give the profile of $\hat{T}_e$ after $\hat{t} = 0.84$ and $\hat{t} = 5.87$ from the start of avalanche. We see from Fig. 1 of the curves (a) and (b), that the pivot point of the profile of $\hat{T}_e$ appears at the every early stage of avalanche, and that $\hat{T}_e$ is decreased inside of this point and $\hat{T}_e$ is increased outside by the enhanced heat flux from the inner region. By the passing of time, when the energy is exhausted enough by the enhanced transport, then the pivot point disappears, that is shown (a) and (c). After the back transition completes, the profile (a) is again realized.

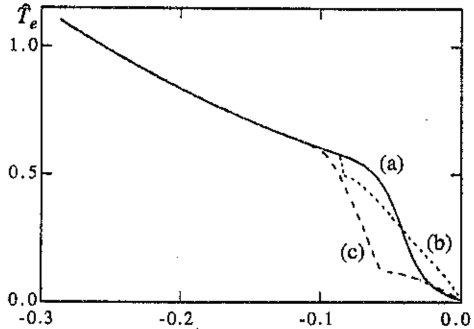

Fig. 1 The time slise of the electron temperature. (a) $\hat{t} = 0$, (b) $\hat{t} = 0.84$, (c) $\hat{t} = 5.87$.

III. Non-local Model Analysis of Heat Pulse Propagation

Recently, the experimental data for the transient transport of the heat pulse propagation has been accumulated. There reported an interesting observation that the heat flux changes faster than the change of the temperature profile in the experiment of the on-off of the central heating. The rapid change of the plasma state and its hysteresis nature were successfully modeled by a transport coefficient depending directly on the global heating power [2]. The purpose of this study is to propose another way, i.e., to consider the non-local effect in the heat flux and to investigate its dependence.

III-1 Non-local Model of The Heat Flux

To take the non-local effect in the heat flux into account, we propose the following model,

$$q(r) = -\int_0^a n_e \chi_e(T(r'), \nabla T(r')) K_f(r - r') \nabla T(r') dr' \ , \tag{2}$$

where q is the electron heat flux, n_e is the electron density, χ_e is the electron diffusivity, $K_f(r - r') = r / r' \left[C_{local} \delta(r - r') + C_{global} 1 / (\sqrt{\pi} l) \cdot \exp\{-(r - r')^2 / l^2\} \right]$ is the kernel which includes the non-local effects and T is the electron temperature. The weighting function r / r' is introduced to impose the boundary condition $q(0) = 0$. C_{local} and C_{global} are the numerical constants which represent the local effect and the non-local effect, and l is the half width of the non-local effect. The kernel becomes the delta function when $l \to 0$. Then the heat flux reduces to that in the local transport model. By changing these parameters, we solve the energy transport equation numerically. We firstly assume $\chi_e = const$ to elucidate the non-local effect and the deposition profile is modeled as $P = P_0 \exp\{-100(r/a)^2\}$.

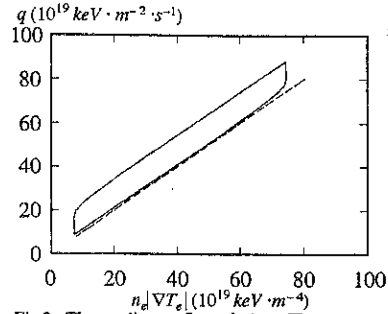

Fig.2 The gradient – flux relation. The parameters are $r/a = 0.5$, $l/a = 0.5$, $t_c = 0$, $C_{local} = 0.9$, $C_{global} = 0.1$. When $l/a = 0$, the hysteresis disappears (dashed line).

III-2. Simulation of Non-local Model

We use the following plasma parameters. Major and minor radii are $R = 2.0\ m$ and $a = 0.2\ m$. The constant electron density and diffusivity are $n_e = 4.0 \times 10^{19}\ m^{-3}$ and $\chi_e = 1.0\ m^2/\mathrm{sec}$ and $\tau_E \sim a^2/\chi_e = 0.04$ sec. The non-local effect little affects the stationary profile, $T_e(r)$, for the wide range of parameter l. The hysteresis is obtained in gradient-flux relation in the transient response, which is shown in Fig.2. A single linear line is obtained in the stationary case. When the central heating power changes from 0.1 to 1.0 MW (step function), the temperature gradient increases slowly but the heat flux increases rapidly because of the non-local effect which is included in our model. When the central heating power changes from 1.0 to 0.1 MW, the temperature gradient decreases slowly but the heat flux decreases rapidly. So the hysteresis is clockwise away from the region of heat deposition. On the other hand, the hysteresis is counter-clockwise at the region heated directly ($r/a \le 0.1$). The width of the hysteresis increases with l and has the peak where the value of r/a is nearly equal to l/a which is the case shown in Fig.2. If $l \to 0$, the width goes to zero because the heat flux is reduced to the local transport model. The width also depends on the time t_c, which is the switching time of the heating power. When $t_c = 0$ (the power is changed as the step function), the width takes the maximum value, and for the t_c larger than the energy confinement time, the width behaves as $1/t_c$. The hysteresis is prominent when $t_c \ll \tau_E$.

To investigate the model of diffusivity dependent on temperature or temperature gradient is left for our future work.

IV. Summary

In this article we study the characteristics of the heat transport including two model, i.e., the model of the hysteresis characteristics of the heat conductivities and that of the non-local effect in the heat flux. And we obtain the avalanche at the energy transport and the hysteresis in gradient-flux relation, respectively

References

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