The influence of Debye plasma on the ground state energies of exotic systems

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Abstract

The effect of plasma environment on the ground state energies of exotic systems $pp\mu$, $dd\mu$ and $tt\mu$ has been analyzed within a generalized three-body formalism using multi-term correlated basis sets. The Debye screening model of the plasma has been adopted for such a study. The binding energies of p with $p\mu$, d with $d\mu$ and t with $t\mu$ have been estimated for a range of values of the Debye screening parameters. The systems tend towards instability for increased screening. The effect of particle correlation has been investigated in detail and is found to play important role for the stability in these systems.

Key words: exotic systems, Debye plasma, three-body system, quantum confinement
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1 Introduction

Study of the effect of external environment like that of a plasma on the energy levels and other structural properties of atomic, ionic and exotic systems has become a subject of extensive investigations in recent years [1–12]. A broad

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discussion on the subject matter was given by Griem [13], Fujimoto [14], Ichimaru [15] and Rajagopal [16]. Depending on the density and temperature of the external plasma, one can adopt various models of plasma-atom interaction whose effect is to alter the potential energy compared to that of the free atom. For low density and high temperature one usually considers the Debye screening model [17] while for high density and low temperature the ion sphere model [18] is usually adopted.

In this work we are interested in studying the ground state energy of the Coulombic three-body muonic molecular ions $pp\mu$, $dd\mu$ and $tt\mu$ under an external plasma environment. These systems are particularly important in the muon catalyzed fusion processes [19–21]. Muonic molecular ions with protonic substitutions are less adiabatic than the corresponding hydrogen molecular ions due to the mass difference between muon and proton. Such systems are of general interest theoretically [22, 23]. During the passage of particles through matter most of these exotic hadronic systems are formed, albeit, their low mean lives [20, 24, 25]. One can consider the background to mimic a plasma. Hence one can apply a plasma model for estimating the properties of the exotic systems in such an environment, particularly for a fusion plasma. Although a number of highly accurate theoretical estimates are available for the bound state properties of such free exotic three-body systems [26–29], calculations predicting the behavior in presence of plasma are still scanty [30–33].

In the current communication we use the methodology adopted earlier [32] for the estimation of the energy of the exotic systems $pp\mu$, $dd\mu$ and $tt\mu$ in their spherically symmetric ground states in presence of plasma. Coupling to the plasma is included by the Debye screening model [17] in which the potential of the interaction between the charged particles is represented by screened Coulomb potential. We use the Ritz variational method in which the trial wave function is a linear combination of product basis functions. The particle correlation is taken care of by introducing explicitly the interparticle coordinate into the basis functions. The behavior of the energy of the system is analyzed with respect to the Debye screening constant. We aim at predicting an overall but unambiguous behavior of the ground state energies by using reasonably good basis sets such that the computation time is reasonable.

2 Method

The system consisting of a muon and two heavier particles is displayed in Fig. 1. The particles are placed along the three corners of a triangle with the muon at the origin. Their masses are $m_3 = m_\mu = 206.7682657 m_e$ and $m_1 = m_2 = \{m_p, m_d \text{ or } m_t\}$ depending on the system under study with $m_p = 1836.1526675 m_e$, $m_d = 3670.4829550 m_e$ and $m_t = 5496.92158 m_e$. We consider nonrelativistic Hamiltonian of such a system embedded in a Debye plasma. Thus the Coulomb interaction between particles is screened by plasma and reads

$$V = -\frac{e^{-\lambda r_1}}{r_1} - \frac{e^{-\lambda r_2}}{r_2} + \frac{e^{-\lambda r_{12}}}{r_{12}}. \tag{1}$$

The Debye screening constant is represented by [17]

$$\lambda = \left[\frac{4\pi(1+Z)n}{\kappa T}\right]^{\frac{1}{2}} , \tag{2}$$

where n is the density number, T is the temperature of the plasma and Z is the nuclear charge (which is unity in the present case). The screening parameter can be adjusted by using suitable values of plasma density and temperature.

For the spherically symmetric ground state of a three-body system, momentum conservation leads to a Hamiltonian expressible in terms of relative coordinates r_1, r_2 and r_{12} and the expectation value of the Hamiltonian with respect to a real and normalized wavefunction Ψ can be represented as

$$\begin{aligned}
\langle\Psi|H|\Psi\rangle = \int \Bigg\{ & \frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_3}\right)\left(\frac{\partial\Psi}{\partial r_1}\right)^2 + \frac{1}{2}\left(\frac{1}{m_2}+\frac{1}{m_3}\right)\left(\frac{\partial\Psi}{\partial r_2}\right)^2 \\
& + \frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\left(\frac{\partial\Psi}{\partial r_{12}}\right)^2 + \frac{1}{m_3}\left(\frac{r_1^2+r_2^2-r_{12}^2}{2r_1r_2}\right)\left(\frac{\partial\Psi}{\partial r_1}\right)\left(\frac{\partial\Psi}{\partial r_2}\right) \\
& + \frac{1}{m_2}\left(\frac{r_2^2-r_1^2+r_{12}^2}{2r_2r_{12}}\right)\left(\frac{\partial\Psi}{\partial r_2}\right)\left(\frac{\partial\Psi}{\partial r_{12}}\right) \\
& + \frac{1}{m_1}\left(\frac{r_1^2-r_2^2+r_{12}^2}{2r_1r_{12}}\right)\left(\frac{\partial\Psi}{\partial r_1}\right)\left(\frac{\partial\Psi}{\partial r_{12}}\right) \\
& + \left(-\frac{e^{-\lambda r_1}}{r_1} - \frac{e^{-\lambda r_2}}{r_2} + \frac{e^{-\lambda r_{12}}}{r_{12}}\right)\Psi^2 \Bigg\} \\
& r_1r_2r_{12}dr_1dr_2dr_{12}
\end{aligned} \tag{3}$$

To minimize the expectation value, the wavefunction Ψ is expanded in terms of the interparticle coordinates which takes care of the electron correlation effect explicitly

$$\Psi(r_1, r_2, r_{12}) = \sum_{klj} C_{klj}\chi_{kl}(1,2)\eta_j(1,2) , \tag{4}$$

where

$$\chi_{kl}(1,2) \sim (e^{-\rho_k r_1 - \rho_l r_2} + e^{-\rho_l r_1 - \rho_k r_2})$$

constructed of Slater type orbitals are to take care of the radial correlation whereas

$$\eta_j(1,2) \sim r_{12}^{n_j} e^{-\beta_j r_{12}}$$

describe the angular proton-proton (d-d or t-t) correlations dependent on the distance between them, r_{12}. In actual computations we used eight different exponent parameters ρ which resulted in 36 different χ_{kl} functions. For the angular expansion we used nine different η's leading to altogether $36 \times 9 = 324$ linear variation coefficients for the fully correlated calculations. The linear expansion, Eq. (3), leads to the matrix generalized eigenvalue problem

$$\underline{\underline{\mathbf{H}}}\,\underline{\mathbf{C}} = E\,\underline{\underline{\mathbf{S}}}\,\underline{\mathbf{C}} \;, \tag{5}$$

where $\underline{\underline{\mathbf{H}}}$ and $\underline{\underline{\mathbf{S}}}$ are the Hamiltonian and overlap matrices built in terms of basis functions $\chi_{kl}\eta_j$. In general, the nonlinear parameters ρ and β can be suitably adjusted so as to minimize the ground state eigenvalue from Eq. (4). We have optimized them for the plasma-free case.

3 Results

We have considered here the plasma confined exotic systems $pp\mu$, $dd\mu$ and $tt\mu$ bound by Coulomb interactions. The ground state energies computed as described above are given in Tables 1, 2 and 3, respectively for $pp\mu$, $dd\mu$ and $tt\mu$. The energies of the plasma free systems ($\lambda = 0$) can be compared with very accurate results due to Frolov [27]. Reasonable agreement is observed: seven, six and five figures respectively for $pp\mu$, $dd\mu$ and $tt\mu$.

To decide whether the system is bound or not one needs to compare the three-particle energy to the ground state level of a corresponding two-particle system ($p\mu$, $d\mu$, and $t\mu$ respectively for $pp\mu$, $dd\mu$ and $tt\mu$). We computed the ground state energies of the two-body systems variationally in a basis of eight Slater type orbitals (which is adequate to the basis described above, used for three-body computations). They also are given in Tables 1–3. The results for both two- and three-particle systems containing protons are plotted in Fig. 2. For two other cases (d and t) the picture is very similar. As one can see the ground state energies of both two- and three-particle systems increase as the screening parameter λ is increased, meaning that stability of the systems is weakened by the plasma influence. So that eventually, for large screening, the systems become unbound. By scaling the result of Gomez, Chacham and Mohallem [34] to the actual masses of our systems, we find that the critical values of λ, λ_c, at which the two-body systems become unbound are about 221, 233 and 237, respectively for $p\mu$, $d\mu$ and $t\mu$.

By taking the difference between the two-body and the corresponding three-body energies at each value of λ we find the proton affinity of $p\mu$, deuteron affinity of $d\mu$ and triton affinity of $t\mu$. These hadronic affinities are plotted in Fig. 3, and given in Tables 1–3. They decrease with λ increasing, indicating that the binding of the second proton (deuteron or triton) also weakens with the plasma density increasing. However, this tendency is much weaker than for the first proton (deuteron or triton). The extreme manifestation of such a behavior is that for large values of the screening parameter the proton affinity of the $p\mu$ system is bigger than the binding energy of $p\mu$ itself. The same happens for $d\mu$ and $t\mu$.

Apparently the effect described above is the Thomas collapse [35]: the binding energy per particle is larger for a three-body system than for its two-body counterpart. Such an effect can result in binding of the three-body system even if the two-body subsystems are not bound, i.e. for $\lambda \geq \lambda_c$. Checking whether such a possibility can be physically realized for $pp\mu$, $dd\mu$ and $tt\mu$ systems would be of great interest and importance. However this is not the aim of present investigation. The quality of our trial function is good for $\lambda < \lambda_c$ but not good enough to represent properly the $\lambda \geq \lambda_c$ region.

We would like to understand where such a strong three-body binding comes from in terms of our computation. For this purpose we computed the dipole polarizabilities of the two-particle systems, using the standard linear response theory with a perturbed function chosen as linear combination of eight STO's [12]. The quality of results is again checked against the standard second order perturbation theory results for the plasma-free hydrogen-like systems (see Tables 1–3). As the Debye screening increases the polarizability increases very rapidly, reaching extremely large values at large λ. This indicates that the strong binding of three-particle systems is via polarization effects. This is confirmed by three-particle computations: when computed by including only radial correlations (36-linear-parameter computation; all functions $\eta(r_{12})$ in Eq. 3 set equal to 1) the total three-body energy is very poor (see Tables 1–3 and Fig. 3). For large screening the radial correlation is not enough to bind the system. It is the angular correlation which describes the polarization effects properly so that the system turns out to be bound. The effect of the angular correlation on the ground state energy is presented in Fig. 4. For all the systems under consideration the absolute value of the angular correlation contribution decreases when the screening is increased, Fig. 4a. However, this is just a manifestation of the fact that all the Coulomb interaction effects are weakened by the screening. In fact the role of angular correlation relatively increases with λ, which can be seen from Fig. 4b.

4 Conclusion and synopsis

In summary, we have considered the effect of plasma as envisaged by the Debye model on the energy of exotic hadronic systems $pp\mu$, $dd\mu$ and $tt\mu$. We have explicitly demonstrated the importance of the inclusion of particle correlations in evaluating connected properties. For strong screening the hadronic affinity is larger than the binding energy of the related two-particle system. We have found that the effect is due to an extremely strong polarization of two-particle systems, increasing when the screening is increased. Fact that the binding of three-particle system is stronger the binding of two-particle subsystems is known as the Thomas collapse [35]. In the case of short-range attractive two-body interactions it can lead to the binding of a three-body system even if the two-body subsystems are not bound [35]. We believe this can be the case for the systems considered in this paper and this shall be a subject of our further investigation.

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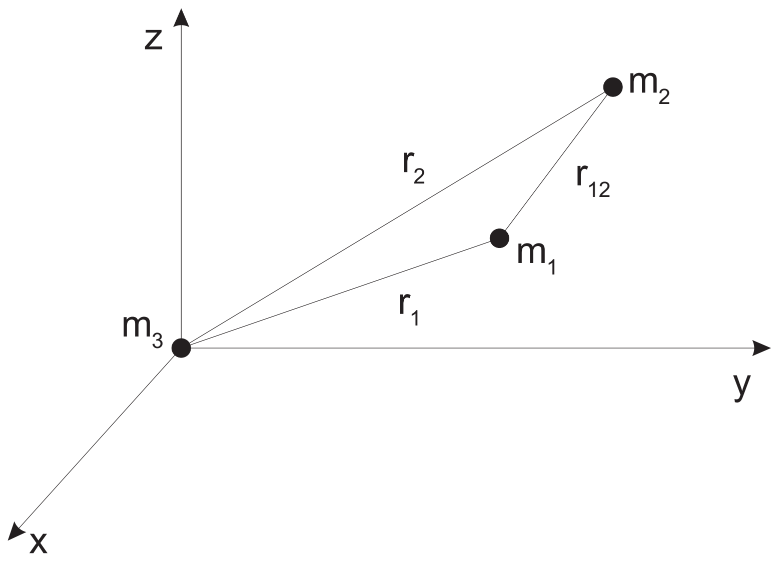

Fig. 1. Coordinates for a three-body system.

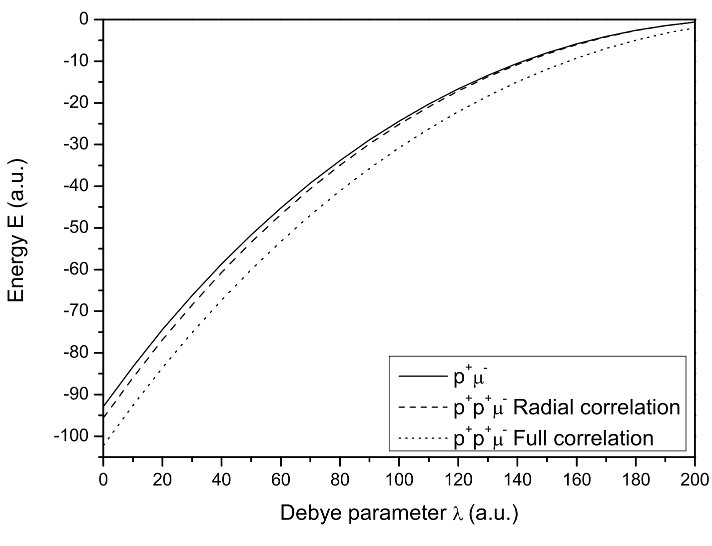

Fig. 2. Energy of $p\mu$ and $pp\mu$ against Debye parameter.

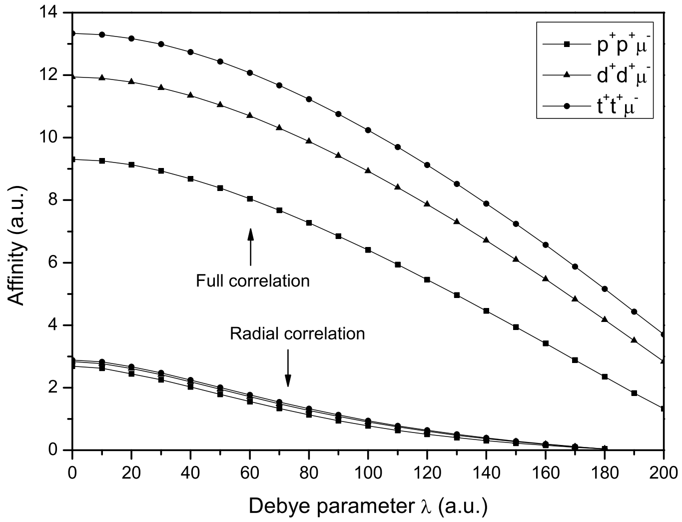

Fig. 3. Hadronic affinity against Debye screening parameter.

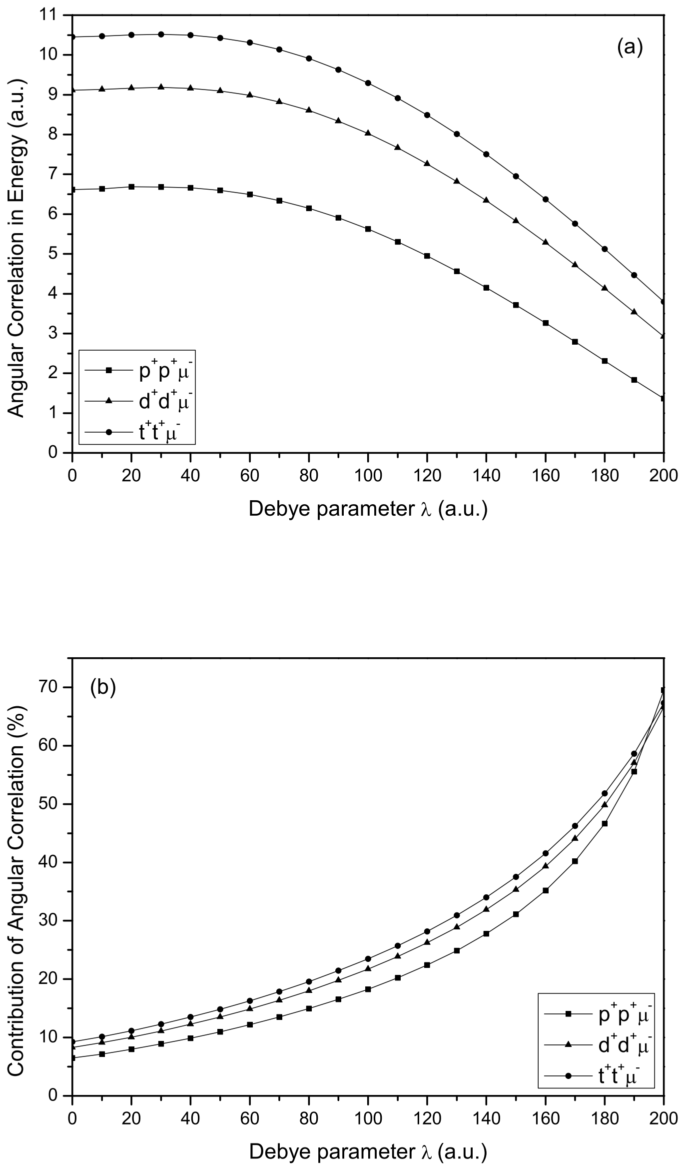

Fig. 4. Contribution of the angular correlation effects to the ground state energy against the Debye screening parameter: a) absolute energy contribution in a.u., b) percentage contribution with respect to the total energy.

Table 1

Structural properties of $pp\mu$ in Debye plasma.

Debye parameter[a] λ (a.u.)	Plasma density[a] n/c.c.	-E($p^{\pm}\mu^{\mp}$) (a.u.)	Dipole polarizability $\alpha * 10^6$ (a.u.)	-E ($p^+p^+\mu^-$) (a.u.) Radially correlated	Fully correlated	Affinity (a.u.)
0.0		92.9204	0.701	95.6048	102.2235	9.3031
		92.9204[b]	0.701[b]			
10.0	9.87(+26)	83.3105	0.710	85.9294	92.5682	9.2577
20.0	3.95(+27)	74.4332	0.737	76.8751	83.5641	9.1309
30.0	8.88(+27)	66.2287	0.780	68.4834	75.1649	8.9362
40.0	1.58(+28)	58.6487	0.841	60.6724	67.3333	8.6846
50.0	2.47(+28)	51.6528	0.924	53.4389	60.0386	8.3858
60.0	3.55(+28)	45.2067	1.034	46.7601	53.2540	8.0473
70.0	4.84(+28)	39.2808	1.180	40.6136	46.9562	7.6754
80.0	6.32(+28)	33.8491	1.376	34.9780	41.1240	7.2749
90.0	7.99(+28)	28.8886	1.642	29.8327	35.7385	6.8499
100.0	9.87(+28)	24.3787	2.012	25.1580	30.7825	6.4038
110.0	1.19(+29)	20.3009	2.541	20.9352	26.2403	5.9394
120.0	1.42(+29)	16.6384	3.326	17.1465	22.0973	5.4589
130.0	1.67(+29)	13.3759	4.540	13.7749	18.3403	4.9644
140.0	1.93(+29)	10.4993	6.523	10.8046	14.9573	4.4579
150.0	2.22(+29)	7.9957	9.996	8.2203	11.9369	3.9412
160.0	2.52(+29)	5.8529	16.630	6.0077	9.2691	3.4162
170.0	2.85(+29)	4.0596	30.600	4.1533	6.9447	2.8851
180.0	3.19(+29)	2.6044	62.370	2.6435	4.9559	2.3515
190.0	3.56(+29)	1.4722	136.100	1.4643	3.2960	1.8238
200.0	3.94(+29)	0.6365	299.200	0.5974	1.9603	1.3238
210.0	4.35(+29)	0.0508	651.900		0.9471	0.8963

[a]We have chosen here a typical case of 1000 eV plasma. The plasma screening parameter λ chosen here gives the value of the possible electron density from Eq. (2).
[b]Standard second order perturbation theory result for the hydrogen-like system.

Table 2

Structural properties of $dd\mu$ in Debye plasma.

Debye parameter[a] λ (a.u.)	Plasma density[a] n/c.c.	-E($d^{\pm}\mu^{\mp}$) (a.u.)	Dipole polarizability $\alpha * 10^6$ (a.u.)	-E ($d^+d^+\mu^-$) (a.u.) Radially correlated	Fully correlated	Affinity (a.u.)
0.0		97.8708	0.600	100.7020	109.8165	11.9457
		97.8708[b]	0.600[b]			
10.0	9.87(+26)	88.2417	0.607	91.0103	100.1456	11.9038
20.0	3.95(+27)	79.3114	0.627	81.9278	91.0954	11.7840
30.0	8.88(+27)	71.0249	0.660	73.4384	82.6204	11.5955
40.0	1.58(+28)	63.3377	0.708	65.5224	74.6842	11.3465
50.0	2.47(+28)	56.2122	0.771	58.1591	67.2567	11.0445
60.0	3.55(+28)	49.6164	0.853	51.3277	60.3122	10.6958
70.0	4.84(+28)	43.5227	0.961	45.0079	53.8281	10.3054
80.0	6.32(+28)	37.9064	1.103	39.1799	47.7844	9.8780
90.0	7.99(+28)	32.7461	1.292	33.8247	42.1632	9.4171
100.0	9.87(+28)	28.0222	1.548	28.9242	36.9481	8.9259
110.0	1.19(+29)	23.7172	1.901	24.4609	32.1243	8.4071
120.0	1.42(+29)	19.8152	2.405	20.4181	27.6781	7.8629
130.0	1.67(+29)	16.3016	3.148	16.7802	23.5971	7.2955
140.0	1.93(+29)	13.1630	4.290	13.5319	19.8695	6.7065
150.0	2.22(+29)	10.3871	6.141	10.6593	16.4848	6.0977
160.0	2.52(+29)	7.9624	9.344	8.1486	13.4331	5.4707
170.0	2.85(+29)	5.8781	15.310	5.9870	10.7054	4.8273
180.0	3.19(+29)	4.1238	27.250	4.1618	8.2936	4.1698
190.0	3.56(+29)	2.6883	52.290	2.6597	6.1904	3.5021
200.0	3.94(+29)	1.5558	104.700	1.4656	4.3895	2.8337
210.0	4.35(+29)	0.6999	209.600	0.5585	2.8861	2.1862
220.0	4.77(+29)	0.0786	415.200		1.6768	1.5982

[a]We have chosen here a typical case of 1000 eV plasma. The plasma screening parameter λ chosen here gives the value of the possible electron density from Eq. (2).
[b]Standard second order perturbation theory result for the hydrogen-like system.

Table 3

Structural properties of $tt\mu$ in Debye plasma.

Debye parameter[a] λ (a.u.)	Plasma density[a] n/c.c.	-E($t^{\pm}\mu^{\mp}$) (a.u.)	Dipole polarizability $\alpha * 10^6$ (a.u.)	-E ($t^+t^+\mu^-$) (a.u.) Radially correlated	Fully correlated	Affinity (a.u.)
0.0		99.6363	0.569	102.5203	112.9718	13.3355
		99.6363[b]	0.569[b]			
10.0	9.87(+26)	90.0009	0.575	92.8231	103.2953	13.2944
20.0	3.95(+27)	81.0528	0.594	83.7245	94.2289	13.1761
30.0	8.88(+27)	72.7389	0.625	75.2091	85.7272	12.9883
40.0	1.58(+28)	65.0157	0.667	67.2580	77.7540	12.7383
50.0	2.47(+28)	57.8467	0.728	59.8511	70.2793	12.4326
60.0	3.55(+28)	51.2006	0.780	52.9684	63.2776	12.0770
70.0	4.84(+28)	45.0504	0.897	46.5902	56.7266	11.6762
80.0	6.32(+28)	39.3721	1.024	40.6972	50.6066	11.2345
90.0	7.99(+28)	34.1444	1.192	35.2710	44.8999	10.7555
100.0	9.87(+28)	29.3484	1.418	30.2939	39.5905	10.2421
110.0	1.19(+29)	24.9668	1.727	25.7489	34.6639	9.6971
120.0	1.42(+29)	20.9839	2.161	21.6198	30.1065	9.1226
130.0	1.67(+29)	17.3855	2.792	17.8911	25.9061	8.5206
140.0	1.93(+29)	14.1583	3.744	14.5483	22.0512	7.8928
150.0	2.22(+29)	11.2904	5.251	11.5773	18.5315	7.2410
160.0	2.52(+29)	8.7702	7.766	8.9649	15.3370	6.5668
170.0	2.85(+29)	6.5870	12.210	6.6984	12.4590	5.8720
180.0	3.19(+29)	4.7303	20.460	4.7653	9.8892	5.1589
190.0	3.56(+29)	3.1881	36.210	3.1527	7.6201	4.4320
200.0	3.94(+29)	1.9442	66.080	1.8459	5.6452	3.7010
210.0	4.35(+29)	0.9737	120.600	0.8258	3.9589	2.9852
220.0	4.77(+29)	0.2401	217.400	0.0638	2.5569	2.3168

[a]We have chosen here a typical case of 1000 eV plasma. The plasma screening parameter λ chosen here gives the value of the possible electron density from Eq. (2).
[b]Standard second order perturbation theory result for the hydrogen-like system.