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Polarized Radiative Transfer of Electron Cyclotron Wave in LHD

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1. Introduction

The polarization transfer of the propagating electromagnetic wave through the plasma is mainly treated with two methods. The first method is based on the coupled differential equations of the electromagnetic wave. The coupled equations for the plasma with the sheared magnetic field were described for the normal incidence to the magnetic field to analyze the issues of Electron Cyclotron Heating (ECH)[1]. Coupled equations were also used to investigate hot plasma effects on the polarization state for the arbitrary incident angle, for shearless cases[2]. At these treatments, two eigenmodes of the shearless magnetized plasma were used. The two eigenmodes in the uniformly sheared magnetic field, called helical modes, were obtained for the strong shear case[3]. The equations using the helical modes were derived for the arbitrary incident angle[4]. For polarimetry measurement, another differential equation was given for the reduced Stokes parameters. In this equation, the polarization states or the Stokes parameters are described by the rotation on the Poincaré sphere due to the effect of the birefringence. This method with the reduced Stokes parameters was reviewed by Segre[5]. Recently, the absorption of the propagating wave and the effect of the dichroism are taken into account for the evolution of the reduced Stokes parameters[6]. The full Stokes parameters are used to describe the polarization state. Using the evolution equations of the full Stokes parameters, a single path absorption rate in second harmonic electron cyclotron heating(ECH) for the perpendicular injection is investigated on the Large Helical Device(LHD).

2. Evolution equations of full Stokes parameters

In general, there are no orthogonal eigenpolarizations when the absorption or the dichroism and the birefringence are taken into account. In the cases of the circular and linear dichroism, there are two orthogonal eigenpolarizations. The Mueller matrix that includes the effects of both the dichroism and the birefringence is dealt with in these cases. In a magnetized plasma, there is a longitudinal component of the electric field in general. This component is negligible in the lowest order of $(\omega_{\rm p}^2\omega_{\rm c}/\omega^3)$, compared to the transverse components[4]. Here, $\omega_{\rm p}$, $\omega_{\rm c}$ and ω are the plasma frequency, the electron cyclotron frequency and the wave frequency, respectively. The wave electric field is elliptically polarized in the normal plane, that is expressed with the coordinates of (x, y), to the propagating z direction. The two orthogonal eigenpolarizations have the azimuth angle of the ellipse and the ellipticity of (θ, ϵ) and $(\theta + \pi/2, -\epsilon)$, respectively. The transverse fields of the two eigenmodes with the initial phase of $\delta_0 = 0$ and the normalized intensity initially are described as

$$\begin{pmatrix} E_{x\rm f} \\ E_{y\rm f} \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\epsilon - {\rm i}\sin\theta\sin\epsilon \\ \sin\theta\cos\epsilon + {\rm i}\cos\theta\sin\epsilon \end{pmatrix}, \begin{pmatrix} E_{x\rm s} \\ E_{y\rm s} \end{pmatrix} = \begin{pmatrix} -n^* \\ m^* \end{pmatrix}, mm^* + nn^* = 1,$$

where the subscripts f, s indicate the indexes of fast and slow characteristic eigenmodes and the asterisk mark (*) means to take the complex conjugate. The form of the Jones matrix can be written as

$$\begin{pmatrix} d_{\rm f}mm^* + d_{\rm s}n^*n & (d_{\rm f} - d_{\rm s})mn^* \\ (d_{\rm f} - d_{\rm s})nm^* & d_{\rm s}mm^* + d_{\rm f}n^*n \end{pmatrix}$$

where $d_{\rm f}$ and $d_{\rm s}$ are the eigenvalues corresponding to the fast and slow eigenmodes, respectively. When the complex refractive indexes are described as $(n_{\rm f,s} - {\rm i}k_{\rm f,s})$, the retardance δ between the fast and slow eigenmodes and each transmittance $p_{\rm f,s}$ are written as $\delta = (n_{\rm s} - n_{\rm f})(\omega/c)z$ and $p_{\rm f,s} = \exp(-k_{\rm f,s}(\omega/c)z)$. Here, c is a speed of light, and z is the propagating path length. The two eigenvalues $d_{\rm f,s}$ in the matrix are taken as $d_{\rm f,s} = p_{\rm f,s}\exp(\pm{\rm i}\delta/2)$, where the signs of plus and minus correspond to the fast and slow eigenmodes, respectively.

The Mueller matrix can be derived from the Jones matrix using the coherency vector[7]. In the homogeneous plasma, the full Stokes parameters $\mathbf{S}(z)$ can be written, using the Mueller matrix $\mathbf{M}(z)$ and the initial Stokes parameters $\mathbf{S}_{\rm init}$, as $\mathbf{S}(z) = \mathbf{M}(z) \cdot \mathbf{S}_{\rm init}$. Only the linear dependencies of the retardance and the transmittance on the path length are taken into account here. The first order differential equation for the full Stokes parameters can be described as

$$\frac{{\rm d}\mathbf{S}(z)}{{\rm d}z} = \mathbf{T}(z) \cdot \mathbf{S}(z),\ \mathbf{T} = \frac{{\rm d}\mathbf{M}(z)}{{\rm d}z} \cdot \mathbf{M}^{-1}(z).$$

where the matrix $\mathbf{M}^{-1}(z)$ is the inverse matrix of $\mathbf{M}(z)$. Since the derivation of the matrix $\mathbf{T}$ is straightforward, but tedious, only the result obtained is shown as follows:

$$\mathbf{T} = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{pmatrix}, t_{11} = t_{22} = t_{33} = t_{44} = -\frac{\omega}{c}(k_{\rm s} + k_{\rm f}),$$

$$
\begin{aligned}
t_{12} &= t_{21} = \frac{\omega}{c}(k_{\rm s} - k_{\rm f}) \cos 2\theta \cos 2\epsilon, t_{13} = t_{31} = \frac{\omega}{c}(k_{\rm s} - k_{\rm f}) \sin 2\theta \cos 2\epsilon, \\
t_{14} &= t_{41} = \frac{\omega}{c}(k_{\rm s} - k_{\rm f}) \sin 2\epsilon, t_{23} = -t_{32} = \frac{\omega}{c}(n_{\rm s} - n_{\rm f}) \sin 2\epsilon, \\
t_{24} &= -t_{42} = -\frac{\omega}{c}(n_{\rm s} - n_{\rm f}) \sin 2\theta \cos 2\epsilon, t_{34} = -t_{43} = -\frac{\omega}{c}(n_{\rm s} - n_{\rm f}) \cos 2\theta \cos 2\epsilon.
\end{aligned}
$$

Each component of the full Stokes parameters is expressed as S_i ($i = 0, 1, 2, 3$), and a three-dimensional vector $\mathbf{s} = (S_1, S_2, S_3)$ is defined here. The unit vectors of the fast and slow characteristic eigenmodes can be written in terms of θ and ϵ, as $\mathbf{s}_{\rm f} = -\mathbf{s}_{\rm s} = (\cos 2\theta \cos 2\epsilon, \sin 2\theta \cos 2\epsilon, \sin 2\epsilon)$ here. Using these definitions, the differential equations can be expressed in the form of vector notation,

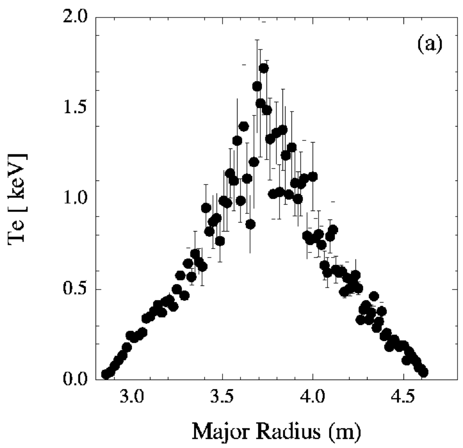
$$
\begin{aligned}
\frac{{\rm d}S_0}{{\rm d}z} &= -2\frac{\omega}{c}k_{\rm s}\frac{S_0 + \mathbf{s}_{\rm s}\cdot\mathbf{s}}{2} - 2\frac{\omega}{c}k_{\rm f}\frac{S_0 + \mathbf{s}_{\rm f}\cdot\mathbf{s}}{2}, \\
\frac{{\rm d}\mathbf{s}}{{\rm d}z} &= -\frac{\omega}{c}k_{\rm s}(S_0 + \mathbf{s}\cdot\mathbf{s}_{\rm s})\mathbf{s}_{\rm s} - \frac{\omega}{c}k_{\rm f}(S_0 + \mathbf{s}\cdot\mathbf{s}_{\rm f})\mathbf{s}_{\rm f} \\
&+ \frac{\omega}{c}(k_{\rm s} + k_{\rm f})\mathbf{s}_{\rm f}\times(\mathbf{s}_{\rm f}\times\mathbf{s}) - \frac{\omega}{c}(n_{\rm s} - n_{\rm f})\mathbf{s}_{\rm f}\times\mathbf{s}.
\end{aligned}
$$

where the scalar and vector products are taken for the components of $i = 1, 2$ and 3. When the effect of wave absorption or the dichroism is not taken into account with $k_{\rm f,s}$=0, the differential equations can be reduced to that of the reduced Stokes parameters obtained previously[5]. The terms $(S_0 + \mathbf{s}_{\rm s,f}\cdot\mathbf{s})/2$ indicate the mode components of the slow and fast eigenmodes that are contained in the propagating wave. The equation on S_0 is consistent with the general description, that the absorption coefficient can be expressed as twice the imaginary part of the propagating wave vector. In the equation of the vector $\mathbf{s}$, the effects of the intensity reduction and the rotation on the Poincaré sphere due to the dichroism are described in addition to the rotation by the birefringence.

3. Application to Electron Cyclotron Heating Study on LHD

For the perpendicular injection, the eigenvalues and polarization under the uniform shear should be defined, including the shear terms in the orders of $[(c/\omega)\alpha]^2$ and $[(c/\omega)\alpha]$ respectively, where α is a shear of the magnetic field[4]. In our case, a pure perpendicular injection is not available, and the effect of the magnetic field component along the wave vector was discussed[8]. Here, the equations are used as a first step to investigate a single path absorption rate in ECH for the perpendicular injection, without the shear term. The complex refractive indexes are written by the function F_q introduced by Dnestrovskii with the lowest order of thermal corrections[9].

Figure 1(a) shows the electron temperature profile for the ECH target plasma to investigate the dependence of the single path absorption on the polarization state. The target plasma is produced with the 82.6GHz gyrotrons. The electron density is about $1.0 \times 10^{19}{\rm m}^{-3}$, and the magnetic field is set to 1.5T. This target plasma is additionally heated with the 84GHz gyrotron at the 2nd harmonic on-axis heating. The change of the diamagnetic signal when the 84GHz gyrotron is turned on is used to evaluate the absorption power. Figure 1(b) shows the absorption power and the calculated single path absorption rate when the $\lambda/4$ polarizer is rotated. The absorption rate is evaluated as the term of $[1 - S_0]$ after the single path. Since the $\lambda/4$ and $\lambda/8$ plates are set at the miterbends, the incident angles of the beam are 45 degree. When the $\lambda/4$ polarizer is rotated, not only the azimuth angle θ but also the ellipticity ϵ are changed. This effect on the ellipticity ϵ is included in the calculation. The absorption power deduced from the diamagnetic signal is not a direct measure of the single path absorption rate, because the wave may be absorbed after multiple reflections at the wall. Nevertheless, the dependence of the power is qualitatively consistent with the single path absorption rate calculated using the obtained equations.



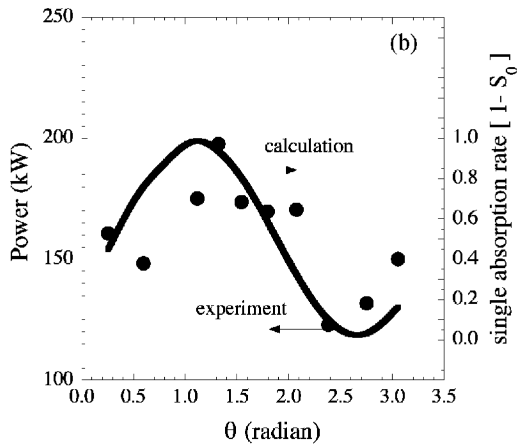


Figure 1: (a) The electron temperature profile of the target plasma. (b) The absorption power deduced from the diamagnetic signal and the calculated absorption rate.

4.Summary

The evolution equations of the polarization state for the full Stokes parameters are obtained from the Mueller matrix with the effects of the circular or linear dichroism and the birefringence. The dependence of the absorption power deduced from the diamagnetic signal on the polarization state may be explained with the calculation using the equations. In the calculation, the effects of the magnetic shear, the magnetic field component along the ray path and the refraction should be included correctly.

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