Fast recovery of vacuum configuration of WEGA stellarator with error field effects

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1. INTRODUCTION

Fast recovery of magnetic configurations is a crucial issue for all fusion devices. Being inherently steady state devices stellarators need rapid and reliable techniques, which would monitor the evolution of different physics parameters in real time, to be implemented with them.

The WEGA stellarator is in operation at the Max-Planck-Institut für Plasmaphysik in Greifswald, Germany, since July 2001. A classical stellarator meant primarily for educational purpose, it has a major radius R=72 cm, a plasma minor radius $a_{eff} \sim 11$ cm and a planar magnetic axis. The natural configuration, generated with 40 toroidal field (TF) coils and 2 pairs of helical field (HF) coils, has a five-fold toroidal periodicity and stellarator symmetry. Two pairs of vertical field (VF) coils of the Helmholtz type, one pair above and the other below the torus, provide the horizontal shift of the magnetic axis. The plasma (resonant) start-up is by means of a 2.45 GHz ECRH system at a magnetic field of 87.5 mT, while with a recently implemented 28 GHz ECRH the machine is operated at 0.5 Tesla.

However, real configurations do not enjoy the periodicity and the symmetry as mentioned above. In case of WEGA, due to an assumed horizontal misalignment between the TF and the HF coils [1], the magnetic field has error fields superimposed on it. As a result the stellarator symmetry breaks down and the toroidal periodicity becomes n=1, meaning unique features for the entire torus.

Encouraging results were obtained with Function Parametrization (FP) for W7-X parameter recovery where cubic polynomials were used to model vacuum configurations [2] in terms of external coil current ratios (CCR), and the finite-beta case [3] in terms of CCR, plasma pressure and toroidal plasma currents. Since error field effects were always neglected, it was now decided to test the applicability of the method on a smaller device including the perturbations due to these effects. The confining magnetic field of a stellarator is generated entirely by currents in external coils, and since this does not require a plasma a magnetic configuration exists in vacuum. We analyzed the vacuum configurations of WEGA with magnetic islands having perturbed modes, and recovering the important islands formed an important part of this work.

2. PRINCIPLES OF FP

The basic principle of FP consists in getting a simple representation of a certain (dependent or response) variable Y as a function of a set of independent (or predictor) variables $\bar{x}$ (which are assumed to be known *a priori*), resulting from a statistical analysis on a large dataset containing these variables. The ranges of variables over what is also called the training dataset should encompass those expected in the experiment as these statistical models learn the trends of variation within the data and are usually poor extrapolators. A typical FP model has the form $y_i = F(\bar{x}) + \epsilon$ where the function F is an estimate of the dependency of the true value y of Y on $\bar{x}$, and ϵ is a random error term which shows that this representation is only an approximate one. Also known as "regression" in statistics, the above function is set up using the principles of least squares whereby the coefficients of F are estimated from the minimisation of a mean squared error of the estimate $\tilde{y}$ of Y. An error analysis shows the kind of

function that best fits a particular y_i to the data. The quality of the fit can then be tested by another error analysis on an independent subset of the training dataset. These time-consuming offline steps precede the ultimate application of the above equation, when new data for $\bar{x}$ are fed in to calculate unknown y_i. This process is very fast as it simply involves evaluating F. Finally, a regression is termed as linear (non-linear) depending on whether the coefficients of F are linear (non-linear) in the equation for y_i.

3. THE DATABASE OF WEGA CONFIGURA-TIONS

As described above, setting up of an FP model re-lies on analysing a dataset showing the trends of varia-tion of all relevant variables that the model is to approx-imate. Physical parameters describing the WEGA con-figurations, such as the rotational transform on the mag-netic axis (ℓ_{ax}) and at the boundary (ℓ_b), the axis position R_{ax}, the on-axis magnetic field strength B_{ax}, the position $r_{is}^{(n/m)}$ and the width $w_{is}^{(n/m)}$ of important magnetic is-

Figure 1: WEGA configuration space in CCR.

lands (in particular, those with toroidal to poloidal mode number ratios of n/m = 1/5, 1/4 and 1/3), and the profile of ℓ as a function of r_{eff}^2 (r_{eff} = an effective minor radius for labelling flux surfaces) were listed as the response variables for the regression. Definitions of r_{is} and w_{is} were the same as those in [2]. Since magnetic configurations essentially depend on CCR (except for a scaling of the magnetic field whose strength varies explicitly with the current), the predictor variables (components of $\bar{x}$) were $i_{HF} = I_{HF}/I_{TF}$ and $i_{VF} = I_{VF}/I_{TF}$, where i and I denote CCR and the coil currents, respectively. The TF coil is maintained at a constant current in the experiment, and so it was in the dataset and formed the normalisation parameter. For CCR, the ranges $0.5 \leq i_{HF} \leq 2.7$, and $-0.05 \leq i_{VF} \leq 0.05$ were used in generating the data.

Using a field line tracing code [4] whose inputs were the currents in the TF, HF and VF coils of WEGA, we generated a dataset of 250 configurations. Figure 1 shows the configuration space for CCR. Three aspects of the scatter plot are worth mentioning. First, the data points are not random but were generated systematically in a 2-D grid defined by i_{HF} and i_{VF}. Second, there is a void in the bottom-left corner. This region, at small i_{HF}, cor-responds to ultra-low values of rotational transform ℓ so that flux surfaces are not well-formed due to insufficient twist in the magnetic field lines. As seen from figure 2,

Figure 2: Variation of ℓ_{ax} vs i_{VF} at two fixed values of i_{HF}.

ℓ increases as i_{VF} (from negative) tends to zero and positive values at fixed i_{HF}, so the void does not continue upwards. Third, a clustering of points is seen in the region with $1.4 \leq i_{HF} \leq 2.0$. These points were generated in order to get magnetic islands of the desired modes in the configuration.

4. STATISTICAL ANALYSIS RESULTS

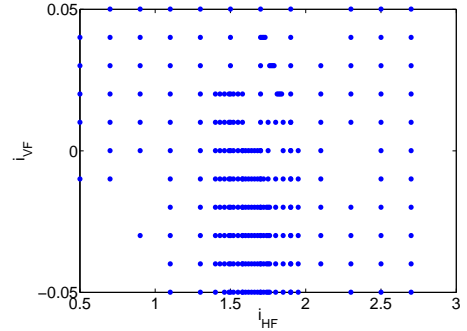As explained in section 2, the purpose of statistical analysis on a dataset is to develop global repre-
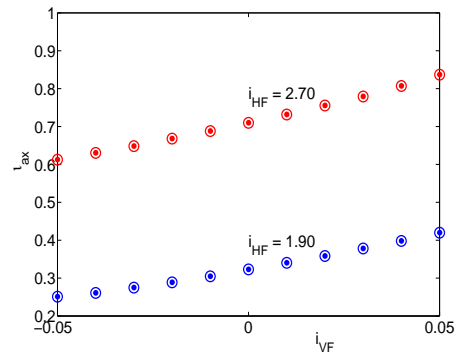
sentations of physical quantities in terms of the predictors, and a study of the error statistics leads to a decision on the best fit. These will now be described for the physical parameters of WEGA. From the database, 150 configurations were used for setting up the FP models, making sure the parameter values covered all relevant experimental cases, and another 75 to test the quality of fit.

a) Scalar parameter recovery

Table 1: Recovery statistics of scalar parameters of WEGA vacuum configurations using linear regression models.											
Parameter	mean	σ	R^2 measure of fit from models				100 (RMS error)$/\sigma$				Remarks
			q-FP	c-FP	4-FP	5-FP	q-FP	c-FP	4-FP	5-FP	
t_{ax}	0.31	0.22	0.9991	0.9997	0.9999	0.9999	2.79	1.63	0.93	0.80	1) For island
t_b	0.33	0.22	0.9927	0.9967	0.9973	0.9976	7.34	5.96	5.50	5.05	parameters
a_{eff}	8.91 cm	4.40 cm	0.8534	0.9020	0.9448	0.9544	38.29	30.00	23.50	21.40	q-FP outputs
R_{ax}	70.58 cm	3.95 cm	0.8230	0.9360	0.9785	0.9931	42.00	25.30	14.94	7.59	anticipated to
B_{ax}	0.30 T	0.018 T	0.8311	0.9509	0.9752	0.9889	41.10	22.16	15.75	11.11	be poor.
$r_{is}^{(1/3)}$	8.09 cm	2.44 cm	—	0.9606	0.9766	0.9934	—	19.85	15.30	8.12	2) For 1/4, 1/5
$w_{is}^{(1/3)}$	2.05 cm	1.15 cm	—	0.5207	0.8572	0.9099	—	69.23	37.79	30.00	regressing r_{is}^2
$r_{is}^{(1/4)}$	8.58 cm	2.06 cm	—	0.9527	0.9924	0.9974	—	21.74	8.74	5.10	improved fit.
$w_{is}^{(1/4)}$	1.08 cm	0.45 cm	—	0.8690	0.9323	0.9790	—	36.19	26.03	14.50	3) Fit of t_b
$r_{is}^{(1/5)}$	7.64 cm	2.57 cm	—	0.9979	0.9991	0.9991	—	4.28	2.61	2.49	saturated with
$w_{is}^{(1/5)}$	0.59 cm	0.41 cm	—	0.9353	0.9431	0.9449	—	24.39	21.95	19.51	c-FP model.

The regressions were done and tested with a quadratic (q-FP), a cubic (c-FP), a 4th order (4-FP) and a 5th order (5-FP) polynomial in (i_{HF}, i_{VF}) including interaction terms, that involved 6, 10, 15 and 21 model coefficients, respectively. In our previous studies [3, 4] on W7-X configurations, a c-FP model was always found to be necessary and sufficient. For WEGA a significant improvement in the regression accuracy was observed, for all the configuration parameters regressed, when a 4-FP model was used. The error statistics are tabulated in Table 1, from which it is also clear that for R_{ax}, B_{ax}, and the parameters of at least two island modes there was a further enhancement in the quality of regression with a 5-FP model. The necessity of higher order regression terms in the models suggests quite a complex dependence of the physical parameters of WEGA configuration on the coil currents. This may be attributed to the error field effects, when the lack of symmetry and a reduction of the natural periodicity introduced the non-linearities.

However, for some of the parameters other causes were also likely. For example, from figure 3 we find the behaviour of R_{ax} with i_{HF} for positive, negative and zero vertical field as being quite complicated in an overall sense, so a single model that approximates this variation is also expected to be complex. For this and the other parameters which showed progressively better fits even up to 5-FP, we then decided to test the feasibility of non-linear regression using rational functions. The purpose was to study the usefulness of these functions from the viewpoint of accuracy as compared to, e.g., the 5-FP

Figure 3: Variation of magnetic axis position with i_{HF} at three fixed values of i_{VF}.

model. Non-linear regressions involve iterative convergence of the error function to its minimum in the hyperspace of the model coefficients, and the Levenberg-Marquardt scheme was used for that. The results were very encouraging, and the rational functions were finally settled with as the best fit.
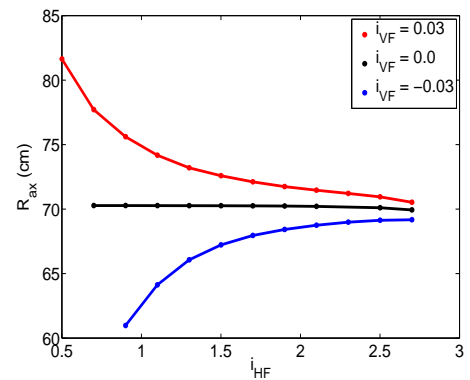
Denoting i_{HF} by x_1, i_{VF} by x_2, root-mean-squared error by ϵ_{rms} and standard deviation by σ, the

best fit rational functions for some of the relevant configuration parameters were

$$R_{ax} = \frac{(a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1^2 + a_4 x_2^2)}{(a_5 + a_6 x_1 + a_7 x_2 + a_8 x_1^2 + a_9 x_2^2 + a_{10} x_1 x_2 + a_{11} x_1^3 + a_{12} x_2^3)}$$ with 13 coefficients, giving $\epsilon_{rms} = 1.1$ mm, an R^2 measure of fit of 0.9992 (defined as the fraction of the total variance in the regressed parameter explained by the chosen model) and a percentage spread error ($\epsilon_{perc} = 100 \, \epsilon_{rms}/\sigma$) of 2.84,

$$B_{ax} = \frac{(b_0 + b_1 x_1 + b_2 x_2 + b_3 x_1^2 + b_4 x_2^2)}{(b_5 + b_6 x_1 + b_7 x_2 + b_8 x_1^2 + b_9 x_2^2 + b_{10} x_1^3)}$$ with 11 coefficients, giving $\epsilon_{rms} = 0.0007$ T, an R^2 measure of fit of 0.9988 and $\epsilon_{perc} = 3.48$,

$$r_{is}^{(1/3)} = \frac{(c_0 + c_1 x_1 + c_2 x_2 + c_3 x_1^2 + c_4 x_2^2)}{(c_5 + c_6 x_1 + c_7 x_2 + c_8 x_1^2 + c_9 x_2^2)}$$ with 10 coefficients, giving $\epsilon_{rms} = 1.8$ mm, an R^2 measure of fit of 0.9932 and $\epsilon_{perc} = 8.27$, and

$$w_{is}^{(1/3)} = \frac{(d_0 + d_1 x_1 + d_2 x_2 + d_3 x_1^2 + d_4 x_2^2 + d_5 x_1^3 + d_6 x_2^3)}{(d_7 + d_8 x_1 + d_9 x_2 + d_{10} x_1^2 + d_{11} x_2^2 + d_{12} x_2^3)}$$ with 13 coefficients, giving $\epsilon_{rms} = 2.1$ mm, an R^2 measure of fit of 0.9515 and $\epsilon_{perc} = 22$.

For R_{ax} and B_{ax} there was a remarkable improvement in accuracy over 5-FP, while for the island parameters the 5-FP results were reproduced, by the use of rational functions with 8-10 coefficients less.

b) Profile parameter recovery

As already listed, the only relevant profile quantity for WEGA is that of ι which was considered to be a function of r_{eff}^2 (apart from CCR) because physically the profile quantities are functions of the toroidal flux which varies as r_{eff}^2. The ι-profile was best fitted with a radial polynomial of the form $\iota(i_{HF}, i_{VF}, r_{eff}^2) = p_0 + p_1 r_{eff}^2 + p_2 (r_{eff}^2)^2$ where CCR (i_{HF}, i_{VF}) are contained within the coefficients p_i. When CCR were combined in a "mixed" cubic form, the uncertainty of ι over the entire profile was ± 0.0025; when the combination was in a "mixed" 4th order form, the uncertainty was ± 0.0015. So the latter was chosen as the best fit, consistent with the scalar parameter results. It may be noted that the radial function is smooth and so cannot represent the discontinuity in the ι-profile due to islands. Also, it allows extrapolations beyond the minor radius of an island chain in a given configuration and will return a consistent value of ι for an r_{eff} even if no good flux surfaces exist in the real configuration for that r_{eff}. To avoid this problem the regression of ι-profile must be combined with that of r_{is} and/or a_{eff} in order to restrict the valid range of r_{eff} for the regression functions.

5. CONCLUSIONS

The FP on WEGA, performed on a small device with a very modest number of independently variable predictors, has shown the inherent non-linearities in the dependencies of the physical state of the WEGA magnetic configurations, with error field perturbations, on the external coil currents. The best-fit statistical models have been either 4th order polynomials with linear regression, or rational functions with non-linear regression. That the latter improved upon the fit further demonstrated the complexities of some of the dependencies, even though the model sizes remained reasonably small. Our results were very encouraging, including the challenging issue of the island width whose determination in the database may itself be erroneous due to problems in detecting the outer separatrix. The FP functions are now in the process of being implemented in software to run with the control system.

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