MONTE CARLO SIMULATION OF THE PARTICLE FLUX IN STELLARATOR MAGNETIC FIELD.

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Introduction

A complex inhomogeneous magnetic field in stellarator type devices creates a large fraction of particles trapped in a local magnetic well and a fast drift across magnetic surfaces. If drift time is shorter then the collision time, i.e.

$$\tau_{drift} = \frac{(a_{pl} - r_0)}{\mathrm{v}_{drift}} \approx \frac{(a_{pl} - r_0)R}{\rho_L} < 1/2\tau_{90} \ , \ \mathrm{v}_{drift} \approx cK_0 / eBR \ ,$$

such particles undergo a large deviation from the starting magnetic surface and can leave the plasma region. Such particles create a loss region in velocity space. By scattering particles into this region Coulomb collisions produce a convective particle flux which can exceed diffusive one. In such a case neoclassical theory does not correctly describe transport processes in plasma and full particle flux should be considered.

Basic formalism

For description of free particle motion we use drift approximation, in which by introducing magnetic moment $\mu = m\mathrm{v}_\perp^2/2B$ phase space is decreased to five coordinates.

$$\frac{\partial f}{\partial t} + \dot{\psi}\frac{\partial f}{\partial \psi} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\varphi}\frac{\partial f}{\partial \varphi} + \dot{\rho}_\parallel \frac{\partial f}{\partial \rho_\parallel} = \Lambda(f) \ , \quad \dot{\mu} = 0 \ .$$

The collision process is described by Lorentz collision operator $\Lambda(f)$.

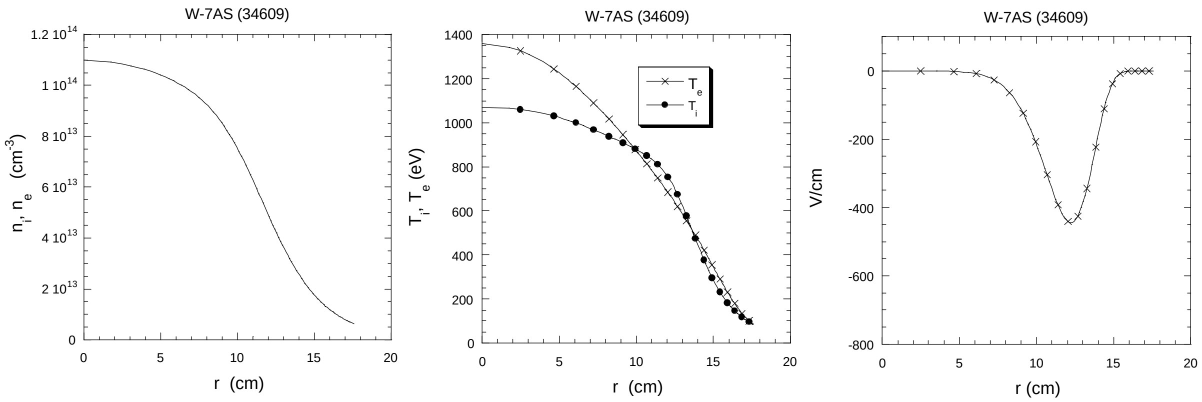
We are interested in steady state $\partial f/\partial t = 0$, when number of particles and total particle flux across magnetic surface are conserved

$$N_{part} = \int f d\mathbf{x} d\mathbf{v} = const \ , F_\psi = \int dS_\psi \cdot \int \mathbf{v} f d\mathbf{v} = const \, , \ dS_\psi = J\nabla\psi d\theta d\varphi \ .$$

The system of characteristic equations coincides with Hamiltonian equations for particle trajectory, derived from the guiding center Lagrangian [1,2]

$$L(\psi,\theta,\varphi,\rho_\parallel) = \mathbf{p}\dot{\mathbf{q}} - H = \frac{e}{c}(\rho_\parallel \mathbf{B} + \mathbf{A}) \cdot \dot{\mathbf{q}} - \left(1/2m(\omega_c \rho_\parallel)^2 + \mu B + e\Phi\right) .$$

Particle orbits are evaluated numerically with total particle energy and magnetic moment keeping constant. After each integrating step the collisions between test particles and Maxwellian background are simulated by random variation of pitch angle $\lambda = \mathrm{v}_\parallel/\mathrm{v}$ and kinetic energy K_0 according to the binomial distribution model [3]. The background plasma is assumed to be a mixture of protons and electrons with equal densities. Electric field E_r is taking into account as an external field, established due to the ambipolarity condition of the neoclassical particle fluxes. The profiles of plasma (*Fig.1*) are modeled according to the experimental data in the advanced stellarator Wendelstein7-AS [4].



a) *b)* *c)*

Fig.1. Profiles of density (*a*), temperature (*b*) and ambipolar electric field E_r (*c*).

Numerical scheme

In Monte Carlo simulations distribution function is represented by the ensemble of the test particles

$$f = \sum_{i=1}^{N_{part}} \delta(\mathbf{x} - \mathbf{x}_i) \quad .$$

All particles start on the one magnetic surface ψ_0 with fixed kinetic energy K_0 and random poloidal θ, toroidal φ and pitch $\lambda = \mathrm{v}_{\|}/\mathrm{v}$ angles. To keep number of particles constant in the volume, every time when a particle crosses starting magnetic surface ψ_0 (inner boundary) or last closed magnetic surface (outer boundary), we replace the particle with a new one. To ensure the steady state condition, the total particle flux across magnetic surface should be statistically constant in radial coordinate and in time. We introduce space $\psi_j = \psi_0 + j\Delta\psi$ and time $t = \tau_{drift} + i\Delta t$ discretesation, and calculate number of particles $N_i(\psi_j)$, which cross each control magnetic surface ψ_j during time Δt. By standard definition of statistics, averaging of the series of values $F_{ij} = N_i(\psi_j)/\Delta t$ gives mean value $\langle F_j \rangle$ and relative statistical error ξ_j of total particle flux on each control magnetic surface

$$\left\langle F_j \right\rangle = \frac{1}{n_j} \sum_{i=1}^{n_j} F_{ij} \;\; , \;\; \xi_j = \frac{1}{n_j} \sqrt{\sum_{i=1}^{n_j} \left| F_{ij} - \left\langle F_j \right\rangle \right|^2} \quad .$$

While the total number of particle and total particle flux are known, one can estimate particle confinement time $\tau = N_{part}/\langle F \rangle$, which does not depend from number of particles in simulation.

Results

1.Effect of magnetic configuration (Fig.2a). In our investigations we are interested mainly in three types of stellarator fields:

1) classical stellarator model, which corresponds to the standard configuration of LHD,

$$B/B_0 = 1 - r/R\cos\theta - \varepsilon_h (r/a_{pl})^2 \cos(l\theta - m\varphi) \quad .$$

2) drift optimized stellarator model, which corresponds to the inner shifted configuration of LHD, (using this analytical model, we simulate also same aspect ratio tokamak (ε_b=0,ε_m=0) and ripple tokamak (ε_b=0) like magnetic fields).

$$B/B_0 = 1 - r/R\cos\theta(1 - \varepsilon_b \cos m\varphi) - \varepsilon_m \cos m\varphi$$

3) real configuration of Wendelstein7-AS (standard case with rotational transform *ι=1/3*), which is presented by Fourier expansion with about 40 harmonics.

$$B / B_0 = 1 + b_{0,0} + \sum_{k=1}^{\infty} b_{0,k} \cos km\varphi + \sum_{l=1}^{\infty} \sum_{k=-\infty}^{\infty} b_{l,k} \cos(km\varphi - l\theta))$$

For parameters $a_{pl}\approx 17.5cm$, $R_{ax}\approx 200cm$, $B_0\approx 2{,}5T$, $\iota\approx 0.3$ and plasma profiles as in *Fig.1.a),b)*, we consider ions starting at radius *12cm* with energy $K_0=1.2keV$, for which time of the drift across the magnetic surfaces τ_{drift} is smaller that time of considerable change of pitch angle due to collisions $1/2\tau_{90}$. For comparison with previous results [5], at the beginning we take into account only pitch angle scattering operator and $E_r=0$. One can see (*Fig.2a*), that a confinement time in drift optimized stellarator is by a factor *2.33* larger than that in ripple tokamak and by a factor *0.76* smaller than that in tokamak (corresponding factors in [5] are *2.5* and *0.76*). Among stellarator configurations the best confinement is observed in drift optimized stellarator, which has confinement time by a factor *2.2* large than in classical one. Standard configuration of Wendelstein7-AS has improving factor *2* in comparison with classical stellarator.

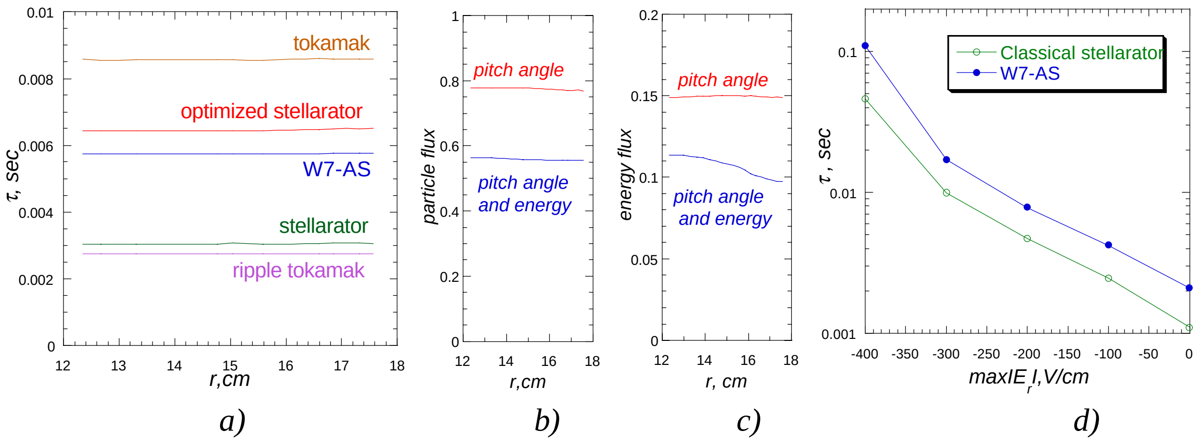


Fig.2. Effect of magnetic configuration *(a)*, energy scattering and slowing down operator *(b,c; relative units)*, ambipolar electric field *(d)*.

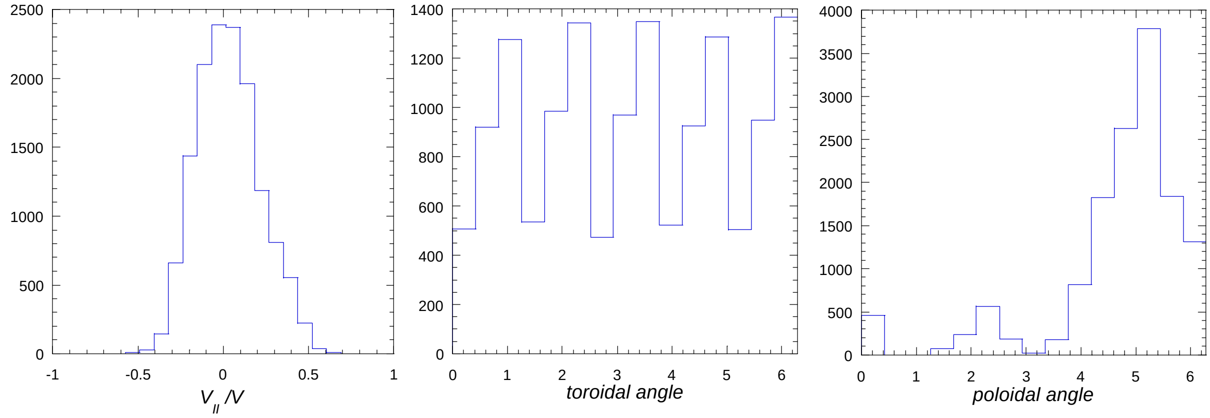
2.Effect of energy scattering and slowing down operator (Fig.2b,c). For the particle fluxes produced by the velocity space diffusion the main role is played by the pitch angle part of the collision operator, while energy scattering and slowing down operator describes the loss of the energy of high energetic particles to the Maxwellian background. As a result of this process, the particle and energy fluxes of ions are decreased by about *20%* and energy flux declines with radius (while particle flux is still statistically constant on control surfaces).

3.Effect of ambipolar electric field (Fig.2d). The guiding center orbits of the localized particles follow the constant contours of the second (longitudinal) adiabatic invariant, which is dependent on electric field potential Φ.

$$J = \oint v_{\parallel} dl = \oint \sqrt{W - \mu B - e\Phi}\, dl = const$$

By closing these contours electric field reduces the direct orbit losses of localized particles across the plasma volume. The ambipolar electric field with peak value *–400V/cm*, which was observed in experiments in Wendelstein7-AS [4] increases confinement time of ions with staring energy $K_0=1.7keV$ by a factor of *50*.

4.Distribution function of the lost particles (Fig.3). The analysis of the distribution function of the lost particles shows, that losses are produced mainly by the direct drift of the localized particles. The pitch angle distribution of lost particles has a large peak in the region of trapped particles $|v_{\|}/v|<0.5$. The periodical structure of losses in toroidal angle coincides with the number of field periods. In poloidal angle most of the test ions are lost in the bottom of the torus, which corresponds to the toroidal drift direction.



a) *b)* *c)*

Fig.3. Pitch angle *(a)*, toroidal *(b)* and poloidal *(c)* distribution of lost particles in Wendelstein7-AS.

Summary

The convective particle flux due to the scattering into the loss region in velocity space and particle confinement time are calculated in the stationary plasma with 5D Monte Carlo method. The magnetic configurations of classical, drift optimized and realistic magnetic field of advanced stellarator Wendelstein7-AS with low rotational transform ($\iota=1/3$) were analyzed with the experimental profiles of densities, temperatures and ambipolar electric field taking into account. It is shown, that due to the optimization of the equilibrium magnetic configuration particle confinement time can be increased by a factor *2*. Additional improvement of the confinement can be achieved by the effect of the ambipolar electric field. It is demonstrated that the electric field, which was simulated according to the experimental data, with peak value *-200V/cm* decreases ion flux by a factor of 4, and one with a peak value *-400V/cm* by a factor of 50. The analyses of the distribution function of the lost ions show that most of the lost particles were trapped ones and escaped from the plasma mainly in the bottom of the torus.

References

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