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MANY FACES OF D-BRANES: FROM FLAT SPACE, VIA ADS TO PP-WAVES*

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We review recent studies of branes in AdS $\times$ S and pp-wave spaces using effective action methods (probe branes and supergravity). We also summarise results on an algebraic study of D-branes in these spaces, using extensions of the superisometry algebras which include brane charges.

1. Introduction

Since the discovery in 1995, D-branes have become a center of intensive research of the string theory community. In this process a lot has been learnt about various manifestations of these objects. It has become clear that, depending on the regime in which one works, D-branes can be described using a variety of techniques. In situations with a small number of branes and weak string coupling, methods of open string conformal field theory are appropriate. These techiques however, have been applied mainly to the study of D-branes in very restricted classes of backgrounds, the main obstacle beeing one's inability to quantise strings in arbitrary backgrounds.

This problem can be partially circumvented by restricting one's interest to low-energy processes in the target space and the worldvolume of branes. In this case a description of branes using effective actions becomes viable. The low-energy dynamics of the *closed* strings is governed by various supergravity actions (which are just various supersymmetric generalisations of the Einstein-Hilbert action). The effective action for *open* strings is the Dirac-Born-Infeld action, which (as the name suggests) is a generalisation of the Dirac action for the relativistic membrane to higher dimensional ob-

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jects, modified to include the brane's "inner" degrees of freedom, i.e. gauge fields. The latter are included in a fashion proposed a long time ago by Born and Infeld in order to regularise the infinite electromagnetic energy of a classical electron. The total (bulk plus brane) effective action is quite complicated due to the non-trivial coupling between the brane and supergravity fields. Hence, in order to use this action, one is often forced to simplify the problem further. Increasing the number of branes, for example, leads to the regime where gravitational back-reaction of the branes cannot be neglected, while the gauge theory on the worldvolume of the branes becomes strongly coupled. In this regime branes can be described as purely gravitational solutions, using only the bulk effective action. On the other side, when the number of branes is very small, the *probe brane* approach is appropriate: only the worldvolume excitations (i.e. scalars and gauge fields) are dynamical fields, while the background is "frozen". In the first two parts of this report we will partially survey our recent study of D-branes in AdS and plane-wave (pp-wave) geometries using the supergravity and Dirac-Born-Infeld effective actions.

Finally, a very powerful method for classifying possible brane configurations in arbitrary backgrounds is the so-called *algebraic method*. The full information about the *non-perturbative* spectrum of string theory (in a given background) is encoded in the "central" extensions of the appropriate superisometry algebra of the background. Unfortunately, the explicit forms of the algebras are essentially not known beyond flat space. Recently, however, we have made an important step in understanding the construction of central extensions of AdS and pp-wave superisometry algebras. In the last part of this survey we report on these results.

2. The probe brane approach

2.1. *From flat space to AdS*

The problem of understanding the full brane-background system is simplified dramatically by making a restriction to the effective actions, and by further restricting to the probe brane approach. However, this still leaves one with a generically complicated action which has to be solved in order to find the exact embedding of the brane surface in the target space. In generic backgrounds, and in cases of supersymmetric brane configurations, one often uses kappa-symmetry or calibration methods in order to replace the second-order differential equations with first order, BPS-like equations. In practice however, both of these methods are ad hoc, since they both

require good intuition about the ansatze for the brane embeddings.

In special backgrounds, such as AdS or pp-waves, the situation is simpler: *supersymmetric* brane configurations can be "derived" from the configurations of branes in flat space. Namely, given a brane configuration in flat space one first replaces some of the branes in the configuration by their supergravity solutions and subsequently focuses on the corresponding near-horizon geometries, while keeping the remaining branes as probes. The actual equations for the embedding of the probe brane can usually be deduced directly from the flat space equations, using Poincaré coordinates for the AdS background. This is due to the fact that in this coordinate system, the relation of AdS coordinates to flat space Cartesian coordinates is direct. Moreover, it turns out that the equations that describe the embedding of the brane in flat space also describe the solutions of the *DBI equations of motion* in the near horizon geometry in Poincaré coordinates. The essential reason why this inheritance property holds is that the brane configurations in question are supersymmetric. In contrast, if the brane system is *non-supersymmetric*, this logic does not hold. For example, consider a circular $D1$ string in the space transverse to a $D3$ brane. In flat space one can easily derive the solution that describes shrinking of the string, and then one can show that this solution (in Cartesian coordinates), when interpreted in Poincaré coordinates, *does not* solve the DBI equations of motion of the $D1$ string in the $AdS_5 \times S^5$ space.

Instead of listing all possible brane configurations which one can derive using this method, lets us illustrate it on a very simple example of two $D3$ branes intersecting over a string,

$$
\begin{aligned}
&D3\ 0\ 1\ 2\ \ 3\ -\ -\ -\ -\ -\ -\\
&D3\ 0\ 1\ -\ -\ 4\ \ 5\ -\ -\ -\ -\ .
\end{aligned} \tag{1}
$$

We take the first brane to create the background, while the second brane is treated as a probe. In flat space the embedding equations of the second brane are given by

$$
x_i = c_i = \text{const.}, \quad (i = 2, 6, 7, 8, 9). \tag{2}
$$

Next, we replace the first brane with the near horison limit of its supergravity solution (i.e. the $AdS_5 \times S^5$ space), which in Poincaré coordinates yields

$$
\begin{aligned}
&ds^2 = R^2 u^2 \big(-\,dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \big) + \frac{R^2}{u^2} \Big(dx_4^2 + \ldots + dx_9^2 \Big),\\
&u^2 = x_4^2 + \ldots + x_9^2 \quad \Rightarrow \quad dx_4^2 + \ldots + dx_9^2 = du^2 + u^2 d\Omega_5^2\,.
\end{aligned} \tag{3}
$$

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When all coefficients c_i in (2) are zero, we see that the equations (2) define a maximal-curvature $AdS_2 \times S^1$ submanifold of (3) [1,2]. In the case when some of the $c_i \neq 0$, the equations (2) solve the $D3$ DBI action in the (full) $D3$ brane supergravity background. However, when taking the near horizon limit, one additionally needs to scale the parameters c_i to zero in order for the solution to survive this limit. As we focus on the region near the $D3$ brane that becomes the background, we simultaneously have to bring the probe $D3$ brane closer and closer to the horizon. The resulting geometry of the $D3$ brane probe describes a brane which starts at the AdS boundary, extends in the u-direction up to some point and then folds back to the boundary [3].

Recently, [4] we have extended this analysis to the cases of supersymmetric brane configuration intersecting under an *angle* in flat space. As in the previous situation, the inheritance property goes through due to supersymmetry. However, the resulting brane geometries are qualitatively different from the one previously discussed, since the branes now non-trivially mix the AdS and sphere submanifolds: the worldvolume surfaces are not factorisable into a product of AdS and sphere submanifolds.

Another new type of brane that has appeared in [4] is a brane with mixed worldvolume flux (i.e. where the worldvolume two-form has one index in the sphere part and one index in the AdS part). This brane wraps *non-supersymmetric* target space cycles and is stabilised only after the mixed worldvolume flux is turned on. To construct this brane, one starts from the flat space configuration of branes intersecting under an angle and performs a T-duality transformation in such a way that the brane which will be replaced with the background *does not* carry any worldvolume flux, while the brane which will become a probe carries flux. Then, as before, one takes the near horizon limit of this configuration.

2.2. *From AdS to pp-waves*

It was realised a long time ago by Penrose that an infinitely boosted observer in an *arbitrary* spacetime, in a neighborhood of its geodesic, observes a very simplified background geometry: the geometry of a gravitational wave. This dramatic simplification has recently been used extensively for a direct check of the gauge-gravity (AdS/CFT) correspondence [5], avoiding the standard strong-weak coupling problems.

On the gravity side, the Penrose limit amounts to a suitable rescaling of the coordinates and parameters characterising the (super)gravity solu-

tion, in such a way that one focuses on the region close to an arbitrary null geodesic. In the same way in which the background undergoes simplification, so do different objects present in the initial space. The geometry of the resulting branes can easily be derived by rescaling the embedding equations of the branes in the same way as the target space coordinates. Since the pp-wave space is *homogeneous* but *not isotropic*, there are three basic families of D-branes which appear in the limit, depending on the relative orientation of the brane and the wave: [6] *longitudinal D-branes* for which the pp-wave propagates along the worldvolume of the D-brane, *transversal D-branes* for which the pp-wave propagates in a direction transverse to the D-brane but the timelike direction is along its worldvolume, and *instantonic D-branes* for which both the direction in which the pp-wave propagates and the timelike direction are transverse to the D-brane. The first class of branes originates from AdS branes for which the geodesic along which the boost was performed belongs to the worldvolume of the brane (before the limit). For the second class, the brane is co-moving with the observer along the the geodesic (i.e. it was infinitely boosted). The third class of branes can be obtained from the first class by a formal T-duality in the timelike direction of the wave.

Based on isometries, the pp-wave coordinates can be split into three groups: the "lightcone coordinates" u and v, and two four-dimensional subspaces with $SO(4) \times SO(4)$ isometry group. The split of the transverse coordinates is due to the nonvanishing 5-form flux. In the case of longitudinal branes, the worldvolume coordinates split accordingly into three sets: the "lightcone coordinates" u and v, m coordinates along the first $SO(4)$ subspace and n along the second $SO(4)$. A Dp brane $(m+n=p-1)$ with such orientation is denoted with $(+,-,m,n)$. The number of preserved supersymmetries depends on the values for (n,m):[6]

- 1/2-BPS D-branes with embedding $(+,-,m+2,m)$,
  for $m = 1, \ldots, 4$,
- 1/4-BPS D-string with embedding $(+,-,0,0)$,
- non-supersymmetric D-branes with embedding $(+,-,m,m)$,
  for $m = 1, 2, 3$.

All these results are valid for the brane placed at the "origin" of the pp-wave. If we *rigidly* move the first or second type of brane outside the origin (without turning worldvolume fluxes), supersymmetry is always reduced to 1/4.

However, the previous three classes do not capture all the branes which

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can appear in pp-waves [4,7]. In the process of Penrose rescaling, not all objects of the initial space will be inherited by the final wave geometry. It is usually said that in order to have a nontrivial Penrose limit of a brane in some background, one needs to take the limit along a geodesic which belongs to the brane. This statement is intuitively understandable: in the Penrose limit an infinitesimal region around the geodesic gets zoomed out. Hence, those parts of the brane which are placed at some nonzero distance from the geodesic get pushed off to infinity. However, this reasoning can be circumvented if the distance between the geodesic and the brane is determined by free parameters of the solution [4]. In that case one can take the Penrose limit along a geodesic that *does not* belong to the brane, as long as the parameter labeling the brane in a family of solutions is appropriately scaled.

For example, let us consider the family of solutions corresponding to two intersecting D-branes and let us take the Penrose geodesic to lie on the first of the two branes. Then the Penrose limit of the second brane can be nontrivial if, while taking the Penrose limit of the target space metric, we simultaneously scale the angle between the two branes to zero. It should be emphasized that the final configuration obtained in this way is different from the one which is obtained by first sending the angle to zero and then taking the Penrose limit of the metric. Namely, if we first set the angle to zero, the problem will reduce to taking the limit along a geodesic that belongs to the worldvolume of the brane, which has been discussed already. However, if we follow the procedure outlined above, the Penrose limit of the second D-brane is a brane with a relativistic pulse propagating on its worldvolume (i.e. with some of $x_i = $ const. getting replaced by $x_i(x^+)$). The precise form of this worldvolume wave carries information about the position of the brane with respect to the geodesic before the limit was taken [4].

3. The supergravity approach

When the number of branes in some space becomes very large, the probe brane approach is inadequate and a supergravity description takes over. Finding supergravity solutions for D-branes in AdS spaces is, however, still essentially an open problem; the explicit constructions have been carried out only in a few specific cases. One of the reasons for this is that fully localised supergravity solutions for D-brane intersections in *flat space* are generically not known. Hence in order to construct the brane solutions in

asymptotically AdS and pp-wave spaces, one has to start from scratch. We will present here the construction of the (extremal) D-brane solutions in asymptotically pp-wave spaces.

The main difficulty in constructing these solutions consists of identifying a coordinate system where the description of the D-brane is the simplest. This is similar to the problem that one would face if one would only know about Minkowski space in spherical coordinates and would try to describe flat D-branes in these coordinates. Cartesian coordinates are the natural coordinates to describe infinitely extending D-branes in flat space. So the question that one should first ask is "what are the analogues of the Cartesian coordinates for D-branes in pp-wave backgrounds?". The answer to this question is more complicated than in flat space, as it depends very much on what kind of D-branes one considers. It was shown [8,9] that Brinkman coordinates are the natural coordinates for a description of 1/4-BPS and nonsupersymmetric D-branes, while the natural coordinates for the 1/2-BPS D-branes are the Rosen coordinates.

For the metric part of the ansatz, one writes a simple standard metric for a superposition of D-branes with waves,

$$
ds^2 = H(y, y')^{-\frac{1}{2}}\Big(2du\big(dv + S(x, x', y, y')du\big) - d\vec{x}^2 - d\vec{x}'^2\Big)
$$
$$
- H(y, y')^{\frac{1}{2}}(d\vec{y}^2 + d\vec{y}'^2). \quad (4)
$$

The metric is given in the string frame, and the D-brane worldvolume coordinates are $(u, v, x^i = x^1, \ldots, x^m, x'^I = x'^1, \ldots, x'^n)$, while the directions transverse to the D-brane are $(y^a = y^1, \ldots, y^{(4-m)}, y'^A = y'^1, \ldots, y'^{(4-n)})$. The function H characterising the D-brane is at this stage allowed to depend on all transverse coordinates y, y'. The ansatz for the RR field strength and the dilaton reads

$$
F_{[p+1]} = du \wedge dv \wedge dx^1 \wedge \cdots \wedge dx^m \wedge dx'^1 \cdots \wedge dx'^n \wedge dH^{-1}, \quad (5)
$$
$$
F_{[5]}^{\mathrm{w}} = F_{[5]}^{(1)} + *F_{[5]}^{(1)},
$$
$$
F_{[5]}^{(1)} = W(z^\mu) du \wedge \Big(dx^1 \wedge \cdots \wedge dx^m \wedge dy^1 \wedge \cdots \wedge dy^{(4-m)}\Big), \quad (6)
$$
$$
e^\phi = H^{\frac{3-p}{4}}, \quad (7)
$$

where '*' in $F_{[5]}$ denotes Hodge duality with respect to the metric (4) and $W(z)$ is an undetermined function which can depend on all directions transverse to the pp-wave. In the case of the $D3$ brane, one also has to add to the form (5) its Hodge dual.

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One of the main characteristics and perhaps limitations of this ansatz is that the metric is diagonal in Brinkman coordinates. This property forces one to delocalise the *supersymmetric* solutions along some directions transverse to the brane when solving the equations of motion.[a] The smearing procedure physically means that one is constructing an array of D-branes of the same type with an infinitesimally small spacing. However, as we have seen before, the probe brane results tell us that, unless we turn on additional bulk fluxes (sourced by the worldvolume fluxes of the 1/2-BPS D-branes), a periodic array of *rigid* D-branes in *Brinkman* coordinates with orientation $(+, -, n + 2, n)$ is only one quarter supersymmetric. Hence the supersymmetric solutions that we find due to the smearing procedure are only 1/4 BPS. However, these restrictions have to be imposed only on the harmonic function characterising the D-brane, and not on the function characterising the pp-wave. Hence, all our solutions asymptotically tend to the *unmodified* Hpp-wave. Also, despite the simplicity of the ansatz, the *nonsupersymmetric* solutions, describing branes with $(+, -, m, m)$ orientation, are fully localised.

Plugging the ansatz (4)-(7) into the equations of motion and the Bianchi identities, one obtains solutions with the following characteristics, which depend on the orientation of the branes. The presence of the D-brane modifies the function S which characterises the pp-wave, while the function H (which specifies the D-brane) is completely unmodified by the presence of the wave. Therefore, this ansatz does not catch the *back-reaction of the pp-wave on the D-brane*. For a generic embedding of the D-brane, one expects that the (fully localised) D-brane is modified by the wave. However, as our fully localised, nonsupersymmetric solution demonstrates, this does not have to hold for specific embeddings. By examining the behavior of the radially infalling geodesics, one discovers that if the pure pp-wave was focusing the geodesics, this attractive behaviour is strengthened in the presence of a supersymmetric brane, as one would expect. Surprisingly, however, the non-supersymmetric geometries exhibit repulsion behavior.

4. The algebraic approach

Rather surprisingly, a modification of the superalgebra of anti-de-Sitter backgrounds which accounts for the presence of D-branes in the string

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[a]This is the same type of restriction that one faces when constructing supergravity solutions for intersecting D-branes, with a simple diagonal ansatz.

spectrum is still unknown. At an algebraic level, D-branes manifest themselves through non-zero expectation values of bosonic tensorial charges. There exists a widespread, but incorrect, belief that the inclusion of these brane charges into the anti-de-Sitter superalgebras follows the well-known flat-space pattern. In flat space, the inclusion of brane charges leads to a rather minimal modification of the super-Poincaré algebra: the bosonic tensorial charges appear on the right-hand side of the anti-commutator of supercharges, transform as tensors under the Lorentz boosts and rotations, while they commute with all other generators. The brane charges are therefore often loosely called "central", and the resulting algebra is referred to as the maximal bosonic "central" extension of the super-Poincaré algebra. However, despite several attempts to construct a similar modification of anti-de-Sitter superalgebras, a *physically satisfactory* solution is as of yet unknown.

There are two basic physical requirements which have to be satisfied by an anti-de-Sitter algebra which is modified to include brane charges. The algebra has to include at least the brane charges which correspond to all D-branes that are already known to exist, and it also has to admit at least the supergraviton multiplet in its spectrum. Mathematically consistent modifications of anti-de-Sitter superalgebras can be constructed, but all existing proposals fail to satisfy one or both of these physical criteria. [10] In [11] we have identified a simple reason why previous attempts to extend anti-de-Sitter superalgebras with brane charges have failed: such extensions are *only* physically acceptable when one adds new *fermionic* brane charges as well.

The necessity of including new fermionic brane charges into the modified algebra can be understood from a very simple argument based on Jacobi identities, in combination with the two physical requirements just mentioned [11]. Consider an anti-de-Sitter superisometry algebra, or a pp-wave contraction of it. The bracket of supercharges can, very symbolically, be written in the form

$$\{Q_\alpha, Q_\beta\} = (\Gamma^{AB})_{\alpha\beta} M_{AB} \,, \tag{8}$$

where Q and M are the supercharges and rotation generators respectively (we have grouped together momentum and rotation generators by using a notation in the embedding space). Suppose now that we add a bosonic tensorial brane charge Z on the right-hand side of this bracket. This extension has to be made consistently with the Jacobi identities. Consider the

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(Q, Q, Z) identity, which takes the symbolic form

$$(Q_\alpha, Q_\beta, Z) = [\{Q_\alpha, Q_\beta\}, Z] - \{[Q_\alpha, Z], Q_\beta\} - \{[Q_\beta, Z], Q_\alpha\}$$
$$= (\Gamma^{AB})_{\alpha\beta}[M_{AB}, Z] - 2\{[Q_{(\alpha}, Z], Q_{\beta)}\}. \tag{9}$$

As the brane charge Z is a tensor charge, it will transform non-trivially under the rotation generators. This implies that the first term of (9) will not vanish. The Jacobi identity can then only hold if Z *also* transforms non-trivially under the action of the supersymmetry generators! (In flat space, only the vanishing bracket $[P, Z]$ appears in the first term of the Jacobi identity (9), because in that case the $\{Q, Q\}$ anti-commutator closes on the translation generators). The simplest option is to assume that *no new fermionic charges* should be introduced, and that therefore symbolically

$$[Q_\alpha, Z] = Q_\alpha. \tag{10}$$

Although it is possible to construct an algebra based on (10) which satisfies all Jacobi identities, it is physically unsatisfactory [10]. The essential reason is that brackets like (10) are incompatible with multiplets on which the brane charge is zero (the left-hand side would vanish on all states in the multiplet, while the right-hand side is not zero). In other words, one cannot "turn off" the brane charges. The only other way out is to add *new fermionic charges* Q'_α to the algebra, such that (10) is replaced with

$$[Q_\alpha, Z] = Q'_\alpha. \tag{11}$$

In this case it becomes possible to find representations in which both Z and the new charge Q'_α are realised trivially, as expected for e.g. the super-graviton multiplet, while still allowing for multiplets with non-zero brane charges.

    This formal argument based on Jacobi identities may come as a surprise, and one would perhaps find it more convincing to see new fermionic brane charges appear in *explicit* models. In [11] we have shown that such charges indeed do appear. In order to show this, we have analysed the world-volume superalgebras of the supermatrix model and the supermembrane in a pp-wave limit of the anti-de-Sitter background. These models exhibit, in the absence of brane charges, a world-volume version of the superisometry algebra of the background geometry. When bosonic winding charges are included, the algebra automatically exhibits fermionic winding charges as well. Moreover, configurations on which these charges are non-zero can be found explicitly, or can alternatively be generated from configurations on which the fermionic winding charges are zero. On the basis of these results

we have briefly discussed a D-brane extension of the osp*(8|4) superisometry algebra with bosonic as well as fermionic brane charges, which avoids the problems with purely bosonic modifications as first observed in [10]. A partial construction of this algebra has been carried out in [12].

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