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Critical Times for Overcritical Relativistic Electron Beams
Injected into Plasmas

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Abstract

The theory of beam injection into unmagnetized plasma is extended by introducing, and discussing the relevance of, several criteria for propagation of cold, overcritical electron beams. Estimates of the criteria are given in terms of a simple beam-plasma model derived from linear injection theory. The influence of the radial beam profile upon the net magnetic field and, hence, on propagation, is taken into account. The validity of neglecting, as is customary in injection theory, the nonlinear magnetic force upon the plasma response is also studied. The parameter range of validity is estimated at the violation time of the ALFVÉN criterion.

1. INTRODUCTION

The purpose of this paper is to discuss and extend the theory of beam injection into unmagnetized plasma. The injection theory we have in mind (COX and BENNETT, 1970; HAMMER and ROSTOKER, 1970; ROSINSKII and RUKHADZE, 1972; RUKHADZE and RUKHLIN, 1972; ROSINSKII et al., 1972; KÜPPERS et al., 1973 a) assumes a rigid, parallel, cold electron beam with a density small compared with the plasma density n . Beam instabilities, in particular macroscopic ones, are neglected. A linear theory of plasma response including inertia and collisions of plasma electrons is used; hence heating of the plasma by the beam is not taken into account.

Roughly speaking, the plasma response does two things:

1. Magnetic shielding of the beam current.
2. Neutralization of the beam charge.

Electric neutralization is more effective than magnetic shielding (see KÜPPERS et al., 1973 c) and persists in time. Magnetic shielding disappears by magnetic diffusion, i.e. because of collisions; consequently, the magnetic field strength and the region of nonvanishing magnetic field both grow with time $\tilde{t} = t - z/c\beta'$, as counted from the transit of the beam front through the plane $z = \text{const}$. Here β' is the beam velocity in units of c . Anomalous values of the collision frequency ν may be used, which amounts in effect to taking into account a certain amount of nonlinear plasma response.

The behavior of the parallel, cold beam is usually estimated in the following way: For $\tilde{t} \lesssim t_D$, where t_D is the characteristic magnetic diffusion time,

$$t_D = \nu^{-1} \left(\frac{R \omega_p}{c} \right)^2, \quad (1.1)$$

one assumes unperturbed propagation of the beam as long as the ALFVÉN criterion (ALFVÉN, 1939) for the net current is satisfied. On the contrary, for $\tilde{t} \gtrsim t_D$ the beam is assumed to be destroyed if the beam current I' is overcritical, i.e. greater than the ALFVÉN current

$$I_A = \frac{m c^3}{e} \beta' \gamma' . \quad (1.2)$$

The notation in eqs. (1.1) and (1.2) is standard except for $\gamma' = 1/\sqrt{1-\beta'^2}$ and for R , the beam radius. From t_D a characteristic magnetic diffusion length $l_D = c \beta' t_D$ is sometimes derived.

LOVELACE and SUDAN (1971) appear to be the first to have estimated the "ALFVÉN time" t_A , at which the ALFVÉN criterion for the net current gets violated. They have not considered the influence of various radial profiles of the beam's current density, however. LOVELACE and SUDAN (1971) have also pointed out that the energy loss of the beam, owing to plasma heating, must be considered in discussing the behavior of the beam.

In this paper we do the following:

1. Introduce, and discuss the relevance of, several criteria for propagation of overcritical beams, i.e. with $I' > I_A$, through unmagnetized plasma. Estimates of the criteria are given in terms of a simple beam-plasma model derived from linear injection theory.
2. Study the validity of neglecting the nonlinear magnetic force upon the plasma electrons, as is customary in injection theory.

For simplicity the investigation is restricted to injection into an infinite, homogeneous plasma with infinitely heavy ions and without an external magnetic field \underline{B}_0 .

In an earlier paper (KÜPPERS et al., 1973 a) we have shown that, in the inertial phase of injection, the magnetic shielding of the beam current depends very strongly upon the radial profile of the beam current density. The influence of the beam profile was also considered by ROSINSKII et al. (1972). We are interested in the question how the propagation criteria depend on the radial beam profile. Because the difference in magnetic shielding for various beam profiles decreases in time owing to magnetic diffusion, the difference in the propagation properties of the beam will be particularly large if the time of violation of the relevant propagation criterion is small in units of the magnetic diffusion time t_D , i.e. if I'/I_A is large. In this paper we shall use a model that distinguishes between square profiles and smooth profiles of beam current density. Smooth profiles are defined by a current density that varies slowly over one skin distance c/ω_p (compare KÜPPERS et al., 1973 a).

From linear injection theory and from the propagation criteria critical beam times can be calculated numerically. We shall publish them together with analytical results in another paper (KÜPPERS et al., 1973 c).

2. PROPAGATION CRITERIA

We have chosen five criteria that we think are relevant in discussing the question of beam propagation. They are:

1. The ALFVÉN criterion (ALFVÉN, 1939).
2. The LAWSON criterion (LAWSON, 1957).
3. The magnetic-energy criterion.
4. The virial criterion.
5. The heating criterion.

Criteria 1 to 4 concern the interaction of the net magnetic field and the beam and must be discussed with special regard to the radial profile of the beam current density. It will turn out that the diffusion time t_D is to an order of magnitude an upper bound to the actual beam destruction times derived from criteria 1 to 4, but that these destruction times may be much earlier than t_D if I'/I_A is sufficiently large. Criterion 5 concerns the energy loss of the beam due to plasma heating.

We shall assume that return current heating is the dominant loss process (LOVELACE and SUDAN, 1971).

An important point is the following. Injection theory shows that for all practical purposes the net magnetic field \underline{B} depends only on r and $\tilde{t} = t - z/c\beta'$. Hence criteria 1 to 4 are really conditions on \tilde{t} , i.e. they give maximal effective pulse times, or break-off times, \tilde{t}_{\max} of an injected beam. In other words, criteria 1 to 4 have to do with beam interruption at the beam's point of entry into the plasma. On the contrary, the heating criterion determines a

maximum free path L_H of the beam in the sense that the beam will have lost its total energy by plasma heating after transversing the distance L_H .

Under certain conditions one may calculate maximum beam lengths l_{\max} from the break-off times \tilde{t}_{\max} . This is possible if the relevant break-off times are smaller than $L_H/c\beta'$ (Fig. 1). Otherwise the maximum beam length is given by L_H (Fig. 2). Figures 1 and 2 are oversimplified in that the path L_H is assumed to be constant in time. In reality the return current and the energy loss by return-current heating both decrease with $\tilde{t} = t - z/c\beta'$; hence L_H will be an increasing function of time. This increase will be enhanced if, in addition, the electrical resistivity decreases with plasma temperature increasing in time.

Criterion 1

The ALFVÉN criterion (ALFVÉN, 1939) for propagation of a cold, parallel electron beam is defined by

$$R_e' > \delta r. \quad (2.1)$$

Here R_e' is the beam's gyroradius formed with the average beam velocity:

$$R_e' = \frac{m c^2 \beta \gamma'}{e B}, \quad (2.2)$$

where B is a characteristic value of the net self-field of the beam-plasma system. The quantity δr describes the radial extent of the magnetic field within the beam volume, i.e. $\delta r \lesssim R$. Theory shows (KÜPPERS et al, 1973 a)

that injection of a beam into plasma initially yields $\delta r \sim R$ for smooth beam profiles, and $\delta r \sim \frac{c}{\omega_p} < R$ for square beam profiles. We exclude more exotic possibilities, such as hollowed-out beams, from our discussion. In the case of the square profile δr grows with time until, finally, $\delta r \sim R$.

First we shall express the ALFVÉN criterion by suitable beam parameters; then its physical content will be discussed.

The ALFVÉN criterion can be expressed as

$$\frac{v'}{y'} \equiv \frac{I'}{I_A} < \frac{s R}{2 \delta r} \quad , \quad (2.3)$$

where $I', I_A, R, \delta r$ have been explained above, and s is the magnetic shielding factor:

$$s = \frac{B^*}{B} \sim \frac{I'}{I} \geq 1. \quad (2.4)$$

Here B^* is the magnetic field of the unshielded beam, and I is the net current, i.e. the sum of the beam and plasma currents, taken, for example, where $B = B_{\max}$. The ratio I'/I_A is, of course, identical with the well-known beam parameter v'/y' , where

$$v' = \frac{N' e^2}{m c^2} \quad (2.5)$$

is the line density of the beam in units of the inverse classical electron radius.

During the inertial phase of beam injection, $v\tilde{t} < 1$, the ALFVÉN criterion is satisfied (KÜPPERS et al., 1973 a) because $n' < n$,

$$\frac{sR}{\delta r} \sim \left(\frac{R\omega_p}{c}\right)^2, \quad (2.6)$$

and

$$\frac{v'}{y'} \equiv \frac{I'}{I_A} \sim \left(\frac{R\omega_p}{c}\right)^2 \frac{n'}{4ny'}. \quad (2.7)$$

This holds for arbitrary beam currents I' according to linear theory. At later times \tilde{t} the right-hand side of eq. (2.3) decreases until, finally, the ALFVÉN criterion will be violated. To an order of magnitude the corresponding ALFVÉN time t_A will be earlier than the characteristic magnetic diffusion time t_D , at which $s = 1$, $\delta r \sim R$ may be assumed. At the ALFVÉN time magnetic shielding still prevails, i.e.

$$s(t_A) > 1. \quad (2.8)$$

The larger I'/I_A at given values of $c/R\omega_p$ and v the smaller the ratio t_A/t_D will be.

We now discuss the significance of the ALFVÉN criterion. It is generally assumed that when the ALFVÉN criterion is satisfied this means that a parallel, cold electron beam can propagate unperturbed by the magnetic self-field of the beam-plasma system. Examples of the validity of this assumption are given by selfconsistent equilibrium configurations of beams in vacuo that are partially neutralized by imbedded ions (BENFORD and BOOK, 1971). If, on the contrary, the ALFVÉN criterion is violated and

the electric forces are small compared with the magnetic forces, as is the case for beam injection (see KÜPPERS et al., 1973 c), then clearly a propagating beam can no longer be parallel and cold, but must assume a more complicated structure. Again, selfconsistent equilibrium configurations lend themselves as examples, viz. warm beams with nonvanishing transverse velocities (BENFORD and BOOK, 1971), or force-free, nonaxisymmetric beams (YOSHIKAWA, 1971).

In the case of an injected beam with smooth current profile (ROSINSKII et al., 1972; KÜPPERS et al., 1973 a) the magnetic field is distributed over the whole beam volume ($\delta r \sim R$) and one expects the ALFVÉN criterion soon to be violated for a whole range of radii, once it is violated at all. Then the development of a different beam structure is necessary over much of the beam volume if propagation is to be maintained. If such a new state cannot be reached, for instance owing to instabilities, then the beam will be destroyed upon violation of the ALFVÉN criterion, probably with a time delay of the order of the beam's gyroperiod. We therefore expect that an injected beam with smooth radial profile will alter its state considerably upon violating the ALFVÉN criterion and will probably be broken off.

The picture is more complex for an injected beam with square profile of its current density (COX and BENNETT, 1970; HAMMER and ROSTOKER, 1970; ROSINSKII and RUKHADZE, 1972; RUKHADZE and RUKHLIN, 1972). If the ALFVÉN criterion is violated at early times when $t_A \ll t_D$ and $\delta r \ll R$ (this happens for I'/I_A sufficiently large) then the ALFVÉN criterion is

violated only in the thin magnetic layer of thickness δr and the bulk of the beam will not be perturbed. If the ALFVÉN criterion is violated at later times, when $t_A \lesssim t_D$ and $\delta r \lesssim R$, then the whole beam will be affected as in the case of the smooth radial profile of the beam. Hence we expect violation of the ALFVÉN criterion to be important for a beam with square profile only when it occurs at sufficiently late times $t_A \lesssim t_D$ such that δr is of the order of the beam radius.

Criterion 2

The LAWSON criterion (LAWSON, 1957; KÜPPERS et al., 1973 b) generalizes the ALFVÉN criterion by including the effect of electric forces on the beam particles. For beams injected into nonrelativistic (unmagnetized) plasma the LAWSON criterion virtually coincides with the ALFVÉN criterion because $E/B \sim (v_e^{th}/c)^2 \ll 1$ for smooth beam profiles, $E/B \sim v_e^{th}/c \ll 1$ for square beam profiles (see KÜPPERS et al., 1973 c).

Criterion 3

If the magnetic energy contained in a unit length of the beam's volume, viz.

$$W_{\text{mag}} = 2\pi \int_0^R \frac{B^2}{8\pi} r dr, \quad (2.9)$$

is greater than the beam's kinetic energy per unit length, viz.

$$W_{kin} = N' m c^2 (\gamma' - 1), \quad (2.10)$$

then a drastic change of the state of the beam, e.g. strong pinching, is energetically possible. Hence we define

$$W_{mag} < W_{kin} \quad (2.11)$$

as the "magnetic-energy criterion" for unperturbed propagation of a parallel beam. We shall discuss the significance of this criterion together with that of the virial criterion below.

With the use of the simple beam-plasma injection model introduced above the magnetic-energy criterion can be expressed approximately in the form

$$\frac{R_e'}{\delta r} > \frac{\gamma' + 1}{2s\gamma'} \quad , \quad (2.12)$$

or, in terms of beam parameters,

$$\frac{\gamma'}{\gamma'} \equiv \frac{I'}{I_A} < \frac{s^2 R}{\delta r} \frac{\gamma'}{\gamma' + 1} \quad . \quad (2.13)$$

At the ALFVÉN time the magnetic-energy criterion is satisfied because of equation (2.8); it will be violated at a later time $\tilde{t} = t_M$, which is earlier than a time of the order t_D . In short, $t_A < t_M \lesssim t_D$. Thus the magnetic-energy criterion is less restrictive than the ALFVÉN criterion for injected beams, in contrast to beams in vacuo. The larger I'/I_A at given values of $R\omega_p/c$ and ν the smaller the ratio t_M/t_D will be.

Criterion 4

The virial equation (CAP, 1972; KÜPPERS et al., 1973 b) for a beam-plasma system of cylindrical symmetry, with total electric charge Q zero, connects the total net current

$$I(\infty) = 2\pi \int_0^{\infty} j(r) r dr \quad (2.14)$$

with the space integral of the total, radial kinetic pressure increment of the beam and the plasma

$$\Delta P(\infty) = 2\pi \int_0^{\infty} [p(r) - p(\infty)] r dr \quad (2.15)$$

by

$$I^2(\infty) = 2c^2 \Delta P(\infty). \quad (2.16)$$

For deriving a propagation criterion another form of the virial equation in which integrals over r are extended only from 0 to R is more advantageous, viz.

$$I^2(R) = c^2 Q^2(R) + 2c^2 \Delta P(R) - 2\pi r^2 c^2 [p(R) - p(\infty)]. \quad (2.17)$$

Here $Q(R)$ is the electric charge in the volume considered. The reason for choosing equation (2.17) rather than equation (2.16) is that $I(\infty) = 0$ for beam injection into plasma (KÜPPERS et al., 1973 c).

In order to estimate whether catastrophic beam pinching is excluded by the radial kinetic pressure the beam can acquire by radial deflection of its particles, we neglect the contributions of the plasma and of electric charge and use as an upper bound of the radial beam pressure the value obtainable by radial particle deflection, viz.

$$\max \Delta P'(R) = m c^2 N' \left(\gamma' - \frac{1}{\gamma'} \right). \quad (2.18)$$

Hence we may define

$$I^2(R) < 2 m c^4 N' \left(\gamma' - \frac{1}{\gamma'} \right) \quad (2.19)$$

as the "virial criterion" for unperturbed propagation of a parallel beam. The criterion presupposes that gradients in the direction of beam propagation may be neglected as small. The criterion can be expressed in terms of beam parameters thus:

$$\frac{\gamma'}{\gamma'} \equiv \frac{I'}{I_A} < 2 s^2, \quad (2.20)$$

or

$$I^2 < 2 I' I_A \quad (2.21)$$

where $s \gtrsim 1$ is again the magnetic shielding factor. The magnetic-energy criterion and the virial criterion are approximately equal for $\delta r \sim R$, but differ for $\delta r \ll R$, in which case the virial criterion is more restrictive than the magnetic-energy criterion.

It is easy to show that the virial criterion is satisfied in the inertial regime. If t_V is the time \tilde{t} of violation of the virial criterion one derives $t_V \lesssim t_M$ and $t_V \lesssim t_D$. The larger I'/I_A at given values of ν and $R\omega_p/c$ the smaller the ratio t_V/t_D will be. Contrary to t_M , $t_V \gtrsim t_A$ cannot be shown by simple model estimates, but it turns out to be true numerically in the parameter range considered (see KÜPPERS et al., 1973 c). One would not believe beam pinching to occur for a $t_V < t_A$ because strong deflection of beam particles should not happen, if the ALFVÉN criterion were satisfied at $\tilde{t} = t_V$.

We propose that violation of the magnetic-energy criterion and/or the virial criterion approximately determine the time \tilde{t} at which catastrophic beam pinching occurs in all cases such that violation of the ALFVÉN criterion has not led to interrupting the beam. As mentioned, we think this is true of beams with square profiles and I'/I_A so large that $t_A \ll t_D$ and $\delta r \ll R$. In all other cases we think that the beam is interrupted already at $\tilde{t} = t_A$ and that Criteria 3 and 4 do not apply.

Criterion 5

The maximum free path of the injected beam is limited by the energy loss of the beam to the plasma, which we assume to be determined by return current heating (LOVELACE and SUDAN, 1971). Because electric and magnetic shielding is good for $n' < n$, $\tilde{t} < t_D$, the total beam energy can be equated

with its kinetic energy, and the maximum free path L_H of the beam is determined by

$$\int_0^{L_H} \eta j_e^2(z, \tilde{t}) dz = n' m c^3 (\gamma' - 1) \beta'. \quad (2.22)$$

Here η is the resistivity of the plasma and j_e is the plasma return current density. We define the condition $z < L_H(\tilde{t})$ as the "heating criterion".

While magnetic shielding is good, equation (2.22) may be evaluated with the approximation $j_e^2 \sim j'^2$. Then

$$L_H \sim \nu^{-1} \frac{c n (\gamma' - 1)}{n' \beta'}, \quad (2.23)$$

where again ν is the collision frequency of the plasma electrons. Comparison with the characteristic magnetic diffusion length $l_D = \beta' c t_D$ yields

$$\frac{L_H}{l_D} \sim \frac{\gamma'}{4(\gamma' + 1)} \left(\frac{I'}{I_A} \right)^{-1}. \quad (2.24)$$

Hence the maximum free path L_H of the beam is smaller than the diffusion length l_D whenever $I' > \frac{1}{4} I_A$. In these cases l_D has no direct physical significance. Generally, the maximum beam path will be L_H while the maximum beam length will be determined by the minimum of L_H and the break-off length $l_{\max} = \beta' c \tilde{t}_{\max}$ as derived from the pertinent one of Criteria 1 to 4 (see Figures 1 and 2).

A more accurate evaluation of L_H is in principle possible by numerical evaluation of injection theory; however, this will be significant only for late beam times \tilde{t} in cases where $\tilde{t}_{\max} \sim t_D$.

3. VALIDITY OF LINEAR INJECTION THEORY

Among the approximations made in linear injection theory neglecting the nonlinear magnetic force on the plasma electrons is particularly restrictive and, at the same time, its validity can be estimated to some degree. This we shall do now.

LEE (1971), LEE and SUDAN (1971) have investigated beam injection into plasma perpendicular to an external magnetic field \underline{B}_0 . They found deviations from the case $\underline{B}_0 = 0$ for magnetic shielding at small times whenever the electron gyrofrequency Ω_{e0} was greater than the plasma frequency, viz. for $\Omega_{e0} > \omega_{pe}$. We conclude that a necessary condition for the linearized injection theory to be valid is

$$\Omega_e < \omega_p \quad (3.1)$$

where now Ω_e is the gyrofrequency of the plasma electrons in the magnetic self-field \underline{B} of the beam-plasma system (no \underline{B}_0 present). We shall refer to this condition as "the first LEE condition" for brevity.

We shall at least require that equation (3.1) still be satisfied at the ALFVÉN time in order that the result of the calculation be valid. For a smooth beam profile ($\delta r \sim R$) the quantity Ω_e/ω_p at the ALFVÉN time can be obtained from the ALFVÉN criterion, equation (2.1), viz.

$$\frac{\Omega_e}{\omega_p}(t_A) \sim \frac{\beta' \gamma' c}{\omega_p R} \sim \frac{\beta'}{2} \left(\frac{n' \gamma'}{n} \right)^{\frac{1}{2}} \left(\frac{I'}{I_A} \right)^{-\frac{1}{2}}, \quad (3.2)$$

such that for a wide range of parameters equation (3.1) is satisfied. The evaluation for a square profile requires knowing $\delta r(t_A)$ and $\Omega_e(t_A)$.

From KÜPPERS et al. (1973 c) it follows that at least in the range q between 1 and 100, where

$$q = \frac{R\omega_p}{2c} \frac{1}{\sqrt{\nu t}} \quad (3.3)$$

δr is approximately given by

$$\delta r \sim \frac{R}{3q}, \quad (3.4)$$

while $\Omega_e(t_A)$ can be obtained from

$$\Omega_e \sim \frac{\omega_p \beta' n'}{2n} \sqrt{\nu t} \quad (3.5)$$

for $\nu t > 1$. From equations (2.1), (3.3) to (3.5) it follows that for a square beam profile Ω_e/ω_p at the ALFVÉN time is given by

$$\frac{\Omega_e}{\omega_p}(t_A) \sim \beta' \sqrt{\frac{3\gamma' n'}{4n}}, \quad (3.6)$$

which, again, satisfies equation (3.1) in a wide range of parameters.

LEE (1971) has shown that deviations from the case $\underline{B}_0 = 0$ still exist for $\Omega_{e0} < \omega_p$ at larger times. Whenever $t_{DL} < t_D$, where

$$t_{DL} = \frac{R\omega_p}{c\beta'\gamma'\Omega_{e0}} \sqrt{\frac{m_i}{m_e}}, \quad (3.7)$$

t_D being defined by equation (1.1), then the characteristic magnetic diffusion time is actually given by t_{DL} rather than by t_D (LEE, 1971).

If one assumes that this result can also be applied to the present case of the magnetic self-field one should require that t_{DLA} be at least larger than the ALFVÉN time t_A , where

$$t_{DLA} = \frac{R \omega_p}{c \beta' \gamma' \Omega_e(t_A)} \sqrt{\frac{m_i}{m_e}} \quad (3.8)$$

Evaluation of this stricter condition ("second LEE condition") requires explicit knowledge of t_A . We restrict the discussion to beams of square radial profiles; from equations (2.1), (3.3) to (3.5) one obtains

$$t_A \sim \frac{3 \gamma' m}{v n'} \quad (3.9)$$

Hence

$$\frac{t_{DLA}}{t_A} \sim \frac{4}{3\sqrt{3}} \frac{1}{\beta'^2 \gamma'^2} \frac{v}{\omega_p} \sqrt{\frac{m_i I'}{m_e I_A}} \quad (3.10)$$

For hydrogen plasma and even for a large anomalous collision frequency $\nu \sim \omega_p / 100$ (GUILLORY and BENFORD, 1973) the validity condition $t_{DLA} > t_A$ is difficult to satisfy for energetic beams with $\gamma' \gg 1$. For instance $\gamma' = 10$ would require $I'/I_A \gtrsim 10^5$. Higher ion masses somewhat improve the situation.

In a previous paper (KÜPPERS et al., 1973 a) it was shown that during the inertial phase of injection the macroscopic gyroradius R_e of the plasma electrons is of the order of δr . Hence a sufficient validity condition for omitting the nonlinear magnetic force is given by the "gyro condition":

$$\Omega_e(\tilde{t}) \cdot \tilde{t} < 1, \quad (3.11)$$

i.e. the time \tilde{t} counted from the transit of the beam front should be shorter than the gyroperiod, calculated at time \tilde{t} , of the plasma electrons.

In the inertial regime, $v\tilde{t} < 1$, the gyro condition is satisfied only for $\Omega_e < \nu$. Even for large anomalous collision frequencies, $\nu \sim \omega_p / 100$ (GUILLORY and BENFORD, 1973) this requires

$$\frac{\beta' m'}{2n} < \begin{cases} 10^{-2} \frac{R\omega_p}{c} & \text{for smooth profiles,} \\ 10^{-2} & \text{for square profiles.} \end{cases} \quad (3.12)$$

For the square beam profile this is a serious restriction.

At the ALFVÉN time the gyro condition is satisfied only if

$$t_A < \frac{\delta r}{c \beta' \gamma'} \quad (3.13)$$

This requires the ALFVÉN criterion to be violated at equivalent distances shorter than the beam radius. It follows that the gyro condition is a very strict condition that will be seldom satisfied at break-off times t_A , t_M , or t_V . However the results of LEE (1971), LEE and SUDAN (1971) suggest that the gyro condition is actually too severe.

A necessary and sufficient validity condition of the linear injection theory is unknown. The results by LEE (1971), LEE and SUDAN (1971), employed above, are restricted to plane geometry, a uniform and time-constant external \underline{B}_0 -field and a square profile of the beam. It is therefore not

completely clear whether these results can indeed be applied to the present case. It appears that a nonlinear injection theory including the magnetic force and the response of the ions would be very desirable in order to clear up these problems.

4. SUMMARY AND CONCLUSION

We have discussed the relevance of several propagation criteria for overcritical, relativistic electron beams injected into unmagnetized plasma. Estimates of the criteria were given in terms of a simple beam-plasma model derived from linear injection theory. Numerical results will be published elsewhere (KÜPPERS et al., 1973 c). Criteria 1 to 4 of Section 2 limit the effective pulse time of the beam, while Criterion 5 determines the maximum free path of the beam (see Figures 1 and 2). We have shown that the propagation criteria depend, in principle, on the radial profile of the beam's current density. Moreover we suggest that beams with smooth profiles, and beams with square profiles at sufficiently small I'/I_A , such that $\delta r \sim R$, will probably be interrupted at the ALFVÉN time t_A , while beams with square profiles at sufficiently large I'/I_A , such that $\delta r \ll R$, will probably be interrupted by strong pinching at the time when the magnetic-energy criterion or the virial criterion is violated. These times are later than the ALFVÉN time. Whenever I'/I_A is sufficiently large, beam interruption is expected to occur at times that are small compared with the characteristic magnetic diffusion time t_D .

Numerical results (KÜPPERS et al., 1973 c) for beams with parabolic profiles and with square profiles, respectively, for $I'/I_A \sim 1$ to 100, $n'/n \sim 10^{-1}$ to 10^{-3} , and $v/\omega_p = 10^{-2}$ indicate that $\omega_p t_A \sim 10^4$ to 10^6 , and $\omega_p t_V \sim 10^5$ to 10^7 . Beam interruption should be observable if these times are shorter than the pulse time t_p of the beam, i.e. for plasma densities n greater than a critical density n_c :

$$n > n_c = \frac{m}{4\pi e^2} \left(\frac{\omega_p \tilde{t}_{max}}{t_p} \right). \quad (4.1)$$

For example $t_p = 50$ ns and $\omega_p \tilde{t}_{max} = 10^4$ or 10^7 gives $n_c = 1.2 \times 10^{13}$ cm⁻³ or 1.2×10^{19} cm⁻³, respectively. To our knowledge such limitations of the pulse time of the beam have not been found in experiments so far. In fact up to now few experiments have been made that conform to the assumptions of the theory considered by us, and of which the relevant parameters have all been published. An experiment relevant for us is perhaps the one by LEVINE et al. (1971), where $I'/I_A \sim 2.5$, $n'/n \sim 10^{-3}$, $n_e \sim 10^{14}$ cm⁻³, $t_p \sim 50$ ns. The theoretical break-off times of the beam are, in the case of a parabolic profile, $t_A \sim 1800$ ns, and in the case of a square profile, $t_A \sim 3600$ ns, which are both very long compared with the pulse time and, hence, could not be measured experimentally. However, at higher plasma densities n and/or higher values of n'/n the break-off time can become shorter than a beam pulse time of the order of 100 ns.

A second result concerns the validity of neglecting the nonlinear magnetic force upon the plasma electrons in injection theory. We think that a necessary condition of validity is given by the first LEE condition, $\Omega_e < \omega_p$, while a sufficient condition is given by the gyro condition, $\Omega_e(\tilde{t}) \tilde{E} < 1$. We have estimated these conditions at the ALFVÉN time, the result being that the first LEE condition is satisfied in a wide range of parameters, while the gyro condition is not satisfied in situations of practical interest. A validity

condition more restrictive than the first LEE condition, the second LEE condition, was also investigated, although its significance in the present case is not altogether clear. All three conditions of validity depend on the radial profile of the beam current density. They should be considered when beam propagation is investigated with the aid of linear injection theory and propagation criteria. Our estimates seem to show that truly reliable results on beam propagation can often be obtained only by developing a nonlinear injection theory including the magnetic force and the response of the ions.

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FIGURE CAPTIONS

Figure 1. Schematic drawing of beam propagation in the case

$$\tilde{t}_{max} < L_H / c\beta'.$$

Figure 2. Schematic drawing of beam propagation in the case

$$\tilde{t}_{max} > L_H / c\beta'.$$

Fig. 1

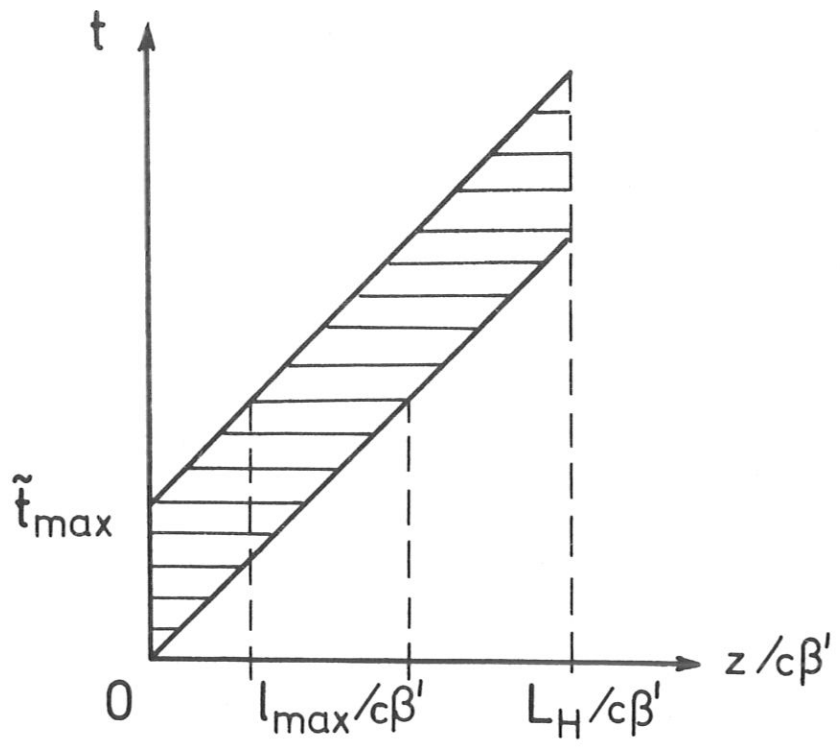
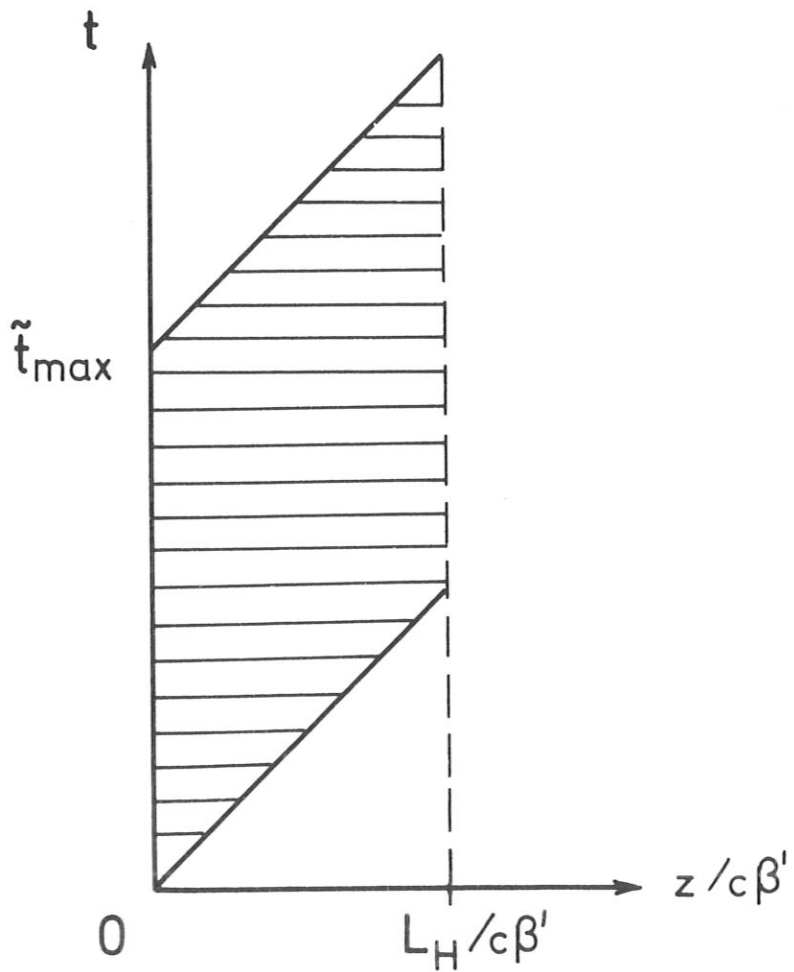


Fig. 2



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