

Calculation of magnetic coordinates for stellarator fields

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We present an extension of the Nemov algorithm [1] to compute magnetic coordinates in stellarators and their associated metric coefficients using only field line tracing, and covering not only the core of the plasma, but also island chains and scrape-off layer regions in the plasma edge. The algorithm is applied to the full W7-X vacuum field geometry. The method has been optimized for numerical accuracy by minimizing recourse to derivatives of derived quantities. The only requirements are the foreknowledge of the magnetic field and its topological structure.

1 Introduction

The magnetic coordinate system for a given magnetic field plays a fundamental role in the study of confined plasmas in stellarator devices. Very often, physics problems of interest are only tractable provided one works in a suitable magnetic coordinate system, for example one in which the field lines are straight [2, 3], or where some symmetry or quasi-symmetry of the system is made apparent [4]. Several methods have been developed over the years to establish relevant coordinates and compute geometrical quantities such as metric coefficients for different magnetic field configurations [1, 3, 5, 6, 7]. These are best adapted to cases with nested closed flux surfaces. We present here the extension of one such method [1] to the plasma edge of stellarators such as W7-X [8], generalized so as to be able to treat the presence of separatrices and of diverted islands. The method requires only a foreknowledge of the (total) magnetic field, and is optimized for numerical accuracy. It is however limited to non-ergodic fields. Ultimately, it delivers the full set of magnetic coordinates and metric coefficients. These can then be further used for plasma modelling, for example in the 3-D finite-volume plasma fluid code BoRiS [9].

2 General algorithm

Our goal is to obtain, starting from the magnetic field information only, and with good numerical accuracy, a set of Boozer-like coordinates, applicable throughout the edge plasma of complicated devices like the W7-X stellarator, including the treatment of

separatrices and island chains. The method described here involves only field line tracing, and minimizes the need for computing spatial derivatives of derived quantities. We refer to our end coordinates as the BoRiS coordinate system, as the algorithm was initially developed for use by the BoRiS code, but without loss of generality. We use both a Cartesian (x, y, z) and a cylindrical (R, φ, z) coordinate system for the spatial variables. The Boozer coordinate system is noted (ψ, θ, ζ) and the BoRiS coordinate system (s, ϑ, ϕ) . We assume the (non-ergodic) magnetic field is given on a cylindrical grid within an annular box domain $[\mathcal{B}]$ ranging over $[R_{min} \dots R_{max}; 0 \dots 2\pi; Z_{min} \dots Z_{max}]$. In essence, we will be mapping the spatial box $[\mathcal{B}]$ to $m + 1$ boxes in Boozer space, with m domains ranging over $[0 \dots F_T(\Psi_{sep}^{island}); 0 \dots 2\pi; 0 \dots 2\pi]$ for the m islands of the plasma edge and one domain, covering $[0 \dots F_T(\Psi_{SOL}); 0 \dots 2\pi; 0 \dots 2\pi]$, which includes both the core and SOL regions (beyond the island chain), where $F_T(\Psi_{sep}^{island})$ and $F_T(\Psi_{SOL})$ represent the total toroidal flux enclosed within the island separatrix or the last usable flux surface in the SOL, respectively. These domains will then get renormalized into $m + (1 + \varepsilon_{SOL})$ unit cubes $[0 \dots 1; 0 \dots 1; 0 \dots 1]$ in BoRiS coordinates, where ε_{SOL} denotes the normalized thickness of the SOL in flux coordinates. This mapping and the calculation of the metric coefficients describing it are the goal of this work. We will thus compute:

- The rotational transform: $\iota(\psi)$
- The flux surface function: $F_T \equiv \psi \rightarrow s$ and its spatial radial derivatives $\left. \frac{\partial F_T}{\partial R} \right|_{\substack{z=0 \\ \varphi=0}}^{\pm}$
- The flux surface derivatives: $F'_T \equiv \frac{dF_T}{ds}$ and $F'_P \equiv \frac{dF_P}{ds}$
- The toroidal and poloidal currents: $J(\psi)$ and $I(\psi)$, respectively
- The Boozer and BoRiS Jacobians, $\mathcal{J}(\psi, \theta, \zeta)$ and $\mathcal{J}(s, \vartheta, \phi) \equiv \sqrt{g}$ respectively
- The angle-like coordinates: $\theta = 2\pi\vartheta$ and $\chi \equiv \int \vec{B} \cdot d\vec{\ell} \rightarrow \zeta = 2\pi\phi$
- The Clebsch vectors: $\nabla\psi$ and $(\nabla\theta - \iota\nabla\zeta)$
- Secondary metric coefficients: $\tilde{\beta}$, $\tilde{\sigma}$, $g_{\vartheta s}$, g^{ss} , $g^{\vartheta s}$ and $g^{\phi s}$.

The algorithm proceeds as follows. We begin with a first pass at field-line tracing, in which we identify the location of the magnetic axes, as well as the islands and SOL structure. A longer field line trace yields the field-line integral χ . We then re-arrange the field-line data so as to define the flux surfaces. Next, we compute the rotational transform ι , the toroidal flux function F_T and its radial derivatives. The toroidal (J) and poloidal (I) currents follow. The Jacobians for both the Boozer and the BoRiS coordinate systems can now be calculated. We continue with the angle-like coordinates θ , ϑ , ζ and ϕ . We resort to the Nemov algorithm [1] in order to determine the Clebsch components of the field: $\nabla\psi$ and its conjugate $(\nabla\theta - \iota\nabla\zeta)$. This also yields ∇s and g^{ss} information. Lastly, for non-vacuum fields, one must determine the covariant component of the magnetic field, $\tilde{\beta}$. Appropriate constitutive relations complete the set of needed metric coefficients and physical quantities.

3 The W7–X vacuum field case

We consider the standard ι case, where ι tends to 1 at the separatrix, and the edge is inhabited by a 5/5 chain of large islands, which will partly intercept divertor structures. Beyond the chain of islands, there also exists a so-called “laminar” zone, where flux surfaces are still well-defined. We refer to this laminar zone as the far Scrape-Off Layer (SOL), in extension of the tokamak nomenclature.

We show in Figures contour plots of g^{ss} for the two W7–X symmetry planes. It is clearly seen that g^{ss} goes to zero at all X- and O-points, but can exhibit rapid spatial variations.

References

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4 Figures

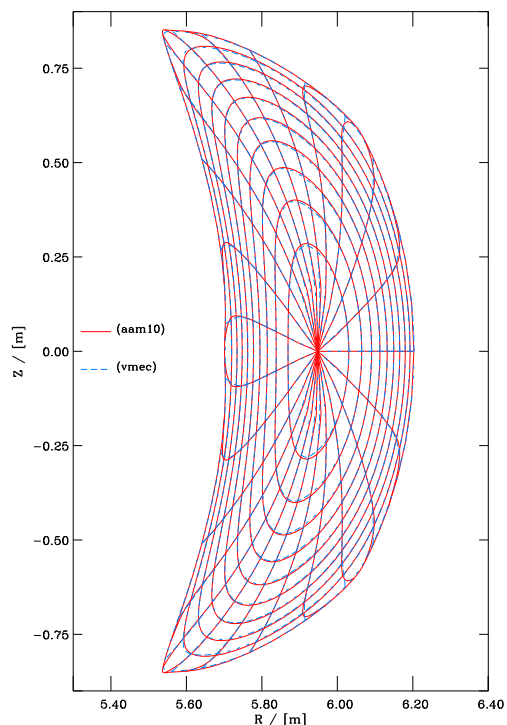


Figure 1: The (s, ϑ) gridding of the $(\varphi = 0)$ plane, according to `aam10` (in red), and `VMEC` (in dashed blue).

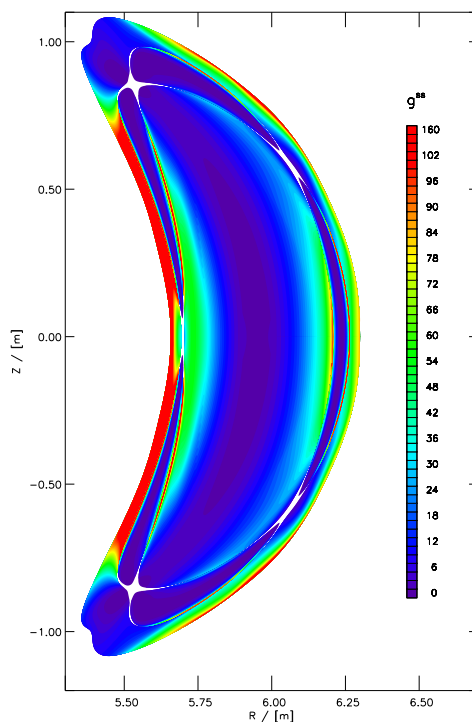


Figure 2: The g^{ss} variation on the $(\varphi = 0)$ symmetry plane, for the W7-X standard case vacuum field. The island values were scaled down by a factor of 100.

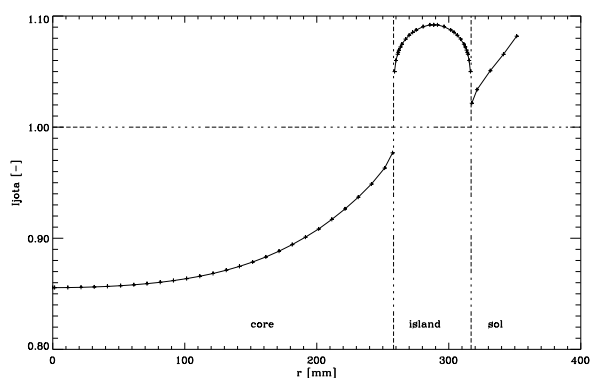


Figure 3: The rotational transform profile for the W7-X vacuum field case, including the island and outer scrape-off regions.

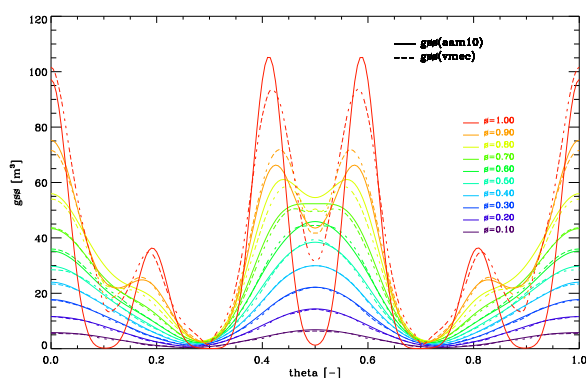


Figure 4: The variation of g^{ss} as a function of angle and flux surface index, on the $(\varphi = 0)$ symmetry plane for the W7-X case. The results of a `VMEC` run are shown for comparison.