Calculations of ECCD with parallel momentum conservation and finite collisionality

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Introduction. The adjoint approach [1] is an advanced and convenient method for calculation of the ECCD in plasmas which is applicable for both tokamaks and stellarators. A key point of the adjoint approach is the choice of model for the corresponding Spitzer function, which should conserve parallel momentum in like-particle collisions and must be valid for desired range of collisionality. The highly collisional (classical) limit, $\nu_e^* \to \infty$ (here, $\nu_e^* = \nu_e R/\iota v$ is the monoenergetic collisionality, $\nu_e(v) = \nu_{ee}(v) + \nu_{ei}(v)$ is the collision frequency, R and ι are the major radius and the rotational transform, respectively), where trapped particles do not play any role, gives the upper limit for CD efficiency, while the opposite collisionless limit, $\nu_e^* \to 0$, which accounts for trapped particles, tends to underestimate it. The latter, for which rapid and accurate solvers have been developed recently (see [2] and references therein), is appropriate for calculations of ECCD in hot plasmas. The intermediate collisional regime, with small but finite collisionality, $\nu_e^\star \ll 1$, where the contribution of barely trapped electrons can also be non-negligible, requires special attention [3].

In this work, we consider the generalized Spitzer problem in view of tools for calculation of ECCD efficiency suitable for implementation in ray-tracing codes.

Description of the model. For calculations of ECCD efficiency, the (adjoint) drift kinetic equation must be solved [1],

$$v_\parallel \nabla_\parallel (\chi f_{eM}) + C^{\rm lin}(\chi f_{eM}) = -\nu_{e0}\, \frac{v_\parallel}{v_{\rm th}}\, b\, f_{eM}. \qquad (1)$$

the general solution of which describes all necessary physics including the finite collisionality. Here, $\chi(\mathbf{r};\mathbf{v})$ is the local response function and $C^{\rm lin}$ is the linearized collision operator with conservation of parallel momentum. This problem is 4D (3D for tokamaks) and to date there are no well-tested solvers which are generally applicable. The NEO-2 code (see [4] and references therein) solves this problem by the field-line-tracing technique for all collisionalities, but the present version is applicable only for 3D problem and is not yet suitable for stellarators.

As an alternative approximation, the momentum correction technique (mct) [5] can be applied. This model is based on the concept of a collisionality-dependent "effective trapped particle fraction", $f_{\rm tr}^{\rm eff}(\nu_e^\star(x))$ with the limits $f_{\rm tr}^{\rm eff}(\nu_e^\star \to \infty) = 0$ and $f_{\rm tr}^{\rm eff}(\nu_e^\star \to 0) = f_{\rm tr}$, which describes the integral effect of finite collisionality (here, $f_{\rm tr}$ the "geometrical" fraction of trapped particles). This function can be constructed from the neoclassical monoenergetic transport coefficients, calculated by DKES (for details, see [5]). In this model, the solution of Eq.(1) is represented (similar to the collisionless solution) as a product of functions only of the global variables, $x = v/v_{\text{th}}$ and $\lambda = v_\perp^2/(v^2 b)$ with $b = B/B_{\max}$,

$$\chi^{\text{eff}}(x,\lambda) = \text{sign}(v_\parallel)\mathcal{H}^{\text{eff}}(x;\lambda)\, K^{\text{eff}}(x). \tag{2}$$

In contrast to the collisionless solution, $\mathcal{H}^{\text{eff}}(x;\lambda)$ contains also the monoenergetic collisionality (i.e. velocity) as a parameter,

$$\mathcal{H}^{\text{eff}}(x;\lambda) = \frac{1}{1 - f_{\text{tr}}^{\text{eff}}(\nu_e^\star(x))}\left((1-f_{\text{tr}})\mathcal{H}(\lambda) + h(1-\lambda)\delta f_{\text{tr}}^{\text{eff}}(x)\right) \tag{3}$$

(here, $\delta f_{\text{tr}}^{\text{eff}}(x) = f_{\text{tr}}^{\text{eff}}(\nu_e^\star(x)) - f_{\text{tr}}$, $\mathcal{H}(\lambda)$ is the collisionless pitch-dependence (see [2]), and $h(y)$ is the Heaviside function). Whereas $\mathcal{H}(1) = 0$, at finite but low $\nu_e^\star$ a trapped-passing-particle boundary layer of width $\propto \sqrt{\nu_e^\star}$ appears allowing for an additional nearly constant contribution only in the passing domain. This simple "off-set" modeling is very reasonable outside of this boundary layer, but fails within the layer. The contribution within the layer scales as $\nu_e^\star$ and is negligible compared to the "off-set" modeling with $\delta f_{\text{tr}}^{\text{eff}} \propto \sqrt{\nu_e^\star}$. The generalized Spitzer function, $K^{\text{eff}}(x)$, which takes into account also the monoenergetic viscosity due to the finite collisionality, can be caclulated from a 1D integro-diffential equation (for details, see [5]).

This approximate model can be easily applied for ray-tracing calculations. Despite its limited applicability (contrary to NEO-2, the model is applicable only for low collisionality, $\nu_e^\star \ll 1$), this model is very attractive, being applicable for arbitrary configurations. This approximate model can be easily applied for ray-tracing calculations. Despite its limited applicability (contrary to NEO-2, the model is applicable only for low collisionality, $\nu_e^\star \ll 1$), this model is very attractive, being applicable for arbitrary configurations.

Qualitatively, the impact of finite collisionality in ECCD arises in two ways. The first is a reduction of drag due to the barely trapped electrons and the second is a direct contribution of barely trapped electrons (if allowed by the cyclotron resonance condition). The leading order finite collisionality correction in ECCD scales as [3] $\delta j_\parallel \propto \sqrt{\nu_e^\star} \cdot j_\parallel^{\text{coll}}$ (here, $j_\parallel^{\text{coll}}$ is the current in collisional limit). Both these effects are described by the local solution of Eq.(1) and partly represented in the approximate solution Eq.(2). Below, we apply the "off-set" mct-model for circular tokamaks and benchmark it against the NEO-2 code.

Numerical results. In Fig. 1, the ECCD efficiency for X2-mode with fixed obliqueness as function of (normalized) magnetic field, $nY = n\omega_{ce}/\omega$ with $n = 2$ is shown. Two points of location of the RF-source with $N_\parallel = 0.256$ ($\beta = 15°$) are chosen, which correspond to the minimum (left) and maximum (right) of B. The plasma parameters chosen guarantee a rather large collisionality effects. Apart from the models with finite collisionality effects (labeled as

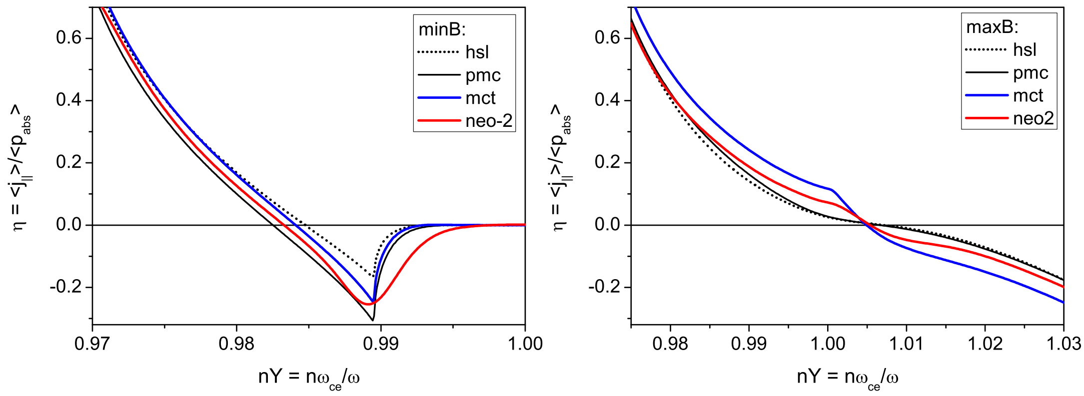

Figure 1: The local ECCD efficiency, $\eta = \langle j_{\|} \rangle / \langle p_{\rm abs} \rangle$, calculated by the different models for X2-mode with $N_{\|} = 0.256$ for a tokamak with $\epsilon = r/R = 0.2$ in $B_{\min}$ and $B_{\max}$ points (left and right, respectively). Plasma parameters: $n_e = 10^{20}$ m^{-3}, $T_e = 1$ keV and $Z_{\rm eff} = 1$.

"neo-2" and "mct"), also the collisionless model with parallel momentum conservation ("pmc") and its high-speed-limit ("hsl") were applied as reference.

When only passing electrons are in resonance, both the "off-set" mct-model and NEO-2 give a similar deviation from the collisionless solution. If, however, the resonance line touches the passing/trapped boundary, current diffusion from passing to barely trapped domain should appear. In Fig.1, this point corresponds to a sharp change of the slope in collisionless and mct-solutions. At the same time, the local solution from NEO-2 correctly describes the current diffusion and this is well seen from the smoothed transition. For the maximum of B, the "off-set" mct-model somewhat overestimates the ECCD efficiency in comparison with the NEO-2 results. Nevertheless, the qualitative dependence is the same as with NEO-2.

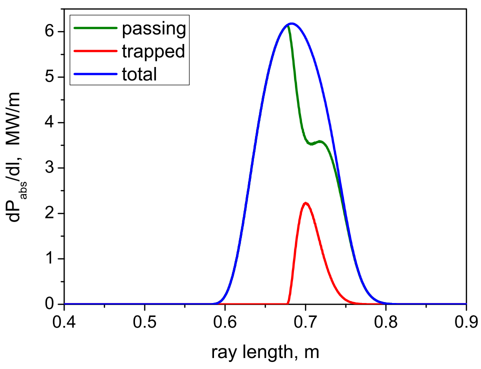

Figure 2: $dP_{\rm RF}/dl$ for 1 MW O2-mode, 140 GHz, $B_0 = 2.5$ T, in W7-X for $n_e = 10^{20}$ m^{-3}, $T_e = 2$ keV, $Z_{\rm eff} = 1.5$.

The role of finite-collisionality effects in W7-X was checked for both the X2- and O2-scenarios. The launch-port is located near the "bean-shaped" plane and the RF-beam is directed to the inner-wall mirror with $N_{\|} = 0.25$. The calculations were performed by the ray-tracing code TRAVIS (see [3])with the "off-set" mct-model implemented (the NEO-2 code is not yet applicable for stellarators).

For the O2-mode, the plasma is optically "gray" and participation of trapped electrons in the cyclotron interaction is unavoidable. Despite launching near the maximum of B, trapped electrons can absorb a significant part of the injected power. As example, the deposition along the ray is shown in Fig.2, where approximately 13% of the power is absorbed by trapped electrons.

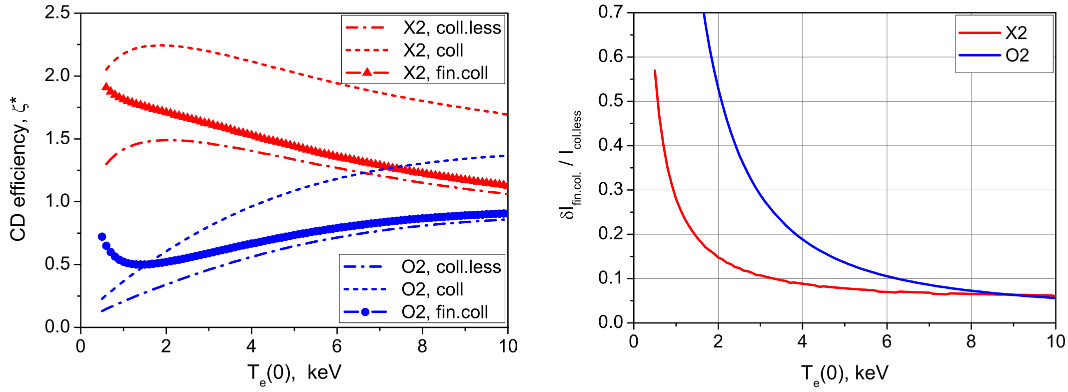

Figure 3: Left: T_e-dependence of the total ECCD efficiency, $\xi^* = 32.7\times10^{-20} n_e(0) R/T_e(0)\cdot I_{\rm tor}/P_{\rm RF}$, calculated for both X2- and O2-modes with $N_\parallel = 0.25$ in W7-X with $n_e = 10^{20}$ m^{-3} and $Z_{\rm eff} = 1.5$. Right: relative contribution to ECCD of the finite collisionality. Only single pass is considered.

In order to check the importance of finite collisionality, the electron temperature is scanned with all other parameters fixed. For comparison, both the X2-mode and the O2-mode were considered. In Fig.3 (left), the total ECCD efficiency calculated with finite collisionality is plotted. For reference, both analytical collisional and collisionless limits are depicted. In Fig.3 (right), the impact of finite collisionality in ECCD, $\delta I_{\rm fc}/I_{\rm cl}$, is shown (here, $\delta I_{\rm fc} = \delta I_{\rm fc} - I_{\rm cl}$ with $I_{\rm fc}$ and $I_{\rm cl}$ are the toroidal current values calculated with and without finite collisionality, respectively).

The different electron temperatures correspond to different locations of the cyclotron resonance line in velocity-space. Analysing the energy of electrons responsible for the dominant contribution to ECCD, one can estimate the (monoenergetic) collisionality for the resonance electrons. The effect of finite collisionality contributes to the ECCD practically up to $T_e = 10\,\mathrm{keV}$. Nevertheless, since the applicability of the “off-set” mct-model is limited only by the range $\nu_e^\star \ll 1$, the results for $T_e < 2$ keV are rather doubtful.

Summary. Finite collisionality effects in ECCD are important not only for low but also for high electron temperature plasmas, especially in stellarators. Both the NEO-2 code (slow & precise) and the “off-set” mct-model (“quick & dirty”) produce qualitatively similar results. Large ECCD can be obtained only for low collisionality, and here the leading order collisionality correction is calculated.

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