Class. Quantum Grav. **19** (2002) 5017–5024 PII: S0264-9381(02)37048-5

Galactic collapse of scalar field dark matter

Miguel Alcubierre[1,5], **F Siddhartha Guzmán**[1], **Tonatiuh Matos**[2], **Darío Núñez**[4,5], **L Arturo Ureña-López**[2,6] **and Petra Wiederhold**[3]

[1] Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1, D-14476 Golm, Germany
[2] Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, AP 14-740, 07000 México, DF, Mexico
[3] Departamento de Control Automático, Centro de Investigación y de Estudios Avanzados del IPN, AP 14-740, 07000 México, DF, Mexico
[4] Centre for Gravitational Physics and Geometry, Penn State University, University Park, PA 16802, USA
[5] Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, AP 70-543, 04510 México, DF, Mexico
[6] Astronomy Centre, University of Sussex, Brighton BN1 9QJ, UK

Received 15 May 2002, in final form 19 July 2002
Published 17 September 2002
Online at stacks.iop.org/CQG/19/5017

Abstract
We present a scenario for core galaxy formation based on the hypothesis of scalar field dark matter. We interpret galaxy formation through the collapse of a scalar field fluctuation. We find that a cosh potential for the self-interaction of the scalar field provides a reasonable scenario for the formation of a galactic core plus a remnant halo, which is in agreement with cosmological observations and phenomenological studies in galaxies.

PACS numbers: 0425D, 9530S, 9535, 9862A, 9880

In the last few years, the quest concerning the nature of the dark matter in the universe has received much attention and has become of great importance for understanding the structure formation in the universe. Some candidates for dark matter have been discarded and some others have recently appeared. The standard candidates of the cold dark matter (CDM) model are axions and WIMP'S (weakly interacting massive particles), which are themselves not free of problems. Axions are massive scalar particles with no self interaction. In order for axions to be an essential component of the dark matter content of the universe, their mass should be $m \sim 10^{-5}$ eV. With this axion mass, the scalar field collapses forming compact objects with masses of the order of $M_{\rm crit} \sim 0.6\, m_{\rm Pl}^2/m \sim 10^{-6}\, M_\odot$ [1, 2], which corresponds to objects with the mass of a planet. Since the dark matter mass in galaxies is ten times higher than the luminous matter, we would need tenths of millions of such objects around the solar system, which is clearly not the case. On the other hand, there are many viable particles with nice features in super-symmetric theories, such as WIMP'S, which behave just like standard CDM. However, a central debate nowadays is whether CDM can explain the observed scarcity of dwarf galaxies and the smoothness of the galactic-core matter densities, since high resolution numerical simulations with standard CDM predict an excess of dwarf galaxies and density profiles with cusps [3]. Even when there are some attempts to find a solution to these problems inside the CDM paradigm [4, 5], the debate is still open because new observations in galactic centres of dwarf galaxies do not show a real correspondence with CDM predictions [6, 7]. This is the reason why we need to look for alternative candidates that can explain both the structure formation at cosmological level, the observed amount of dwarf galaxies, and the dark matter density profile in the core of galaxies.

In a recent series of papers, we have proposed that the dark matter in the universe is of a scalar field nature with a strong self-interaction [8–14]. The scalar field has been proposed as a viable candidate, since it mimics standard CDM above galactic scales very well, reproducing most of the features of the standard lambda cold dark matter (ΛCDM) model [10, 11, 15, 16]. However, at galactic scales, the scalar field model presents some advantages over the standard ΛCDM model. For example, it can explain the observed scarcity of dwarf galaxies since it produces a sharp cut-off in the mass power spectrum. Also, its self-interaction can, in principle, explain the smoothness of the energy density profile in the core of galaxies [11, 12]. Nevertheless, when a new dark matter candidate is proposed, it is important to study the final object that would be formed as a result of a gravitational collapse of such candidate.

The formation of galaxies through gravitational collapse of dark matter is not an easy problem to understand. A good model for galaxy formation has to take into account all the observed features of real galaxies. For example, it seems that many disk galaxies contain a black hole in their centre, but others do not [17]. Typical galaxies are spiral, elliptical or dwarf galaxies (irregular galaxies may be galaxies still evolving). In most spiral and elliptical galaxies the luminous matter extends to $\sim$10–30 kpc, and the total content of matter (including dark matter) is of the order of 10^{10}–10^{12} $M_\odot$, with about ten times more dark matter than luminous one. The central density profile of the dark matter in galaxies should not be cusp [18]. Even though the luminous matter represents only a small fraction of the total amount of matter in galaxies, it plays an important role in galaxy formation and stability [8]. On the other hand, it is still not well established if the mass of the central black hole and the mass of the halo are correlated [19], etc.

There are some ideas in this respect when dealing with a scalar field. It is known that the final stage of a collapsed scalar field could be a massive object made of scalar field particles in quantum coherent states, such as boson stars (for a complex scalar field, see [20] and references therein) or oscillatons (for a real scalar field) [1, 2, 14, 21]. It is thus important to investigate whether the scalar field would collapse to form structures of the size of galaxies and provide the correct properties of any galactic dark matter candidate, such as growing or flat rotation curves and appropriate dark matter distribution functions. Also, we need to know which are the conditions that must be imposed on the scalar field particles.

Our main aim in this paper is to present a plausible scenario for the formation of a core in the galaxy with a remnant halo under the scalar field dark matter (SFDM) hypothesis. Through a gravitational cooling process [1, 2], a cosmological fluctuation of the scalar field collapses to form a compact oscillaton by ejecting part of the field. The key idea consists precisely in assuming that such final object could distribute as galactic dark matter does [14]. The final configuration then should consist of a central object (a core), i.e. an oscillaton, surrounded by a diffuse cloud of scalar field, both formed at the same time due to the same collapse process.

At the cosmological scale, one finds that the mass of the boson is not the only parameter that determines the power spectrum. The self-interaction of the scalar field is also very important. Following an analogous procedure to the one used in particle physics, we may write a phenomenological Lagrangian with all the terms we need in order to reproduce the observed universe. In particular, if one uses a minimally coupled real scalar field Φ with a self-interaction potential of the form [10, 22] ($\kappa_0 = 8\pi G$, we use natural units such that $\hbar = c = 1$)

$$V(\Phi) = V_0[\cosh(\lambda\sqrt{\kappa_0}\Phi) - 1], \tag{1}$$

then one can show [10, 11] that the SFDM model reproduces well all the successes of the standard ΛCDM above galactic scales. The free parameters of the scalar potential, V_0 and the scalar field mass $m_\Phi = \lambda\sqrt{V_0\kappa_0}$, can be fitted by cosmological observations. Doing this one finds that [11]

$$\lambda \simeq 20.28, \tag{2}$$

$$V_0 \simeq (3 \times 10^{-27} m_{\text{Pl}})^4, \tag{3}$$

$$m_\Phi \simeq 9.1 \times 10^{-52} m_{\text{Pl}} = 1.1 \times 10^{-23}\,\text{eV}, \tag{4}$$

with $m_{\text{Pl}} \equiv G^{-1/2}$ the Planck mass.

If at the centre of a galaxy there is an oscillaton, i.e. an oscillating solitonic object with coherent scalar oscillations around the minimum of the scalar potential (1), then the scalar field Φ and the metric coefficients (considering the spherically symmetric case) would be time dependent. It has been shown that such a configuration can be stable, non-singular and asymptotically flat [1]. For the scalar field collapse, the critical value for the mass of an oscillaton (the maximum mass for which a stable configuration exists) will depend on the mass of the boson. Roughly speaking, if we take $m_\Phi = 1.1 \times 10^{-23}$ eV, and use the formula for the critical mass of the oscillaton corresponding to a scalar field with a Φ^2 potential (i.e. just a mass term), we expect the critical mass to be [1, 2]

$$M_{\text{crit}} \sim 0.6\frac{m_{\text{Pl}}^2}{m_\Phi} \sim 10^{12}\,M_\odot. \tag{5}$$

This is a surprising result: the critical mass of the model shown in [10, 11] is of the order of magnitude of the dark matter content of a standard galactic halo.

In order to study this situation for the case of a potential of the form (1), we present a numerical simulation of Einstein's equations in which the energy–momentum tensor is that of a real scalar field. The scenario of galactic formation we assume is as follows: a sea of scalar field particles fills the universe and forms localized primordial fluctuations that could collapse to form stable objects, which we will interpret as the dark matter halos of galaxies.

We evolve the spherically symmetric line element

$$\mathrm{d}s^2 = -\alpha^2(r,t)\,\mathrm{d}t^2 + a^2(r,t)\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2, \tag{6}$$

with $\alpha(r,t)$ the lapse function and $a(r,t)$ the radial metric function. We choose the polar-areal slicing condition (i.e. we force the line element to have the above form at all times, so that the area of a sphere with $r = R$ is always equal to $4\pi R^2$). This choice of gauge will force the lapse function $\alpha(r,t)$ to satisfy an ordinary differential equation in r (see below).

The energy–momentum tensor of the scalar field is

$$T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \frac{g_{\mu\nu}}{2}[\Phi^{,\alpha}\Phi_{,\alpha} + 2V(\Phi)]. \tag{7}$$

We now introduce the first-order variables $\Psi = \Phi_{,r}$ and $\Pi = a\Phi_{,t}/\alpha$. Using these new variables, the Hamiltonian constraint becomes

$$\frac{a_{,r}}{a} = \frac{1-a^2}{2r} + \frac{\kappa_0 r}{4}[\Psi^2 + \Pi^2 + 2a^2 V], \tag{8}$$

and the polar-areal slicing condition takes the form

$$\frac{\alpha_{,r}}{\alpha} = \frac{a_{,r}}{a} + \frac{a^2-1}{r} - \kappa_0 r a^2 V. \tag{9}$$

All other components of Einstein's equations either vanish, or are a consequence of the last two equations.

The Klein–Gordon (KG) equation now reads

$$\Phi_{,t} = \frac{\alpha}{a}\Pi, \tag{10}$$

$$\Pi_{,t} = \frac{1}{r^2}\left(\frac{r^2\alpha\Psi}{a}\right)_{,r} - a\alpha\frac{\mathrm{d}V}{\mathrm{d}\Phi}, \tag{11}$$

$$\Psi_{,t} = \left(\frac{\alpha\Pi}{a}\right)_{,r}. \tag{12}$$

Equations (8)–(12) form the complete set of differential equations to be solved numerically. For numerical purposes, the evolution equation for Π above is further transformed into the equivalent form

$$\Pi_{,t} = 3\frac{\mathrm{d}}{\mathrm{d}r^3}\left(\frac{r^2\alpha\Psi}{a}\right) - a\alpha\frac{\mathrm{d}V}{\mathrm{d}\Phi}. \tag{13}$$

Note that the first term on the right-hand side of this equation includes now a first derivative with respect to r^3 (and not a third derivative). The reason for doing this transformation has to do with the numerical regularization near the origin of the $1/r^2$ factor in equation (11) above (see [23]).

In order to deal with non-dimensional units, we define $x = lr$. A natural scale for the potential is given by $l^{-1} = 1/\sqrt{\kappa_0 V_0} = 12\,\mathrm{pc} = 40\,\mathrm{yr}$. The parameter l also gives the time scale $\tau = tl$. The scalar field has an initial Gaussian profile

$$\sqrt{\kappa_0}\Phi(x, t = 0) = A\,\mathrm{e}^{-x^2/s^2}, \tag{14}$$

with A the amplitude and s the width of the Gaussian. The physical properties describing the state of the system are the energy density of the scalar field $\rho_\Phi = \left(m_{\mathrm{Pl}}^2 l^2 / 8\pi\right)\rho_s$, with ρ_s the dimensionless quantity

$$\rho_s = \frac{1}{2a^2}(\Psi^2 + \Pi^2) + V, \tag{15}$$

and the integrated mass

$$M(x) = \frac{1}{2}\frac{m_{\mathrm{Pl}}^2}{l}\int_0^x \rho_s(X) X^2\,\mathrm{d}X. \tag{16}$$

Some details about our numerical implementation are in order. To integrate the KG equation numerically we use a method of lines with standard centred second-order finite differences in space, and a third order in time integrator. At the outer boundary we impose a condition for simple outgoing radial waves. We deal with the singularity at $x = 0$ by straddling the origin and imposing the adequate parity conditions for each function on a auxiliary point at $x = -\Delta x/2$. The ordinary differential equations for a and α are solved using a second-order Runge–Kutta method. The third order in time integration has been chosen to reduce as much as possible the numerical dissipation in our code, which we have found to be crucial in order to obtain reliable results for the long time runs we have studied.

The numerical simulations suggest that the critical mass for the case considered here, using the scalar potential (1), is approximately [21, 24]

$$M_{\mathrm{crit}} \simeq 0.1\frac{m_{\mathrm{Pl}}^2}{\sqrt{\kappa_0 V_0}} = 2.5 \times 10^{13}\,M_\odot. \tag{17}$$

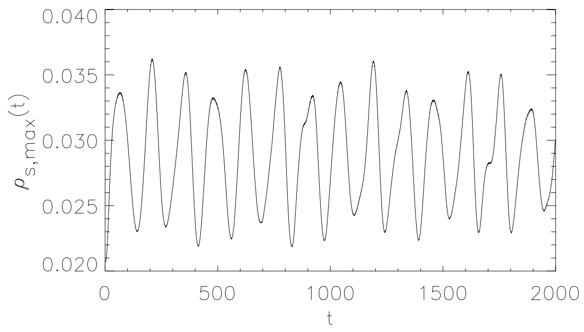

Figure 1. Temporal evolution of the maximum value of the energy density. The parameters for the initial configuration are $A = 0.01$, $s = 2.0$, see (14), which correspond to an initial mass of $M_i = 3.25 \times 10^{12}\, M_\odot$ (see also figure 3).

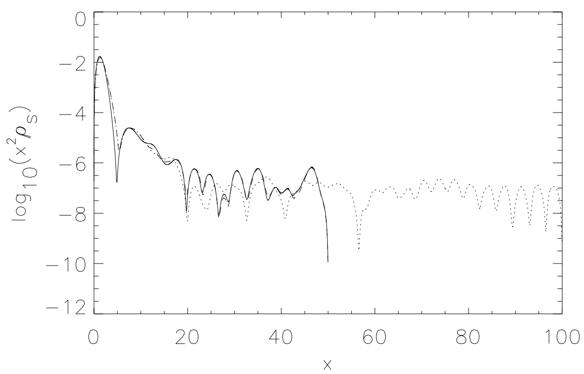

Figure 2. Energy density of the object at $t = 2000$. In the plot we show $x^2\rho_s(x)$ using a logarithmic scale. The solid line corresponds to a run with $\Delta x = 0.005$ and boundaries at $x = 50$, the dashed line to a run with $\Delta x = 0.01$ and boundaries at $x = 50$, and the dotted line to a run with $\Delta x = 0.01$ and boundaries at $x = 100$.

The results of the numerical simulations are as follows. Essentially, we have found three different types of behaviour for the scalar field collapse. In the first case, a generic feature is that scalar field distributions with an initial mass slightly larger than the critical mass collapse very violently and form a black hole. In the second type of behaviour, fluctuations with an initial mass significantly smaller than the critical mass cannot form stable oscillatons: the scalar field is completely ejected out as the system evolves [24]. The third behaviour corresponds to a case where a fraction of the initial density is spread out, leaving an oscillating object that appears to be stable. This situation happens in a narrow window of initial conditions, between 0.05–1 $\times$ $M_{\rm crit}$ [24].

In figure 1 we show the evolution in time of the maximum value of the energy density for an initial configuration that results in the formation of an oscillaton. In this case, we have taken an initial configuration with $A = 0.01$, $s = 2.0$, which implies an initial mass of $M_i = 3.25 \times 10^{12}\, M_\odot$. For this run we used $\Delta x = 0.005$, $\Delta t = 0.1\Delta x$ and 10 000 grid points, which puts the outer boundaries at $x = 50$. The run was followed until $t = 2000$, some 40 light crossing times.

In figure 2 we plot the energy density $x^2\rho_s(x)$ of the object at $t = 2000$, using a logarithmic scale, for three different runs: the run mentioned above with $\Delta x = 0.005$ and boundaries at

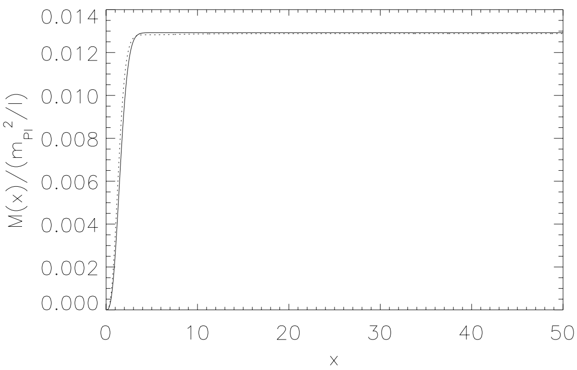

Figure 3. Initial (solid line) and final (dotted line) integrated masses.

$x = 50$, and two runs with half the resolution ($\Delta x = 0.01$) and with the boundaries $x = 50$ and $x = 100$, respectively. Note how the two runs with the boundaries at the same location coincide very well throughout the computational domain. The run with the boundaries twice as far agrees well with the other runs for $x < 20$, but differs significantly outside where $x^2 \rho_s < 10^{-6}$. This indicates that at such low levels the solution is dominated by boundary effects. Our boundary condition is clearly introducing numerical noise at such late times.

Figure 3 shows the integrated mass (16) for the initial and final ($t = 2000$) stages of the evolution. A small drop of $\sim$0.5% in the total integrated mass can be observed, but convergence tests suggest that most (if not all) of this mass loss is caused by a small amount of numerical dissipation still present in our numerical method. This implies that the system does not radiate any significant amount of energy during the time of the simulation, which indicates that the object is very stable (see figure 1).

From the cosmological point of view, the narrow window of initial conditions means that not all fluctuations will collapse into stable objects. Moreover, the collapsed objects will have masses of the same order of magnitude $M_{\rm final} \sim 10^{12}\, M_{\odot}$, as it seems to be precisely the case for galaxies.

At this point, we would like to mention that it has been shown before that dark halos of galaxies could be scalar solitonic objects, even in the presence of baryonic matter [25–28]. Actually, the boson mass estimated in all these different approaches roughly coincides with the value (4), even if the latter was estimated from a cosmological point of view [11]. However, previous studies [26, 28] have considered *complex* scalar fields endowed with a quadratic scalar potential, for which the dark halos are indeed boson stars. Boson stars and oscillatons are close relatives [1, 21], and the encouraging results at the galactic level of boson stars can be obtained with oscillatons as well. But the cosmological scenario for real scalar fields (in particular, using a cosh potential [11, 13, 22]) is not as troublesome as that for complex scalar fields [28]. It is here where we can appreciate the non-trivial properties of potential (1): its strong self-interaction provides a reliable cosmological scenario, while at the same time it has the desired properties of a quadratic potential at galactic scales. This means that the results presented could fill the gap between the success at cosmological and galactic levels.

In summary, from the results of the numerical simulations of the collapse of the real scalar field with a cosh potential we find many similarities with the structure of the halos of galaxies. The scalar field density profile is not singular at the centre. This fact, and the values of the final masses obtained using the cosmological values (3) and (4) for the parameters of the self-interaction potential, show that these objects could correspond to realistic galaxies. Moreover, our results are in agreement with the observational constraints related to the phenomenological maximum galactic mass pointed out by Salucci and Burkert [29]. Therefore, we expect that cosmological fluctuations of this scalar field, due to Jeans instabilities, will in general collapse to form objects with masses of the order of the mass of the halo of a typical galaxy. In this sense, we would be working within a top-down hierarchical model of structure formation.

We have shown before [10, 11] that the SFDM model could be a good model for the universe at cosmological level, here we see that the scalar field could also be a good candidate for the dark matter content of individual galaxies (as suggested in [8, 9, 14]).

Acknowledgments

We would like to thank Erasmo Gómez, Aurelio Espíritu and Kenneth Smith for technical support, and Edward Seidel for useful discussions. This work was partly supported by CONACyT México, under grants 119259 (LAU), 32138-E and 34407-E. DN acknowledges the Conacyt and DGAPA-UNAM grants.

References

[1] Seidel E and Suen W 1991 *Phys. Rev. Lett.* **66** 1659
[2] Seidel E and Suen W 1994 *Phys. Rev. Lett.* **72** 2516
[3] Moore B, Governato F, Quinn T, Stadel J and Lake G 1998 *Astrophys. J.* **499** L15
Jing Y P and Suto Y 2000 *Astrophys. J.* **529** L69
[4] Dalal N and Kochanek C S 2002 *Astrophys. J.* at press (*Preprint* astro-ph/0111456)
[5] Primack J R 2001 *Preprint* astro-ph/0112255
Primack J R 2002 *Preprint* astro-ph/0205391
[6] Binney J J and Evans N W 2001 *Mon. Not. R. Astron. Soc.* **327** L27
Blais-Ouellettes, Carignan C and Amram P 2002 *Preprint* astro-ph/0203146
Trott C M and Wesbster R L 2002 *Preprint* astro-ph/0203196
Salucci P, Walter F and Borriello A 2002 *Preprint* astro-ph/0206304
[7] de Blok W J G, MacGaugh S S, Bosma A and Rubin V C 2001 *Astrophys. J.* **552** L23
MacGaugh S S, Rubin V C and de Blok W J G 2001 *Astron. J.* **122** 2381
de Blok W J G, MacGaugh S S and Rubin V C 2001 *Astron. J.* **122** 2396
[8] Guzmán F S and Matos T 2000 *Class. Quantum Grav.* **17** L9
Matos T and Guzmán F S 2000 *Ann. Phys., Lpz* **9** SI-133
[9] Matos T, Guzmán F S and Núñez D 2000 *Phys. Rev.* D **62** 061301
[10] Matos T and Ureña-López L A 2000 *Class. Quantum Grav.* **17** L75
[11] Matos T and Ureña-López L A 2001 *Phys. Rev.* D **63** 63506
[12] Matos T and Ureña-López L A 2002 *Phys. Lett.* B **538** 246
[13] Lidsey J E, Matos T and Ureña-López L A 2002 *Phys. Rev.* D **66** 023514 (*Preprint* astro-ph/0111292)
[14] Matos T and Guzmán F S 2001 *Class. Quantum Grav.* **18** 5055
[15] Goodman J 2000 *Preprint* astro-ph/0003018
[16] Peebles P J E 2000 *Preprint* astro-ph/0002495
[17] Merritt D, Ferrarese L and Joseph C L 2001 *Science* **293** 1116
[18] Moore B 1994 *Nature* **370** 629
Burkert A 1995 *Astrophys. J.* **477** L25
Tyson J A, Kochanski G P and Dell'Antonio I P 1998 *Astrophys. J.* **498** L107
[19] Ferrarese L and Merritt D 2000 *ApJL* **539** L9
Gebhardt K *et al* 2000 *ApJL* **539** L13
[20] Mielke E W and Schunck F E 1999 *Proc. 8th Marcel Grossmann Meeting (Jerusalem, Israel)* (Singapore: World Scientific) (*Preprint* gr-qc/9801063)
[21] Ureña-López L A 2002 *Class. Quantum Grav.* **19** 2617
Ureña-López L A, Matos T and Becerril R in preparation
[22] Sahni V and Wang L 2000 *Phys. Rev.* D **62** 103517
[23] Hawley S H and Choptuik M W 2000 *Phys. Rev.* D **62** 104024
Honda E P and Choptuik M W 2001 *Preprint* hep-ph/0110065

[24] Alcubierre M, Guzmán F S, Matos T, Núñez D, Ureña López L A and Wiederhold P in preparation
[25] Sin S J 1994 *Phys. Rev.* D **50** 3650
Ji S U and Sin S J 1994 *Phys. Rev.* D **50** 3655
[26] Lee J W and Koh I G 1996 *Phys. Rev.* D **53** 2236
[27] Hu W, Barkana R and Gruzinov A 2000 *Phys. Rev. Lett.* **85** 1158
[28] Arbey A, Lesgourgues J and Salati P 2001 *Phys. Rev.* D **64** 123528
Arbey A, Lesgourgues J and Salati P 2002 *Phys. Rev.* D **65** 083514
[29] Salucci P and Burkert A 2000 *Preprint* astro-ph/0004397