Numerical studies of zonal flow turbulence interaction

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Introduction

Numerical ITG-turbulence studies in an electrostatic two-fluid framework show, that the structure and intensity of zonal flows, poloidal $\mathbf{E} \times \mathbf{B}$-flows with zero poloidal and toroidal mode numbers, affects radial heat transport in confinement plasmas. At the same time the zonal flow evolution is governed by the turbulence generated Reynolds stress pattern setting up an equilibrium state between the zonal flows and the turbulence. A stress response functional that predicts the evolution and scale of zonal flows is derived and validated from synthetic flow measurements.

Time-evolution of zonal flows

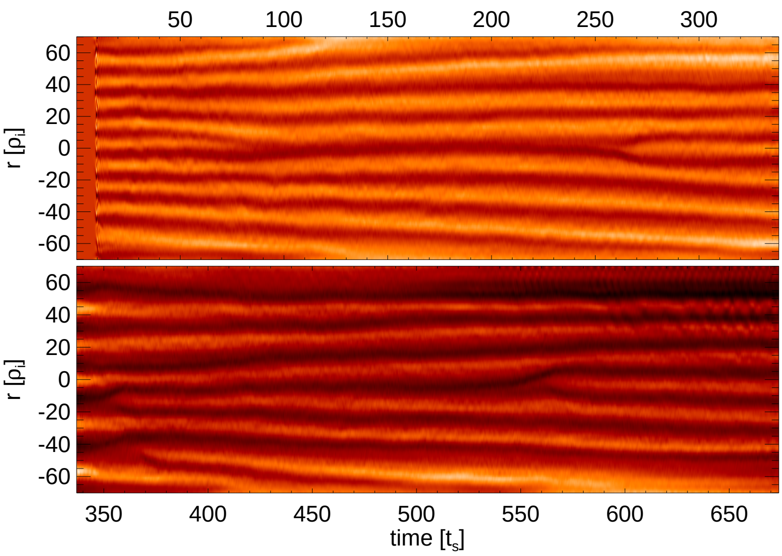

Figure 1: Zonal flow patterns. Light/Dark color represents flow in/against the ion-diamagnetic drift direction and $t_s \equiv 2\pi qR/c_s$. Upper/lower plot: self-consistent flow/initially stretched flow.

The framework for the ITG-turbulence studies is an electrostatic, large aspect ratio, circular geometry, two-fluid model with the approximation of adiabatic electrons where parallel heat conductivities are chosen to agree with damping rates of kinetic phase mixing [1]. Since gyrokinetic turbulence studies qualitatively reproduced the zonal flow behavior of the fluid approach it was justified to do further analysis in the fluid framework which is computionally more practical.

A characteristic zonal flow pattern can be observed in self consistent turbulence studies of the core region, fig. 1. The time-development of the zonal flow velocity is shown in the color coded upper figure. As the flows appear to repel each other and boundary conditions allow propagation out of the computational domain, a change in radial scale can be observed but only within highly constrained limits. Below a certain threshold new flows grow maintaining the characteristic pattern. The lower figure, fig. 1, shows the zonal flow evolution of a modified turbulent state with a stretched self-consistent initial flow pattern. As this radial scale appears not to be one of a zonal-flow-turbulence equilibrium state, the flow immediately starts to decay into a pattern of characteristic scale length. Turbulent states modified with initial flow ramps, eliminating immediate harmonic generation, self-consistently decay into the characteristic pattern indicating instability growth as the primary scale generating mechanism.

Generation of zonal flow scale

The force balance for the zonal flow velocity $\overline{v}_\theta$ driven by a total Reynolds stress R is

$$\partial_t \overline{v}_\theta = -\partial_r R. \tag{1}$$

The deterministic zonal flow behavior observed in fig. 1 suggests, that the stress is a functional of the flow and the intensity I of the turbulence. Since the stress R is the result of a flux surface average of the turbulence equations it must retain the form invariances of the original equations. This restricts the set of possible terms in the functional to the shearing rate $u \equiv \partial_r \overline{v}_\theta$, combinations of powers of u and higher derivatives thereof with an odd total number of flow derivatives per term.

Based on observations a total stress response functional must describe the growth and saturation of zonal flows as well as a dominant robust radial scale. A candidate functional with constants $\alpha, \beta, \gamma, \delta > 0$ is

$$R = I\left[\alpha u\left(1-\beta u^2\right) - \gamma \partial_r^2 u - \delta \partial_r^4 u\right]. \tag{2}$$

Rescaling with the turbulence intensity I is attributed to the proportionality of the local turbulence level and the stress. For small shearing rates only the linear term is dominant and thus α describes the linear growth rate of the zonal flows. As the shearing rate increases, the non-linear second term becomes more dominant leading to a flow saturation when $\beta u^3 \gtrsim u$. Both the higher derivatives of the shearing rate are necessary to incorporate the damping of smallest scales as well as the characteristic zonal flow scale.

An estimate for growth rate $\Gamma(k_r, u)$ of the zonal flows can be derived using a mean-field theory approximation of $\partial_r u^3 \approx 3\left\langle u^2\right\rangle \partial_r u$.

$$\Gamma(k_r, u) = I k_r^2\left(\alpha(1-3\beta\left\langle u^2\right\rangle) + \gamma k_r^2 - \delta k_r^4\right) \tag{3}$$

An appropriate radial average is denoted by $\langle \ldots \rangle$. For small values of $\left\langle u^2\right\rangle$ the growth rate Γ has only two positive real roots in k_r. They define the region of initial zonal flow growth. The lower limit for growth is zero and the upper limit is defined by the δk_r^4 term setting the smallest scale limit beyond which no zonal flows driven by ITG-turbulence exists. At the threshold of $\langle u^2 \rangle = 1/3\beta$ another real root appears pushing the lower limit for zonal flow growth away from zero towards higher k_r with any further increase of the shearing rate.

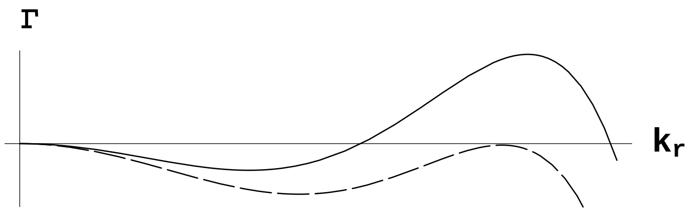

Figure 2: Proposal of zonal flow growth rate Γ with respect to k_r. The dashed line depicts a higher shearing rate u than the continuous line.

Figure 2 illustrates the saturation mechanism at a finite zonal flow scale. With an increasing shearing rate the region of growth gets more and more confined as the upper and lower growth limits move closer together until saturation at $\langle u^2 \rangle^{\text{sat}} = (\gamma^2 + 4\alpha\delta)/12\alpha\beta\delta$ with a finite zonal flow scale $k_r^{\text{sat}} = \sqrt{\gamma/2\delta}$. For amplitudes $\langle u^2 \rangle > \langle u^2 \rangle^{\text{sat}}$ all flows are damped. The stability and scale generation of the momentum balance, eq. (1), with the stress R, eq. (2), has further been verified numerically.

A stress response functional based on wave-kinetic theories and discussed in [3] is quite similar to (2) with $\delta = 0$ and $\gamma < 0$.

$$R = I\left[\alpha\partial_r\overline{v}_\theta\left(1-\beta\left(\partial_r\overline{v}_\theta\right)^2\right)+\gamma\partial_r^2\overline{v}_\theta\right] \quad (4)$$

This functional, eq. (4), already reproduces stress patterns for supplied self-consistent flow patterns. However, numerical solutions show that the only stable solution is a flow with the largest scale accommodated by the domain. The one scale contained in eq. (4) is therefore insufficient to describe the zonal flow saturation at a characteristic scale. This behavior is explained by the missing third real positive root in a growth rate approximation for eq. (4) using the aforementioned approximation. For the stress, eq. (4), the lower limit of growth remains at $k_r = 0$ for any flow amplitude indicating that the second scale in eq. (2) is indeed necessary.

To validate the response functional, eq. (2), turbulence studies with two superimposed fixed synthetic flows are used. The primary flow has a ramp profile with a constant shearing rate, located in the self-consistently damped region, to set up a turbulence level close to a zonal-flow-turbulence equilibrium state. The secondary sinusoidal flows of different radial scales are perturbatory with comparatively small amplitudes such that their stress contributions from their shearing rates u_s are $\alpha_s(k_r)u_s$. The intend is to measure the k_r-dependence of their growth rate $\gamma_m = k_r^2\alpha_s(k_r)$ and compare it with the approximation Γ, eq. (3).

Common analysis techniques, e.g. least-squares approximations, proved to be inadequate to separate and extract the deterministic fractions of the contributions. However, analysis of stress responses in single flow studies allow construction of an ”a priori” covariance estimator for unwanted contributions including random fluctuations. In the two-flow studies this estimator is now used to minimize the variance of the stress response caused by u_s with respect to the unwanted contributions produced by other sources.

The measurements of $\alpha_s(k_r)$ confirm that two scales are indeed necessary. Initially zonal flows of all k_r grow but with increasing shearing rate small k_r are damped. Any further increase of the primary shearing rate pushes the lower limit of the growth region towards higher k_r while, at the same time, the upper limit decreases. For an even higher primary shearing rate $\alpha_s < 0$, thus all k_r are damped. The overall behavior of $\alpha_s(k_r)k_r^2$ is very similar to the growth rate estimate (3). This shows that functional (2) indeed describes the zonal flow evolution and intrinsic scale.

Conclusions

Based on form invariance requirements and self-consistent flow observations a response functional incorporating a characteristic radial scale for the total Reynolds stress was constructed, eq. (2). The functional generates the scale by further and further constraining the region of zonal flow growth from both the higher and lower k_r side with increasing shearing rate until saturation at a characteristic scale. Turbulence studies with synthetic flows were used to measure the k_r-dependence of the stress response. The measurements verified that the constructed response functional indeed describes the zonal flow time-evolution and scale generation. Coefficient dependency analysis on plasma parameters will lead to further insights on the formation of internal transport barriers in fusion devices.

References

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