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Free Boundary Three-Dimensional Anisotropic Pressure Equilibria

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The VMEC2000 code has been extended to model anisotropic pressure three-dimensional equilibria. A variant of a bi-Maxwellian distribution function is applied to describe the energetic particle species such that it satisfies the constraint imposed by the lowest order solution of the Fokker-Planck equation that the hot particles are uniformly distributed along the magnetic field lines. This particular anisotropic pressure model has been previously adapted to a fixed boundary version of the VMEC code [1]. The implementation in VMEC2000, which we have labelled as the ANIMEC (Anisotropic Neumann Inverse Moments Equilibrium Code) also treats free boundary conditions previously considered in an earlier version of the VMEC code [2].

We have tested the energetic particle driven anisotropic pressure model that has been developed on a 2-field period quasiaxisymmetric stellarator (QAS) [3] with off-axis fast particle deposition which provides very stringent conditions for convergence. Choosing $\langle\beta\rangle \simeq 4.5\%$, with $\langle\beta\rangle \equiv \int d^3x\mu_0\left(p_{||}+p_\perp\right)/\int d^3xB^2$ where $p_{||}\left(p_\perp\right)$ corresponds to the parallel (perpendicular) pressureof the entire plasma, we specifically consider the following four cases:

1. High field (HF) side deposition of energetic particles about a layer around a critical field $B_C = 4.9T$ with $p_\perp > p_{||}$ $\left(p_\perp/p_{||} \sim 3.4\right)$,

2. Low field (LF) side deposition of fast particles about a layer around a critical field $B_C = 4.2T$ with $p_\perp > p_{||}$ $\left(p_\perp/p_{||} \sim 3.4\right)$,

3. High field (HF) side deposition of energetic particles about a layer around a critical field $B_C = 4.9T$ with $p_{||} > p_\perp$ $\left(p_{||}/p_\perp \sim 3.4\right)$,

4. Low field (LF) side deposition of fast particles about a layer around a critical field $B_C = 4.2T$ with $p_{||} > p_\perp$ $\left(p_{||}/p_\perp \sim 3.4\right)$.

Under vacuum conditions, the equilibrium calculations closely recover the vacuum magnetic flux surfaces obtained from Poincaré plots resulting from magnetic field line tracing. At finite β $(\langle\beta\rangle \simeq 4.5\%)$, the entire plasma column shifts significantly away from the major axis. The flux surface shapes change with pressure, but are not significantly affected by $p_{||}/neqp_{\perp}$. In fact the cases we have examined are indistinguishable except for the LF $p_{\perp} > p_{||}$ example which balloons slightly further away from the major axis. This is particularly noticeable in the bean-shaped up-down symmetric cross-section.

The convergence towards an equilibrium state is measured by following the residual horizontal force. Typically when this force reaches a level of 1×10^{-9}, the matric preconditioner is activated. Under vacuum conditions, this is achieved in under 1000 iterations and the preconditioner reduces the residual force to below 1×10^{-17} within 20 iterations. At finite $\langle\beta\rangle \simeq 4.5\%$, it takes about 3400 iterations before reaching the preconditioner triggering level of a residual force of 1×10^{-9} after which the force level decreases below 1×10^{-16} in less than 10 iterations. The exception is the LF $p_{\perp} > p_{||}$ case which converges much more slowly requiring 6440 iterations to trigger the preconditioner. Nevertheless, we still reach after that a comparably small residual horizontal force under action of the preconditioner. This compares favourably with a previously developed poorly conditioned anisotropic pressure version of the VMEC code where it is difficult to obtain residual horizontal force levels of 1×10^{-9}.

To verify the quality of the equilibrium state achieved, we calculate aposteriori the flux surface averaged radial force balance (appropriately normalised). For axisymmetric systems, this corresponds to the Grad-Shafranov equation. At $\langle\beta\rangle \simeq 4.5\%$, the poorly conditioned solution obtained with the earlier anisotropic pressure VMEC code version only reaches levels of 10^{-3}-10^{-5} for the four cases investigated. With the current fully preconditioned ANIMEC code, the radial force balance error ranges around 10^{-8}-10^{-11}. The absolute values of the flux surface averaged radial force balance error profiles for the non-preconditioned and the preconditioned finite β equilibria we have computed are shown in Fig. 2). Specifically depicted are the radial force balance error profiles with hot particle deposition on the HF side with $\left(p_{\perp}/p_{||} \sim 3.4\right)$ (blue curves), on the LF side with $\left(p_{\perp}/p_{||} \sim 3.4\right)$ (red curves), on the HF side with $\left(p_{||}/p_{\perp} \sim 3.4\right)$ (magenta curves) and on the LF side with $\left(p_{||}/p_{\perp} \sim 3.4\right)$ (green curves). The solid (dotted) lines correspond to the fully preconditioned (non-preconditioned) results. That is, the preconditioned result achieves error levels that are 5 orders of magnitude than the poorly conditioned result of the previously developed code. The improvements realised enhances the confidence of the reliability and accuracy of computed ANIMEC equilibria for further stability and transport analyses.

Under current-free conditions, the rotational transform decreases significantly at finite β $(\langle\beta\rangle \simeq 4.5\%)$. This decrease is more important for the cases with $p_{||} > p_{\perp}$ at least in the outer $3/4$ of the plasma volume. For $p_{||} > p_{\perp}$, the rotational transform profile with HF fast particle deposition almost identically matches that for LF hot particle deposition. However, for $p_\perp > p_{||}$, the LF energetic particle deposition case has a transform profile larger (smaller) in magnitude than that of HF fast particle deposition in the inner (outer) third of the plasma volume.

The magnetic well at $\langle\beta\rangle \simeq 4.5\%$ becomes stronger in the outer 80% of the plasma volume, but develops a magnetic hill in the inner 20%. For $p_{||} > p_\perp$, the differential volume is indistinguishable for LF versus HF energetic particle desposition. The differential volume profile of the $p_\perp > p_{||}$ cases is larger than those for $p_{||} > p_\perp$ (except very close to the magnetic axis). Furthermore, the magnetic well (hill) for the LF $p_\perp > p_{||}$ case is stronger than that of the HF $p_\perp > p_{||}$ case in the outer 80% (inner 20%) of the plasma volume.

For $p_{||} > p_\perp$, the hot particle pressure contours do not differ significantly whether high field or low field fast particle deposition is applied and the pressures remainmore or less uniform on a flux surface. On the other hand, for $p_\perp > p_{||}$, the energetic particle perpendicular pressure contours concentrate in the region of deposition, for example on the high field side for HF deposition (see Fig. 2). This can be understood by the fact that the trapped hot particles spend most of their orbit time around their deposition region locally enhancing the perpendicular pressure. For $p_\perp > p_{||}$, the energetic particle parallel pressure contours localise in the low field region.

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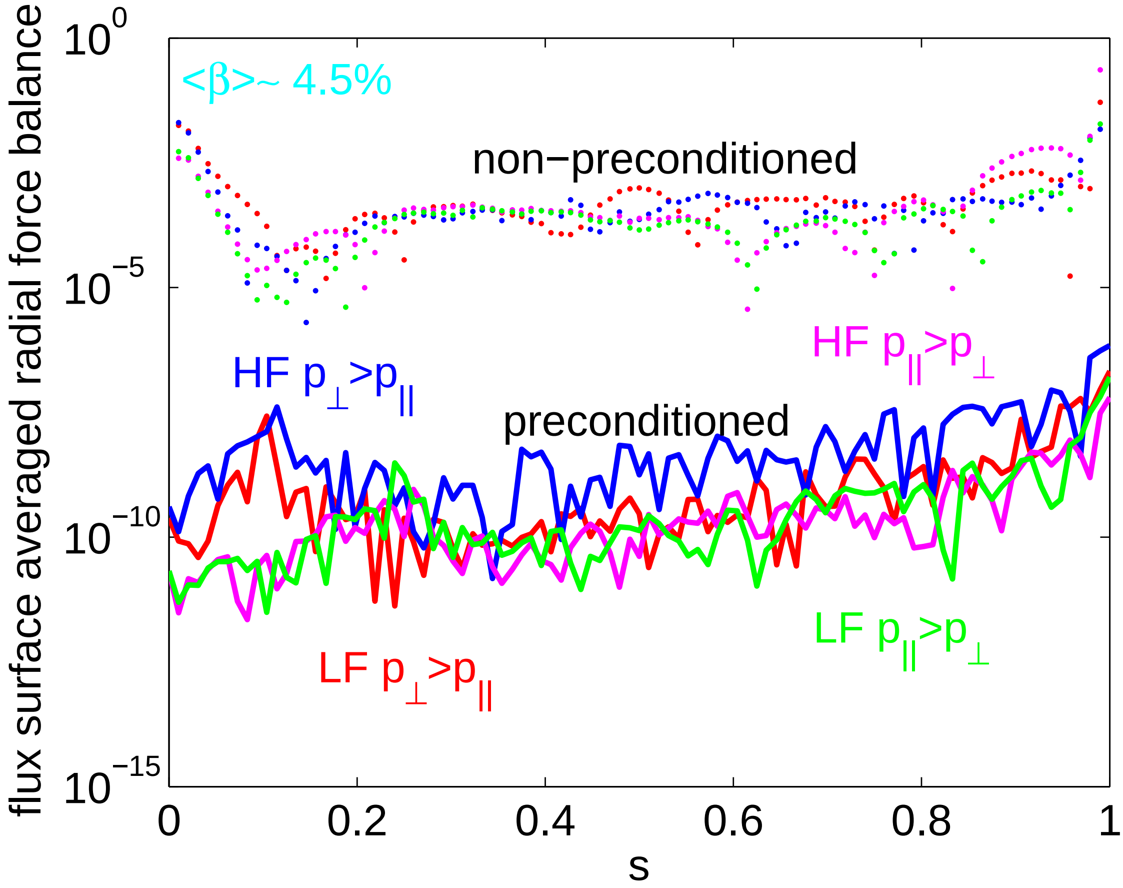

Figure 1: The absolute values of the flux surface averaged radial force balance error profiles

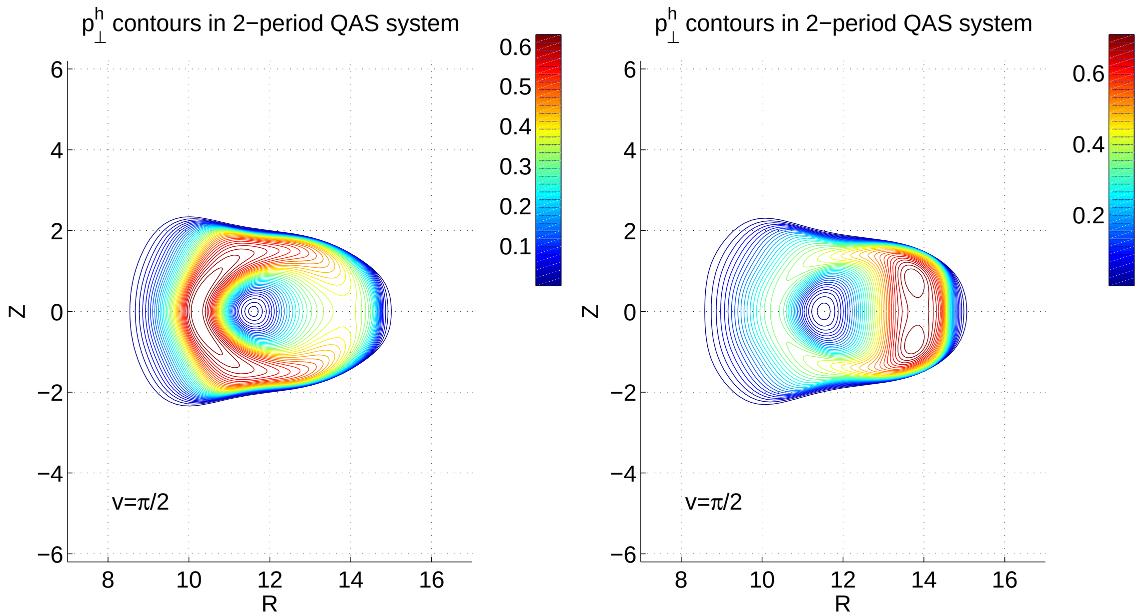

Figure 2: The hot particle perpendicular pressure contours for HF (left) and LF (right) deposition