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A scan of universal breathing-mode reductions

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The near horizon limits of M2 and D3 branes probing special holonomy cones give rise to supersymmetric $AdS_4 \times M_7$ and $AdS_5 \times M_5$ solutions of $D = 11$ and type IIB supergravity, respectively, where M_7 and M_5 are special types of Einstein spaces. Here we will show the existence of consistent Kaluza-Klein truncations of $D = 11$ and type IIB supergravity on such M_7 and M_5 down to matter-coupled supergravities in $D = 4, 5$. We explicitly discuss some details of the particular cases when M_7 and M_5 are Sasaki-Einstein.

1 Introduction

Consistent Kaluza-Klein (KK) truncations provide a very useful and powerful technique to construct string/M-theory backgrounds. By definition, if a lower dimensional (super)gravity theory arises from consistent truncation of a higher-dimensional, parent (super)gravity, any solution of the former is guaranteed to be a solution of the latter. Well-known examples of consistent KK truncations include those associated to the maximally supersymmetric $D = 10, 11$ backgrounds of the form $AdS_4 \times S^7$, $AdS_7 \times S^4$ and $AdS_5 \times S^5$: $D = 11$ supergravity on S^7 (respectively, S^4) truncates consistently to $D = 4$ $N = 8$ $SO(8)$-gauged supergravity [1] (respectively, $D = 7$ $N = 2$ $SO(5)$-gauged supergravity [2]), while type IIB supergravity on S^5 is expected to truncate consistently to $D = 5$ $N = 8$ $SO(6)$-gauged supergravity.

The above examples should be framed into a more general picture. For all N-supersymmetric $D = 10, 11$ backgrounds of string/M-theory of the form $AdS_d \times_w M_{D-d}$, where $\times_w$ denotes warped product and M_{D-d} is a Riemannian space endowed with a precise G-structure specified by supersymmetry, a consistent KK truncation down to the N-extended, pure gauged supergravity (that containing the graviton multiplet only) is conjectured to exist [3]. This conjecture is in fact a theorem for the maximally supersymmetric examples discussed above, and has been shown to also hold for less supersymmetric examples. For instance, the most general $N = 1$ supersymmetric $AdS_5 \times_w M_6$ solution in $D = 11$ [4] is characterised by an $SU(2)$-structure on M_6, whose intrinsic torsion constrains the form of the possible metrics and fluxes on M_6 compatible with supersymmetry. It is now known that $D = 11$ supergravity on M_6 truncates consistently to $N = 2$ pure gauged supergravity [5]. Other cases where the conjecture has been verified include the class of solutions of the form $AdS_5 \times SE_5$ in type IIB and $AdS_4 \times SE_7$ in $D = 11$, where SE_{2n+1} is a $(2n+1)$-dimensional Sasaki-Einstein space: consistent truncations of type IIB and $D = 11$ supergravity on SE_5 and SE_7 exist to $N = 2$ pure gauged supergravity in $D = 5$ [6] and $D = 4$ [3]. See [3,7] for other classes of geometries where the conjecture has been verified.

This general result about consistent KK truncations down to lower-dimensional pure gauged supergravities is quite remarkable, in as much as consistency has been traditionally regarded as a highly unusual feature of a KK truncation. The less supersymmetry an $AdS_d \times_w M_{D-d}$ string/M-theory background

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preserves, the less constrained the internal geometry on M_{D-d} is. For the lower-supersymmetric cases, the previous theorem thus provides truly remarkable examples where consistent KK truncations on wide classes of internal geometries exist, challenging the conventional lore. At a more practical level, this translates into a wealth of higher dimensional solutions that can be constructed out of a single lower-dimensional one: any solution of, say, minimal $D = 5$ $N = 2$ pure gauged supergravity gives rise to different type IIB solutions depending on the SE_5 space (*e.g.* S^5, $T^{1,1}$, $Y^{p,q}$, $L^{p,q,r}$, etc.) used to perform the uplift.

The downside of the theorem, especially in cases of low supersymmmetry, is that pure supergravity (the lower-dimensional theory recovered from consistent KK truncation under the assumptions of the theorem), typically has a very simple field content: *e.g.* just the metric and the graviphoton in the SE truncations mentioned above. Of course, this would not be a problem if one just wanted to embed, for example, the $D = 4$ or $D = 5$ Reissner-Norsdtrom-AdS (RNAdS) black hole, which is a solution to minimal gauged supergravity, into $D = 11, 10$ supergravity via uplift on SE_7 or SE_5. But one would be missing the chance of exactly embedding into $D = 10, 11$ supergravity other solutions involving extra matter fields, that describe extremely interesting physics. For instance, consider an enlarged model containing the metric and $U(1)$ gauge vector corresponding to the bosonic sector of minimal $D = 4, 5$ $N = 2$ gauged supergravity, and also a $U(1)$-charged scalar. In this setting, the RNAdS black hole can develop scalar hair below certain critical temperature, and this has been argued to holographically correspond to a superconducting phase transition in the boundary of AdS space [8,9]. Other interesting gravitational models including matter fields have been phenomenologically cooked up in order for them to admit solutions with potential applications in the holographic description of other condensed matter systems, like quantum critical points displaying non-relativistic (Schrödinger [10] or Lifshitz [11]) symmetry.

Various applications of AdS/CFT to condensed matter theory (AdS/CMT) have been very actively investigated over the last couple of years. An interesting problem is, in fact, to determine whether the gravitational toy models used for AdS/CMT can be derived from consistent truncation of $D = 10, 11$ supergravity. The existence of exact embeddings of those models into string/M-theory is not of mere scholastic interest, since a precise holographic dictionary is only guaranteed to exist if the corresponding duality can be suitably embedded into string theory. Unlike their lower-supersymmetric pure supergravity counterparts, the maximal supergravities have a sufficiently rich field content to be able to potentially accommodate the effective gravity plus matter theories used in AdS/CMT. Actually, the maximal supergravities are *so rich* that they turn out too complicated and cumbersome to try and make contact with AdS/CMT models.

The obvious question arises: for the low-supersymmetry examples, can the consistent truncations to pure supergravity discussed above be extended to include matter couplings? The answer to this question is not obvious a priori, given the difficulty to obtain consistent truncations to pure supergravity in the first place. Also, retaining matter couplings is inconsistent in some known cases: see *e.g.* [12]. Here we will show that, associated to configurations of M2 and D3 branes probing the apex of special holonomy cones $C(M)$ over Riemanian seven and five-dimensional Einstein spaces M, consistent truncations exist down to matter-coupled supergravity in four and five spacetime dimensions. In particular, we will show how the consistent truncations of $D = 11$ and type IIB supergravity on SE_7 and SE_5 mentioned above can be extended to include matter couplings. One of the massive fields that survives the truncation is the scalar *breathing mode*, parametrising homogeneous deformations of the volume of M. Thus, we collectively call these *breathing mode truncations.* Another noteworthy feature of the truncations we will consider is their universality: they are valid for any member of the relevant class of Einstein spaces. For an early discussion of breathing mode reductions see [13].

2 M2 and D3 branes on top of special holonomy cones and consistent truncations

As is well known, the near horizon geometry corresponding to a stack of M2 branes probing the tip of a metric cone $C(M_7)$ over a seven-dimensional Einstein space M_7, is $AdS_4 \times M_7$. If $C(M_7)$ has special holonomy, then Killing spinors can be shown to exist on M_7 and the $D = 11$ $AdS_4 \times M_7$ solution is supersymmetric. Aside from the maximally supersymmetric case, corresponding to $C(M_7) = \mathbb{R}^8$ and $M_7 = S^7$, there are three classes of supersymmetric solutions, corresponding to the three possible types of special holonomy that the eight-dimensional manifold $C(M_7)$ may have: $Spin(7)$, $SU(4)$ and $Sp(2)$. The relevant $C(M_7)$ is thus $Spin(7)$-holonomy, Calabi-Yau (CY), or Hyper-Kähler (HK), and the associated M_7 manifolds are weak-G_2, Sasaki-Einstein (SE) and tri-Sasaki. The supersymmetry preserved by the $D = 11$ $AdS_4 \times M_7$ background in each case is $N = 1$, $N = 2$ and $N = 3$.

For D3-branes probing the apex of a metric cone $C(M_5)$, and their associated near horizon geometries, a similar type IIB story unfolds. Now, if supersymmetry is to be preserved, $C(M_5)$ can only be a CY three-fold, and M_5 a SE five-dimensional manifold. The corresponding $AdS_5 \times M_5$ background preserves $N = 2$ supersymmetry.

It can be explicitly checked that both $D = 11$ supergravity and type IIB admit consistent truncations on those classes of internal geometries down to matter-coupled gauged supergravity in $D = 4$ and $D = 5$:

1. $D = 11$ supergravity on any weak-G_2 manifold truncates consistently to $D = 4$ $N = 1$ gauged supergravity coupled to a chiral multiplet [14];
2. $D = 11$ supergravity on any SE_7 manifold truncates consistently to $D = 4$ $N = 2$ gauged supergravity coupled to a vector multiplet and a hypermultiplet [14];
3. $D = 11$ supergravity on any tri-Sasaki manifold truncates consistently to $D = 4$ $N = 4$ gauged supergravity coupled to three vector multiplets: see [15];
4. Type IIB supergravity on any SE_5 manifold truncates consistently to $D = 5$ $N = 4$ gauged supergravity coupled to two vector multiplets [15, 16] (see also [17]).

It is particularly interesting to note the supersymmetry preserved by truncations 3 and 4 above: from the supersymmetry of the $AdS_4 \times M_7$ (with M_7 tri-Sasaki) and $AdS_5 \times M_5$ (with M_5 Sasaki-Einstein) vacua (see above), one might have naively concluded that the gauged supergravities arising from consistent truncations should be $N = 3$ in $D = 4$ and $N = 2$ in $D = 5$, rather than the correct $N = 4$. In fact, unlike the situation in ungauged supergravity, the maximally (super)symmetric vacuum of a gauged supergravity typically breaks supersymmetry. In cases 3 and 4 above, the maximally supersymmetric vacuum turns out to break $N = 4 \to N = 3$ and $N = 4 \to N = 2$, respectively, compatible with the supersymmetry of the corresponding $D = 11$ and type IIB backgrounds. The supersymmetry of the above truncations can be pinned down by simply replacing the internal spaces by the special holonomy manifolds to which they reduce when the fluxes are turned off (which effectively *ungauges* the lower-dimensional theory). M-theory on G_2-holonomy, $CY_3 \times S^1$ and $HK_4 \times T^3$ (where T^3 is the three-dimensional torus), preserves $N = 1$, $N = 2$ and $N = 4$ in $D = 4$, respectively. Type IIB strings on $HK_4 \times S^1$ preserve $N = 4$ in $D = 5$. These are the supersymmetries quoted in 1–4 above.

3 Breathing-mode reductions on SE manifolds

Let us now explicitly work out the examples 2 and 4, corresponding to $D = 11$ and type IIB on SE_7 and SE_5 spaces. Recall that a SE_{2n+1} manifold is charactersied by an $SU(n)$-structure specified by a real one-form η, a real two-form J and a complex n-form Ω subject to the differential constraints

$$d\eta = 2J\,, \qquad dJ = 0\,, \qquad d\Omega = i(n+1)\eta \wedge \Omega\,, \tag{1}$$

and algebraic conditions that can be found in *e.g.* [14, 15]. Locally, a SE space can be regarded as a circle fibration over a Kähler-Einstein (KE) $2n$-dimensional manifold and thus admits a local metric of the form

$$ds^2(SE_{2n+1}) = ds^2(KE_{2n}) + \eta \otimes \eta \ . \tag{2}$$

The Kähler form and holomorphic $(n, 0)$-form of KE_{2n} are related to J and Ω, and η is seen here to correspond to the one-form along the S^1 fiber, dual to the corresponding (Reeb) Killing vector.

Let us now explicitly show how $D = 11$ and type IIB supergravity can be consistently reduced on SE_7 and SE_5, respectively. For the metric, we write

$$ds^2_D = ds^2_{D-2n-1} + e^{2U} ds^2(KE_{2n}) + e^{2V}(\eta + A_1) \otimes (\eta + A_1) \tag{3}$$

where $(D, n) = (11, 3),\ (10, 2)$ for $D = 11$ and IIB supergravity, respectively. Accordingly, ds^2_4 or ds^2_5 is a pseudo-Riemannian line element on the external four- or five-dimensional spacetime. A_1 is a spacetime $U(1)$ gauge field, with two-form field strength $F_2 = dA_1$, arising from the Reeb isometry of the internal SE space and U, V are spacetime-dependent scalar deformations of the SE space: suitable linear combinations of U and V contain the *breathing mode* of the SE space, parametrising overall volume deformations of the SE space, and a *squashing mode*, stretching the S^1 fibers with respect to the KE base while preserving the volume of SE.

The above ansatz for the metric should be supplemented by ansatze for the fluxes. In order to do this, we take all possible wedge products of spacetime form fields with the tensors (η, J, Ω) defining the $SU(n)$-structure on SE_{2n+1}. For the $D = 11$ supergravity four-form we accordingly take

$$\begin{aligned} G_{(4)} =\ & f\mathrm{vol}_4 + H_3 \wedge (\eta + A_1) + H_2 \wedge J + dh \wedge J \wedge (\eta + A_1) + 2hJ \wedge J \\ & + \sqrt{3}\left[\chi(\eta + A_1) \wedge \Omega - \tfrac{i}{4} D\chi \wedge \Omega + \text{c.c.}\right] \ . \end{aligned} \tag{4}$$

Here, H_3 and H_2 are real three and two-form field strengths, h is a real scalar, χ is a complex scalar charged under the $U(1)$ gauge field A_1 descending from the eleven-dimensional metric; its covariant derivative is given by $D\chi \equiv d\chi - 4iA_1\chi$. Finally, f is a spacetime scalar, required by the Bianchi identity satisfied by G_4 to be given by a combination of the above scalar fields.

For the type IIB case, we supplement the $D = 10$, $n = 2$, metric (3) with the following choice for the NSNS fields (the dilaton Φ and three-form field strength $H_{(3)}$) and RR field-strengths ($F_{(5)}$, $F_{(3)}$, $F_{(1)}$):

$$\begin{aligned} F_{(5)} =\ & 4e^{-4U-V+Z}\mathrm{vol}_5 + 2e^{Z} J \wedge J \wedge \hat{e}^5 - 2e^{-4U+V} * K_1 \wedge \hat{e}^5 + e^{-V} * K_2 \wedge J \\ & + K_2 \wedge J \wedge \hat{e}^5 + K_1 \wedge J \wedge J + \Big(e^{-V} * L_2 \wedge \Omega + L_2 \wedge \Omega \wedge \hat{e}^5 + c.c.\Big) \\ F_{(3)} =\ & G_3 + G_2 \wedge \hat{e}^5 + G_1 \wedge J + \big(N_1 \wedge \Omega + N_0 \Omega \wedge \hat{e}^5 + c.c.\big) \\ H_{(3)} =\ & H_3 + H_2 \wedge \hat{e}^5 + H_1 \wedge J + \big(M_1 \wedge \Omega + M_0 \Omega \wedge \hat{e}^5 + c.c.\big) \\ F_{(1)} =\ & da \ , \qquad \Phi = \phi \end{aligned} \tag{5}$$

Here, vol_5 and $*$ are the volume form and Hodge dual corresponding to the five-dimensional metric ds^2_5 in (3), Z, a, ϕ, are real scalars, M_0, N_0 complex scalars, G_3, H_3, G_2, H_2, K_2, K_1 real forms, and L_2, M_1, N_1, complex forms, all of them defined on the external five-dimensional spacetime. We have also ensured the selfduality, with respect to the ten-dimensional metric (3), of the five-form $F_{(5)}$.

4 $D = 4$ $N = 2$ $(U(1) \times \mathbb{R})$-gauged supergravity from M-theory on SE_7

Here we will argue that (3), (4) provide a consistent truncation of $D = 11$ supergravity. Full details can be found in [14]. The Bianchi identity imposed on the four-form (4), translates into Bianchi identities for

H_3 and H_2. These can be then solved by suitably introducing two-form B_2 and one-form B_1 potentials. A straightforward counting of degrees of freedom then shows that the $D = 11$ ansatz (3), (4) thus incorporates two $D = 4$ vectors, A_1, B_1, five (real) scalars U, V, h, χ, and one two-form B_2, along with the four-dimensional metric. Substituting the ansatz (3), (4) into the equations of motion of $D = 11$ supergravity, one can show that all SE_7 dependence drops out, leaving equations of motion for the four-dimensional fields. This shows the consistency of the truncation.

This $D = 4$ field content is compatible with $D = 4$ $N = 2$ supergravity coupled to an $N = 2$ vector multiplet and a tensor multiplet. As discussed in [14], the latter contains (B_2, U, χ, χ^*). This tensor multiplet can be dualised into a hypermultiplet, by dualising B_2 into a scalar a. Performing this dualisation, we find that the resulting six scalars parametrise the homogeneous, symmetric moduli space

$$\frac{SU(1,1)}{U(1)} \times \frac{SU(2,1)}{SU(2) \times U(1)} \tag{6}$$

where the first factor parametrises the special-Kähler manifold corresponding to the two (real) scalars in the universal vector multiplet and the second factor is associated to the quaternionic-Kähler space appropriate to the universal hypermultiplet. Our model can be thought of as particular gauged version of the $D = 4$ theory arising from consistent truncation of $D = 11$ supergravity on $CY_3 \times S^1$ and can, accordingly, be expected to preserve $N = 2$ supersymmetry. Indeed, the supersymmetry of our model can be made manifest at the level of the action by rewriting the (dualised) action into the canonical form of $D = 4$ $N = 2$ gauged supergravity [18].

Finally, the curvature of the internal SE_7 space and the background four-form flux are responsible for a gauging in our model. With only two vectors A_1, B_1 available, we can only have an Abelian gauging. Only the scalars in the hypermultiplet turn out to be charged. At the level of the lagrangian, this gives rise to a fibration over spacetime of the hypermultiplet non-linear sigma model, with fibers defining a non-compact gauge group $U(1) \times \mathbb{R} \subset SU(2,1)$. The compact factor is related to the $U(1)$ R-symmetry, while the non-compact factor $\mathbb{R}$ reflects the symmetry of the lagrangian under shifts of the scalar a dual to the original two-form B_2.

5 $D = 5$ $N = 4$ $(U(1) \times H_3)$-gauged supergravity from type IIB on SE_5

Here we will argue that (3), (5) provide a consistent truncation of IIB supergravity, referring to [15] for the details. Just as we did for the M-theory case, imposing the Bianchi identities on the $D = 10$ forms (5) is useful in order to derive Bianchi identities for the five-dimensional fields and to subsequently perform a counting of degrees of freedom. It is not difficult to show that we are now dealing with four $D = 5$ (real) two-forms L_2, L_2^*, B_2, C_2; four vectors A_1, B_1, C_1, E_1; and eleven scalars U, V, a, ϕ, b, c, h, χ, χ^*, ξ, ξ^*. The explicit expressions of the field strengths appearing in (5) in terms of these potentials can be found in [15]. As in the $D = 11$ example of the previous section, direct substitution into the type IIB equations of motion now shows that these $D = 5$ fields satisfy $D = 5$ equations of motion in their own right, with all dependence on the internal SE_5 dropping out. This is, again, proof of the consistency of the truncation.

This $D = 5$ theory is moreover, locally supersymmetric. This is more clearly discussed in the ungauged limit, obtained by setting the background fluxes to zero and untwisting and flattening SE_5 into $HK_4 \times S^1$. In this ungauged theory, one can safely dualise the four two-forms into four vectors. The resulting dual theory contains, besides the $D = 5$ metric, eight vectors, eleven scalars, and is thus compatible with $D = 5$ (ungauged) supergravity coupled to two $N = 4$ vector multiplets. In fact, the dualised lagrangian can be cast into canonical $N = 4$ form [19]. In particular, the eleven scalars can be shown to parametrise the moduli space

$$SO(1,1) \times \frac{SO(5,2)}{SO(5) \times SO(2)} \tag{7}$$

appropriate for $N = 4$.

Finally, an explicit analysis shows that the four vectors of our original gauged theory are gauge fields for the non-abelian gauge group $U(1) \times H_3 \subset SO(5,2)$. The $U(1)$ factor corresponds to the R-symmetry of our model, and H_3 is the three-dimensional Heisenberg group.

6 Conclusions

We have shown the existence of consistent truncations of $D = 11$ and type IIB supergravity on supersymmetric Einstein spaces M_7 and M_5, related to planar M2 and D3 configurations with reduced supersymmetry. The resulting theories in $D = 4$ and $D = 5$ dimensions are gauged supergravities, including full matter supermultiplets and containing, in particular, the breathing mode of the internal space. We have given details of two particular examples, arising from truncations on Sasaki-Einstein spaces. In this case, the theories we obtain fully supersymmetrise other known consistet truncations on SE spaces [13, 20, 21], and reduce to the truncations to minimal supergravity of [3,6] when the matter couplings are turned off. We have discussed these truncations at the level of the bosonic fields, but some of these have now been explicitly checked to be consistent also in the fermionic sector up to four-fermion terms [22, 23] (see also [24]). We have emphasised that these truncations are universal: valid for all internal manifolds in the Einstein class under consideration. The type IIB SE_5 truncation has been particularised for $SE_5 = T^{1,1}$ in [25, 26], where an extra $N = 4$ vector multiplet could be kept, and for $SE_5 = S^5$ in [27], where the inconsistency of keeping simultaneously the breathing mode and other modes particular to S^5 was shown.

The inclusion of matter couplings in these theories make them ideal playgrounds to study exact embeddings of AdS/CMT systems into string theory. For instance, holographic superconductors have been embedded into type IIB [28] and $D = 11$ [29, 30] by means of related consistent truncations. Solutions displaying Schrödinger symmetry were first been embedded into type IIB using, among other techniques, consistent truncations on SE_5 spaces [20]. The elusive embedding of Lifhitz-invariant backgrounds into the higher dimensional supergravities has now been also achieved [31–34] using, among other methods, consistent truncations.

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References

[1] B. de Wit and H. Nicolai, Nucl. Phys. B **281**, 211 (1987).
[2] H. Nastase, D. Vaman, and P. van Nieuwenhuizen, Phys. Lett. B **469**, 96 (1999) [arXiv:hep-th/9905075].
[3] J. P. Gauntlett and O. Varela, Phys. Rev. D **76**, 126007 (2007) [arXiv:0707.2315 [hep-th]].
[4] J. P. Gauntlett et al., Class. Quantum Gravity **21**, 4335 (2004)
[5] J. P. Gauntlett, E. O. Colgain, and O. Varela, J. High Energy Phys. **0702**, 049 (2007) [arXiv:hep-th/0611219].
[6] A. Buchel and J. T. Liu, Nucl. Phys. B **771**, 93 (2007) [arXiv:hep-th/0608002].
[7] J. P. Gauntlett and O. Varela, J. High Energy Phys. **0802**, 083 (2008) [arXiv:0712.3560 [hep-th]].
[8] S. S. Gubser, Phys. Rev. D **78**, 065034 (2008) [arXiv:0801.2977 [hep-th]].
[9] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008) [arXiv:0803.3295 [hep-th]].
[10] D. T. Son, Phys. Rev. D **78**, 046003 (2008) [arXiv:0804.3972 [hep-th]].
[11] S. Kachru, X. Liu, and M. Mulligan, Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].
[12] P. Hoxha, R.R. Martinez-Acosta, and C.N. Pope, Class. Quantum Gravity **17**, 4207 (2000) [arXiv:hep-th/0005172].

[13] J. T. Liu and H. Sati, Nucl. Phys. B **605**, 116 (2001) [arXiv:hep-th/0009184].

[14] J. P. Gauntlett et al., J. High Energy Phys. **0904**, 102 (2009) [arXiv:0901.0676 [hep-th]].

[15] J. P. Gauntlett and O. Varela, J. High Energy Phys. **1006**, 081 (2010) [arXiv:1003.5642 [hep-th]].

[16] D. Cassani, G. Dall'Agata, and A. F. Faedo, J. High Energy Phys. **1005**, 094 (2010) [arXiv:1003.4283 [hep-th]].

[17] J. T. Liu, P. Szepietowski, and Z. Zhao, Phys. Rev. D **81**, 124028 (2010) [arXiv:1003.5374 [hep-th]].

[18] L. Andrianopoli et al., J. Geom. Phys. **23**, 111 (1997) [arXiv:hep-th/9605032].

[19] J. Schon and M. Weidner, J. High Energy Phys. **0605**, 034 (2006) [arXiv:hep-th/0602024].

[20] J. Maldacena, D. Martelli, and Y. Tachikawa, J. High Energy Phys. **0810**, 072 (2008) [arXiv:0807.1100 [hep-th]].

[21] K. Skenderis, M. Taylor, and D. Tsimpis, J. High Energy Phys. **1006**, 025 (2010). [arXiv:1003.5657 [hep-th]].

[22] I. Bah et al., Fermions and $D = 11$ Supergravity On Squashed Sasaki-Einstein Manifolds [arXiv:1008.1423 [hep-th]].

[23] I. Bah et al., J. High Energy Phys. **1101**, 100 (2011). [arXiv:1009.1615 [hep-th]].

[24] J. T. Liu, P. Szepietowski, and Z. Zhao, Phys. Rev. D **82**, 124022 (2010) [arXiv:1009.4210[hep-th]].

[25] D. Cassani and A. F. Faedo, Nucl. Phys. B **843**, 455 (2011) [arXiv:1008.0883 [hep-th]].

[26] I. Bena et al., Supersymmetric Consistent Truncations of IIB on T(1,1), arXiv:1008.0983 [hep-th].

[27] J.T. Liu and C.N. Pope, Inconsistency of Breathing Mode Extensions of Maximal Five-Dimensional arXiv:1005.4654 [hep-th].

[28] S. S. Gubser et al., Phys. Rev. Lett. **103**, 141601 (2009) [arXiv:0907.3510 [hep-th]].

[29] J. P. Gauntlett, J. Sonner, and T. Wiseman, Phys. Rev. Lett. **103**, 151601 (2009) [arXiv:0907.3796 [hep-th]].

[30] J. P. Gauntlett, J. Sonner, and T. Wiseman, J. High Energy Phys. **1002**, 060 (2010) [arXiv:0912.0512 [hep-th]].

[31] K. Balasubramanian and K. Narayan, J. High Energy Phys. **1008**, 014 (2010) [arXiv:1005.3291 [hep-th]].

[32] A. Donos and J. P. Gauntlett, J. High Energy Phys. **1012**, 002 (2010) [arXiv:1008.2062 [hep-th]].

[33] R. Gregory et al., J. High Energy Phys. **1012**, 047 (2010) [arXiv:1009.3445 [hep-th]].

[34] A. Donos et al., J. High Energy Phys. **1012**, 003 (2010) [arXiv:1009.3805 [hep-th]].