Counting supersymmetric branes

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ABSTRACT: Maximal supergravity solutions are revisited and classified, with particular emphasis on objects of co-dimension at most two. This class of solutions includes branes whose tension scales with $g_\mathrm{s}^{-\sigma}$ for $\sigma > 2$. We present a group theory derivation of the counting of these objects based on the corresponding tensor hierarchies derived from E_{11} and discrete T- and U-duality transformations. This provides a rationale for the wrapping rules that were recently discussed for $\sigma \leq 3$ in the literature and extends them. Explicit supergravity solutions that give rise to co-dimension two branes are constructed and analysed.

Dedicated to the memory of Laurent Houart

Contents

1 Introduction

It is by now well-established that maximal supergravity in $D \geq 3$ dimensions admits a systematic hierarchy of p-form potentials with $0 \leq p \leq D$. This structure has been termed tensor hierarchy [1–3] and has been derived from the properties of gauged maximal supergravity theories. Tantalizingly, the tensor hierarchy derived in this way agrees very well with a 'spectral analysis' of the Kac-Moody Lie algebra E_{11} [4, 5].[1] The compatibility of the tensor hierarchy with supersymmetry was also checked in many cases, most notably the occurence of ten-forms in type IIB supergravity in $D = 10$ [6, 7]. The various p-forms that appear in the tensor hierarchy transform under the continuous hidden symmetry groups $E_{11-D}(\mathbb{R})$. For this paper we will use the tensor hierarchy as predicted by E_{11} and summarized in table 1.

In supergravity, there typically exist relatively simple solutions that couple to a p-form potential. These are called $(p-1)$ branes and have a p-dimensional world-volume and a string frame tension with fixed scaling with respect to the string coupling constant $g_{\rm s}$. For

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[1]There are small (potential) differences in $D = 3$ that we will argue in footnote 5 are not important for the purposes of this paper.

$D\backslash p$	1	2	3	4	5	6	7	8	9	10
IIA	**1**	**1**	**1**		**1**	**1**	**1**	**1**	**1**	**2×1**
IIB		**2**		**1**		**2**		**3**		**4⊕2**
9	**2⊕1**	**2**	**1**	**1**	**2**	**2⊕1**	**3⊕1**	**3⊕2**	**4⊕2×2**	
8	$(\mathbf{3},\mathbf{2})$	$(\bar{\mathbf{3}},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2})$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})$	$(\bar{\mathbf{6}},\mathbf{2})\oplus(\mathbf{3},\mathbf{2})$	$(\mathbf{15},\mathbf{1})\oplus(\bar{\mathbf{3}},\mathbf{3})$ $\oplus 2\times(\bar{\mathbf{3}},\mathbf{1})$		
7	$\overline{\mathbf{10}}$	**5**	$\bar{\mathbf{5}}$	**10**	**24**	$\mathbf{40}\oplus\overline{\mathbf{15}}$	$\mathbf{70}\oplus\overline{\mathbf{45}}\oplus\mathbf{5}$			
6	$\mathbf{16_c}$	**10**	$\mathbf{16_s}$	**45**	$\mathbf{144_s}$	$\mathbf{320}\oplus\mathbf{126_+}$ $\oplus\mathbf{10}$				
5	$\overline{\mathbf{27}}$	**27**	**78**	**351**	$\mathbf{1728}\oplus\mathbf{27}$					
4	**56**	**133**	**912**	$\mathbf{8645}\oplus\mathbf{133}$						
3	**248**	$\mathbf{3875}\oplus\mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}\oplus\mathbf{248}$							

Table 1. The tensor hierarchy of p-forms for $3 \leq D \leq 10$ as predicted by E_{11}. For type IIA, there is no non-abelian U-duality, for type IIB it is $\mathrm{SL}(2,\mathbb{R})$. We omit the scalars that form the (non-linear) coset $E_{11-D}/K(E_{11-D})$ with $K(E_{11-D})$ the maximal compact subgroup. In $D = 8$, we take $E_3 = \mathrm{SL}(3) \times \mathrm{SL}(2)$. See also [4, 5].

example, in the line for IIA in table 1 one can see the standard supersymmetric D0, F1, D2, D4, NS5, D6 and D8 branes of the theory. In addition, there are a $p = 8$ potential and two $p = 10$ potentials. The question which of these couple to *supersymmetric* branes has been answered for example in [7, 8] where it was found that none of them gives rise to a supersymmetric brane in type IIA.

Looking at table 1, the question arises more generally: *Out of the p-form potentials in a given dimension D, which ones are sources for supersymmetric branes of maximal supergravity and what is the scaling of their tension with g_s?* This is the main question we will answer fully and completely using E_{11} in this paper. We note that quite a number of results in this direction have already been obtained in the recent past [9–12]. The analysis rested on analysing the structure of the Wess-Zumino terms as a necessary criterion for supersymmetry and resulted in a number of wrapping rules that permit the derivation of the number of supersymmetric solitons of a given scaling $g_s^{-\sigma}$ for $\sigma = 0, 1, 2, 3$ from the known branes in IIA and IIB. An additional outcome of this analysis was that, in order to get the right number of solutions, it is not sufficient to just study the dimensional reduction of the IIA/IIB branes but a supplemental 'inflow' of objects was needed for $\sigma \geq 2$ from so-called generalised Kaluza-Klein monopoles. This only happens for branes of co-dimension at most two. Branes of co-dimension less than two couple to p-forms with $p = D - 1$ or $p = D$ and hence not to fields with local degrees of freedom.

That not all p-forms give rise to supersymmetric solutions can be seen for example in the type IIB case. There one obtains an $\mathrm{SL}(2,\mathbb{R})$ triplet of 8-forms [6, 13] that would naïvely suggest a triplet of supersymmetric 7-branes. One supersymmetric 7-brane is certainly the D7 (with $\sigma = 1$), another one is its S-dual partner (with $\sigma = 3$). However, it turns out that one cannot associate a supersymmetric 7-brane with the third 8-form potential [10, 14, 15]. The reason is basically that these are dual to only the two scalars (ϕ, χ) parametrising the

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA	1	1	1		1	1	1		1	
IIB		2		1		2		2		2
9	3	2	1	1	2	3	2	2	2	
8	6	3	2	3	6	8	6	6		
7	10	5	5	10	20	25	20			
6	16	10	16	40	80	96				
5	27	27	72	216	432					
4	56	126	576	2016						
3	240	2160	17280							

Table 2. Total number of supersymmetric p-branes of maximal supergravity in D space-time dimensions as predicted by E_{11}. Note that p is shifted by one compared to table 1 as it now labels the p-branes (coupling to a $(p+1)$-form potential). The table contains all g_s scalings together.

$SL(2,\mathbb{R})/SO(2)$ scalar manifold. This kind of reasoning will not apply to p-forms with $p = D-1$ or $p = D$ in D space-time dimensions since the corresponding forms are not dual to any of the physical fields.[2] U-duality is compatible with supersymmetry, so all the members of the U-duality orbit of a given supersymmetric brane will be supersymmetric. This is what happens in the example above: the third potential is not in the orbit of the D7-brane.

With this reasoning we could already determine a lower bound on the number of supersymmetric p-branes in D space-time dimensions by looking at the size of the U-duality orbits of the standard branes.[3] We claim that this already gives the full correct answer rather than just a lower bound. As will be explained in more detail below, the reason is that from an E_{11} point of view, this orbit corresponds entirely to *real* roots of the algebra, whereas all other orbits correspond to *imaginary* roots. That the solutions associated to imaginary roots are not supersymmetric was verified in detail in the null case in [21].[4] What has to be done therefore from an E_{11} logic is to select among the potentials of table 1 those that correspond to real roots. This can be done since one knows for all the potentials the corresponding roots. What one arrives at is table 2. For consistency we note that U-duality preserves the norm.[5]

In the table, all possible g_s scalings are still grouped together but it is desirable to differentiate the different types of non-perturbative behaviour in terms of string frame tensions. This can be done by recalling that U-duality is composed out of T- and S-duality, see for instance. [23, 24]. T-duality does not change the power of g_s [24] and therefore one

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[2]The $(D-1)$-forms correspond to possible (gauge) deformations of maximal supergravity, the D-forms are related to constraints on these deformations; this will not be of importance in this paper.

[3]The counting of supersymmetric branes presented here only deals with 'pure' or 'elementary' branes and does not consider intersecting branes nor dyonic [16] or more complicated bound states. These can be also analysed by algebraic methods [17–20].

[4]The difference between real and imaginary also played an important role in [22].

[5]The difference in the 3-form spectrum in $D = 3$ in the tensor hierarchy analysis of [2] and the E_{11} analysis of [4, 5] consists of the **248** of E_8 in table 1. It lies entirely in the imaginary root sector and so does not source any supersymmetric branes. Its presence or not does not alter table 2.

needs to consider the breaking

$$E_{11-D}(\mathbb{R}) \supset \mathrm{SO}(10-D, 10-D; \mathbb{R}) \times \mathbb{R}_+. \tag{1.1}$$

The weight under $\mathbb{R}_+$ corresponds to σ when normalized correctly. We will discuss this breaking in more detail below and there will give tables counting the supersymmetric objects for all values of p, D and σ. Our results represent the first complete count of supersymmetric branes in all dimensions $D \geq 3$ and agree with completely independent derivations where available.

Viewing the supersymmetric branes as associated to real roots of E_{11} makes it very simple to follow them through the various dimensions and obtain higher-dimensional origins of the generalised KK-monopoles and similar 'obscure' non-standard objects necessary for the correct counting.

Besides counting the solutions, it is of interest to construct them. This can be done by performing U-duality transformations on known solutions and in this way generalised KK monopoles of co-dimension two have already been constructed in the past [25, 26]. These solutions require a fair number of isometry directions when viewed as ten-dimensional solutions because one needs to perform many T-duality transformations on the known solutions.

We will not address here the construction of supersymmetric generalised Kaluza-Klein monopole solutions with co-dimension less than two but leave this to future work. The reason is that they require the extension of the U-duality transformation rules to deformed supergravity, most notably to massive type IIA supergravity where the D8-brane is already a solution only of the mass deformed theory [27]. Some solutions can be constructed without such generalizations of e.g. [28] and which ones can easily be decided by studying the corresponding E_{11} root vectors.

The rest of this note starts by giving a more detailed description of the E_{11} derivation of the counting of branes and tables for all values of p, D and σ in section 2. There we will also discuss the higher-dimensional $(D = 11)$ origin of the solutions. In section 3, we explicitly construct local solutions using T-duality transformations. Appendix A gives the type IIA and type IIB origin of all the branes and appendix B contains our conventions.

2 E_{11} derivation

We start this section by recalling the usefulness of E_{11} [29] as a bookkeeping device for the field content of maximal supergravity in various space-time dimensions [4–6].

The Dynkin diagram of E_{11} is given in figure 1. If one is interested in maximal supergravity in D dimensions one has to 'cut' the diagram at node D. To the left one will have a diagram of type A_{D-1}, corresponding to the so-called 'gravity line'. To the right of the cut one obtains the diagram of the hidden symmetry E_{11-D}.[6] In other words one has

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[6]For $D = 9$, one has to also remove node 11, for $D = 10$, there are two choices, corresponding to type IIA and type IIB [6].
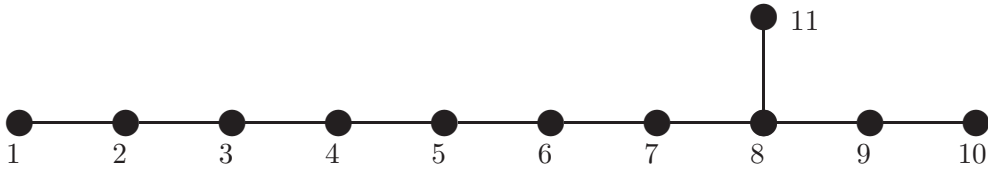
Figure 1. Dynkin diagram of E_{11} with labelling of nodes.

a decomposition[7]

$$E_{11} \supset GL(D, \mathbb{R}) \times E_{11-D}. \tag{2.1}$$

One obtains the spectrum of fields in D space-time dimensions by decomposing the adjoint representation of E_{11} under this subgroup. As one deals with the adjoint representation one can label the elements by the roots α of the algebra. Any such root can be expanded in a basis of simple roots α_i as

$$\alpha = \sum_{i=1}^{11} m^i \alpha_i. \tag{2.2}$$

As E_{11} is an infinite-dimensional Kac-Moody algebra, α can be either real or imaginary, depending on whether its norm $|\alpha|^2$, computed in the standard Killing metric, is greater than zero or not. The simplest example of an imaginary root is the affine null root

$$\delta = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 6\alpha_8 + 4\alpha_9 + 2\alpha_{10} + 3\alpha_{11}. \tag{2.3}$$

Imaginary roots can have multiplicities; the multiplicity of δ is equal to 8. Real roots always have multiplicity equal to 1.

The result of the decomposition of the adjoint of E_{11} under (2.1) can be ordered by 'level', that is by the number of times the deleted simple node α_D appears in the expansion. The results of this decomposition at low levels for all D can be found in [4–6]. In this way one can associate to any given field in maximal supergravity a root α *et vice versa*.[8]

As explained in the introduction, we are interested in the breaking of U-duality to T-duality. The U-duality group E_{11-D} is constructed from the nodes from $D + 1$ up to 11; the $SO(10 - D, 10 - D)$ T-duality subgroup is obtained by ignoring the node 10 of the E_{11-D} diagram. The $\mathbb{R}_+$ factor in (1.1) is therefore associated with the scaling under node 10. More precisely, we have [6][9]

$$\sigma = m^{10}, \tag{2.4}$$

i.e., the string frame tension dependence σ is the eigenvalue under the fundamental weight generator λ_{10} acting on a given generator of E_{11}.

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[7]The enhancement from $SL(D, \mathbb{R}) \cong A_{D-1}$ to $GL(D, \mathbb{R})$ is due to the additional Cartan generator of the node where one cut the diagram.

[8]Since only Borel elements are associated with fields in a non-linear realization, we always have $\alpha > 0$. Due to our restriction to real roots in the following we are not displaying multiplicity indices.

[9]See also [30].

Let us view this in an example from type IIB. According to [6], there are three 8-form potentials, these correspond to the three roots

$$\delta - \alpha_{10}, \quad \delta \quad \text{and} \quad \delta + \alpha_{10}. \tag{2.5}$$

Referring back to (2.3) the scaling factors of the three potentials are therefore

$$\sigma = 1, \quad \sigma = 2 \quad \text{and} \quad \sigma = 3. \tag{2.6}$$

The first potential is the one that couples to the D7 of type IIB, the last one is its S-dual. Both the corresponding roots are real. The middle potential with $\sigma = 2$ corresponds to the null root δ and does not give rise to a supersymmetric solution [21]. Even though the three potentials together form the **3** of $\text{SL}(2, \mathbb{R})$ one cannot map the $\sigma = 1$ potential fully on $\sigma = 2$ potential.

Now, we have all the material at hand to find the number of supersymmetric p-brane solutions of a given scaling σ in D space-time dimensions. It is straight-forward to implement this on the computer and we merely give the results in table form in tables 3 to 11. One computes all real E_{11} roots (up to height 161) and selects all those that correspond to antisymmetric tensor, groups them by rank, weight under λ_{10} and dimension.[10] Alternatively, one can take the tensor hierarchy representations of table 1, compute the Weyl orbits of the dominant (real) weights (this gives table 2) and then branch these Weyl orbits under the T-duality Weyl group.[11] Note that we have chosen to display the tables with σ held fix. In this way, one does not immediately see the feature that for $D = 3, 4, 6, 10$, the numbers of p-branes with p fixed and varying σ are always symmetric, reflecting the underlying $\mathbb{Z}_2$ S-duality. For the remaining values of D there is of course also a $\mathbb{Z}_2$ S-duality that acts on the representations but in these cases the branched orbits arrange themselves asymmetrically. The restriction to real roots we have employed here also follows from the fact that U-duality (the Weyl group) preserves the norm of the roots.

The higher-dimensional origin of a given object can also be recovered easily using the root α corresponding to the potential and its meaning in the various dimensions. We have performed this for the higher dimensional origin in type IIA, type IIB and M-theory.

It turns out that there is quite a number of mixed symmetry tensors that occur in this analysis. Here, we only give the list of tensors in $D = 11$ (M-theory) that contribute to the counting and refer to appendix A for type IIA and type IIB. We denote by $T_{(M_1, M_2, M_3, \dots)}$ an irreducible tensor of $GL(11, \mathbb{R})$ with Young tableau consisting of columns of lengths M_1, M_2, etc. We restrict to $11 \geq M_1 \geq M_2 \geq \dots$, and columns of length 11 are needed to keep track of the overall weight. In this notation, the three-form of $D = 11$ supergravity is therefore written as $T_{(3)}$. The graviton is in the adjoint of $GL(11, \mathbb{R})$ and is listed as $T_{(10,1)}$ below.

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[10]The identification of the root associated with a given potential can for example be done with the tables of [6] or with the help of the simpLie software of [5].

[11]In some cases there are contributions from more than one irreducible U-duality representation.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA		1								
IIB		1								
9	2	1								
8	4	1								
7	6	1								
6	8	1								
5	10	1								
4	12	1								
3	14	1								

Table 3. Supersymmetric p-branes in D dimensions that scale like $g_{\rm s}^{-0}$, i.e., $\sigma = 0$. The table agrees with that of [11] and is in accord with the 'fundamental wrapping rule'. The table contains the fundamental string and its reductions as well reductions of KK momentum states.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA	1		1		1		1		1	
IIB		1		1		1		1		1
9	1	1	1	1	1	1	1	1	1	
8	2	2	2	2	2	2	2	2		
7	4	4	4	4	4	4	4			
6	8	8	8	8	8	8				
5	16	16	16	16	16					
4	32	32	32	32						
3	64	64	64							

Table 4. Supersymmetric p-branes in D dimensions that scale like $g_{\rm s}^{-1}$, i.e., $\sigma = 1$. The table agrees with that of [11] and is in accord with the 'D-brane wrapping rule'. The table contains the usual D-brane spectrum.

With these conventions the full list of mixed symmetry tensors in $D = 11$ necessary for the counting in table 2 is

$$T_{(10,1)}, T_{(3)}, T_{(6)}, T_{(8,1)}, \tag{2.7a}$$

$$T_{(9,3)}, T_{(9,6)}, T_{(9,8,1)}, \tag{2.7b}$$

$$T_{(10,1,1)}, T_{(10,4,1)}, T_{(10,6,2)}, T_{(10,7,4)}, T_{(10,7,7)}, T_{(10,8,2,1)}, T_{(10,8,5,1)}, T_{(10,8,7,2)},$$
$$T_{(10,8,8,4)}, T_{(10,8,8,7)}, \tag{2.7c}$$

$$T_{(11,4,3)}, T_{(11,5,1,1)}, T_{(11,6,3,1)}, T_{(11,6,6,1)}, T_{(11,7,4,2)}, T_{(11,7,6,3)}, T_{(11,7,7,5)}, T_{(11,8,3,1,1)},$$
$$T_{(11,8,4,4)}, T_{(11,8,5,2,1)}, T_{(11,8,6,4,1)}, T_{(11,8,7,2,2)}, T_{(11,8,7,5,2)}, T_{(11,8,7,7,3)},$$
$$T_{(11,8,8,1,1,1)}, T_{(11,8,8,4,1,1)}, T_{(8,8,5,4)}, T_{(11,8,8,6,2,1)}, T_{(11,8,8,7,4,1)}, T_{(11,8,8,7,7,1)},$$
$$T_{(11,8,8,8,2,2)}, T_{(11,8,8,8,5,2)}, T_{(11,8,8,8,7,3)}, T_{(11,8,8,8,8,5)}, T_{(11,8,8,8,8,8)}. \tag{2.7d}$$

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA						1				
IIB						1				
9				1	2					
8				1	4	4				
7			1	6	12	8				
6		1	8	24	32	16				
5	1	10	40	80	80					
4	12	60	160	240						
3	84	280	560							

Table 5. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-2}, i.e., $\sigma = 2$. The table agrees with that of [11] and is in accord with the 'dual wrapping rule'. This table contains the NS-branes and reductions of (generalised) Kaluza-Klein monopoles.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA										
IIB								1		
9							1	1		
8						2	4	2		
7					4	12	12			
6				8	32	48				
5			16	80	160					
4		32	192	480						
3	64	448	1344							

Table 6. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-3}, i.e., $\sigma = 3$. The table agrees with that of [12] and is in accord with the 'exotic wrapping rule'.

We have grouped the generators into four classes in a specific way that arises from the form of their extremal (lowest) root vectors. The first line belongs to E_8, the second line to E_9, the third line to E_{10} and the last line to E_{11}. Solutions corresponding to the first two lines with co-dimension at most two can be written in terms of $D = 11$ supergravity by using the techniques of [25, 26]. The last two lines are associated with deformations of supergravity [4, 5, 31].

The real root components of these mixed symmetry tensors are always obtained when the maximal number of indices is identical. This can be seen by considering the action of the lowering operators of $GL(11, \mathbb{R})$ on the tensors. The lowest root component of the multiplet is the one where all indices take their maximum value. For example, if we write the tensor $T_{(10,1,1)}$ explicitly with indices as

$$T_{(10,1,1)} \; : \; T_{M_1 \dots M_{10}, N, P}, \tag{2.8}$$

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
IIA										
IIB										1
9									1	
8								2		
7						1	4			
6					8	16				
5				40	96					
4		1	160	512						
3	14	574	2304							

Table 7. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-4}, i.e., $\sigma = 4$. There is no obvious wrapping rule associated with this table.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
6					8					
5				80						
4			32	480						
3		448	2688							

Table 8. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-5}, i.e., $\sigma = 5$. Dimensions in which there are no branes have been eliminated from the table.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
4				240						
3		280	3360							

Table 9. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-6}, i.e., $\sigma = 6$. Dimensions in which there are no branes have been eliminated from the table.

$D\backslash p$	0	1	2	3	4	5	6	7	8	9
4				32						
3		64	2688							

Table 10. Supersymmetric p-branes in D dimensions that scale like g_{s}^{-7}, i.e., $\sigma = 7$. Dimensions in which there are no branes have been eliminated from the table.

then the lowest root vector component is $(M, N, P = 1, \ldots, 11)$

$$T_{2\ldots11,11,11}. \tag{2.9}$$

We see that this tensor, reduced along the direction 11 will give rise to a pure nine-form. In fact, it is the nine-form that the mass of massive type IIA couples to [28, 32]. We note that the highest $GL(11, \mathbb{R})$ level needed in E_{11} to accommodate all the tensors of (2.7) is $\ell = 17$.

$D = 3$:

$\sigma\backslash p$	0	1	2	3	4	5	6	7	8	9
8		1	2304							
9			1344							
10			560							
11			64							

Table 11. Supersymmetric p-branes in 3 dimensions that scale like $g_{\rm s}^{-\sigma}$ for $\sigma = 8, 9, 10, 11$. This table is different from the preceding ones as only $D = 3$ appears and we have therefore chosen a more compact presentation.

One can also analyse the T-duality representations that occur for the different values of σ. For $\sigma \leq 3$, these were already given in [10, 12]. For $\sigma = 4$ we find that for $D \leq 7$ the following representations of $SO(10 - D, 10 - D; \mathbb{R})$ arise[12]

$(D-2)$-brane	$(7-D)$-form
$(D-1)$-brane	self-dual $(10-D)$-form and $(7-D,1)$ hook

In addition, there is a singlet $(D-3)$-brane in $D = 4$.

For $\sigma = 5$ one finds the following representations of $SO(10 - D, 10 - D; \mathbb{R})$ in $D \leq 7$.

$(D-2)$-brane	$(4-D)$ tensor spinor
$(D-1)$-brane	$(6-D)$ tensor spinor and $(4-D,1)$ hook spinor

For $\sigma > 5$ one can also easily determine the T-duality representations but since there are very few cases that appear, we do not list them here. For even σ one obtains tensors and for odd σ (general) tensor spinors. To recover the correct counting from the T-duality representations one again has to restrict to real roots; alternatively, the correct counting is given by the size of the $SO(10 - D, 10 - D; \mathbb{R})$ Weyl orbit of the lowest weight.

3 Co-dimension two supergravity solutions

The IIA/IIB solutions that are required as additional sources for the supersymmetric branes discussed in section 2 can be constructed from (discrete) U-duality. In the language of E_{11} introduced above this corresponds to Weyl transformations. In this section we exemplarily construct some such solutions of co-dimension two.

3.1 $D = 7$ and $1/g_{\rm s}^2$

As the first example we take 4-branes in $D = 7$ maximal supergravity with tension $1/g_{\rm s}^2$ in string frame. There are two transverse directions that we take to be in directions 1 and 2. The counting predicts a total of 12 such objects. They have the following ten-dimensional origin

- Three singly wrapped NS5-branes by choosing one of the world-volume directions to lie along one of the three compact directions.

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[12]If $D = 7$ the hook tableau does not exist and the representation is absent. Similarly, in the table for $\sigma = 5$.

- Six wrapped Kaluza-Klein 6-monopoles. The NUT has to be in one of the three compact directions, and another world-volume direction is along one of the other two compact directions, leading to 3×2 possibilities.

- Three 'non-standard' branes. These are the ones that we will discuss in more detail now.

Considered as real roots of E_{11} the three 'non-standard' branes correspond to

$$\beta_1 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 6\alpha_8 + 4\alpha_9 + 2\alpha_{10} + 4\alpha_{11}, \tag{3.1a}$$

$$\beta_2 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 7\alpha_8 + 4\alpha_9 + 2\alpha_{10} + 4\alpha_{11}, \tag{3.1b}$$

$$\beta_3 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 7\alpha_8 + 5\alpha_9 + 2\alpha_{10} + 4\alpha_{11}. \tag{3.1c}$$

From a IIA perspective they are all at level $(2,4)$ since the coefficients of α_{10} and α_{11} are 2 and 4, respectively. They all belong to the $GL(10,\mathbb{R})$ representation with an $(8,2)$ Young tableau. Considering the affine null root δ that has level $(2,3)$ in the IIA decomposition, we expect that all solutions are more complicated versions of the fundamental string that sits at level $(0,1) = (2,4) - (2,3)$ in the level decomposition. Additionally, we expect from the form of the roots that this string is extended along the directions 9 and 10.

Let us construct a solution corresponding to β_1. The fastest way of doing this is by realising β_1 as a short Weyl transformation acting on a known solution. We choose

$$\beta_1 = w_8 w_{11} w_8(\beta_0) \tag{3.2}$$

for $\beta_0 = \delta - \alpha_8$. The BPS seed solution corresponding to β_0 is a (smeared) KK6-monopole that is aligned as follows in IIA:

1	2	$\hat{3}$	4	5	6	7	8	9	10
		×	×	×	×	×		N	×

The hat on the direction 3 indicates that we choose this direction to be the time direction, 'N' indicates the NUT. The IIA metric of this solution is given in string frame by

$$ds^2_{\beta_0} = H\left(\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^8\right)^2\right) + H^{-1}\left(dx^9 - Bdx^8\right)^2$$
$$- \left(dx^3\right)^2 + \left(dx^4\right)^2 + \left(dx^5\right)^2 + \left(dx^6\right)^2 + \left(dx^7\right)^2 + \left(dx^{10}\right)^2. \tag{3.3}$$

All other fields vanish. The equations of motion are satisfied if H is harmonic and B is its conjugate harmonic[13]

$$\partial_i H = -\sum_{j=1,2} \epsilon_{ij} \partial_j B. \tag{3.4}$$

We furthermore assume H to be positive.

The Weyl transformation w_8 corresponds to an interchange of direction 8 and 9; the Weyl transformation w_{11} is a double T-duality in directions 9 and 10 with subsequent

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[13]We use $\epsilon_{12} = +1$ and a sign choice was made in writing (3.4). The equations are also solved for the opposite choice.

interchange of the two directions [24]. Carrying out the transformation dictated by (3.2) using the Buscher rules one obtains the following string frame configuration[14]

$$ds^2_{\beta_1} = \tilde{H}\left(\left(dx^9\right)^2 + \left(dx^{10}\right)^2\right) + H\left(\left(dx^1\right)^2 + \left(dx^2\right)^2\right)$$
$$- \left(dx^3\right)^2 + \left(dx^4\right)^2 + \left(dx^5\right)^2 + \left(dx^6\right)^2 + \left(dx^7\right)^2 + \left(dx^{10}\right)^2, \tag{3.5a}$$

$$B_{9\,10} = \tilde{B}, \tag{3.5b}$$

$$\phi = \frac{1}{2}\log \tilde{H}. \tag{3.5c}$$

All other field components vanish. The functions $\tilde{H}$ and $\tilde{B}$ are defined by

$$\tilde{H} = \frac{H}{H^2 + B^2}, \qquad \tilde{B} = \frac{B}{H^2 + B^2} \tag{3.6}$$

and are also conjugate

$$\partial_i \tilde{H} = \sum_{j=1,2} \epsilon_{ij} \partial_j \tilde{B}. \tag{3.7}$$

We see that our expectations regarding the configuration are borne out. The similarity to the fundamental string (B.10) can be seen after rescaling to Einstein frame.

This type IIA solution is best interpreted as a solution of $D = 7$ supergravity where one has reduced along the isometry directions 8, 9 and 10. In this case it couples not to an $(8, 2)$ mixed Young tableaux but to a pure 5-form. The tension of this object in string frame can be shown to be $1/g_s^2$ by standard U-duality techniques and we note that this solution has already appeared in [25].

3.2 $D = 7$ and g_s^{-3}

Table 6 predicts four 4-branes in $D = 7$ with tension proportional to g_s^{-3}. One of them is obtained by wrapping the S-dual of the D7 brane of type IIB, the origin of the other three branes is not immediate from the standard string theory branes. They could be obtained by $SL(5, \mathbb{Z})$ U-duality from the wrapped S-dual in $D = 7$. We instead construct them directly in type IIA by considering the corresponding E_{11} roots.

The four roots are

$$\beta_1 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 6\alpha_8 + 4\alpha_9 + 3\alpha_{10} + 3\alpha_{11}, \tag{3.8a}$$

$$\beta_2 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 6\alpha_8 + 5\alpha_9 + 3\alpha_{10} + 3\alpha_{11}, \tag{3.8b}$$

$$\beta_3 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 7\alpha_8 + 5\alpha_9 + 3\alpha_{10} + 3\alpha_{11}, \tag{3.8c}$$

$$\beta_4 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 4\alpha_6 + 5\alpha_7 + 7\alpha_8 + 5\alpha_9 + 3\alpha_{10} + 4\alpha_{11} \tag{3.8d}$$

From the form of the roots, we can guess their type IIB intepretation. The first root is the one corresponding to the S-dual of the type IIB 7-brane; the other three resemble more complicated versions of the D1-branes. Similarly, we can give their origin in $D = 11$. Here,

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[14]This is essentially the same calculation as in [26].

the first three arise from wrapping the KK7-monopole in specific ways, whereas β_4 is a more complicated 3-brane.

In type IIA theory, the first three are similar to D0-particles whereas the last one is a kind of D2-brane. Let us construct the solution for β_3 and β_4 explicitly. For β_3 we do this by reduction of a suitable (smeared) KK7-monopole of $D = 11$. From the root vector β_3 one deduces that the KK7-monopole in $D = 11$ has to be

$$ds_{11}^2 = H\left(\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^{11}\right)^2\right) - \left(dx^3\right)$$
$$+ \sum_{k=4,5,6,7,9,10} \left(dx^k\right)^2 + H^{-1}\left(dx^8 - Bdx^{11}\right)^2 . \tag{3.9}$$

The NUT is in the direction 8 and time is in direction 3. The functions B and H depend on x^1 and x^2 and satisfy (3.4). This solution has to be reduced along the M-theory direction x^{11}. The result in IIA Einstein frame is

$$ds_{\beta_3}^2 = \tilde{H}^{-1/8}H\left(\left(dx^1\right)^2 + \left(dx^2\right)\right)^2 - \tilde{H}^{-1/8}\left(dx^3\right)^2$$
$$+ \tilde{H}^{-1/8} \sum_{k=4,5,6,7,9,10} \left(dx^k\right)^2 + \tilde{H}^{7/8}\left(dx^{10}\right)^2 , \tag{3.10a}$$

$$\phi = -\frac{3}{4}\log\tilde{H}, \tag{3.10b}$$

$$A_8 = -\tilde{B}, \tag{3.10c}$$

where $\tilde{H}$ and $\tilde{B}$ are as in (3.6). As expected, the solutions couples to the 1-form potential and is hence similar to a D0-brane.

Since $\beta_4 = w_{11}(\beta_3)$ we can construct the solution for β_4 by performing a double T-duality in directions 9 and 10 and subsequent exchange of directions 9 and 10. We need to use the Buscher rules extended to the RR sector since the IIA vector potential is non-vanishing. These have been studied for example in [28]. A version sufficient for our purposes is derived in (B.9) in the appendix. The resulting solution of type IIA in Einstein frame is

$$ds_{\beta_4}^2 = \tilde{H}^{-3/8}H\left(\left(dx^1\right)^2 + \left(dx^2\right)\right)^2 - \tilde{H}^{-3/8}\left(dx^3\right)^2$$
$$+ \tilde{H}^{-3/8} \sum_{k=4,5,6,7,9,10} \left(dx^k\right)^2 + \tilde{H}^{5/8}\left(\left(dx^{10}\right)^2 + \left(dx^9\right)^2 + \left(dx^{10}\right)^2\right) , \tag{3.11a}$$

$$\phi = -\frac{1}{4}\log\tilde{H}, \tag{3.11b}$$

$$A_{8\,9\,10} = -\tilde{B}. \tag{3.11c}$$

The solution couples to the 3-form potential of type IIA as expected and also displays a metric characteristic of a D2-type object. This solution has appeared in slightly different form in [25], where it was also shown that its tension in string frame scales like $1/g_s^3$.

3.3 $D = 4$ and g_{s}^{-4}

As one example for string frame tension $1/g_{\mathrm{s}}^4$ we study a co-dimension two brane in $D = 4$. According to table 7 there is a single such object. Its root vector is

$$\beta = \alpha_3 + 2\alpha_4 + 4\alpha_5 + 6\alpha_6 + 8\alpha_7 + 10\alpha_8 + 7\alpha_9 + 4\alpha_{10} + 5\alpha_{11}. \tag{3.12}$$

The origin of this solution in type IIA and type IIB is that of a more complicated NS5-brane, whereas in $D = 11$ it is related to the M5-brane. We construct the solution in type IIA by starting from a known solution and performing an appropriate series of duality transformations.

One way of arriving at β is by starting from

$$\beta_0 = \alpha_3 + 2\alpha_4 + 3\alpha_5 + 5\alpha_6 + 7\alpha_7 + 9\alpha_8 + 6\alpha_9 + 3\alpha_{10} + 4\alpha_{11} \tag{3.13}$$

(which agrees with $w_8 w_9 w_7 w_8 w_6 w_7(\beta_4)$ with β_4 of (3.8d)) and then considering the following chain of E_{11} Weyl transformations

$$\beta = w_{10} w_9 w_8 w_7 w_6 w_5 w_{11}(\beta_0). \tag{3.14}$$

The only transformations that do not correspond simply to a permutation of space-like directions in type IIA are the ones in with indices 10 and 11; we will deal with them by lifting the solution corresponding to β_0 to $D = 11$ where the reflection in 10 is also a simple permutation.[15] The reflection in 11 is a, as before, a double T-duality in type IIA in directions 9 and 10 with interchange of the two directions.

The type IIA Einstein frame solution corresponding to β_0 (obtained as a permutation variant of (3.11)) is

$$
\begin{aligned}
ds^2_{\beta_0} = {}& H\tilde{H}^{-3/8}\left(\left(dx^1\right)^2 + \left(dx^2\right)^2\right) - \tilde{H}^{-3/8}\left(dx^3\right)^2 \\
& + \tilde{H}^{-3/8}\sum_{k=4,5,9,10}\left(dx^k\right)^2 + \tilde{H}^{5/8}\left(\left(dx^6\right)^2 + \left(dx^7\right)^2 + \left(dx^8\right)^2\right),
\end{aligned} \tag{3.15a}
$$

$$\phi = -\frac{1}{4}\log\tilde{H}, \tag{3.15b}$$

$$A_{6\,7\,8} = -\tilde{B}. \tag{3.15c}$$

The w_{11} reflection gives the metric IIA Einstein frame solution

$$
\begin{aligned}
ds^2_{w_{11}(\beta_0)} = {}& H\tilde{H}^{-5/8}\left(\left(dx^1\right)^2 + \left(dx^2\right)^2\right) + \tilde{H}^{3/8}\sum_{k-6,7,8,9,10}\left(dx^k\right)^2 \\
& + \tilde{H}^{-5/8}\left(-\left(dx^3\right)^2 + \left(dx^4\right)^2 + \left(dx^5\right)^2\right),
\end{aligned} \tag{3.16a}
$$

$$\phi = \frac{1}{4}\log\tilde{H}, \tag{3.16b}$$

$$A_{3\,4\,5} = -\frac{1}{\tilde{H}}. \tag{3.16c}$$

---

[15]Note that is the final reflection in 10 that changes the scaling of the tension with g_{s}.

Performing now the remaining transformations of (3.14) one arrives finally at the solution for β in type IIA Einstein frame

$$ds^2_\beta = H\tilde{H}^{-3/4}\left(\left(dx^1\right)^2 + \left(dx^2\right)^2\right) - \tilde{H}^{-3/4}\left(dx^3\right)^2 + \tilde{H}^{-3/4}\left(dx^4\right)^2$$

$$+ \tilde{H}^{1/4}\sum_{k=5}^{10}\left(dx^k\right)^2, \tag{3.17a}$$

$$\phi = -\frac{1}{2}\log\tilde{H}, \tag{3.17b}$$

$$B_{34} = -\frac{1}{\tilde{H}}. \tag{3.17c}$$

We see that it indeed couples to the NS-NS 2-form as expected and has a metric similar to that of an NS 5-brane. By the usual U-duality arguments its tension in string frame scales like $1/g_s^4$. This solution has been already listed in [25].

3.4 $\quad D = 3$ and g_s^{-4}

As a final example we consider another $1/g_s^4$ brane, this time a 0-brane in $D = 3$. Out of the 14 such objects in table 7, 7 correspond more complicated NS 5-branes of type IIA (obtained by reduction of solutions of the type studied just now) whereas there are 7 purely gravitational solutions that we now aim to construct. They correspond to tensors of shape $(8, 7, 1)$ in type IIA and are hence generalised Kaluza-Klein monopoles.

One E_{11} root that corresponds to such an object is

$$\beta_{\mathrm{G}} = \alpha_3 + 3\alpha_4 + 5\alpha_5 + 7\alpha_6 + 9\alpha_7 + 11\alpha_8 + 7\alpha_9 + 4\alpha_{10} + 6\alpha_{11}. \tag{3.18}$$

It can be reached from β of (3.12) by the following chain of Weyl transformations:

$$\beta_{\mathrm{G}} = w_{11}w_8w_7w_6w_5w_4(\beta). \tag{3.19}$$

All of these operations we can carry out easily as permutations and (double) T-duality transformation in type IIA. What one arrives at is the following Einstein frame metric[16]

$$ds^2_{\beta_{\mathrm{G}}} = H\tilde{H}^{-1}\left(\left(dx^1\right)^2 + \left(dx^2\right)^2\right) + \sum_{k=4}^{9}\left(dx^k\right)^2 + \tilde{H}\left(dx^{10}\right)^2 + 2dx^3dx^{10}. \tag{3.20}$$

All other fields are zero as this solution is purely gravitational. As we only performed T-dualities on the solution (3.17), the solution also has a string frame tension $1/g_s^4$. This solution was given before in [25].

3.5   Relation to Geroch group

The proper language for analysing solutions that depend on two (or less) transverse direction is that of the Geroch group [33–35]. The Geroch group is the infinite-dimensional

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[16]Note the absence of $-\left(dx^3\right)^2$ in the metric which stems from the cancellation of terms when applying the T-duality rules (B.9).

solution generating group of supergravity solutions that have $D - 2$ commuting Killing vectors. It is an affine symmetry group [35, 36]; in the present case, it is affine E_8, i.e., E_9 [36, 37]. All the solutions relevant here are in fact described by an affine $SL(2, \mathbb{R})$ subgroup of E_9.

A convenient quantity for parametrising such solutions is the complex Ernst potential that we here define as [26][17]

$$\mathcal{E} = B + iH. \tag{3.21}$$

The conjugate harmonic equation (3.4) then becomes the Cauchy-Riemann condition for $\mathcal{E}$ to be holomorphic in $\xi = x^1 + ix^2$. The transformed Ernst potential $\tilde{\mathcal{E}} = \tilde{B} + i\tilde{H}$ then is anti-holomorphic in ξ according to (3.7). Furthermore, the two Ernst potentials are related by

$$\tilde{\mathcal{E}} = \frac{1}{\bar{\mathcal{E}}}. \tag{3.22}$$

Thinking of $\mathcal{E}$ as parametrising the upper half plane, this corresponds to the action of the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \in PGL(2, \mathbb{Z}) \tag{3.23}$$

on it, where a negative determinant amounts to an additional complex conjugation of $\mathcal{E}$.

Any holomorphic Ernst potential $\mathcal{E}$ corresponds to a BPS solution [26] and hence any holomorphic transformation of $\mathcal{E}$ will give a new BPS solution. Infinite towers of supersymmetric solutions were constructed using this approach in [26]. However, the way of constructing these solutions required going to higher and higher level in the affine group. The mixed symmetry tensors in the affine group embedded in E_{11} are of the form [31, 38, 39]

$$T_{(9,9,\ldots,9,3)}, T_{(9,9,\ldots,9,6)}, T_{(9,9,\ldots,9,8,1)}, \tag{3.24}$$

where we have used the $GL(11, \mathbb{R}) \subset E_{11}$ decomposition that is appropriate for $D = 11$ supergravity. The number of repetitions of columns of length 9 is equal to the affine level ℓ_{aff}. We see that only up to affine level equal to $\ell_{\text{aff}} = 1$ it is possible to obtain forms in $D \geq 3$ dimensions. The other supersymmetric solutions of co-dimension two in the infinite towers of [26] are brane solutions only in $D = 2$ and therefore do not contribute to table 2.[18]

4 Co-dimension less than two solutions?

In this section, we address the issue of constructing supergravity solutions with co-dimension equal to one or zero. We will see by means of an example that the construction

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[17]This complex function of two real fields is sufficient as long as one stays in an $SL(2, \mathbb{R})$ subgroup as we are doing for the elementary BPS solutions.

[18]An additional complication beyond $\ell_{\text{aff}} = 1$ is that one also has to perform T-dualities in a time-like direction. Such T-dualities can change the signature of the theory [40–42] and some of the infinitely many solutions of [26] are therefore solutions of theories with different signature.

of these objects is not always possible without also considering deformations of maximal supergravity.

The example we consider are space-filling branes in $D = 8$ with string frame tensions scaling like g_{s}^{-3}. Table 6 predicts two such objects, the corresponding real roots are

$$\beta_1 = \alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 5\alpha_5 + 6\alpha_6 + 7\alpha_7 + 8\alpha_8 + 4\alpha_9 + 3\alpha_{10} + 4\alpha_{11}, \tag{4.1a}$$

$$\beta_2 = \alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 5\alpha_5 + 6\alpha_6 + 7\alpha_7 + 8\alpha_8 + 7\alpha_9 + 3\alpha_{10} + 4\alpha_{11}. \tag{4.1b}$$

They only differ in the α_9 coefficient. Viewed from $D = 11$ they are part of the level four tensor with Young tableau $(10, 1, 1)$, i.e., the Romans mass of (2.8). This representation does not correspond to any of the gradient representations of [38] or to the the dual fields of [31]. But it is known to be the one that gives rise to the Romans mass [43] upon dimensional reduction to $D = 10$ [32]. Viewed from a type IIA perspective the roots (4.1) belong to a similar $(9, 1, 1)$ representation of $GL(10, \mathbb{R})$.

However, when viewed as coming from type IIB, the two roots are very different due to the different α_9 coefficient. The root β_1 belongs to a pure eight-form potential in IIB, whereas β_2 belongs to a tableau of shape $(10, 2, 2)$ at level seven. This is indeed what is to be expected since we know that there is a space-filling g_{s}^{-3} brane in type IIB already in $D = 10$, namely the S-dual of the D7-brane. Its dimensional reduction to $D = 8$ will give a similar brane in $D = 8$ and it is the one associated with β_1. β_2, however, does not have such a simple explanation and it represents the new generalised KK-monopole required for making the counting correct.

Attempting to construct the local solution corresponding to β_2 we consider a sequence of Weyl transformations that relate β_2 to known solutions. The following relation using a fundamental Weyl reflection gives the solution

$$\beta_2 = w_9(\beta_1). \tag{4.2}$$

Since we know what IIB solution β_1 corresponds to, and w_9 is represented as the exchange of directions 9 and 10 in IIA language, all we would need to do is to take the β_1 type IIB solution (S-dual of D7), translate it into IIA variables and then exchange directions 9 and 10.

When trying to do this in practice one encounters the problem that the necessary IIA version of the D7 brane (or its S-dual) is the D8-brane (or its 'S-dual'). Whereas the D7-brane is a bona fide solution of type IIB supergravity with two transverse directions on which all quantities depend holomorphically, the D8 brane is not a solution of the usual type IIA: One needs to consider the massive deformation of Romans to accommodate the D8-brane [27]. That the D8-brane does not arise in type IIA can be seen as follows. In order to T-dualise one needs an isometry direction in the transverse space. However, since all everything depends holomorphically on the transverse space, turning one of the two directions isometric will render all functions constant and then the IIB solution is diffeomorphic to flat space. This can be circumvented in massive IIA theory [28]. Therefore, we see that in order to describe all the supersymmetric branes of table 2, we need to include all possible deformations (massive/gaugings) of maximal supergravity as well. We leave the construction of these solutions in gauged maximal supergravity or massive supergravity to future work.

5 Conclusions

In this paper we have determined the number of supersymmetric p-branes associated with a $(p+1)$-form potential in $3 \leq D \leq 10$ dimensional maximal supergravity. The rule for finding the correct number rested on identifying supersymmetric branes as associated with real roots of E_{11} as imaginary roots do not correspond to supersymmetric configurations [21]. The numbers we have obtained in this way coincide perfectly with the counting done by completely different means for the cases that were analysed in [11, 12]. Our results are complete and give predictions for supersymmetric branes that become very heavy at weak string coupling. For all supersymmetric branes we have determined their origin in ten-dimensional IIA and IIB theory and M-theory in terms of mixed symmetry tensors.

All the new branes that are found in this way have co-dimension at most two. For the co-dimension two case we have explained how to identify the local supergravity solutions and have derived them by analysing Weyl orbits in E_{11}. The solutions thus obtained had already been derived differently in the literature in [25]. We have not addressed the issue of constructing finite energy solutions by suitable superpositions, i.e., choices of the Ernst potential in (3.21) but suspect that this will go very much along the lines of [44]. We also see how to associate the co-dimension two solutions to mixed symmetry tensors of maximal supergravity. This we did for the example of type IIA and have exhausted the list of affine generators in (A.1). Clearly, the same can be done for type IIB and $D = 11$ supergravity.

For solutions with co-dimension at most one, the treatment given here in terms of massless or ungauged supergravity is generally not sufficient and the formalism will have to be extended to construct the new solutions. The role of all the new solutions for microstate counting remains to be clarified.

Acknowledgments

Note added. After this paper was finished, the preprint [45] appeared that extends the Wess-Zumino analysis of [9–12] for $D \geq 6$. Where comparable those results are also in agreement with ours. Similarly, the paper [46] appeared after first release of this paper and contains some overlap with the results discussed in section 3.

A Type IIA and IIB origin of all solutions

In this appendix, we give the complete list of all tensors in type IIA and type IIB necessary for accommodating all supersymmetric solutions summarised in table 2.

The type IIA mixed tensor fields necessary for accommodating all the solutions are:

$$T_{(9,1)}, T_{(1)}, T_{(2)}, T_{(3)}, T_{(5)}, T_{(6)}, T_{(7)}, T_{(7,1)}, \tag{A.1a}$$

$$T_{(8,1)}, T_{(8,2)}, T_{(8,3)}, T_{(8,5)}, T_{(8,6)}, T_{(8,7)}, T_{(8,7,1)}, \tag{A.1b}$$

$$T_{(9)}, T_{(9,1,1)}, T_{(9,3)}, T_{(9,3,1)}, T_{(9,3,3)}, T_{(9,4,1)}, T_{(9,5,1)}, T_{(9,5,2)}, T_{(9,6)}, T_{(9,6,2)}, T_{(9,6,3)},$$

$$T_{(9,6,4)}, T_{(9,6,5,2)}, T_{(9,6,6)}, T_{(9,7,1)}, T_{(9,7,1,1)}, T_{(9,7,2,1)}, T_{(9,7,4)}, T_{(9,7,4,1)}, T_{(9,7,5,1)},$$

$$T_{(9,7,6,1)}, T_{(9,7,6,2)}, T_{(9,7,7)}, T_{(9,9,7,2)}, T_{(9,7,7,3)}, T_{(9,7,7,6)}, T_{(9,7,7)}, \tag{A.1c}$$

$$T_{(10,1,1)}, T_{(10,3,2)}, T_{(10,3,3)}, T_{(10,4)}, T_{(10,4,1)}, T_{(10,4,1,1)}, T_{(10,4,3)}, T_{(10,5,1,1)}, T_{(10,5,2)},$$

$$T_{(10,5,2,1)}, T_{(10,5,3,1)}, T_{(10,5,5)}, T_{(10,5,5,1)}, T_{(10,6,2)}, T_{(10,6,3,1)}, T_{(10,6,3,2)},$$

$$T_{(10,6,4,2)}, T_{(10,6,5,2)}, T_{(10,6,5,3)}, T_{(10,6,6,1)}, T_{(10,6,6,3)}, T_{(10,6,6,4)}, T_{(10,6,6,5)},$$

$$T_{(10,7,2)}, T_{(10,7,2,1,1)}, T_{(10,7,3,1,1)}, T_{(10,7,3,3)}, T_{(10,7,4)}, T_{(10,7,4,1)}, T_{(10,7,4,1,1)},$$

$$T_{(10,7,4,2)}, T_{(10,7,4,2,1)}, T_{(10,7,4,4)}, T_{(10,7,5,2,1)}, T_{(10,7,5,3)}, T_{(10,7,5,3,1)},$$

$$T_{(10,7,5,4,1)}, T_{(10,7,6,1,1)}, T_{(10,7,6,2,2)}, T_{(10,7,6,3)}, T_{(10,7,6,4,1)}, T_{(10,7,6,4,2)},$$

$$T_{(10,7,6,5,2)}, T_{(10,7,6,6,3)}, T_{(10,7,7)}, T_{(10,7,7,1,1,1)}, T_{(10,7,7,2,2)}, T_{(10,7,7,3)},$$

$$T_{(10,7,7,3,1,1)}, T_{(10,7,7,4,1,1)}, T_{(10,7,7,4,3)}, T_{(10,7,7,4,4)}, T_{(10,7,7,5)}, T_{(10,7,7,5,1)},$$

$$T_{(10,7,7,5,1,1)}, T_{(10,7,7,5,2)}, T_{(10,7,7,5,2,1)}, T_{(10,7,7,5,4)}, T_{(10,7,7,6,2,1)}, T_{(10,7,7,6,3)},$$

$$T_{(10,7,7,6,3,1)}, T_{(10,7,7,6,4,1)}, T_{(10,7,7,6,6)}, T_{(10,7,7,6,6,1)}, T_{(10,7,7,7,1,1)}, T_{(10,7,7,7,2,2)},$$

$$T_{(10,7,7,7,3)}, T_{(10,7,7,7,4,1)}, T_{(10,7,7,7,4,2)}, T_{(10,7,7,7,5,2)}, T_{(10,7,7,7,6,2)}, T_{(10,7,7,7,6,3)},$$

$$T_{(10,7,7,7,7,1)}, T_{(10,7,7,7,7,3)}, T_{(10,7,7,7,7,4)}, T_{(10,7,7,7,7,5)}, T_{(10,7,7,7,7,7)}. \tag{A.1d}$$

As in (2.7), we have grouped them into four classes according to the origin of their lowest root vectors in E_8, E_9, E_{10} or E_{11}.

The higher-dimensional of all the branes of table 2 can be found in type IIB by considering the following set of tensors:

$$T_{(9,1)}, T_{(0)}, T_{(2)}, T_{(4)}, T_{(6)}, T_{(7,1)}, \tag{A.2a}$$

$$T_{(8)}, T'_{(8)}. T_{(8,2)}, T_{(8,4)}, T_{(8,6)}, T_{(8,7,1)}, \tag{A.2b}$$

$$T_{(9,2,1)}, T_{(9,3)}, T_{(9,4,1)}, T_{(9,5,2)}, T_{(9,6,1)}, T_{(9,6,3)}, T_{(9,6,5)}, T_{(9,7,1,1)}, T_{(9,7,3,1)},$$

$$T_{(9,7,4)}, T_{(9,7,5,1)}, T_{(9,7,6,2)}, T_{(9,7,7,1)}, T_{(9,7,7,3)}, T_{(9,7,7,5)}, T_{(9,7,7,7)} \tag{A.2c}$$

$$T_{(10)}, T'_{(10)}, T_{(10,2,2)}, T_{(10,4)}, T_{(10,4,1,1)}, T_{(10,4,2)}, T_{(10,4,4)}, T_{(10,5,2,1)}, T_{(10,5,4,1)},$$

$$T_{(10,6,2)}, T_{(10,6,2,2)}, T_{(10,6,3,1)}, T_{(10,6,4,2)}, T_{(10,6,5,1)}, T_{(10,6,5,3)}, T_{(10,6,6)},$$

$$T_{(10,6,6,2)}, T_{(10,6,6,4)}, T_{(10,6,6,6)}, T_{(10,7,1,1,1)}, T_{(10,7,3,1,1)}, T_{(10,7,4,1)}, T_{(10,7,4,2,1)},$$

$$T_{(10,7,4,3)}, T_{(10,7,5,1,1)}, T_{(10,7,5,3,1)}, T_{(10,7,5,5,1)}, T_{(10,7,6,3)}, T_{(10,7,6,3,2)}, T_{(10,7,6,4,1)},$$

$$T_{(10,7,6,5,2)}, T_{(10,7,6,6,3)}, T_{(10,7,7,1,1)}, T_{(10,7,7,2,1,1)}, T_{(10,7,7,3,3)}, T_{(10,7,7,4,1,1)},$$

$$T_{(10,7,7,5,1)}, T_{(10,7,7,5,2,1)}, T_{(10,7,7,5,3)}, T_{(10,7,7,5,5)}, T_{(10,7,7,6,1,1)}, T_{(10,7,7,6,3,1)},$$

$$T_{(10,7,7,6,5,1)}, T_{(10,7,7,7,3)}, T_{(10,7,7,7,3,2)}, T_{(10,7,7,7,4,1)}, T_{(10,7,7,7,5,2)}, T_{(10,7,7,7,6,3)},$$

$$T_{(10,7,7,7,7)}, T_{(10,7,7,7,7,2)}, T_{(10,7,7,7,7,4)}, T_{(10,7,7,7,7,6)}. \tag{A.2d}$$

B Conventions

We use the mostly plus convention for the metric.

B.1 Type IIA and type IIB

Our conventions for type IIA and type IIB supergravity (in a non-democratic formulation) are as follows. The NS-NS sector action in string frame is

$$S_{\text{NS-NS}} = \int d^{10}x \sqrt{-g^{\text{s}}} e^{-2\phi} \left(R^{\text{s}} + 4(\partial\phi)^2 - \frac{1}{12}H_{(3)}^2 \right), \tag{B.1}$$

where we have eliminated Planck's constant and the pre-factor. Upon changing to the Einstein frame metric $g_{mn}^{\text{E}} = e^{-\phi/2}g_{mn}^{\text{s}}$, the action becomes

$$S_{\text{NS-NS}} = \int d^{10}x \sqrt{-g^{\text{E}}} \left(R^{\text{E}} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{-\phi}H_{(3)}^2 \right). \tag{B.2}$$

Here, $H_{(3)}^2 = H_{mnp}H^{mnp}$. The NS-NS sector is common to type IIA and type IIB, however, it will be useful to distinguish the fields in the two theories. We put hats on all type IIB quantities.

The string frame action for the R-R sector of IIA is given by

$$S_{\text{R-R}}^{(\text{IIA})} = \int d^{10}x \sqrt{-g^s} \left(-\frac{1}{4}F_{(2)}^2 - \frac{1}{48}F_{(4)}^2 \right). \tag{B.3}$$

For type IIB it is given by the pseudo-action (of course, to be supplemented by self-duality of the five-form)[19]

$$S_{\text{R-R}}^{(\text{IIB})} = \int d^{10}x \sqrt{-\hat{g}^s} \left(-\frac{1}{2}\hat{F}_{(1)}^2 - \frac{1}{12}\hat{F}_{(3)}^2 - \frac{1}{480}\hat{F}_{(5)}^2 \right). \tag{B.4}$$

In both actions, $F_{(p+1)}$ denotes a field strength with $p+1$ anti-symmetric indices arising from a p-form potential; its square is constructed by full contraction with the metric. We do not give the Chern-Simons terms as they will vanish for all solutions considered in this paper. Similarly, we will not give the 'transgression' and Chern-Simons type terms that appear in the definition of the various field strengths.

In the presence of an isometry, the two theories become equivalent and one can map one set of variables to the other.[20] Assume that $z = 10$ is the isometry direction. The map

---

[19]In string frame, the $SL(2,\mathbb{R})$ symmetry of type IIB is not manifest since the metric transforms. In Einstein frame, the kinetic terms for the two scalars combine as usual into $-\frac{1}{2}\tau_2^{-2}\partial\tau\partial\bar{\tau}$, where $\tau = \hat{\chi} + ie^{-\hat{\phi}}$ and $\partial\hat{\chi} = \hat{F}_{(1)}$ and $SL(2,\mathbb{R})$ acts by fractional linear transformations on τ. The modified two 3-form field strengths $(\hat{F}_{(3)} + \hat{\chi}\hat{H}_{(3)}, \hat{H}_{(3)})$ form an $SL(2,\mathbb{R})$ doublet. It is the modified field strengths that satisfy trivial Bianchi identities (and hence derive directly from 2-form potentials that also transform as a linear doublet). The 5-form potential $\hat{F}_{(5)}$ is invariant but satisfies a non-trivial Bianchi identity [47]. All this also follows from the structure of the exceptional algebra [48, 49].

[20]This is clear since there is only one maximal ungauged supergravity theory in $D = 9$. Relations of the type below have already been derived before, see for instance [50].

between the two reduced theories is most easily done in the Einstein frame. We write the reduction ansatz in terms of the zehnbein decomposed as a block diagonal matrix[21]

$$
e_m{}^a = \begin{pmatrix} e^{c\varphi} e_\mu{}^\alpha & e^{-7c\varphi} \mathcal{A}_\mu \\ 0 & e^{-7c\varphi} \end{pmatrix}, \qquad \text{where} \quad c = \frac{1}{4\sqrt{7}}. \tag{B.5}
$$

The field strength of the Kaluza-Klein vector $\mathcal{A}$ will be denoted by $\mathcal{F}$ in order to distinguish it from the forms arising from the reduction of the various p-form fields. With this ansatz the NS-NS sector action (B.1) reduces to

$$
S_{\text{NS-NS}} = \int d^9 x \sqrt{-g_{(9)}} \left(R_{(9)} - \frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} (\partial\phi)^2 \right.
$$
$$
\left. - \frac{1}{4} e^{-\phi + \frac{3}{\sqrt{7}}\varphi} H_{(2)}^2 - \frac{1}{4} e^{-\frac{4}{\sqrt{7}}\varphi} \mathcal{F}_{(2)}^2 - \frac{1}{12} e^{-\phi - \frac{1}{\sqrt{7}}\varphi} H_{(3)}^2 \right), \tag{B.6}
$$

where we have set the volume of the isometry direction to one. Again, we do not give the full definitions of the various field strengths as they will not matter in our solutions. The reduced action is valid for both type IIA and IIB; one has to put hats on *all* variables for type IIB.

The type IIA R-R sector (B.3) reduces in Einstein frame to

$$
S_{\text{R-R}}^{\text{(IIA)}} = \int d^9 x \sqrt{-g_{(9)}} \left(-\frac{1}{2} e^{\frac{3}{2}\phi + \frac{\sqrt{7}}{2}\varphi} F_{(1)}^2 - \frac{1}{4} e^{\frac{3}{2}\phi - \frac{1}{2\sqrt{7}}\varphi} F_{(2)}^2 \right.
$$
$$
\left. - \frac{1}{12} e^{\frac{1}{2}\phi + \frac{5}{2\sqrt{7}}\varphi} F_{(3)}^2 - \frac{1}{48} e^{\frac{1}{2}\phi - \frac{3}{2\sqrt{7}}\varphi} F_{(4)}^2 \right), \tag{B.7}
$$

whereas the type IIB R-R sector action (B.4) reduces to[22]

$$
S_{\text{R-R}}^{\text{(IIB)}} = \int d^9 x \sqrt{-\hat{g}_{(9)}} \left(-\frac{1}{2} e^{2\hat{\phi}} \hat{F}_{(1)}^2 - \frac{1}{4} e^{\hat{\phi} + \frac{3}{\sqrt{7}}\hat{\varphi}} \hat{F}_{(2)}^2 \right.
$$
$$
\left. - \frac{1}{12} e^{\hat{\phi} - \frac{1}{\sqrt{7}}\hat{\varphi}} \hat{F}_{(3)}^2 - \frac{1}{48} e^{\frac{2}{\sqrt{7}}\hat{\varphi}} \hat{F}_{(4)}^2 \right). \tag{B.8}
$$

The map between the two theories (in Einstein frame) is now easily deduced to be[23]

$$
g_{(9)} = \hat{g}_{(9)}, \qquad\qquad H_{(3)} = \hat{H}_{(3)}, \tag{B.9a}
$$
$$
F_{(1)} = \hat{F}_{(1)}, \qquad\qquad F_{(4)} = \hat{F}_{(4)}, \tag{B.9b}
$$
$$
F_{(2)} = \hat{F}_{(2)}, \qquad\qquad F_{(3)} = \hat{F}_{(3)}, \tag{B.9c}
$$
$$
\mathcal{F}_{(2)} = \hat{H}_{(2)}, \qquad\qquad H_{(2)} = \hat{\mathcal{F}}_{(2)}, \tag{B.9d}
$$

together with the (orthogonal) transformation of dilatons

$$
\phi = \frac{3}{4}\hat{\phi} + \frac{\sqrt{7}}{4}\hat{\varphi}, \qquad\qquad \varphi = \frac{\sqrt{7}}{4}\hat{\phi} - \frac{3}{4}\hat{\varphi}. \tag{B.9e}
$$

---

[21]We omit the Einstein frame superscript in order to keep the expressions more legible. Our reduction ansatz gives the Einstein frame in $D = 9$.

[22]Note that the pseudo-action is transformed into a proper action as there is no longer any problem with self-duality in nine dimensions. The five-form in nine dimensions was dualised into a four-form.

[23]The relations for the field strengths can be directly integrated up to the corresponding potentials, ignoring Chern-Simons and transgression terms.

B.2 (Some) known solutions

The fundamental string extended in directions 9 and 10 and smeared such that the only remaining transverse directions are 1 and 2 is written in Einstein frame as [51]

$$ds_{\rm F1}^2 = H^{-3/4}\left(\left(dx^9\right)^2 - \left(dx^{10}\right)^2\right) + H^{1/4}\sum_{i=1,\ldots,8}\left(dx^i\right)^2, \tag{B.10a}$$

$$\phi = -\frac{1}{2}\log H, \tag{B.10b}$$

$$B_{9\,10} = 1/H, \tag{B.10c}$$

where H is harmonic. As it only uses the NS-NS fields, it is a solution of type IIA and IIB identical in form.

Interpreting it as a solution of type IIB we can perform an S-duality transformation. The result is the D1 string solution. Its metric is identical to (B.10a) but the dilaton ϕ changes sign and $B_{9\,10}$ is replaced by the Ramond potential $A_{9\,10}$.[24]

The D7 brane of type IIB is given by the following expression in Einstein frame

$$ds_{\rm D7}^2 = \tau_2\left(\left(dx^9\right)^2 + \left(dx^{10}\right)^2\right) - \left(dx^3\right)^2 + \sum_{i=1,2,4,\ldots,8}\left(dx^i\right)^2, \tag{B.11a}$$

$$\tau = \chi + ie^{-\phi} \tag{B.11b}$$

for any holomorphic (or anti-holomorphic) $\tau = \tau(x^1 + ix^2)$. We have aligned this solution such that the transverse space is in directions 9 and 10 and time is in direction 3. In this way it corresponds to the E_{11} root

$$\beta = \alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 5\alpha_5 + 6\alpha_6 + 7\alpha_7 + 8\alpha_8 + 4\alpha_9 + \alpha_{10} + 4\alpha_{11}. \tag{B.12}$$

As is well-known, this solution is not of finite energy and one has to use the modular j-function to 'restrict' it to the fundamental domain of the τ plane such that the energy density becomes proportional to the finite volume of the fundamental domain rather than to the infinite volume of the full upper half plane [15, 44]. One also has to change the conformal factor of the two-dimensional transverse space appropriately. This makes the configuration $SL(2,\mathbb{Z})$ invariant. If one uses 24 D7 branes the transverse space becomes smooth.

Performing an S-duality on the D7 solution produces the S7 brane which has the same Einstein frame metric but τ replaced by $-1/\tau$. On β of (B.12) it has the effect of increasing the coefficient of α_{10} from 1 to 3 and therefore one obtains the root β_1 of (4.1a).

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