REACTION-DIFFUSION PROCESSES IN IMPURITY SEEDED RADIATIVE PLASMAS

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1. Introduction

The aim of this paper is to describe impurity seeded radiative plasmas by a single reaction diffusion equation (RDE) resulting from multi-fluid equations (cp. [1]). The starting point of [1] are the one-dimensional, time-dependent hydrodynamic equations describing the self-consistent dynamics of the plasma hydrogen ions (i), the impurity ions (j) with the charge state Z_j, and the electrons (e) along the magnetic field lines. We start here with the currentless plasma model equations of [1] which result in the RDE when the Lagrangian mass variable and the equation of state are included. The influence of the impuries carbon and beryllium on the RDE is treated by solving steady and time-dependent problems.

2. Reaction Diffusion Equation

Applying the average ion approximation with the average charge $< Z_j >$, assuming all velocities to be equal, $v_e = v_i = v_j \equiv v$, introducing the mass density $\rho_m = m_i n_i + m_j n_j$, the total density $N = n_e + n_i + n_j = 2n_i + (1 + < Z_j >)n_j$, the pressure $p = N \cdot T$ (T - temperature) we obtain [1]:

$$\frac{dN}{dt} + N\frac{\partial v}{\partial x} = 0, \tag{1}$$

$$\rho_m \frac{dv}{dt} + \frac{\partial p}{\partial x} = 0, \tag{2}$$

$$\frac{3}{2}\frac{dp}{dt} + \frac{5}{2}p\frac{\partial v}{\partial x} - \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H(x) - (n_i + < Z_j > n_j)n_j L_{rad}(T). \tag{3}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x},\ n_i = \frac{1}{2}[N - (1 + < Z_j >)n_j]; \tag{4}$$

$n_{e,i,j}$ - densities, $m_{e,i,j}$ - masses, H - external heat source. To close the system of equations (1) - (3) one needs the impurity density n_j in dependence of the other model functions.

Introducing *Lagrangian coordinates* τ, y (y - mass variable):

$$\tau = t,\ y(x,t) = \int_{x_1(t)}^{x} dx' N(x',t),\ \ \frac{d}{dt} = \frac{\partial}{\partial \tau},\ \frac{\partial}{\partial x} = N(x,t)\frac{\partial}{\partial y}, \tag{5}$$

$x_1(t) = \{x | v(x,t) = 0\}$, and the equation of state, $p = NT = p(T),\ p(T)$ - given function, leads to the Lagrangian RDE for the temperature:

$$\frac{\partial T}{\partial \tau} - \frac{2}{5}\xi_p \frac{\partial}{\partial y}\frac{p(T)\kappa_e}{T}\frac{\partial T}{\partial y} = \frac{2}{5}\frac{T}{p(T)}\xi_p \left\{ H(y) - \frac{1}{2}\left[N + (< Z_j > -1)n_j\right] n_j L_{rad}(T) \right\}, \tag{6}$$

$\xi_p^{-1} = 1 - (2/5)\partial \ln p(T)/\partial \ln T$. n_j is expressed either by (i) $n_j = \xi_j N,\ \xi_j = const$ (simplified approximation - SA) or (ii) as a function of $T:\ n_j = n_j(T)$.

The impurity affects (i) the radiation loss term $Q_R = \frac{1}{2}\left[N + (< Z_j > -1)n_j\right] n_j L_{rad}(T)$ (L_{rad} - radiation loss function) and (ii) the electron heat conduction coefficient $\kappa_e = C_3(Z_{eff}) n_e T / (m_e \nu_{ee}) = C_3(Z_{eff}) \cdot \kappa_0 T^\delta,\ \delta = 5/2$ (ν_{ee} - ee collision frequency, $\kappa_0 = const$); $C_3(x) = 3.9(1 + 1.7x)/[(1 + 2.65x)(1 + 0,28x)]$, $Z_{eff} = (n_i/n_e)(1 + Z_0)$, $Z_0 = (n_j/n_i)$ $< Z_j^2 >$. n_j is determined by the differential equation

$$\frac{d}{dT} \ln\ n_j + < Z_j > \frac{d}{dT} \ln\left[\frac{p(T)}{T} + (< Z_j > -1)n_j\right] = \frac{G_j - 1}{T}, \tag{7}$$

derived from the impurity balance equation (cp. [1]); $G_j = \alpha - \beta - < Z_j >,\ \alpha = [C_2(Z_{eff}) + C_2(Z_0)] < Z_j^2 >,\ \beta = C_2(Z_{eff}) Z_{eff} < Z_j >,\ C_2(x) = 2.2(1 + 0.52x)/[(1 + 2.65x)(1 + 0.28x)],\ < Z_j^2 >= (1/n_j) \sum_{Z_j} n_{Z_j} Z_j^2$. Solutions of Eq. (7) under the condition of an *isobaric change* $p = p_0$ will be included.

3. Equilibrium

The possible equilibrium states both in Lagrangian and Eulerian coordinates are determined by very similar equations, i.e. the Lagrangian representation has no advantage in investigating steady states. Therefore we will consider *equilibrium solutions in Eulerian coordinates*:

$$\frac{\partial}{\partial x} \kappa \frac{\partial T}{\partial x} + H - Q_R = 0,\ Q_R = \frac{1 + (< Z_j > -1)\xi_j}{2} \xi_j \left(\frac{p_0}{T}\right)^2 L_{rad},\ \xi_j = \frac{n_j}{N}, \tag{8}$$

$x \in X = [x_{min}, x_{max}]$. ξ_j is either (i) calculated or (ii) estimated by introducing averaged values (SA): $< \xi_j >= a,\ \ < C_3 >= b;\ a, b = const.$

We apply *sheath boundary conditions*:

$$\kappa(T) \frac{\partial T}{\partial x}\bigg|_{x=x_{min}} = \alpha_0 T(0,t)^{\beta_0},\ \ \kappa(T) \frac{\partial T}{\partial x}\bigg|_{x=x_{max}} = -\alpha_n T(1,t)^{\beta_n},\ \alpha_{0,n} \propto p_0. \tag{9}$$

Parameters: $X = [0, L]$, L = 60 m - connection length; $n_e \tau$ = 10^{16} m^{-3}s; κ_0 = 1.5 10^{22} $(eV)^{-5/2}$/(ms); $H(x) = H_0$ = const = 6 10^{25} eV/(m^3s) - *symmetrical task*; p_0 = 10^{21} eV/m^3, $n_{j0} = n_j(T = 1$ eV) = $10^{18}$$cm^{-3}$; $\alpha_0 = \alpha_n$ = 2 10^{26} $(eV)^{1/2}$/(m^2s), $\beta_{0,n}$ = 0.5 ; For $L_{rad},\ < Z_j >$ the ADPAK data are used.
Impurity data for C: Fig. 1. $\Rightarrow$ a_C=0.1, b_C=1.7. Impurity data for Be: Fig. 3. $\Rightarrow$ a_{Be}=0.15, b_{Be}=1.85.

We compute *possible solutions* for the *symmetrical task*, i.e. we solve *initial value problems* at the maximum temperatures $T_{max} = T(L/2)$ 80 to 120 eV with vanishing derivatives and compare these with SA results with the estimated constants $a_C, b_C,\ a_{Be}, b_{Be}$. The exact solutions are displayed in Figs. 2 (a), 4 (a) as profiles $T(x)$, and as phase space portraits in the phase plane (T, T_x) where the "boundary value curves" (9) as dashed lines are also displayed. Solutions to our *boundary value problem* are curves whose beginning and ending points are on the curves (9) in the phase plane. There exists only one solution, i.e. no bifurcation occurs for the parameters used. SA results: Figs. 2 (b), 4 (b). Comparison: Both the phase space portraits and the boundary value curves are changed. The resulting solution to our boundary value problem (dashed line) shows that the SA gives only a qualitative agreement with the exact solution, but it cannot be used for higher or lower temperatures.

4. Time-Dependent Solution

Considering the SA equilibrium solution for carbon (dashed line in Fig. 2 (b)) in *Eulerian coordinates*, transforming it to *Lagrangian coordinates* (18) with $x_1 = L/2 \Rightarrow T(y)$ (full line in Fig. 5); $y(0) = -y(L)$=-0.334 p_0 m^{-2}; modulating this state, and solving the Lagrangian RDE (6) for this initial temperature distribution with Dirichlet's boundary condition $T(y(0)) = T(y(L)) = 80$ eV, proves this state to be the only existing steady state which is *stable* (Fig. 5).

5. Summary

The impurities affect, with respect to their n_j dependence, the reaction diffusion process via the radiation loss term and the electron heat conductivity. This is demonstrated for carbon and beryllium: For both impurities the density behaves non-monotonically for temperatures lower than 10 eV. Mean quantities are estimated that can be used in a simplified model (SA) to solve the RDE. *Equilibrium*: We consider the symmetrical task to compute possible solutions to sheath boundary value problems (phase space portraits and temperature profiles) in the laboratory frame. *Transformation to Lagrangian coordinates*: The boundary value problems in Eulerian coordinates lead to Dirichlet problems in Lagrangian coordinates. The solution (for carbon) of the RDE shows the time evolution to the above mentioned steady state which thus is proved to be stable.

References

[1] P. Bachmann and D. Sünder: "1D Multi-Fluid Plasma Models."
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P. Bachmann, D. Sünder: Contrib. Plasma Phys. **38** (1998) 290.

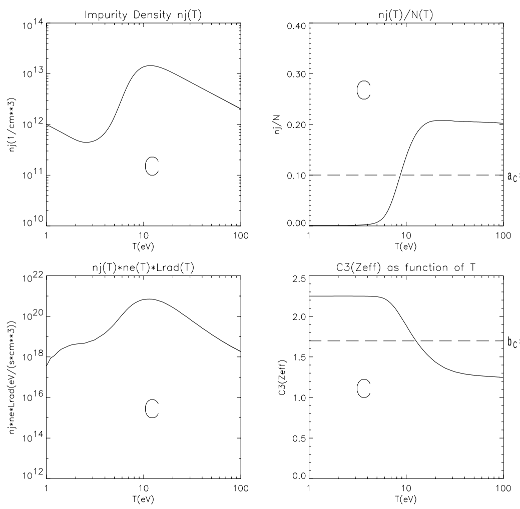

Figure 1. n_j, n_j/N, $n_j n_e L_{rad}$, $C_3(Z_{eff})$ as functions of T for C.

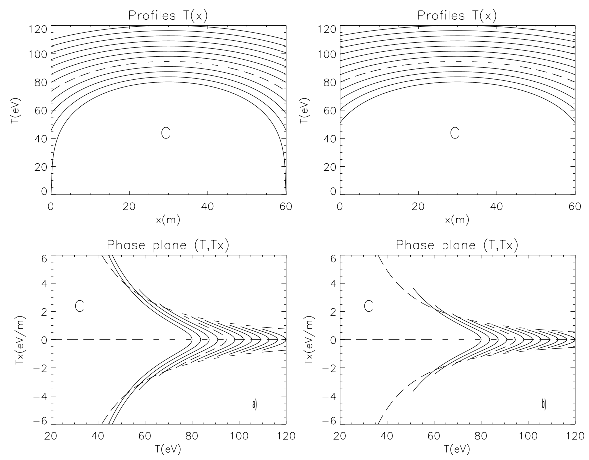

Figure 2. Profiles $T(x)$ and phase plane portraits (T, T_x) for C; (a) - exact solution, (b) - SA with a_C=0.1, b_C=1.7.

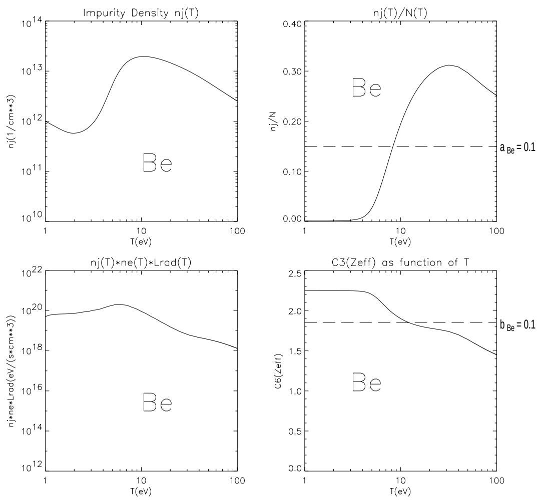

Figure 3. n_j, n_j/N, $n_j n_e L_{rad}$, $C_3(Z_{eff})$ as functions of T for Be.

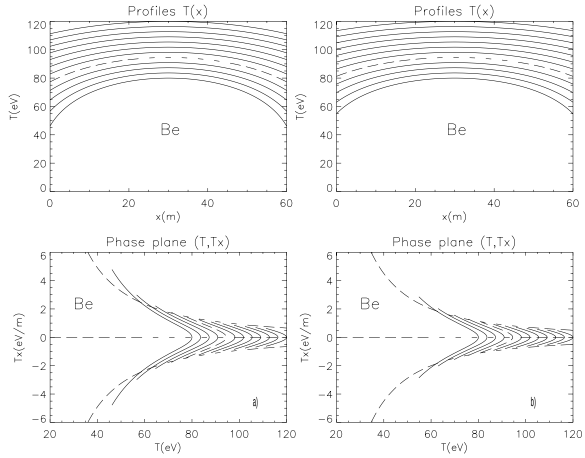

Figure 4. Profiles $T(x)$ and phase plane portraits (T, T_x) for Be; (a) - exact solution, (b) - SA with a_{Be}=0.15, b_{Be}=1.85.

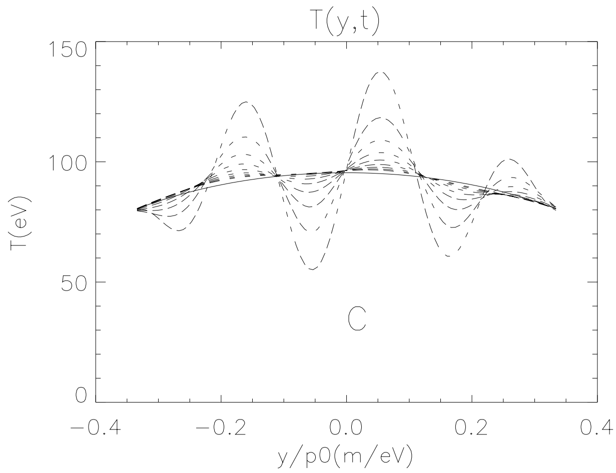

Figure 5. Time evolution of the temperature profile in Lagrangian coordinates to the steady state (full line) which corresponds the equlibrium solution in Fig. 6 (b) in Eulerian coordinates.