TOKAMAK REFUELING VIA INDUCED CONVECTION

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This paper explores the possibility of tokamak refueling by inducing inward radial convection through the conversion of externally injected toroidal mechanical momentum to the canonical momentum of the trapped population. The longitudinal adiabatic invariant for a trapped particle is given by $J = m \oint v_{\|} dl$, where

$$v_{\|} = \left[\frac{2}{m}(W - \mu B - e\Phi)\right]^{1/2} \tag{1}$$

is the parallel velocity, W is the total particle energy, $\mu = mv_{\perp}^2/2B$ is the magnetic moment, and $\Phi(r)$ is the electrostatic plasma potential. Along the banana orbit, approximating $B(r,\theta) = B_0(1-\varepsilon\cos\theta)$ and $\Phi(r,\theta) = \Phi(r_0) \pm .5wE_r(\cos\theta - \cos\theta_0)/(1-\cos\theta_0)$, where $\pm\theta_0$ is the poloidal angle of reflection located at radius r_0, $E_r = -\partial\Phi/\partial r$ is the radial electric field assumed constant over the banana width $w \approx \sqrt{r_0/R}(B/B_\theta)r_c$, R is the tokamak major radius and $\varepsilon = r/R$ gives

$$v_{\|} = \left[\frac{2}{m}\left\{W - \mu B_0(1-\varepsilon\cos\theta) - e\Phi(r_0) \mp \frac{weE_r}{2}\frac{\cos\theta - \cos\theta_0}{1-\cos\theta_0}\right\}\right]^{1/2} . \tag{2}$$

For $|weE_r| \ll 2\varepsilon\mu B_0$; i.e., for $|E_r| \ll |\varepsilon^{1/2} v_t B_\theta|$, the θ-dependence of $v_{\|}$ contributed by E_r in (2) is negligibly small, so that (2) may be approximated as

$$v_{\|} \approx \left[\frac{2}{m}\left\{W - \mu B_0(1-\varepsilon\cos\theta) - e\Phi(r_0)\right\}\right]^{1/2} . \tag{3}$$

Following Ref. [1], the trapped-particle precession velocity is given by

$$v_{\phi,\,prec} \approx -\frac{eT_{eV}}{e_j B_\theta R}\left[\frac{E(\kappa^2)}{K(\kappa^2)} - \frac{1}{2}\right] = -2v_{\phi,\,prec}^{max}\left[\frac{E(\kappa^2)}{K(\kappa^2)} - \frac{1}{2}\right], \tag{4}$$

where

$$\kappa^2 = \frac{W - \mu B_0(1-\varepsilon) - e\Phi(r_0)}{2\varepsilon\mu B_0} \approx \frac{1}{2\varepsilon}\frac{v_{\|0}^2}{v_{\perp 0}^2} \leq 1 , \tag{5}$$

$K(\kappa^2)$ and $E(\kappa^2)$ are the complete elliptic integrals of the first and second kind, and $v_{\|0}$ and $v_{\perp 0}$ are the components of the mean particle velocity at $\theta = 0$. The magnitude of the maximum precession velocity $v_{\phi,\,prec}^{max}$ occurs for $\kappa^2 = 0$ ($E/K = 1$), and for $\kappa^2 = 1$ ($E/K = 0$), corresponding to the minimum and maximum permissible trapped-particle parallel velocities, $|v_{\|0}/v_{\perp 0}| = 0$ and $|v_{\|0}/v_{\perp 0}| = \sqrt{2\varepsilon}$. The trapped particles precess in the opposite toroidal directions at these two extreme values of $|v_{\|0}/v_{\perp 0}|$. The magnitude of the average precession velocity $|\langle v_{\phi,\,prec}\rangle| \ll v_{\phi,\,prec}^{max}$. This result is consistent with the fact that the trapped electrons are unable to share the bulk toroidal motion of the free electrons during Ohmic heating, leading to the well-known neoclassical correction to Spitzer resistivity. The average trapped-particle precession would also be small in comparison with the average toroidal velocity

$$< v_{\phi j}^{free} > \sim \mathcal{O}\left[\frac{T_{eV}(0)}{aB_\theta}\right], \tag{6}$$

of free electrons and ions needed to balance the radial pressure gradient [2]. The rotation velocity in (6) is co-directed for ions and contra-directed for electrons. In either case the current contribution is co-directed. In addition to the toroidal precession velocity given by (4), the trapped particle suffers a toroidal $\mathbf{E} \times \mathbf{B}$ drift $v_\phi^{drift} \sim E_r B_\theta / B^2$. This small drift makes only a minor modification to the precession result of (4).

Additional forces acting on trapped particles arise from (i) radial pressure gradients ∇p, (ii) interparticle collisions, and (iii) the Ohmic-heating toroidal electric field E_ϕ. ∇p effects can be treated in a manner similar to the E_r effects; they do not materially affect trapped-particle precession.

The force due to the interparticle collisions is by far the subtlest and the most pertinent in the present context. During one complete bounce period $\tau_b = \omega_b^{-1}$, a trapped particle traverses a distance $\sim 2\pi q R$, where q is the safety factor. The trapped particle collides with $(2\pi q R)(\pi \lambda_D^2) n_e \gtrsim 2 \times 10^{13}$ individual electrons and ions during this interval. The interparticle collisions exert a smooth force $\mathcal{F}$ on the trapped particle with two distinct components $\mathcal{F} =< \mathcal{F}_\phi > + \mathcal{F}^{random}$, where $< \mathcal{F}_\phi >$ is a steady toroidal force on the trapped electrons (ions) given by

$$< \mathcal{F}_{\phi e(\phi i)} > \approx \nu_{ee(ei)} m_e < v_{\phi e}^{free} > + \nu_{ie(ii)} m_i < v_{\phi i}^{free} > \tag{7}$$

The toroidal equation of motion for a trapped particle in a tokamak under the combined influence of the collisional and electric-field forces may be written as

$$\frac{d}{dt}(m_j v_\phi) = e_j(\mathbf{E} + \mathbf{v} \times \mathbf{B})_\phi + < \mathcal{F}_{\phi j} > + \mathcal{F}_{\phi j}^{random}. \tag{8}$$

Integrated over one complete bounce period, starting and ending at the banana tip ($v_\phi = 0$), the left-hand side vanishes. One obtains from (8)

$$< (\mathbf{v} \times \mathbf{B})_\phi > \Delta\tau = -E_\phi \Delta\tau - \frac{< \mathcal{F}_{\phi j} >}{e_j} \Delta\tau - \frac{< \mathcal{F}_{\phi j}^{random} >}{e_j} \Delta\tau \tag{9}$$

where $\Delta\tau = \tau_b$ and angle brackets denote the average over the bounce cycle. For the integration over one bounce cycle to be meaningful, it is imperative that $\omega_b \gg \nu$ so that the particle remains trapped during the interval τ_b. The last term in (9) containing the random collisional contribution will vanish upon averaging over the trapped-particle distribution function. Since $(\mathbf{v} \times \mathbf{B})_\phi = v_r B_\theta$, one obtains from (9)

$$< v_r > = \frac{\Delta r}{\Delta\tau} = -\frac{E_\phi}{B_\theta} - \frac{< \mathcal{F}_{\phi j} >}{e_j B_\theta}, \tag{10}$$

as the mean trapped particle radial convection velocity and $\Delta r =< v_r > \Delta\tau$ as the average convection per bounce period. The first term on the right-hand side of (10) is the Ware pinch. The second term represents additional pinch due to the steady toroidal force $< \mathcal{F}_{\phi j} >$ exerted by the free particles on the trapped particles due to their differential toroidal rotation.

The toroidal momentum $(e_j E_\phi + < \mathcal{F}_{\phi j} >)\Delta\tau$ lost by the free particles plus the momentum contributed by E_ϕ during one bounce cycle of the trapped particle exactly equals $e_j B_\theta \Delta r$, which is the canonical toroidal angular momentum gained by the trapped particle during its radial displacement Δr. Since the entire toroidal momentum lost by the free particles has been accounted for, none remains to alter the toroidal precession velocity of the trapped fraction. Hence, the trapped particle precession result of (4) continues to be valid when all the principal forces acting on the trapped particles have been included.

Since only the fraction $\varepsilon^{1/2}$ of the particles is trapped, the net effective radial convection velocity of species j in the banana regime is obtained by multiplying the result of (10) by $\varepsilon^{1/2}$, giving

$$< v_{rj} >_{banana}= -\varepsilon^{1/2}\left[\frac{E_\phi}{B_\theta}+\frac{< \mathcal{F}_{\phi j} >}{e_j B_\theta}\right]. \tag{11}$$

(10)-(11) show that the steady toroidal force $< \mathcal{F}_{\phi j} >$ acts on the trapped particles in a manner identical to the toroidal electric field force $e_j E_\phi$ in its ability to induce radial convection in the banana regime. Extending the result of (11) by analogy with the neoclassical Ware pinch results for the plateau regime gives

$$< v_{rj} >_{plateau}= -\frac{\varepsilon^2 v_{te}}{qR\nu_{ei}}\left[\frac{E_\phi}{B_\theta}+\frac{< \mathcal{F}_{\phi j} >}{e_j B_\theta}\right]. \tag{12}$$

With increasing ν_{ei} in the plateau regime, Ware pinch caused by E_ϕ in (12) diminishes steadily due to the ever-decreasing number of effectively trapped particles, disappearing altogether in the high-collisionality Pfirsch-Schlüter regime. Unlike the force due to E_ϕ, which remains unaffected by collisionality, the toroidal force $< \mathcal{F}_\phi >$ on trapped electrons and ions given by (7) scales with collisionality, so that the inward convection due to $< \mathcal{F}_\phi >$ in (12) remains constant throughout the plateau regime. Like the Ware pinch, $< \mathcal{F}_\phi >$-induced pinch disappears in the Pfirsch-Schlüter regime owing to the absence of trapped particles. The sudden cessation/inception of $< \mathcal{F}_\phi >$-induced radial convection at the plateau/Pfirsch-Schlüter boundary is reminiscent of abrupt transport changes accompanying L-H transitions in tokamak plasmas.

Force $< \mathcal{F}_\phi >$ in (7) is dominated by terms containing like-particle collisions. One obtains from (6) and (7)

$$< \mathcal{F}_{\phi i} >\approx \nu_{ii} m_i < v_{\phi i}^{free} >\approx \frac{\nu_{ii} m_i T_{eV}(0)}{aB_\theta}\ . \tag{13}$$

From (11) and (13) one obtains the rotation-induced ion convection velocity

$$< v_{ri} >_{banana}\approx -\frac{\varepsilon^{1/2} < \mathcal{F}_{\phi i} >}{eB_\theta} \approx -\frac{\varepsilon^{1/2}\nu_{ii} m_i T_{eV}(0)}{eaB_\theta^2} \sim \mathcal{O}(-1\,\mathrm{ms}^{-1})\ . \tag{14}$$

Significantly, the experimentally observed inward-convection velocities lie in the range consistent with the result of (14). The ubiquitous toroidal force $< \mathcal{F}_\phi >$ experienced by trapped particles assumes added significance during neutral-beam injection or in the presence of steep density gradients associated with transport barriers and H-mode plasmas, where toroidal precession velocities in excess of tens of kilometers per second have been observed. Such large toroidal velocities would cause radial convection velocities exceeding several meters per second. (14) also shows that the inward convection would become extremely enhanced in regions of low B_θ; e.g., for radii inside the transition region between positive and negative shear. This could have bearing upon the formation of the experimentally observed particle and energy transport barriers.

The inward convection induced by toroidal rotation may be used for tokamak refueling. It suffices to feed neutral particles in the form of pellets close to the plasma periphery. The ionized particles are then driven inward using a combination of current drive for electron momentum and neutral beam injection to supply the ion momentum.

Although radial convection is implicit in neoclassical theory and follows directly as the Onsager symmetric counterpart of bootstrap current [3], its pivotal role in tokamak transport has never been recogonized. To this day, inward convection is referred to as *anomalous inward convection.* In steady state, the toroidal momentum released (as bootstrap current) by the outward plasma diffusion exactly equals the momentum reabsorbed through inward convection. Thus, there is no net steady-state bootstrap current [4] (except the small contribution due to neutral particles injected directly into the plasma interior). In steady state, the entire momentum lost through Ohmic-current dissipation must be supplied by external sources in the form of current drive and neutral-beam momentum injection. This can be realized owing to the high efficieny of low-phase-velocity Alfven-wave current drive [4] via the ν_{ee} enhancement resulting from radiative collisionality contributed by Kirchhoff radiation [4]. The ν_{ee} enhancement would also result in inward convection of the electron component comparable in magnitude to the ion convection result of (14).

Radiative effects are invariably ignored in existing neoclassical formulations; Balescu-Lenard-like equations include only the ballistic effects of plasma waves while neglecting the much larger radiative contributions. The sole rationale for neglecting radiative contributions rests on the faster time scale of the plasma waves compared with collisions ($\omega \gg \nu$); high-frequency radiation is discarded as innately irrelevant, irrespective of its intensity. Using this assumption, the troublesome time-dependent term in Eq.(10.7) of Ref. [6] is dropped, facilitating the derivation of the Balescu-Lenard equation. The sweeping assumption involving only the time scales makes no allowance for the relative strengths of the radiative and ballistic contributions. In a more complete treatment one would be obliged to include radiation effects through higher-order correlations (up to fourth order) for all ω in the manner described for the case of low ω for an unmagnetized plasma in Chapter V of Ref. [6]. Pending similar effort for a magnetized plasma, the Kirchhoff law approach of Ref. [5] is currently the sole recourse available for estimating radiative collisionality.

Omission of rotation-induced convection and radiative collisionality exposes fundamental flaws in the current practice of neoclassical theory. These flaws are responsible for the profusion of so-called transport anomalies. *Supraclassical theory* [4], comprised of neoclassical theory modified to include the radiative and convective contributions, is able to resolve not only the problem of tokamak refueling but also the bulk of the remaining anomalies of tokamak transport [4,5].

[1] K. Miyamoto, *Plasma Physics for Nuclear Fusion*, MIT Press (1976); p.65.

[2] Presence of E_r in the radial-force-balance equation would alter the result of (6). Such detail, however, has no primary bearing on the arguments of this paper.

[3] H. Wobig (private communication)

[4] S. Puri, 27th EPS Conf. on Control. Fusion and Plasma Physics, Budapest (2000).

[5] S. Puri, Phys. Plasmas **5**, 2932 (1998).

[6] Y. M. Klimontovich in *The Statistical Theory of Non-Equilibrium Plasmas* (1967).